EE 457 HW 4

Yagiz Savas

Question 1)

i)

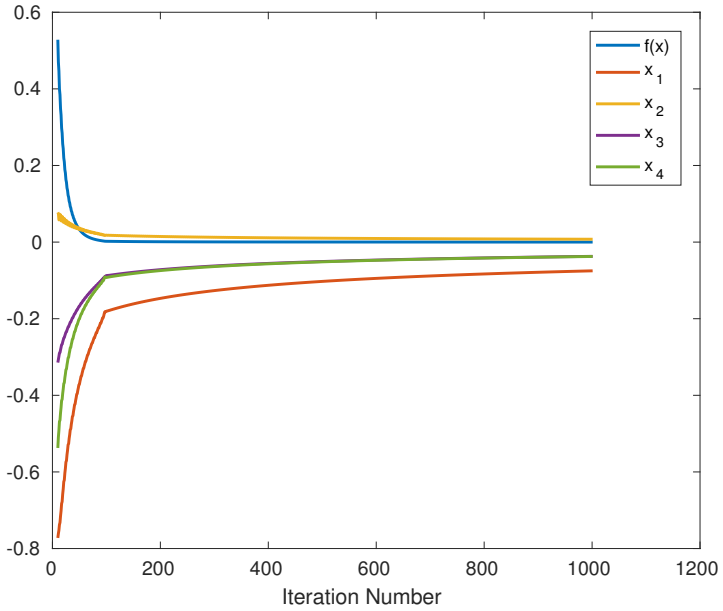

Figure 1: Minimization results for Powell function with the use of conjugate direction algorithm. Initial point of $x = [3, -1, 0, 1]^T$ and secant method as exact line search method are used. Stopping criteria is $k > 1000$, where k is the number of iterations.

ii)

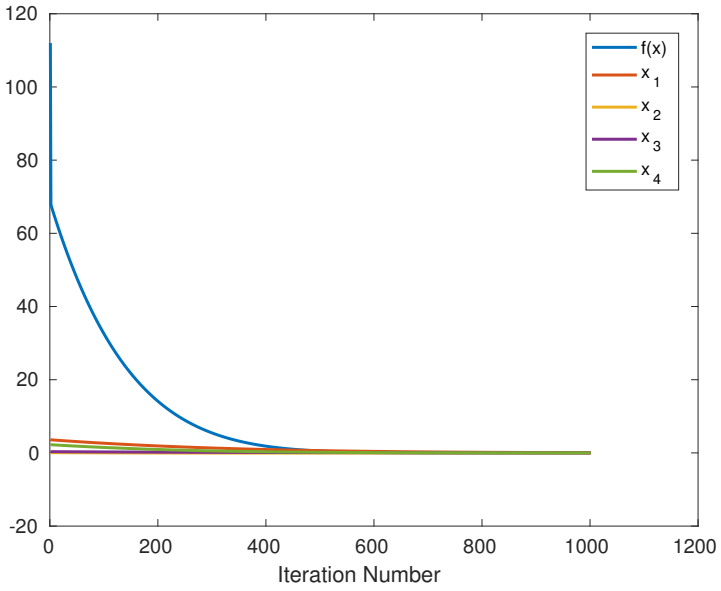

Figure 2: Minimization results for Powell function with the use of Quasi-Newton method. **Rank one correction algorithm** is used for the calculation of estimate inverse Hessian. Initial point of $x = [3, -1, 0, 1]^T$ and secant method as exact line search method are used. Stopping criteria is $k > 1000$, where k is the number of iterations.

iii)

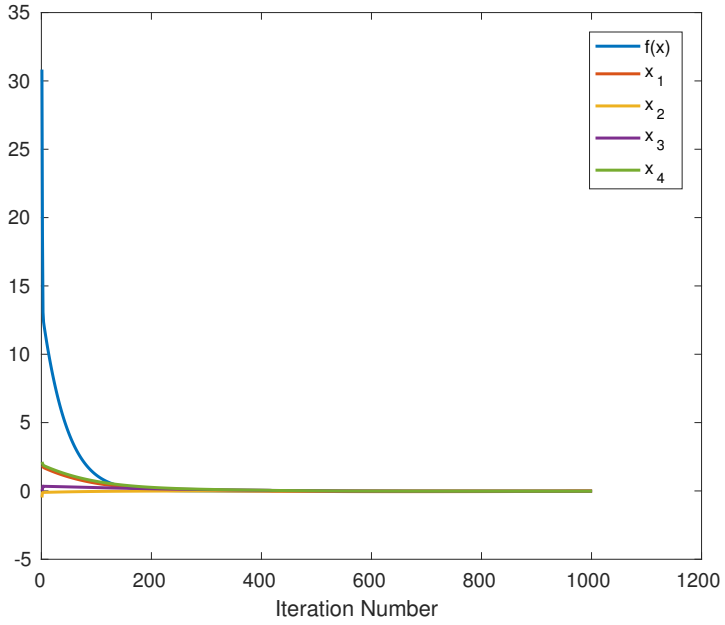

Figure 3: Minimization results for Powell function with the use of Quasi-Newton method. **DFP algorithm** is used for the calculation of estimate inverse Hessian. Initial point of $x = [3, -1, 0, 1]^T$ and secant method as exact line search method are used. Stopping criteria is $k > 1000$, where k is the number of iterations.

