Name:

Date:

Week

Homework

7. **The Fundamental Theorem of Calculus does not apply to the integral $\int_0^5 (x-3)^{-4/3}\,dx$; why not? What would the Fundamental Theorem of Calculus calculate for this integral if we applied it anyway? Would it give the correct answer?**

The Fundamental Theorem of Calculus does not apply to $(x-3)^{-4/3}$ because it has an infinite discontinuity at $x=3$, which is in the range of integration.

FTC: $\int_0^5 (x-3)^{-4/3}\,dx = \left[-3(x-3)^{-1/3}\right]_0^5 = -3(2^{-1/3} - 3^{1/3})$

This doesn't give a correct answer because $x-3$ is x shifted 3 units to the right.

25. $\int_0^1 x^{-0.99}\,dx = \lim_{c\to 0^+} \int_c^1 x^{-0.99}\,dx = \lim_{c\to 0^+} \left[100x^{0.01}\right]_c^1 = \lim_{c\to 0^+} (100 - 100c^{0.01}) = 100$

27. $\int_1^\infty x^{-0.99}\,dx = \lim_{c\to\infty} \int_1^c x^{-0.99}\,dx = \lim_{c\to\infty} \left[100x^{0.01}\right]_1^c = \lim_{c\to\infty} (100c^{0.01} - 100) = \infty$

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39. $\int_0^{\infty} \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx$ $\quad u=\sqrt{x}$, $2du = x^{-1/2}dx$

$\int x^{-1/2}e^{-\sqrt{x}}dx = 2\int e^{-u}du = -2e^{-u} = -2e^{-\sqrt{x}}$

$= \lim_{c\to 0^+}\int_c^1 \frac{1}{\sqrt{x}e^{\sqrt{x}}}dx + \lim_{c\to\infty}\int_1^c \frac{1}{\sqrt{x}e^{\sqrt{x}}}dx$

$= \lim_{c\to 0^+}\left[-2e^{-\sqrt{x}}\right]_c^1 + \lim_{c\to\infty}\left[-2e^{-\sqrt{x}}\right]_1^c$

$= \lim_{c\to 0^+}(-2e^{-1}+2e^{-\sqrt{c}}) + \lim_{c\to\infty}(-2e^{-\sqrt{c}}+2e^{-1})$

$= -2e^{-1}+2+0+2e^{-1} = 2$

53. $\int_1^2 \frac{1}{x(x-1)}\,dx$ $\quad \frac{1}{x(x-1)} = \frac{A}{x}+\frac{B}{x-1} = \frac{(A+B)x-A}{x(x-1)}$ $\quad A=-1$, $B=1$

$\int\left(\frac{1}{x-1}-\frac{1}{x}\right)dx = \ln|x-1| - \ln|x| = \ln\left|\frac{x-1}{x}\right|$

$= \lim_{c\to 1^+}\int_1^2 \frac{1}{x(x-1)}dx$

$= \lim_{c\to 1^+}\left[\ln\left|\frac{x-1}{x}\right|\right]_c^2$

$= \ln\frac{1}{2} - \lim_{c\to 1^+}\ln\left|\frac{c-1}{c}\right| = \infty$

Optional:

73. Frank is evaluating electric motors to drive automated mixing for some waste tanks that he must maintain. One pump is advertised to have a probability of failure that follows the exponential distribution

$$f(t) = 0.31e^{-0.31t},$$

where the time $t > 0$ is measured in years. Frank knows that the expected time of failure for something following this distribution is

$$\int_0^{\infty} tf(t)dt.$$

How long can he expect one of these pumps to last?