Direct Position Determination of Narrowband Radio Frequency Transmitters

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***Abstract*—The most common methods for location of communications or radar transmitters are based on measuring a specified parameter such as signal angle of arrival (AOA) or time of arrival (TOA). The measured parameters are then used to estimate the transmitter location. Since the AOA/TOA measurements are done at each base station separately, without using the constraint that all measurements must correspond to the same transmitter, they are suboptimal. We propose a technique that uses exactly the same data as the common methods, except that the estimation of location is based on exact maximum likelihood, and the location determination is direct. Although there are many stray parameters, including the attenuation coefficients and the signal waveform, the method requires only a two-dimensional search. Monte Carlo simulations indicate that the accuracy is equivalent to AOA, TOA, and their combination for high SNR, while for low SNR, the accuracy of the proposed method is superior.**

***Index Terms*—Angle of arrival (AOA), array processing, emitter localization, matched-field processing, maximum likelihood, time of arrival (TOA).**

I. Introduction

THE PROBLEM of emitter location attracts much interest in the signal processing, vehicular technology, and underwater acoustics literature. Defense-oriented location systems have been reported since World War I. Perhaps the first paper on the mathematics of emitter location, using angle of arrival (AOA), is due to Stansfield [1]. Many other publications followed, including a fine review paper by Torrieri [2]. The papers by Krim and Viberg [3] and Wax [4] are comprehensive review papers on antenna array processing for location by AOA. Recently, Van-Trees [5] published a book that is fully devoted to array processing. Positioning by time-of-arrival (TOA) is well known in radar systems [6], and in underwater acoustics [7]. In underwater acoustics, matched-field processing (MFP) is viewed as a promising procedure for source localization [8]. MFP can be interpreted as the maximum *a posteriori* (MAP) estimate of location given the observed signal at the output of an array of sensors [8], [9]. Another interpretation of MFP is the well-known beamforming extended to wide-bandwidth signals, nonplanar wave fields, and unknown environmental parameters.

In this letter, we discuss a method that has some similarities with MFP. While the concept is similar, the details are different. The models of underwater acoustic propagation are usually more complex than the models used in most AOA/TOA electromagnetic emitter location papers and systems. Hence, the required processing for traditional matched-field processing is rather heavy. Moreover, the underwater source's distance from the sensors is usually the same order of magnitude as the sensor array size. Hence, the far-field assumption that is usually used in electromagnetic AOA does not hold for MFP.

The direct position determination (DPD) method that we propose takes advantage of the rather simple propagation assumptions that are usually used for radio frequency (RF) signals. This enables us to obtain a simple closed-form cost function. The cost function can be maximized using a two-dimensional (2-D) search for an emitter known to be located on a plane or a three-dimensional (3-D) search in general. The DPD belong to the least squares family if the noise statistics are unknown. If the noise is Gaussian, DPD is the exact maximum-likelihood estimate of location. We demonstrate that DPD outperforms AOA, TOA, and the combination of AOA and TOA. The DPD technique requires the transmission of the received signals (possibly sampled) to a central processing location. However, AOA and TOA require only the transmission of the measured parameters to the central processing location. This is the cost of employing DPD. This letter focuses on the single signal case. Extensions to multiple signals will be published in the near future.

II. Problem Formulation and a Potential Algorithm

Consider a transmitter and L base stations intercepting the transmitted signal. Each base station is equipped with an antenna array consisting of M elements. Denote the transmitter position by the vector of coordinates $\mathbf{p}$ and the lth base station position by the vector of coordinates $\mathbf{q}_l$. The signal observed by the lth base station array is given by

$$\mathbf{r}_l(t) = b_l \mathbf{a}_l(\mathbf{p}) s\left(t - \tau_l(\mathbf{p}) - t_0\right) + \mathbf{n}_l(t) \tag{1}$$

where $\mathbf{r}_l(t)$ is a time-dependent $M \times 1$ vector, b_l is an unknown complex scalar representing the channel effect (attenuation), $\mathbf{a}_l(\mathbf{p})$ is the lth array response to signal transmitted from position $\mathbf{p}$, and $s(t - \tau_l(\mathbf{p}) - t_0)$ is the signal waveform, transmitted at time t_0 and delayed by $\tau_l(\mathbf{p})$. The vector $\mathbf{n}_l(t)$ represents noise and interference, including multipath observed by the array.

