A Simple and Accurate TDOA-AOA Localization Method Using Two Stations

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***Abstract*—This letter focuses on locating passively a point source in the three-dimensional (3D) space, using the hybrid measurements of time difference of arrival (TDOA) and angle of arrival (AOA) observed at two stations. We propose a simple closed-form solution method by constructing new relationships between the hybrid measurements and the unknown source position. The mean-square error (MSE) matrix of the proposed solution is derived under the small error condition. Theoretical analysis discloses that the performance of the proposed solution can attain the Cramér-Rao bound (CRB) for Gaussian noise over the small error region where the bias compared to variance is small to be ignored. The proposed solution can be extended directly to more than two observing stations with CRB performance maintained theoretically. Simulations validate the performance of the proposed method.**

***Index Terms*—Closed-form solution, Cramér-Rao bound (CRB), hybrid measurements, passive source localization.**

I. Introduction

PASSIVE source localization is a fundamental problem for many applications in radar, sonar, and communications. Various source localization methods, e.g., [1]–[14], have been proposed based on different types of measurements, such as time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), and their combinations. The source localization problem is nontrivial in general since the direct relationships between the position of a source and the measurements are nonlinear and nonconvex. The maximum likelihood estimator (MLE) is asymptotically efficient. Yet an initial solution guess is required using iterative implementation to avoid numerical global search. It needs to be in a small (convex) neighborhood of the actual solution to ensure global convergence. Trying several different initial guesses followed by goodness-of-fit evaluations may improve the chance of obtaining a global solution, it would undoubtedly increase the computational costs. To improve the robustness and reduce the complexity, [1] and [2] developed closed-form TDOA localization solutions using TDOAs (and their derivatives), which can attain the the Cramér-Rao bound (CRB) under mild conditions. Using the generalized trust region subproblems technique, [3] obtains an excellent positioning solution by analyzing the necessary optimality conditions of the squared range difference least squares (LS) cost function. [4] approached the problem by applying semidefinite-programming relaxation to the nonconvex range difference maximum likelihood (ML) cost function.

Regarding the passive source localization using AOAs, most existing methods, e.g., [6]–[9], are in the two-dimensional (2D) space using one-dimensional AOAs, viz., bearing measurements. In three-dimensional (3D) space, 2D AOA represented by the pair of azimuth and elevation angles is used instead. Many estimators are available for 2D AOA estimation, such as [15]. Recently, [12] proposed a pseudolinear estimator for 3D target motion analysis using the azimuth and elevation angles, and showed good localization performance under large sample condition. Rather than using only TDOA or AOA measurements, [13] presented a hybrid TDOA and bearing location scheme that gives better accuracy than using TDOA alone. [14] proposed an algorithm using hybrid bearing and TDOA measurements that exploits a geometrically derived constraint to improve performance. The constrained method applies to both 2D and 3D cases, it requires, however, line search and is not in closed-form.

The interest in locating an emitter with the low probability of interception (LPI) has become more and more apparent in practice. An LPI source often utilizes a beamforming technique or a power control strategy. Thus it could be hardly observed by many stations, probably at most two stations. It seems difficult for some existing source localization methods to deal with an LPI source, as their successful implementations require the minimum number of stations or a large number of measurements. For example, a TDOA-based method in the 2D or 3D scenario needs at least 4 or 5 stations, respectively, to ensure a unique solution. Therefore, it is meaningful to study a source localization problem with only two stations and hybrid measurements, although the problem seems a little specific at a glimpse. The situation of limited number of observers can also occur in the ocean environment.

