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XISAAB

Fasalka Afraad

4

DUGSIGA SARE

XISAAB

Fasalka Afraad

4

DUGSIGA SARE

H O R D H A C

Buuggan xisaabta ah waxa loogu talagalay Fasalka afraad ee Dugsiga Sare, waxana la filayaa in uu si habsan u fuliyo fikradaha xisaabta ah ee ardayga marxaladani u baahan yahay.

Buuggani sal ahaan waa kii horey loogu isticmaali jiray dugsiyadeena ee ku qoraan jirey afka ingiriiska, hase ahaatee cutub kasta waa la ballaariyay, cutubyo dhawr ahna waa lagu kordhiyay.

Buuggan waxa wadajir u qoray jaallayaasha kala ah Xasan Daahir Obsiye, Xuseen Max'd Xaaji Cumar (Xanaan) Max'd Cali Muuse, Ibraahim Aw Aaden. Muuse Cabdi Cilmi, Cusmaan Aaden (Badawi) Max'd Cabdulle Bariir, Max'd Saciid Samatar, iyo Max'd Cabdiraxmaan Yuusuf. Sawirrada waxa sameeyey Cabdiraxmaan Cali Maxamed. Max'd Cali iyo Cabdullaahi Riyaale. Waxa mahad gaar ah leh Max'd Cali iyo Cabdullaahi Riyaale. Waxa mahad gaar ah leh Max'd Aw Daahir Cabdi (Gallan) oo isku dubbariday aadna u lafaguray buugan.

Waxaa kale oo aan u mahad naqaynaa Jaalle Aadan Maxamed Macallin Qaasim (maato) oo saxay buuggan una diyaariyey daabacaadda labaad.

Mahad lamo ilaawaan ah waxa leh Madbacadda Qaranka oo suuragelisay soo bixit-aanká buuggan,

*Maamulaha Xafiiska Manaahijta
Xasan Daahir Obsiye*

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CUTUB I

TRIGNOOMETERI

XAGLO ISKU DHISAN

Waa lays moodsin karaa in $\cos(A - B)$ oggoshahay astaanta kala dhigga oo markaa

$$\cos(A - B) = \cos A - \cos B.$$

Aan is tusaalayno in aan arrintaasi run ahayn. Bal ka soo qaad in $A = 90^\circ$; B -na = 30° .

$$\text{Markaa } \cos(A - B) = \cos(90 - 30) = \cos 60 = \frac{1}{2}.$$

$$\text{Laakiinse } \cos A - \cos B = \cos 90 - \cos 30 = -\frac{\sqrt{3}}{2}. \text{ Sidaan aragno,}$$

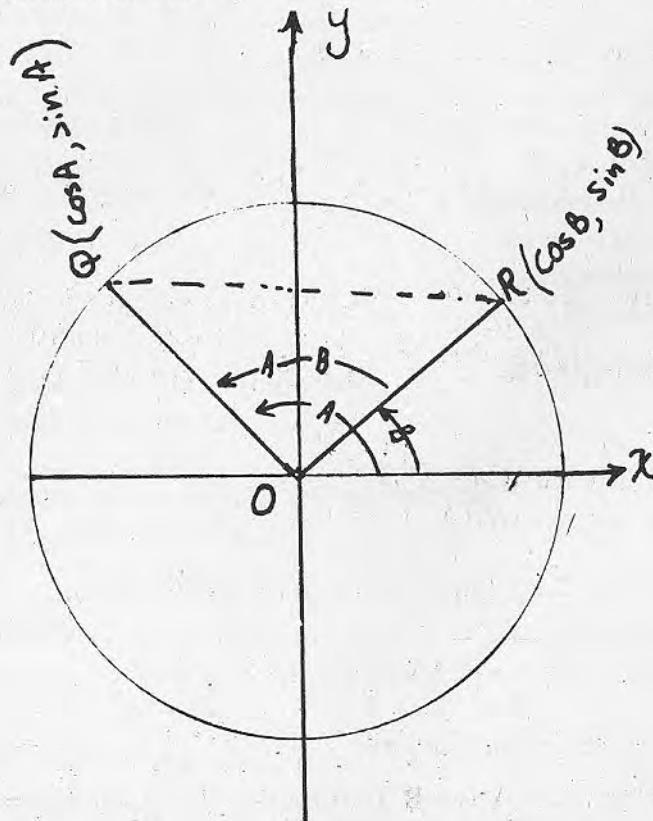
$$\cos(A - B) \neq \cos A - \cos B.$$

Aragtiin 1

Xaglo kasta oo A iyo B ah, $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

Caddeyn :

Sawir goobo gacan halbeegga leh; xuddunteeduna unugga O tahay. A iyo B ha ahaadeen sida sawirkha hoose muujinayo oo kale.



Shax 1

Mar haddii gacanka goobadu 1 yahay markaa kulannada R iyo Q waa $(\cos B, \sin B)$ iyo $(\cos A, \sin A)$ sida ay u kala horreeyaan.

Marka hore aan midiidsanno jidka fogaanta, waxaan heleynaa

$$\begin{aligned} RQ^2 &= (\cos A - \cos B)^2 + (\sin A - \sin B)^2 \\ &= \cos^2 A - 2 \cos A \cos B + \cos^2 B + \sin^2 A - 2 \sin A \sin B + \sin^2 B \\ &= 2 - 2 \cos A \cos B - 2 \sin A \sin B \end{aligned}$$

Tixgeli :

Marka xiga, \triangle ROQ, waxaan aragnaa in \angle ROQ = $\angle A - \angle B$.

Midiidsiga jidka kosaynu wuxuu ina siinaya, in $RQ^2 = OR^2 + OQ^2 - 2 OR \cdot OQ \cos(A - B)$. Laakiinse $OR = OQ = 1$:

$$\therefore RQ^2 = 2 - 2 \cos(A - B).$$

$$\therefore 2 - 2 \cos A \cos B - 2 \sin A \sin B = 2 - 2 \cos(A - B) \\ - 2 \cos(A - B) = -2 \cos A \cos B - 2 \sin A \sin B$$

$$\therefore \cos(A - B) = \cos A \cos B + \sin A \sin B$$

(1)

Aragtiin 2

Xaglo kasta oo A iyo B ah, $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

Caddeyn :

Isle'egta (1) haddii aan B ku beddolo — B, waxan heleyntaa $\cos[(A - (-B))] = \cos A \cos(-B) + \sin A \sin(-B)$. Waxaanse ognahay in $\cos(-B) = \cos B$ iyo $\sin(-B) = -\sin B$. Sidaa awgeed.

$$\cos[(A - (-B))] = \cos A \cos B + \sin A (-\sin B) \text{ ama}$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

(2)

Aragtiin 3

Xaglo kasta oo A iyo B ah, $\sin(A - B) = \sin A \cos B - \cos A \sin B$.

Caddeyn :

Isle'egta (1), A ku beddel $(90^\circ + A)$. Markaa isle'egta (1) waxay noqonaysaa, $\cos[(90^\circ + A) - B] = \cos(90^\circ + A) \cos B + \sin(90^\circ + A) \sin B$.

[Laakiinse, $\cos(90^\circ + A) = -\sin A$, waajiba $\sin(90^\circ + A) = \cos A$

$$\cos[90^\circ + A - B] = \cos[(90^\circ + A) - B] \\ = -\sin(A - B).$$

$$\therefore -\sin(A - B) = -\sin A \cos B + \cos A \sin B \text{ ama}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

(3)

Aragtiin 4

Xaglo kasta oo A iyo B ah, $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

Caddeyn :

Isle'egta (1), A ku beddel $(90^\circ - A)$. Markaa

$$\cos[(90^\circ - A) - B] = \cos(90^\circ - A) \cos B + \sin(90^\circ - A) \sin B.$$

Laakiinse, waxaan naqaan in $\cos(90^\circ - A) = \sin A$, iyo in $\sin(90^\circ - A) = \cos A$.

$$\cos[(90^\circ - A) - B] = \sin A \cos B + \cos A \sin B.$$

Waxaan kaloon naqaan in

$$\cos [90^\circ - A] = \cos [90^\circ - (A + B)] = \sin (A + B).$$

$$\sin (A + B) = \sin A \cos B + \cos A \sin B \quad (4)$$

Tusaale 1 :

Adoon tusaha trignoometeriga la kaashan, raadi :

b. $\cos 75^\circ$ t. $\sin 75^\circ$

Furfuris :

b. $\cos 75^\circ = \cos (30^\circ + 45^\circ) = \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ.$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

t. $\sin 75^\circ = \sin (30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

Tusaale 2 :

Adoo isticmaalayaa qiimayaashan seebidda ah ee $\cos 37^\circ = 0.80$, $\sin 37^\circ = 0.60$, $\cos 40^\circ = 0.77$, iyo $\sin 40^\circ = 0.64$. Raadi : b) $\cos 77^\circ$ t) $\sin 3^\circ$

Furfuris :

b. $\cos 77^\circ = \cos (37 + 40)$
 $= \cos 37 \cos 40 - \sin 37 \sin 40$
 $= (0.80)(0.77) - (0.60)(0.64)$
 $= 0.616 - 0.384 = 0.232$

t. $\sin 3^\circ = \sin (40 - 37)$
 $= \sin 40 \cos 37 - \cos 40 \sin 37$
 $= (0.64)(0.80) - (0.77)(0.60)$
 $= 0.5120 - 0.4620 = 0.05$

Tusaale 3 :

Haddii $\sin(x + a) = \cos(x - b)$, ku raadi tan x xaglaha a iyo b.

Furfuris :

Mar haddii $\sin(x + a) = \cos(x - b)$ waxaan haynaa in
 $\sin x \cos a + \cos x \sin a = \cos x \cos b + \sin x \sin b$.

$$\therefore \sin x (\cos a - \sin b) = \cos x (\cos b - \sin a)$$

$$\therefore \frac{\sin x}{\cos x} = \frac{\cos b - \sin a}{\cos a - \sin b}$$

$$\tan x = \frac{\cos b - \sin a}{\cos a - \sin b}$$

Aragtiin 5

Xaglo kastao A iyo B ah,

$$b. \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$t. \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Caddeyn :

$$b. \text{Waxaan naqaan in } \tan x = \frac{\sin x}{\cos x} \text{ sidaa awgeed,}$$

$$\tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)}$$

$$= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B + \sin A \sin B}$$

Hooseeyaha iyo sarreeyahaba u qaybi $\cos A \cos B$, oo midna eber ahayn. Waxaynu heleynaa.

$$\frac{\sin A \cos B}{\cos A \cos B} = \frac{\cos A \sin B}{\cos A \cos B}$$

$$\tan(A - B) = \frac{\sin A \cos B}{\cos A \cos B}$$

$$= \frac{\cos A \cos B + \sin A \sin B}{\cos A \cos B}$$

$$= \frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}$$

$$= \frac{1 + \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}}{1}$$

$$= \frac{\tan A - \tan B}{1 + \tan A \tan B} \quad (5)$$

Caddeyn :

t. Ardeyda laylis ahaan looga tegey.