The sampled version of the signal in (1) is given by

$$\begin{aligned}
\mathbf{r}_l(j) &= b_l \mathbf{a}_l(\mathbf{p}) s_l(j) + \mathbf{n}_l(j), \qquad 0 \le j \le N_s - 1 \\
s_l(j) &\triangleq s\left(t - \tau_l(\mathbf{p}) - t_0\right)\big|_{t=jT} \\
\mathbf{r}_l(j) &\triangleq \mathbf{r}_l(t)\big|_{t=jT} \\
\mathbf{n}_l(j) &\triangleq \mathbf{n}_l(t)\big|_{t=jT}.
\end{aligned} \tag{2}$$

Manuscript received July 31, 2003; revised October 20, 2003. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Dimitris A. Pados.

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Digital Object Identifier 10.1109/LSP.2004.826501

We observe that information about the transmitter position is embedded in the observed signal in two different ways. The first is the array response. If the signal is in the far field (its distance from the station is many times the array aperture), the array response becomes a function of the angle of arrival only. The position is also reflected by the TOA of the signal at the array $\tau_l(\mathbf{p})$, which depends on the distance between the transmitter and the station.

In order to obtain an attractive algorithm it is desirable to separate the delay and the transmit time from the signal waveform. This happens naturally in the frequency domain representation of the problem. Taking the discrete Fourier transform (DFT) of (2) we get

$$\begin{aligned}\bar{\mathbf{r}}_l(k) &= b_l \mathbf{a}_l(\mathbf{p})\bar{s}(k)e^{-j\omega_k[\tau_l(\mathbf{p})+t_0]} + \bar{\mathbf{n}}_l(k)\\ \omega_k &\triangleq \frac{2\pi k}{N_s T}, \qquad 0 \le k \le N_s - 1\end{aligned} \tag{3}$$

where the overbar indicate the DFT coefficient of the corresponding time samples.

The least squares estimate of the position is given by minimizing the cost function

$$Q(\mathbf{p}) = \sum_{l=1}^{L}\sum_{k=0}^{N_s-1}\left\|\bar{\mathbf{r}}_l(k) - b_l\mathbf{a}_l(\mathbf{p})\bar{s}(k)e^{-j\omega_k[\tau_l(\mathbf{p})+t_0]}\right\|^2 \tag{4}$$

where $\|\bullet\|$ stands for the Frobenius norm. Note that the cost function can be represented by a sum over L terms as follows:

$$\begin{aligned}Q(\mathbf{p}) &= \sum_{l=1}^{L} Q_l(\mathbf{p})\\ Q_l(\mathbf{p}) &\triangleq \sum_{k=0}^{N_s-1}\left\|\bar{\mathbf{r}}_l(k) - b_l\mathbf{a}_l(\mathbf{p})\bar{s}(k)e^{-j\omega_k[\tau_l(\mathbf{p})+t_0]}\right\|^2.\end{aligned} \tag{5}$$

Define the following vectors:

$$\begin{aligned}\bar{\mathbf{r}}_l &\triangleq \left[\bar{\mathbf{r}}_l^T(0), \bar{\mathbf{r}}_l^T(1), \cdots, \bar{\mathbf{r}}_l^T(N_s-1)\right]^T\\ \bar{\mathbf{s}}_l &\triangleq \left[\bar{s}(0)e^{-j\omega_0[\tau_l(\mathbf{p})+t_0]}, \cdots \bar{s}(N_s-1)e^{-j\omega_{N_s-1}[\tau_l(\mathbf{p})+t_0]}\right]^T\\ \mathbf{c}_l &\triangleq \bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\end{aligned} \tag{6}$$

where $\otimes$ stands for the Kronecker product. Now (5) can be represented by

$$Q(\mathbf{p}) = \sum_{l=1}^{L}\|\bar{\mathbf{r}}_l - \mathbf{c}_l b_l\|^2. \tag{7}$$

The estimate of b_l that minimizes the cost function is given by

$$\begin{aligned}\hat{b}_l &= \left(\mathbf{c}_l^H\mathbf{c}_l\right)^{-1}\mathbf{c}_l^H\mathbf{r}_l\\ &= \left(\left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]^H\left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]\right)^{-1}\left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]^H\bar{\mathbf{r}}_l\\ &= \left(\left[\bar{\mathbf{s}}_l^H\bar{\mathbf{s}}_l \otimes \mathbf{a}_l^H(\mathbf{p})\mathbf{a}_l(\mathbf{p})\right]\right)^{-1}\left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]^H\bar{\mathbf{r}}_l\\ &= \frac{1}{\|\bar{\mathbf{s}}_l\|^2\|\bar{\mathbf{a}}_l(\mathbf{p})\|^2}\left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]^H\bar{\mathbf{r}}_l\end{aligned} \tag{8}$$

where $[\bullet]^H$ stands for the Hermitian transpose operation.

