


Efficient range-free localization using elliptical distance correction in heterogeneous wireless sensor networks

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Abstract

In this article, a novel range-free localization algorithm is proposed based on the modified expected hop progress for heterogeneous wireless sensor networks where all nodes' communication ranges are different. First, we construct the new cumulative distribution function expression of expected hop progress to reduce the computational complexity. Then, the elliptical distance correction method is used to improve the accuracy of the estimation distance and simultaneously decrease overhead. Finally, using the modified distance, the coordinate of the unknown node can be obtained by maximum likelihood estimation. Compared with other algorithms for heterogeneous wireless sensor network, the proposed algorithm is superior in the localization accuracy and efficiency when used in random and uniform placement of nodes.

Keywords

Heterogeneous wireless sensor networks, range-free localization, multihop, expected hop progress, elliptical distance correction

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Introduction

Wireless sensor network (WSN) comprises many sensors randomly/spatially distributed over a certain area with the purpose of monitoring a spatial physical phenomenon, tracking and locating mobile targets.¹⁻⁴ WSN has attracted keen attentions in various applications due to their densely distributed, self-adaption, lost-cost, and possibly heterogeneous sensors. Some researchers have made in-depth researches on WSN and have applied some new methods such as neural network, Laplacian eigenmap, and Bayesian method to solve the problems existing in WSNs.⁵⁻⁷

According to the way of information processing, the WSN can be divided into the distributed algorithm and centralized algorithm. With the purpose of monitoring

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and controlling, the algorithm data should be collected and processed in the data center. Therefore, most of the centralized algorithms with high precision are used. However, the centralized algorithm has the disadvantage of large calculation quantity.⁸ In the distributed algorithm, signal processing is advantageous in terms of scalability to sensor networks, so this article mainly studies the distributed WSN.^{9,10}

The localization information of sensor nodes is very critical to the performance of WSN. In recent years, a wide variety of localization methods have been proposed, according to the diverse application requirements which can be roughly divided into two types: range based and range free, based on whether or not the network needs to measure the actual distances between nodes. Range-based algorithms, such as the received signal strength indication (RSSI),¹¹ time difference of arrival (TDOA),¹² and angle of arrival (AOA),¹³ exploit the measurements of angle and distance. Then, the location of the unknown nodes (UNs) can be calculated by the trilateral measurement method and triangulation method. Therefore, it needs extra hardware supporting, large computing, and communicating with high costs. The range-free algorithms are based on hop count information to estimate the distance between anchor nodes (ANs) and UNs without any extra hardware supporting. It includes distance vector hop (DV-Hop),¹⁴ Amorphous,¹⁵ localization using expected hop progress (LAEP),¹⁶ and so on. However, most existing range-free algorithms are based on the unrealistic assumption that all nodes have the same communication radius, that is, WSN is homogeneous. In fact, heterogeneity problem of WSN commonly exists in practical application since sensors are designed using various technologies to achieve different tasks, and their sensing as well as communication capabilities are commonly different. Thus, the approaches, which assume the same communication capability throughout the network, make the localization accuracy poor in heterogeneous wireless sensor networks (HWSNs) and are unsuitable for such networks. The expected hop progress (EHP) algorithm for the HWSN localization is proposed.¹⁷ The algorithm depends not only on the communication radius of ANs but also on the communication radius of intermediate nodes (INs) using those radii that are closer to a real Euclidean distance generated by any two selected nodes. However, its conditional cumulative distribution function (CDF) has high computing complexity. To solve the problem, Assaf et al.¹⁸ presented a modified EHP algorithm by redefining new CDF. This approach achieves satisfactory localization results and shows remarkable communication performance for HWSN, but compared with the original EHP, the algorithm has additional overhead and energy consumption because it needs some training before redefining CDF. To the best of our knowledge, there are very few sufficient and

efficacious algorithms in HWSN localization. However, to further improve the estimation distance or localization accuracy, researchers have proposed various correction methods such as residual correction method, weighted least squares (WLS), and weighted centroid method. However, these correction methods either increase computations due to multiple iterations such as Assaf et al.^{17,19} or assume that all nodes have the same communication radius. Therefore, up to now, no method can obtain the satisfying localization results for HWSN.

