ME303 TERM PROJECT REPORT

QUESTION-2

Course's Code	**: ME303**
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a) Hand calculations of Questıon-2 are below

TERM PROJECT – QUESTION 2

$$y_1''(t) = y_1(t) + 2y_2'(t) - \mu^* \frac{y_1(t)+\mu}{D_1} - \mu \frac{y_1(t)-\mu^*}{D_2}$$

$$y_2''(t) = y_2(t) - 2y_1'(t) - \mu^* \frac{y_2(t)}{D_1} - \mu \frac{y_2(t)}{D_2}$$

$$D_1 = \sqrt{((y_1+\mu)^2 + y_2^2)}^{\,3} \qquad \mu^* = 1-\mu$$

$$D_2 = \sqrt{(y_1-\mu^*)^2 + y_2^2}^{\,3}$$

→ we should write the followings in terms of $\boxed{x' = Ax + B}$

lets say ⇒ $k = \begin{bmatrix} y_1 \\ y_1' \\ y_2 \\ y_2' \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} \Rightarrow k' = \begin{bmatrix} y_1' \\ y_1'' \\ y_2' \\ y_2'' \end{bmatrix} = \begin{bmatrix} k_1' \\ k_2' \\ k_3' \\ k_4' \end{bmatrix}$

so ⇒ $y_1 = k_1$, $y_1' = k_2$ | $y_1' = k_1'$, $y_1'' = k_2'$

$y_2 = k_3$, $y_2' = k_4$ | $y_2' = k_3'$ $y_2'' = k_4'$

→ $\boxed{y_1' = k_2 = k_1'}$ → (1) $\boxed{y_2' = k_4 = k_3'}$ → (2)

now we should create a new form of

$$\boxed{X' = Ax + B}$$

$$x = k = \begin{bmatrix} y_1 \\ y_1' \\ y_2 \\ y_2' \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 - \frac{\nu^*}{D_1} - \frac{\nu}{D_2} & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & m & 0 \end{bmatrix}$$

let say "m"

$$\downarrow$$

$$X' = k' = \begin{bmatrix} y_1' \\ y_1'' \\ y_2' \\ y_2'' \end{bmatrix} = \begin{bmatrix} k_1' \\ k_2' \\ k_3' \\ k_4' \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & \left(\frac{-\nu^* \cdot \nu}{D_1} + \frac{\nu^* \nu}{D_2} \right) & 0 & 0 \end{bmatrix}^{-1}$$

$$\boxed{X' = Ax + B}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ m & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & m & 0 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ \left(\frac{-\nu^* \nu}{D_1} + \frac{\nu^* \nu}{D_2} \right) \\ 0 \\ 0 \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} k_1' \\ k_2' \\ k_3' \\ k_4' \end{bmatrix}}_{x'}$$

If we do matrix multiplication and addition

$$\begin{bmatrix} k_2 \\ mk_1 + 2k_4 \\ k_4 \\ -2k_2 + mk_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-\mu^* \mu}{D_1} + \frac{\mu^* \mu}{D_2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} k_1' \\ k_2' \\ k_3' \\ k_4' \end{bmatrix}$$

