Assignment 8

AKKASANI YAGNESH REDDY
CS21BTECH11003

Question: Show that if $F(x, y, z)$ is a joint distribution. then for any $x_1 \leq x_2, y_1 \leq y_2, z_1 \leq z_2$:

$$\begin{aligned} &F(x_2, y_2, z_2) + F(x_1, y_1, z_1) + F(x_1, y_2, z_1) \\ &+F(x_2, y_1, z_1) - F(x_1, y_2, z_2) - F(x_2, y_1, z_2) \\ &-F(x_2, y_2, z_1) - F(x_1, y_1, z_1) \geq 0 \end{aligned} \tag{1}$$

Solution: We know that,

$$F(x) = \Pr\left(X \leq x\right) \tag{2}$$

Using this instead of F, We also know that,

$$0 \leq \Pr\left(x_1 \leq x \leq x_2, y_1 \leq y \leq y_2, z_1 \leq z \leq z_2\right) \tag{3}$$

$$\begin{aligned} &= \Pr\left(x \leq x_2, y_1 \leq y \leq y_2, z_1 \leq z \leq z_2\right) \\ &- \Pr\left(x \leq x_1, y_1 \leq y \leq y_2, z_1 \leq z \leq z_2\right) \end{aligned} \tag{4}$$

$$\begin{aligned} &= \Pr\left(x \leq x_2, y \leq y_2, z_1 \leq z \leq z_2\right) \\ &- \Pr\left(x \leq x_2, y \leq y_1, z_1 \leq z \leq z_2\right) \\ &- \Pr\left(x \leq x_1, y \leq y_2, z_1 \leq z \leq z_2\right) \\ &+ \Pr\left(x \leq x_1, y \leq y_1, z_1 \leq z \leq z_2\right) \end{aligned} \tag{5}$$

$$\begin{aligned} &= \Pr\left(x \leq x_2, y \leq y_2, z \leq z_2\right) \\ &- \Pr\left(x \leq x_2, y \leq y_2, z \leq z_1\right) \\ &- \Pr\left(x \leq x_2, y \leq y_1, z \leq z_2\right) \\ &+ \Pr\left(x \leq x_2, y \leq y_1, z \leq z_1\right) \\ &- \Pr\left(x \leq x_1, y \leq y_2, z \leq z_2\right) \\ &+ \Pr\left(x \leq x_1, y \leq y_2, z \leq z_1\right) \\ &+ \Pr\left(x \leq x_1, y \leq y_1, z \leq z_2\right) \\ &- \Pr\left(x \leq x_1, y \leq y_1, z \leq z_1\right) \end{aligned} \tag{6}$$

Hence proved because the above is ≥ 0 and is same as given in question that is,

$$\begin{aligned} &F(x_2, y_2, z_2) + F(x_1, y_1, z_1) + F(x_1, y_2, z_1) \\ &+F(x_2, y_1, z_1) - F(x_1, y_2, z_2) - F(x_2, y_1, z_2) \\ &-F(x_2, y_2, z_1) - F(x_1, y_1, z_1) \geq 0 \end{aligned} \tag{7}$$