Layli :

- 1) Adoo isticmaalaya qiimayaashan seebidda ah, ee $\cos 27^\circ = 0.89$, $\sin 27^\circ = 0.45$, $\cos 44^\circ = 0.72$, $\sin 44^\circ = 0.89$, Raadi qiimaha ;
 - b) $\sin 170^\circ$
 - t) $\cos 71^\circ$
 - j) $\cos 54^\circ$
- 2) Adeegso aragtiinyadii wadarta iyo faraqyada xaglahaa soona saar qiimaha runta ah ee mid kasta oo soo socda :

b. $\tan 75^\circ$	t. $\sin 15^\circ$
j. $\sin 75^\circ$	x. $\tan 15^\circ$
kh. $\cos 285^\circ$	d. $\sin 285^\circ$
r. $\tan 255^\circ$	s. $\cos 165^\circ$
sh. $\sin 195^\circ$	dh. $\cos 165^\circ$

- 3) Haddii A tahay xagal waaxda I ku taalla $\sin A = \frac{4}{5}$, B-na tahay xagal waaxda II ku taal oo $\cos B = -\frac{51}{149}$, raadi mid kastoo kuwaa soo socda ah :
- b. $\sin(A + B)$
 - t. $\cos(A + B)$
 - j. $\sin(A - B)$
 - x. $\cos(A - B)$
 - kh. $\tan(A + B)$
 - d. $\tan(A - B)$
- 4) Haddii α tahay xagal ku taal waaxda III, oo $\sin \alpha = -\frac{8}{17}$, β -na tahay xagal ku taal waaxda I oo $\sec B = \frac{5}{3}$, raadi mid kastoo kuwan soo sooda ah :
- b. $\tan(\alpha + \beta)$
 - t. $\cot(\alpha + \beta)$
 - j. $\cos(\alpha + \beta)$
 - x. $\sin(\beta - \alpha)$
 - kh. $\tan(\alpha - \beta)$
 - d. $\cot(\alpha - \beta)$
 - r. $\cos(\beta + \alpha)$
 - s. $\sin(\beta + \alpha)$
- 5) **Kala bixi kuwan :**
- b. $\sin(A - 30^\circ)$
 - t. $\cos(0^\circ + 120^\circ)$
 - j. $\sin(B + B)$
 - x. $\cos(B + B)$
 - kh. $\tan(B + B)$
- 6) **Raadi sansaan oo u dhigma :**
- b. $\cos 2A \cos A + \sin 2A \sin A$
 - t. $\cos 3A \sin A + \sin 3A \cos A$
 - j. $\sin 3B \cos B - \cos 3B \sin B$
 - x. $\sin 2B \sin B - \cos 2B \cos B$
 - kh. $\cos 20^\circ \cos 10^\circ - \sin 20^\circ \sin 10^\circ$
 - d. $\cos 140^\circ \cos 20^\circ + \sin 140^\circ \sin 20^\circ$
 - r. $\sin(A + B) \cos A - \cos(A + B) \sin A$
 - s. $\cos(A + B) \cos(A - B) + \sin(A + B) \sin(A - B)$
 - sh. $\sin(A - B) \sin B - \cos(A - B) \cos B$
- 7) Bayaani in $\tan(120^\circ + x) = \frac{\sin x - \sqrt{3} \cos x}{\cos x + \sqrt{3} \sin x}$
- Caddee mid kasta kuwan soo socda ka mid ah.
- 8) $\tan(135^\circ - x) = \frac{\sin x + \cos x}{\sin x - \cos x}$
- 9) $\frac{\cos(A + B)}{\cos(A - B)} = \frac{1 - \tan A \tan B}{1 + \tan A \tan B}$
- 10) $\frac{\sin(A + B)}{\cos(A - B)} = \frac{\sin A + \cos A \tan B}{\cos A + \sin A \tan B}$
- 11) Haddii $\sin(x - a) = \cos(x + a)$, caddee in $\tan x = 1$.
- 12) Caddee in $\tan(x + 45^\circ) \tan(x - 45^\circ) = -1$.
- 13) Caddee in $\tan(A + B) - \tan A = \frac{\sin B}{\cos A \cos(A + B)}$
- 14) Kala bixi $\sin(A + B + C)$
- 15) Kala bixi $\cos(A - B - C)$

$$16) \text{ Haddii tan A} = \frac{q \sin B}{R + q \cos B}$$

$$\text{Caddee in tan}(B-A) = \frac{R \sin B}{q + R \cos B}$$

LABALAAB IYO BADHKA XAGALEED

Aan ka bilawno jidodka kala bixinta ee 1—5; waxaan gaari doonaa xiriiro badan oo ay labalaabka iyo kala barka xagluhu leeyihii.

JIDADKA XAGLO LABALAABAN.

$$\begin{aligned} b) \cos 2A &= \cos(A+A) = \cos^2 A - \sin^2 A \\ &= (1 - \sin^2 A) - \sin^2 A = 1 - 2 \sin^2 A. \\ &= \cos^2 A - (1 - \cos^2 A) = 2 \cos^2 A - 1. \end{aligned}$$

$$\therefore \cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1 \quad (6)$$

Haddaynnu rabno inaan cos A u furfurro isle'egta $\cos 2A = 2 \cos^2 A - 1$, waxaan heleynaa,

$$2 \cos^2 A = 1 + \cos 2A$$

$$\therefore \cos A = \pm \sqrt{\frac{1 + \cos 2A}{2}}$$

$$\begin{aligned} t) \sin 2A &= \sin(A+A) \\ &= \sin A \cos A + \cos A \sin A \end{aligned}$$

$$\text{ama } \sin 2A = 2 \sin A \cos A \quad (7)$$

$$\text{Isle'egta, } \cos 2A = 1 - 2 \sin^2 A, \text{ waxay ina siisaa } \sin A = \pm \sqrt{\frac{1 - \cos 2A}{2}}$$

$$j) \tan 2A = \tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$\text{ama } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \quad (8)$$

JIDADKA XAGAL BADHEED

Jidodka xaglo kala badhan waxa hawl yari laga helikaraa xiriiraha aan soo gudubnay, ama waxaa toos looga soo diri karaa jidakii kala bixinta.

Haddaan A ku beddello $\frac{A}{2}$, waxaan heleynaa :

$$\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} = 1 - 2 \sin^2 \frac{A}{2} = 2 \cos^2 \frac{A}{2} - 1. \quad (9)$$

Welib :

$$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2} \quad (10)$$

iyoo

$$\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}} \quad (11)$$

Tusaale 1 :

Haddii $\cos \phi = \frac{3}{5}$ oo $0 < \phi < \frac{\pi}{2}$; raadi

b) $\cos 2\phi$ t) $\sin 2\phi$ j) $\cos \frac{\phi}{2}$.

Furfuris :

b) Haddaan la kaashanno jidka (6), waxaan hellaa,
 $\cos 2\phi = 2 \cos^2 \phi - 1$,

$$\cos 2\phi = 2 \left[\frac{3}{5} \right]^2 - 1 = -\frac{7}{25}$$

t) Jidka (7) wuxuu ina siinayaa,
 $\sin 2\phi = 2 \sin \phi \cos \phi$.

$$\text{Laakiin } \sin \phi = \pm \sqrt{1 - \cos^2 \phi}$$

$$= \pm \sqrt{1 - \left[\frac{3}{5} \right]^2} = \pm \frac{4}{5}$$

Haddaba, waxaan ognahay in $0 < \phi < \frac{\pi}{2}$, taas oo ah, $\sin \phi > 0$,

$$\therefore \sin \phi = \frac{4}{5}$$

$$\therefore \sin 2\phi = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

j) Jidka (9) wuxuu ina siinayaa,

$$2 \cos^2 \frac{\phi}{2} = 1 + \cos \phi$$

$$\cos \frac{\phi}{2} = \sqrt{\frac{1 + \cos \phi}{2}} = \sqrt{\frac{1 + 3/5}{2}} = \sqrt{\frac{8}{5 \times 2}} = \sqrt{\frac{4}{5}} = 2 \sqrt{\frac{1}{5}} = \frac{2\sqrt{5}}{5}$$

O g o w : Waxaan qaadanay xidid labajibbaarka togan, taana waxay muujinaysaa in-

$$0 < \phi < \frac{\pi}{2} \longrightarrow 0 < \frac{\phi}{2} < \frac{\pi}{2}$$

Sida awgeed $\cos \frac{\phi}{2} > 0$.

T u s a a l e 2 :

$$\text{Caddee midaalka } \tan^2 \phi = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Caddeyn :

$$\begin{aligned}\frac{1 - \cos 2\phi}{1 + \cos 2\phi} &= \frac{1 - (1 - 2\sin^2\phi)}{1 + (1 - 2\sin^2\phi)} = \frac{2\sin^2\phi}{2(1 - \sin^2\phi)} = \frac{\sin^2\phi}{1 - \sin^2\phi} \\ &= \frac{\sin^2\phi}{\cos^2\phi} = \tan^2\phi\end{aligned}$$

Layli :

- 1) Hadduu $\sin\phi = \frac{4}{5}$, oo $0 < \phi < \pi$, raadi
- b. $\cos 2\phi$ t. siin $\frac{\phi}{2}$
- 2) Raadi qimayaasha, $\sin 2\phi$, $\cos 2\phi$ iyo $\tan 2\phi$ adoon tuse isticmaalin. Marka
- b. $\sin\phi = \frac{3}{55}$ x. $\tan\phi = \frac{8}{15}$
- t. $\cos\phi = \frac{12}{13}$ kh. $\cos\phi = \frac{-5}{12}$
- j. $\sin\phi = \frac{\sqrt{3}}{2}$
- 3) Qiimee:
- b. $1 - 2\sin^2 75^\circ$ t. $\cos^2 165^\circ - \sin^2 165^\circ$
- j. $2\sin 225^\circ$ x. $\frac{2\tan 105^\circ}{1 - \tan^2 105^\circ}$
- 4) U furfur ϕ min 0° ilaa 360° oo ku jirid ah.
- b. $\cos 2\phi + \cos\phi + 1 = 0$
 t. $\sin 2\phi = \sin\phi$
 j. $\cos 2\phi = \sin\phi$
 x. $\cos 2\phi - 2 = 0$
 kh. $\sin 2\phi \cos\phi + \sin^2\phi = 1$
 sh. $2\sin\phi(5\cos 2\phi + 1) = 3\sin 2\phi$
 dh. $3\tan\phi = \tan 2\phi$
 d. $3\cos 2\phi + \cot\phi = 1$
 r. $2\tan\phi \tan 2\phi = 1$
- 5) Ka saar ϕ isle'egyada.
- 1) $x = \cos\phi$, $y = \cos 2\phi$
 2) $x = 2\sin\phi$, $y = 3\cos 2\phi$
 3) $x = \tan 0$, $y = \tan 2\phi$
 4) $x = 2\sec$, $y = \cos 2\phi$
- 6) Fududee :
- 1) $\frac{1 + \cos A}{2}$ t. $\frac{\cos B}{1 + \cos 2B}$
- b. _____ j. $\cos^4 A - \sin^4 A$
- 1) $\frac{1 - \cos \frac{A}{2}}{2}$ x. $2 - \sin x \tan \frac{x}{2}$

Caddee mid kasta oo kuwaan soo socda ka mid ah.

