Target Localization from Bistatic Range Measurements in Multi-Transmitter Multi-Receiver Passive Radar

Article *in* IEEE Signal Processing Letters · December 2015

DOI: 10.1109/LSP.2015.2491961

2 authors:

Ali Noroozi
Khaje Nasir Toosi University of Technology

Mohammad Ali Sebt
Khaje Nasir Toosi University of Technology

Target Localization from Bistatic Range Measurements in Multi-Transmitter Multi-Receiver Passive Radar

Ali Noroozi and Mohammad Ali Sebt

***Abstract*—A closed-form weighted least squares method for finding a target position in the presence of multiple transmitters and multiple receivers from time difference of arrival (TDOA) measurements is proposed. The method is based on the intersection of the ellipsoids defined by bistatic range (BR) measurements from a number of transmitters and receivers. The localization formula is derived from the minimization of the weighted equation error energy. To produce a substantial improvement in the performance of the method, a weighting matrix is derived in two different conditions, which one of them leads to an approximate maximum likelihood (ML) and the other one results in a best linear unbiased estimator (BLUE). Numerical simulations are included to support and corroborate the theoretical developments.**

***Index Terms*—Bistatic range (BR), target position, TDOA, weighted least squares (WLS), weighting matrix.**

I. INTRODUCTION

THE problem of determining the location of a radiating or reflecting object by measuring time difference of arrival (TDOA) of received signals has drawn considerable attention in the last few decades especially in radar and sonar. In these systems, generally there are multiple receivers located at different spatial coordinates to collect the signals emitted or reflected by a target. According to the type of collecting information, there exists a variety of localization techniques for passive radar systems, including received signal strength (RSS) [1], angle of arrival (AOA) [2]–[4], and time difference of arrival (TDOA). The performance of TDOA technique in terms of accuracy and simplicity can be, for the most part, better than that of the others.

In passive radar systems, the relative delay can be determined by calculating the cross-ambiguity function (CAF) between the reference signal and the reflected target echo. This delay is related to the bistatic range (BR) which is the sum of transmitter-target and target-receiver ranges [5]. The locus of points of the constant BR is an ellipsoid with a known transmitter and a known receiver as its foci. When the BRs corresponding to multiple transmitter-receiver pairs are calculated, the ellipsoids result. The intersection of these curves is taken as the estimated target position.

There is a rapidly growing literature on the BR-based method [5]–[9], which indicates the importance of this issue. Nevertheless, most studies in target localization have been conducted with the RD-based methods. The range difference (RD) between the target and two receivers (or transmitters) is obtained by subtracting the measured BR of one of the receivers (or transmitters) from the other. The conventional RD-based method, called hyperbolic line of position (LOP) method [10]–[13], was firstly proposed to obtain the target position. Moreover, there are several solutions using simple closed-form, which tend to use least squares (LS) solution, namely spherical-interpolation (SI) [14]–[16], spherical-intersection (SX) [17], [18], Plane Intersection (PX) [16], [19], and divide and conquer (DAC) methods [20]. In [21], a novel approach to geolocation with RDs using feasible bivectors was developed. Furthermore, in [22][23] a two-step LS estimator was proposed, which approximates the maximum likelihood (ML) estimator when the noise is small.

So far, however, little attention has been paid to determine the target position in the literature when there exist multiple transmitters and multiple receivers. In [7], a LS solution was presented in which only a few number of the BR equations were exploited. Next, these BR equations were converted to the RD ones and the target location was obtained from these RD equations. But, the accuracy of the BR-based method is better than that of RD-based one [9]. Recently, a two-step weighted least squares (WLS) algorithm was proposed, which approximates the ML estimator under the conditions that the noise is small and the same in all the measurements. Unfortunately, in practical applications the noise values in the measured BRs are not equal because the noise in the measurements is dependent on the signal to noise ratio (SNR) [15] and the SNR is also proportional to the product of the square of the distances from the target to the transmitter and receiver [24]. Thus, the presented weighted matrix in [8] is not optimal.

In this study, the issue of target location estimation in the presence of multiple receivers and multiple transmitters is dealt with and a weighted least squares solution for this problem in the general case of noise is presented in two different conditions—when the measurement noise is small, which leads to the ML estimator, and when it is not, which results in the best linear unbiased estimator (BLUE) [25]. It should be noted that the BLUE is frequently more suitable for practical implementation because complete knowledge of the probability density function (PDF) is not necessary.

