$$y_1'' = y_1 + 2y_2' - u^* \frac{y_1 + u}{D_1} - u \frac{y_1 - u^*}{D_2}$$

$$y_2'' = y_2 - 2y_1' - u^* \frac{y_2}{D_1} - u \frac{y_2}{D_2}$$

$$D_1 = ((y_1 + u)^2 + y_2^2)^{3/2}$$

$$D_2 = ((y_1 - u^*)^2 + y_2^2)^{3/2}$$

$u \rightarrow$ constants,
$u^* \rightarrow$

$$y_1'' = x_1' \qquad y_2'' = x_2'$$

$$y_1' = x_1 \qquad y_2' = x_2$$

2 second degree differential equation

$$\begin{cases} y_1' = x_1 \\[2mm] x_1' = y_1 + 2x_2 - \dfrac{u_2(y_1 + u_1)}{((y_1 + u_1)^2 + y_2^2)^{3/2}} - \dfrac{u(y_1 - u_2)}{((y_1 - u_2) + y_2^2)^{3/2}} \\[5mm] y_2' = x_2 \\[2mm] x_2' = y_2 - 2x_1 - \dfrac{u_2(y_2)}{((y_1 + u_1)^2 + y_2^2)} - \dfrac{u(y_2)}{((y_1 - u_2)^2 + y_2^2)^{3/2}} \end{cases}$$

$\rightarrow$ 4 first degree differential equation.

Best Method

In problem we had used 3 differential equation solving method.

1) Euler method

2) Runge-kutta4 method

3) ode45 solver

The result of tic-toc function shows us that:

Required time for Euler method ≈ 0.79

Required time for Runge-kutta4 method ≈0.41

Required time for ode45 ≈0.25

If we compare the graphs, best solution is given by ode45 solver but if we decrease the h values, we start the approach true graphs in Euler and Runge-kutta4 methods but since iteration number increased, elapsed times which is already bigger than time of ode45, increase. That's why, I think best method is using ode45 solver.
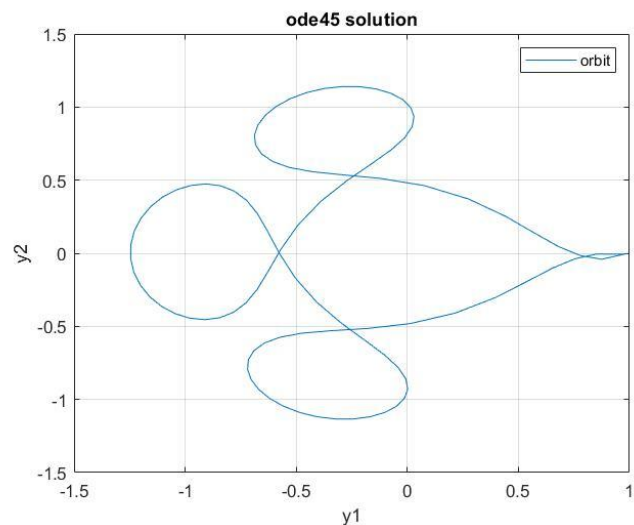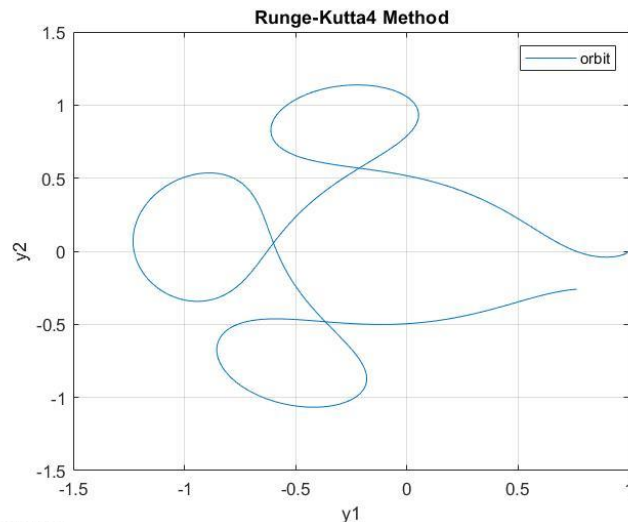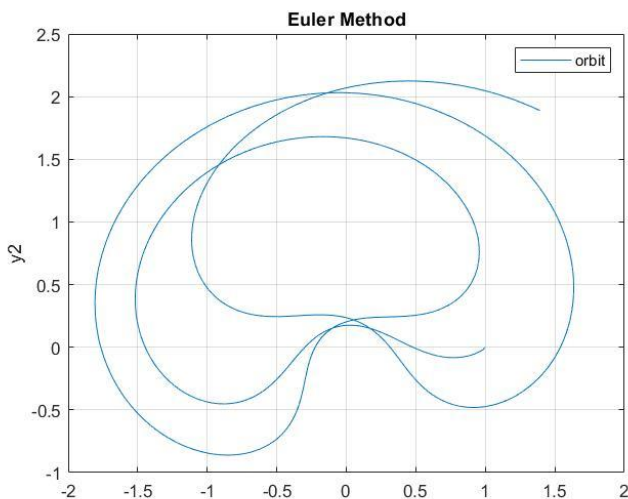
About Orbit

As the question explanation stated, two body are orbiting around each other. The solution that we found indicate the third object that orbit with respect to center of gravity of these two object. I conclude that moon and earth is not stable so the orbit that we found has relative motion also. As the moon orbit around earth, spacecraft's orbit also changes its position according to position of moon but the shape does not change since position of center of gravity does not change according to spacecraft's orbit. Also since the spacecraft move according to gravitational field, it doesn't use any fuel.

Since all the parameters are normalized if we change the parameters, graphs become inconsistent. As T multiplied by 1.5,2 and 3.