$$7) \frac{\sec A \cos 2A}{\cos A + \sin A} = 1 - \tan A$$

$$8) \frac{\cos 2B + \cos B + 1}{\sin 2B + \sin B} = \cot B$$

$$\sin^2 \frac{A}{2} + \cos A$$

$$9) \frac{\cos \frac{A}{2}}{2} = \cos \frac{A}{2}$$

$$10) 2 \cot x = \frac{\sin 2x}{1 - \cos^2 x}$$

$$11) \cot^2 \frac{x}{2} - \tan^2 \frac{x}{2} = 4 \cot x \cdot \csc x$$

$$12) \frac{\sin 2A}{1 + \cos 2A} = \tan A = \frac{1 - \cos 2A}{\sin 2A}$$

$$13) \cos 3A = 4 \cos^3 A - 3 \cos A$$

$$14) \tan x = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$$

$$15) 2 \cos(45^\circ - \theta) \cdot \sin(45^\circ + \theta) = 1 + \sin 2\theta$$

$$16) (1 + \cos 2A) \cdot (2 \cos 2A - 1) = 2 \cos A \cos 3A$$

$$17) \text{Haddii } x = \cos \theta + \cos 2\theta, y = \sin \theta + \sin 2\theta \text{ bayaami in }$$

$$x^2 - y^2 = \cos 2\theta + 2 \cos 3\theta + \cos 4\theta \text{ iyo in } 2xy = \sin 2\theta + 2 \sin 3\theta + \sin 4\theta$$

$$18) \sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$19) \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$20) \text{Haddii } t = \tan \frac{\theta}{2}, \text{kuwaan soo socda mid kasta u tibaax t.}$$

$$\text{b. } 1 + \sin \theta$$

$$\text{t. } \sec^2 \frac{\theta}{2} (3 \sin \theta + 4 \cos \theta - 1)$$

$$\text{j. } 1 + \cos \theta$$

$$\text{x. } \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}}$$

$$\text{kh)} \frac{3 \cos \theta + 1}{1 - 2 \sin \theta}$$

MIDAALLADA WADAR IYO TARAN

Aragtiinyadii (1) iyo (2), go'aannadoodu waxay ahaayeen :

$$\begin{aligned}\cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B\end{aligned}$$

Labada isugeyntoodu waa :

$$\cos(A - B) + \cos(A + B) = 2 \cos A \cos B \quad (12)$$

Kalagayntooduna waa :

$$\cos(A - B) - \cos(A + B) = 2 \sin A \sin B \quad (13)$$

Jidka (3) iyo (4) ee

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \sin(A - B) &= \sin A \cos B - \cos A \sin B\end{aligned}$$

Isugeyntooduna :

$$\sin(A + B) + \sin(A - B) = 2 \sin A \cos B \quad (14)$$

Kalagoynthooduna :

$$\sin(A + B) - \sin(A - B) = 2 \cos A \sin B \quad (15)$$

Tusaale :

U tibaax wadar ama faraq kosaynyo iyo saynyo :

- b. $2 \cos 70^\circ \cos 15^\circ$
- t. $\cos 70^\circ \sin 15^\circ$
- j. $2 \sin 70^\circ \sin 15^\circ$

Furfuris :

- b. Istimaal (12), waxaad heli

$$\begin{aligned}2 \cos 70^\circ \cos 15^\circ &= \cos(70^\circ + 15^\circ) + \cos(70^\circ - 15^\circ) \\ &= \cos 85^\circ + \cos 55^\circ\end{aligned}$$

- t. Iisticmaal (15), waxaad heli,

$$\begin{aligned}\cos 70^\circ \sin 15^\circ &= \frac{1}{2} [\sin(70^\circ + 15^\circ) - \sin(70^\circ - 15^\circ)] \\ &= \frac{1}{2} \sin 85^\circ - \frac{1}{2} \sin 55^\circ\end{aligned}$$

- j. Iisticmaal (13), waxaad heli,

$$\begin{aligned}2 \sin 70^\circ \sin 15^\circ &= \cos(70^\circ - 15^\circ) - \cos(70^\circ + 15^\circ) \\ &= \cos 55^\circ - \cos 85^\circ\end{aligned}$$

WADARO IYO FARAQYO TARANEED

Aragtiinyadii (1) iyo (2) waxay oranayeen

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

Haddaba, ka soo qaad in $A + B = Q$ oo $A - B = R$.

$$\therefore \cos Q = \cos \frac{Q+R}{2} \cos \frac{Q-R}{2} - \sin \frac{Q+R}{2} \sin \frac{Q-R}{2}$$

$$\therefore \cos R = \cos \frac{Q+R}{2} \cos \frac{Q-R}{2} + \sin \frac{Q+R}{2} \sin \frac{Q-R}{2}$$

Labada u dambeeyaa isugeyntooda waa :

$$\cos Q + \cos R = 2 \cos \frac{Q+R}{2} \cos \frac{Q-R}{2} \quad (16)$$

Kalagoynthoodana waa :

$$\cos Q - \cos R = -2 \sin \frac{Q+R}{2} \sin \frac{Q-R}{2} \quad (17)$$

Sidoo kale haddaan ka bilowno aragtiinyadii (3) iyo (4) waxaan heleynaa :

$$\sin Q + \sin R = 2 \sin \frac{Q+R}{2} \cos \frac{Q-R}{2} \quad (18)$$

iyoo

$$\sin Q - \sin R = 2 \cos \frac{Q+R}{2} \sin \frac{Q-R}{2} \quad (19)$$

Tusaale :

U tibaax sansaan taran :

b. $\cos 40^\circ + \cos 28^\circ$

t. $\cos 73^\circ - \cos 23^\circ$

j. $\sin 50^\circ - \sin 26^\circ$

Furfuris :

b. Midaalka (16) wuxuu inoo bannaynayaan

$$\begin{aligned} \cos 40^\circ + \cos 28^\circ &= 2 \cos \frac{(40^\circ + 28^\circ)}{2} \cos \frac{(40^\circ - 28^\circ)}{2} \\ &= 2 \cos 34^\circ \cos 6^\circ \end{aligned}$$

t. Midaalka (17) wuxu oggol yahay in

$$\begin{aligned} \cos 73^\circ - \cos 13^\circ &= -2 \sin \frac{(73^\circ + 13^\circ)}{2} \sin \frac{(73^\circ - 13^\circ)}{2} \\ &= -2 \sin 43^\circ \sin 30^\circ. \end{aligned}$$

j. Midaalka (19) wuxuu ina siinayaan

$$\begin{aligned}\sin 50^\circ - \sin 26^\circ &= 2 \cos \frac{(50^\circ + 26^\circ)}{2} \sin \frac{(50^\circ - 26^\circ)}{2} \\ &= 2 \cos 38^\circ - \sin 12^\circ.\end{aligned}$$

Tusaale :

Furfur

- b. $\sin 7x - \sin x = \sin 3x, 0 < x \leq \pi$
- t. $\cos 4x - \cos 6x = 2 \sin 3x \sin x, -\pi \leq x \leq 0$
- j. $\cos 5A = 2 \cos 3A \cos 2A, 0 \leq A \leq 2\pi$

Furfuris :

$$\begin{aligned}b. \quad \sin 7x - \sin x &= 2 \cos \frac{(7x + x)}{2} \sin \frac{(7x - x)}{2} \\ &= 2 \cos \frac{8x}{2} \cdot \sin \frac{6x}{2} \\ &= 2 \cos 4x \sin 3x \\ \therefore 2 \cos 4x \sin 3x &= \sin 3x \\ \sin 3x (2 \cos 4x - 1) &= 0 \\ \sin 3x = 0 \text{ ama } 2 \cos 4x - 1 &= 0\end{aligned}$$

$$\begin{aligned}3x = \sin^{-1} 0 \text{ ama } 4x = \cos^{-1} \frac{1}{2} \\ 3x = \pi \text{ ama } 4x = \frac{\pi}{3} \\ x = \frac{\pi}{3} \text{ iyo } x = \frac{\pi}{12}\end{aligned}$$

$$\begin{aligned}t. \quad \cos 4x - \cos 6x &= -2 \sin \frac{(4x + 6x)}{2} \sin \frac{(4x - 6x)}{2} \\ &= -2 \sin 5x \sin (-x) \\ \therefore -2 \sin 5x \sin (-x) &= 2 \sin 3x \sin x \\ 2 \sin 5x \cdot \sin x - 2 \sin 3x \cdot \sin x &= 0 \\ 2 \sin x (\sin 5x - \sin 3x) &= 0\end{aligned}$$

Laakiin

$$\begin{aligned}\sin 5x - \sin 3x &= 2 \cos \frac{(5x + 3x)}{2} \sin \frac{(5x - 3x)}{2} \\ &= 2 \cos 4x \sin x\end{aligned}$$

$$\begin{aligned}\therefore 2 \sin x (\sin 5x - \sin 3x) &= 0 \\ 2 \sin x (2 \cos 4x \sin x) &= 0 \\ 4 \sin^2 x \cos 4x &= 0, \sin^2 x \cos 4x = 0\end{aligned}$$

$$\sin^2 x = 0 \text{ ama } \cos 4x = 0$$

$$x = \sin^{-1} 0 \text{ ama } 4x = \cos^{-1} 0,$$

waxaan ognahay in $-\pi \leq x \leq 0$,

$$x = -\pi \text{ ama } 0;$$

$$4x = -\frac{\pi}{2} \text{ ama } x = -\frac{\pi}{8}.$$

j. $2 \cos 3A \cos 2A = +\cos(3A + 2A) + \cos(3A - 2A)$
 $= \cos 5A + \cos A$
 $\therefore \cos 5A = \cos 5A + \cos A$
 $\cos A = 0; A = \cos^{-1} 0$

waxase aan ognahay in $0 \leq A \leq 2\pi$

$$\therefore A = \frac{\pi}{2}, \text{ ama } \frac{3\pi}{2}.$$

Layli:

Kuwan soo socda, mid walba u tibaax wadar ama faraq kosayno iyo saynayo :

- | | |
|-----------------------------------|-----------------------------------|
| 1) $2 \sin 13^\circ \sin 5^\circ$ | 6) $\cos 77^\circ \cos 8^\circ$ |
| 2) $\sin 25^\circ \cos 11^\circ$ | 7) $\cos(-8)^\circ \sin 13^\circ$ |
| 3) $\sin 6^\circ \sin 24^\circ$ | 8) $\sin 43^\circ \sin 13^\circ$ |
| 4) $\cos 13^\circ \sin 48^\circ$ | 9) $\sin 13^\circ \cos 43^\circ$ |
| 5) $\cos 43^\circ \sin 13^\circ$ | 10) $\sin 15^\circ \sin 66^\circ$ |

U tibaax sansaan taran :

- | | |
|-------------------------------------|-------------------------------------|
| 11) $\cos 77^\circ + \cos 2^\circ$ | 14) $\cos 15^\circ - \cos 55^\circ$ |
| 12) $\cos 46^\circ - \cos 23^\circ$ | 15) $\sin 16^\circ + \sin 33^\circ$ |
| 13) $\sin 14^\circ - \sin 6^\circ$ | 16) $\sin 6^\circ - \sin 28^\circ$ |

Further isle'egta kasta :

- 17) $\cos 2\theta + \cos 4\theta + \cos 3\theta = 0$
- 18) $\sin 5\theta - \sin 3\theta - \sin \theta = 0$
- 19) $2 \cos 6x \sin 3x = -\sin 12x$
- 20) $\sin 4x = \sin 2x \cos x$

Caddee midaalladan mid kasta :