This letter focuses on the passive localization of a 3D point source with one TDOA and two azimuth-elevation pairs observed at two stations. While several estimators are available from the literature, e.g., [13], [14] for localization using TDOAs and AOAs, most of them are either for 2D only or require numerical search. Indeed, to the best of our knowledge, we have not come across an estimator that is closed-form to handle a

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3D localization scenario with two stations and hybrid measurements. We propose a simple method by constructing new relationships between the hybrid measurements and the unknown source position, which leads to a simple and yet closed-form solution for the source location. Theoretical analysis shows that the mean-square error (MSE) of the proposed solution can attain the CRB accuracy under Gaussian noise when measurement error is not large and the bias is negligible relative to the variance. Simulations validate the performance of the proposed method. The proposed estimator can be directly extended to more than two stations with the CRB performance maintained theoretically.

II. Data Model and CRB

The localization scenario consists of a source and two observation stations in a 3D right-handed rectangular coordinate system. The unknown source position is represented by $\boldsymbol{u} = [u_\mathrm{x}, u_\mathrm{y}, u_\mathrm{z}]^T \in \mathbb{R}^3$ whereas the observation stations are at known positions $\boldsymbol{s}_m = [s_{\mathrm{x},m}, s_{\mathrm{y},m}, s_{\mathrm{z},m}]^T \in \mathbb{R}^3$, $m = 1, 2$, and $(\cdot)^T$ is the transpose operation. As shown in Fig. 1, the source signal is observed by the two stations to produce one TDOA and two AOA pairs (θ_m, ϕ_m), where the azimuth $\theta_m \in (-\pi, \pi]$ and the elevation $\phi_m \in (0, \pi/2)$.

The measurement model in vector form is

$$\hat{\boldsymbol{\kappa}} = \boldsymbol{\kappa} + \boldsymbol{\varepsilon}, \tag{1}$$

where $\hat{\boldsymbol{\kappa}} = [\hat{\tau}_{21}, \hat{\boldsymbol{\kappa}}_1^T, \hat{\boldsymbol{\kappa}}_2^T]^T \in \mathbb{R}^5$ is the measurement vector, $\boldsymbol{\kappa} = [\tau_{21}, \boldsymbol{\kappa}_1^T, \boldsymbol{\kappa}_2^T]^T \in \mathbb{R}^5$ is its actual counterpart whose elements are related to the source position by

$$\tau_{21} = r_2 - r_1, \tag{2}$$

$$\boldsymbol{\kappa}_m = \begin{bmatrix} \mathrm{atan2}(u_\mathrm{y} - s_{\mathrm{y},m}, u_\mathrm{x} - s_{\mathrm{x},m}) \\ \mathrm{atan2}\left(u_\mathrm{z} - s_{\mathrm{z},m}, \sqrt{(u_\mathrm{x} - s_{\mathrm{x},m})^2 + (u_\mathrm{y} - s_{\mathrm{y},m})^2}\right) \end{bmatrix} = \begin{bmatrix} \theta_m \\ \phi_m \end{bmatrix}, m = 1, 2. \tag{3}$$

τ_{21} is the true range difference between the two observation stations, and $r_2 = \|\boldsymbol{u} - \boldsymbol{s}_2\|$ and $r_1 = \|\boldsymbol{u} - \boldsymbol{s}_1\|$ are the actual ranges of the source at the two stations. $\|\cdot\|$ is the l_2-norm. The terms TDOA and range difference are used interchangeably here since they differ from each other by a constant. atan2$(\cdot, \cdot)$ denotes the arctangent function that takes into account the appropriate quadrant of an angle. The measurement error vector $\boldsymbol{\varepsilon} = [e_{21}, \boldsymbol{\varepsilon}_1^T, \boldsymbol{\varepsilon}_2^T]^T \in \mathbb{R}^5$ is assumed to be zero-mean Gaussian with a covariance matrix $\boldsymbol{Q} \in \mathbb{R}^{5\times5}$.