Without loss of generality we assume that

$$\begin{aligned}\|\bar{\mathbf{s}}_l\|^2 &= 1 \quad \forall l\\ \|\mathbf{a}_l(\mathbf{p})\|^2 &= 1 \quad \forall l.\end{aligned} \tag{9}$$

Substituting (8) and (9) in (7) we get

$$\begin{aligned}Q(\mathbf{p}) &= \sum_{l=1}^{L}\left\|\bar{\mathbf{r}}_l - \left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]\left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]^H\bar{\mathbf{r}}_l\right\|^2\\ &= \sum_{l=1}^{L}\bar{\mathbf{r}}_l^H\bar{\mathbf{r}}_l - \bar{\mathbf{r}}_l^H\left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]\left[\bar{\mathbf{s}}_l \otimes \mathbf{a}_l(\mathbf{p})\right]^H\bar{\mathbf{r}}_l\\ &= \sum_{l=1}^{L}\|\bar{\mathbf{r}}_l\|^2 - \sum_{l=1}^{L}\left|\mathbf{a}_l^H(\mathbf{p})\sum_{k=0}^{N_s-1}e^{j\omega_k[\tau_l(\mathbf{p})+t_0]}\bar{s}^*(k)\bar{\mathbf{r}}_l(k)\right|^2.\end{aligned} \tag{10}$$

Instead of finding the minimum of $Q(\mathbf{p})$, we can find the maximum of $\tilde{Q}(\mathbf{p})$ defined by

$$\tilde{Q}(\mathbf{p}) = \sum_{l=1}^{L}\left|\mathbf{a}_l^H(\mathbf{p})\sum_{k=0}^{N_s-1}e^{j\omega_k[\tau_l(\mathbf{p})+t_0]}\bar{s}^*(k)\bar{\mathbf{r}}_l(k)\right|^2. \tag{11}$$

Define the vectors

$$\begin{aligned}\mathbf{d}_l &\triangleq \left[d_l(0), \cdots, d_l(N_s-1)\right]^T\\ d_l(k) &\triangleq e^{j\omega_k\tau_l(\mathbf{p})}\mathbf{a}_l^H(\mathbf{p})\bar{\mathbf{r}}_l(k);\\ \bar{\mathbf{s}} &\triangleq \left[\bar{s}(0)e^{-j\omega_0 t_0}, \cdots, \bar{s}(N_s-1)e^{-j\omega_{N_s-1}t_0}\right]^T.\end{aligned} \tag{12}$$

Using these definitions, we can rewrite (11) as

$$\begin{aligned}\tilde{Q}(\mathbf{p}) &= \sum_{l=1}^{L}|\bar{\mathbf{s}}^H\mathbf{d}_l|^2\\ &= \sum_{l=1}^{L}\bar{\mathbf{s}}^H\mathbf{d}_l\mathbf{d}_l^H\bar{\mathbf{s}}\\ &= \bar{\mathbf{s}}^H\left(\sum_{l=1}^{L}\mathbf{d}_l\mathbf{d}_l^H\right)\bar{\mathbf{s}}\\ &= \bar{\mathbf{s}}^H\mathbf{D}\bar{\mathbf{s}}.\end{aligned} \tag{13}$$

Under the common assumption that the signal waveform is not known to the receivers, the cost function in (13) is maximized by selecting the vector $\bar{\mathbf{s}}$ as the eigenvector corresponding to the largest eigenvalue of the matrix $\mathbf{D}$. Hence, (13) reduces to

$$\tilde{Q}(\mathbf{p}) = \lambda_{\max}(\mathbf{D}) \tag{14}$$

where the right side of (14) denotes the largest eigenvalue of $\mathbf{D}$, and the matrix $\mathbf{D}$ is a function of the data, the array response at each base station, the location of the base stations, and the unknown emitter location $\mathbf{p}$. It is clear that the maximization of (14) requires only a 2-D (or 3-D) search, although the estimator knows neither the channel response nor the signal. It is interesting to note that the dimensions of the matrix $\mathbf{D}$ are $N_s \times N_s$, which might be rather large for some cases. However, we can replace $\mathbf{D}$ with the $L \times L$ matrix $\tilde{\mathbf{D}}$ where

$$\mathbf{U} \triangleq [\mathbf{d}_1, \cdots \mathbf{d}_L] \quad \mathbf{D} = \mathbf{U}\mathbf{U}^H \quad \tilde{\mathbf{D}} \triangleq \mathbf{U}^H\mathbf{U}. \tag{15}$$