The rest of the study is organized as follows. Section II formulates the problem based on the BR values. In order to further improve the performance of the estimator, Section III introduces

Manuscript received June 19, 2015; revised September 06, 2015; accepted October 12, 2015. Date of publication October 16, 2015; date of current version October 26, 2015. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Parv Venkitasubramaniam.

The authors are with the Department of Electrical Engineering, K. N. Toosi University of Technology, Tehran, Iran (e-mail: ali_noroozi@ee.kntu.ac.ir; sebt@kntu.ac.ir).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LSP.2015.2491961

a weighting matrix in two different cases. Section IV gives a performance study by simulations and conclusions are drawn in Section V.

II. PROBLEM FORMULATION

This paper will focus on finding an individual target position in the general case in which there exist multiple transmitters and receivers in three-dimensional space. It is assumed that M transmitters are present, sending out orthogonal waveforms and these signals are reflected by a target at an unknown position $\mathbf{x}_t = [x_t \quad y_t \quad z_t]^T$. The ith transmitter is placed at known position $\mathbf{x}_{Tx_i} = [x_{Tx_i} \quad y_{Tx_i} \quad z_{Tx_i}]^T$ for $i = 1, \ldots, M$. In addition, there are N receivers at known positions $\mathbf{x}_{Rx_j} = [x_{Rx_j} \quad y_{Rx_j} \quad z_{Rx_j}]^T$, $j = 1, \ldots, N$, collecting the direct signals from a specified transmitter and the reflected signals from the target.

We first formulate the problem for ith transmitter and then extend it to multiple transmitters. It should also be noted that the superscript $\hat{}$ is used to denote the noisy or estimated value of a variable.

The Euclidean distance between the target and Transmitter i is

$$\begin{aligned} R_{tTx_i} &= \|\mathbf{x}_{Tx_i} - \mathbf{x}_t\| \\ &= \sqrt{(x_{Tx_i} - x_t)^2 + (y_{Tx_i} - y_t)^2 + (z_{Tx_i} - z_t)^2} \end{aligned} \tag{1}$$

The Euclidean distance between the target and the jth receiver is

$$\begin{aligned} R_{tRx_j} &= \left\|\mathbf{x}_{Rx_j} - \mathbf{x}_t\right\| \\ &= \sqrt{\left(x_{Rx_j} - x_t\right)^2 + \left(y_{Rx_j} - y_t\right)^2 + \left(z_{Rx_j} - z_t\right)^2} \end{aligned} \tag{2}$$

The Euclidean distance from Transmitter i and the jth receiver to the origin can be calculated, respectively, as

$$R_{Tx_i} = \|\mathbf{x}_{Tx_i}\| = \sqrt{x_{Tx_i}^2 + y_{Tx_i}^2 + z_{Tx_i}^2} \tag{3}$$

$$R_{Rx_j} = \left\|\mathbf{x}_{Rx_j}\right\| = \sqrt{x_{Rx_j}^2 + y_{Rx_j}^2 + z_{Rx_j}^2} \tag{4}$$

The TDOA measurement multiplied by the speed of light is the range difference between the direct and indirect paths. The BR is defined as the sum of receiver-target range R_{tRx_j} and target-transmitter range R_{tTx_i} as well [5]. According to this definition, the BR is

$$\begin{aligned} R_{bTx_iRx_j} &= R_{tTx_i} + R_{tRx_j} \\ &= \sqrt{(x_{Tx_i} - x_t)^2 + (y_{Tx_i} - y_t)^2 + (z_{Tx_i} - z_t)^2} \\ &\quad + \sqrt{\left(x_{Rx_j} - x_t\right)^2 + \left(y_{Rx_j} - y_t\right)^2 + \left(z_{Rx_j} - z_t\right)^2} \end{aligned} \tag{5}$$

By rearranging the above equation, we have

$$\begin{aligned} &R_{bTx_iRx_j} - \sqrt{(x_{Tx_i} - x_t)^2 + (y_{Tx_i} - y_t)^2 + (z_{Tx_i} - z_t)^2} \\ &= \sqrt{\left(x_{Rx_j} - x_t\right)^2 + \left(y_{Rx_j} - y_t\right)^2 + \left(z_{Rx_j} - z_t\right)^2} \end{aligned} \tag{6}$$