- 21) $\frac{\sin A + \sin B}{\cos A + \cos B} = \tan \frac{(A + B)}{2}$
- 22) $\frac{\sin A - \sin B}{\sin A + \sin B} = \cot \frac{(A + B)}{2} \tan \frac{(A - B)}{2}$
- 23) $\frac{\cos A + \cos B}{\cos A - \cos B} = \cot \frac{(A + B)}{2} \cot \frac{(A - B)}{2}$
- 24) $\frac{\sin A - \sin B}{\cos A + \cos B} = \tan \frac{(A - B)}{2}$
- 25) $\frac{\sin(A + B) + \sin(A - B)}{\cos(A + B) - \cos(A - B)} = \cot B$
- 26) $\frac{\cos(2x - y) - \cos(2y - x)}{\sin(2y - x) - \sin(2x - y)} = \frac{\tan(x + y)}{2}$
- 27) $\cos^2(A + B) - \cos^2(A - B) = -\sin 2A \sin 2B$
 $= \sin^2(A - B) - \sin^2(A + B)$

$$28) \frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \tan 2A$$

$$29) \frac{\sin M + \sin (M+2)}{\cos M + \cos (M+2)} = \tan (M+1)$$

$$30) \frac{\cos Q + \cos 3Q}{\cos Q - \cos 3Q} = \frac{1}{2} (\cot^2 Q - 1)$$

$$31) \cos 31^\circ - \sin 61^\circ = -\sin 1^\circ [\sin(90^\circ - \theta)] \\ = \cos \theta$$

$$32) \sin 48^\circ - \cos 48^\circ = \sqrt{2} \sin 2$$

$$33) \frac{\sin(B+T+J) + \sin(B-T-J)}{\cos(B+T+J) - \cos(B-T-J)} = \frac{\tan T \tan J - 1}{\tan T + \tan J}$$

$$34) \frac{\cos 3A + \cos A}{\cos - 2 \sin 3A \cos A} = 2$$

MIDIISI (Dhaqangelin)

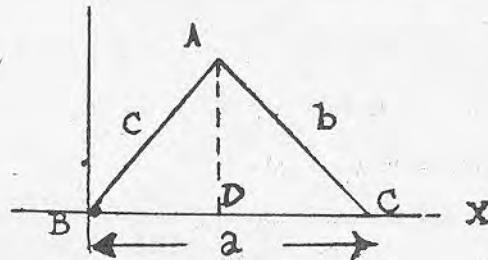
b. Bed Saddexagal.

Ka soo qaad in saddexagal u dhigan yahay sida shaxanka 2aad tusayo oo kale.
Markaa bedka saddexagalku waa :

$$S = \frac{1}{2} a \cdot AD$$

$$\text{Laakiinse } AD = C \sin B$$

$$\therefore S = \frac{1}{2} ac \sin B.$$



Shaxa 2

Waxa intaa xiga, haddaynnu saddexagalka u dhigno si A iyo B midba markeeda unugga ugu taallo, markaa bedka S wuxuu noqonayaa

$$S = \frac{1}{2} bc \sin A \text{ ama } S = \frac{1}{2} ab \sin C$$

Sidaa awgeed

$$\text{Bedka } \triangle ABC = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B = \frac{1}{2} cb \sin A \quad 20$$

Jidka bedka saddexagal waxa la adeegsan karaa marka laba dhinac iyo xagasha u dhexeysa la ogyahay.

Haddiise saddexda dhinac uun la ogyahay, waxa bedka saddexagal lagu raadshaa jidkan soo socda ee loo yaqaan **Jidka «Hero»**.

Jidka Hero.

Haddii bedka saddexagalku S yahay, markaa sida jidka (20) bannaynaya

$$S = \frac{1}{2} ab \sin C$$

$$S^2 = \frac{a^2 b^2}{4} \sin^2 C = \frac{a^2 b^2}{4} (1 - \cos^2 C)$$

Isla markaa xeerka Kosaynu wuxuu dhigaa in

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}, \cos^2 C = \left[\frac{a^2 + b^2 - c^2}{2ab} \right]^2$$

$$1 - \cos^2 C = 1 - \left[\frac{a^2 + b^2 - c^2}{2ab} \right]^2$$

$$= [1 + \frac{a^2 + b^2 - c^2}{2ab}] \cdot [1 - \frac{a^2 + b^2 - c^2}{2ab}]$$

$$= \frac{2ab + a^2 + b^2 - c^2}{2ab} \cdot \frac{2ab - a^2 - b^2 + c^2}{2ab}$$

$$= \frac{(a+b)^2 - c^2}{2ab} \cdot \frac{c^2 - (a-b)^2}{2ab}$$

$$= \frac{(a+b+c)(a+b-c)(c+a-b)(c-a+b)}{4 a^2 b^2}$$

$$S^2 = \frac{a^2 b^2}{4} \cdot \frac{(a+b+c)(a+b-c)(c+a-b)(c-a+b)}{(a+b+c)(a+b-c)(c+a-b)(c-a+b)}$$

$$= \frac{4 a^2 b^2}{16}$$

Ka soo qaad in : $a + b + c = 2s$, markaa

$$a + b - c = 2s - 2c, \quad c + a - b = 2s - 2b.$$

$$c - a + b = 2s - 2a.$$

$$S^2 = \frac{2s(2s - 2c)(2s - 2b)(2s - 2a)}{16}$$

$$S = \sqrt{s(s-a)(s-b)(s-c)}$$

Tusaale :

Soo saar bedka saddexagalka leh :

- 1) $a = 32$ sm. $b = 48$ sm. $C = 30^\circ$
- 2) $a = 14$ sm. $c = 9$ sm. $B = 150^\circ$.
- 3) $a = 5$, $b = 6$, $c = 9$.

Furfuris :

- 1) Bedka $S = \frac{1}{2} ab \sin C$
 $= \frac{1}{2} 32 \cdot 48 \cdot \sin 30^\circ$
 $= 384 \text{ sm}^2$

- 2) Bedka $S = \frac{1}{2} a c \sin B$
 $= \frac{1}{2} 14 \cdot 9 \sin 150^\circ$
 $= \frac{1}{2} 14 \cdot 9 \sin 30^\circ$
 $= \frac{1}{2} 14 \cdot 9 \cdot \frac{1}{2} = 63 \frac{1}{2} \text{ sm}^2$

3) Istimmaal «Jidka Hero» iyada oo

$$s = \frac{5 + 6 + 9}{2} = 10$$

$$\text{Bedka } S = \sqrt{10(10-5)(10-6)(10-9)}$$

$$= 10\sqrt{2}$$

RAADINTA XAGLAHA SADDEXAGAL
MARKA SADDEXDA DHINAC LA OGYAHAY

Sida jidodka xagal badh dhigayaan.

$$\sin^2 \frac{A}{2} = \frac{1}{2}(1 - \cos A),$$

$$\cos^2 \frac{A}{2} = \frac{1}{2}(1 + \cos A)$$

Xeerka kosaynkuna wuxuu sheegayaan in

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

Sidaa awgeed

$$1 - \cos A = 1 - \frac{b^2 + c^2 - a^2}{2bc} = \frac{a^2 - (b - c)^2}{2bc}$$

$$= \frac{(a + b - c)(a - b + c)}{2bc}$$

Sidaan horeba yeellay, ka soo qaad in

$$a + b + c = 2s, \text{ markaa } a - b + c = 2(s - b)$$

$$a + b - c = 2(s - c), \quad b + c - a = 2(s - b)$$

$$\therefore 1 - \cos A = \frac{4(s - b)(s - c)}{2bc}$$

$$1 + \cos A = \frac{4s(s - a)}{2bc}$$

$$\sin^2 \frac{A}{2} = \frac{(s - b)(s - c)}{bc}$$

$$\cos^2 \frac{A}{2} = \frac{s(s - a)}{bc}$$

$$\therefore \sin \frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{bc}}$$

$$\cos \frac{A}{2} = \sqrt{\frac{s(s - a)}{bc}}$$

$$\therefore \sin \frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{bc}}$$

weliba $\cos \frac{A}{2} = \sqrt{\frac{s(s - a)}{bc}}$

$$\tan \frac{A}{2} = \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}}$$

$$= \sqrt{\frac{(s-b)(s-c)}{bc}} \div \sqrt{\frac{s(s-a)}{bc}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

Fiiro :

Jidodka waxa lagu soo dheegey $\frac{A}{2}$, sidoo kalese waa lagu soo dheegi karaa $\frac{B}{2}$ iyo $\frac{C}{2}$

Raadinta xaglaha saddexagal marka a, b iyo c la ogyahay waxaa ku hawl yar jidka tanjentka, sababtoo ah, waxaynu u baahanaynaa oo keli ah logardamka afarta tibaxood ee s, $s - a$, $s - b$ iyo $s - c$.

Tusaale :

Raadi xaglaha saddexagalka leh $a = 13.2$, $b = 16.7$, $c = 22.5$

Furfuris :

$$s = \frac{a+b+c}{2} = \frac{13.2 + 16.7 + 22.5}{2} = 26.2$$

$$\log(s-a) = \log 13 = 1.1139$$

$$\log(s-b) = \log 9.5 = 0.9777$$

$$\log(s-c) = \log 3.7 = 0.5682$$

$$\log s = \log 26.2 = 1.4183$$

$$\begin{aligned}\log \tan \frac{A}{2} &= \frac{1}{2} [\log(s-b) + \log(s-c) - \log s - \log(s-a)] \\ &= \frac{1}{2} [0.9777 + 0.5682 - 1.4183 - 1.1139] \\ &= 9.5018 - 10\end{aligned}$$

$$\begin{aligned}\therefore \frac{A}{2} &= \tan^{-1}(\text{lid } \log 9.5018 - 10) \\ &= 17^\circ 37\end{aligned}$$

$$\therefore A = 35^\circ 14.$$

$$\begin{aligned}\tan \frac{B}{2} &= \frac{1}{2} [\log(s-a) + \log(s-c) - \log s - \log(s-b)] \\ &= \frac{1}{2} [1.1139 + 0.5682 - 1.4183 - 0.9777] \\ &= 9.6431 - 10\end{aligned}$$

$$\begin{aligned}\frac{B}{2} &= \tan^{-1}(\text{lid } \log 9.6431 - 10) \\ &= 23^\circ 44\end{aligned}$$

$$\therefore B = 47^\circ 28$$

$$\text{Markaa } C = 180 - (35^\circ 14 + 47^\circ 28) = 97^\circ 18$$

JOOG IYO FOGAAN

Xagga sahaminta shey kasta jooggisa iyo fogaantiisa waxaa la heli karaa marka meel dhulka ah laga ilaaleeyo. Halkan waxaynu dhawr tusaale ku muujinaynaa noocyada laylisyada imaan kara marka aan soo saarayno joogga bar fog, guud ahaan.

T u s a a l e :

Daar jooggeedu 20 m. yahay baa waxa ka taagan tiir. Xagasha tiirka madaxiisu la samaynayo aqalka saqafkiisa waa $10^{\circ} 40'$. Aqalka guntiisa iyo tiirka madaxiisuna waxay isla sameeyaan xagal $22^{\circ} 12'$. Soo saar joogga tiirka.

