Assignment 7

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Outline

Question: Let X and Y be independent normal random variables with zero mean and unit variances.
(a)Find the p.d.f of $\frac{X}{|Y|}$ (b)Find the p.d.f of $\frac{|X|}{|Y|}$

Solution: Let the random variables Z and W be such that,

$$Z = \frac{X}{|Y|} \tag{1}$$

$$W = \frac{|X|}{|Y|} \tag{2}$$

Given X and Y are normal distribution variables with mean 0 and variance 1. So their p.d.f are,

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \tag{3}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} \tag{4}$$

So since X and Y are independent variables,

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x).f_Y(y)dy \tag{5}$$

$$f_Z(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{\frac{-(x^2+y^2)}{2}} dy \tag{6}$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(zy).f_Y(y)dy \tag{7}$$

$$f_Z(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{\frac{-((zy)^2+y^2)}{2}} dy \tag{8}$$

$$f_Z(z) = \frac{1}{\pi}\int_{0}^{\infty} e^{\frac{-y^2((z)^2+1)}{2}} dy \tag{9}$$

$$f_Z(z) = \frac{\sqrt{1+z^2}}{\pi} \int_0^\infty e^{\frac{-y^2}{2}} dy \tag{10}$$

$$f_Z(z) = \frac{\sqrt{1+z^2}}{\pi} \sqrt{\frac{\pi}{2}} \tag{11}$$

$$f_Z(z) = \sqrt{\frac{1+z^2}{2\pi}} \tag{12}$$

For W, X can take negative values too so our integral of (a) becomes double

$$f_W(w) = 2(f_Z(z)) \tag{13}$$

$$f_W(w) = \sqrt{\frac{2(1+w^2)}{\pi}} \tag{14}$$