Assignment 1

AKKASANI YAGNESH REDDY
cs21btech11003
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Question: If $\mathbf{a} = i - 2j + 3k$,$\mathbf{b} = 2i + 3j - 5k$. prove that $\mathbf{a}$ and $\mathbf{a} \times \mathbf{b}$ are perpendicular.

Solution: Given vectors

$$\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix} \tag{2}$$

Cross product between vectors is defined by,If

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \tag{3}$$

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{4}$$

Then,

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} \tag{5}$$

By following the definition and substituting values we get

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} (-2)(-5) - (3)(3) \\ (3)(2) - (-5)(1) \\ (1)(3) - (-2)(2) \end{pmatrix} \tag{6}$$

$$\Rightarrow \mathbf{a} \times \mathbf{b} = \begin{pmatrix} 1 \\ 11 \\ 7 \end{pmatrix} \tag{7}$$

$\mathbf{a}$ and $\mathbf{a} \times \mathbf{b}$ are perpendicular if dot product between them is 0. Dot product between two vector $\mathbf{a}$ and $\mathbf{b}$ is, If

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \tag{8}$$

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{9}$$

Then,

$$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \tag{10}$$

Substituting values,

$$\mathbf{a}.(\mathbf{a} \times \mathbf{b}) = (1)(2) + (-2)(3) + (3)(-5) \tag{11}$$

$$\Rightarrow \mathbf{a}.(\mathbf{a} \times \mathbf{b}) = 0 \tag{12}$$

Hence both $\mathbf{a}$ and $\mathbf{a} \times \mathbf{b}$ are perpendicular to each other.

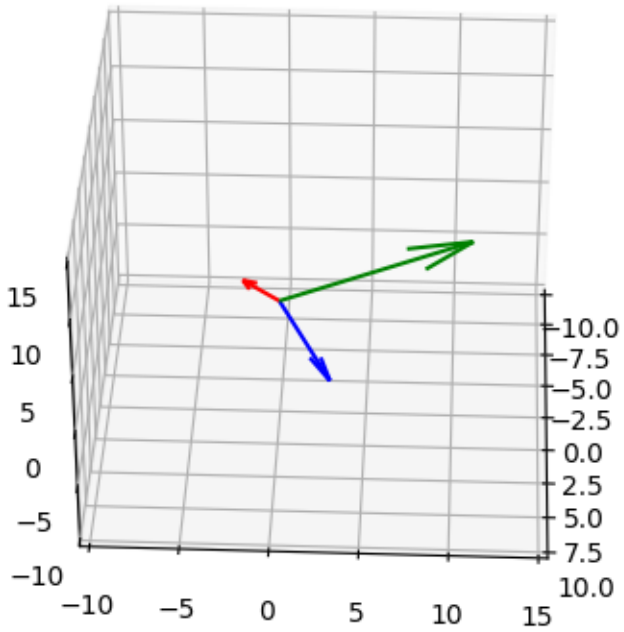

Fig. 1. Python generated figure