The CRB matrix of $\boldsymbol{u}$ is [16]

$$\mathrm{CRB}\,(\boldsymbol{u}) = \mathrm{FIM}^{-1}\,(\boldsymbol{u}) \tag{4}$$

where FIM is the Fisher information matrix. From the Gaussian density model of $\hat{\boldsymbol{\kappa}}$, it is given by [16]

$$\mathrm{FIM}\,(\boldsymbol{u}) = \left(\frac{\partial \boldsymbol{\kappa}}{\partial \boldsymbol{u}^T}\right)^T \boldsymbol{Q}^{-1} \frac{\partial \boldsymbol{\kappa}}{\partial \boldsymbol{u}^T}, \tag{5}$$

where

$$\frac{\partial \boldsymbol{\kappa}}{\partial \boldsymbol{u}^T} = \left[\boldsymbol{c}, \boldsymbol{D}_1^T, \boldsymbol{D}_2^T\right]^T \in \mathbb{R}^{5\times3}, \tag{6}$$

$$\boldsymbol{c} = (\boldsymbol{u} - \boldsymbol{s}_2)/r_2 - (\boldsymbol{u} - \boldsymbol{s}_1)/r_1, \tag{7}$$

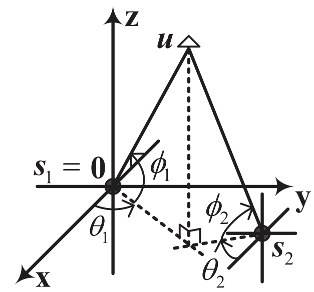

Fig. 1. Localization geometry.

$$\boldsymbol{D}_m = \begin{bmatrix} \frac{-(u_\mathrm{y} - s_{\mathrm{y},m})}{l_m^2} & \frac{u_\mathrm{x} - s_{\mathrm{x},m}}{l_m^2} & 0 \\ \frac{-(u_\mathrm{x} - s_{\mathrm{x},m})(u_\mathrm{z} - s_{\mathrm{z},m})}{r_m^2 l_m} & \frac{-(u_\mathrm{y} - s_{\mathrm{y},m})(u_\mathrm{z} - s_{\mathrm{z},m})}{r_m^2 l_m} & \frac{l_m}{r_m^2} \end{bmatrix}, \tag{8}$$

$l_m = [(u_\mathrm{x} - s_{\mathrm{x},m})^2 + (u_\mathrm{y} - s_{\mathrm{y},m})^2]^{1/2}$, $m = 1, 2$.

The CRB will be used as a benchmark for performance evaluation in Section IV. Note that the CRB is the bound for an unbiased estimator. As a result, the CRB is adequate to indicate the best possible accuracy of a source position estimator over the small error region only in which the bias is small to be negligible as compared to the estimation variance.

III. The Proposed Method

We are interested in estimating the unknown source position $\boldsymbol{u}$ via $\hat{\boldsymbol{\kappa}}$. It is difficult to do the estimation using the data model (1) directly as $\boldsymbol{\kappa}$ is nonlinear and nonconvex with respect to $\boldsymbol{u}$. New relationships shall be constructed as shown below to circumvent this difficulty.

The localization geometry (see Fig. 1) shows that

$$\boldsymbol{u} - \boldsymbol{s}_m = r_m \boldsymbol{b}_m, \quad m = 1, 2, \tag{9}$$

where $\boldsymbol{b}_m = [\cos\phi_m \cos\theta_m, \cos\phi_m \sin\theta_m, \sin\phi_m]^T \in \mathbb{R}^3$ is the unit vector of the actual source location with respect to the mth station. It means the vector $\boldsymbol{u} - \boldsymbol{s}_m$ lies in the one-dimensional subspace in $\mathbb{R}^3$ spanned by $\boldsymbol{b}_m$. Let $\boldsymbol{G}_m \in \mathbb{R}^{3\times2}$ be

$$\boldsymbol{G}_m = \begin{bmatrix} \sin\theta_m & \sin\phi_m \cos\theta_m \\ -\cos\theta_m & \sin\phi_m \sin\theta_m \\ 0 & -\cos\phi_m \end{bmatrix}, \quad m = 1, 2. \tag{10}$$