To solve those problems, in this article, by defining CDF as Poisson distribution and an elliptical distance, a novel range-free localization algorithm is proposed based on EHP for HWSN where all nodes' communication ranges are different. First, we assume that the degree of irregularity (DOI) of the communication radius is equal to zero, that is, transmission coverage of each node is circular. And there is no path loss in the communication process. Then, the new CDF is defined as Poisson distribution to reduce the computational complexity. Finally, we propose an elliptical distance correction method to calculate the distance between nodes and use maximum likelihood estimation (MLE) method to compute the UN location information without increasing overhead.

The remainder of this article is organized as follows: In section "Network model," we elaborate the localization model of HWSN in the two-dimensional (2D) space and introduce hop progress model for different communication ranges and node densities. Section "The proposed algorithm" describes the proposed localization algorithm and distance correction method in HWSN. Experimental results analysis is given in section "Experimental results and analysis." We finally conclude this article in section "Conclusion."

Network model

Figure 1 illustrates the HWSN scenario of N sensor nodes randomly and uniformly deployed in a 2D square area $S = L \times L$, where all sensors are of the different communication radius. It is also assumed that a few special sensors commonly known as ANs, which are marked with red asterisks, are aware of their positions. UNs marked with red circles are oblivious to this information and need to obtain their positions by ANs. The link between any two nodes represents one hop, which is able to reach or communicate with other sensors. Clearly, partial connectivity among sensors is present, and distance estimates between unknown sensors and anchors might be multiple hops.

In a multihop range-based HWSN localization, the goal is to estimate the location of all unknown sensors using sensors with known locations and partial

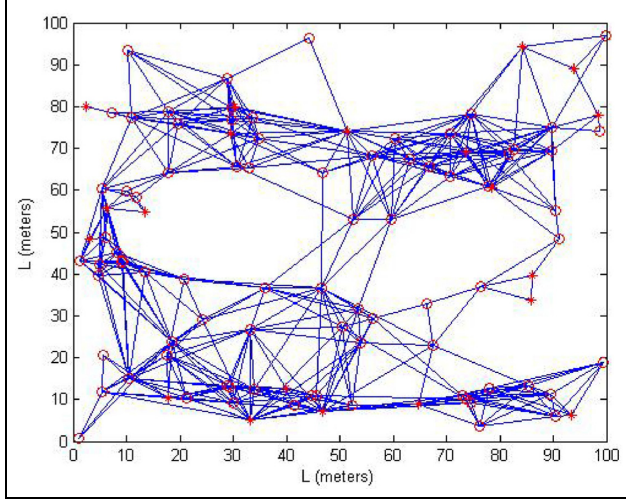


Figure 1. Heterogeneous wireless sensor network model.

information of the distances between some pair of sensors.²⁰ To keep the generality, we suppose that the i -th AN broadcasts data packet containing its position and communication radius in a densely distributed HWSN. Then, the j -th UN receives the data packet through multihop communication. For the sake of simplicity, we exploit the shortest path method to obtain the distance between sensor nodes as $d_{i-j} \approx d_{i-k_1} + d_{k_1-k_2} + \dots + d_{k_{m-1}-k_m} + d_{k_m-j}$, where d_{i-k} (d_{k-j}) denotes the distance between ANs (INs) and INs (UNs). Unlike previous homogeneous algorithms, the hop distance of IN possesses huge discrepancy due to the different sensor communication radii, as shown in Figure 2. It makes the distance estimation far from actual distance. To solve this problem, the minimum mean square error (MMSE) distance estimation method is given as follows

$$\hat{d}_{i-j} = E(d_{i-j}) \approx E(d_{i-k_1}) + E(d_{k_1-k_2}) + \dots + E(d_{k_{m-1}-k_m}) + E(d_{k_m-j}) \quad (1)$$

where k_m denotes m -th IN. $E(d_{i-k_1})$, $E(d_{k_{m-1}-k_m})$, and $E(d_{k_m-j})$ represent first-hop distance $d_{i-k_1}^E$, IN-hop distance $d_{k_{m-1}-k_m}^E$, and the last-hop distance $d_{k_m-j}^E$, respectively. Therefore, the formula for calculating the measure distances \hat{d}_{i-j} between ANs and UNs is

$$\begin{cases} \hat{d}_{i-j} = d_{i-k_1}^E + \sum_{m=1}^{H-2} d_{k_m-k_{m+1}}^E + d_{k_H-j}^E, & H > 1 \\ \hat{d}_{i-j} = d_{i-j}^E, & H = 1 \end{cases} \quad (2)$$

where H is the minimum number of hops between ANs and UNs. d_{i-j}^E denotes EHP with only one hop. In the next section, the novel method is developed to accurately derive the expressions of both distance estimation and distance correction.

