Assignment 8

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Outline

Question: Show that if $F(x, y, z)$ is a joint distribution. then for any $x_1 \leq x_2, y_1 \leq y_2, z_1 \leq z_2$:

$$\begin{aligned} F(x_2, y_2, z_2) + F(x_1, y_1, z_1) + F(x_1, y_2, z_1) \\ +F(x_2, y_1, z_1) - F(x_1, y_2, z_2) - F(x_2, y_1, z_2) \\ -F(x_2, y_2, z_1) - F(x_1, y_1, z_1) \geq 0 \end{aligned} \tag{1}$$

Solution: We know that, Using this instead of F,

$$F(x) = \Pr(X \leq x) \tag{2}$$

$$0 \leq \Pr(x_1 \leq x \leq x_2, y_1 \leq y \leq y_2, z_1 \leq z \leq z_2) \tag{3}$$

$$0 \leq \Pr(x \leq x_2, y_1 \leq y \leq y_2, z_1 \leq z \leq z_2)$$

$$- \Pr(x \leq x_1, y_1 \leq y \leq y_2, z_1 \leq z \leq z_2) \tag{4}$$

$$= \Pr(x \leq x_2, y \leq y_2, z_1 \leq z \leq z_2) - \Pr(x \leq x_2, y \leq y_1, z_1 \leq z \leq z_2)$$
$$- \Pr(x \leq x_1, y \leq y_2, z_1 \leq z \leq z_2) + \Pr(x \leq x_1, y \leq y_1, z_1 \leq z \leq z_2) \quad (5)$$

$$
\begin{aligned}
&= \Pr(x \le x_2, y \le y_2, z \le z_2) - \Pr(x \le x_2, y \le y_2, z \le z_1) \\
&- \Pr(x \le x_2, y \le y_1, z \le z_2) + \Pr(x \le x_2, y \le y_1, z \le z_1) \\
&- \Pr(x \le x_1, y \le y_2, z \le z_2) + \Pr(x \le x_1, y \le y_2, z \le z_1) \\
&+ \Pr(x \le x_1, y \le y_1, z \le z_2) - \Pr(x \le x_1, y \le y_1, z \le z_1) \qquad (6)
\end{aligned}
$$

Hence proved because the above is ≥ 0 and is same as given in question that is,

$$\begin{aligned} &F(x_2, y_2, z_2) + F(x_1, y_1, z_1) + F(x_1, y_2, z_1) \\ &+F(x_2, y_1, z_1) - F(x_1, y_2, z_2) - F(x_2, y_1, z_2) \\ &-F(x_2, y_2, z_1) - F(x_1, y_1, z_1) \geq 0 \end{aligned} \tag{7}$$