In Collaboration With:

The Micro-scale and Interfacial Fluid Physics Laboratory

The Soft Materials and Matter Transport Group

PAPER-BASED MEMS SENSORS AND ASSOCIATED MATHEMATICAL MODELS

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Contents

Abstract

High-efficiency (i.e. performance/cost ratio) in devices becomes harder to achieve as fabrication costs escalate and the assemblies of elegant high-performance devices require more sophisticated material-manufacturing systems. Conceptually, however, devices such as piezo transducers simply create connections between mechanical stimuli and electrical resistance changes, which can be achieved through simple, well-designed device architectures. This allows for development of low-cost instrumentation-measurement systems which can be tailored to capture any quantity of interest with which it is interfaced. Piezoresistive micro-electro-mechanical systems (MEMS) sensor devices are such examples of elegant, simple, efficient architectures which are highly-tunable for any function. We achieve these devices via an intuitive and frugal approach: prepared paper-base as substrate material which is green and abundant; bonded a carbon-graphite ink resistor pattern (strain gauge) to paper-base; applied thin-film electrodes via stencil process. Utilizing the fundamentals of kirigami/origami, piezoresistive theory, and heat-free soldering, such paper-based devices can prove to be a highly efficient and facile option for electronic devices while minimizing global impact on waste and energy. Furthermore, use of a soft meta-material (liquid-metal ink) and the deformability of papers leads to easier access to wider, more unique form factors and scalable designs, without increasing the cost of fabrication significantly, leading to simple assemblies of highly-tunable electronic devices that are mechanochemically-processed and disposable.

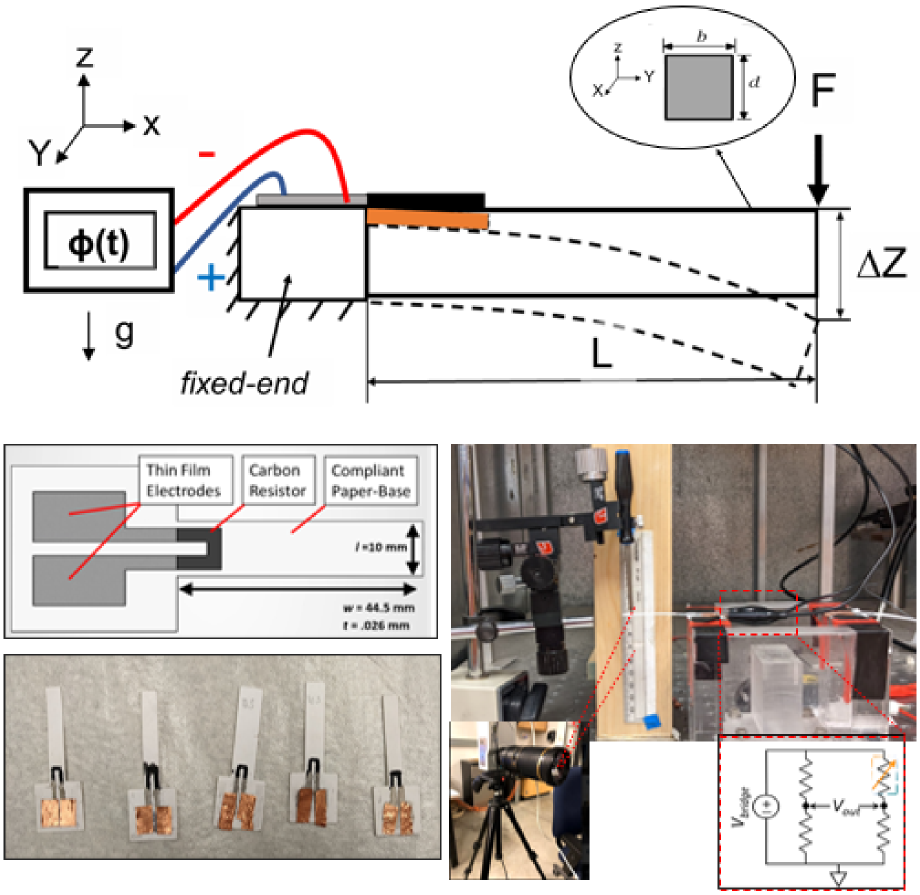

Figure 1: Cantilevered Beam Model with beam cross-section and power signal (i.) top-down view of physical model (ii.) rapid prototypes with varying geometric form-factors (iii.) experimental set-up.