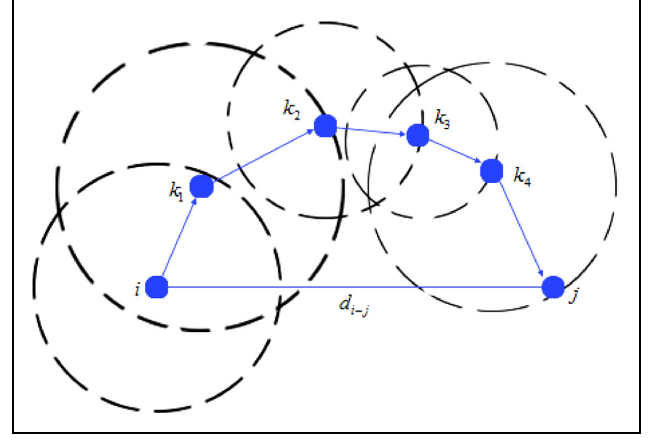


Figure 2. Multihop communication.

If we consider that UNs have the capacity to receive the information from ANs for distance estimation, then each unknown sensor would estimate its own location using distance estimates and absolute positions to at least three anchors by MLE method.

Notations are as follows: i , k , and j indicate ANs, INs, and UNs, respectively. Similarly, r_i , r_k , and r_j correspond to their communication radius, respectively. N , N_a , and N_u represent the number of all nodes, ANs, and UNs, respectively. λ is node density. $area(i, r_i)$ is the i -th node's coverage area having the i -th sensor as a center and r_i as a circle of radius.

The proposed algorithm

Equation (2) shows that in order to obtain the estimate distance, four distance formulae need to be derived, that is, first-hop distance $d_{i-k_1}^E$, intermediate-hop distance $d_{k_{m-1}-k_m}^E$, the last-hop distance $d_{k_m-j}^E$ (especially only one-hop distance d_{i-j}^E). First-hop distance $d_{i-k_1}^E$, intermediate-hop distance $d_{k_{m-1}-k_m}^E$, and the last-hop distance $d_{k_m-j}^E$ (or only one-hop distance d_{i-j}^E) are given in the next section, respectively.

For the sake of clarity, this article is only to discuss the two-hop distance, in what follows, we denote by X , Y , and Z the random variables that represent d_{i-j} , d_{k-j} , and d_{i-k} , respectively. Then, the probabilistic distribution mode for HWSN is developed using the conditional CDF $F_{Z|X}(z) = P(Z \leq z|x)$ of Z with respect to the random variable X .

First-hop and intermediate-hop distance derivation

As can be seen from Figure 3, $Z \leq z$ is guaranteed only if there are no nodes in the dashed area A . Therefore, the conditional CDF can be defined as follows²¹

$$F_{Z|X}(z) = P(Z \leq z|x) = P(A_0) \quad (3)$$

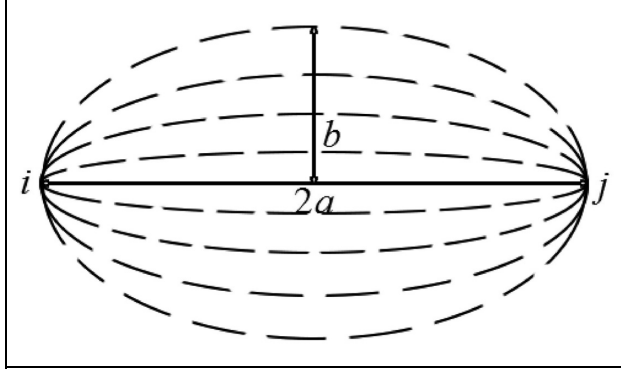


Figure 4. Modeling the multihop path with a set of ellipses.

location estimation method, such as trilateration, triangulation, and MLE. Position correction method has been proposed via exploiting residual correction method in Assaf and colleagues,^{17,18,21} which greatly increases the computational complexity because of iteration computations. To avoid the excessive iterations, enlightened by Sivanageswararao et al.²² and Golestanian and Poellabauer,²³ we propose a novel distance correction method, which can adapt to the heterogeneous nature of WSNs without adding any overhead, based on elliptic properties.