By squaring both sides of the above equation and using (1), (3) and (4), it can be obtained

$$\begin{aligned} &\left(x_{Rx_j} - x_{Tx_i}\right) x_t + \left(y_{Rx_j} - y_{Tx_i}\right) y_t \\ &\quad + \left(z_{Rx_j} - z_{Tx_i}\right) z_t - R_{bTx_iRx_j} R_{tTx_i} \\ &= \frac{1}{2}\left(R_{Rx_j}^2 - R_{bTx_iRx_j}^2 - R_{Tx_i}^2\right) \end{aligned} \tag{7}$$

Doing this for the set of N receivers yields, in matrix form,

$$\mathbf{A}_i \mathbf{x}_i = \mathbf{b}_i \tag{8}$$

where (see (9)–(11), shown at the bottom of the page). Now we generalize the equations to M transmitters. After accomplishing the above procedure for each of the transmitter and then combining all the resulting equations into matrix form, the problem can be represented similar to (8) as

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{12}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{S}_1 & -\mathbf{r}_{b1} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{S}_2 & \mathbf{0} & -\mathbf{r}_{b2} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_M & \mathbf{0} & \mathbf{0} & \ldots & -\mathbf{r}_{bM} \end{bmatrix}_{(M \times N) \times (M+3)} \tag{13}$$

$$\mathbf{x} = [x_t \quad y_t \quad z_t \quad R_{tTx_1} \quad R_{tTx_2} \quad \cdots \quad R_{tTx_M}]^T \tag{14}$$

$$\mathbf{b} = [\mathbf{b}_1^T \quad \mathbf{b}_2^T \quad \ldots \quad \mathbf{b}_M^T]^T \tag{15}$$

$$\mathbf{S}_i = \begin{bmatrix} x_{Rx_1} - x_{Tx_i} & y_{Rx_1} - y_{Tx_i} & z_{Rx_1} - z_{Tx_i} \\ \vdots & \vdots & \vdots \\ x_{Rx_N} - x_{Tx_i} & y_{Rx_N} - y_{Tx_i} & z_{Rx_N} - z_{Tx_i} \end{bmatrix} \tag{16}$$

and $\mathbf{r}_{bi} = [R_{bTx_iRx_1} \quad \ldots \quad R_{bTx_iRx_N}]^T$ is the noise-free BRs corresponding to the ith transmitter.

Owing to measurement errors and noise in the BRs, the noise-free measurements in $\mathbf{A}$ and $\mathbf{b}$ are not available, but we have the noisy values, $\hat{\mathbf{A}}$ and $\hat{\mathbf{b}}$. By replacing these two values into (12),

$$\mathbf{A}_i = \begin{bmatrix} x_{Rx_1} - x_{Tx_i} & y_{Rx_1} - y_{Tx_i} & z_{Rx_1} - z_{Tx_i} & -R_{bTx_iRx_1} \\ \vdots & \vdots & \vdots & \vdots \\ x_{Rx_N} - x_{Tx_i} & y_{Rx_N} - y_{Tx_i} & z_{Rx_N} - z_{Tx_i} & -R_{bTx_iRx_N} \end{bmatrix}_{N \times 4} \tag{9}$$

$$\mathbf{x}_i = [x_t \quad y_t \quad z_t \quad R_{tTx_i}]^T \tag{10}$$

$$\mathbf{b}_i = \frac{1}{2}\begin{bmatrix} R_{Rx_1}^2 - R_{bTx_iRx_1}^2 - R_{Tx_i}^2 \\ \vdots \\ R_{Rx_N}^2 - R_{bTx_iRx_N}^2 - R_{Tx_i}^2 \end{bmatrix}_{N \times 1} \tag{11}$$

the error vector, which is the difference between the right-hand and left-hand sides of the equation, is expressed as

$$\boldsymbol{\varepsilon} = \hat{\mathbf{A}}\mathbf{x} - \hat{\mathbf{b}} \tag{17}$$

