Journal of Applied Physics

Lumped model of bending electrostrictive transducers for energy harvesting

Cite as: J. Appl. Phys. **116**, 124106 (2014); https://doi.org/10.1063/1.4896185
Submitted: 11 March 2014 • Accepted: 10 September 2014 • Published Online: 25 September 2014

Mickaël Lallart, Liuqing Wang, Claude Richard, et al.

JOURNAL OF APPLIED PHYSICS **116**, 124106 (2014)

Lumped model of bending electrostrictive transducers for energy harvesting

Mickaël Lallart,[a] Liuqing Wang, Claude Richard, Lionel Petit, and Daniel Guyomar
Université de Lyon, INSA-Lyon, LGEF EA 682, 8 rue de la Physique, Villeurbanne, F-69621, France

(Received 11 March 2014; accepted 10 September 2014; published online 25 September 2014)

Electroactive polymers, and more particular dielectric electrostrictive polymers, have been of great interest over the last decade thanks to their flexibility, easy processing, conformability, and relatively low cost. Their application as actuators, sensors, or energy harvesters suits very well to systems that require high strain. In particular, bending devices are an important application field of such materials, especially when dealing with devices subjected to air or liquid flows. Nevertheless, the design of such devices and their associated electrical interface still requires starting from the local aspects of the electrostrictive effect. In order to provide a simple yet efficient design tool, this paper exposes a simple lumped model for electrostrictive dielectric devices working under flexural solicitation. Based on the analysis of the converted energy with respect to the provided energy, it is shown that electrostrictive systems can easily be reduced to a simple spring-mass-damper system with a quadratic dependence to the applied voltage on the mechanical side and to a current source controlled by the applied voltage with a capacitive internal impedance on the electrical side. Experimental measurements carried out to evaluate the mechanical to electrical conversion effect as well as the energy harvesting abilities in such systems also validate the proposed approach. [http://dx.doi.org/10.1063/1.4896185]

I. INTRODUCTION

Along with the development of smart materials for actuation, sensing, or energy harvesting, an increased attention has been placed on electroactive polymers (EAPs) over the last few years.[1–8] Electrostrictive polymers are a particular branch of electroactive polymers that feature very low Young's modulus, have an easy fabrication process, and are conformable and low cost. Electrostrictive polymers are divided into two categories: ionic[9,10] and dielectric.[11–18] While the former can exhibit very large deformation, its limits in terms of frequency range and high losses make the latter more adapted to applications involving energy harvesting at frequencies higher than 1 Hz.

Energy harvesting is the process of converting energy from a source directly available to the system (for instance mechanical, thermal) to useful electrical energy that can be used by the connected electrical system. The spreading of energy harvesting products has been enabled by the dramatic decrease of electronic device consumption as well as the maintenance issue and environmental concerns raised by primary batteries.[19,20] In case of mechanical energy harvesting, energy source can be found in various nature in terms of strain and stress levels and frequencies. While many works were devoted to piezoelectric,[21] electromagnetic,[22] or electrostatic[23] devices for mechanical energy harvesting, addressing low frequency, high strain systems is better suited to electrostrictive polymers for the above-mentioned reasons. When using such materials, energy harvesting techniques may consider charge/discharge cycles based on electrostatic energy harvesting,[24–26] or the use of pseudo-piezoelectric operations.[27–29]

The principles of the electromechanical conversion effect in dielectric electrostrictive polymers originate from the Maxwell stress induced by charges on the electrodes of the material and by the strain-dependent orientability of the dipoles.[30–35] Nevertheless, while local constitutive equations of electrostriction have been derived from formal analysis (using phenomenological approach[30,36–38] or physical interpretation[34,35]), general models linking macroscopic quantities usually rely on numerical integration.[39,40] No simple high-level model (such as electromechanically coupled spring-mass-damper for piezoelectric element) linking macroscopic quantities (force, displacement, voltage, and current) has been proposed in order to be able to efficiently design the structure and the associated interface (e.g., electrical circuit), although some studies empirically deduced the possible law using analogy with piezoelectricity when the polymer is subjected to a bias voltage.[41]

Hence, this work aims at giving a lumped model for electrostrictive beam devices working under flexural solicitations. While previous works established the dependency of the flexural displacement with the electric field,[42] no real energy conversion analysis has been performed so far, preventing linking macroscopic and local parameters for the considered structure type. Hence, the objective of this paper is to give a simple yet effective formulation of the macroscopic electromechanical equations (motion equation and current generation equation) for 1D structures based on the total converted energy (both from mechanical to electrical and from electrical to mechanical), allowing the derivation of a set of mechanical and electrical macroscopic equations that permit both quickly evaluating the performance of a given device or optimizing the global system, for instance, for energy harvesting purposes. Hence, while in Ref. 42 the aim was to investigate and optimize the strain/displacement

[a]mickael.lallart@insa-lyon.fr

for a given applied electric field, therefore focusing on actuator only (the displacement was arising from the application of an electric field—no external force was considered), without any energy conversion analysis and concerns, and without a simplified lumped model, the present paper is more concerned with the energy and conversion analysis, especially from the mechanical to electrical aspect (externally applied force—although the proposed model can also consider the electrical to mechanical conversion), and with a final objective to set up a high-level, simplified lumped model that can be easily used for the design of the transducer environment, like electrical interfaces.

The paper is organized as follows. Section II exposes the derivation of this simple lumped model, based on the electrical and mechanical energy densities for obtaining the simplified global response of the system. The particular case of a cantilever beam is discussed in Sec. III to give an illustrative example of the model. Then, the proposed approach is experimentally assessed in Sec. IV. Finally, a brief conclusion is presented in Sec. V, summarizing the main findings of this work.