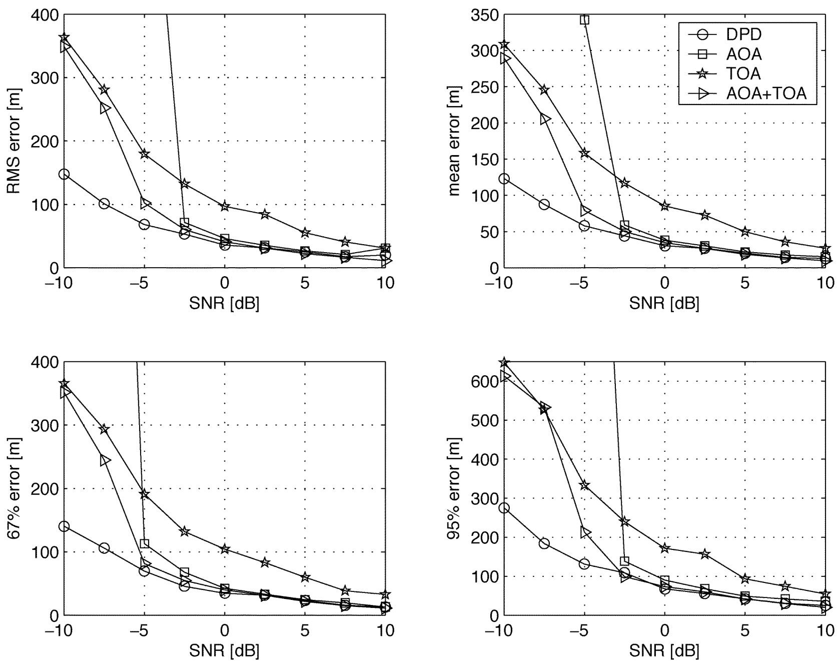

Fig. 1. RMS, mean, 67%, 95% of miss distance for four different methods, unknown signal.

Thus, (14) becomes

$$\tilde{Q}(\mathbf{p}) = \lambda_{\max}(\tilde{\mathbf{D}}). \tag{16}$$

This result holds for a single observation of the signal for a period equivalent to N_s samples. Extension to multiple observations of the signal is straightforward.

The case of known signal waveform (e.g., training signal or a synchronization signals are known to the receivers) is of great interest as was shown by Li and Compton in [10]. In this case, we return to (13) and rewrite it as follows:

$$\begin{aligned}
\tilde{Q}(\mathbf{p}) &= \overline{\mathbf{s}}^H \mathbf{D} \overline{\mathbf{s}} = \mathbf{z}^H \mathbf{S}^H \mathbf{U} \mathbf{U}^H \mathbf{S} \mathbf{z} = \mathbf{z}^H \tilde{\mathbf{U}} \tilde{\mathbf{U}}^H \mathbf{z} \\
\mathbf{S} &\triangleq \mathrm{diag}\left\{[\overline{s}(0), \overline{s}(1), \cdots, \overline{s}(N_s - 1)]\right\} \\
\tilde{\mathbf{U}} &\triangleq \mathbf{S}^H \mathbf{U} \\
\mathbf{z} &\triangleq \left[1, e^{-j\omega_1 t_0}, e^{-j2\omega_1 t_0}, \cdots, e^{-j(N_s-1)\omega_1 t_0}\right]^T \\
\omega_1 &\triangleq \frac{2\pi}{N_s T}.
\end{aligned} \tag{17}$$

The unknowns are the transmit time t_0, and the emitter position, $\mathbf{p}$. For any given $\mathbf{p}$, we can estimate t_0 by a one-dimensional search or by FFT of the columns of $\tilde{\mathbf{U}}$. If we choose the later method, we get the following cost function:

$$\begin{aligned}
\mathbf{w} &\triangleq \sum_{l=1}^{L} \left| \mathrm{FFT}\{\tilde{\mathbf{U}}_l\} \right|^2 \\
\tilde{\mathbf{U}}_l &\triangleq \left[\tilde{\mathbf{U}}(1,l), \tilde{\mathbf{U}}(2,l), \cdots \tilde{\mathbf{U}}(N_s, l)\right]^T \\
\tilde{Q}(\mathbf{p}) &= \max_k \{\mathbf{w}_k\}.
\end{aligned} \tag{18}$$

Stated in words, perform FFT on each of the columns of $\tilde{\mathbf{U}}$, and sum the squared absolute value of the Fourier coefficient over the L results to obtain the vector $\mathbf{w}$. The length of $\mathbf{w}$ corresponds to the FFT length, which may be a multiplicity of N_s, depending on the desired resolution. The maximum element of $\mathbf{w}$ is the desired cost function.