iv)

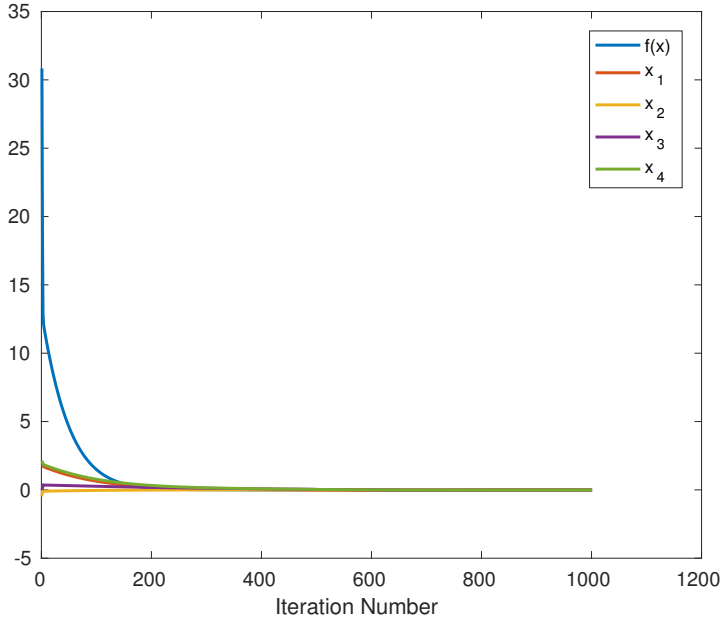

Figure 4: Minimization results for Powell function with the use of Quasi-Newton method. **BFGS algorithm** is used for the calculation of estimate inverse Hessian. Initial point of $x = [3, -1, 0, 1]^T$ and secant method as exact line search method are used. Stopping criteria is $k > 1000$, where k is the number of iterations.

v)

Table 1: Results for algorithms employed to find minimum of Powell function. Stopping criteria of $k > 1000$ is used for all algorithms.

Problem number:	**Function Value:**
i)	6.5179e-05
ii)	7.2296e-22
iii)	6.0591e-06
iv)	2.7843e-09

One important note on the use of Quasi-Newton methods is that initial point for the estimate of inverse Hessian dramatically change the results. For example, in Table 1 it can be seen that rank one correction algorithm provides by far the best result. However, it is because of the use of random initial symmetric matrices for that algorithm. When identity matrix is used as initial matrix, as it is the case for (ii) and (iii), algorithm does not even converge! Therefore, by taking convergence issues into account, it can be deduced that BFSG algorithm is a better algorithm to use for this case. For conjugate direction method, better final function values with less effort can be obtained with the use of inexact line search methods, i.e. Armijo-Goldstein conditions.

Question 2)

i)

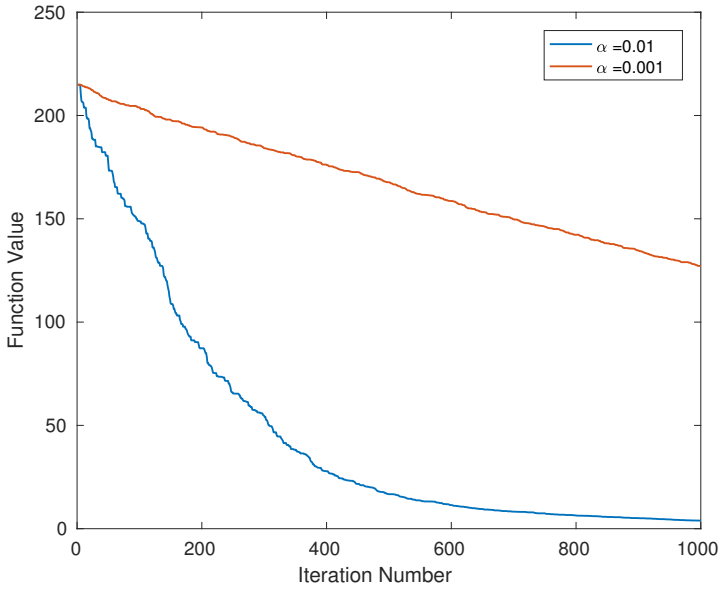

Figure 5: Minimization results for Powell function with the use of **Naive random search**. Uniformly distributed neighborhood of $x_k - \alpha \leq x_{k+1} \leq x_k + \alpha$ is used.