F u r f u r i s :

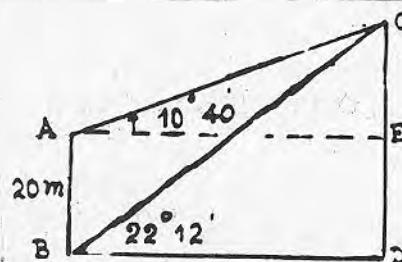
S h a x 3

Sida shaxanku tusayo AB waxay u joogtaa aqalka gidaarkiisa. CD waxay u joogtaaa tiirka. Haddaba

$$\angle ABC = 67^{\circ} 48'$$

$$\angle ACB = 11^{\circ} 32'$$

$$\triangle ABC, AC = \frac{20 \sin 67^{\circ} 48'}{\sin 11^{\circ} 32'}$$



$$\triangle ACE, CE = AC \sin 10^{\circ} 40'$$

Shax 3

$$CE = \frac{20 \sin 67^{\circ} 48' \sin 10^{\circ} 40'}{\sin 11^{\circ} 32'}$$

$$\begin{aligned} \log CE &= \log 20 + \log \sin 67^{\circ} 48' + \log \sin 10^{\circ} 40' - \log \sin 11^{\circ} 32' \\ &= 1.3010 + 9.9666 - 10 + 9.2674 - 10 - (9.3009 - 10) \\ &= 1.2341 \end{aligned}$$

$$CE = 17.15$$

$$\text{Joogga tiirku} = 20 + 17.15 = 37.15 = 37.16\text{m.}$$

L a y l i :

Raadi bedka saddexagalka leh.

- 1) $a = 12, b = 20 \quad \angle C = 48^{\circ}$
- 2) $b = 20, c = 12 \quad \angle A = 150^{\circ}$
- 3) $a = 24, c = 15.9 \quad \angle B = 120^{\circ}$
- 4) $b = 4.9, c = 11.3 \quad \angle A = 135^{\circ}$
- 5) $a = 6.8, c = 19.6 \quad \angle B = 39^{\circ}$

Raadi bedka saddexagalka leh.

- 6) $a = 14, b = 22, c = 28$
- 7) $a = 4.8, b = 7.7, c = 12.5$
- 8) $a = 11.86, b = 14.13, c = 19.17$
- 9) $a = 127.2, b = 158.5, c = 193.5$
- 10) $a = 91, b = 89, c = 25$
- 11) Saddexagalka leh dhinacyada $a = 41.00$, iyo $c = 5.67$ baa bedkiisu 15.9 sm^2 . yahay. Soo saar B.
- 12) Barbaare ayaa dhinacyadiisa deriska ihi, 28 sm. iyo 41 sm. kala yihiin. Xagal ka mid ahina waa 135° . Soo saar bedka barbarroolaha.

- 13) Barbarroole ayaa xagla gooyaashiisu 100 sm. iyo 120 sm. kala yihiin, waxaanay iska jaraan xagal 42° ah. Raadi bedka barbarroolaha.
- 14) Doon xubnaha maqan ee saddexagalka leh
- b. $\text{bed} = 710, \quad a = 34, \quad c = 49$
t. $\text{bed} = 1840, \quad b = 67, \quad c = 25$
- Raadi xaglaha saddexagalka leh
- 15) $a = 9 \text{ sm.}, \quad b = 9 \text{ sm.}, \quad c = 14 \text{ sm.}$
16) $a = 26, \quad b = 48, \quad c = 30$
17) $a = 15.3, \quad b = 13.9, \quad c = 19.2$
18) $a = 35, \quad b = 17, \quad c = 12$
19) $a = 13.14, \quad b = 11.64, \quad c = 10.36$
20) Saddexagal ABC caddee in

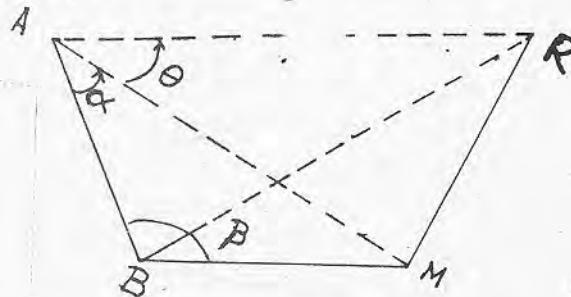
$$\text{b. } \sin \frac{B}{2} = \sqrt{\frac{(s - c)(s - a)}{ca}}$$

$$\text{t. } \cos \frac{B}{2} = \sqrt{\frac{s(s - b)}{ca}}$$

$$\text{j. } \tan \frac{B}{2} = \sqrt{\frac{s(s - b)}{s(s - a)}}$$

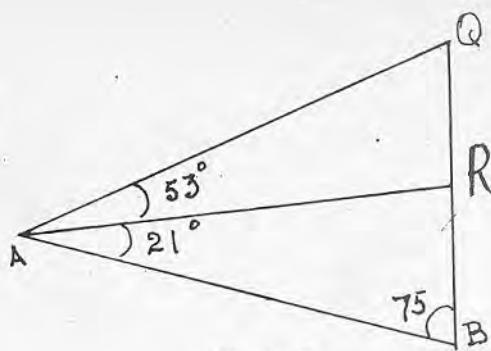
- 21) Nin dhulka maraya ayaa wuxuu dhaadey in xagasha kacsan ee tiir $26^\circ 30$ tahay. Markuu 300 m. ku soo dhowaadey tiirkiina wuxu dhaadey in xagasha kacsan ee tiirku $67^\circ 14$ tahay. Soo saar joogga tiirka.
- 22) Laba meelood A iyo B oo buur 200 m. ah ka mid ah ayaa xaglaha ka dhacsani yihiin $37^\circ 26$ iyo $17^\circ 35$ siday u kala horreeyaan. Intee bay kala fog yihiin labada meelood.
- 23) Dayuurad majiiro jiif ah haysata baa xawaaraheedu 400 m / saac yahay. Nin dayuuradda la socda ayaa wuxuu arkay in xagasha ku dhacsan meel G tahay $19^\circ 37$, saddex sikind ka dib ayaa la arkay in xagasha ku dhacsan isla meeshii G tahay $47^\circ 40$. Soo saar joogga dayuuraddu marayso.
- 24) Shaxanka hoose, AB waa xariiq sal jiifta oo dhererkeeduna 1 m. yahay. Xarriiqda AB kuma taal sallaxa qotoma ee R ku taal. Muuji in joogga RM ee R ka sarreeyso A tahay

$$\frac{1 \times \sin \theta \tan \theta}{\sin(\alpha + \beta)}$$



Shax 4

- 25) A iyo B waa laba meelood oo bannaan siman ka mid ah, waxana ay isu jiraan 3 mayl. R iyo Q waa laba balli oo ku toosan B. Xaglaha ka sameysana waa sida shaxanka hoose tusaayo. Raadi foganta RQ.



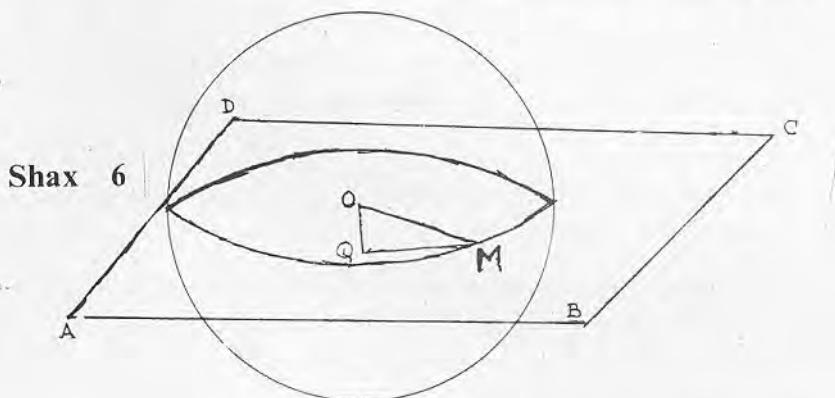
Shax 5

- 26) X iyo Y waa laba meelood oo 300 waار isu jira oo waqooyi qumman ka xigta tiirka RN, kuna wada yaal sallax jiifa. R waxay X ka kacsan tahay $70^{\circ} 38$, isla markaa NXY = $64^{\circ} 20$, NYX = $57^{\circ} 35$, soo saar joogga tiirka. (Ku soo isticmaal m.)

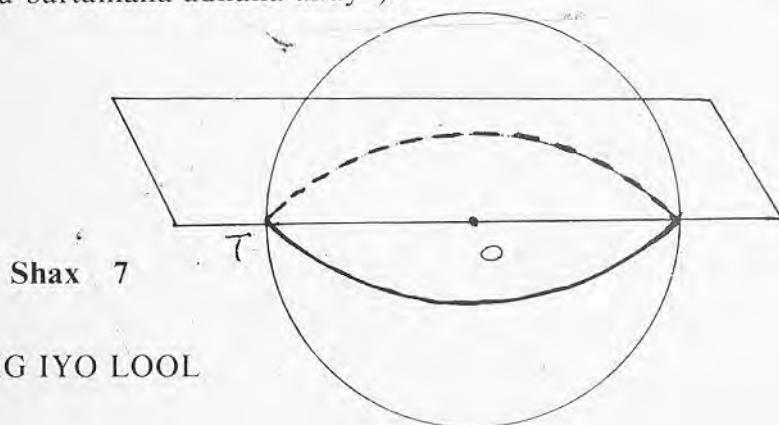
GOOBO WEYN IYO GOOBO YAREY

Ka soo qaad in sallax jaray adke, (eeg shax-6) Isgoyska labada waa goobo :

Shax-6 waxa ka muuqda in sallaxa ABCD goynaya adkaha, ka soo qaad in bartamaha adkuhu yahay 0; kana soo qaad in M tahay bar ku taal meelaha sallaxa iyo adkuhu iska sallaxa. Haddaba OM waa gacanka adkaha oo ah R. Haddii la jeexo qotomaha OQ ee yar yahay ka adkaha. Goobada isgoyskaas oo kale waxa la yiraahdaa **Goobo Yarey**.

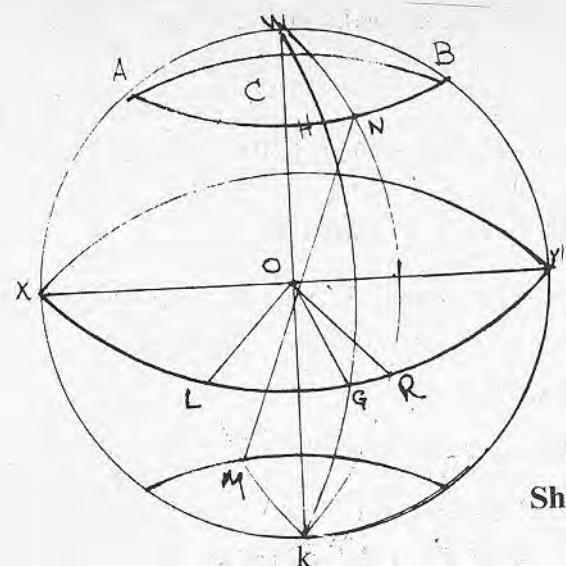


Haddiise sallax marayo bartamaha O ee adkaha (eeg shax 7) markaa ON = QM = R. Markaa gacanka goobada isgoyskana wuxuu le'eg yahay ka adkaha goobadaas oo kalena waxa la yiraahdaa **Goobo weyn** (gacanka adkuhu waa ka weyn ka goobo isgoys kaloo kasta oo aan xudunteedu bartamaha adkaha ahayn).



DHIG IYO LOOL

Ka soo qaad in O yahay xuddunta dhulka, kana soo qaad in W iyo K kala yihiin cirifyada dhulka ee waqooyi iyo koonfur. Haddaba xarriiqda toosan ee WOK waxaa la yiraa **Dhidib cirifeedka dhulka**.



Shax 8

Qeexda Dhulbadhe.