It follows that $\boldsymbol{G}_m^T \boldsymbol{G}_m = \boldsymbol{I}_2$ and $\boldsymbol{G}_m^T \boldsymbol{b}_m = \boldsymbol{0}_2$, where $\boldsymbol{I}_2 \in \mathbb{R}^{2\times2}$ is the identity matrix and $\boldsymbol{0}_2 \in \mathbb{R}^2$ is the zero vector. That is to say, the columns of $\boldsymbol{G}_m$ are an orthonormal basis of the plane orthogonal to $\boldsymbol{b}_m$. As a result, we have $\boldsymbol{G}_m^T(\boldsymbol{u} - \boldsymbol{s}_m) = \boldsymbol{0}_2$, or equivalently,

$$\boldsymbol{G}_m^T \boldsymbol{u} = \boldsymbol{G}_m^T \boldsymbol{s}_m, \quad m = 1, 2. \tag{11}$$

Note that the relationships in (11) were derived for the first time in [10] and later appeared in [11], we arrive at the same relationships from the subspace viewpoint that will facilitate the analysis for our source localization problem.

On the other hand, we construct an identity by realizing that $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ are unit norm vectors,

$$r_2(\boldsymbol{b}_2 - \boldsymbol{b}_1)^T(\boldsymbol{b}_2 + \boldsymbol{b}_1) = 0. \tag{12}$$

From (2) we have $r_2(\boldsymbol{b}_2 + \boldsymbol{b}_1) = r_2\boldsymbol{b}_2 + (r_1 + \tau_{21})\boldsymbol{b}_1$. Upon using (9), (12) becomes

$$2(\boldsymbol{b}_2 - \boldsymbol{b}_1)^T\boldsymbol{u} = (\boldsymbol{b}_2 - \boldsymbol{b}_1)^T(\boldsymbol{s}_1 + \boldsymbol{s}_2 - \tau_{21}\boldsymbol{b}_1). \tag{13}$$

Putting together (11) and (13) in matrix form yields

$$\boldsymbol{h} = \boldsymbol{G}^T\boldsymbol{u}, \tag{14}$$

where $\boldsymbol{h} = [(\boldsymbol{b}_2 - \boldsymbol{b}_1)^T(\boldsymbol{s}_1 + \boldsymbol{s}_2 - \tau_{21}\boldsymbol{b}_1), \boldsymbol{s}_1^T\boldsymbol{G}_1, \boldsymbol{s}_2^T\boldsymbol{G}_2]^T \in \mathbb{R}^5$ and $\boldsymbol{G} = [2(\boldsymbol{b}_2 - \boldsymbol{b}_1), \boldsymbol{G}_1, \boldsymbol{G}_2] \in \mathbb{R}^{3\times 5}$.

Equation (14) only holds approximately in practice due to the measurement errors in the TDOA and AOAs. Expressing in (14) the TDOA and AOAs in terms of their noisy observations from the model (1) and then approximating up to first order noise terms yield

$$\hat{\boldsymbol{h}} \approx \hat{\boldsymbol{G}}^T\boldsymbol{u} + \boldsymbol{T}\boldsymbol{\varepsilon}, \tag{15}$$

where $\hat{\boldsymbol{h}}$ and $\hat{\boldsymbol{G}}$ are $\boldsymbol{h}$ and $\boldsymbol{G}$ with the true values replaced by the measurements,

$$\boldsymbol{T} = \begin{bmatrix} -(\boldsymbol{b}_2 - \boldsymbol{b}_1)^T\boldsymbol{b}_1 & r_1\boldsymbol{b}_2^T\boldsymbol{L}_1 & -r_2\boldsymbol{b}_1^T\boldsymbol{L}_2 \\ \boldsymbol{0}_2 & \boldsymbol{T}_1 & \boldsymbol{0}_{2\times 2} \\ \boldsymbol{0}_2 & \boldsymbol{0}_{2\times 2} & \boldsymbol{T}_2 \end{bmatrix}, \tag{16}$$

