AER-4027 Design of Nozzles and Intakes Problem Sheet

1. Introduction and Review of Internal Flows (10)

[1] Review and Coding of Gas Dynamics Procedures (See Appendix A).

Appendix A includes Keys used to distinguish various cases in each gas dynamics procedure. The first procedure (that for isentropic flow is given together with its results coded in Maple). You are required to code the normal shock, oblique shock, Prandtl Meyer angle, and Prandtl-Meyer flow procedures. You are encouraged to use Maple, Mathematica, or Matlab. Team work is allowed.

[2] Compare between diffusers/nozzles using the following criteria:

Function, Action and Goal, Flow Stability, Design Objectives, Type (using $M_{Inlet} < 1$ or > 1), Area Change, Sketch, Velocity/Mach Number Changes, Pressure/Temperature/Density Changes, Shock Waves, Flow Separation, Design Complexity, Efficiency, and Applications. Present your comparison in table format.

[3] Compare between aerodynamic efficiency/effectiveness.

Present your comparison in table format.

[4] Discuss each of the following loss measures in terms of its use and application.

#	Objective Function	#	Objective Function
1	$\Delta S\left[\mathrm{J/K}\right] \equiv$ Entropy Change	**9**	$\eta_N, \eta_D \equiv$ Nozzle / Diffuser Efficiency
2	$\dot{S}_{\mathrm{gen}}\left[\mathrm{W/K}\right] \equiv$ Entropy Generation Rate	**10**	$f \equiv$ Friction Factor
3	$\Phi, \Theta\left[\mathrm{W/m^3}\right] \equiv$ Vis Dissipation Fn, $\Phi = \vec{\tau} : \vec{\nabla}\vec{v}$	**11**	$h_L, h_f\left[\mathrm{m}\right] \equiv$ Major Head/Friction Loss of Energy $h_m\left[\mathrm{m}\right] \equiv$ Minor Head/Friction Loss of Energy
4	$\bar{P}_0 \equiv$ Stagnation Pressure Ratio	**12**	$K_L,\ K,\ \xi \equiv$ Loss Coefficient / Factor for a compt
5	$\bar{P}_{\mathrm{Total}} \equiv$ Total Pressure Ratio	**13**	$\Delta P = P_1 - P_2\left[\mathrm{Pa}\right] \equiv$ Pressure Loss, $\Delta P = K_L\left(\frac{1}{2}\rho \mathrm{v}^2\right)$
6	$\bar{P}_{\mathrm{Static}} \equiv$ Static Pressure Ratio	**14**	L/D $\equiv$ Lift to Drag Ratio
7	$n \equiv$ Polytropic Index	**15**	$C_{\mathrm{D}} \equiv$ Drag Coefficient
8	$\eta \equiv$ Efficiency	**16**	

NB: $h_L = f\dfrac{L}{D}\dfrac{\mathrm{v}^2}{2g},\ h_m = \dfrac{\Delta P}{\rho g}$

[5] Pressure Recovery

Illustrate the pressure recovery phenomenon using the following flows;

(a) Internal Flow in a Venturi
(b) External Flow over a Cylinder

[6] Gradual Enlargement

[7] Gradual Contraction

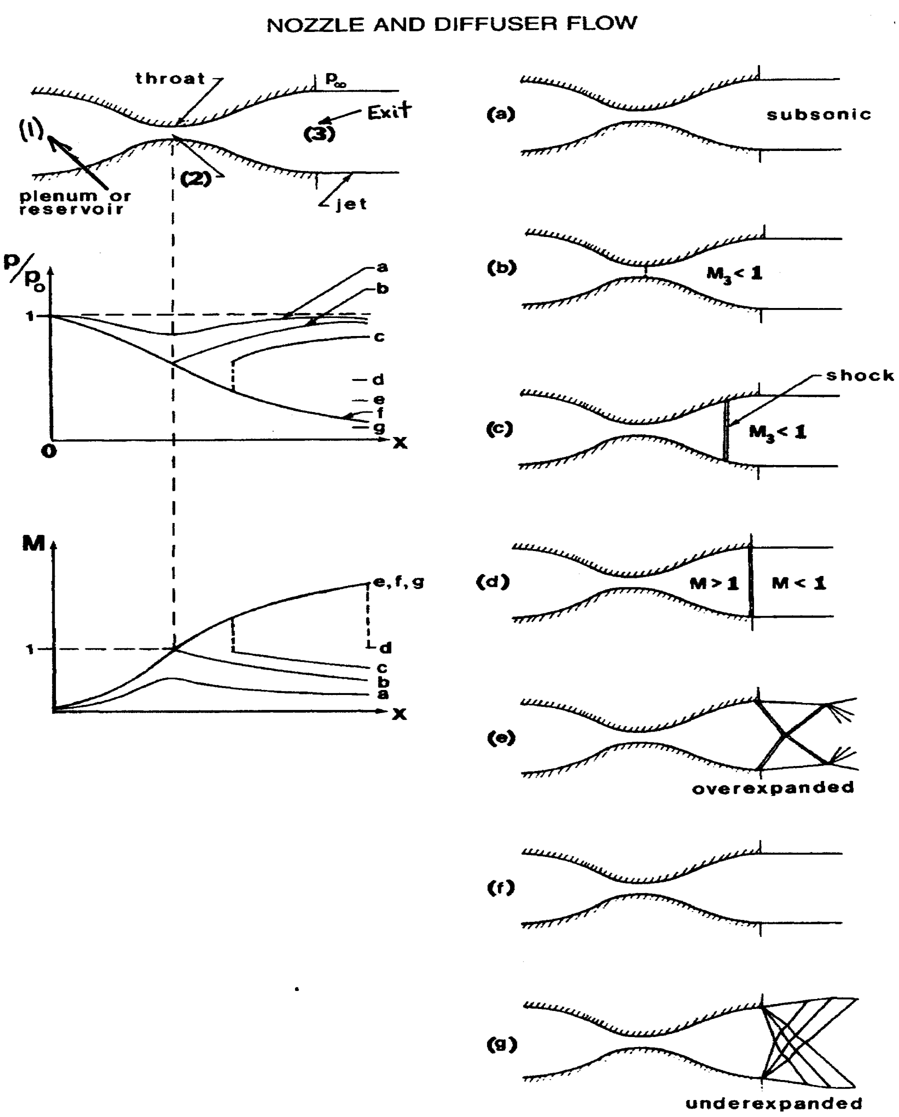
[8] Sudden Enlargement

[9] Sudden Contraction

[10]

2. Analytical and Semi-Empirical Methods (6)

[1] Quasi One-Dimensional Flow in a Convergent-Divergent Duct



Nozzle flow regimes.

Consider the shown symmetric convergent-divergent nozzle. The nozzle has an area per unit width given by,

$$A(x) = 11.9774x^2 - 15.4774x + 6, \quad 0 \le x \le 1$$

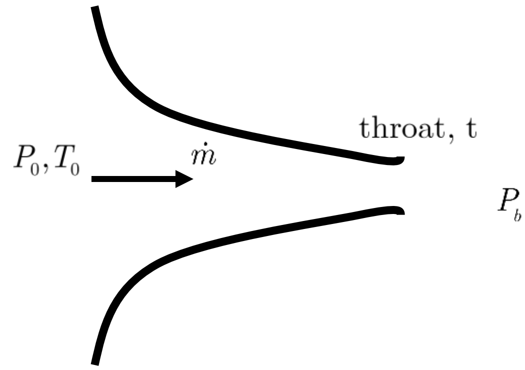
Plot the nozzle contour, find the limiting values of P_∞ / P_0, then write a computer program to calculate / plot the following, M versus x, (b) P / P_0 versus x, (c) Each of $\dot{m}, M_{\text{Inlet}}, M_{\text{Throat}}, M_{\text{Exit}}, P_{\text{Exit}} / P_0$ versus P_∞ / P_0.

[Use x=0, 1, Step 0.05, and P_∞ / P_0 = 1, 0.98, 0.97, 0.96085, 0.95, 0.9, 0.8, 0.7, 0.6, 0.5, 0.435, 0.4, 0.3, 0.2, 0.1, 0.04, 0]

[2] Quasi One-Dimensional Flow in a Convergent Duct

(a) Show that the mass flow-rate through a Laval nozzle maybe expressed as,

$$\dot{m} = \sqrt{\frac{\gamma}{R}} \frac{P_0}{\sqrt{T_0}} A \frac{M}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}$$



(b) A converging nozzle with a throat area $A_t = 0.0645\ \mathrm{m}^2$ is supplied with air at low velocity and at a pressure and temperature of 8 atm and 470 K respectively. Plot the mass flow-rate, $\dot{m}$, through the nozzle versus back pressure, P_b, assuming the flow to be isentropic.

[3] Loss Coefficients for Internal Flow Components (See Appendix B)

Obtain values for the loss coefficient, K_L, for each of the components and cases shown in Appendix B (consult various fluid mechanics references). Organize your data into tables, charts, and/or equations.

[4] Friction Factor (See Appendix C)

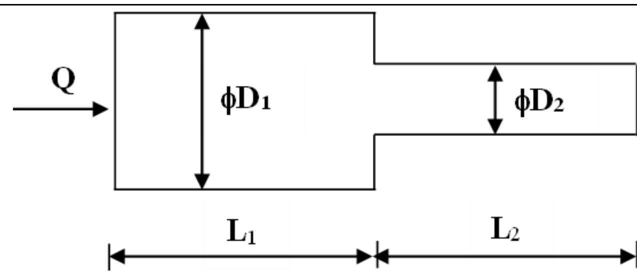
Collect various equations for the friction factor, f, for the flat plate and circular ducts.