The solution of (17) in the WLS sense, which minimizes the weighted equation error energy $\boldsymbol{\varepsilon}^T\mathbf{W}\boldsymbol{\varepsilon}$ with respect to $\mathbf{x}$, can be found as

$$\hat{\mathbf{x}} = \left(\hat{\mathbf{A}}^T\mathbf{W}\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{A}}^T\mathbf{W}\hat{\mathbf{b}} \tag{18}$$

where $\mathbf{W}$ is a symmetric positive definite matrix of dimension $(MN) \times (MN)$, which is used to improve the performance of the estimation. Finally, the target position can be found using

$$\hat{\mathbf{x}}_t = \begin{bmatrix} \hat{x}_t \\ \hat{y}_t \\ \hat{z}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \hat{\mathbf{x}} \tag{19}$$

III. Weighting Matrix

Generally speaking, it is far from easy to determine the weighting matrix required in the LS method. So far, there has been little discussion about an appropriate weighting matrix used in the LS-based method to find the target position.

Let $\hat{\mathbf{r}}_b$ be all the BR measurements. These measurements can be represented as

$$\hat{\mathbf{r}}_b = \mathbf{r}_b + \mathbf{n} \tag{20}$$

where $\mathbf{r}_b = [\,\mathbf{r}_{b1}^T \quad \mathbf{r}_{b2}^T \quad \ldots \quad \mathbf{r}_{bM}^T\,]^T$ denotes all the noise-free BR measurements, $\mathbf{n} = [\,\mathbf{n}_1^T \quad \mathbf{n}_2^T \quad \ldots \quad \mathbf{n}_M^T\,]^T$ is the noise vector and $\mathbf{n}_i = [\,n_{Tx_iRx_1} \quad \ldots \quad n_{Tx_iRx_N}\,]^T$ is the noise present in the measurements of ith transmitter. It is assumed that the BRs are corrupted by additive mutually uncorrelated zero mean white Gaussian noise with covariance matrix

$$\mathbf{C}_\mathbf{n} = \mathrm{E}\left\{\mathbf{n}\mathbf{n}^T\right\} = \mathrm{diag}\left(\left[\,\boldsymbol{\sigma}_1^{2T} \quad \boldsymbol{\sigma}_2^{2T} \quad \ldots \quad \boldsymbol{\sigma}_M^{2T}\,\right]^T\right) \tag{21}$$

where $\boldsymbol{\sigma}_i^2 = [\,\sigma_{i1}^2 \quad \sigma_{i2}^2 \quad \ldots \quad \sigma_{iN}^2\,]^T$ for $i = 1, \ldots, M$. Because of the noise in the BR measurements given by (20) we have $\hat{\mathbf{b}} = \mathbf{b} + \tilde{\mathbf{b}}$ and $\hat{\mathbf{A}} = \mathbf{A} + \tilde{\mathbf{A}}$ where the error term is denoted by $\sim$ and we obtain

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{O} & -\mathbf{n}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{O} & \mathbf{0} & -\mathbf{n}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{0} & \mathbf{0} & \ldots & -\mathbf{n}_M \end{bmatrix}_{(M\times N)\times(M+3)} \tag{22}$$

$$\tilde{\mathbf{b}} = -\mathbf{r}_b \odot \mathbf{n} - \frac{1}{2}\mathbf{n} \odot \mathbf{n} \tag{23}$$

where $\mathbf{O}$ represents a zero matrix of size $N \times 3$ and the symbol $\odot$ represents the Schur product (element-by-element product). Substituting (22) and (23) into the error vector $\boldsymbol{\varepsilon}$ (17), yields

$$\boldsymbol{\varepsilon} = \hat{\mathbf{A}}x - \hat{\mathbf{b}} = \tilde{\mathbf{A}}x - \tilde{\mathbf{b}} = (\boldsymbol{\Lambda} - \boldsymbol{\Gamma})\,\mathbf{n} + \frac{1}{2}\mathbf{n} \odot \mathbf{n} \tag{24}$$

where

$$\boldsymbol{\Lambda} = \mathrm{diag}\,(\mathbf{r}_b) \tag{25}$$

$$\boldsymbol{\Gamma} = \mathrm{diag}\left(\left[\,R_{tTx_1}\mathbf{1}^T \quad R_{tTx_2}\mathbf{1}^T \quad \ldots \quad R_{tTx_M}\mathbf{1}^T\,\right]^T\right) \tag{26}$$

and $\mathbf{1}$ is a vector of all ones of length N. When the measurement noise is small, the second term on the right in (24) can be ignored. Therefore, $\boldsymbol{\varepsilon}$ becomes a Gaussian random vector with covariance matrix given by

$$\mathbf{C}_{\boldsymbol{\varepsilon}} = \mathrm{E}\left\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\right\} \approx (\boldsymbol{\Lambda} - \boldsymbol{\Gamma})\,\mathbf{C}_\mathbf{n}\,(\boldsymbol{\Lambda} - \boldsymbol{\Gamma}) \tag{27}$$

