Vv186 Recitation

Week 11

By Yahoo

Outline

- Real and Complex Power Series
- Exponential and Trigonometric Function

Real and Complex Power Series

Formal Power Series

For any complex sequence (a_k), the expression

$$\sum_{k=0}^{\infty} a_k z^k$$

is called a (formal) complex power series.

Convergent Power Series

Let $\sum_{k=0}^{\infty} a_k z^k$ be a complex power series. Then there exists a unique number $\varrho \in [0,\infty]$ such that

1. $\sum_{k=0}^{\infty} a_k z^k$ is absolutely convergent if $|z| < \varrho$
2. $\sum_{k=0}^{\infty} a_k z^k$ diverges if $|z| > \varrho$.

Example

The radius of convergence of $\sum_{k=0}^{\infty} z^k$ is 1. When $|z|<1$, $\sum_{k=0}^{\infty} z^k$ converges. When $|z|>1$, $\sum_{k=0}^{\infty} z^k$ diverges.

Remark

When $|z| = \varrho$, the series be either convergent or divergent.

Example: $\sum_{k=0}^{\infty} z^k$ diverges to infinity when z=1.
($\sum_{k=0}^{\infty} z^k$ converges to 1/2 when z=-1.)

Hadamard's formula

The radius of convergence of the power series is given by

$$\varrho = \begin{cases} \dfrac{1}{\overline{\lim} \sqrt[k]{|a_k|}}, & 0 < \overline{\lim\limits_{k\to\infty}} \sqrt[k]{|a_k|} < \infty \\ 0, & \overline{\lim\limits_{k\to\infty}} \sqrt[k]{|a_k|} = \infty, \\ \infty, & \overline{\lim\limits_{k\to\infty}} \sqrt[k]{|a_k|} = 0. \end{cases}$$

Practice

1. Find the radius of convergence of the power series $\sum_{\mathrm{k}=0}^{\infty} \frac{1}{2^k} \mathrm{z}^{\mathrm{k}}$.
2. Find the radius of convergence of the power series $\sum_{\mathrm{k}=0}^{\infty} \frac{1}{2^k} \mathrm{z}^{k^2}$.

Uniform Convergence of Power Series

If $\sum_{k=0}^{\infty} a_k z^k$ is a complex power series with radius of convergence ϱ, then for any $R < \varrho$ the series converges uniformly on $|z| < R$.

Example

Radius of convergence of $\sum_{k=0}^{\infty}(-1)^k x^k$ is 1 and $\sum_{k=0}^{\infty}(-1)^k x^k$ converges when $|x| < 1$. Therefore, $\sum_{k=0}^{\infty}(-1)^k x^k$ converges uniformly on $[-R, R]$ for any $0 < R < 1$.

However, $\sum_{k=0}^{\infty}(-1)^k x^k$ does not converge uniformly on $(-1, 1)$.

Exponential and Trigonometric Function

Power series definition

- Exponential Function
- Trigonometric Function

Useful results

Derivative

1. Sin’(x)=Cos(x)
2. Exp’(x)=Exp(x)

sin(x+y)=sin(x)cos(y)+cos(x)sin(y)

Thank you for your attention!