

Homework 3

This homework is due by the end of the day on November 5th 2018. Solutions should appear as a vignette in your package called “homework-3”. If you do not want to write your solution for questions 2 - 4 in L^AT_EX, you may provide a solution using paper and pencil and include an image in your write-up.

1. CASL page 117, question 7.
 2. CASL page 200, question 3
 3. CASL page 200, question 4
 4. CASL page 200, question 5
 5. CASL page 200, question 6
-

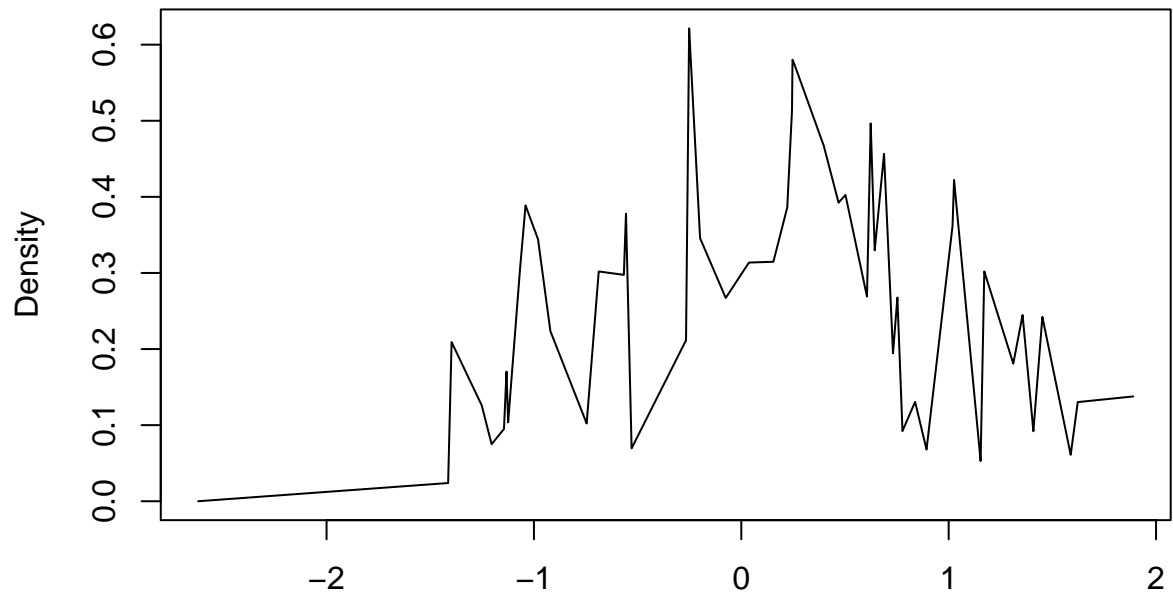
1.

```
set.seed(6909)
#write out the Epanechnikov kernel function
kernel_epan <- function(x) {
  ran <- as.numeric(abs(x) <= 1)
  val <- (3/4) * ( 1 - x^2 ) * ran
  return(val)
}

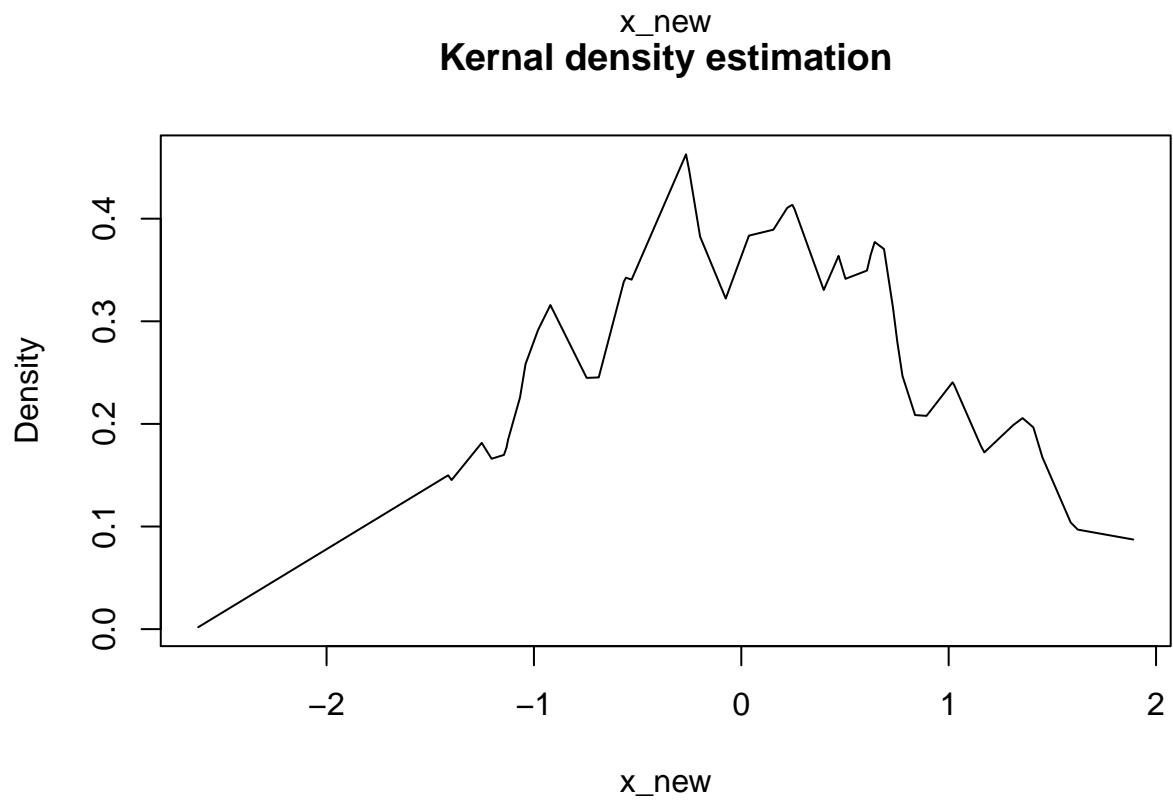
#write out the Epanechnikov density function
i = 1
h = 1
kern_density <- function(x, h, x_new){
  dst_est <- numeric()
  for (i in 1:length(x_new)){
    dst_est[i] <- mean(kernel_epan((x_new[i]-x)/h))/h
  }
  dst_est
}

#visualize different bandwidths
x <- rnorm(1000, 0, 1)
x_new <- sort(rnorm(50, 0, 1))
h = c(0.01, 0.1, 0.5, 1, 2, 3)
for (i in h){
  plot(x_new, kern_density(x,i,x_new), ylab = "Density", main = "Kernal density estimation", type = "l")
}
```