I Introduction

In this manuscript I introduce guiding principals for probing the response of a power signal passed through a Piezoresistive Cantilevered Beam Sensor interfaced with a control volume. By transforming and processing signal from the transducer or sensing element, we visit the fundamental mathematical models for our paper-based scientific instrument and measurement system. The experimental method consists of a sensor calibration (To account for E-M coupling) and subsequent spectral analysis. While a power signal is passed through the sensor, the sensor tip position and deflection is also recorded. Next we further define our model and apply constraints to our problem. We can then derive beam properties and define a load in the controlled environment. After the signal has been anti-aliased, we start to build intuition about the response of our paper-based MEMS sensor by assuming ideal conditions in order to understand the dynamics taking place. Ultimately, this mathematical perspective serves as the foundation for elevating a frugally developed paper-based device to a highly-efficient measurement instrument.

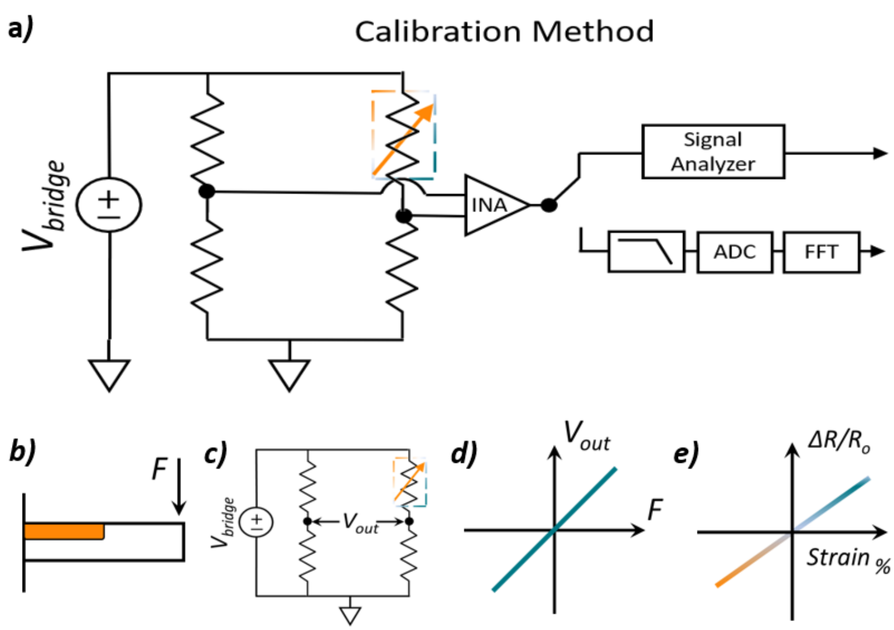

Figure 2: (a) Example Calibration Measurement System. (b) Load applied to top surface of sensor with PZR Strain Gauge (orange) while tip position and deflection was recorded. (c) The PZR power signal ϕ is transduced using a RL circuit with with a wheatstone bridge configuration. (d) Calibration of the electro-mechanical coupling of PZR Sensor. (e) Derivation of dimensionless sensor properties (Gauge Factor = slope).

