

# Assignment 9

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# Outline

1 Question

2 Solution:

**Question:** The random variable  $X$  has the Erlang density  $f(x) = c^4 \cdot x^3 \cdot e^{-cx}$ . We observe the samples  $X_i = 3.1, 3.4, 3.3$ . Find the ML estimate  $c$ .

**Solution:** Lets generalize and find ML. Lets take for  $n$  random values the p.d.f will be

$$f(x, c) = c^4 \cdot x^3 \cdot e^{-cx} \quad (1)$$

$$f(x_1, x_2, x_3, \dots, x_n, c) = c^{4n} \cdot (x_1 \dots x_n)^3 \cdot e^{-nc\hat{x}} \quad (2)$$

Where  $\hat{x}$  is the mean of random variable  $X$ .

Now to find ML of this function partially differentiate it w.r.t  $c$ .

$$\frac{\partial f(X, c)}{\partial c} = \frac{\partial c^{4n} \cdot (x_1 \dots x_n)^3 \cdot e^{-n^c \hat{x}}}{\partial c} \quad (3)$$

$$\frac{\partial f(X, c)}{\partial c} = n \cdot c^{4n-1} \cdot x^3 e^{-cn\hat{x}} (4 - c\hat{x}) \quad (4)$$

for ML equate partial differentiation to zero and that value is the estimate of  $c$ ,

$$n \cdot c^{4n-1} \cdot x^3 e^{-cn\hat{x}} (4 - c\hat{x}) = 0 \quad (5)$$

$$4 - c\hat{x} = 0 \quad (6)$$

$$c = \frac{4}{\hat{x}} \quad (7)$$

given  $X_i = 3.1, 3.4, 3.3,$

$$\hat{x} = \frac{3.1 + 3.3 + 3.4}{3} \quad (8)$$

$$\hat{x} = 3.27 \quad (9)$$

Now,

$$c = \frac{4}{\hat{x}} \quad (10)$$

$$c = \frac{4}{3.27} \quad (11)$$

$$c = 1.224. \quad (12)$$