As shown in Figure 2, multihop paths between i -th AN and j -th UN have links with different lengths. The shape of multihop paths is similar to ellipse. Hence, we present the possible multihop paths between the nodes in the shape of ellipses with a large diameter of $2a$ and a small diameter of $2b$, as illustrated in Figure 4.²³ Ellipse perimeter formula, by conventional approximation, can be written as follows

$$L = 2\pi b + 4(a - b) \quad (10)$$

The purpose of distance correction is to obtain the straight-line distance from node i to node j , that is, elliptical major axis. Thus, equation (10) can be seen as follows

$$2a = \frac{L}{2} + (2 - \pi)b \quad (11)$$

where $2a$ is elliptical major axis, $L/2$ is half of the ellipse circumference, and b is the semi-short axis of ellipse. As seen from Figures 2 and 4, the semi-short axis in the multihop HWSN gradually decreases with the decrease in hop counts n_{hop} . To determine the value of b , we utilize $n_{hop,i,j}$ instead of b . On the other side, compared with the traditional multihop distance correction expressions in homogeneous WSN, elliptical major axis $2a$ and semi-perimeter $L/2$ indicate the real distance d_{i-j} and the estimation distance \hat{d}_{i-j} in equation (11), respectively. Meanwhile, the second part of the equation (i.e.

Algorithm 1. Proposed localization algorithm for HWSN.

Input: initialization N, N_a , anchor node coordinates (x_i, y_i) , and r_i , where $i = 1, 2, \dots, N$, parameters λ ;
Output: unknown node coordinates (x_j, y_j)
1: Compute \hat{d}_{i-j} by equations (1) to (9)
2: **for** $i = 1, 2, \dots, N_a$
3: Compute $\delta_{i,j}$ by equation (14);
4: **end for**
5: **for** $j = N - N_a, \dots, N$ **do**
6: Compute d_{i-j} according to equation (12);
7: Updating \hat{d}_{i-j} with d_{i-j} ;
8: **end for**
9: Compute (x_j, y_j) with MLE;

$(2 - \pi)b$) is the correction distance, in which the coefficient of b is regarded as a modifying factor. Since the modifying factor is varied in HWSN, the coefficient of b is represented by symbol η . Therefore, equation (11) can be written as follows

$$d_{i-j} = \hat{d}_{i-j} + \eta \cdot n_{hop,i,j} \quad (12)$$

Unfortunately, we have only knowledge of the path length \hat{d}_{i-j} . The key to deriving the correction distance by equation (12) is to select an appropriate parameter η . According to the analysis of traditional distance correction process, a new calculation method of correction factor is developed for multihop HWSN. Exploiting the average hop distance of ANs, the expression of parameter η is as

$$\eta = \frac{1}{N_a^2} \cdot \sum_{i=1}^{N_a} \sum_{j=1}^{N_a} \frac{\delta_{i,j}}{n_{hop,i,j}} \quad (13)$$

in which

$$\delta_{i,j} = d_{i-j} - \hat{d}_{i-j} \quad (14)$$

In equation (14), $\delta_{i,j}$ is the estimation error for any two ANs, namely, the difference between the real distance and the estimation distance. The main steps of our proposed algorithm are given in Algorithm 1.

Experimental results and analysis

In this section, to validate the performance of the proposed method, we will mainly compare our results with the results of Amorphous,¹⁵ DV-Hop,¹⁴ and LAEP,¹⁶ in which their parameters are tuned for the sake of contrast under the same condition, as shown in Table 1. In the simulation, sensor nodes randomly and uniformly deployed in a 2D square area $S = 100 \times 100 \text{ m}^2$. We assume that the number of anchors N_a is set to 20, and that the number of all nodes N is set to 100, 200, ..., 500, 600. We always assume that the transmission

Table 1. Simulation parameters.