II. SYSTEM MODELING

This section aims at exposing the general theoretical development for the lumped model of electrostrictive-based beam devices. Starting from the mechanical behavior of a structure with a bonded electrostrictive element on its surface and subjected to flexural vibration, an energy analysis is then performed in the electrical domain to get the electrical expression and then to the mechanical aspect for obtaining the motion equation.

The global investigated system is presented in Figure 1, where h_p and h_b are the respective thickness of the polymer and the substrate beam, L_p and L_b their length. Finally, the width of the device will be denoted w in the following.

A. Mechanical behavior

The mechanical behavior of the structure featuring flexural bending beam can be obtained starting from the constitutive equations of electrostrictive polymers assuming transverse isotropic material and polarization along axis 3 given as, considering Voigt's notation

$$\begin{cases} S_1 = s_{11}^E T_1 + s_{12}^E T_2 + s_{13}^E T_3 + M_{31} E_3{}^2 \\ S_2 = s_{12}^E T_1 + s_{11}^E T_2 + s_{13}^E T_3 + M_{31} E_3{}^2 \\ D_3 = \epsilon_{33}^T E_3 + 2M_{31} T_1 E_3 + 2M_{31} T_2 E_3 + 2M_{33} T_3 E_3, \end{cases} \tag{1}$$

with S, T, E, and D, respectively, referring to the strain, stress, electric field, and electric displacement. s, M, and ϵ are the compliance, electrostrictive coefficient, and absolute permittivity of the material. Superscripts denote the constant parameter. Then, assuming Euler-Bernoulli behavior ($S_2 = 0$, $T_3 = 0$, i.e., plane sections remain plane), it is possible to find the expression of the stress along axis 1 as a function of the strain along the same direction and the electric field as

$$T_1 = c_p^E S_1 - \left(1 - \nu_p^E\right) c_p^E M_{31} E_3{}^2 \quad \text{with} \quad c_p^E = \frac{Y^E}{1 - \nu_p^{E2}}, \tag{2}$$

with Y^E and ν_p^E the Young modulus and Poisson's ratio of the short-circuited polymer, respectively.

However, in order to have a non-null response in the case of bending, the neutral axis should not be in the middle of the polymer. Hence, a substrate is added to the structure. For the latter and according to Hooke's law, the relationship between the stress along axis 1 T_1 and strain S_1 along the same axis is given by

$$T_1 = c_b S_1 \quad \text{with} \quad c_b = \frac{Y_b}{1 - \nu_b{}^2}, \tag{3}$$

where Y_b and ν_b, respectively, refer to the Young modulus and Poisson's ratio of the substrate.

Finally, from the stress-strain expressions of the polymer and substrate layers (Eqs. (2) and (3) respectively), and considering the neutral axis h_n

$$h_n = \frac{h_b{}^2 c_b + 2h_b h_p c_p^E + h_p{}^2 c_p^E}{2\left(h_b c_b + h_p c_p^E\right)}, \tag{4}$$

it is possible to get the expression of the strain as a function of the flexural displacement $u_3(x_1)$

$$S_1 = -(x_3 - h_n)\frac{\partial^2 u_3(x_1)}{\partial x_1{}^2}. \tag{5}$$

B. Electrical aspect

Now the mechanical expression of the strain (which depends on the stress and electric field) obtained, and using the electrical constitutive equation in Eq. (1), it is possible to derive the electric displacement as a function of electric field E_3 and strain S_1 in the active layer as

$$\begin{aligned} D_3 &= \left[\epsilon_{33}^T - 2\left(\frac{1}{s_{11}^E} + \left(1 - \nu_p^E\right)^2 c_p^E\right) M_{31}{}^2 E_3{}^2\right] E_3 \\ &\quad + 2M_{31}\left(1 - \nu_p^E\right) c_p^E S_1 E_3 \\ &\approx \epsilon_{33}^T E_3 + 2M_{31}\left(1 - \nu_p^E\right) c_p^E S_1 E_3. \end{aligned} \tag{6}$$

From this equation and from the strain-flexural displacement expression in the polymer given by Eq. (5), the variation of the provided electrical energy to the polymer (only the latter layer receives the electrical energy, so that the thickness ranges from $h_b - h_n$ to $h_b + h_p - h_n$ in the following expression) is given by, assuming that the thickness h_p is small enough to approximate the relationship between the electric field E_3 and voltage V as $E_3 = -V/h_p$

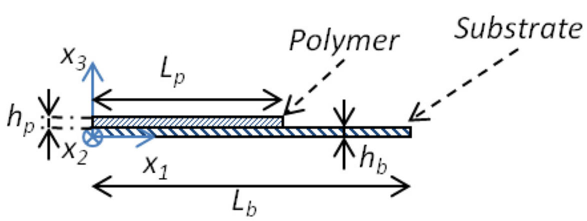

FIG. 1. Schematic of the considered structure.

$$dW_{prov,elec} = \int_0^w \int_0^{L_b} \int_{h_b-h_n}^{h_b+h_p-h_n} E_3 dD_3 dx_3 dx_1 dx_2$$
$$\approx \frac{wL_p\epsilon_{33}}{h_p} VdV - \frac{M_{31}\left(1-\nu_p^E\right)h_b c_b c_p^E w}{h_p} \times \left(\frac{h_p+h_b}{h_p c_p^E + h_b c_b}\right) V^2 d\left(\left.\frac{\partial u_3}{\partial x_1}\right|_{x_1=L_p}\right)$$
$$- \frac{M_{31}\left(1-\nu_p^E\right)h_b c_b c_p^E w}{h_p}\left(\frac{h_p+h_b}{h_p c_p^E + h_b c_b}\right) \times \left(\left.\frac{\partial u_3}{\partial x_1}\right|_{x_1=L_p}\right) VdV. \quad (7)$$