Under this circumstance, the solution of weighted LS estimation is the same as the ML estimation, given by

$$\hat{\mathbf{x}} = \left(\hat{\mathbf{A}}^T\mathbf{C}_{\boldsymbol{\varepsilon}}^{-1}\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{A}}^T\mathbf{C}_{\boldsymbol{\varepsilon}}^{-1}\hat{\mathbf{b}} \tag{28}$$

According to (18) and (28), $\mathbf{W}$ is obtained as

$$\mathbf{W} = \mathbf{C}_{\boldsymbol{\varepsilon}}^{-1} \tag{29}$$

On the other hand, when the noise term in range measurements is not small, the second term in (24) is considerable and cannot be neglected. To minimize the weighted equation error energy under this condition, the inverse of the error covariance matrix can be a good candidate for selecting the weighting matrix (that is, $\mathbf{W} = \mathbf{C}_{\boldsymbol{\varepsilon}}^{-1}$) the same as the previous case. By defining $\boldsymbol{\eta} = \frac{1}{2}\mathbf{n} \odot \mathbf{n}$, the exact value of $\mathbf{C}_{\boldsymbol{\varepsilon}}$ is given by

$$\begin{aligned} \mathbf{C}_{\boldsymbol{\varepsilon}} &= (\boldsymbol{\Lambda} - \boldsymbol{\Gamma})\,\mathbf{C}_\mathbf{n}\,(\boldsymbol{\Lambda} - \boldsymbol{\Gamma}) + (\boldsymbol{\Lambda} - \boldsymbol{\Gamma})\,\mathbf{C}_{\mathbf{n}\boldsymbol{\eta}} \\ &\quad + \mathbf{C}_{\mathbf{n}\boldsymbol{\eta}}^T\,(\boldsymbol{\Lambda} - \boldsymbol{\Gamma}) + \mathbf{C}_{\boldsymbol{\eta}} \\ &= (\boldsymbol{\Lambda} - \boldsymbol{\Gamma})\,\mathbf{C}_\mathbf{n}\,(\boldsymbol{\Lambda} - \boldsymbol{\Gamma}) + \mathbf{C}_{\boldsymbol{\eta}} \end{aligned} \tag{30}$$

where $\mathbf{C}_{\mathbf{n}\boldsymbol{\eta}} = \mathrm{E}\{\mathbf{n}\boldsymbol{\eta}^T\}$ and $\mathbf{C}_{\boldsymbol{\eta}} = \mathrm{E}\{\boldsymbol{\eta}\boldsymbol{\eta}^T\}$. Note that the odd moments of $\mathbf{n}$ are zero because its distribution function is even. Therefore, $\mathbf{C}_{\mathbf{n}\boldsymbol{\eta}}$ and its transpose contained the sum of odd moments are zero. By considering $\boldsymbol{\eta}$ as $[\,\boldsymbol{\eta}_1^T \quad \boldsymbol{\eta}_2^T \quad \ldots \quad \boldsymbol{\eta}_M^T\,]^T$, where $\boldsymbol{\eta}_i$, $i = 1, \ldots, M$, has dimension $N \times 1$, we have

$$\left[\mathrm{E}\left\{\boldsymbol{\eta}_i\boldsymbol{\eta}_i^T\right\}\right]_{kl} = \begin{cases} \frac{3}{4}\sigma_{ik}^4 & k = l \\ \frac{1}{4}\sigma_{ik}^2\sigma_{il}^2 & k \neq l \end{cases}, \quad i = 1, 2, \ldots, M \tag{31}$$

$$\left[\mathrm{E}\left\{\boldsymbol{\eta}_i\boldsymbol{\eta}_j^T\right\}\right]_{kl} = \frac{1}{4}\sigma_{ik}^2\sigma_{jl}^2, \quad i, j = 1, 2, \ldots, M \,\&\, i \neq j \tag{32}$$

Finally, it is important to note that the weighting matrix requires the knowledge of the values of the range (i.e. R_{tTx_i}). In other words, the error covariance matrix $\mathbf{C}_{\boldsymbol{\varepsilon}}$ is not known since $\boldsymbol{\Gamma}$ in this matrix contains the unknown distance between the transmitters and the target R_{tTx_i}. Thus, we can first use (18) with $\mathbf{W} = \mathbf{I}$, which is the least squares solution of (17), to obtain an initial solution for estimating $\boldsymbol{\Gamma}$ and then derive the final answer from (18) with $\mathbf{W} = \mathbf{C}_{\boldsymbol{\varepsilon}}^{-1}$.