Parameter	Value
S	$100 \times 100 \text{ m}^2$
N_a	20
λ	0.01:0.01:0.06
N	100:100:600
r_i and r_j	6–30 m

capabilities of all nodes (i.e. the communication radius of the ANs r_i and the communication radius of the UNs r_j) are set between 6 and 30 m, except in Figure 8. And r_i and r_j are uniformly distributed random numbers in the range of [6, 30]. In order to make the comparison more convincing, all experimental results are obtained by averaging over 200 trials. Note that the central processing unit (CPU) times were obtained by running the MATLAB code on a DELL computer with Intel(R) Core (TM) i7-6700 CPU, 3.40 GHz, 16.0 GB RAM, with MATLAB 2014(a) on Windows 10 (64-bit operating system) in all our experiments.

Evaluation metrics definitions

In this article, we propose the evaluating indicator of localization algorithms, that is, mean distance error (MDE), distance estimation error (DER), and normalized root mean square error (NRMSE) defined by

$$\text{MDE} = \frac{\sum_{i=1}^{N_a} \sum_{j=1}^{N_u} |d_{i-j} - \hat{d}_{i-j}|}{N_a \cdot N_u} \quad (15)$$

$$K_i = \begin{cases} 1, & \text{if } i = 0 \\ K_{i-1} \pm \text{Rand} \times \text{DOI}, & \text{if } 0 < i < 360 \cap i \in N, \text{ where } |K_0 - K_{359}| \leq \text{DOI} \end{cases} \quad (19)$$

$$\text{DER} = \frac{|d_{i-j} - \hat{d}_{i-j}|}{d_{i-j}} \quad (16)$$

and

$$\text{NRMSE} = \frac{\sum_{j=1}^{N_u} \sqrt{\frac{(x_j - \hat{x}_j)^2 + (y_j - \hat{y}_j)^2}{r_j^2}}}{N_u} \quad (17)$$

where (\hat{x}_j, \hat{y}_j) is the estimation value of coordinates (x_j, y_j) of the UN, r_j is the communication radius of the j -th UN, and N_u is the number of the UNs.

Transmission model

In this article, the DOI transmission model is used, and the specific formula is as follows

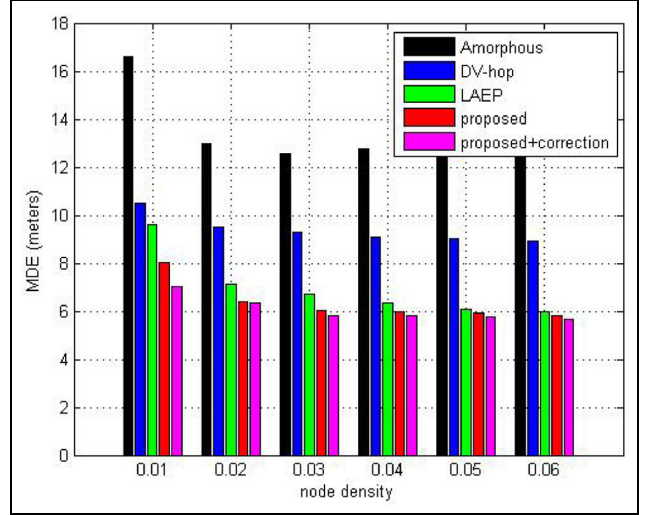


Figure 5. Relationship between MDE and node density for different localization methods in HWSN.

$$P_R(d) = P_T - PL(d_0) - 10\sigma \log_{10} \left(\frac{d}{d_0} \right) \times K_i \quad (18)$$

where $P_R(d)$ is the received signal power, $P_T = 0 - 4 \text{ dBm}$ is the transmission signal power, $\sigma(2 \sim 4) = 4(\text{indoor}, \text{outdoor})$ is the path loss exponent, $PL(d_0) = 55 \text{ dB}$ ($d_0 = 1 \text{ m}$) is the path loss for a reference distance of d_0 , d is the distance between the transmitter and the receiver, and K_i is a coefficient to represent the difference in path loss in different directions and is calculated as follows

where DOI is the maximum path loss percentage variation per unit degree change in the direction of radio propagation. DOI model becomes the regular model (RM) when DOI equals zero. And the specific formula is as follows

$$P_R(d) = P_T - PL(d_0) - 10\sigma \log_{10} \left(\frac{d}{d_0} \right) \quad (20)$$

All the experimental results are obtained on the premise that DOI equals to zero, that is, transmission coverage of each node is circular.