From the previous expression, it is possible to decompose the energy variation into a purely dielectric energy and a coupling energy (part of the provided electrical energy is converted into mechanical energy)

$$dW_{prov,elec} = dW_{elec} + dW_{coupling,elec}, \quad (8)$$

which can be identified as

$$\begin{aligned} dW_{elec} &= C_0 VdV \\ dW_{coupling,elec} &= \alpha(x)Vd(u_3(x)V) \\ &= \alpha(x)V^2 du_3(x) + \alpha(x)Vu_3(x)dV, \end{aligned} \quad (9)$$

with C_0 the equivalent clamped capacitance and $\alpha(x_1)$ the coupling coefficient (called "force factor" in the following in a similar manner than piezoelectric systems[43]), which actually depends on the considered position x along the x_1 axis of the flexural displacement $u_3(x)$

$$C_0 = \frac{wL_p\epsilon_{33}}{h_p}, \quad (10)$$

$$\alpha u_3(x) = \frac{M_{31}\left(1-\nu_p^E\right)h_b c_b c_p^E w}{h_p}\left(\frac{h_p+h_b}{h_p c_p^E + h_b c_b}\right)\left(\left.\frac{\partial u_3}{\partial x_1}\right|_{x_1=L_p}\right). \quad (11)$$

The provided electrical energy being given by the voltage and charge Q provided by the source, the lumped expression of the electrical relationship is then given as

$$Q = C_0 V - \alpha(x)Vu_3(x). \quad (12)$$

C. Mechanical aspect

Although the scope of this paper focuses on the mechanical to electrical conversion, this subsection proposes to investigate the mechanical aspect of bending electrostrictive systems. It is now considered that the energy is provided by a mechanical source. Starting from the stress expressions in both the polymer and substrate (Eqs. (2) and (3)), it is possible to derive the provided mechanical energy variation as

$$dW_{prov,mech} = \int_0^w \int_0^{L_b} \int_{-h_n}^{h_b+h_p-h_n} T_1 dS_1 dx_3 dx_1 dx_2$$
$$= c_b I_b \int_0^{L_b} \frac{\partial^2 u_3(x_1)}{\partial x_1{}^2} d\left(\frac{\partial^2 u_3(x_1)}{\partial x_1{}^2}\right) dx_1 + A\int_0^{L_p} \frac{\partial^2 u_3(x_1)}{\partial x_1{}^2} d\left(\frac{\partial^2 u_3(x_1)}{\partial x_1{}^2}\right) dx_1$$
$$+ \frac{1}{2} M_{31}\left(1-\nu_p^E\right)h_b c_b c_p^E w\left(\frac{h_p+h_b}{h_b c_b + h_p c_p^E}\right) \times \frac{1}{h_p} V^2 d\left(\left.\frac{\partial u_3}{\partial x_1}\right|_{x_1=L_p}\right), \quad (13)$$

where I_b denotes the second moment of area of the substrate

$$I_b = \frac{h_b{}^3 w}{12} \quad (14)$$

and A is defined as

$$A = \frac{w}{12}\left(\frac{c_b{}^2 h_b{}^4 + 4c_b c_p^E h_b{}^3 h_p + 6c_b c_p^E h_b{}^2 h_p{}^2 + 4c_b c_p^E h_b h_p{}^3 + c_p^{E2} h_p{}^4}{c_b h_b + c_p^E h_p}\right). \quad (15)$$

Again, this expression can be decomposed into available mechanical energy variation and part of the energy variation converted into electricity

$$dW_{prov,mech} = dW_{mech} + dW_{coupling,mech}, \quad (16)$$

which can be further identified as, with $u_3(x)$ the flexural displacement at the considered point x

$$\begin{aligned} dW_{mech} &= K(x)u_3(x)du_3(x) \\ dW_{coupling,mech} &= \frac{1}{2}\alpha(x)V^2 du_3(x), \end{aligned} \quad (17)$$

with $K(x)$ and $\alpha(x)$ are defined as the short circuit stiffness and force factor, respectively

$$K(x)u_3{}^2(x) = c_b I_b \int_0^{L_b}\left[\frac{\partial^2 u_3(x_1)}{\partial x_1{}^2}\right]^2 dx_1 + A\int_0^{L_p}\left[\frac{\partial^2 u_3(x_1)}{\partial x_1{}^2}\right]^2 dx_1, \quad (18)$$

$$\alpha u_3(x) = \frac{M_{31}\left(1-\nu_p^E\right)h_b c_b c_p^E w}{h_p}\left(\frac{h_p+h_b}{h_p c_p^E + h_b c_b}\right)\left(\left.\frac{\partial u_3}{\partial x_1}\right|_{x_1=L_p}\right). \quad (19)$$

It can be noted that the stiffness K depends not only on the displacement at the considered position x but also on the flexural shape of the structure. Additionally, this parameter can also be found from the short-circuit response of the device. It is also possible to verify that the expression of α is identical to the one obtained in Sec. II B.

Once the static mechanical behavior is obtained, and by considering that the dynamic deformed shape is similar, it is also possible to derive the dynamic mass M as

$$M(x) = \frac{1}{{u_3}^2(x)} \int_0^{L_b} m(x_1){u_3}^2(x_1)dx_1, \tag{20}$$

with $m(x_1)$ the lineic mass of the beam, defined from the material dimensions and the respective densities ρ_b and ρ_p of the substrate and polymer

$$\begin{cases} m(x_1) = w(h_b\rho_b + h_p\rho_p) & \text{for} \quad x_1 \in [0, L_p] \\ m(x_1) = wh_b\rho_b & \text{for} \quad x_1 \in]L_p, L_b]. \end{cases} \tag{21}$$