$$\boldsymbol{L}_m = \begin{bmatrix} -\cos\phi_m \sin\theta_m & -\sin\phi_m\cos\theta_m \\ \cos\phi_m\cos\theta_m & -\sin\phi_m\sin\theta_m \\ 0 & \cos\phi_m \end{bmatrix}, \tag{17}$$

$$\boldsymbol{T}_m = -r_m\begin{bmatrix} \cos\phi_m & 0 \\ 0 & 1 \end{bmatrix}, \tag{18}$$

$m = 1, 2$, and $\boldsymbol{0}_{2\times 2}$ is the zero matrix of size 2.

It follows from (1) that $\boldsymbol{T}\boldsymbol{\varepsilon}$ in (15) is zero-mean Gaussian with the covariance matrix $\boldsymbol{W} = \boldsymbol{T}\boldsymbol{Q}\boldsymbol{T}^T$. As a result, we obtain from (15) a weighted LS estimate of $\boldsymbol{u}$:

$$\begin{aligned} \hat{\boldsymbol{u}} &= \arg\min \left(\hat{\boldsymbol{h}} - \hat{\boldsymbol{G}}^T\boldsymbol{u}\right)^T \boldsymbol{W}^{-1}\left(\hat{\boldsymbol{h}} - \hat{\boldsymbol{G}}^T\boldsymbol{u}\right) \\ &= \left(\hat{\boldsymbol{G}}\boldsymbol{W}^{-1}\hat{\boldsymbol{G}}^T\right)^{-1}\hat{\boldsymbol{G}}\boldsymbol{W}^{-1}\hat{\boldsymbol{h}}. \end{aligned} \tag{19}$$

Using $\boldsymbol{u} = (\hat{\boldsymbol{G}}\boldsymbol{W}^{-1}\hat{\boldsymbol{G}}^T)^{-1}\hat{\boldsymbol{G}}\boldsymbol{W}^{-1}\hat{\boldsymbol{G}}^T\boldsymbol{u}$ and approximating $\hat{\boldsymbol{u}} - \boldsymbol{u}$ up to the linear noise terms, the MSE matrix is

$$\begin{aligned} \mathrm{MSE} &= \mathrm{E}\left\{(\hat{\boldsymbol{u}} - \boldsymbol{u})(\hat{\boldsymbol{u}} - \boldsymbol{u})^T\right\} \approx \left(\boldsymbol{G}\boldsymbol{W}^{-1}\boldsymbol{G}^T\right)^{-1} \\ &= \left[\left(\boldsymbol{T}^{-1}\boldsymbol{G}^T\right)^T\boldsymbol{Q}^{-1}\boldsymbol{T}^{-1}\boldsymbol{G}^T\right]^{-1}. \end{aligned} \tag{20}$$

Remark 1: The proposed method will give a unique solution if $\boldsymbol{G}$ is full column rank and $\boldsymbol{T}$ is invertible. From (10), (18) and the definitions of $\boldsymbol{G}$ below (14) and $\boldsymbol{T}$ in (16), this is ensured if $\boldsymbol{b}_1 \neq \boldsymbol{b}_2$. That is, the source is not on the same side of both stations when the source and the stations are on a straight line. This condition is usually satisfied in practice, since the chance that $\boldsymbol{b}_1 = \boldsymbol{b}_2$ is nearly zero.

Remark 2: The matrix $\boldsymbol{W}$ in (19) is not known in practice since it depends on the actual source position[1]. We can use the identity matrix instead of $\boldsymbol{W}$ in (19) to obtain an initial position estimate of the source first, and then apply the initial position estimate, together with the AOA observations, to computing $\boldsymbol{W}$ via (16)–(18). Simulation results show that one or two repetitions are sufficient. Note that unlike an iterative ML method that usually needs a good initial solution from another (suboptimal) estimator, the proposed method is self-contained with no help of other methods. If the estimate of $\boldsymbol{W}$ has poor rank or bad condition number and we do not replicate, the resulting solution reduces back to the LS solution that could act as an initial value for the ML method.