Comparing with classical methods

Figure 5 depicts the localization MDE achieved by Amorphous, DV-Hop, LAEP, and the proposed algorithm for different node densities. As seen from Figure 5, the proposed algorithm can adapt to low or high node density due to complying with the

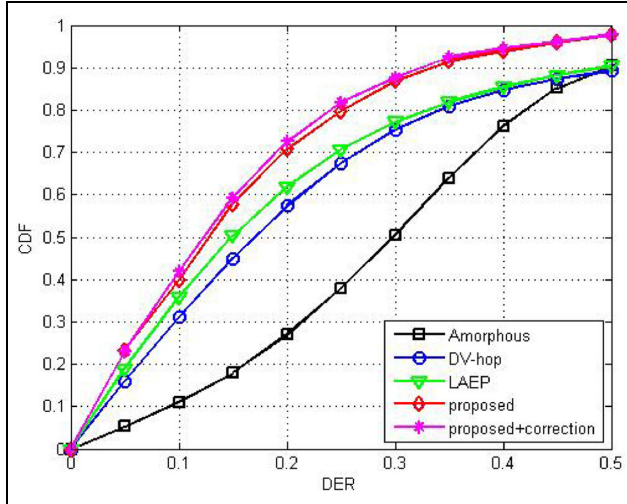


Figure 6. Distribution of DER for different localization methods in HWSN.

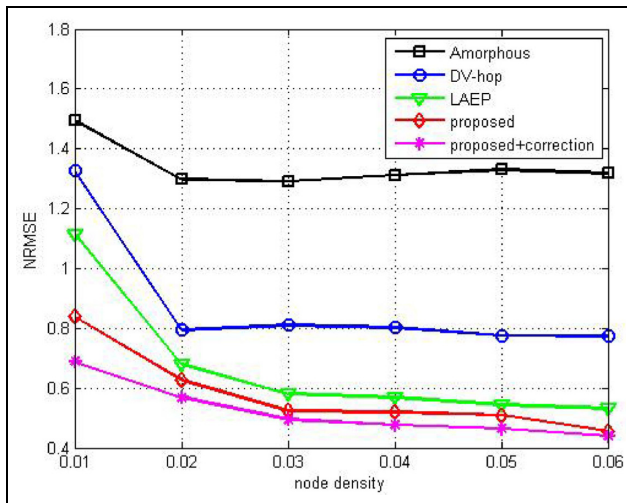


Figure 7. NRMSE versus node density in HWSN.

heterogeneous nature of the WSN. The performance of the proposed correction algorithm is optimal compared to others, especially for low node density.

The comparison of DER distribution for different localization methods in HWSN is shown in Figure 6. Figure 6 shows that the distribution of the proposed-correction algorithm's DER is 0.73 in the interval of 0.2. And the other algorithms (including the proposed algorithm, LAEP, DV-Hop and Amorphous) are 0.71, 0.62, 0.59 and 0.28 respectively. Meanwhile, the proposed algorithm improves slightly by means of its modified algorithm. This further demonstrates the superiority of the proposed algorithm. A comparison between the results from Figures 5 and 6 indicates that the proposed algorithm is more suitable for HWSN.

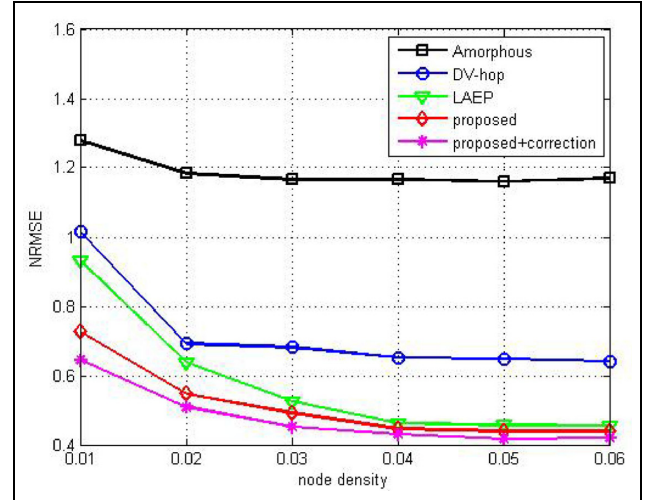


Figure 8. NRMSE versus node density in semi-heterogeneous WSN.