IV. Simulations

In this section, simulation results on the performance of the proposed method to the problem of localizing a single target in a two-dimensional space are presented. Here, we first examine the effect of increasing the number of transmitters on the performance of the proposed method. Then, we draw a comparison between the proposed method and others during the same experiment. The performance of the BR-based method is evaluated according to the root mean square (RMS) error of the target location estimation.

In the simulation studies, the noise in the measured BR is modeled as a zero-mean Gaussian random variable with a known variance which is dependent only on the signal to noise ratio (SNR) at each pair transmitter-receiver [15]. As a consequence, the measurements of the BR were corrupted by additive Gaussian noise with the standard deviation $\sigma_{ij} = kR_{tTx_i}R_{tRx_j}$ for $i = 1, \ldots, M \,\&\, j = 1, \ldots, N$, where k is a constant [24].

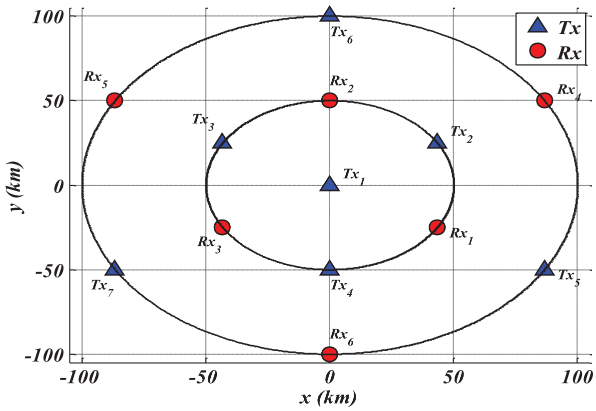

Fig. 1. Position of transmitters and receivers.

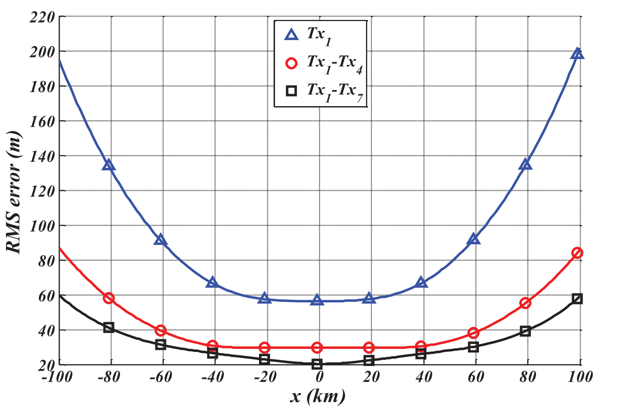

Fig. 2. Effect of increasing the number of transmitters on the performance of the method.

The sample variance and bias were generated using 1000-trial Monte-Carlo runs. The RMS error, defined by

$$\begin{aligned}\mathrm{RMSE}\left(\hat{\mathbf{p}}\right) &= \sqrt{\mathrm{E}\left\{\left\|\mathbf{p}-\hat{\mathbf{p}}\right\|^2\right\}} \\ &= \sqrt{\mathrm{Var}(x)+\mathrm{Bias}_x^2+\mathrm{Var}(y)+\mathrm{Bias}_y^2},\end{aligned}$$

is our criterion for assessing the performance of the algorithm.