[5] Pipes in Series

Calculate the pressure losses in the shown pipe-line, given:

$Q = 1.25 \times 10^{-4}\ \mathrm{m^3/s}$ $L_1 = L_2 = 4$ m $D_1 = 13$ mm $D_2 = 8$ mm

$\rho = 850\ \mathrm{kg/m^3}$ $\nu = 1.95 \times 10^{-5}\ \mathrm{m^2/s}$



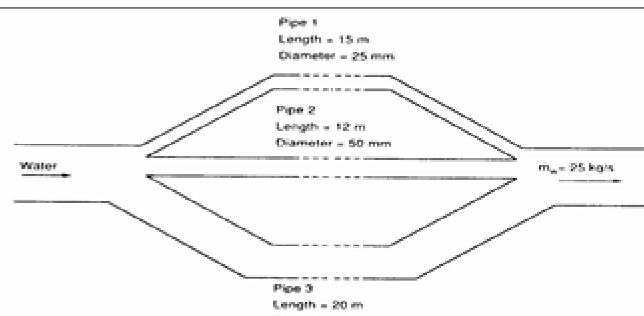
[6] Pipes in Parallel

A horizontal pipe carries water of density 1000 kg/m^3 at the rate of 25 kg/s. At a junction in the pipe, the flow is divided into three other pipes running in parallel, as shown in Fig 3. Determine,

(a) the head lost to friction

(b) the flow rate of water through each pipe in parallel, neglecting minor losses

NB: $f_1 = 0.005,\ f_2 = 0.0075,\ f_3 = 0.01$



- water flowing through three pipes in parallel.

3. Nozzles / Outlets (8)

Prandtl-Meyer Flow and 2-D Method of Characteristics

[1] Governing Equations and Waves (Choose the Correct Answer)

(a) Linearized (small perturbation) supersonic flow is governed by,
[i] the wave equation, $\left(M_\infty^2 - 1\right)\varphi_{xx} - \varphi_{yy} = 0$, [ii] Laplace equation, $\left(1 - M_\infty^2\right)\varphi_{xx} + \varphi_{yy} = 0$

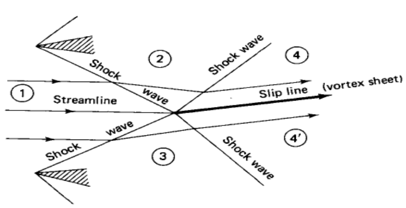
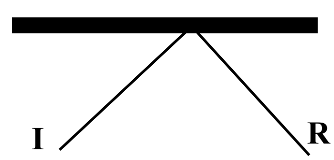
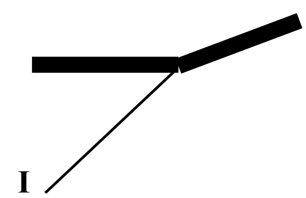
(b) A left running expansion wave,
[i] turns the flow in the c.w. θ direction, [ii] turns the flow in the c.c.w. θ direction

(c) A compression Mach wave,
[i] turns the flow into itself, [ii] turns the flow away from itself

(d) For the shown Oblique Shock Wave Intersection,
[i]the intersected waves are of the same family [ii] the intersected waves are of opposite family

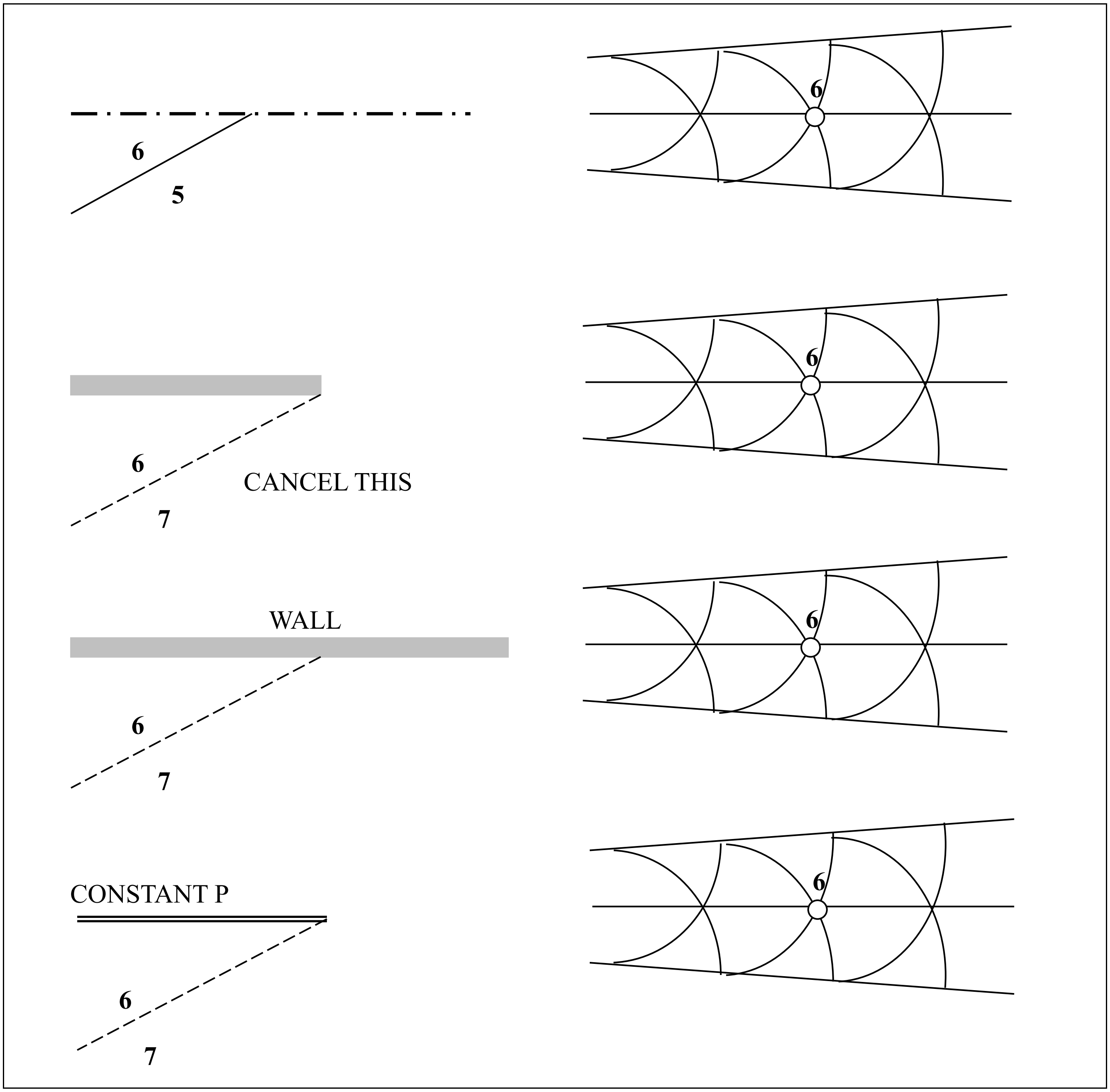
(e) For the Shown Oblique Shock Wave Reflection, given $M_1 = 2$, $\beta_{\mathrm{I}} = 40^{\circ}$, hence
[i] $\beta_{\mathrm{R}} \approx 42^{\circ}$ [ii] $\beta_{\mathrm{R}} \approx 39.7^{\circ}$

(f) For the shown Oblique Shock Wave Cancellation given, $M_1 = 3.5$, $\beta_{\mathrm{I}} = 45^{\circ}$, to cancel the OSW,
[i] $\theta \approx 28.2^{\circ}$ [ii] $\theta \approx 32.2^{\circ}$

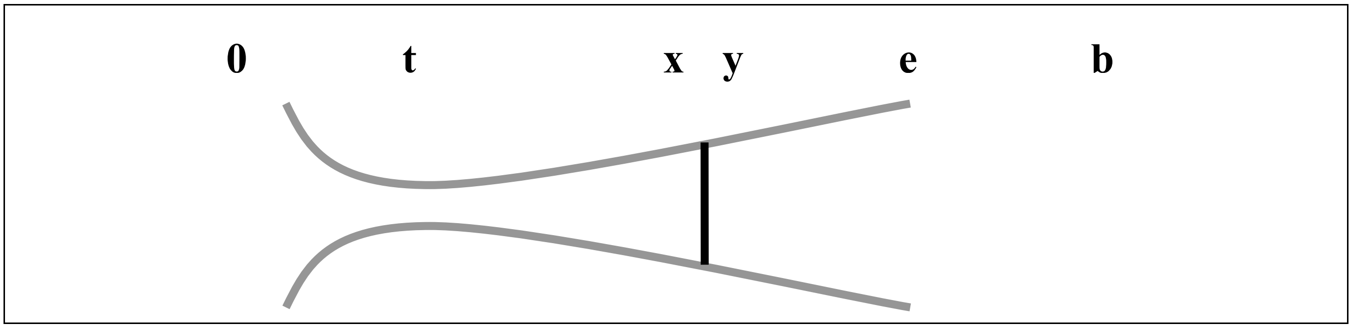
OSW Intersection	**OSW Reflection**	**OSW Cancellation**
		

[2] Wave Interaction in Physical and Hodograph Planes

Two-dimensional steady supersonic flow occurs in state 6 where $M_6 = 2.14$. For the shown four cases, indicate the states on the hodograph and complete the physical plane sketch. The Mach numbers at states 5 and 7 are $M_5 = 2.06$ and $M_7 = 2.21$.



