Assignment 3

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Outline

Question

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Question: We have four boxes. Box 1 contains 2000 components of which 5% are defective.Box 2 contains 500 components of which 40% are defective. Boxes 3 and 4 contain 1000 each with 10% defective. We select at random one of the boxes and we remove at random a single component.
a) What is the probability that the selected component is defective?
b)We examine the selected component and we find it defective. On the basis of this evidence, we want to determine the probability that it came from box 2.

Solution

Let the random variable Y take values,

Variable	Value	description
Y	0	If the item is found to defective
Y	1	If the item is found to be non-defective.

Let the random variable X take values,

Variable	Value	description
X	0	If the item is from box 1
X	1	If the item is from box 2
X	2	If the item is from box 3
X	2	If the item is from box 4

a) By total probability theorem;

$$\begin{aligned}\Pr(Y=0) = &\Pr(X=0) * \Pr(Y=0|X=0) \\ &+ \Pr(X=1) * \Pr(Y=0|X=1) \\ &+ \Pr(X=2) * \Pr(Y=0|X=2) \\ &+ \Pr(X=3) * \Pr(Y=0|X=3)\end{aligned} \tag{1}$$

$$\Pr(Y=0) = \frac{1}{4}(0.05 + 0.4 + 0.1 + 0.1) \tag{2}$$

$$\Rightarrow \Pr(Y=0) = 0.1624 \tag{3}$$

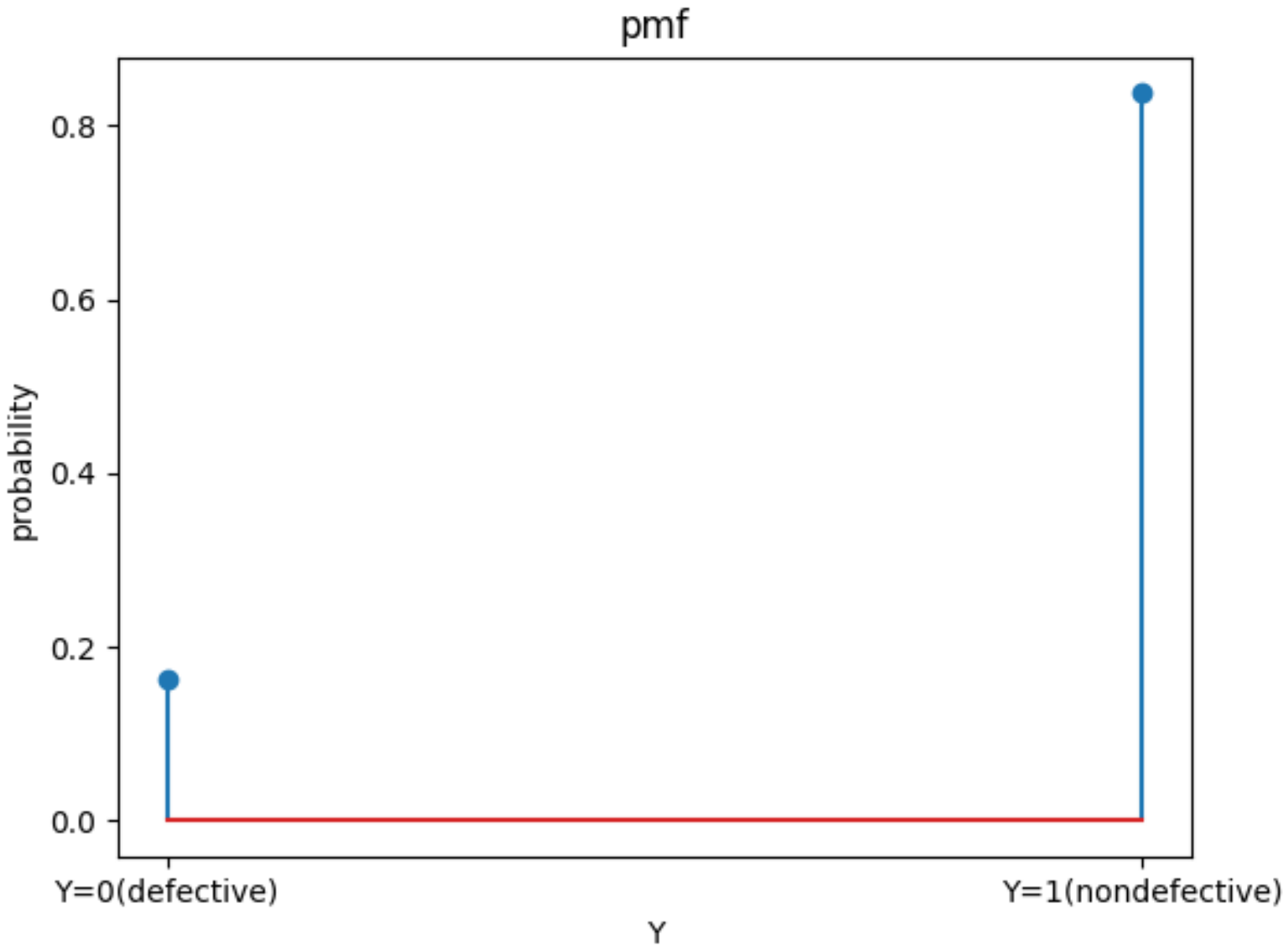
pmf
0.8
0.6
0.4
0.2
0.0
probability
Y=0(defective)
Y=1(nondefective)
Y