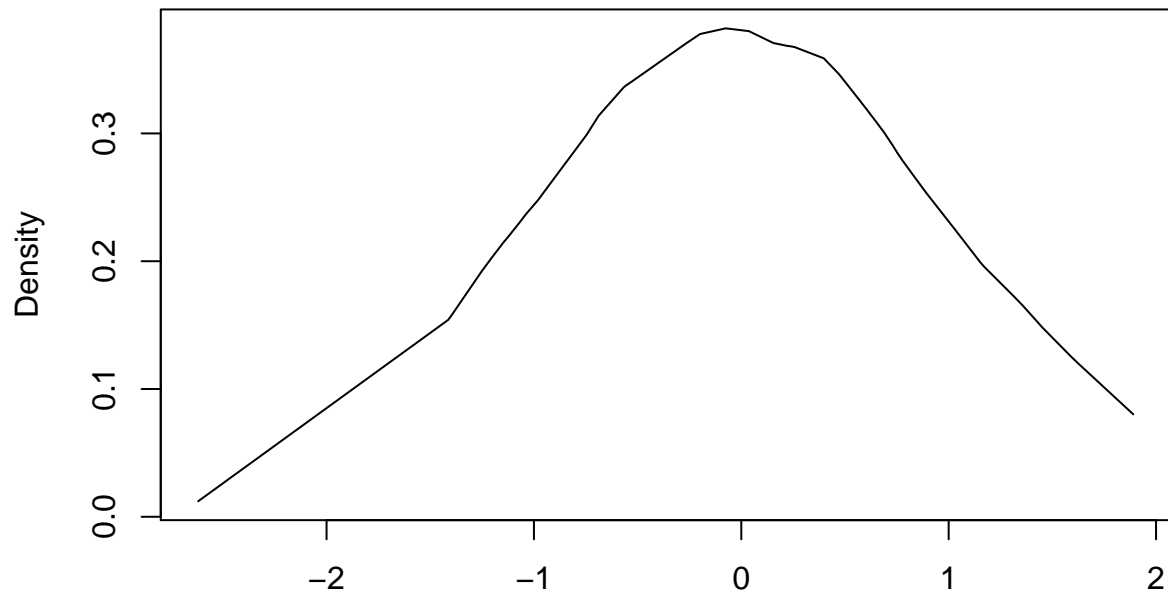
Kernal density estimation



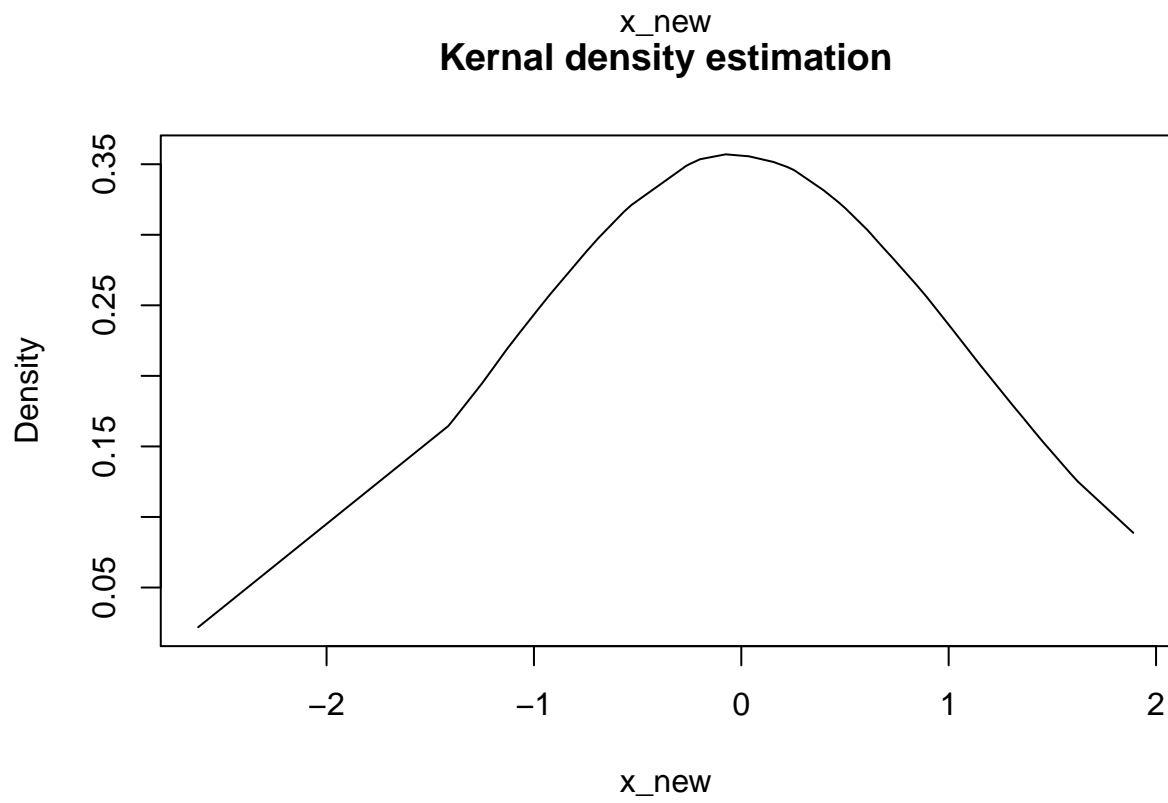
Kernal density estimation



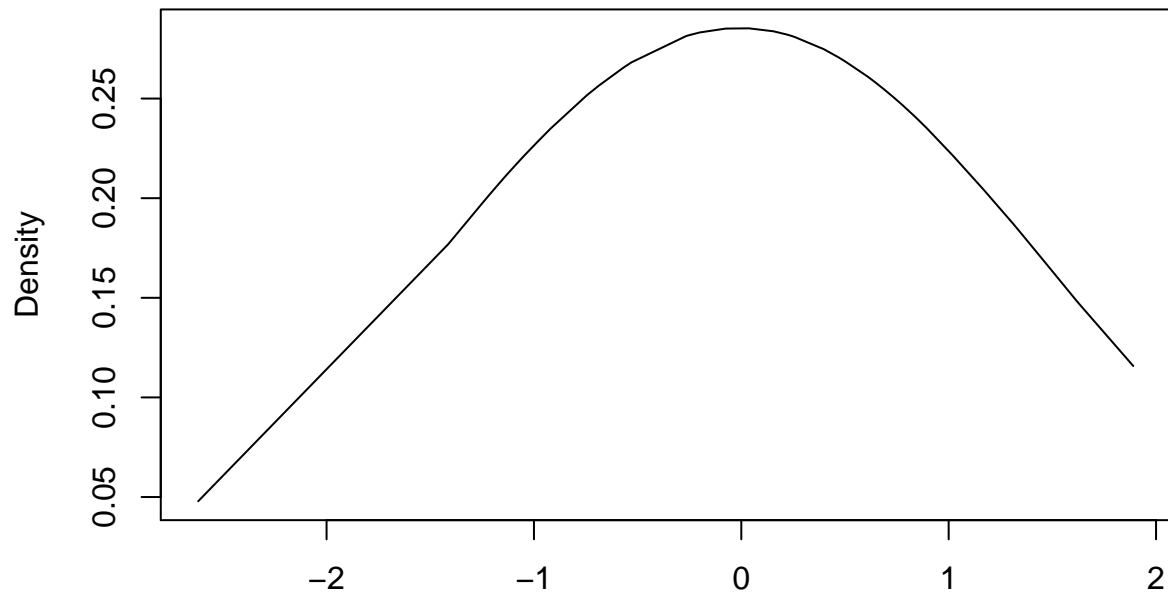