$$k_1' = k_2 + 0 = k_2$$

$$k_2' = mk_1 + 2k_4 + \frac{\mu^* \mu}{D_2} - \frac{\mu^* \mu}{D_1}$$

$$k_3' = k_4 + 0 = k_4$$

$$k_4' = -2k_2 + mk_3$$

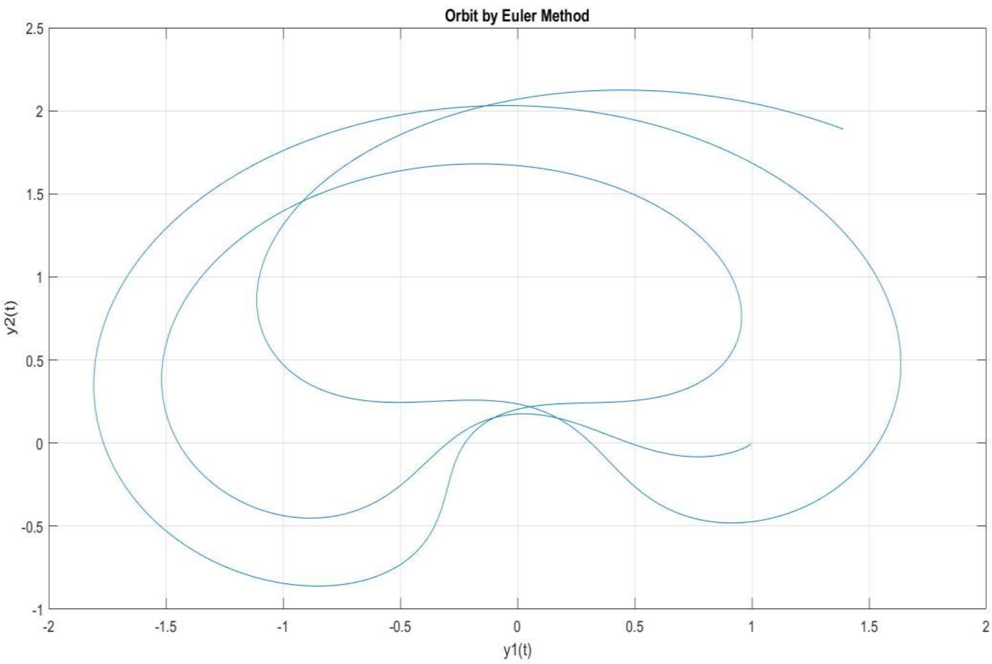
now we should complete the equations 1 and 2

$$y_1'' = k_2' = \left(1 - \frac{\mu^*}{D_1} - \frac{\mu}{D_2}\right) k_1 + 2k_4 - \mu^* \mu \left(\frac{1}{D_1} - \frac{1}{D_2}\right)$$

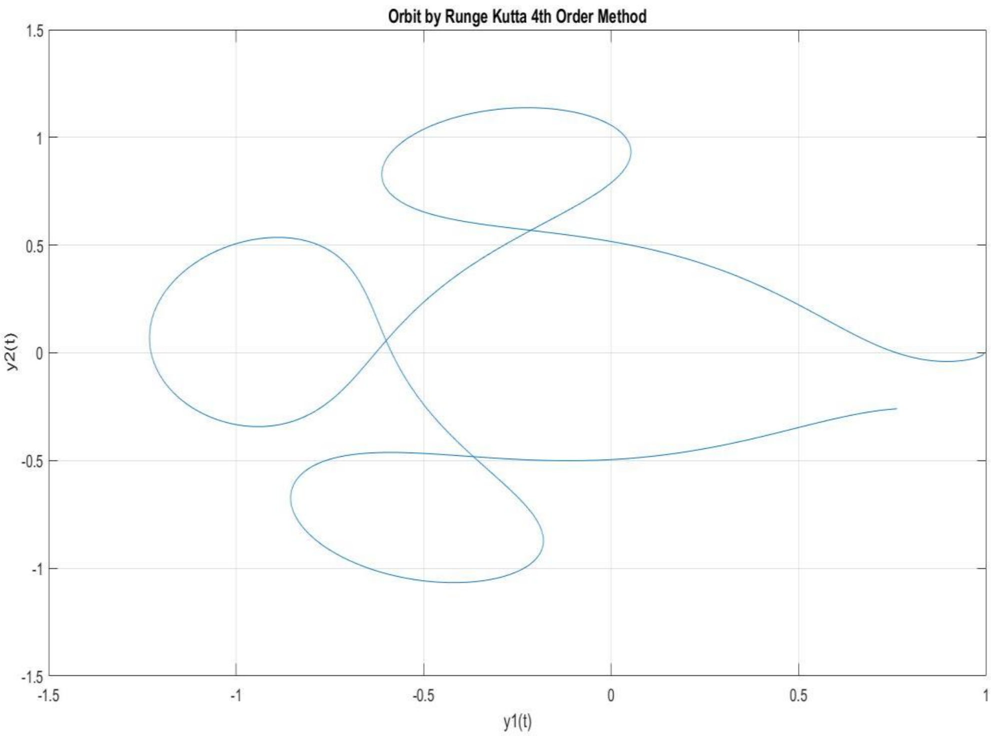
$$y_2'' = k_4' = -2k_2 + \left(1 - \frac{\mu^*}{D_1} - \frac{\mu}{D_2}\right) k_3$$