Figure: pmf

b) By bayes theorem;

$$\Pr(X = 1|Y = 0) = \frac{\Pr(X = 1) * \Pr(Y = 0|X = 1)}{\Pr(Y = 0)} \tag{4}$$

$$\Pr(X = 1|Y = 0) = \frac{0.4 * 0.25}{0.1625} \tag{5}$$

$$\Pr(X = 1|Y = 0) = 0.6153 \tag{6}$$

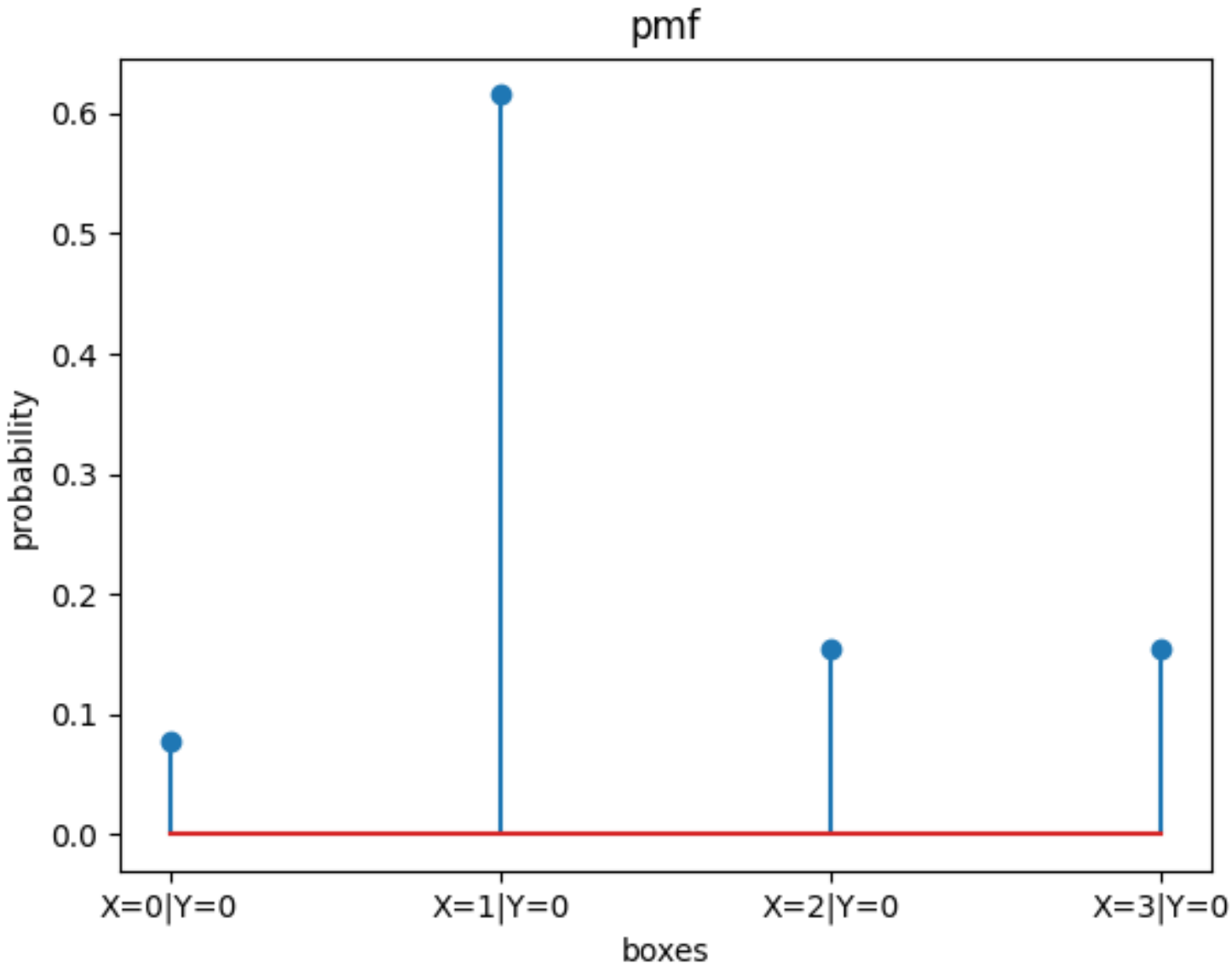

Figure: pmf