Kernal density estimation



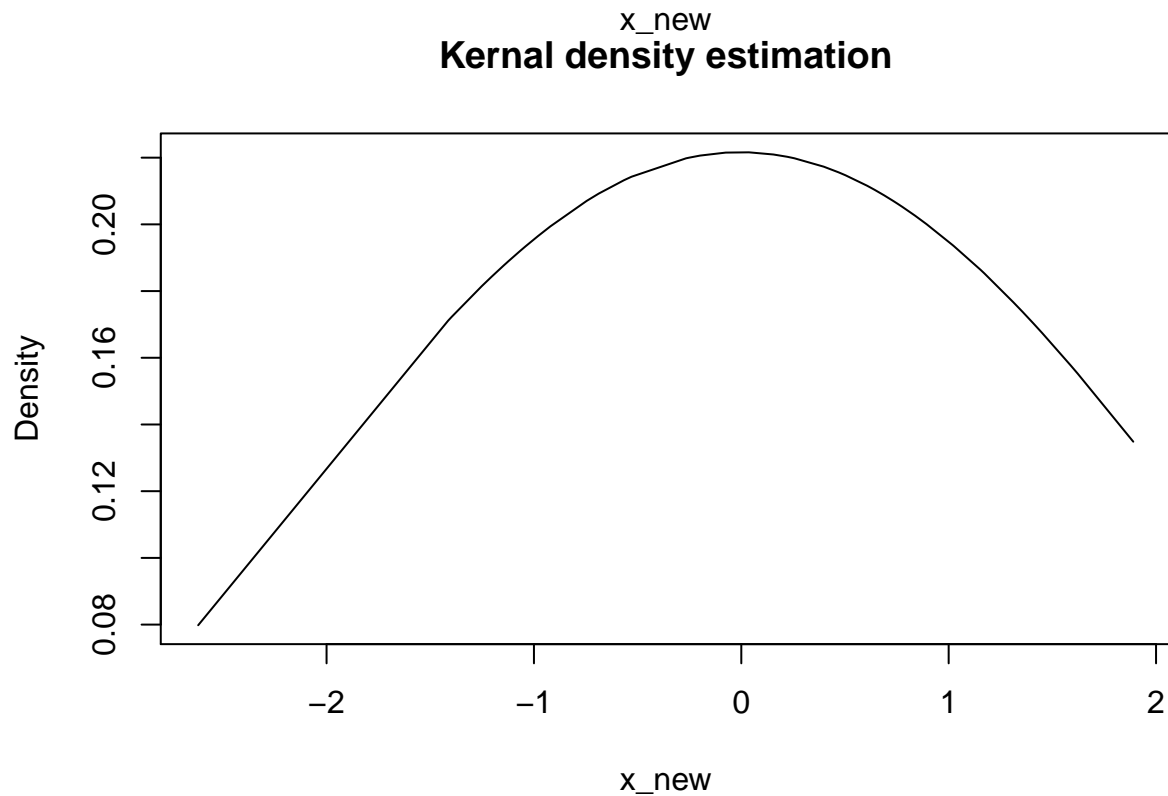
Kernal density estimation



Kernal density estimation



Kernal density estimation



From the plots above, as bandwidth increases, the estimations become smoother and closer to normal distribution. And when bandwidth increases to a certain value, the estimation remains similar no matter how bandwidth increases.

