EE 457 - Homework 1

Due 02/03/2017

The objective of this homework is to review basic linear algebra concepts. Solution should be done by hand, but you are encouraged to use MATLAB to check whether your answer is correct. You may deliver a hard copy of your work or upload a soft copy to the moodle system. In case you have any questions, please feel free to ask via e-mail to the teaching assistant.

P1. Determine whether the following set of vectors are linearly independent or not? Please justify your answer.

$$\left\{ \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$$

P2. Let $x = \begin{bmatrix} 1 & -3 & 2 \end{bmatrix}^T$, $v_1 = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}^T$, $v_2 = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix}^T$ and W be the subspace spanned by v_1 and v_2. Find the closest point to x in the subspace W. (Hint: You may use orhogonal projections after applying Gram–Schmid process on W.)

P3. Let $x \in \mathbb{C}^n$ be a complex vector of dimension n. Show that $\|x\|_p = (\sum_{i=1}^{n} |x_i|^p)^{1/p}$ is a valid norm for any p where $1 \leq p < \infty$.

P4. Let $A \in \mathbb{R}^{m \times n}$ be an $m \times n$ real matrix. Show that $\|A\|_F = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 \right)^{1/2}$, which is referred to as the Frobenius norm, is a valid norm.

P5. Given

$$A = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 5 & 3 \\ 6 & 3 & 9 \end{bmatrix} \quad b = \begin{bmatrix} 14 \\ 15 \\ 21 \end{bmatrix}$$

a) Find the null space of A.

b) Find the range space of A.

c) Check whether $Ax = b$ has a solution without explicitly solving it.

d) If it exists, find the solution set for $Ax = b$.

P6. Given

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

a) Find the eigenvalues of A.

b) Is A diagonalizable? If so, diagonalize it. Otherwise, explain why you can not diagonalize it.

P7. Given

$$Q_1 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad Q_2 = \begin{bmatrix} 2 & -4 & 3 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{bmatrix}$$

Determine if the following matrices are positive definite, positive semidefinite, negative definite, negative semidefinite or indefinite.

a) Q_1

b) Q_2

c) $Q_1 + Q_2$

d) $Q_1 Q_2$

e) $Q_2 Q_1$

P8. Let $A \in \mathbb{R}^{m \times n}$ be an $m \times n$ real matrix with $rank(A) = m$. Show that the set $\{x \in \mathbb{R}^n : Ax = b\}$ is convex.