Dhulbadhuu waa isgoyska dhulka iyo sallax mara xuddunta O ee dhidib cirifeedka WOK ku qotoma.

Shax 8, dhulbadhuu waa XLY. Sallax kasta oo la barbarro ah dhulbadhaha waxay dusha iska gooyaan goobo-yarey, oo la oran karo waa la barbarro lool (sida AHB).

Qeeex :

Goobo weynayaasha isu mara cirifyada waqooyi iyo koonfur waxaa la yiraa **Dhigo**. Gaar ahaan ta Igilan mareysa waxaa loogu yeeraa «Dhigta giriinwij». Waana ta lagu heshiiyey in kuwaa kale oo idil looga cabbir qaado, galbeed iyo bariba.

Shax 13, WHGK waa dhigta giriinwij, N-na waa bar dhulka dushiisa ku taal. Sallax dhigta WNRK (ee mara N) wuxuu la sameeyaa sallaxa WHGK (ee mara giriinwijkha) xagalsha GOR (= NCH).

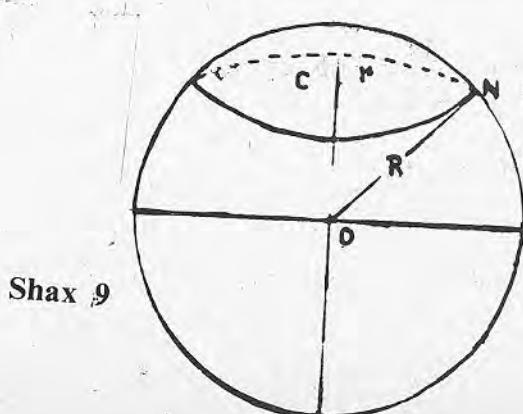
Xagasha GOR waa dhigta N ee bariga giriinwij. Sidoo kale xagasha GOL waxay u joogtaa dhigta M ee galbeedka giriinwij. Dhiguuhu waxay u kala yaalliiin min 180° bari ilaa 180° galbeed.

Loolka bar waa lagu cabbiraa intaas digirii Waqqoyiga Koonfurta dhulbadhaha. Loolka waqooyiga dhulbadhaha waxaa ina siinayaa xagasha RON, maadaama sallax WNRK oo ku qotoma sallax dhulbadhaha, LGR. Loolka N waxa kaloo nasiinaya xagasha ONC, maadaama ONC iyo NOC ay talantaali yihiin sidoo kale loolka M ee koofurta dhulbadhaha waa laga cabbiri xagasha LOM.

Loolashu waxay u loollaan karaan min $90^{\circ}W$ ilaa $90^{\circ}K$.

GACANKA BARBARRAHA-LOOL

Shax 9 O waa xuddunta dhulka R waa gacan dhulka, C waa xuddunta



Shax 9

barbarraha-loolka mara N, r-na waa gacanka barbarraha. Haddaba $r = R \cos \alpha$ ONC.

Tusaale 1 :

Soo saar fogaanta meel lool 57° W ihi u jirto dhidib cirifkeeda. (Qaado in gacanka dhulku 3960 mayl yahay).

Furfuris :

Eeg shax. 10, AB waa dhulbadhihi, O waa xuddunta dhulka, C-na waa xuddunta barbarraha loolka mara N.

$$\text{Haddaba } CN = R \cos 57^\circ$$

$$= 3960 \times \cos 57^\circ$$

$$= 2157 \text{ mayl}$$

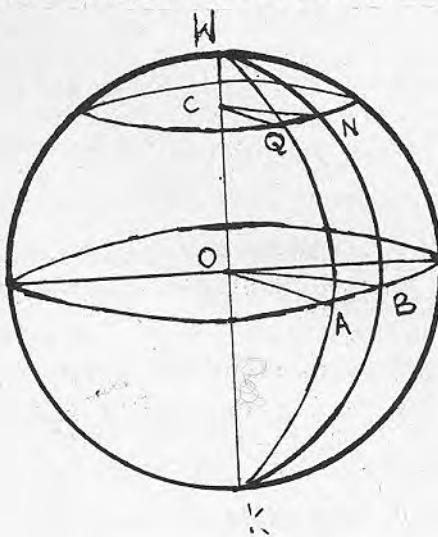
Fogaanta meeshu u jirto dhidibka WK = 2157 mayl.

Tusaale 2 :

Soo saar fogaanta laga cabbiray barbarraha lool ee u dhexeeya laba barood oo loolkoodu yahay 64° W, mid kasta, dhigahooduna kala yihii 74° B iyo 112° B. (U qaado in $2\pi R = 4 \times 10^4$ Km.).

Furfuris :

Shax. 11 N waa barta leh lool 64° W, dhig 112° B. Q-na waa ta lool 64° W, dhig 74° B. B. O. waa xuddunta dhulka, d-na waa xuddunta barbarraha loolka mara N iyo Q. Barbarraha meerkiisu waa $2\pi R$.



Shax 10

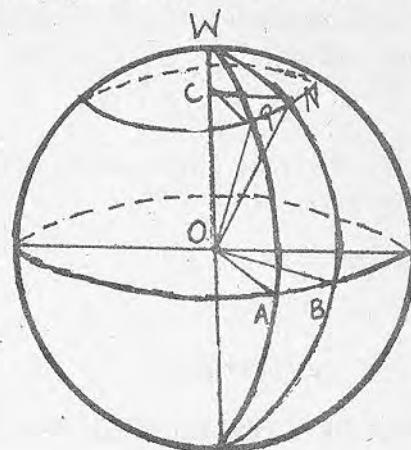
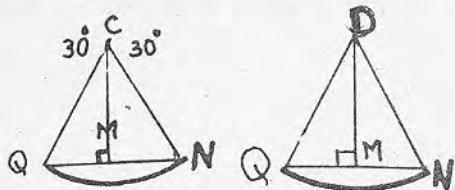
Faraqa dhigaha N iyo Q maraa waa $112^\circ - 74^\circ = 38^\circ$

$$\begin{aligned} \text{Boqon } NQ &= \frac{38}{360} \text{ ee ka barbarraha loolka N iyo Q} \\ &= \frac{38}{360} \cdot 2\pi r = \frac{38}{360} \cdot 2\pi R \cos 64^\circ \\ &= \frac{38}{360} \cdot 4 \cdot 10^4 \cos 64^\circ \\ &= \frac{38}{9} (10^3) (0.4384) \\ &= 1851 \text{ km.} \end{aligned}$$

Tusale 3 :

P iyo Q waa laba meelood oo dhulka dushiisa ka mid ah waxay kala leeyihii; lool 48° W, dhig 15° B, iyo lool 48° W, dhig 95° B. Soo saar fogaanta ay labada meelood isu jiraan haddii laga cabbiro :

- b. Barbarraha lool.
- t. Goobo weyn..



Shax 11

K (b)

Furfuris :

- b. Fogaanta ay labada meelood isu jiraan marka laga cabbiro barbarraha lool

Shaxan

$$\begin{aligned}
 &= \text{qaanso } PQ \\
 &= \frac{80}{360} \times 2\pi r \\
 &= 2/9 \times 2\pi R \cos 48^\circ \\
 &= 2/9 \times 4 \times 10^4 \cos 48^\circ \text{ Km.} \\
 &= 8/9 \times 10^4 \cos 48^\circ \text{ Km.} \\
 &\approx 5950 \text{ Km.}
 \end{aligned}$$

Shaxan "c"

$$t. PM = r \sin 40^\circ = R \cos 48^\circ \sin 40^\circ$$

$$PM = R \sin \theta$$

Shaxan "d"

$$\therefore \sin \theta = \cos 48^\circ \sin 40^\circ$$

$$\therefore \theta = 25^\circ 28'$$

$$\therefore \text{POQ} = 2\theta = 50^\circ 56'$$

$$= 50.93^\circ$$

(Shaxan "d")

- ∴ Qaansada PQ ee Goobo weyn

$$\begin{aligned}
 &= \frac{50.93}{360} \times 2\pi R \\
 &= \frac{50.93}{360} \times 4 \times 10^4 \text{ Km.} \\
 &= \frac{50.93}{9} \times 10^3 \text{ Km.} \\
 &= 5659 \text{ Km.}
 \end{aligned}$$

MAYL NOOTIKEED

Qeex :

Mayl nootikeed waa dhererka qaanso dhig ka sameyso xagal 1 ah xuddunta dhulka. Haddaynu gacanka dhulka u qaadanno 3960 mayl, waxaan heleyntaa in 1

$$\text{mayl nootikeed} = \frac{1}{60.360} : 2\pi \cdot 3960$$

$$= 1.15 \text{ mayl} = 1.84 \text{ Km.} = 1840 \text{ m.}$$

Layli :

Laylisyadan u qaado in

$$2\pi R = 4 \cdot 10^4 \text{ iyo in } \log \pi = 0.4971.$$

- 1) Raadi dhererka barbarraha lool 58° K.
- 2) Raadi fogaanta meel loolka 64° K lihi ay u jirto dhidib WK.
- 3) Kubbad-adduun baa gacankeedu 90 sm. yahay . Raadi gacanka barbarraha lool 85° W.
- 4) Laba meelood baa waxay leeyihin lool 64° K. Dhigahooduna waa 79° B iyo 71° G siday u kala horreyaan. Soo saar fogaanta u dhixeyesa labada meelood.
- 5) Laba meelood baa hal dhig ku wada yaal. Loolashooduna waa 64° W iyo 43° W. Soo saar fogaanta u dhixeyesa labada meelood marka cabbirkooda lala raaco dhigtaa.
- 6) Laba dhicood oo dhulbaraha ku yaal baa 300 km. isu jira. Waa maxay faraqa loolashoodu ?
- 7) Washir kubbad-adduun baa gacan 21 sm. ah leh. Raadi :
 - b. Dhererka barbarraha lool 67° G
 - t. Fogaanta u dhixeyesa xudduntiisa iyo barbarraha lool 67° G.
- 8) Washir kubbad adduun baa gacankeedu yahay 21 sm. Laba meelood A iyo C oo dushiisa ku yaal, ayaa A lool 45° W dhig 75° B leedahay, C-na lool 45° W dhig 75° B. laba xarig baa A iyo C ku kala giig tiran. Labada xarig mid wuxuu raacaa barbarraha loolka min A ilaa C, midna wuxuu dul maraa ciriska waqooyi. Raadi dhererka labada xarig.
- 9) Markab geeddi ah oo 400 km. waqooyi toos ugu socday baa ogaadey in oo dhigtuu haystay 10° ka durkay. Waa maxay loolkoo joogaa hadda ?
- 10) M iyo N waa laba meelood oo ka mid ah lool 36° W. Labadooda dhigoodna faraqadu waa 24° Raadi :
 - b. Fogaanta M iyo N u dhixeyesa marka lala raaco barbarraha lool.
 - t. Xagasha M iyo N ay la sameyaan xuddunta dhulka.
 - j. Fogaanta u dhixeyesa M iyo N marka lala raaco goobo-weyn.
- 11) Laba meelood oo dhigi wada marto ayaa 345 km. kala fog markii cabbirkfa fogaantaas lala raacay dhigta. Soo saar faraqa labadooda lool.
- 12) Laba meelood oo ku wada yaal lool 52° K. Ayaa 300 km. kala fog marka cabbirkfa fogaanta lala raaco barbarraha lool. Soo saar faraqa labadooda dhigood.
- 13) A waa dhobicda 75° K iyo 120° B, R-na waa dhobicda 75° K iyo 120° G. Raadi fogaanta u dhixeyesa A iyo R marka la raaco b) Goobo-weyn t) Xarriiqda toosan ee isku xirta A iyo R
- 14) Raadi dhererka kulaalaha Kaabrikoorno (lool 23° 30 k).