The simulation results shown in Figure 7 illustrate that the positioning error decreases as node density increases. This is mainly because the accuracy of the hop distance estimation is improved with the increase in node density. Specifically, the line chart of the proposed algorithm shows a major decline, and then a slow downtrend when node density is greater than 0.03. And the positioning error of the proposed algorithm is less than LAEP under various node densities. Furthermore, the proposed modified algorithm obviously improves the performance of the proposed algorithm and its accuracy and, therefore, provides a better adaptability of nodes-density-variety, especially for sparse networks.

To confirm the feasibility of the proposed algorithm, we also made some experiments, mainly on the variety in the communication radius and the number of ANs. Figure 8 shows the NRMSE of five algorithms under the condition that the communication radius of UNs ranges from 6 to 30 m, the transmission capability of ANs is set to 22 m, and the remaining initial conditions remain unchanged. Figure 9 plots the localization NRMSE versus number of ANs in HWSN. As seen from Figure 9, the positioning error reduces as the number of ANs increases. A comparison of the results from Figures 7 to 9 indicates that it is important to reasonably determine the communication radius of ANs for improving the location accuracy.

Generally, the result of MATLAB simulation proves the feasibility of the proposed algorithm. The proposed algorithm not only has less error than other algorithms in dense HWSN but also can be utilized in a sparsely distributed HWSN. Moreover, its correction algorithm is proposed to solve the constraint of low node density.

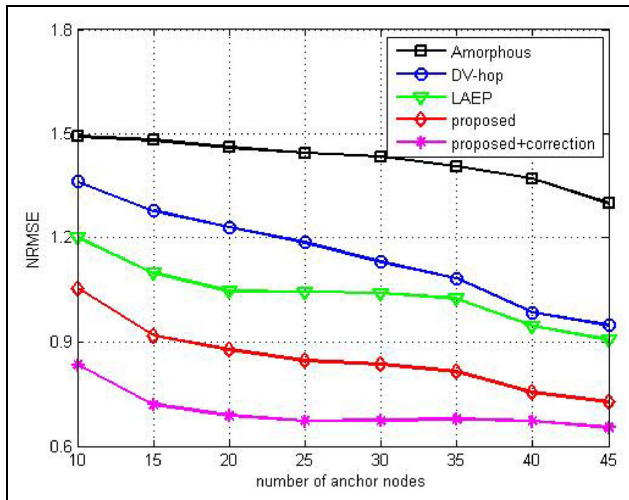


Figure 9. NRMSE versus number of ANs in HWSN.

Conclusion

In this article, a novel range-free localization algorithm is proposed for the HWSN. Furthermore, a distance correction mechanism which complies with the heterogeneous nature of WSNs is developed to improve localization accuracy without incurring additional costs. Our algorithm, whether applied with or without correction, outperforms the state-of-the-art range-free ones in terms of localization accuracy and adaptability to HWSN.