II Experiment 1: MEMS Sensor Calibration

Piezoresistor (PZR) Sensor Calibration Method. The Wheatstone Bridge is balanced using a single potentiometer where the PZR strain gauge is treated as the variable resistor. The power signal output is amplified using an instrumentation amplifier (INA) and the spectral content of the power or noise signal is measured using a signal analyzer function. The signal is analyzed alternatively by passing the signal through an antialiasing filter, sampled with an analog-to-digital converter (ADC) and analyzed using the fast Fourier transform (FFT) method. The VBridge input into the wheatstone bridge was subject to DC (Direct Current) and AC (Alternating Current) bias throughout the measurements. The electro-mechanical coupling of the measurement system is accomplished through the PZR strain gauge bonded to the top surface of the table-top or cantilever beam.

II.I Sensor Properties

The corresponding mathematical equations for this process or operations are explained from the Direct Piezoresistive Effect with the following properties. Hence we arrive at the following dimensional and dimensionless properties for the sensor in equations 1-4.

Piezoresistor Properties

δ = Beam-Body Deflection
R = Strain Gauge or Resistor Signal
ε = Strain
ρ = Electrical Resistivity
L = Resistor Length
w = Resistor Width
A = Resistor Conducting Area
$\Delta R/R_0$ = Relative Change in Resistance
K = Strain Gauge Factor/Electromechanical Coupling
ν = Poission's Ratio

$$\varepsilon = \frac{\delta}{L} \tag{1}$$

$$R = \frac{\rho L}{w^2} = \rho\frac{L}{A} \tag{2}$$

$$R = \frac{\Delta R}{R_0} = (1 + 2\nu)\varepsilon + \frac{\Delta\rho}{\rho_0}; R = K * \varepsilon \tag{3}$$

$$K = \frac{\Delta R/R_o}{\varepsilon} = 1 + 2\nu + \frac{\Delta\rho/\rho_0}{\varepsilon} \tag{4}$$

After the above equations have been correlated with the calibration data and spectral content, the strain gauge sensing element, or reciever, has now characterized the piezoresistive electro-mechanical coupling coefficient "K" of the sensing element (i.e., Gauge Factor). To summarize, this calibration method accounts for any variation or error from the piezoresistor or strain gauge itself across measurements.

III Experiment 2: Spectral Analysis

In our system we define our time-series input $X(t)$ and power-energy signal as $\phi(t)$ with output signal $Y(t)$ and voltage drop $R(t)$. The signal has a frequency (f) and bandwidth "B".

We assume Parseval's Theorem [CITE] and arrive at the following:

$$X(t) = S_x(f) = \pi(\frac{f}{B}) \equiv |X\hat{(f)}|^2 \tag{5}$$

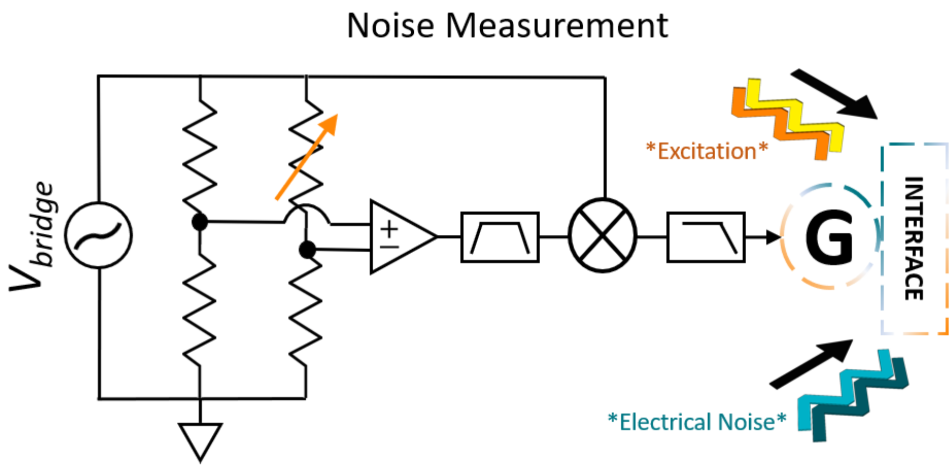

Figure 3: Noise Measurement System and Signal Conditioning. The experimental set-up from the calibration method is modified with a AC biased Vbridge, a bandpass filter, and a low-pass filter in series with the sensor interface. The sensor at the interface is sensitive to electrical and any observable excitation in the environment (i.e. mechanical).