- 16) Raadi fogaanta u dhexaysa meesha X ($26^{\circ} 17' W$, $25^{\circ} 14' B$) iyo meesha Y ($49^{\circ} 6' W$; $25^{\circ} 15' G$)

AMIN MEELEED

Waniinka dunidu wuxuu iska rogaa min Galbeed ilaa Bari. Sidaa awgeed haddii M iyo N ay yihiin laba barood oo dhulka dushiisa ah, isla markaa dhigta N martaa galbeed ka xigto dhigta M marta, markaa M baa cadceedda uga horreynaysa N muddo la ogaan karo.

Weliba, mar haddaan ognahay in 24kii saacba dhulku hal waniin (360°) dhammeeyo, waxaan aragnaa in faraqa hal saac uu ku yimid faraq $\frac{360^{\circ}}{24} = 15^{\circ}$ ah. Haddii 15° oo dhigeed

ku beegan tahay hal saac, markaa 1° waxaa ku beegan 4 minit oo ammineed. Sidaa awgeed laba meelood oo kasta waa la xisaabin karaa ammin meeleydyadooda; waxaase lagama maarmaan ah in la ogaado digahooda ?. Si la isu garab dhigo ammin meeleydyada loogana baxsado qas, ammin meeledka Giriinwij (dhig 0°) ayaa loo qaataa beegal oo aamminaha saacadaha sal u ah.

T u s a a l e :

Dhigaha Muqdisho iyo New York waa $45^{\circ} B$ iyo $75^{\circ} G$ siday u kala horreyaan. Haddaba faraqa dhigahoodu waa $45^{\circ} + 75^{\circ} = 120^{\circ}$. Faraqa amminahooduna waa $\frac{120}{15} = 8$ saac.

∴ Markay 12ka duhur Muqdisho tahay. New York waa 4-tii subaxnimo.

L a y l i :

Soo saar faraqa ammin meeleydyada (ku seeb minityo) laba meelood oo dhigadoodu kala yihiin :

- 1) $47^{\circ} 13' B$ iyo $95^{\circ} 37' G$
- 2) $45^{\circ} 25' B$ iyo $125^{\circ} 40' G$
- 3) $47^{\circ} 13' B$ iyo $47^{\circ} 13' G$
- 4) $50^{\circ} B$ iyo $25^{\circ} G$
- 5) $113^{\circ} 27' B$ iyo $179^{\circ} 18' G$
- 6) $85^{\circ} B$ iyo $125^{\circ} 40' G$
- 7) $35^{\circ} B$ iyo $75^{\circ} 15' G$
- 8) Ankoraage ($150^{\circ} G$) waa ka immisadii marka ay Muqdisho ka tahay 12kii duhur ee Jimce ?
- 9) Axad 12kii habeennimo ayaa markab ka dhoofay meesha A ($60^{\circ} W$, $150^{\circ} B$) ilaa meesha R ($60^{\circ} W$, $150^{\circ} Cap$). Haddii markabku raaco barbarraha lool kuna socdo xawaare 200 km./saac; goormuu gaari R ?
- 10) Dhigta Qaahira martaa waa $30^{\circ} B$, ta Belfaas martaana waa $5^{\circ} 56' G$. Haddii qorraxdu 19.15 ka dhacdo Belfaas goormay ka dhacdaa Qaahira ?

TIROOYIN KAKAN

Sida aynu wada ognahay, isku beegnaan $1 - 1$ ayaa ka dhexeysha baraha xarriiq tiro iyo tirooyinka maangalka ah, taasoo inoo fududeysay ka shaqaynta Aljebraha. Haddaba, waxa mararka qaarkood dhacda inaan tirooyin maangal ah loo helin falafka aljebraha. Tusaale ahaan, ma jirto tiro maangal ah oo raalligelineysa isle'egta: $x^2 + 1 = 0$. Isle'egta iyo wixii la mid ah si aynnu u furfurro, waxaynnu adeegsaneynaa ururka tirooyinka Kakan.

Tirooyinka habdhiskaa kū jiraa waxay ka kooban yihin, lammaanayaal horsan oo maangal ah (a, b). Sidaa daraadeed, way inoo suuragelaysaa inaynnu isku beegnaan $1 - 1$ ka dhexaysiinno tirooyinka kakan iyo dhibcaha sallax ka kooban yahay. Guud ahaan Zi waxay u taagan tahay lammaanayaal horsan oo tirooyin maangal ah, K-na waxay u taagan tahay ururka tirooyinkaa oo dhan.

Q E E X O**Q e e x 1 :**

$K = R \times R$ (R Laanqayr R) = $\{(a, b) / a \in R, b - na \in R\}$ taasu waxay inoo sheegaysaa ururka tirooyinka aynnu ku shaqayn doonno.

Q e e x 2 :

Ka soo qaad in $Z_1 = (a_1, b_1) \in K$, $Z_2 = (a_2, b_2) \in K$, markaa $Z_1 = Z_2$ haddii iyo haddii oo qudha $a_1 = a_2$ isla markaana $b_1 = b_2$ (taasi waxay micno u yeelaysaa isle'ekaanta K).

Q e e x 3 :

Haddii $Z_1 = (a_1, b_1) \in K$, $Z_2 = (a_2, b_2) \in K$, markaa :

$$\begin{aligned} b. \quad Z_1 + Z_2 &= (a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2) \\ t. \quad Z_1 \cdot Z_2 &= (a_1, b_1) \cdot (a_2, b_2) \\ &= (a_1 a_2 - b_1 b_2, a_1 b_2 + a_2 b_1) \end{aligned}$$

Intaasuna waxay muujinaysaa laba xisaabfal, isugeynta iyo iskudhufshada tirooyinka ku jira ururkeenna.

T u s a a l e 1 :

U qor $(3, 5) + (7, 9)$ sida lammaane horsan (a, b).

F u r f u r i s :

Qeex raac.

$$(3, 5) + (7, 9) = (3 + 7, 5 + 9) = (10, 14)$$

T u s a a l e 2 :

U qor $(3, 5) \cdot (7, 9)$ sida lammaane horsan (a, b).

F u r f u r i s :

Qeex raac.

$$\begin{aligned} (3, 5) \cdot (7, 9) &= (3 \cdot 7 - 5 \cdot 9, 3 \cdot 9 + 5 \cdot 7) \\ &= (21 - 45, 27 + 35) \\ &= (-24, 62). \end{aligned}$$

L a y l i :

b. Wadar kasta u qor sida lammaane horsan (a, b).

1) $(3, 6) + (2, 1)$

6) $(-2, -1) + (2, 1)$

- 2) $(7,1) + (3, -5)$ 7) $(2,3) + (1,1)$
 3) $(0,7) + (3,0)$ 8) $(4,5) + (0,0)$
 4) $(-6, -2) + (0,1)$ 9) $(-11,8) + (11, -8)$
 5) $(3, -2) + (-2,0)$

t. Taran kastaba u qor sida lammaane horsan (a, b).

- 1) $(1,1) \cdot (1,1)$ 7) $(3, -2) \cdot (1, -1)$
 2) $(1,0) \cdot (2,1)$ 8) $(0,1) \cdot (1,0)$
 3) $(2,3) \cdot (4,1)$ 9) $(3,4) \cdot (1,0)$
 4) $(3,1) \cdot (0,2)$ 10) $(0,1) \cdot (0,2)$
 5) $(2,2) \cdot (3,4)$ 11) $(0,1) \cdot (0,1)$
 6) $(-2,1) \cdot (1,3)$ 12) $(1,0) \cdot (1,0)$

j. Kala sheeg in lammaanayaashan horsani isle'eg yihin in kale :

- 1) $(1,12)$ iyo $(12,1)$ 4) $(5,1)$ iyo $(10,2)$
 2) $(3,5)$ iyo $(6,5)$ 5) $(6,3)$ iyo $(6,1)$
 3) $(4,2)$ iyo $(4,2)$ 6) $(3,1)$ iyo $(1,3)$

Ururka K, astaamaha soo socda oo ku dhisan xisaabfallada isugeynta iyo iskudhufashada ayuu leeyahay.

I. Xisaabfalka Isugeynta.

1. Oodnaanshaha Isugeynta.

K waxay ku oodan tahay Xisaabfalka Isugeynta. Haddii $Z_1 \in K, Z_2 \in K$. Markaa $Z_1 + Z_2 \in K$

C a d d e y n :

$$\text{Ka soo qaad in } Z_1 = (a_1, b_1), Z_2 = (a_2, b_2).$$

$$\text{Raac qeexda 3. } Z_1 + Z_2 = (a_1 + a_2, b_1 + b_2).$$

Mar haddii R ku oodan tahay xisaabfalka isugeynta, waxaynnu ognahay in $(a_1 + a_2) \in R$, isla markaas $(b_1 + b_2) \in R$. Markaa innaka oo raacaynna Qeexda 1 : $(a_1 + a_2, b_1 + b_2) \in K$, t.a.; $(Z_1 + Z_2) \in K$.

2. Hormogelinta Isugeynta.

Isugeynta K way hormogashaa; astaantaa waxaa innagaga filan qeexda oo ah :

Q e e x d a 4 :

$$\begin{aligned} &\text{Haddii } Z_1 \in K, Z_2 \in K, Z_3 \in K, \text{ markaa} \\ &(Z_1 + Z_2) + Z_3 = Z_1 + (Z_2 + Z_3) \end{aligned}$$

3. Asal Madoorshaha Isugeynta.

Waxaa jira $Z_0 \in K$ oo marka $Z + Z_0 = Z_0 + Z = Z$, rumeynaysana dhammaan $Z \in K$.

C a d d e y n :

Ka soo qaad in

$$Z = (a, b), \text{ kutirsanaha } Z_0 = (0, 0) \in K.$$

$$\begin{aligned} \text{Raac Qeexda 3; } Z + Z_0 &= (a, b) + (0, 0) \\ &= (a + 0, b + 0) \\ &= (a, b) = Z \end{aligned}$$

Halkaa waxaa ka caddaatay in eber yahay asal madoorshaha isugeynta ee ku jira ururka R. Sidaa awgeed waxaynu niraahnnaa eber waa asal madoorshaha isugeynta ee ku jira ururka K. Haddaynu doonnana marka $Z_0 = (0, 0)$ waxaynu niraahnnaa kutirsanaha eber ee ku jira ururka K.

4. Isweydaarka Isugeynta.

Z kasta oo kurtiisane K ah, waxaa u jira —Z ∈ K oo Z + (—Z) = (—Z) + Z = Z₀.

C a d d e y n :

Ka soo qaad in

Z = (a, b), kutirsanaha —Z = (—a, —b) ek. Markaa waxaynu haysanaa :

$$Z + (-Z) = (a, b) + (-a, -b) \quad \text{Sido kale :}$$

$$= [a + (-a), b + (-b)]$$

$$= (0, 0)$$

$$-Z + Z = (-a, -b) + (a + b)$$

$$= [(-a) + a, (-b) + b]$$

$$= (0, 0).$$

5. Kala Hormarinta Isugeynta.

Haddii Z₁ ∈ K, Z₂ ∈ K, Markaa Z₁ + Z₂ = Z₂ + Z₁.