[3] Standing Normal Shock in Divergent Part of Supersonic Nozzle (See Appendix D)



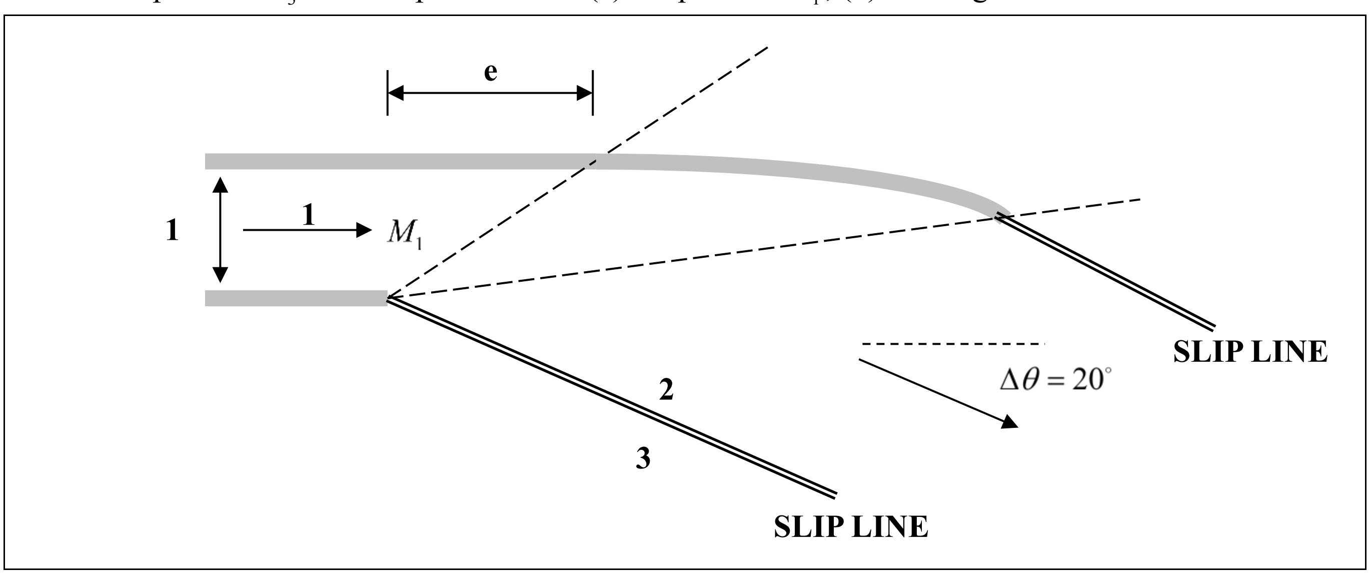
[A] Air is flowing in a converging / diverging nozzle with an area ratio of 10. The inlet stagnation-pressure $P = 7\times10^5$ N/m^2, and the pressure in the exhaust region is $P = 1\times10^5$ N/m^2. A normal shock wave is standing in the nozzle divergence. Determine the area ratio at which the normal shock wave occurs. Assume $\gamma = 1.4$.

[B] Consider a convergent – divergent duct with exit and throat areas of 0.5 m^2 and 0.25 m^2 respectively. The inlet reservoir pressure is 1 atm and the exit static pressure is 0.6 atm. For this pressure ratio, the flow will be supersonic in a portion of the nozzle, terminating with a normal shock inside the nozzle. Calculate the local area ratio $\left(A/A^*\right)$ at which the shock is located inside the nozzle.

[C] Air at a stagnation pressure of 7 bar and a stagnation temperature of 300 K expands through a frictionless convergent-divergent nozzle to the exhaust pressure of 5 bar. The expansion ratio of the nozzle is 2. Calculate:
(a) the exit velocity
(b) the exit Mach number

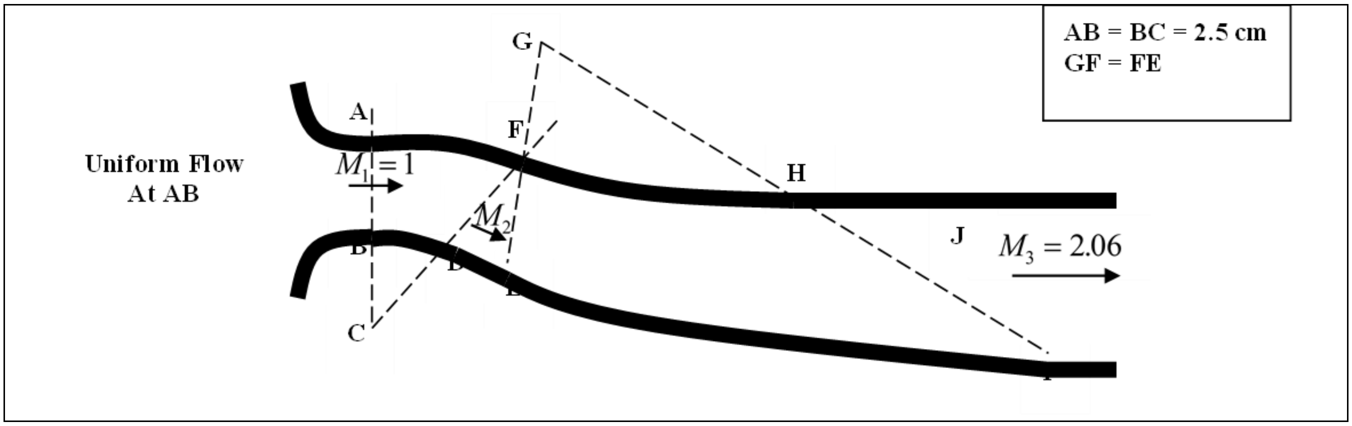
[4] Supersonic Transport Utilizing a Supersonic Nozzle

A supersonic transport utilizes a Prandtl-Meyer nozzle to turn the exit flow by 20° as shown in the sketch. Assume $\gamma = 1.4$ in the uniform flow at state 1 where $M_1 = 2$. At the start of take-off, the velocity of the atmosphere v_3 is zero and the pressure P_3 is atmospheric. Find: (a) the pressure P_1, (b) the length e



[5] Design of a Supersonic Nozzle

A nozzle is to be designed to deliver a parallel uniform stream of air at $M_3 = 2.06$. The general arrangement of the nozzle is as shown below.



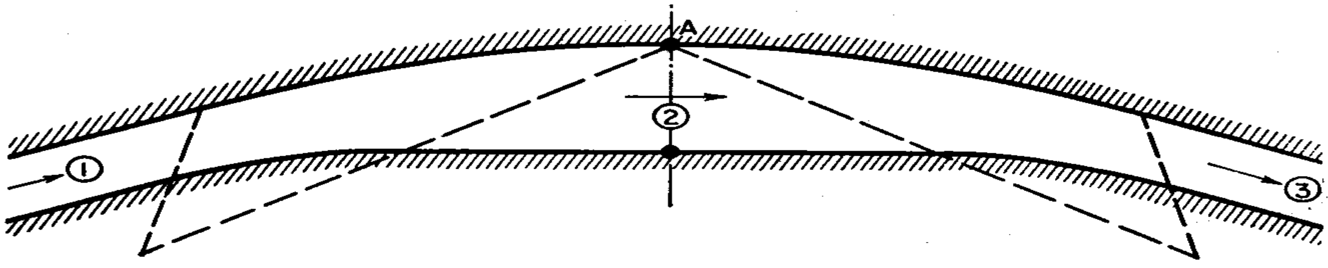
REGION	FLOW
AFDB	Expansive corner flow around corner C
FED	Uniform and parallel
FHIE	Expansive corner flow around corner G
J	Uniform, parallel, and has the same direction as that of the flow in the throat AB

(a) Find the lengths and inclinations to the horizontal of the following lines: FD, FE, DE, and HI.
(b) Using these principal dimensions and directions, make a sketch to scale of the nozzle.

[6] Design of a Supersonic Elbow

The shown two-dimensional supersonic elbow turns a supersonic stream through the angle $\theta_2 - \theta_1$ without any net change in Mach number. Two simple-wave turns are employed. In the first turn, left-running Mach waves <u>expand</u> the flow from 1 to 2, whereas in the second turn, right-running Mach waves <u>compress</u> the flow from 2 to 3. The waves are centred as shown and hence the streamlines are Prandtl-Meyer streamlines.

(a) Sketch the hodograph plane showing states 1, 2, and 3.
(b) Determine the main dimensions.

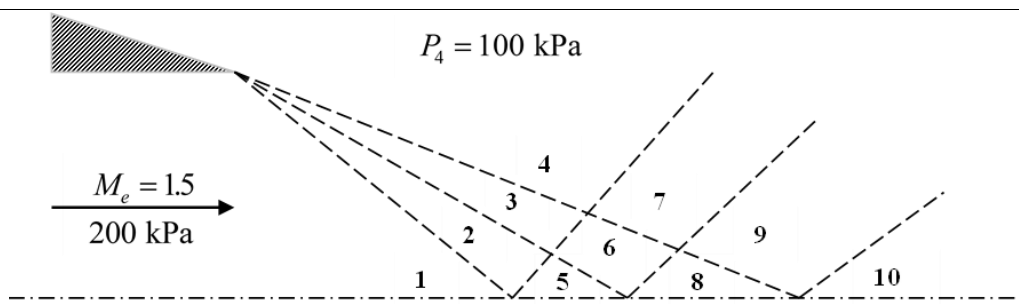


[7] Flow at Supersonic Nozzle Exit

(a) Plot a Prandtl-Meyer Streamline from sonic flow to M = 3 with r^* = any convenient length (eg 1 cm).