Then Fourier Tables:

$$2B * sinc(2\pi Bt) \Leftrightarrow \frac{\pi f}{2B} \tag{6}$$

For a power-energy signal X(t) with auto-correlation function:

$$R_x(\tau) = 2Bsinc(2\pi Bt) \tag{7}$$

we have a anti-aliased Fourier Transform of the power signal $\phi(t)$ where $\delta(t) \equiv \phi(t) \equiv X(t)$ and τ is a standardized time constant.

$$S_x(f) = R_x\hat{(}f) = \frac{f\pi}{2B} \tag{8}$$

Compute total power of $X(t)$:

$$P_X = \int_{-\infty}^{\infty} R_x(f)\,df \;\; = \int_{-B}^{B} R_x(f)\,df \tag{9}$$

$$R_x(f) = X(t) = \frac{V_{out}/2}{V_{in}} = \frac{sin(x)}{sinc(x)} = \frac{Response}{Input} \tag{10}$$

Amplitude Response of Ideal Low Pass Filter (LPF):

$$|H(f)| = Y(t) = sinc(x) = \frac{sin(x)}{X} \tag{11}$$

Where

$$|H(f)|^2 = \frac{\pi f}{B}$$

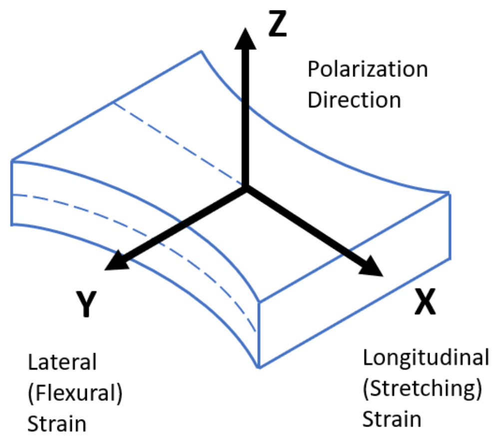

Figure 4: Coordinate System and Deformation Mechanisms at Neutral Axis Principal. Different deformation modes due to longitudinal (stretching) strain ε_x, lateral (flexural) strain ε_y, and polarization direction assumed to be out of plane. X in [mm] is the distance to spot displacement with tilt angle θ_X and spot deflection δ_X.

Fundamental Input-Output relation for Signals and Linear Time In-variant (LTI) Systems

$$S_Y(f) = |H(f)|^2 S_X(f)$$

Power Response:

$$P_Y = \int_{-\infty}^{\infty} S_y(f)\,df = \int_{-\infty}^{\infty} |H(f)|^2 S_X(f)\,df \tag{12}$$

Therefore Power Output of the system:

$$P_Y = \int_{-\infty}^{\infty} S_y(f)\,df = B \tag{13}$$

III.I Assumptions and Boundary Conditions

To construct our ideal model, we make a number of assumptions and assume ideal conditions in all aspects of the experiment. The sensor therefore is a finite cantilevered beam with fixed-free boundary conditions and uniaxial loading (or excitation) on the top surface-place. Assuming an ideal thermo-mechanical system, the response to work-energy input on the MEMS is recorded as total deformation of the body ε and dimensionless power signal R^*. Therefore we define our principal axes in Fig [4] where longitudinal (beam-wise) strain ε_x is measured as well as lateral strain ε_y. The degree of deformation (i.e. Polarization) at a plane/spot is the total body deformation $\delta \propto \varepsilon$ and surface slope of θ. Equation [15] is a spot deflection approximation where the third term is the dynamic deflection coefficient of the surface $(\frac{Q}{\omega_n^3})^{1/2}$ and the Power Spectral Density (PSD) at the natural frequency for the ladder calculated from $\phi(t)$.

