Assignment 4

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LaTeX

Outline

1. Question
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Question

For a Poisson random variable X with parameter λ show that

$$(a)\Pr\left(0 < X < 2\lambda\right) > \frac{(\lambda - 1)}{\lambda} \tag{1}$$

$$(b)E[X(X-1)] = \lambda^2 \tag{2}$$

$$(c)E[X(X-1)(X-2)] = \lambda^3 \tag{3}$$

Solution

Given X is a Poisson random variable so,

$$\Pr(X = x) = \frac{e^{-\lambda}.\lambda^x}{x!} \tag{4}$$

$$E(X) = \lambda \tag{5}$$

$$Var(X) = \sigma_X^2 = \lambda \tag{6}$$

(a)

(a) From Chebyshev's inequality we know that,
for any $\epsilon > 0$

$$\Pr\left(|X - \eta| \geq \epsilon\right) \leq \frac{\sigma_X^2}{\epsilon^2} \tag{7}$$

Also by definition of probability,

$$\Pr\left(|X - \eta| \geq \epsilon\right) + \Pr\left(|X - \eta| < \epsilon\right) = 1 \tag{8}$$

Substituting 8 in 7 the inequality becomes,

$$\Pr\left(|X - \eta| < \epsilon\right) > 1 - \frac{\sigma_X^2}{\epsilon^2} \tag{9}$$

Since this is Poisson distribution,

$$\eta = \lambda \tag{10}$$

$$\sigma_X^2 = \lambda \tag{11}$$

$$\epsilon = \lambda \tag{12}$$

Substituting values in the equation.

$$\Pr\left(|X - \lambda| < \lambda\right) > 1 - \frac{\lambda}{\lambda^2} \tag{13}$$

$$\Rightarrow \Pr\left(|X - \lambda| < \lambda\right) > 1 - \frac{1}{\lambda} \tag{14}$$

$$\Rightarrow \Pr\left(0 < X < 2\lambda\right) > 1 - \frac{1}{\lambda} \tag{15}$$

Hence proved.

(b)

(b) To show,

$$E[X(X-1)] = \lambda^2 \tag{16}$$

Lets use the definitions,

$$E[X] = \sum_{x=0}^{x=\infty} x\frac{e^{-\lambda}.\lambda^x}{x!} \tag{17}$$

Also one the property of mean is that,

$$E[g(x)] = \sum_{x=0}^{x=\infty} g(x)\frac{e^{-\lambda}.\lambda^x}{x!} \tag{18}$$

Here $g(x) = x(x-1)$ substituting it,

$$E[X(X-1)] = \sum_{x=2}^{x=\infty} x(x-1)\frac{e^{-\lambda}.\lambda^x}{x!} \tag{19}$$

$$\Rightarrow E[X(X-1)] = e^{-\lambda}\lambda^2 \sum_{x=2}^{x=\infty} \frac{.\lambda^{x-2}}{(x-2)!} \tag{20}$$

We know that by Taylors expansion,

$$e^x = \sum_{k=0}^{k=\infty} \frac{x^k}{k!} \tag{21}$$

Which is same as

$$e^{\lambda} = \sum_{x=2}^{x=\infty} \frac{.\lambda^{x-2}}{(x-2)!} \tag{22}$$

Substituting it,

$$E[X(X-1)] = e^{-\lambda}\lambda^{2}e^{-\lambda} \tag{23}$$

$$\Rightarrow E[X(X-1)] = \lambda^{2} \tag{24}$$

Hence proved

(c)

(c) using the above results we get

$$E[X(X-1)(X-2)] = \sum_{x=3}^{x=\infty} x(x-1)(x-2)\frac{e^{-\lambda}.\lambda^{x}}{x!} \tag{25}$$

$$E[X(X-1)(X-2)] = e^{-\lambda}\lambda^{3}\sum_{x=2}^{x=\infty} \frac{.\lambda^{x-3}}{(x-3)!} \tag{26}$$

$$E[X(X-1)(X-)2] = e^{-\lambda}\lambda^{3}e^{-\lambda} \tag{27}$$

$$\Rightarrow E[X(X-1)(X-2)] = \lambda^{3} \tag{28}$$

Hence proved