The graphs obtained with the euler, rk4 and ode45 methods are as follows.

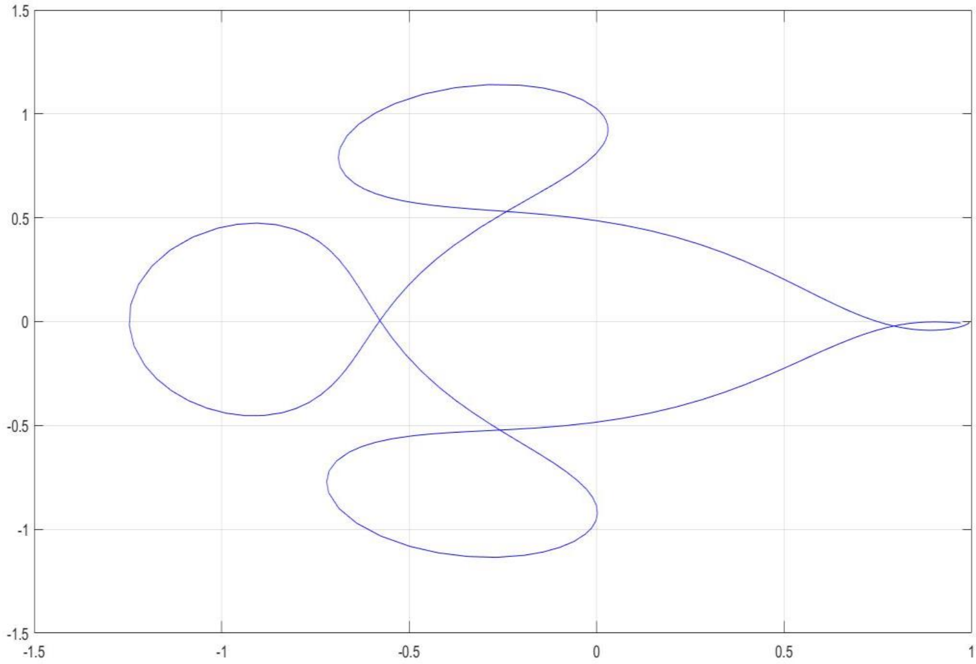
Plotting With Euler Method:



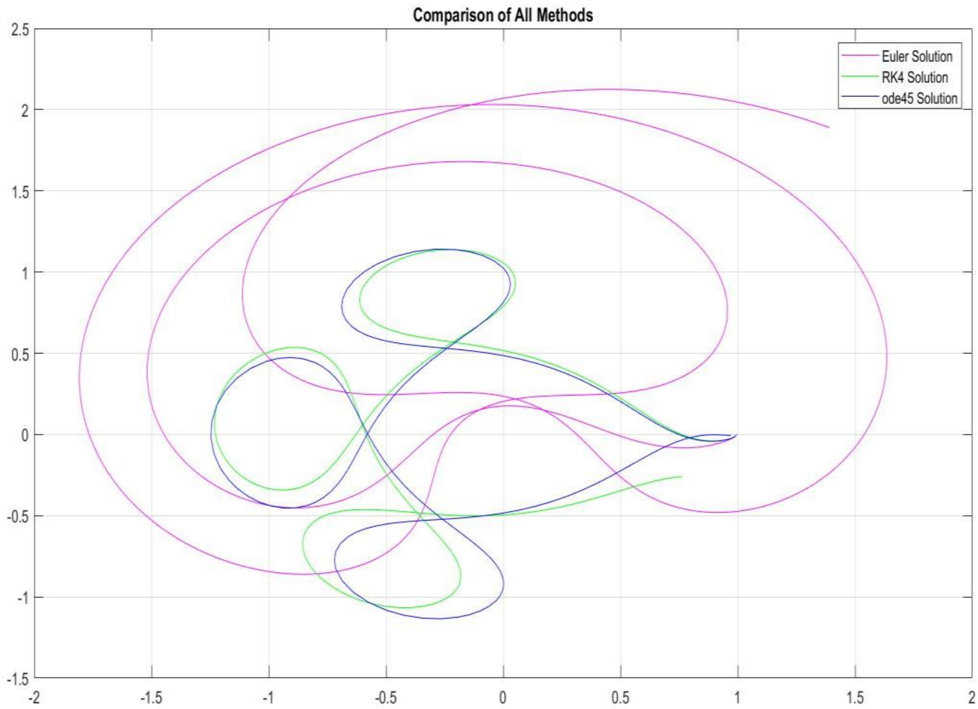
Plotting With RK4 Method:



Plotting With ode45 Method:



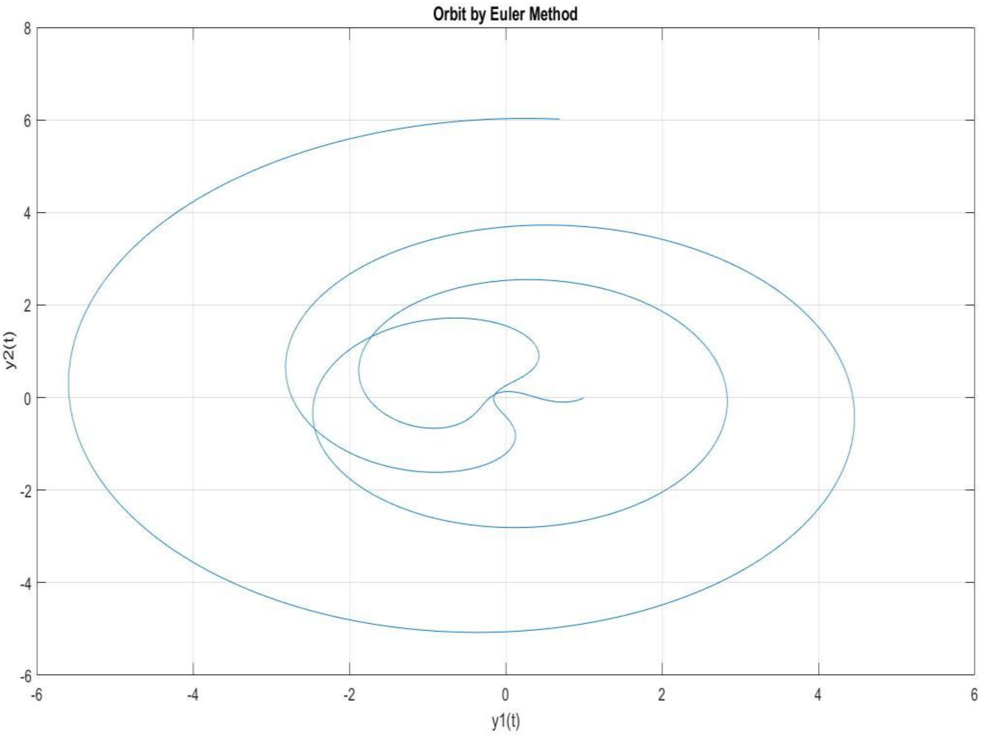
Comparison of 3 Methods:



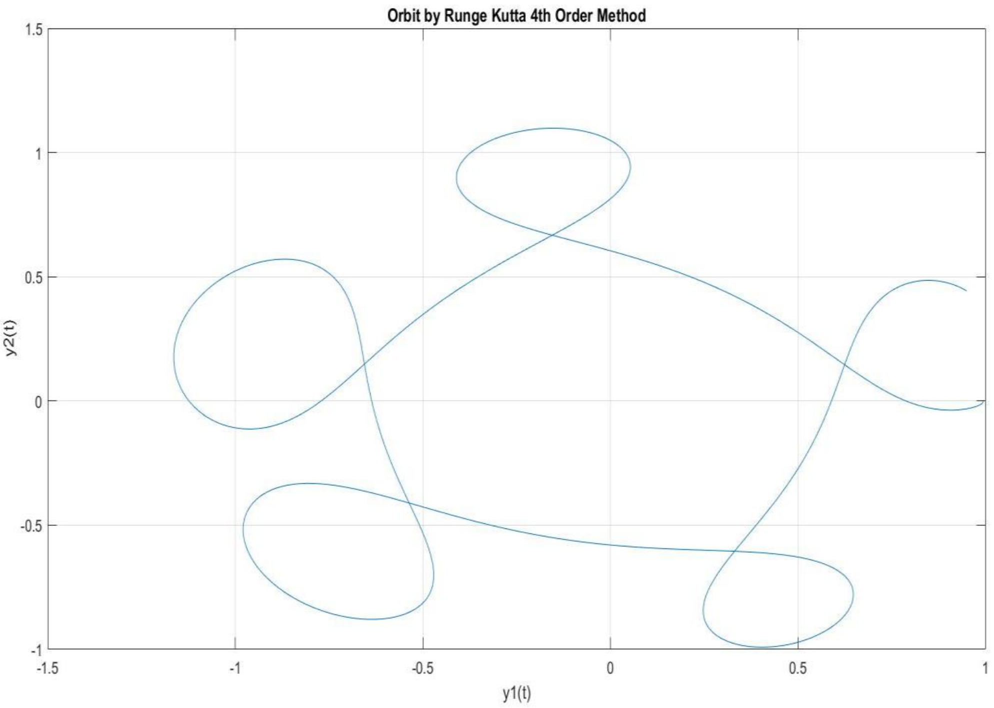
As can be seen from the graphs, the ode45 method has been the closest approach to the real orbit. This means that the margin of error of the ode45 method is less than the others. In addition, according to the code written in matlab, the ode45 method was the shortest and fastest method.

If we **increase the final_time**, the resulting graphs are as follows.
(final_time=   23.06521656015796)

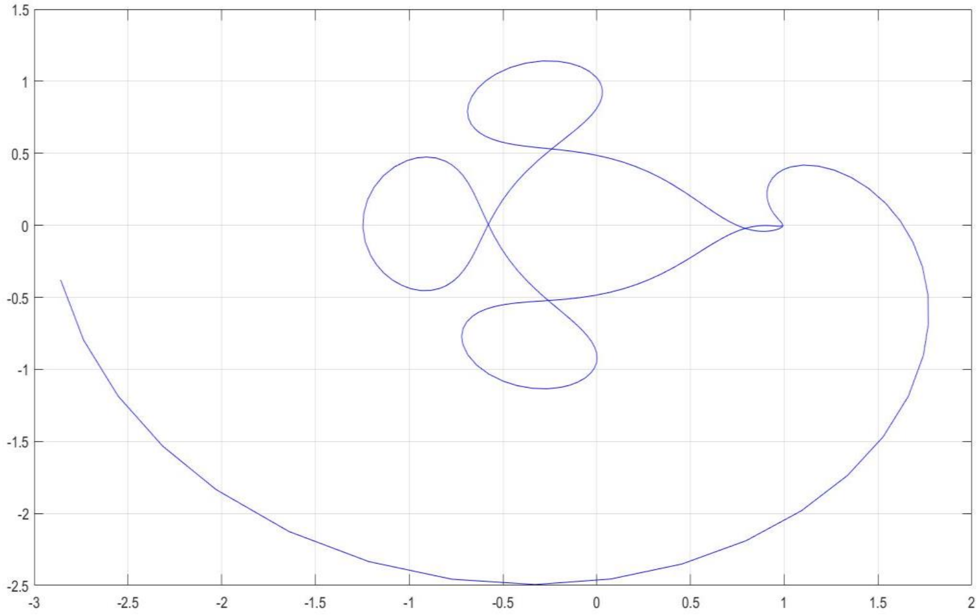
Plotting With Euler Method:



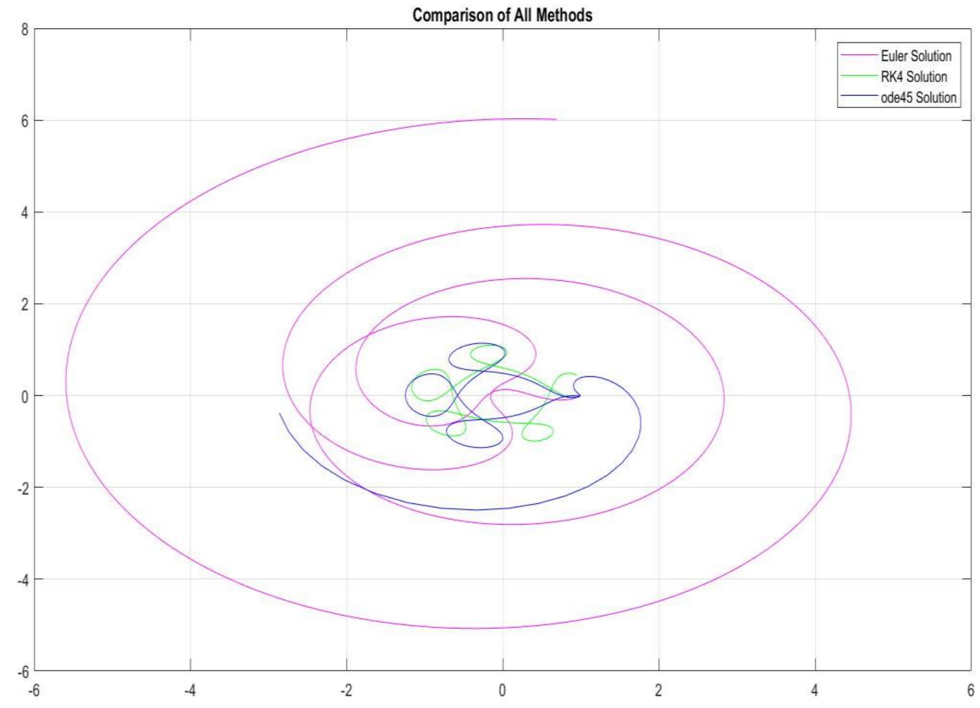
Plotting With RK4 Method:



Plotting With ode45 Method:



Comparison of 3 Methods:



As a result of increasing the final_time in this way, the euler and ode45 graphs have completely changed by taking the shape of an ellipse. In addition, the simulation gives more errors at a larger T value. This change is not physically possible, as the moon and earth have their own orbits. Therefore, it can be said that the first values given in the question are more valid and it is more logical to accept these values.

In order for the simulation to be valid and to compile properly, there must be no force other than gravity on the satellite, and the gravitational force applied by the Earth and Moon must be constant. Therefore, since changing the parameters as we did above changes the forces acting on the satellite, the orbit of the satellite changes and an imbalance occurs between the forces.

Discussion:

In this model, the gravitational forces of the sun, other planets and stars are neglected. These parameters may be necessary for a more realistic simulation. In addition, the mass of the sun is much greater than the mass of the moon, earth and satellite. Therefore, this parameter will have a great influence on the plotted trajectory. On the other hand, since the distance between the Earth and the Moon changes as these two revolve around the Sun, accepting it as constant may cause errors in the simulation. Additionally, asteroids, earth-to-space shuttles, orbiting gases, and frictional forces are other variables to take into account.