Approximating Real-World Beam Deflection:

$$\theta_{max} = \pi * \frac{\delta_{max}}{L} \tag{14}$$

$$\delta_{max} = g * (\frac{1}{32\pi^3})^{1/2}(\frac{Q}{\omega_n^3})^{1/2}(PSD)^{1/2} \tag{15}$$

$$\delta(X) = 77 * (\frac{X}{L})(\frac{Q}{\omega_n^3})^{1/2}(PSD)^{1/2}; \delta_X = \theta_X * X \tag{16}$$

$$\varepsilon = \sqrt{\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2} = \frac{\delta_{max}}{L} \tag{17}$$

With geometry and relevant material,thermal, etc. properties in-hand, we may derive any of the following properties for the MEMS Sensor using the equations below. There are several other ways to approximate real-world beam deflection or force/energy terms which we will discuss later. But for the sake of simplicity, spot deflection $\delta(X)$ will assumed to capture the instantaneous motion of a spot in [mm]. This is explained in the section below.

IV Beam Properties

Cantilevered Beam With Uniaxial Loading

P = Load Applied
b = Base length or beam width
L = Beam length
t = Beam thickness
A = Cross-sectional Area
δ = Deflection
E = Elastic Modulus
G = Shear Modulus
K = Bulk Modulus
ν = Poisson's Ratio

Generalized Hooke's Law :

$$\nu = -\frac{lateral}{axial} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x} \tag{18}$$

$$\sigma = E\varepsilon = \frac{P}{A} = E\frac{\delta}{L} \tag{19}$$

Mechanics of Materials :

$$\varepsilon = \frac{\delta}{L} \tag{20}$$

$$K = \frac{E}{3(1-2\nu)} \tag{21}$$

$$\delta = \frac{PL}{EA} \tag{22}$$

$$\tau = G\gamma \tag{23}$$

$$G = \frac{E}{2(1+\nu)} \tag{24}$$

With all or most of the material properties in hand along with strain readings, we can derive the mechanical behavior of the sensor as well as the stress-strain tensors by probing the deflection term δ (i.e. Strain). Hence we arrive at the stress-strain for normal stress and shear.

Stress-Strain Tensor

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}; \gamma_{yz} = \frac{\tau_{yz}}{G}; \gamma_{xz} = \frac{\tau_{xz}}{G}$$

Continuing on with our derivations, the mechanical or materials properties we are really concerned with are with are the gradient volume change across the sensor or dilation. Dilation is often caused by hydrostatic stress which is simply pressure that acts uniformly across a surface or material. This then defines the direction and hydrostatic load distrubuted throughout the material of the sample. Firstly, we need to transform the dilation vector "e" in terms of strain, to normal stress.

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) \tag{25}$$

Following this definition further, in the case of hydrostatic stress (p):

$$p = \sigma_x = \sigma_y = \sigma_z \tag{26}$$

$$e = \frac{3(1-2\nu)}{E}p \tag{27}$$

IV.I Conservative Equations of Cantilevered Beam

- Euler-Bernoulli and Electric Current Conservation
- E-M Coupling and Tip-Mass
- Conservation of Moments and Force
- Dimensionless Variables for Damping and Free-Vibrations

These equations are the are the same as the "Damping mechanisms in a tip-mass piezoelectric cantilever system" paper

V Discussion

The reviewed equations are the bare minimum for direct measurements from the MEMS sensor. We may further develop this mathematical model by delving deeper into mechanics of materials, thermofluids, fluid mechanics, beam theory, and all other Euler energy equations which may be involved. By transforming the response signal of the sensor, for an ideal interaction, into qualities/quantities we can study to better understand the dynamics at play during these interactions.

VI Appendix

VII Acknowledgements

This project was possible through review of project related works: [1], ...

References

[1] Xinyu Liu et al. "Paper-based piezoresistive MEMS force sensors". In: *Proceedings of the IEEE International Conference on Micro Electro Mechanical Systems (MEMS)* (Feb. 2011), pp. 133–136. DOI: 10.1109/MEMSYS.2011.5734379.