Declaration of conflicting interests

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References

1. Sarkar R and Gao J. Differential forms for target tracking and aggregate queries in distributed networks. *IEEE ACM T Network* 2013; 21(4): 377–388.
2. Du XD, Ji JT and Yan DP. Application research of wireless sensor network in intelligent transportation system. *Adv Mater Res* 2010; 108–111: 1170–1175.
3. Li S, Lou Y and Liu B. Bluetooth aided mobile phone localization: a nonlinear neural circuit approach. *ACM T Embed Comput S* 2014; 13(4): 78.
4. Honeine P, Mourad F, Kallas M, et al. Wireless sensor networks in biomedical: body area networks. In: *International workshop on systems, signal processing and their applications*, Tipaza, Algeria, 9–11 May 2011, pp.388–391. New York: IEEE.
5. Li S and Qin F. A dynamic neural network approach for solving nonlinear inequalities defined on a graph and its application to distributed, routing-free, range-free localization of WSNs. *Neurocomputing* 2013; 117: 72–80.
6. Li S, Wang Z and Li Y. Using Laplacian eigenmap as heuristic information to solve nonlinear constraints defined on a graph and its application in distributed range-free localization of wireless sensor networks. *Neural Process Lett* 2013; 37(3): 411–424.
7. Nguyen TLT, Septier F, Rajaona H, et al. A Bayesian perspective on multiple source localization in wireless sensor networks. *IEEE T Signal Proces* 2016; 64(7): 1684–1699.
8. Zhang Y and Li S. Distributed biased min-consensus with applications to shortest path planning. *IEEE T Automat Contr* 2017; 62: 5429–5436.
9. Li S, Zhou MC, Luo X, et al. Distributed winner-take-all in dynamic networks. *IEEE T Automat Contr* 2017; 62(2): 577–589.
10. Li S, He J, Li Y, et al. Distributed recurrent neural networks for cooperative control of manipulators: a game-theoretic perspective. *IEEE T Neur Net Lear* 2016; 28: 415–426.
11. Mahfouz S, Mourad-Chehade F, Honeine P, et al. Non-parametric and semi-parametric RSSI/distance modeling for target tracking in wireless sensor networks. *IEEE Sens J* 2016; 16(7): 2115–2126.
12. Chen H, Ping D, Xu Y, et al. A robust location algorithm with biased extended Kalman filtering of TDOA data for wireless sensor networks. In: *International conference on wireless communications, networking and mobile computing*, Wuhan, China, 26 September 2005, pp.883–886. New York: IEEE.
13. Stefano GD and Petricola A. A distributed AOA based localization algorithm for wireless sensor networks. *J Comput* 2008; 3(4): 1–8.
14. Yi X, Liu Y, Deng L, et al. An improved DV-Hop positioning algorithm with modified distance error for wireless sensor network. In: *Second international symposium on knowledge acquisition and modeling*, Wuhan, China, 30 November–1 December 2009, pp.216–218. New York: IEEE.
15. Shen S, Yang B, Qian K, et al. An improved amorphous localization algorithm for wireless sensor networks. In: *International conference on networking and network applications (NaNA)*, Hakodate, Japan, 23–25 July 2016, pp.69–72. New York: IEEE.
16. Wang Y, Wang X, Wang D, et al. Range-free localization using expected hop progress in wireless sensor networks. *IEEE T Parall Distr* 2009; 20(10): 1540–1552.
17. Assaf AE, Zaidi S, Affes S, et al. Cost-effective and accurate nodes localization in heterogeneous wireless sensor networks. In: *IEEE international conference on communications*, London, 8–12 June 2015, pp. 6601–6608. New York: IEEE.
18. Assaf AE, Zaidi S, Affes S, et al. Low-cost localization for multihop heterogeneous wireless sensor networks. *IEEE T Wirel Commun* 2015; 15(1): 472–484.

19. Assaf AE, Zaidi S, Affes S, et al. Efficient range-free localization algorithm for randomly distributed wireless sensor networks. In: *Global communications conference (GLOBECOM)*, Atlanta, GA, 9–13 December 2013. New York: IEEE.
20. Anderson BDO, Shames I, Mao G, et al. Formal theory of noisy sensor network localization. *SIAM J Discrete Math* 2010; 24(2): 684–698.
21. Zaidi S, Assaf AE, Affes S, et al. Range-free nodes localization in mobile wireless sensor networks. In: *IEEE international conference on ubiquitous wireless broadband, Montreal, QC, 4–7 October 2015*, pp.1–6. New York: IEEE.
22. Sivanageswararao S, Krishna YKS and Rao KN. An elliptical routing protocol for wireless mesh networks: performance analysis. *Int J Comput Appl* 2014; 102(8): 29–34.
23. Golestanian M and Poellabauer C. Localization in heterogeneous wireless sensor networks using elliptical range estimation. In: *International conference on computing, networking and communications*, Kauai, HI, 15–18 February 2016, pp.1–7. New York: IEEE.