To evaluate the effect of increasing the number of transmitters on the performance of the proposed method with a fixed number of receivers and to compare the performance of the proposed algorithm with others, the simulations have been carried out with six receivers, a number of transmitters and one target. The positions of the transmitters and receivers are shown in Fig. 1. Suppose that the target moves along the x-axis at $\mathrm{y}=70$ km and the distance between two adjacent points on the x-axis is 1 km. The standard deviation of noise σ_{ij} is $10^{-8}R_{tTx_i}R_{tRx_j}$. Fig. 2 gives the RMS error of the estimator in three various cases. In this figure, the inverse of (27) is applied as the weighting matrix since the considered standard deviation of noise σ_{ij} is small. The curve marked with Δ corresponds to the target localization only with one transmitter (that is, Tx_1). The RMS error of target position estimation using four transmitters (i.e. $\mathrm{Tx}_1 - \mathrm{Tx}_4$) and all the existing transmitters (i.e. $\mathrm{Tx}_1 - \mathrm{Tx}_7$) are marked with $\bigcirc$ and $\square$, respectively. The results show that with a fixed number of receivers the more the number of the transmitters is, the more accuracy will be obtained. Furthermore, Figs. 3 and 4 compare the performance of the proposed method to other ones in the presence of all the transmitters and receivers under the low and relatively high noise conditions, respectively. In these two figures, the inverse of (27) and (30) are used as the weighting matrix required in the proposed ML and BLUE estimators, respectively. Note that the standard deviation of noise in Fig. 3 is similar to the previous simulation, whereas this parameter in Fig. 4 is equal to $2\times10^{-6}R_{tTx_i}R_{tRx_j}$. Under the low noise condition, the performance of the proposed ML and BLUE estimator are the same and better than other methods (see Fig. 3). Thus, it is better to apply the inverse of (27) as the weighting matrix, which is simpler. On the contrary, when the noise is relatively high the performance of the BLUE estimator is much better in comparison with other methods as shown in Fig. 4. From these two figures, it is evident that an appropriate weighting matrix generates a substantial improvement in the performance of the proposed method.

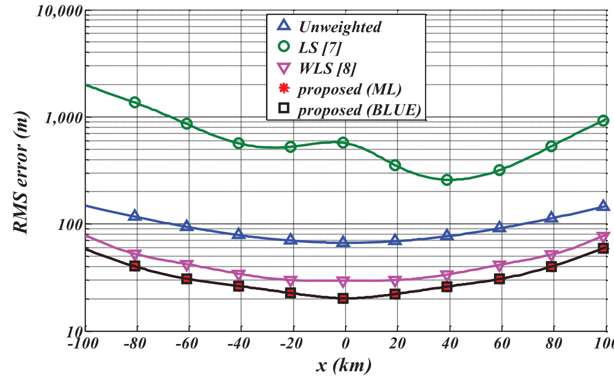

Fig. 3. Comparison among the performance of the proposed method with other ones under the low noise condition.

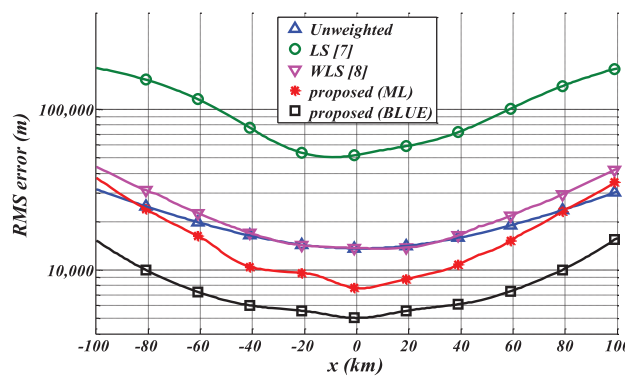

Fig. 4. Comparison among the performance of the proposed method with other ones under the relatively high noise condition.

V. Conclusion

This study proposed a weighted least squares method to determine the location of a target in the presence of several transmitters and several receivers using TDOA measurements. A weighting matrix needed in the method is derived in two various conditions—when the measurement noise is small and when it is not. This weighting matrix was generated to improve the performance of the proposed method. When the noise was small, we neglected the second order noise terms in the error equation. Under this condition, the solution of the problem was the same as the ML estimation. But, when the noise was relatively high, we applied a weighting matrix which led to the BLUE. Computer simulations were included to corroborate the theoretical developments.

References

[1] A. J. Weiss, "On the accuracy of a cellular location system based on RSS measurements," *IEEE Trans. Veh. Technol.*, vol. 52, no. 11, pp. 1508–1518, Nov. 2003.

[2] R. A. Poisel, *Electronic Warfare Target Location Methods*, 2nd ed. Norwell, MA, USA: Artech House, 2012, pp. 139–223.