D. Global lumped model

For the electrical aspect, the differentiation of the expression of the charge (Eq. (12)) allows expressing the outgoing current I from the polymer as

$$I = -\frac{dQ}{dt} = \alpha(x)\dot{V}u_3(x) + \alpha(x)V\dot{u}_3(x) - C_0\dot{V}, \tag{22}$$

while for the mechanical part, the global provided energy, taking into account the kinetic energy (Eqs. (17) and (20)) is given as a function of the externally applied force F

$$\int F du_3(x) = \int M\ddot{u}_3(x)du_3(x) + \int Ku_3(x)du_3(x) + \int \frac{1}{2}\alpha(x)V^2 du_3(x), \tag{23}$$

which allows getting the equivalent lumped model for an electrostrictive-based electromechanical structure operating in bending mode and considering the displacement at the position x

$$\begin{cases} M(x)\ddot{u}_3(x) + Ku_3(x) = F - \frac{1}{2}\alpha(x)V^2 \\ I = \alpha(x)V\dot{u}_3(x) + \alpha(x)u_3(x)\dot{V} - C_0\dot{V}. \end{cases} \tag{24}$$

Furthermore, dynamic losses can also be introduced through a structural damping coefficient C, yielding the global lumped model

$$\begin{cases} M(x)\ddot{u}_3(x) + C\dot{u}_3(x) + K(x)u_3(x) = F - \frac{1}{2}\alpha(x)V^2 \\ I = \alpha(x)V\dot{u}_3(x) + \alpha(x)u_3(x)\dot{V} - C_0\dot{V}, \end{cases} \tag{25}$$

where the expression of the model parameters is summarized as follows:

$$\begin{cases} M(x) = \frac{1}{{u_3}^2(x)} \int_0^{L_b} m(x_1){u_3}^2(x_1)dx_1, \\ K(x) = \frac{1}{{u_3}^2(x)} \left\{ c_b I_b \int_0^{L_b} \left[\frac{\partial^2 u_3(x_1)}{\partial {x_1}^2}\right]^2 dx_1 + A \int_0^{L_p} \left[\frac{\partial^2 u_3(x_1)}{\partial {x_1}^2}\right]^2 dx_1 \right\}, \\ \alpha(x) = \frac{1}{u_3(x)} \left[\frac{M_{31}\left(1 - \nu_p^E\right) h_b c_b c_p^E w}{h_p} \left(\frac{h_p + h_b}{h_p c_p^E + h_b c_b}\right) \left(\left.\frac{\partial u_3}{\partial x_1}\right|_{x_1 = L_p}\right) \right], \\ C_0 = \frac{wL_p\epsilon_{33}}{h_p}. \end{cases} \tag{26}$$

Hence, this globally coupled equation set shows the importance of the electromechanical factor $\alpha(x)$ for the energy conversion process, as the latter fixes the amount of charge generated for a given mechanical excitation of the global structure and/or the electromechanical force induced by the application of a voltage on the electroactive material. More precisely, this parameter is tightly related to the coupling coefficient k whose expression is given as, using definition from Refs. 44 and 45 and assuming $\alpha(x)V^2u \ll Ku^2$

$$k = \frac{\text{coupled energy}}{\sqrt{\text{elastic energy} \times \text{dielectric energy}}} \approx \frac{\alpha(x)V}{\sqrt{C_0K}}. \tag{27}$$

III. CASE STUDY

As an application example of the previous development, this section aims at expliciting the particular case of a cantilever beam clamped at $x_1 = 0$ (so that the polymer is bonded close to the fixed side) driven by a force F at its free end ($x_1 = L_b$).

In this case, the flexural displacement can be derived through the expression of the moment $\mathcal{M}(x_1)$ exerted by the force F given by

$$\mathcal{M}(x_1) = (L_b - x_1)F. \tag{28}$$

As this moment is related to the longitudinal stress T_1 following

$$\mathscr{M}(x_1) = w\int_{-h_n}^{h_b+h_p-h_n} T_1 x_3 dx_3, \tag{29}$$

with the neutral axis h_n given by

$$h_n = \begin{cases} \dfrac{{h_b}^2 c_b + 2h_b h_p c_p^E + {h_p}^2 c_p^E}{2\left(h_b c_b + h_p c_p^E\right)} & \text{in the active zone } (0 < x_1 \leq L_p) \\ \dfrac{h_b}{2} & \text{in the passive zone } (L_p < x_1 \leq L_b), \end{cases} \tag{30}$$

it is possible to derive the expression of the flexural displacement $u_3(x_1)$, using previous expressions in combination with Eq. (5) and considering boundary conditions (null displacement and slope at $x_1 = 0$) as well as the continuity of the displacement and slope at the interface between the active and passive zones as

$$u_3(x) = \begin{cases} \dfrac{1}{A}\left(\dfrac{B}{2}{E_3}^2 - \dfrac{L_b}{2}F\right)x^2 + \dfrac{1}{6A}Fx^3 & \text{in the active zone } (0 \leq x \leq L_p) \\ \dfrac{2}{c_b {h_b}^3 w}(x - 3L_b)x^2 F + a_1 x + a_2 & \text{in the passive zone } (L_p < x \leq L_b), \end{cases} \tag{31}$$

with

$$\begin{cases} A = \dfrac{w}{12}\left(\dfrac{{c_b}^2 {h_b}^4 + 4c_b c_p^E {h_b}^3 h_p + 6c_b c_p^E {h_b}^2 {h_p}^2 + 4c_b c_p^E h_b {h_p}^3 + c_p^{E2} {h_p}^4}{c_b h_b + c_p^E h_p}\right), \\ B = -\dfrac{w}{2}\left(\dfrac{h_b + h_p}{c_b h_b + c_p^E h_p}\right)\left(1 - \nu_p^E\right) c_p^E c_b h_p h_b M_{31}, \\ a_1 = \dfrac{BL_p}{A}{E_3}^2 + \left(\dfrac{12 L_b L_p}{c_b {h_b}^3 w} - \dfrac{6{L_p}^2}{c_b {h_b}^3 w} - \dfrac{L_b L_p}{A} + \dfrac{{L_p}^2}{2A}\right)F, \\ a_2 = -\dfrac{B{L_p}^2}{2A}{E_3}^2 + \left(-\dfrac{6L_b {L_p}^2}{c_b {h_b}^3 w} + \dfrac{4{L_p}^3}{c_b {h_b}^3 w} + \dfrac{L_b {L_p}^2}{2A} - \dfrac{{L_p}^3}{3A}\right)F. \end{cases} \tag{32}$$