Remark 3: Although we restrict the proposed method to the localization problem based on two stations, it can be generalized to the scenarios that have more than two stations. To be specific, let M be the total number of stations that are able to observe the source signal, where the station positions are $\boldsymbol{s}_m$, the AOAs are (θ_m, ϕ_m) and the measurement noise covariance matrix is $\boldsymbol{Q} \in \mathbb{R}^{(3M-1)\times(3M-1)}$. Following the similar derivations in Section III, we have the matrix equation $\tilde{\boldsymbol{h}} = \tilde{\boldsymbol{G}}^T\boldsymbol{u}$, where $\tilde{\boldsymbol{h}} = [(\boldsymbol{b}_2 - \boldsymbol{b}_1)^T(\boldsymbol{s}_1 + \boldsymbol{s}_2 - \tau_{21}\boldsymbol{b}_1), (\boldsymbol{b}_3 - \boldsymbol{b}_1)^T(\boldsymbol{s}_1 + \boldsymbol{s}_3 - \tau_{31}\boldsymbol{b}_1), \ldots, (\boldsymbol{b}_M - \boldsymbol{b}_1)^T(\boldsymbol{s}_1 + \boldsymbol{s}_M - \tau_{M1}\boldsymbol{b}_1), \boldsymbol{s}_1^T\boldsymbol{G}_1, \boldsymbol{s}_2^T\boldsymbol{G}_2, \ldots, \boldsymbol{s}_M^T\boldsymbol{G}_M]^T \in \mathbb{R}^{3M-1}$, $\tilde{\boldsymbol{G}} = [2(\boldsymbol{b}_2 - \boldsymbol{b}_1), 2(\boldsymbol{b}_3 - \boldsymbol{b}_1), \ldots, 2(\boldsymbol{b}_M - \boldsymbol{b}_1), \boldsymbol{G}_1, \boldsymbol{G}_2, \ldots, \boldsymbol{G}_M] \in \mathbb{R}^{3\times(3M-1)}$, and $\boldsymbol{b}_m$ and $\boldsymbol{G}_m$ are defined below (9) and in (10). The resulting closed-form solution is given by (19), where $\hat{\boldsymbol{G}}$ and $\hat{\boldsymbol{h}}$ are now defined according to $\tilde{\boldsymbol{G}}$ and $\tilde{\boldsymbol{h}}$. $\boldsymbol{T} \in \mathbb{R}^{(3M-1)\times(3M-1)}$ has the similar structure as in (16), where the first $M-1$ rows are obtained by the first-order error analysis similar to that for the first row in (16), and the rest are block diagonal with diagonal blocks $\boldsymbol{T}_m$, $m = 1, 2, \ldots, M$.

IV. Performance Analysis

We shall show from the following analysis that the theoretical approximate MSE in (20) is equal to the CRB, viz.,

$$\mathrm{MSE} = \mathrm{CRB}\,(\boldsymbol{u}), \tag{21}$$

which is also equivalent to the claim that $\mathrm{MSE}^{-1} = \mathrm{FIM}(\boldsymbol{u})$. Note that (20) is obtained up to the first order noise terms of the estimate (19). Thus we expect the proposed estimator is able to reach the CRB performance over the small error region in which the higher order noise terms are insignificant to be neglected.