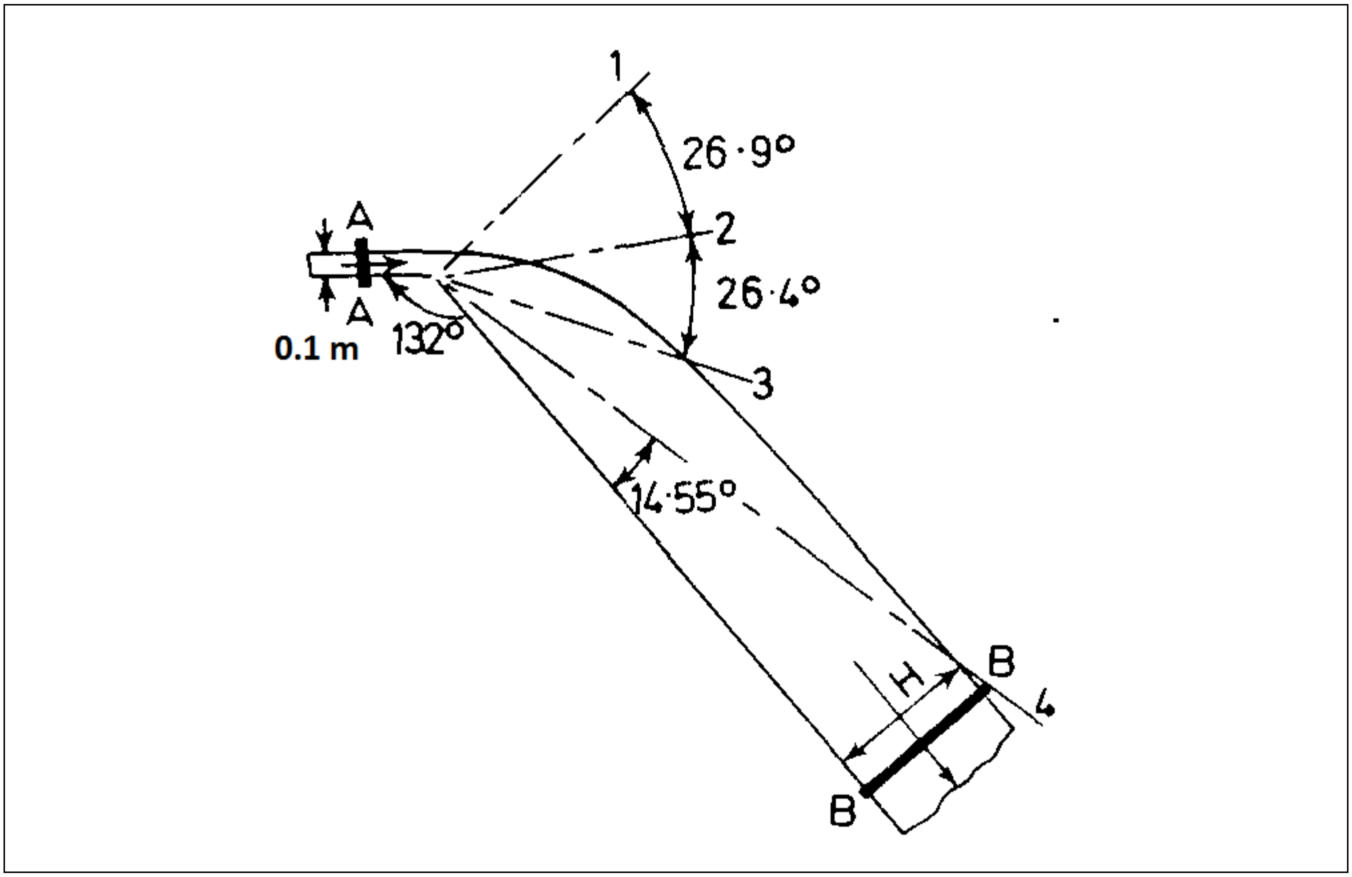
(b) Air-flow at the shown exit of a Mach 1.5 supersonic nozzle is <u>expanded</u> from an exit-plane pressure of 200 kPa to a back-pressure of 100 kPa. Determine the flow just downstream of the nozzle exit using the Region-to-Region method. Show your computations in the form of a table

REGION	θ	ν	M	μ	$\theta+\mu$	$\theta-\mu$
1	0		1.500			
2						
3						
4						
5	0					
6						
7						
8	0					
9						
10	0					



Exit Flow from a Supersonic Nozzle

[8] Flow in a Corner Duct



The figure shows the design of a duct corner round which is expanded a supersonic air flow. The duct is two-dimensional and the upper wall corresponds to a streamline of the flow which passes thru an expansion fan, centred at the corner O and bounded by Mach lines (0-1) and (0-4) as shown.

The duct corner is flanged at A and B at which section the uniform conditions of the flow are P_4=0.7X10^5 N/m^2 and temperature, T_4=290 K.

Determine,

(a) The included angle 1-0-4 of the expansion fan,

(b) The dimension H of the duct if the width is constant,

(c) The Mach number and the local speed of sound at the Mach line positions (0-2) and (0-3),

(d) The mass flow thru the duct (per unit width),

(e) The magnitude and direction of the force acting on the flanged section (per unit width).

4. Diffusers / Intakes (6)

[1] Shock Reflection (See Appendix E)

[2] Design of a Simple Two-Dimensional Supersonic Diffuser

Design a two-dimensional supersonic diffuser for air that uses oblique shocks to increase the pressure by a ratio of 1.69 when the approach Mach number is 2.0. Make a scale drawing or sketch of the diffuser.

[3] Diffuser with Two Oblique Shocks

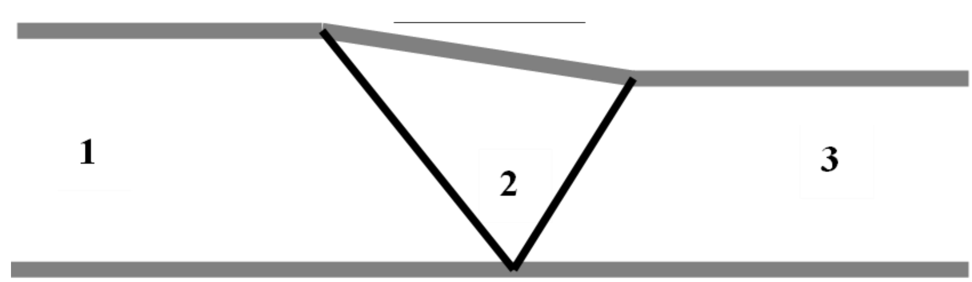
The shown oblique shock wave pattern was obtained from a shadowgraph (a special experimental technique) for the steady adiabatic air-flow through a frictionless, two-dimensional flow passage.

(a) What is the flow direction? Give reasons.

(b) Determine the Mach numbers in regions 1, 2, and 3.

(c) Determine the percentage of the loss in stagnation pressure from inlet to outlet.

(d) Determine the overall static pressure-ratio between outlet and inlet.



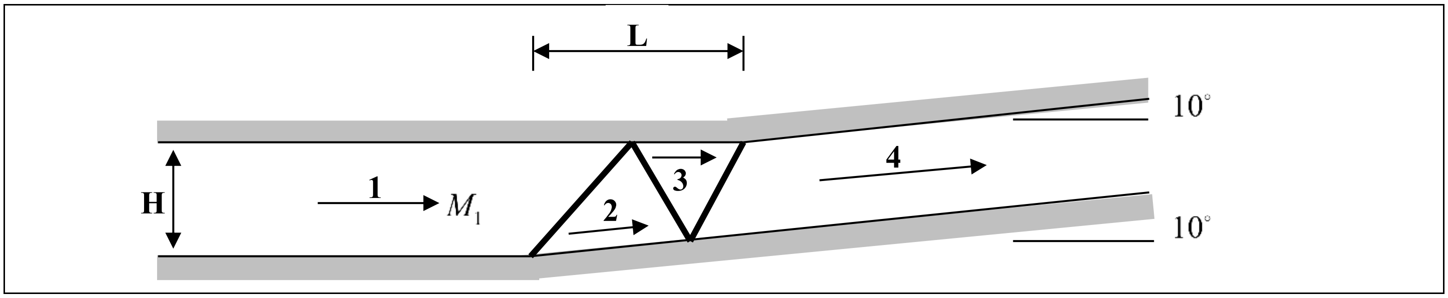
[4] Diffuser with Three Oblique Shocks (Shock Reflections)

Consider the shown inviscid two-dimensional airflow.

(a) Find the minimum Mach number, M_1, for this case to exist.

(b) Find M_2, M_3, M_4, and the ratios P_2/P_1, P_3/P_1, and P_4/P_1.

(c) Find the length L in terms of the height H corresponding to this M_1.