Hence, from the displacement $u_3(x)$, it is possible to get the expression of the force factor $\alpha(x)$ such as

$$\alpha(x) = \begin{cases} \left[\dfrac{M_{31}\left(1 - \nu_p^E\right) h_b c_b c_p^E w}{h_p}\left(\dfrac{h_p + h_b}{h_p c_p^E + h_b c_b}\right)\right] \times \left[\dfrac{2BL_p {E_3}^2 + \left(-2L_b L_p + {L_p}^2\right)F}{Bx^2 {E_3}^2 + \left(\dfrac{1}{3}x^3 - L_b x^2\right)F}\right], \\ \text{in the active zone } (0 \leq x \leq L_p) \\ \left[\dfrac{M_{31}\left(1 - \nu_p^E\right) h_b c_b c_p^E w}{h_p}\left(\dfrac{h_p + h_b}{h_p c_p^E + h_b c_b}\right)\right] \times \left\{\dfrac{\left[2BL_p {E_3}^2 + \left(-2L_b L_p + {L_p}^2\right)F\right] c_b {h_b}^3 w}{2A\left[-(6L_b x^2 - 2x^3)F + (a_1 x + a_2) c_b {h_b}^3 w\right]}\right\}. \\ \text{in the passive zone } (L_p < x \leq L_b) \end{cases} \tag{33}$$

However, the contribution of the electric field is negligible regarding the one from the force with typical and realistic parameter values. Therefore, the force factor can be approximated as follows:

$$\alpha(x) \approx \begin{cases} \dfrac{M_{31}\left(1 - \nu_p^E\right) h_b c_b c_p^E w}{h_p}\left(\dfrac{h_p + h_b}{h_p c_p^E + h_b c_b}\right)\left(\dfrac{-2L_b L_p + {L_p}^2}{\dfrac{1}{3}x^3 - L_b x^2}\right) \\ \text{in the active zone } (0 \leq x \leq L_p) \\ \left[\dfrac{M_{31}\left(1 - \nu_p^E\right) h_b c_b c_p^E w}{h_p}\left(\dfrac{h_p + h_b}{h_p c_p^E + h_b c_b}\right)\right] \\ \times \left\{\dfrac{\left(-2L_b L_p + {L_p}^2\right) c_b {h_b}^3 w}{4Ax^3 - 12AL_b x^2 + \left[(24A - 2c_b {h_b}^3 w)L_b L_p + (c_b {h_b}^3 w - 12A){L_p}^2\right]x + \left[(c_b {h_b}^3 w - 12A)L_b {L_p}^2 + \left(8A - \dfrac{2}{3}c_b {h_b}^3 w\right){L_p}^3\right]}\right\} \\ \text{in the passive zone } (L_p < x \leq L_b), \end{cases} \tag{34}$$

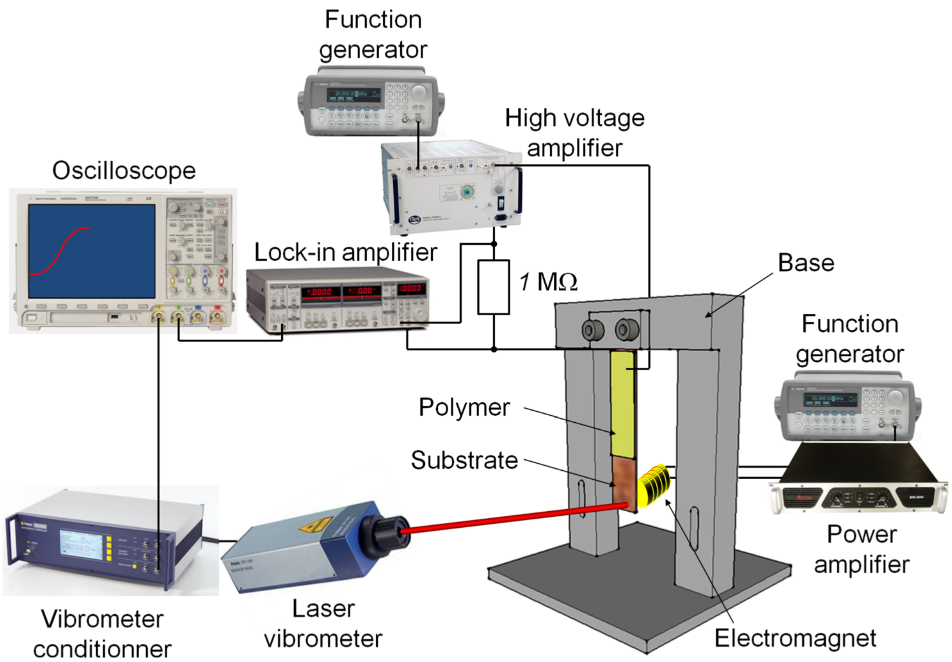

FIG. 2. Schematic of the experimental setup.

TABLE I. Dimensions and material parameters of the experimental structure.

