# Assignment 6

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**Question:**The random variables $X$ and $X$ are independent and $Z = X + Y$.Find $f_Y(y)$.If ,

$$f_X(x) = ce^{-cx}U(x) \tag{1}$$
$$f_Z(z) = c^2 ze^{-cz}U(z) \tag{2}$$

**Solution:**Given,

$$Z = X + Y \tag{3}$$

So we know if $X$ and $Y$ are independent events,

$$f_Z(z) = \int_0^z f_X(x) * f_Y(z-y)dx \tag{4}$$
$$\Rightarrow f_Z(z) = \int_0^z f_X(z-y) * f_Y(y)dy \tag{5}$$

Lets use equation 5 since we have to find $y$,

$$c^2 e^{-cz} zU(z) = \int_0^z ce^{-c(z-y)}U(z) * f_Y(y)dy \tag{6}$$
$$\Rightarrow cz = \int_0^z e^{cy} * f_Y(y)dy \tag{7}$$

Now differentiating on both sides w.r.t $z$ we get,

$$c = e^{cz} f_Y(z) \tag{8}$$
$$\Rightarrow f_Y(z) = ce^{-cz} \tag{9}$$

The above derived equation is $f_Y(y)$