ii)

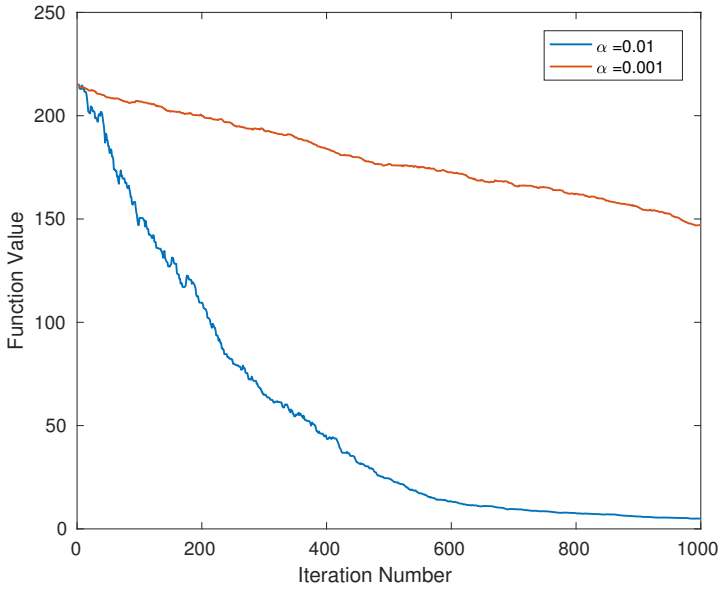

Figure 6: Minimization results for Powell function with the use of **Simulated Annealing Algorithm**. Uniformly distributed neighborhood of $x_k - \alpha \leq x_{k+1} \leq x_k + \alpha$ is used. Probability of choosing an ascent direction is $p = 0.2$ for both cases.

Question 3)

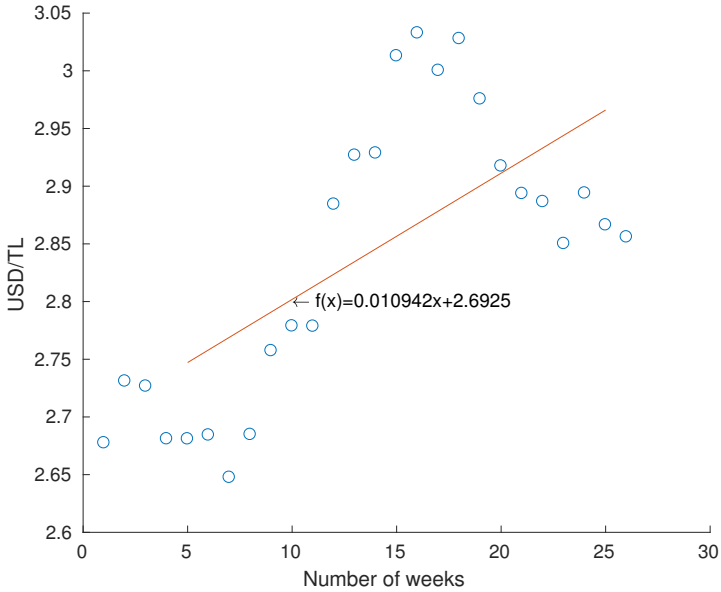

Figure 7: Data fit to USD/TL data by using **Recursive least square algorithm**. Note that $f(x) = mx + c$ with $m = 0.010942$ and $c = 2.6925$. This linear fit is obtained using data 5:25.