Steel substrate		Polyurethane	
Length, L_b	54 mm	Length, L_p	39 mm
Thickness, h_b	80 μm	Thickness, h_p	80 μm
Width, w	17 mm	Width, w	17 mm
Young's modulus, Y_b	210 GPa	Short-circuit Young's modulus, Y_p^E	30 MPa
Poisson's ratio, ν_b	0.29	Short-circuit Poisson's ratio, ν_p^E	0.49
Density, ρ_b	8000 kg m^{-3}	Density, ρ_p	1100 kg m^{-3}
		Electrostrictive coefficient, M_{31}	-1.3×10^{-18} m^2 V^{-2}
		Relative permittivity, $\epsilon_{33}^T/\epsilon_0$	4

which is independent from the force and electric field and only depends on the material properties and dimensions.

IV. EXPERIMENTAL VALIDATION

From the results of the case study, this section aims at experimentally validating the previously exposed approach and model. As the paper scope focuses on the mechanical to electric conversion effect, only the generated current through the coupling coefficient α will be investigated.

The experimental setup, shown in Figure 2, consists of an electromagnet fed by a function generator through a power amplifier, allowing exciting the structure (with a frequency of approximately 22.5 Hz, which is close to the resonance frequency—the theoretical value from the stiffness K and mass M, obtained from Eq. (26), is 33.4 Hz—the difference being possibly explained by losses, discrepancies in the material parameter, gluing layer, and non exact point force excitation at the end of the beam). A laser vibrometer is also used for monitoring the displacement of the structure at a particular location. For the electroactive material, the electric field is applied by a voltage source through a power amplifier. A 1 MΩ load is also connected to the polymer to monitor the current (this load is much less than the impedance of the polymer so that it is almost in short-circuit case) through a lock-in amplifier. Finally, an oscilloscope is used to monitor vibrometer and lock-in amplifier output voltage.

The considered structure consists of a steel substrate bonded with polyurethane film. Dimensions and material parameters are given in Table I.

The first set of measurements consists in evaluating the electroactive coefficient α (force factor) when considering several displacement locations along the length of the beam. Results depicted in Figure 3, obtained for two different bias electric fields applied to the polymer, show very good

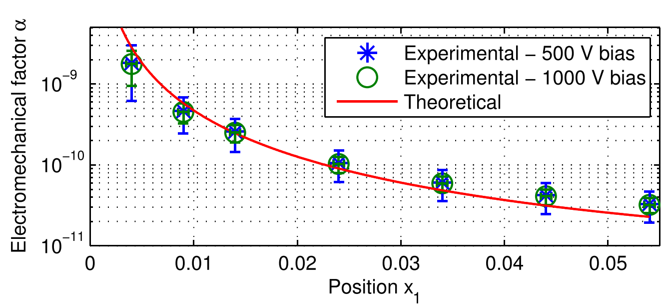

FIG. 3. Theoretical and measured electromechanical coefficient for different bias voltages.

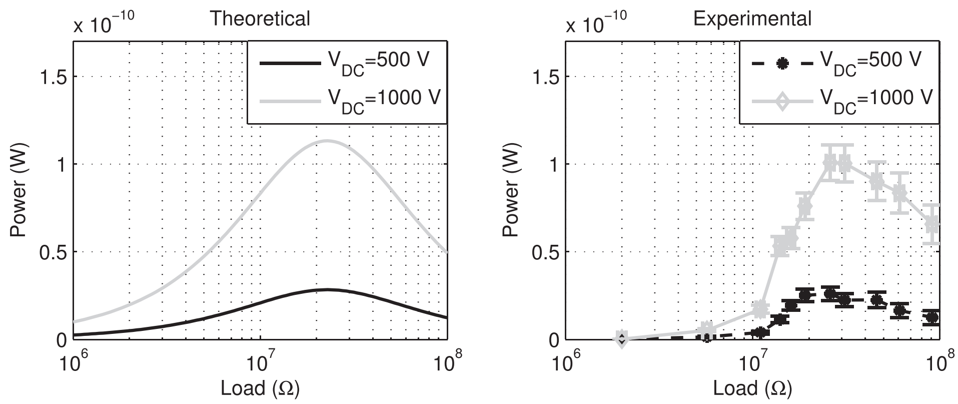

FIG. 4. Theoretical and measured load power for different bias voltages.

agreement with theoretical predictions. As the generated current is constant while the considered displacement position is varying, the value of the force factor varies with the inverse considered position. Furthermore, the approximation of the negligible contribution of the electric field with respect to the force is also validated, as the change of the bias voltage does not alter the value of α.

Then, the second set of measurements consisted in evaluating the power that can be harvested from the bending beam with the considered displacement taken at the free end of the beam. In this case, a resistive load is added in addition to the 1 MΩ resistor that measures the current. Then, from the load value and measured current, it is possible to evaluate the harvested power P as

$$P = R{I_{RMS}}^2, \tag{35}$$

with I_{RMS} the root mean square value of the current and R the resistance obtained by the combination of the 1 MΩ resistor and the load. Additionally, under such a configuration, it is also possible to get the theoretical expression of the power starting from the expression of the current (electrical equation of Eq. (25)—for the sake of clarity, $\alpha(x)$ is simplified as α and $u_3(x)$ as u for the derivation, x being constant as a fixed location is considered)

$$I = \alpha V\dot{u} + \alpha u\dot{V} - C_0\dot{V}, \tag{36}$$

which can be simplified as, considering that $\alpha u \ll C_0$ and that the generated AC voltage V_{AC} is far less than the bias voltage V_{DC}

$$I \approx \alpha V_{DC}\dot{u} - C_0\dot{V}_{AC}. \tag{37}$$

Hence, when the system is loaded with a load R ($I = V_{AC}/R$), the expression of the AC voltage in the frequency domain (with ω the angular frequency) yields

$$V_{AC} = \frac{R\alpha V_{DC}v}{1 + jRC_0\omega}, \tag{38}$$

where v denotes the velocity. Hence, the expression of the RMS power on the load is given by[27]

$$P = \frac{1}{2}\frac{V_{AC}{V_{AC}}^*}{R} = \frac{1}{2}\frac{\alpha^2{V_{DC}}^2R}{1 + (RC_0\omega)^2}v^2, \tag{39}$$

with X^* denoting the complex conjugate of X.