2.

By definition of convex function, for $t \in [0, 1]$

$$\begin{aligned} f(tx + (1-t)y) &\leq tf(x) + (1-t)f(y) \\ g(tx + (1-t)y) &\leq tg(x) + (1-t)g(y) \end{aligned}$$

Then

$$\begin{aligned} h(tx + (1-t)y) &= f(tx + (1-t)y) + g(tx + (1-t)y) \\ &\leq tf(x) + (1-t)f(y) + tg(x) + (1-t)g(y) \\ &= t(f(x) + g(x)) + (1-t)(f(y) + g(y)) \\ &= th(x) + (1-t)h(y) \end{aligned}$$

Therefore, $h = f + g$ is convex.

3.

we want to prove that $f(x) = |x|$ is convex. For $t \in [0, 1]$

$$\begin{aligned} f(x) &= |x| \\ f(tx + (1-t)y) &= |tx + (1-t)y| \\ &\leq |tx| + |(1-t)y| \\ &= f(tx) + f((1-t)y) \\ &= tf(x) + (1-t)f(y) \end{aligned}$$

Thus, $f(x) = |x|$ is convex.

the other question, \mathcal{L}_1 norm:

$$|x|_1 = \sum_{i=1}^n |x_i|$$

By the last question, it's also convex.

4.

l_2 norm

$$\begin{aligned} f(x) &= x^2 \\ f(tx + (1-t)y) &= t^2x^2 + (1-t)^2y^2 + 2t(1-t)xy \\ tf(x) + (1-t)f(y) &= tx^2 + (1-t)y^2 \\ f(tx + (1-t)y) - [tf(x) + (1-t)f(y)] &= t(t-1)x^2 + t(t-1)y^2 + 2t(1-t)xy \\ &= t(1-t)(-x^2 - y^2 + 2xy) \\ &= t(t-1)(x-y)^2 \\ &\leq 0 \quad \text{since } 0 \leq t \leq 1 \\ f(tx + (1-t)y) &\leq tf(x) + (1-t)f(y) \end{aligned}$$

Therefore, $f(x) = x^2$ is convex. Objective function of elastic net is

$$\frac{1}{2n} \|y - Xb\|_2^2 + \lambda[(1-\alpha)\frac{1}{2}\|b\|_2^2 + \alpha\|b\|_1]$$

It's a sum of l_2 norm and l_1 norm, from the previous two exercises, it's convex

5.

According to textbook page 189:

```
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loading required package: foreach
```

```
## Loaded glmnet 2.0-16
```

```
# KKT check function
```

```
#Return:
```

```
#A logical vector indicating where the KKT conditions have been violated by the variables that are curr
```

```
check_kkt <- function(y, X, b, lambda) {
```

```
  resid <- y - X %*% b
```

```
  s <- apply(X, 2, function(xj) crossprod(xj, resid)) / lambda / nrow(X)
```

```
  (b == 0) & (abs(s) >= 1)
```

```
}
```

```
##use iris as dataset
```

```
x <- scale(model.matrix(Sepal.Length ~. -1, iris))
```

```
y <- iris[,1]
```

```
#implement lasso regression
```

```
#Check which variables violate KKT condition
```

```
#Return: logical vector indicating where the KKT conditions have been violated
```

```
lasso_reg_with_screening <- function(x, y){
```

```
  m1 <- cv.glmnet(x,y,alpha=1)
```

```
  #use 1se as the criteria to choose lambda
```

```
  lambda <- m1$lambda.1se
```

```
  b <- m1$glmnet.fit$beta[, m1$lambda == lambda]
```

```
  check_kkt(y, x, b, lambda)
```

```
}
```

```
lasso_reg_with_screening(x,y)
```

```
##      Sepal.Width      Petal.Length      Petal.Width      Speciessetosa
```

```
##              FALSE              FALSE              FALSE              FALSE
```

```
## Speciesversicolor Speciesvirginica
```

```
##              FALSE              FALSE
```

We can observe that all coefficients get a false result for KKT violation, which indicates that no KKT violation (output can be seen in the homework-3.pdf file)