[3] P. Stoica and K. Sharman, "Maximum likelihood methods for direction-of-arrival estimation," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 38, no. 7, pp. 1132–1143, Jul. 1990.

[4] J. A. Cadzow, "Methods of direction-of-arrival," in *IEEE Pacific Rim Conf. Commun., Comput. and Signal Process.*, May 1991, pp. 25–28.

[5] M. Malanowski and K. Kulpa, "Two methods for target localization in multistatic passive radar," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 1, pp. 572–580, Jan. 2012.

[6] L. Rui and K. C. Ho, "Elliptic localization: Performance study and optimum receiver placement," *IEEE Trans. Signal Process.*, vol. 62, no. 18, pp. 4673–4688, Sep. 2014.

[7] M. Dianat, M. R. Taban, J. Dianat, and V. Sedighi, "Target localization using least squares estimation for MIMO radars with widely separated antennas," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 49, no. 4, pp. 2730–2741, Oct. 2013.

[8] M. Einemo and H. Ch. So, "Weighted least squares algorithm for target localization in distributed MIMO radar," *Signal Process.*, vol. 115, pp. 144–150, Oct. 2015.

[9] A. Noroozi and M. A. Sebt, "Comparison between range-difference-based and bistatic-range-based localization in multistatic passive radar," in *16th Int. Symp. Radar (IRS)*, Jun. 2015, pp. 1058–1063.

[10] N. Marchand, "Error distributions of best estimate of position from multiple time difference hyperbolic networks," *IEEE Trans. Aerosp. Electron. Syst.*, vol. ANE-11, no. 2, pp. 96–100, Jun. 1964.

[11] H. B. Lee, "A novel procedure for assessing the accuracy of hyperbolic multilateration systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-11, no. 1, pp. 2–15, Jan. 1975.

[12] J. J. Caffery, Jr., "A new approach to the geometry of TOA location," in *IEEE-VTS Fall VTC Veh. Tech. Conf.*, 2000, pp. 1943–1949.

[13] D. C. Cooper, "Statistical analysis of position-fixing general theory for systems with gaussian errors," *Proc. Inst. Elect. Engineers*, pp. 637–640, Jun. 1972.

[14] B. Friedlander, "A passive localization algorithm and its accuracy analysis," *IEEE J. Oceanic Eng.*, vol. OE-12, pp. 234–245, Jan. 1987.

[15] J. O. Smith and J. S. Abel, "The spherical interpolation method of source localization," *IEEE J. Oceanic Eng.*, vol. 12, no. 1, pp. 246–252, Jan. 1987.

[16] J. O. Smith and J. S. Abel, "Closed-form least-squares source location estimation from range-difference measurements," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-35, pp. 1661–1669, Dec. 1987.

[17] H. C. Schau and A. Z. Robinson, "Passive source localization employing intersecting spherical surfaces from time-of-arrival differences," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-35, pp. 1223–1225, Aug. 1987.

[18] G. Mellen, M. Pachter, and J. Raquet, "Closed-form solution for determining emitter location using time difference of arrival measurements," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 3, pp. 1056–1058, Jul. 2003.

[19] R. O. Schmidt, "A new approach to geometry of range difference location," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-8, pp. 821–835, Nov. 1972.

[20] J. S. Abel, "A divide and conquer approach to least-squares estimation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 26, pp. 423–427, Mar. 1990.

[21] R. O. Schmidt, "Least-squares range difference location," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, no. 1, pp. 234–242, Jan. 1996.

[22] Y. T. Chan and K. C. Ho, "A simple and efficient estimator for hyperbolic location," *IEEE Trans. Signal Process.*, vol. 42, pp. 1905–1915, Aug. 1994.

[23] K. C. Ho, "Bias reduction for an explicit solution of source localization using TDOA," *IEEE Trans. Signal Process.*, vol. 60, no. 5, pp. 2101–2114, May 2012.

[24] J. N. Willis, *Bistatic Radar*, 2nd ed. Raleigh, NC, USA: SciTech, 2005, pp. 65–70.

[25] S. M. Kay, *Fundamentals of Statistical Signal Processing*, ser. Estimation Theory. . Upper Saddle River, NJ, USA: Prentice Hall, 1993, vol. I, pp. 133–141.