The value of C_0 has been evaluated both theoretically from Eq. (10) and experimentally through a multimeter, giving 293 pF theoretically and 292 pF in the experiment, confirming the validity of the model. Results depicted in Figure 4 show a very good agreement between predicted and harvested power, both in terms of power level but also in terms of optimal load, therefore demonstrating the validity of the previously exposed approach.

V. CONCLUSION

This paper provided a simple lumped model of electrostrictive systems working in bending mode, derived from the constitutive equations of electrostriction and based on energy analysis. Such a model allows not only optimizing the system design in order to have optimal working points but also permits designing control laws and/or electrical interface and assess their performance using a simple yet quite accurate model in a quick way. As an example, the design of efficient control interfaces for electrostrictive-based actuators or electric interfaces for electrostrictive-based energy harvester as well as their comparative analysis can be greatly simplified using the proposed approach.

ACKNOWLEDGMENTS

The research of the second author has been partially supported by China Scholarship Council.

[1]Y. Bar-Cohen, "Electroactive polymers as artificial muscles-capabilities, potentials and challenges," *Handbook on Biomimetics*, "Motion" paper No. 134 (NTS Inc., 2000), Sec. 11, Chap. 8.
[2]G. W. Taylor, J. R. Burns, S. A. Kammann, W. B. Powers, and T. R. Welsh, "The energy harvesting Eel: A small subsurface ocean/river power-generator," IEEE J. Oceanic Eng. **26**(4), 539–547 (2001).
[3]S. Ashley, "Artificial muscles," Sci. Am. **289**, 52–59 (2003).
[4]G. H. Kim, "Electroactive polymer composites as a tactile sensor for biomedical applications," Macromol. Res. **12**(6), 564–572 (2004).
[5]F. Carpi and D. De Rossi, "Electroactive polymer-based devices for e-textiles in biomedicine," IEEE Trans. Inf. Technol. Biomed. **9**(3), 295–318 (2005).
[6]A. Nisar, N. Afzulpurkar, B. Mahaisavariya, and A. Tuantranont, "MEMS-based micropumps in drug delivery and biomedical applications," Sens. Actuators, B **130**, 917–942 (2008).
[7]I. A. Anderson, T. A. Gisby, T. G. McKay, B. M. O'Brien, and E. P. Calius, "Multi-functional dielectric elastomer artificial muscles for soft and smart machines," J. Appl. Phys. **112**(4), 041101 (2012).
[8]R. D. Kornbluh, R. Pelrine, H. Prahlad, A. Wong-Foy, B. McCoy, S. Kim, J. Eckerle, and T. Low, "From boots to buoys: Promises and challenges of

