Sondage Stratifié

1:

Mâle: $\bar{Y}_h = 6$, $\sigma_h^2 = 4$ — 60 — 100 — 40 — Femelle: $\bar{Y}_h = 4$, $\sigma_h^2 = 2,25$

1/ $\bar{Y} \longrightarrow \sigma^2 = \sum_{h=1}^{n} \frac{N_h}{N} \cdot \sigma_h^2 + \sum_{h=1}^{n} \frac{N_h}{N} (\bar{Y}_h - \bar{Y})^2$

On a $\bar{Y} = \frac{1}{100} \times (60 \times 6 + 40 \times 4) = 5,2$

alors $\sum_{h=1}^{2} \frac{N_h}{N} \sigma_h^2 = \frac{60}{100} \times 4 + \frac{40}{100} \times 2,25 = 3,3$

$\sum_{h=1}^{2} \frac{N_h}{N} (\bar{Y}_h - \bar{Y})^2 = \frac{60}{100}(6-5,2)^2 + \frac{40}{100}(4-5,2)^2 = 0,96$

d'où $\sigma^2 = 3,3 + 0,96 = 4,26$

2/ $Var(\hat{T}_\pi) = N^2 \cdot Var(\hat{\bar{Y}}_\pi) = N^2 \cdot \frac{(1-f)}{m} \cdot \sigma_c^2 = \frac{1}{m}(1-f) \cdot N^2 \cdot \frac{N}{N-1} \sigma^2$

alors $Var(\hat{T}_\pi) = \frac{1}{m} \cdot N^2 (1-f) \cdot \frac{N}{N-1} \sigma^2 = \frac{1}{10} \cdot 100^2 (1-0,1) \cdot \frac{100}{99} \times 4,26$

$= 3872,73$

3/ $Var(\hat{T}_{prop}) = N^2 \cdot var(\hat{\bar{Y}}_{prop}) = N^2 \cdot \frac{1}{m}(1-f) \cdot \sum \frac{N_h}{N} \cdot \sigma_{hc}^2$

$= N^2 \cdot \frac{1}{m}(1-f) \cdot \sum \frac{N_h}{N} \frac{N_h}{N_h - 1} \sigma_h^2$

alors $Var(\hat{T}_{prop}) = \frac{N^2}{m} \cdot (1-f) \cdot \sum \frac{N_h^2}{N(N_h - 1)} \cdot \sigma_h^2$

$= \frac{100^2}{10}(1-0,1) \cdot \left(\frac{40^2}{100(39)} \cdot 2,25 + \frac{60^2}{100(59)} \cdot 4 \right) = 3027,38$

4/ $m_h = \frac{N_h \cdot \sigma_{hc}}{\frac{1}{m} \sum N_h \cdot \sigma_{hc}}$ où $\sigma_{hc} = \sqrt{\sigma_{hc}^2} = \sqrt{\frac{N_h}{N_h - 1} \cdot \sigma_h^2} = \sqrt{\frac{N_h}{N_h - 1}} \cdot \sigma_h$

$Var(\hat{T}_{opt}) = \frac{1}{m} \cdot \left(\sum N_h \cdot \sigma_{hc} \right)^2 - \sum N_h \cdot \sigma_{hc}^2 = \sum_{h=1}^{n} N_h^2 \cdot \frac{1}{m_h} (1 - f_h)$

valeur réel — valeur approchée

$m_{hF} = \frac{N_{hF} \, \sigma_{hcF}}{\frac{1}{m} \cdot \sum N_h \cdot \sigma_{hc}} = \frac{40 \cdot \sqrt{\frac{40}{39} \times 2,25}}{\frac{\left(60 \cdot \sqrt{\frac{60}{59} \cdot 4} + 40 \sqrt{\frac{40}{39} \cdot 2,25}\right)}{10}} = 3,34 \simeq 3$

$m_{hH} = \frac{60 \sqrt{\frac{60}{59} \times 4}}{*} = 6,66 \simeq 7$

$$\text{Var}(\hat{\bar{T}}_{opt}) = \sum_{h=1}^{2} \frac{N_h^2}{m_h}(1 - f_h)\,\sigma_{hc}^2 = \frac{60^2}{7}\left(1 - \frac{7}{60}\right)\frac{60}{59} \times 4$$
$$+ \frac{40^2}{3}\left(1 - \frac{3}{40}\right)\frac{40}{39} \times 2{,}25 = 2937{,}14$$

Exercice 2 :

Classe 1	Classe 2	Classe 3
$N_1 = 500$	$N_2 = 1000$	$N_3 = 2500$
$\hat{\bar{Y}}_1 = 10$	$\hat{\bar{Y}}_2 = 15$	$\hat{\bar{Y}}_3 = 20$
$S_{1c}^2 = 4$	$S_{2c}^2 = 7$	$S_{3c}^2 = 10$