The analysis starts by rewriting (7) and (8) as

$$\boldsymbol{c} = \boldsymbol{b}_2 - \boldsymbol{b}_1, \tag{22}$$

$$D_m = \frac{1}{r_m}\begin{bmatrix} -\sin\theta_m/\cos\phi_m & \cos\theta_m/\cos\phi_m & 0 \\ -\sin\phi_m\cos\theta_m & -\sin\phi_m\sin\theta_m & \cos\phi_m \end{bmatrix} \tag{23}$$

where (9) and the relationships $l_m = r_m\cos\phi_m$, $m = 1, 2$, have been used. Using (18) and (23), we obtain

$$\boldsymbol{T}_m\boldsymbol{D}_m = \boldsymbol{G}_m^T, \quad m = 1, 2. \tag{24}$$

On the other hand, we have

$$r_m\boldsymbol{L}_m\boldsymbol{D}_m = \boldsymbol{G}_m\boldsymbol{G}_m^T = \boldsymbol{I}_3 - \boldsymbol{b}_m\boldsymbol{b}_m^T, \quad m = 1, 2, \tag{25}$$

where the relationship between two orthogonal projection matrices whose range subspaces are the orthogonal complements of each other has been used in the second equation. From (25),

$$\begin{aligned} 2(\boldsymbol{b}_2 - \boldsymbol{b}_1)^T = &-(\boldsymbol{b}_2 - \boldsymbol{b}_1)^T\boldsymbol{b}_1(\boldsymbol{b}_2 - \boldsymbol{b}_1)^T \\ &+ r_1\boldsymbol{b}_2^T\boldsymbol{L}_1\boldsymbol{D}_1 - r_2\boldsymbol{b}_1^T\boldsymbol{L}_2\boldsymbol{D}_2. \end{aligned} \tag{26}$$

[1]The actual AOAs can be generated from the source position.

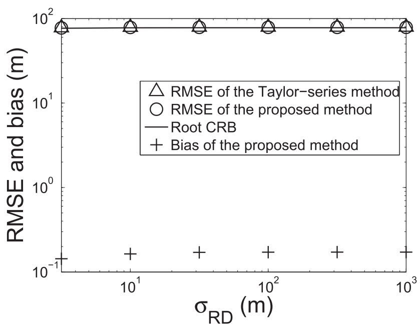

Fig. 2. Source localization RMSE and bias as σ_{RD} increases.

It follows from (24) and (26) and the partial derivative (6) that $\boldsymbol{T}\frac{\partial \boldsymbol{\kappa}}{\partial \boldsymbol{u}^T} = \boldsymbol{G}^T$, or equivalently, $\frac{\partial \boldsymbol{\kappa}}{\partial \boldsymbol{u}^T} = \boldsymbol{T}^{-1}\boldsymbol{G}^T$. Comparing (4)–(5) with (20) immediately results in $\mathrm{MSE}^{-1} = \mathrm{FIM}(\boldsymbol{u})$. Therefore, (21) holds.

V. Simulations

Simulations are presented to validate the performance of the proposed method. $\boldsymbol{Q}$ is modeled as $\mathrm{diag}[\sigma^2_{\mathrm{RD}}, \sigma^2_{\mathrm{AOA}}, \sigma^2_{\mathrm{AOA}}, \sigma^2_{\mathrm{AOA}}, \sigma^2_{\mathrm{AOA}}]$, where σ_{RD} and σ_{AOA} denote the range difference and AOA standard deviations, respectively. Simulation results are displayed in terms of the root-mean-square error (RMSE) given by $\mathrm{RMSE}(\boldsymbol{u}) = \sqrt{\sum_{l=1}^{L} \|\hat{\boldsymbol{u}}_l - \boldsymbol{u}\|^2 / L}$ and the estimation bias obtained from bias $(\boldsymbol{u}) = \sqrt{\left\|\sum_{l=1}^{L} \hat{\boldsymbol{u}}_l / L - \boldsymbol{u}\right\|^2}$, where $\hat{\boldsymbol{u}}_l$ denotes a position estimate of the source at the lth ensemble and $L = 5000$ is the number of ensemble runs. Besides, we use the root CRB as a benchmark for performance evaluation. The theoretical approximate MSE in (20) needs not be shown since it has proved to be equal to the CRB under the small error condition. The two stations are deployed at the endpoints of a diameter on a circle with radius 300 m centered at the coordinate origin, with the diameter at a certain angle from the x-axis. To alleviate the dependency of a particular geometry, the presented simulation results are the average of 10 different station deployments with the angles sampled from uniform distributions.