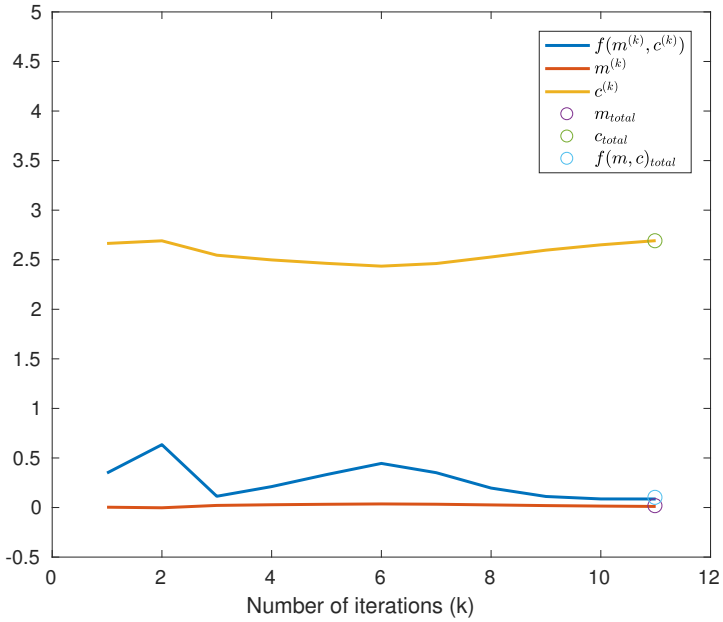

Figure 8: Function value and parameter changes with increasing number of data. Note that $(.)_{total}$ represents values calculated using least square solution with all data.

Table 2: Comparison of recursive least square solution using data 5:25 and least square solution using all data

	Recursive LS	**LS using all data**
$f(m, c)$	0.0862	0.0953
m	0.0109	0.0114
c	2.6925	2.6841

As can be seen from Table 2 there is no significant difference between the values calculated although LS solution uses more data.

Question 4)

i)

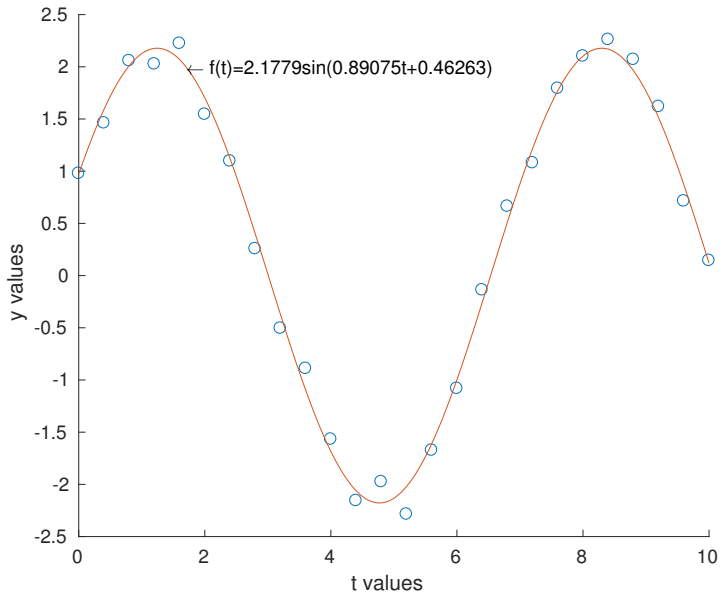

Figure 9: Sinusoid of best fit using **Gauss-Newton algorithm**.

ii)

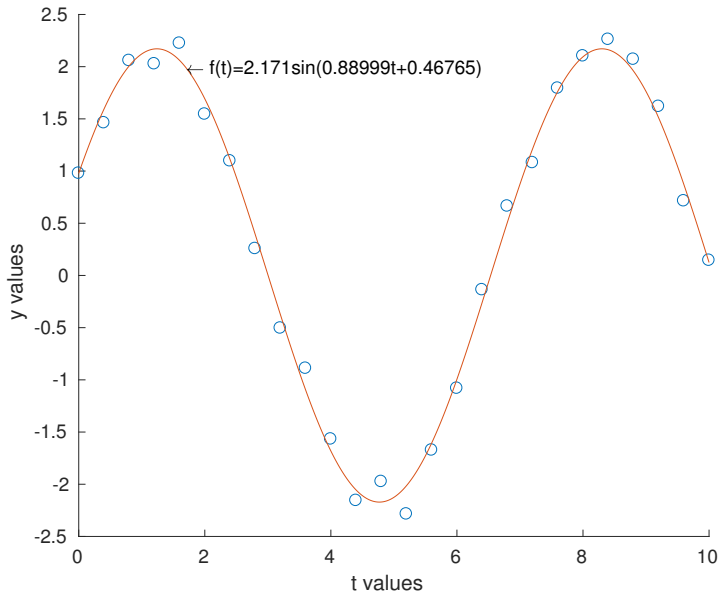

Figure 10: Sinusoid of best fit using **Levenberg-Marquardt algorithm**. Correction term $\mu_k = 100$.

Since these nonlinear fit methods are actually types of Newton's method, inappropriate initial point selections may result in wrong fits! For both algorithms, initial point is chosen $x = [1, 1, 1]^T$. For Gauss-Newton algorithm, minimum eigenvalue of matrix $J^T J$ during iterations was 2.5. Therefore, positive definiteness condition was not violated. As a conclusion, there is no critical difference between the results provided by two algorithms.