dielectric elastomer energy harvesting," in *Electroactivity in Polymeric Materials* (Springer, 2012), Chap. 3, pp. 67–93.
[9]M. Uchida and M. Taya, "Solid polymer electrolyte actuator using electrode reaction," Polymer **42**(22), 9281–9285 (2001).
[10]J. Lin, Y. Liu, and Q. M. Zhang, "Charge dynamics and bending actuation in Aquivion membrane swelled with ionic liquids," Polymer **52**(2), 540–546 (2011).
[11]Q. M. Zhang, J. Su, C. H. Kim, R. Ting, and R. Capps, "An experimental investigation of electromechanical responses in a polyurethane elastomer," J. Appl. Phys. **81**, 2770–2776 (1997).
[12]R. Liu, Q. Zhang, and L. E. Cross, "Experimental investigation of electrostrictive polarization biased direct apparent piezoelectric properties in polyurethane elastomer under quasistatic conditions," J. Appl. Polym. Sci. **73**, 2603–2609 (1999).
[13]Z.-Y. Cheng, V. Bharti, T.-B. Xu, H. Xu, T. Mai, and Q. M. Zhang, "Electrostrictive poly(vinylidene fluoride-trifluoroethylene) copolymers," Sens. Actuators, A **90**, 138–147 (2001).
[14]F. M. Guillot and E. Balizer, "Electrostrictive effect in polyurethane," J. Appl. Polym. Sci. **89**, 399–404 (2003).
[15]C. Huang, Q. M. Zhang, G. De Botton, and K. Bhattacharya, "All-organic dielectric-percolative three-component composite materials with high electromechanical response," Appl. Phys. Lett. **84**, 4391–4393 (2004).
[16]B. Guiffard, D. Guyomar, L. Seveyrat, Y. Chowanek, M. Bechelany, D. Cornu, and P. Miele, "Enhanced electroactive properties of polyurethane films loaded with carbon-coated SiC nanowires," J. Phys. D: Appl. Phys. **42**, 055503 (2009).
[17]V. Bobnar, X. Li, G. Casar, A. Eršte, S. Glinšek, X. Qian, and Q. M. Zhang, "Tailoring electrically induced properties by stretching relaxor polymer films," J. Appl. Phys. **111**, 083515 (2012).
[18]M. Lallart, J.-F. Capsal, A. K. Mossi Idrissa, J. Galineau, M. Kanda, and D. Guyomar, "Actuation abilities of multiphasic electroactive polymeric systems," J. Appl. Phys. **112**, 094108 (2012).
[19]J. Krikke, "Sunrise for energy harvesting products," IEEE Pervasive Comput. **4**(1), 4–8 (2005).
[20]J. Paradiso and T. Starner, "Energy scavenging for mobile and wireless electronics," IEEE Pervasive Comput. **4**, 18–27 (2005).
[21]S. R. Anton and H. A. Sodano, "A review of power harvesting using piezoelectric materials (2003-2006)," Smart Mater. Struct. **16**, R1 (2007).
[22]E. Arroyo, A. Badel, F. Formosa, Y. Wu, and J. Qiu, "Comparison of electromagnetic and piezoelectric vibration energy harvesters: Model and experiments," Sens. Actuators, A **183**, 148–156 (2012).
[23]S. Meninger, J. O. Mur-Miranda, R. Amirtharajah, A. P. Chandrakasan, and J. H. Lang, "Vibration-to-electric energy conversion," IEEE Trans. VLSI Syst. **9**(1), 64–76 (2001).
[24]Y. Liu, K. L. Ren, H. F. Hofmann, and Q. Zhang, "Investigation of electrostrictive polymers for energy harvesting," IEEE Trans. Ultrason., Ferroelectr., Freq. Control **52**(12), 2411–2417 (2005).
[25]K. Ren, Y. Liu, H. Hofmann, Q. M. Zhang, and J. Blottman, "An active energy harvesting scheme with an electroactive polymer," Appl. Phys. Lett. **91**, 132910 (2007).
[26]T. McKay, B. O'Brien, E. Calius, and I. Anderson, "An integrated, self-priming dielectric elastomer generator," Appl. Phys. Lett. **97**, 062911 (2010).
[27]M. Lallart, P.-J. Cottinet, L. Lebrun, B. Guiffard, and D. Guyomar, "Evaluation of energy harvesting performance of electrostrictive polymer and carbon-filled terpolymer composites," J. Appl. Phys. **108**(3), 034901 (2010).
[28]P.-J. Cottinet, M. Lallart, D. Guyomar, B. Guiffard, L. Lebrun, G. Sebald, and C. Putson, "Analysis of AC-DC conversion for energy harvesting using an electrostrictive polymer P(VDF-TrFE-CFE)," IEEE Trans. Ultrason., Ferroelectr., Freq. Control **58**(1), 30–42 (2011).
[29]M. Lallart, P.-J. Cottinet, L. Lebrun, and D. Guyomar, "Review: Electrostrictive polymer for mechanical energy harvesting," J. Polym. Sci., Part B: Polym. Phys. **50**(8), 523–535 (2012).
[30]X. Zhao and Z. Suo, "Electrostriction in elastic dielectrics undergoing large deformation," J. Appl. Phys. **104**, 123530 (2008).
[31]B. Li, L. Liu, J. Wu, Z. Zhu, and H. Chen, "Electrostriction in dielectric elastomer: Effect on electromechanical actuation," Proc. SPIE **7642**, 764212 (2010).
[32]Z. Suo, "Theory of dielectric elastomers," Acta Mech. Solida Sin. **23**, 549 (2010).
[33]A. Ask, A. Menzel, and M. Ristinmaa, "Electrostriction in electro-viscoelastic polymer," Mech. Mater. **50**, 9–21 (2012).
[34]J.-F. Capsal, M. Lallart, P.-J. Cottinet, J. Galineau, G. Sebald, and D. Guyomar, "Evaluation of macroscopic polarization and actuation abilities of electrostrictive dipolar polymers using microscopic Debye/Langevin formalism," J. Phys. D.: Appl. Phys. **45**(20), 205401 (2012).
[35]M. Lallart, J.-F. Capsal, G. Sebald, P.-J. Cottinet, and D. Guyomar, "Converse electrostrictive effect in dielectric polymers," Sens. Actuators, B **190**, 259–264 (2014).
[36]S. Eury, R. Yimnirun, V. Sundar, P. J. Moses, S.-J. Jang, and R. E. Newnham, "Converse electrostriction in polymers and composites," Mater. Chem. Phys. **61**, 18–23 (1999).
[37]K. Mrabet, R. Rahouadj, and C. Cunat, "An irreversible thermodynamic model for semicrystalline polymers submitted to multisequence loading at large strain," Polym. Eng. Sci. **45**(1), 42–51 (2005).
[38]W. Hong, X. Zhao, J. Zhou, and Z. Suo, "A theory of coupled diffusion and large deformation in polymeric gels," J. Mech. Phys. Solids **56**, 1779–1793 (2008).
[39]A. Ask, A. Menzel, and M. Ristinmaa, "On the modelling of electro-viscoelastic response of electrostrictive polyurethane elastomers," IOP Conf. Ser.: Mater. Sci. Eng. **10**, 012101 (2010).
[40]V. Racherla, "An electromechanical model for characterizing sensing and actuating performance of unimorphs based on "plain" dielectric polymers," Sens. Actuators, A **168**, 343–350 (2011).
[41]D. Guyomar, M. Lallart, and P.-J. Cottinet, "Electrostrictive conversion enhancement of polymer composites using a nonlinear approach," Phys. Lett. A **375**(3), 260–264 (2011).
[42]M. Lallart, C. Richard, P. Sukwisut, L. Petit, D. Guyomar, and N. Muensit, "Electrostrictive bending actuators: Modeling and experimental investigation," Sens. Actuators, A **179**, 169–177 (2012).
[43]A. Badel, M. Lagache, D. Guyomar, E. Lefeuvre, and C. Richard, "Finite element and simple lumped modeling for flexural nonlinear semi-passive damping," J. Intell. Mater. Syst. Struct. **18**, 727–742 (2007).
[44]D. A. Berlincourt, D. R. Curren, and H. Jaffee, "Piezoelectric and piezomagnetic materials and their function in transducers," in *Physical Acoustics*, edited by W. P. Mason (Academic, New York, 1964), Vol. I A, Eq. (30), p. 190.
[45]J. C. Piquette, "Quasistatic coupling coefficients for electrostrictive ceramics," J. Acoust. Soc. Am. **110**, 197–207 (2001).