Assignment 4

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Question: Show that

$$\Pr(A) = \Pr(A|X \le x) F(x) + \Pr(A|X > x)[1 - F(x)] \quad (1)$$

Solution: Lets define a random variable Y such that,

Variable	Value	description
Y	1	If event A happens
Y	0	If event A does not happen

Now lets take the given equation R.H.S.

$$\Pr(A|X \le x) F(x) + \Pr(A|X > x)[1 - F(x)] \quad (2)$$

$$\Rightarrow \Pr(Y = 1|X \le x) F(x) + \Pr(Y = 1|X > x)[1 - F(x)] \quad (3)$$

By the definition of conditional probability.

$$\Pr(Y = 1|X \le x) = \frac{\Pr((Y = 1)(X \le x))}{\Pr(X \le x)} \quad (4)$$

$$\Pr(Y = 1|X > x) = \frac{\Pr((Y = 1)(X > x))}{\Pr(X > x)} \quad (5)$$

Also by the definition of cumulative probability,

$$\Pr(X \le x) = F(x) \quad (6)$$

By the definition of probability,

$$\Pr(X \le x) + \Pr(X > x) = 1 \quad (7)$$

Substituting 7 in 6 we get,

$$F(x) + \Pr(X > x) = 1 \quad (8)$$

$$\Rightarrow \Pr(X > x) = 1 - F(x) \quad (9)$$

Substituting equations $4, 5, 6$ and 9 in 3 we get,

$$\frac{\Pr((Y = 1)(X \le x))}{F(x)} F(x) + \frac{\Pr((Y = 1)(X > x))}{(1 - F(x)} [1 - F(x)] \quad (10)$$

$$\Rightarrow \Pr((Y = 1)(X \le x)) + \Pr((Y = 1)(X > x)) \quad (11)$$

$$\Rightarrow \Pr(Y = 1) \quad (12)$$

We showed that R.H.S is the same as L.H.S,

$$\Pr(Y = 1) = \Pr(Y = 1) \quad (13)$$

Hence **proved**.