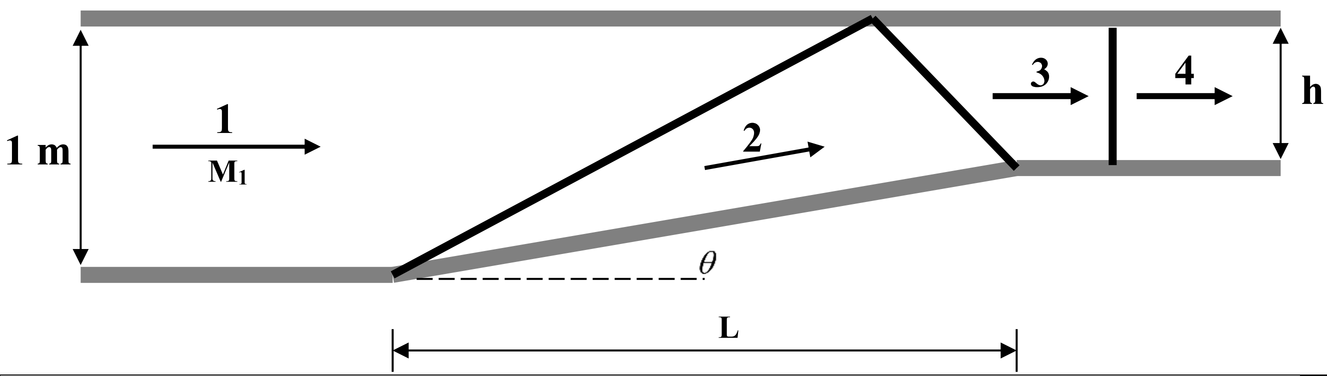
[5] Supersonic Shock Diffuser

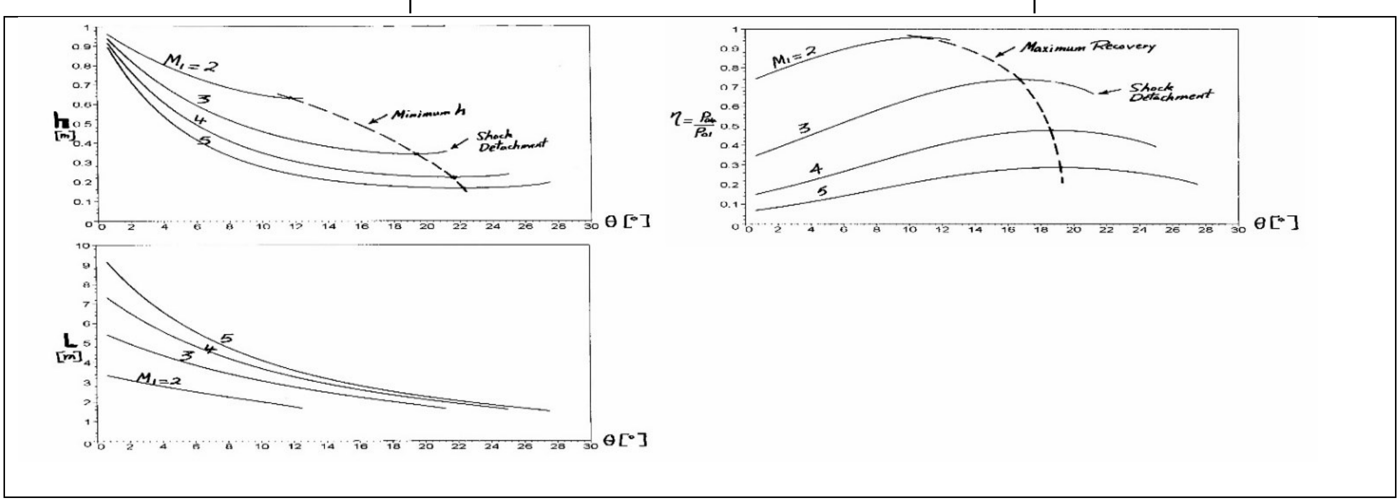
The shown supersonic shock diffuser is based on a method of reducing the Mach number in a channel. The diffuser is carefully designed such that the bend on the lower wall is oriented such that the reflected wave is exactly cancelled by the return bend. For the shown wave pattern, sketch the Hodograph plane showing various states and processes.

(a) For an inlet Mach number M_1=4, find h, L, η for $\theta = 10$ deg.

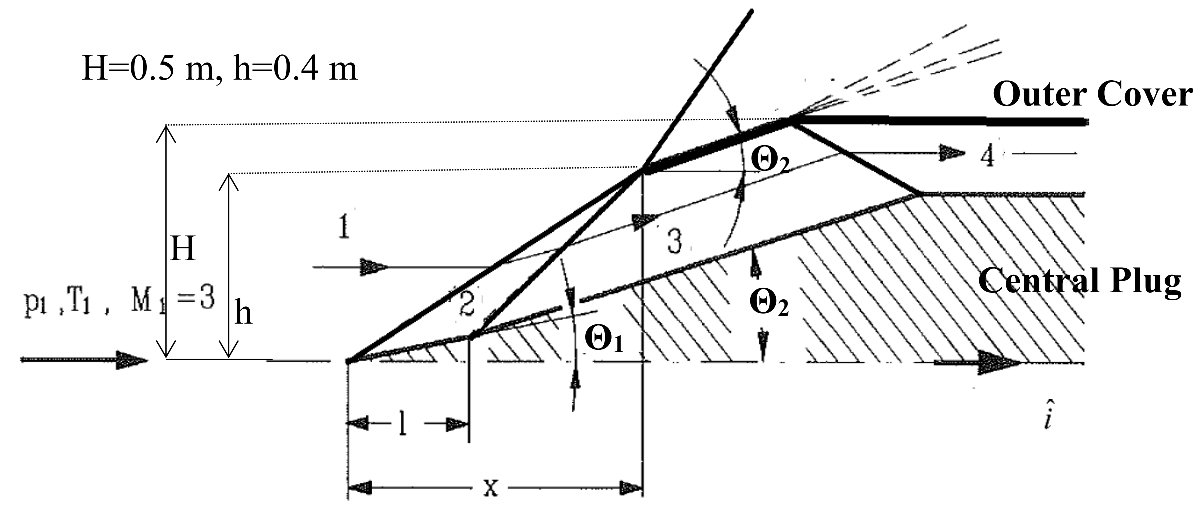
(b) For an inlet M_1=3, design the diffuser for max recovery (min shock losses) i.e. find η, θ, h, L.

(c) For an inlet M_1=3, design the diffuser for an installation restriction on length equal to $L = 2$ m i.e. find η, θ, h.





[6] Analysis of Supersonic Intake



The shown two-dimensional plane Inlet consists of a Central Plug and an Outer Cover. Only the upper half of the Inlet is shown. The Inlet is designed so that at $M_1 = 3$ the shocks from the Central Plug intersect at the tip of the Cover and are not reflected. The Central Plug is a wedge of half angle $\theta_1 = 10^\circ$. At a distance l from the tip of the wedge, the half angle suddenly changes to $\theta_2 = 15^\circ$. The Inlet Cover is infinitely thin and forms an angle θ_2 (equal to the second wedge angle). The approach flow is air with $P_1 = 1$ bar and $T_1 = 273$ K.

(a) Find the distance x from the tip of the wedge to the tip of the cover, such that the first shock just meets the edge of the cover.

(b) Find the distance l from the tip of the wedge to the beginning of the second wedge, such that also the second shock meets with the edge of the cover.

(c) Find the mass flux $\dot{m}$ into the inlet.

(d) Find the pressures P_2, P_3, and P_4.

(e) Compute the force in the $\hat{i}$ direction on the forward tilted part of the cover.

5. Practical Aspects (3)

[1] Novel Designs of Nozzles and Intakes

For each of the following innovative designs for nozzles and intakes, prepare a short presentation to illustrate the design concept.

#	Innovative Design for Nozzles and Intakes	Description
1	**Ejector Air-Intake**	
2	**Oblique Shock Diffuser**	
3	**Scramjet Intake System**	
4	**Variable Ejector Nozzles**	
5	**Iris Nozzles**	

[2] Inlets and Outlets for Various Engines

(a) Describe each of the following Inlets

#	Inlet	Description
1	**Air-Intake Manifold**	
2	**Turbocharger Inlet**	
3	**Supercharger Inlet**	
4	**Fuel Injector**	
5	**Carburetor**	

(b) Describe each of the following Outlets

#	Outlet	Description
1	**Exhaust Manifold**	
2	**Turbocharger Outlet**	
3	**Catalytic Converter**	
4	**Muffler**	
5	**Exhaust Pipe**	

[3] Boundary Layer Control

Appendix A Fundamental Gas Dynamics Procedures

Note: NACA-1135 Report should be consulted for various equations used in the following procedures.

