Assignment 7

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Question: Let X and Y be independent normal random variables with zero mean and unit variances. (a)Find the p.d.f of $\frac{X}{|Y|}$ (b)Find the p.d.f of $\frac{|X|}{|Y|}$

Solution: Let the random variables Z and W be such that,

$$Z = \frac{X}{|Y|} \quad (1)$$

$$W = \frac{|X|}{|Y|} \quad (2)$$

Given X and Y are normal distribution variables with mean 0 and variance 1. So their p.d.f are,

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \quad (3)$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} \quad (4)$$

So since X and Y are independent variables,

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x).f_Y(y)dy \quad (5)$$

$$f_Z(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\frac{-(x^2+y^2)}{2}} dy \quad (6)$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(zy).f_Y(y)dy \quad (7)$$

$$f_Z(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\frac{-((zy)^2+y^2)}{2}} dy \quad (8)$$

$$f_Z(z) = \frac{1}{\pi} \int_{0}^{\infty} e^{\frac{-y^2((z)^2+1)}{2}} dy \quad (9)$$

$$f_Z(z) = \frac{\sqrt{1+z^2}}{\pi} \int_{0}^{\infty} e^{\frac{-y^2}{2}} dy \quad (10)$$

$$f_Z(z) = \frac{\sqrt{1+z^2}}{\pi} \sqrt{\frac{\pi}{2}} \quad (11)$$

$$f_Z(z) = \sqrt{\frac{1+z^2}{2\pi}} \quad (12)$$

For W, X can take negative values too so our integral of (a) becomes double

$$f_W(w) = 2(f_Z(z)) \quad (13)$$

$$f_W(w) = \sqrt{\frac{2(1+w^2)}{\pi}} \quad (14)$$