III. Numerical Results

In order to examine the performance of the advocated method and compare it with the traditional approaches, we performed extensive Monte Carlo simulations. Some examples are shown here. Consider four base stations placed at the corners of a 4 km $\times$ 4 km square. Each base station is equipped with a circular array of five antenna elements. The radius of the array is one wavelength. The transmitter location is selected at random, uniformly, within the square formed by the base stations. Each location determination is based on 32 samples of the signal. The SNR is varied between -10 and $+10$ dB. At each SNR value, we performed 100 experiments in order to obtain the statistical properties of the performance. The path-loss attenuation magnitude is selected at random using normal distribution (mean $= 1$, std $= 0.1$), and the attenuation phase is uniformly distributed in $[-\pi, \pi]$. We applied four different techniques in order to locate the transmitter.

1) AOA estimation using maximum likelihood (also known as beamforming) and maximum-likelihood emitter location estimation using the AOA estimates as the data.
2) TOA estimation using maximum likelihood (under the assumption that the signal waveform is known at the base stations and using all antenna elements) and maximum-likelihood emitter location estimation using the TOA estimates as the data.
3) Maximum-likelihood emitter location estimation using both AOA and TOA as the data.
4) Direct position determination (DPD) according to (16).

The performance evaluation is based on the statistics of the miss distance, i.e., the distance between the true emitter position and the estimated emitter position.

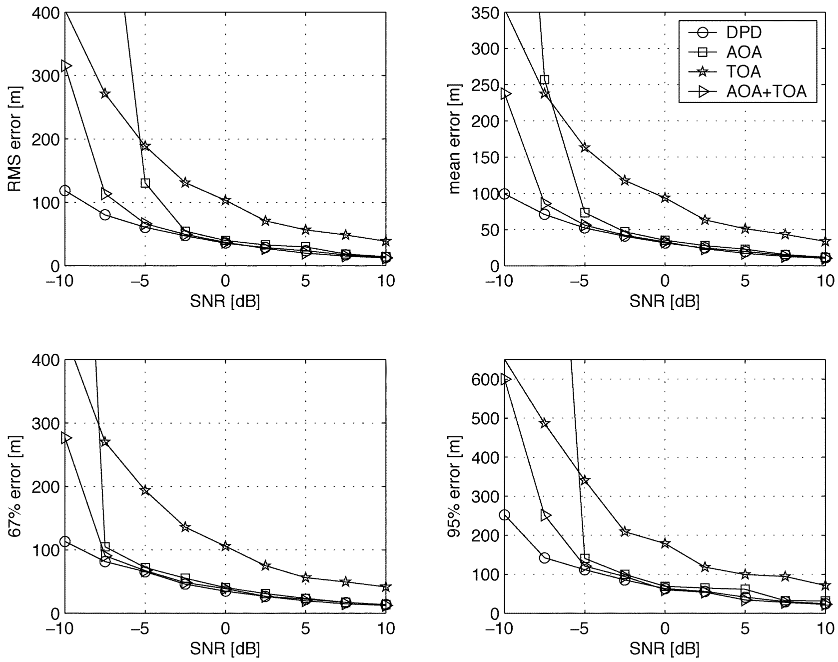

Fig. 2. RMS, mean, 67%, 95% of miss distance for four different methods, known signal.

We used four different criteria:

1) Root mean square (RMS) of miss distance;
2) mean of miss distance;
3) miss distance that upper bounds 67% of the errors;
4) miss distance that upper bounds 95% of the errors.

All the plots in Fig. 1 indicate that DPD is superior to AOA, TOA, and even combined AOA and TOA. The advantage of DPD is at low SNR. At high SNR, all methods give excellent results. Fig. 2 shows similar results for known signals.

IV. Conclusion

We have proposed a direct position determination technique that has the same results as AOA, TOA, and their combination at high SNR but has better accuracy at low SNR. The DPD is closely related to matched-field processing, but it is suitable only for RF signals and not for underwater emitter location. Further research is currently underway that explores the advantages and disadvantages of the proposed method for multiple signals and more complex propagation models. Small-error analysis, threshold prediction, and comparison with the Cramer–Rao bound will be published in the near future.

Acknowledgment

The author would like to thank A. Amar and Y. Isbi for fruitful discussions.

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