#	PROCEDURE	CALL AND ARGUMENTS	N	v / v1	v2	RETURN VARIABLES LIST
1	**Isentropic Process**	**ISN(N,v)**	1	M		[M, PPT, MS, C, TTT, RRT, AAS]
			2	PPT		
			3	MS		
			4	C		
			5	TTT		
			6	RRT		
			7	AAS(SUB)		
			8	AAS(SUP)		
2	**Normal Shock Process**	**NSW(N,v)**	1	M1		[M1, P2P1, M2, V1V2, PT2PT1, P1PT2, T2T1] NB M1>1, M2<1, V1V2=R2R1, T2T1=A2A1^2
			2	P2P1		
			3	M2		
			4	PT2PT1		
3	**Oblique Shock Process**	**OSW(N,v1,v2)**	1	M1	DL	[M1, P2P1, M2, DL, PT2PT1, P1PT2, T2T1, R2R1, TH, DLMAX, THMAX] NB M1>1, M2>1, Angles IN/OUT in radians
			2	M1	P2P1	
			3	M1	M2	
			4	M2	DL	
			5	M2	P2P1	
			6	P2P1	DL	
			7	M1	TH	
			8	M2	TH	
			9	DL	TH	
			10	P2P1	TH	
			11	P2P1	---	
4	**Prandtl Meyer Angle**	**PMA(N,v)**	1	M		[M, NU]
			2	NU		
5	**Prandtl Meyer Fan**	**PMF(N,v1,v2)**	1	M1, DL		[M1, P2P1, M2, DL, NU1, NU2, AL1, AL2]
			2	M1, P2P1		
			3	M1, M2		
			4	M2, DL		
			5	M2, P2P1		
			6	P2P1, DL		

NB: N determines the case, whereas v, v1, v2 are arbitrary variables.

Isentropic Process Procedure Demonstration

[1] Simplified Governing Equations

$$\frac{T}{T_0}=\left(\frac{P}{P_0}\right)^{\frac{1-\gamma}{\gamma}}=\left(\frac{\rho}{\rho_0}\right)^{1-\gamma}=\begin{Bmatrix}\left(1+\frac{\gamma-1}{2}M^2\right)^{-1}\\ 1-\frac{\gamma-1}{\gamma+1}M^{*2}\\ 1-C^2\end{Bmatrix},\quad \frac{A}{A^*}=\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}\frac{1}{M}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} \quad (1)$$

NB1: These are derived from the steady adiabatic energy and quasi one-dimensional flow relations, resp.
NB2: These equations are algebraic and nonlinear.
NB2: In the above equations, $\gamma \equiv$ a parameter $\left(\gamma = 7/5 \text{ for air}\right)$, and the 2's, 1's are constants.
NB3: Knowing any variable, can obtain a value for each of the other variables (and two values for A/A^*).

[2] Problem variables are $M, M^*, C, \frac{P_0}{P}, \frac{\rho_0}{\rho}, \frac{T_0}{T}, \frac{A}{A^*}$. Each of these variables is real with a mathematical domain $\{\text{Var} \mid -\infty \le \text{Var} \le +\infty\} \equiv (-\infty, +\infty)$. From physics, the domains are restricted as follows,

#	Variable	Name	Physical Domain
1	$M=\frac{\text{v}}{a}$	Velocity Ratio	$\{M \mid 0 \le M < \infty\} \equiv [0,\infty)$
2	$M^*=\frac{\text{v}}{a^*}$	Velocity Ratio	$\left\{M^* \mid 0 \le M^* < \sqrt{\frac{\gamma+1}{\gamma-1}}\right\} \equiv \left[0, \sqrt{\frac{\gamma+1}{\gamma-1}}\right)$
3	$C=\frac{\text{v}}{\text{v}_{\text{max}}}$	Velocity Ratio	$\{C \mid 0 \le C < 1\} \equiv [0,1)$
4	$\frac{P}{P_0}$	Pressure Ratio	$\left\{\frac{P}{P_0} \mid 0 < \frac{P}{P_0} \le 1\right\} \equiv (0,1]$
5	$\frac{\rho}{\rho_0}$	Density Ratio	$\left\{\frac{\rho}{\rho_0} \mid 0 < \frac{\rho}{\rho_0} \le 1\right\} \equiv (0,1]$
6	$\frac{T}{T_0}$	Temperature Ratio	$\left\{\frac{T}{T_0} \mid 0 < \frac{T}{T_0} \le 1\right\} \equiv (0,1]$
7	$\frac{A}{A^*}$	Area Ratio	$\left\{\frac{A}{A^*} \mid 1 \le \frac{A}{A^*} < \infty\right\} \equiv [1,\infty)$

[3] Organized Equations with Independent Variable as the Mach number (NTRS: Report 1135)

$$M^{*2}=\frac{\frac{\gamma+1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\ (2),\quad C^2=\frac{\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\ (3),$$

$$\frac{P}{P_0}=\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{1-\gamma}}\ (4),\ \frac{\rho}{\rho_0}=\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{1-\gamma}}\ (5),\ \frac{T}{T_0}=1+\frac{\gamma-1}{2}M^2\ (6), \text{ and}$$

$$\frac{A}{A^*}=\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}\frac{1}{M}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}\ (7)$$

Maple Procedure (Two procedures are given to illustrate programming alternatives)

Given M, it is straight forward to obtain all variables using equations (2) to (7).

Given any one of the other 6 variables, first obtain M from appropriate equation, then the other variables.

```
IsentropicProcess.mws
> restart: with(RealDomain): interface(displayprecision=4):
> Isentropic1 := proc(N,v) local K,PTR,KM1,KP1,GO, M,MS,C,PPT,TTT,RRT,AAS,eq,o ;
   K := G() : KM1   := K - 1  : KP1  := K + 1                            : GO := 1 :
   if     N = 1 then M     := v
   elif   N = 2 then PPT   := v : M := sqrt( 2   *  (PPT^(-KM1/K)-1 )   / KM1 )     :
   elif   N = 3 then MS    := v : M := sqrt( 1   /  (KP1/(2 *MS^2)-KM1/2 )    )     :
   elif   N = 4 then C     := v : M := sqrt( 2   /  (KM1*(1 /(C^2)-1 ))         )     :
   elif   N = 5 then TTT   := v : M := sqrt( 2   *  (1/TTT-1 )            / KM1 )     :
   elif   N = 6 then RRT   := v : M := sqrt( 2   *  (1/RRT^KM1-1 )        / KM1 )     :
   elif   N = 7 then AAS   := v : eq:= AAS-(2*(1+KM1*M^2/2)/KP1)^(KP1/(2*KM1))/M:
                     M     := fsolve(eq,M,0..1)                              : GO := 2 :
   elif   N = 8 then AAS   := v : eq:= AAS-(2*(1+KM1*M^2/2)/KP1)^(KP1/(2*KM1))/M:
                     M     := fsolve(eq,M,1..infinity)                       : GO := 2 :
   fi:if GO<>2 then AAS := (2 *(1 +KM1*M^2/2 )/KP1)^(KP1/(2 *KM1))/M fi             :
   PTR:=(1+KM1*M^2/2) : PPT:=PTR^(-K/KM1) : TTT:=PTR^(-1) : RRT:=PTR^(-1/KM1)        :
   if N <> 3  then MS := sqrt( KP1 / (2 /(    M^2)+KM1) )                      fi :
   if N <> 4  then C  := sqrt(  1  / (2 /(KM1*M^2)+ 1 ) )                      fi :
   o := evalf([M,1./PPT,MS,C,1./TTT,1./RRT,AAS])
  end :
> irp := proc(N, v) local f;
         f:=Isentropic1(N,v) : printf(cat(`% 9.4f `$7,`\n`),f[ii]$ii=1..7) end :
> G :=()-> 1.4:
> for mi from .1 to .5 by .1 do irp(1,mi) od :
   0.1000    1.0070    0.1094    0.0447    1.0020    1.0050    5.8218
   0.2000    1.0283    0.2182    0.0891    1.0080    1.0201    2.9635
   0.3000    1.0644    0.3257    0.1330    1.0180    1.0456    2.0351
   0.4000    1.1166    0.4313    0.1761    1.0320    1.0819    1.5901
   0.5000    1.1862    0.5345    0.2182    1.0500    1.1297    1.3398
> K := 7/5:
> Isentropic2 := proc(Given) local vL,vR,Eq,M2v,EqsVal,lvar;           global K;
   vL   := lhs(Given)     : vR  := rhs(Given)                                     :
   Eq   := table([ (M2 ) = M2                                                     ,
                   (MS2) = (K+1) / (2 /(      M2)+(K-1))                          ,
                   (C2 ) =     1  / (2 /((K-1)*M2)+ 1    )                        ,
                   (PP0) = (1+(K-1)*M2/2)^(-K/(K-1))                              ,
                   (RR0) = (1+(K-1)*M2/2)^(-1/(K-1))                              ,
                   (TT0) = (1+(K-1)*M2/2)^(-1)                                    ,
            (AAS) = (2 *(1 +(K-1)*M2/2 )/(K+1))^((K+1)/(2 *(K-1)))/sqrt(M2)    ]):
   if   vL=M      then M2v:=  solve( M2 -Eq[M2 ], M2 ): M2v:=eval(M2v,M2 =vR^2):
   elif vL=MS     then M2v:=  solve( MS2-Eq[MS2], M2 ): M2v:=eval(M2v,MS2=vR^2):
   elif vL=C      then M2v:=  solve( C2 -Eq[C2 ], M2 ): M2v:=eval(M2v,C2 =vR^2):
   elif vL=PP0    then M2v:=  solve( PP0-Eq[PP0], M2 ): M2v:=eval(M2v,PP0=vR  ):
   elif vL=RR0    then M2v:=  solve( RR0-Eq[RR0], M2 ): M2v:=eval(M2v,RR0=vR  ):
   elif vL=TT0    then M2v:=  solve( TT0-Eq[TT0], M2 ): M2v:=eval(M2v,TT0=vR  ):
   elif vL=AASsub then M2v:= fsolve( vR -Eq[AAS], M2=0..1                     ):
   elif vL=AASsup then M2v:= fsolve( vR -Eq[AAS], M2=1..infinity              ):
   fi:
   EqsVal      := eval(Eq, M2 = M2v)          : EqsVal[M]   := sqrt(EqsVal[M2])   :
   EqsVal[MS]  := sqrt(EqsVal[MS2])           : EqsVal[C]   := sqrt(EqsVal[C2])   :
   lvar := [M, MS, C, PP0, RR0, TT0, AAS]    :
   [seq(lvar[i]=EqsVal[lvar[i]], i=1..nops(lvar))]
  end:
> Egs:=[M=0.5, MS=0.53452, C=0.2182, PP0=0.84302, RR0=0.88517, TT0=0.95238,
                                                   AASsub=1.33985, AASsup=1.33985]:
  for i from 1 to nops(Egs) do print(i,Isentropic2(Egs[i]))                    od:
```

