289 Final Project Proposal

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1 Introduction

Univariate polynomials are polynomials of one indeterminate (variable). A degree-n univariate polynomial is of the form

$$P_n(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0 \quad (a_n \in \mathbb{C})$$

Now, the fundamental theorem of algebra states that

Theorem 1. *Every polynomial function of degree-n, such that $n > 0$, has exactly n complex roots.*

These roots can be integers, non-integer reals, or complex. For example, consider the polynomial

$$f(z) = z^4 - \frac{3}{2}z^3 + \frac{3}{2}z^2 - \frac{3}{2}z + \frac{1}{2}$$

Notice that

$$\begin{aligned} f(z) &= z^4 - \frac{3}{2}z^3 + \frac{3}{2}z^2 - \frac{3}{2}z + \frac{1}{2} \\ &= (z-1)(z-0.5)(z-i)(z+i) \end{aligned}$$

Therefore $f(z)$ has four roots: two complex roots, one integer root, and one non-integer real root.

2 Problem

There exist multiple methods of finding the number of roots. For example, one can use Rational Root Theorem to find the number of rational roots of a polynomial, Descartes' rule of signs to find the number of real roots of a polynomial, etc. In my project, I will train a Feedforward Neural Network (FNN) to find the number of integer, non-integer real, and complex roots of a given polynomial. To my knowledge, there exists a method of using a FNN to find the number of real roots of a polynomial [1]. However, there doesn't seem to be any research on finding the three types I have listed.

3 Generating Dataset

To train my FNN, we will need a dataset of polynomials and its classified roots. However, we don't have to go looking for a dataset. Instead, we can generate our own! A general overview of the method is as follows

1. `Generate a random number n, the degree of our polynomial`
2. `Generate n complex numbers, the roots of our polynomial.`
3. `Compute the polynomial using the n roots.`
4. `Classify the roots into the three categories: integer, non-integer real, and complex roots.`
5. `Repeat.`

References

[1] B. Mourrain, N.G. Pavlidis, D.K. Tasoulis, M.N. Vrahatis, Determining the number of real roots of polynomials through neural networks, *Computers and Mathematics with Applications 51* 527-536 (2006).