For comparison purpose we implemented the Tayor-series method [17], which is an iterative implementation of the MLE. This method takes more computations and requires an initial position guess close to the actual solution. To give the most favorable performance it is initialized with the actual source position although not possible in practice.

Fig. 2 shows the estimation performance of the proposed method as σ_{RD} varies, where the source is set at $\boldsymbol{u} = [1000, 1000, 1000]^T$m and σ_{AOA} is 0.5 degree. The performance of the proposed closed-form method and that of the Taylor-series method are very close to the CRB. Moreover, the estimation bias of the proposed method is relatively insignificant as compared with the RMSEs. We would like to emphasize that the Taylor-series method is invoked by the actual source position and needs many iteration steps.

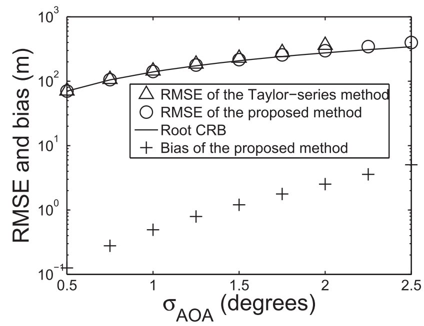

Fig. 3. Source localization RMSE and bias as σ_{AOA} increases.

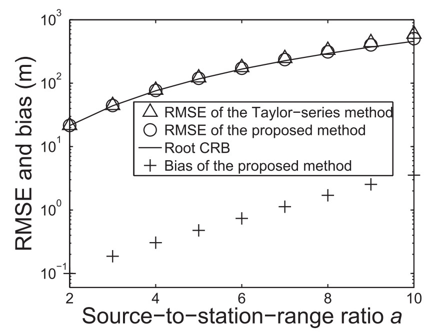

Fig. 4. Source localization RMSE and bias as a increases.

Fig. 3 shows the performance of the proposed method as σ_{AOA} varies, where σ_{RD} is fixed at 10 m. The other settings are the same as above. Fig. 3 reveals that the RMSEs of the proposed method and the Taylor-series method are very comparable, and they both begin to deviate from the root CRB when σ_{AOA} is about 1 degree. Note that the Taylor-series method has the threshold effect when σ_{AOA} is about 2 degrees, while the proposed method remains to provide near CRB performance. We attribute the improvement possibly due to the irregularity of the error surface near the minimum, thereby making the gradient-based Taylor-series method sensitive to noise. Regarding bias we have the similar conclusion as above.

Fig. 4 illustrates the performance of the proposed method as the source range increases, by setting the source location to $\boldsymbol{u} = a\sqrt{3} \times [100, 100, 100]^T$m, where the source-to-station-range ratio a varies from 2 to 10. The noise settings are $\sigma_{\mathrm{RD}} = 10$ m and $\sigma_{\mathrm{AOA}} = 1$ degree. Again, the performance of the proposed method remains promising.

VI. Conclusions

We have proposed a simple method for the 3D passive source localization problem using only two observation stations by exploiting the hybrid TDOA and AOA measurements. The proposed method is closed-form with performance reaching the CRB for Gaussian noise over the small error region where the bias compared to variance is small enough to be ignored. It requires less computations and does not reply on good initial guess as compared to the gradient-based MLE, and may have higher noise tolerance of the threshold effect. The performance of the proposed method is supported by the theoretical analysis and the simulation results. It can be extended to more than two stations directly with the CRB performance maintained theoretically.

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