$\underbrace{m_1 = m_2 = m_3 = 200}_{\text{Sondage à allocation fixe}}$

1/ $\hat{\bar{Y}}_{strat} = \sum_{h=1}^{3} \frac{N_h}{N} \hat{\bar{Y}}_{h\,pers}$ $\qquad N = 4000$

$$\hat{\bar{Y}}_{strat} = \frac{1}{4000}\left(500 \times 10 + 1000 \times 15 + 2500 \times 20\right) = 17{,}5$$

2/ $IC_{0,95}(\bar{Y}) = \left[\hat{\bar{Y}}_{strat} \pm 1{,}96 . \sqrt{\hat{\text{Var}}(\hat{\bar{Y}}_{strat})}\right]$

$$\text{Var}(\hat{\bar{Y}}_{strat}) = \sum_{h=1}^{3} \frac{N_h^2}{N^2}(1 - f_h)\frac{\sigma_{hc}^2}{m_h} = \sum_{h=1}^{3} \frac{N_h^2}{N^2}(1 - f_h)\frac{S_{hc}^2}{m_h} \qquad f_h = \frac{m_h}{N}$$

$$= \frac{500^2}{4000^2}\left(1 - \frac{200}{500}\right) . \frac{4}{200} + \frac{1000^2}{4000^2}\left(1 - \frac{200}{1000}\right) . \frac{7}{200} + \frac{2500^2}{4000^2}\left(1 - \frac{200}{2500}\right)\frac{10}{200}$$

$= 3{,}983$

alors $IC(\bar{Y}) = \left[17{,}5 \pm 1{,}96 . \sqrt{3{,}983}\right] = [13{,}588 \,;\, 21{,}412]$

Exercice 3 :

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Allocations proportionnelles :

$$\sigma^2_{CA1} = \frac{1}{12} \qquad \sigma^2_{CA2} = \frac{9^2}{12} \qquad \sigma^2_{CA3} = \frac{90^2}{12}$$

$$Var(\hat{\bar{Y}}prop) = \sum_{h=1}^{3} \frac{N_h^2}{N^2}(1-f)\frac{\sigma_{hc}^2}{m_h} = \frac{1-f}{mN}\sum_{h=1}^{3} N_h^2 \frac{\sigma_h^2}{(N_h-1)}$$

$$= \frac{1-\frac{111}{1110}}{111.\left(\frac{111}{1110}\right)}\left(\frac{1000^2}{1000-1}\times\frac{1}{12} + \frac{100^2}{(100-1)}\times\frac{9^2}{12} + \frac{10^2}{10-1}\times\frac{90^2}{12}\right)$$

$$= 669,48$$

Cas Optimal :

$$m_h = \frac{N_h.\sigma_{hc}}{\frac{1}{m}\sum_{K=1}^{m} N_h.\sigma_{hc}} \Rightarrow N_1.\sigma_{1c} = 1000\sqrt{\frac{1000}{999}.\frac{1}{12}} = 288,81$$

$$N_2.\sigma_{2c} = 100\sqrt{\frac{100}{99}.\frac{9^2}{12}} = 26,11$$

$$N_3.\sigma_{3c} = 10.\sqrt{\frac{10}{9}.\frac{90^2}{12}} = 273,86$$

$$m_1 = 111 \times \frac{288,81}{261,1 + 288,81 + 273,86} = 38,91$$

$$m_2 = 111 \times \frac{261,1}{261,1 + 288,81 + 273,86} = 35,18$$

$$m_3 = 111 \times \frac{273,86}{261,1 + 288,81 + 273,86} = 36,9$$

parce que la valeur dépasse la taille $\Rightarrow$ max $=10$

on pose $m'_3 = 10 \Rightarrow m' = 111 - 10 = 101$

alors $m'_1 = 101 \times \dfrac{288,81}{261,1 + 288,81} = 53,11$

$$m'_2 = 101 \times \frac{261,1}{261,1 + 288,81} = 48,01$$

on prend $m'_1 = 53 \quad m'_2 = 48 \quad m'_3 = 10$

$$Var(\hat{\bar{Y}}opt) = \sum_{h=1}^{3} \frac{N_h^2}{N^2}\left(1 - \frac{m_h}{N_h}\right)\frac{\sigma_h^2}{m_h}$$

$$= \frac{1}{1110^2}\left(1000^2\left(1 - \frac{53}{1000}\right)\left(\frac{1}{12}\right).\frac{1}{53} + \frac{100^2}{48}\left(1 - \frac{48}{100}\right)\frac{9^2}{12}\right)$$