1, [$M=0.5000$, $MS=0.5345$, $C=0.2182$, $PP0=0.8430$, $RR0=0.8852$, $TT0=0.9524$, $AAS=1.3398$]

2, [$M=0.5000$, $MS=0.5345$, $C=0.2182$, $PP0=0.8430$, $RR0=0.8852$, $TT0=0.9524$, $AAS=1.3398$]

3, [$M=0.5000$, $MS=0.5345$, $C=0.2182$, $PP0=0.8430$, $RR0=0.8852$, $TT0=0.9524$, $AAS=1.3399$]

4, [$M=0.5000$, $MS=0.5345$, $C=0.2182$, $PP0=0.8430$, $RR0=0.8852$, $TT0=0.9524$, $AAS=1.3398$]

5, [$M=0.5000$, $MS=0.5345$, $C=0.2182$, $PP0=0.8430$, $RR0=0.8852$, $TT0=0.9524$, $AAS=1.3398$]

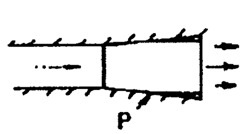
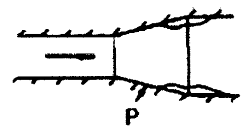
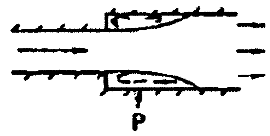
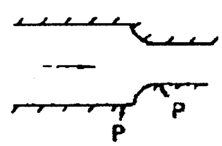
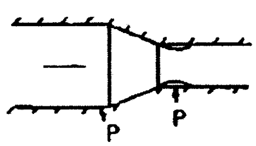
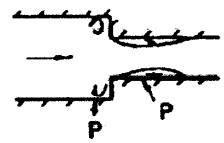
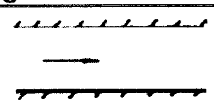
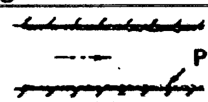
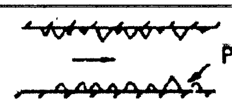
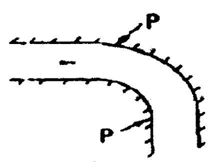
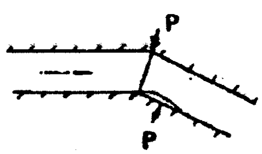
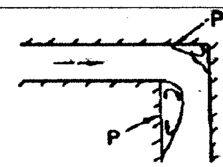
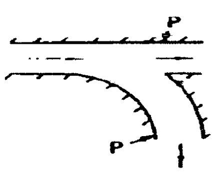
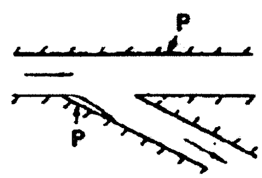
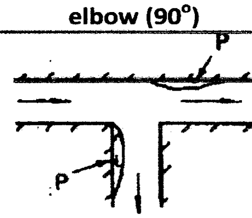
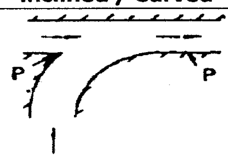
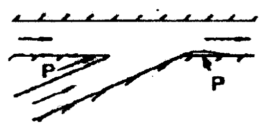
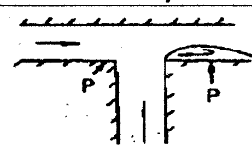
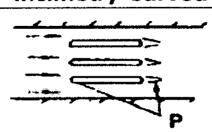
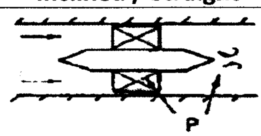
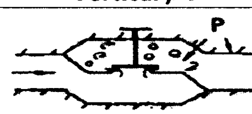
6, [$M=0.5000$, $MS=0.5345$, $C=0.2182$, $PP0=0.8430$, $RR0=0.8852$, $TT0=0.9524$, $AAS=1.3398$]

7, [$M=0.5000$, $MS=0.5345$, $C=0.2182$, $PP0=0.8430$, $RR0=0.8852$, $TT0=0.9524$, $AAS=1.3399$]

8, [$M=1.7024$, $MS=1.4838$, $C=0.6058$, $PP0=0.2019$, $RR0=0.3189$, $TT0=0.6331$, $AAS=1.3398$]

Appendix B Loss Coefficients for Internal Flow Components

[B] Adverse Gradients in Internal Flow Components for Incompressible/Subsonic Inlet Flow
Ref: Internal Flow Systems, Miller pp20

#	Attribute	Flow Process	[A] Low Loss Compts Small Sep Areas	[B] Moderate Loss Compts Moderate Sep Areas	[C] High Loss Compts Large Sep Areas
1	Area Change	Diverging Decelerating (Diffuser)	 gradual enlargement	 gradual enlargement	 sudden enlargement
2		Converging Accelerating (Nozzle)	 gradual contraction	 gradual contraction	 sudden contraction
3	Curvature	Non-Curved Straight	 smooth	 moderate roughness (trans)	 rough
4		Curved Turning	 bend (90° circular)	 elbow (angle)	 elbow (90°)
5	Flow Rate Change	Dividing/ Splitting	 Inclined / Curved	 Inclined / Straight	 Vertical / T
6		Adding/ Combining	 Inclined / Curved	 Inclined / Straight	 Vertical / T
7		Obstructions (Dividing-Combining)			

3

Notes on Major and Minor Losses in a Piping System

1. Major losses in a piping system are the direct result of fluid friction in pipes and ducting. The resulting head losses are usually computed thru the use of friction factors.

2. Minor losses in a pipe system are due to
   (a) Pipe entrance or exit
   (b) Sudden expansion or contraction
   (c) Bends, elbows, tees, and other fittings
   (d) Valves; open or partially closed
   (e) Gradual expansion or contraction

   These losses may not be so minor (for example, a partially closed valve can cause a greater pressure drop than a long pipe). Since the flow pattern in fittings and valves is very complex, the theory is very weak. The losses are commonly measured experimentally and correlated with pipe flow parameters. The loss coefficient, K, is usually given as a ratio of the measured minor head loss thru the device, $h_m = \Delta P/(\rho g)$, to the velocity head, $\mathrm{v}^2/(2g)$ of the associated piping system, i.e.

   $$K = h_m / \left[\mathrm{v}^2/(2g)\right] = \Delta P / \left(\tfrac{1}{2}\rho \mathrm{v}^2\right)$$ (almost all data are reported for turbulent flow conditions)

   A single pipe system may have many minor losses. Since all are correlated with $\mathrm{v}^2/(2g)$, they can be summed into a single total system loss if the pipe has constant diameter i.e.,

   $$\Delta h_{\text{total}} = h_f + \sum h_m = \frac{\mathrm{v}^2}{2g}\left(\frac{fL}{D} + \sum K\right)$$ where L is the total length of pipe axis

   Note, however, that we must sum the losses separately if the pipe size changes so that v^2 changes
   (see White 6th Ed pp 383)

Appendix C Friction Factors

1. Two widely adopted definitions of the friction factor are;

[A] Fanning Friction Factor	**[B] Darcy Friction Factor**
$f_{\mathrm{F}} = \dfrac{\overline{\tau}}{\frac{1}{2}\rho \mathrm{v}^2}$	$f_{\mathrm{D}} = -\dfrac{dP}{dx}\dfrac{D}{\frac{1}{2}\rho \mathrm{v}^2}$

2. For a circular pipe, $\overline{\tau} = -\dfrac{A}{P}\dfrac{dP}{dx} = -\dfrac{D_h}{4}\dfrac{dP}{dx}$ where D_h is the hydraulic diameter used for non-circular ducts or channels and $D_h = 4A_c / P$ (A_c is the cross sectional area and P is the wetted perimeter of the non-circular duct). For a circular duct;
   (a) Fully Developed Laminar Flow, $\mathrm{Re}_{D_h} < 2300$, $f = \dfrac{16}{\mathrm{Re}_{D_h}}$

   For non-circular ducts, $f = \dfrac{c}{\mathrm{Re}_{D_h}}$ (see pp49 in one handout) i.e. $f_D = 4f_F$ and $h_f = 4f_F\dfrac{L}{D_h}\dfrac{\mathrm{v}^2}{2g}$

   (b) Developing Laminar Flow, $\mathrm{Re}_{D_h} < 2300$ and $L < L_e$, see eqs for f

   (c) Fully Developed Turbulent Flow, $\mathrm{Re}_{D_h} > 4000$, see Moody chart etc

