

Assignment 1

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Outline

1 Question

2 solution

Question

Question

If $\mathbf{a} = i - 2j + 3k$, $\mathbf{b} = 2i + 3j - 5k$. prove that \mathbf{a} and $\mathbf{a} \times \mathbf{b}$ are perpendicular.

Solution

Given vectors

$$\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \quad (1)$$

$$\mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix} \quad (2)$$

Cross product between vectors is defined by, If

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad (3)$$

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (4)$$

Solution

Then,

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \quad (5)$$

By following the definition and substituting values we get

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} (-2)(-5) - (3)(3) \\ (3)(2) - (-5)(1) \\ (1)(3) - (-2)(2) \end{pmatrix} \quad (6)$$

$$\Rightarrow \mathbf{a} \times \mathbf{b} = \begin{pmatrix} 1 \\ 11 \\ 7 \end{pmatrix} \quad (7)$$

Solution

a and **a** × **b** are perpendicular if dot product between them is 0. Dot product between two vector **a** and **b** is, If

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad (8)$$

$$\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (9)$$

Then,

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad (10)$$

Solution

Substituting values,

$$\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = (1)(2) + (-2)(3) + (3)(-5) \quad (11)$$

$$\Rightarrow \mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0 \quad (12)$$

Hence both \mathbf{a} and $\mathbf{a} \times \mathbf{b}$ are perpendicular to each other.

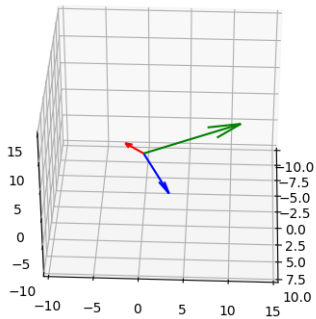


Figure: Python generated figure