Appendix D Balance Across a Standing Normal Shock

$$\frac{P_{0y}}{P_{0x}} = \frac{A_x^*}{A_y^*} = \frac{A_t}{A_e^*} = \frac{A_e}{A_e^*} / \frac{A_e}{A_t} \qquad (1)$$

where

$$\frac{A_e}{A_e^*} = \frac{1}{M_e}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M_e^2\right]^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}} \qquad (2)$$

$$M_e = \left\{\frac{2}{\gamma-1}\left[\left(\frac{P_e}{P_{0e}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]\right\}^{\frac{1}{2}} = \left\{\frac{2}{\gamma-1}\left[\left(\frac{P_e/P_0}{P_{0y}/P_{0x}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]\right\}^{\frac{1}{2}} \qquad (3)$$

Substituting from (3) in (2),

$$\frac{A_e}{A_e^*} = \frac{\left\{\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}\left(\frac{2}{\gamma-1}\right)\left[\left(\frac{P_e/P_0}{P_{0y}/P_{0x}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]\right\}^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}{\left\{\frac{2}{\gamma-1}\left[\left(\frac{P_e/P_0}{P_{0y}/P_{0x}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]\right\}^{\frac{1}{2}}} = \frac{\left\{\frac{2}{\gamma+1}\left(\frac{P_{0y}/P_{0x}}{P_e/P_0}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right\}^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}{\left\{\frac{2}{\gamma-1}\left[\left(\frac{P_{0y}/P_{0x}}{P_e/P_0}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]\right\}^{\frac{1}{2}}} \qquad (4)$$

From (4) in (1) and divide by P_e/P_0,

$$\frac{P_{0y}/P_{0x}}{P_e/P_0} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}\left(\frac{P_{0y}/P_{0x}}{P_e/P_0}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma}\right)}}{\left\{\frac{2}{\gamma-1}\left[\left(\frac{P_{0y}/P_{0x}}{P_e/P_0}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]\right\}^{\frac{1}{2}}} / \frac{A_e}{A_t} / \frac{P_e}{P_0} \qquad (5)$$

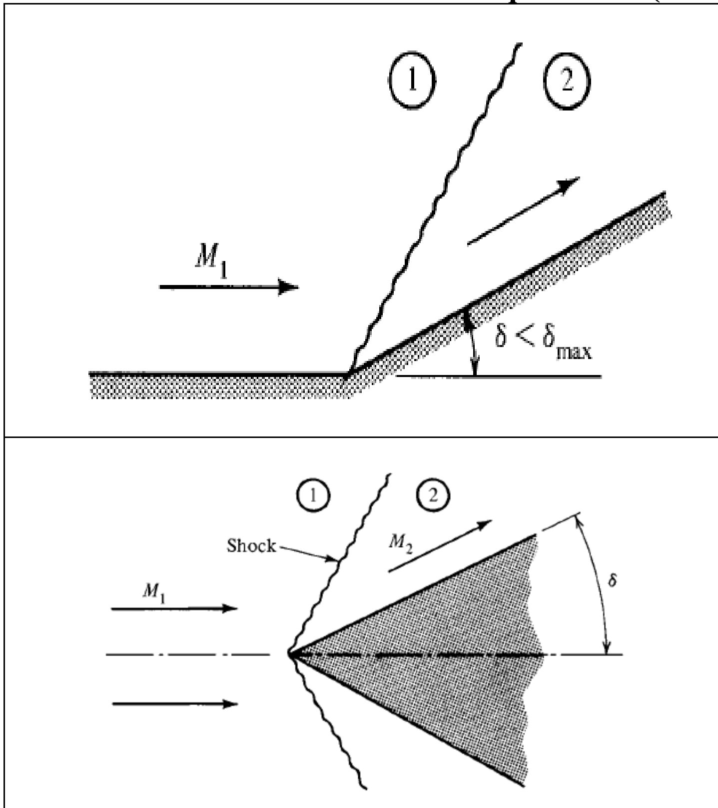
Let $x = \dfrac{P_{0y}/P_{0x}}{P_e/P_0}$, $x^2 = \left(\dfrac{A_t}{A_e}\dfrac{P_0}{P_e}\right)^2\left[\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} x^{\frac{\gamma+1}{\gamma}}\right] / \left[\dfrac{2}{\gamma-1}\left(x^{\frac{\gamma-1}{\gamma}} - 1\right)\right]$, $\dfrac{2}{\gamma-1}\left(x^{\frac{\gamma-1}{\gamma}} - 1\right) = \left(\dfrac{A_t}{A_e}\dfrac{P_0}{P_e}\right)^2\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} x^{-\frac{\gamma-1}{\gamma}}$

Let $y = x^{\left(\frac{\gamma-1}{\gamma}\right)}$, then $y^2 - y = \dfrac{\gamma-1}{2}\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\dfrac{A_t}{A_e}\dfrac{P_0}{P_e}\right)^2 = \alpha \therefore y = \dfrac{1 \pm \sqrt{1+4\alpha}}{2}$

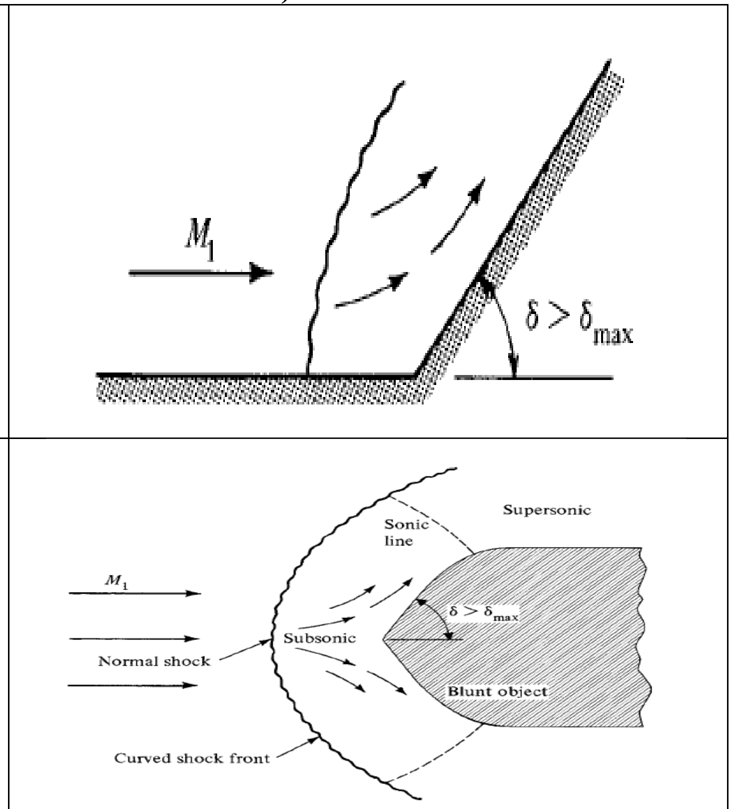
Finally, $\boxed{\dfrac{P_{0y}}{P_{0x}} = \dfrac{P_e}{P_0}\left[\dfrac{1+\sqrt{1+4\alpha}}{2}\right]^{\frac{\gamma}{\gamma-1}} \quad \text{where } \alpha = \dfrac{\gamma-1}{2}\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\dfrac{P_e}{P_0}\dfrac{A_e}{A_t}\right)^{-2}}$

Appendix E Oblique Shock Wave Attachment and Reflection

Oblique Shock (Attachment/Detachment)

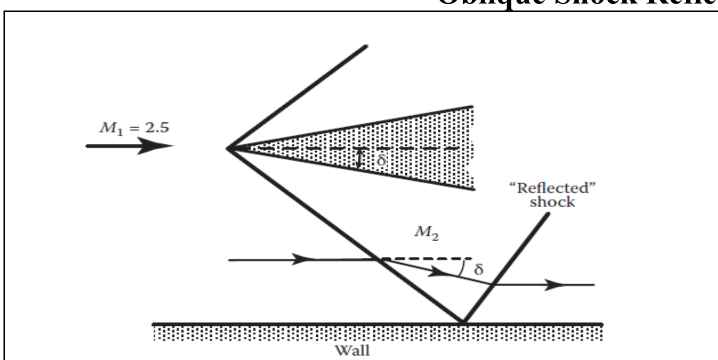


Attached Oblique Shock

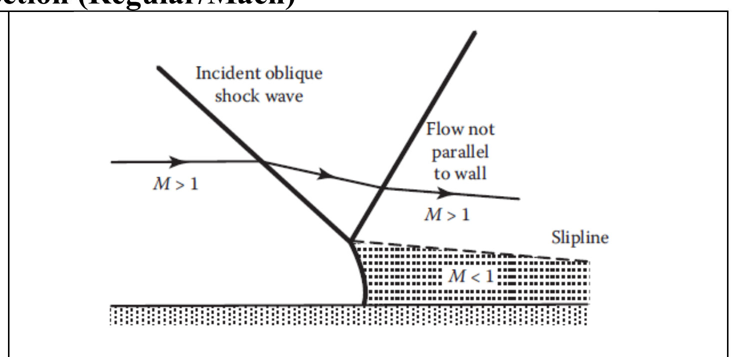


Dettached Oblique Shock

Oblique Shock Reflection (Regular/Mach)



Regular Reflection



Mach Reflection

NB1: In the above $\delta \equiv$ Flow Deflection Angle (Sometimes θ is used for flow deflection angle)

NB2: Have an Oblique Shock Detachment in the following cases;

(a) For given M_1, $\delta > \delta_{max}$, or

(b) For given δ, $M_1 < M_{1min}$