C a d d e y n :

Ka soo qaad in Z₁ = (a, b) kana dhig in Z₂ = (c, d). Innagoo raacayna Qeexda 3-b :

$$\begin{aligned} Z_1 + Z_2 &= (a, b) + (c, d) = (a + c, b + d) \\ &= (c + a, d + b) = (c, d) + (a, b) \\ &= Z_2 + Z_1. \end{aligned}$$

Taasu waxay ina tusaysaa in isugeyntu yeesho Xeerka kala hormarinta ee Ururka K.

II. Xisaabfalka Iskudhufashada.

6. Odnaanshaha iskudhufashada.

K waxay ku oodan tahay Xisaabfalka iskudhufashada. Haddii Z₁ ∈ K, Z₂ ∈ K, Z₁ · Z₂ ∈ K

C a d d e y n :

Ka soo qaad in Z₁ = (a, b), kana dhig in Z₂ = (c, d). Innagoo raacayna Qeexda 3-t :

$$\begin{aligned} Z_1 \cdot Z_2 &= (a, b) \cdot (c, d) \\ &= (ac - bd, ad + bc) \\ &= [ac + (-bd), ad + bc]. \end{aligned}$$

Mar haddii aynu ognahay in R ku oodan yahay xisaabfalka iskudhufashada iyo isugeynta, waxa dhab ah in ac + (—bd) ∈ R, (ad + bc) ∈ R. Markaa innaga oo raacayna Qeexda 1: [(ac + (—bd), (ad + bc)] ∈ K. t.a., Z₁ · Z₂ ∈ K.

7. Iskudhufashadu way kala dhigtaa Isugeynta.

Ururka K iskudhufashadu way kala dhigtaa isugeynta. Haddii Z₁ ∈ K, Z₂ ∈ K, Z₃ ∈ K.

Markaa Z₁ (Z₂ + Z₃) = Z₁ Z₂ + Z₁ Z₃.

C a d d e y n :

Ka soo qaad in Z₁ = (a, b), Z₂ = (c, d), Z₃ = (e, f).

Markaa Z₁ (Z₂ + Z₃) = (a, b) · [(c, d) + (e, f)] Qeexda 3-t.

$$= (a, b) · (c + e, d + f)$$

$$= (ac + ae - bd - bf, ad + af + bc + be)$$

Kaladhigidda R.

$$= (ac - bd, ad + bc) + (ae - bf, af + be). \text{ Qeexda 3-b.}$$

$$= (a, b) · (c, d) + (a, b) · (e, f). \text{ Qeexda 3-t}$$

$$= Z_1 \cdot Z_2 + Z_1 \cdot Z_3.$$

8. Caddee Xeerka kala-hormarinta ee iskudhufashada.

9. Asal Madoorshaha Iskudhufashada.

Waxa jira Z* ∈ K taas oo Z* · Z = Z · Z* = Z.

Waxaa jira taas oo $z \cdot z^* = Z^* \cdot Z = Z$.
Caddeyn:

Ka soo qaad in $Z = (a, b)$, kana dhig kutirsanaha $Z^* = (1, 0)$. Raac Qeexda 3-t.

$$\begin{aligned} &= (a - 0, 0 + b) \\ &= (a, b) = Z \end{aligned}$$

$$\begin{aligned} Z^* \cdot Z &= (1, 0) \cdot (a, b) = (1 \cdot a - 0, 1 \cdot b, 1 \cdot b + 0 \cdot a) \\ &= (a, b) = Z \end{aligned}$$

$$\text{Markaa } Z^* \cdot Z = Z \cdot Z^* = Z.$$

10. Xeerka kala hormarinta Iskudhufashada.

Haddii $Z_1 \in K$, $Z_2 \in K$. Markaa $Z_1 \cdot Z_2 = Z_2 \cdot Z_1$. Tus in astaantaasi dhab tahay.

11. Isweydaar Iskudhufasho.

Z kasta oo kutirsane K ah, waxaa loo helayaa Z^{-1} oo $Z^{-1} \cdot Z = Z \cdot Z^{-1} = (1, 0)$; $Z \neq (0, 0)$

Caddeyn:

Ka soo qaad in $Z = (a, b)$ kana dhig

$$Z^{-1} = \left\{ \frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right\} \in K \dots \dots \text{(1) Raac Qeexda 3-b}$$

$$\begin{aligned} Z \cdot Z^{-1} &= (a, b) \left\{ \frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right\} \\ &= \left\{ \frac{a^2 + b^2}{a^2 + b^2}, \frac{-ab + ab}{a^2 + b^2} \right\} \\ &= (1, 0). \end{aligned}$$

Dhanka kale :

$$\begin{aligned} Z^{-1} \cdot Z &= \left\{ \frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right\} (a, b) \\ &= \left\{ \frac{a^2 + b^2}{a^2 + b^2}, \frac{-ab + ab}{a^2 + b^2} \right\} \\ &= (1, 0). \end{aligned}$$

Tibaaxda (1) waxba yaanay kaa yaabin. Run ahaantii waa furfuriddii isle'egta (a, b) . $(x, y) = (1, 0)$ oo (x, y) la raadiyay. Layli ahaan ayaa loo dhaafay.

Qeex :

Haddii $Z_1 \in K$, $Z_2 \in K$, markaa $Z_1 - Z_2 = Z_1 + (-Z_2)$. Tirada $Z_1 - Z_2$ waxaa la yiraa Faraqa Z_1 iyo Z_2 waana jadeeyada ka soo baxday kalagoynta Z_1 iyo Z_2 .

TIRO KAKAN

Habdhisika lammaanaha horsan ee aynu ilaa imminka ka hadleyney si faa'iido iyo xiisa leh ayaa loogu xiri karaa habdhisika tirada maangalka ah. Bal ugu horreyntii waxaad xusuusataa in garaaf ahaan isku beegnaan $1 - 1$ ahi ka dhexeyso ururka lammaanayaasha horsan (a, b) ee tirooyinka maangalka iyo baraha ku yaal sallaxa (x, y) ee Joometeriga ah, sida isku aaddanaan $1 - 1$ ah uga dhexeyso baraha ku yaal dhidibka $-x$ iyo ururka tirooyinka maangalka ah ee R .

Marka hormada $\{(a, 0) \mid a \in R\}$ e ururka K waxaa loo arki karaa in ay 1 — 1 ahaan isugu aa n yihii ururka R. Waxaa intaa raaca in Qeexihii 3 ilaa 5 ina siinayaan ;

$$(a, 0) + (c, 0) = (a + c, 0)$$

$$(a, 0) \cdot (c, 0) = (ac, 0)$$

$$(a, 0) - (c, 0) = (a - c, 0)$$

Q e e x :

Haddii $Z_1 \in K$, $Z_2 \in K$, oo $Z_2 \neq (0, 0)$; markaa $\frac{Z_1}{Z_2} = Z_1 Z_2^{-1}$. Go'aanka halkaa inooga soo baxayaa waxaa weeye in :

$$\begin{aligned} \frac{(a, 0)}{(c, 0)} &= (a, 0) \cdot (c, 0)^{-1} \\ &= (a, 0) \cdot \left\{ \frac{1}{c}, 0 \right\} \\ &= \left[\frac{a}{c} - 0, a \cdot 0 + 0 \cdot \frac{1}{c} \right] = \left(\frac{a}{c}, \frac{0}{c} \right) \\ &= \left\{ \frac{a}{c}, 0 \right\}, c \neq 0. \end{aligned}$$

Waxaa halkaa ka muuqata in dhaqmaadka xubin horaadka «a» ee tirooyinka $(a, 0) \in K$ iyo tirooyinka $a \in R$ isku mid yihii, marka loo eego afarta xisaabfaal (Ha illoobin in fikrad ahaan a iyo $(a, 0)$ kala duwan yihii).

Q e e x 7 :

Kutirsanaha $(a, 0)$ ee ku jira ururka K waxaa lagu suureeyaa kutirsanaha «a» ee ku jira ururka R; waxaana loo qoraa : $a = (a, 0)$. Markaa taasi waxay tusaysaa in $R \in K$ (R waa hormo K). Haddaba bal waxaad fiirisaa hormada K ee ka kooban dhammaan lammaan-ayaasha horsan ee leh sansaanka $(0, b)$ oo aan $b \neq 0$, $b \in R$. Gaar ahaanna marka aynu labajibbaarro $(0, b)$, waxaynu hellaa : $(0, b)^2 = (0, b) \cdot (0, b) = (-b^2, 0)$

Mar haddii aynu ognahay in $(-b^2, 0)$ lagu suureeyo tirada maangalka ah ee $-b^2$, waxaa xaqiiq ah in $(-b^2, 0)$. Maraka loo eego tiro kasta oo maangal ah oo $b = 0$. Si daa daraadeed way caddahay in $(-b^2, 0)$. Taasi waxay ina tusdaysaa in ururka K inoogu deeqo xidid labajibbaar oo ah $(0, b)$ marka la tiixgelinayo tiro taban oo kasta oo ah $-b^2$. Waxaa intaa raacda, in

$$(0, -b)^2 = (0, -b) \cdot (0, -b) = (-b^2, 0)$$

Sidaas daraadeed K waxay ina siinaysaa laba xidid jibbaar, kuwaas oo ah $(0, b)$ iyo $(0, -b)$. Taas macnaheedu waxa weeye in xididdada labajibbaarka ee $(-b^2, 0)$ ama $-b^2$ ay yihii $(0, b)$ iyo $(0, -b)$. Xididdada labajibbaarka tirooyinka maangalka ah ee aan tabnayn waa $\{(a, 0) \mid a \in R\}$; ka tirooyinka maangalka ah ee tabanina waa $\{(0, b) \mid b \in R, b = 0\}$. Gaar ahaa xididdada labajibbaarka $(2, 1, 0)$ ama -1 waa $(0, 1)$ iyo $(0, -1)$. Haddaba, haddii xidid labajibbaarka -1 uu noqday $(0, 1)$, waxaa halkaa inooga cad inayna $(0, 1)$ tiro maangal ah ahayn.

Q e e x 8 :

Marka la fiiriyo ururka K, $i = (0, 1)$. Haddaynu hore u tusnay in $b i = (b, 0) \cdot (0, 1) = (0, b)$. Tirada b waxaa la yiraa **Tiro maangal soocan ah**.

Q e e x 9 :

Marka la fiiriyo ururka K, $\sqrt{-b^2} = |b| i$, gaar ahaan $\sqrt{-1} = |1| i = 1 \cdot i = i$. Sidaas awgeed lammaane horsan oo kasta :

$$\begin{aligned}(a, b) &= (a, 0) + (0, b) \dots \dots (1) \\&= a + (b, 0) (0, 1) \dots \dots (8) \\&= a + bi\end{aligned}$$

Marka K, sida (1) loo qeexo, waxaa la yiraa Ururka tirooyinka Kakan. Tirada i waxaa la yiraa **Halbeegga maangadka**, a-na waxaa la yiraa **Qaybta maangalka a + bi**-na **Qaybta maangadka**: waxaynnu qorraa $a = M(a + bi)$; $b = Mg(a + bi)$. Halkaa waxaa ka cad in ururka ka kooban tirooyinka maangalka ah uu yahay hormo ka mid ah ururka tirooyinka kakan oo ay marka $b = 0$. Markaa waa caddaatay in tirooyinka Kakan ee $0 + 0i$ iyo $-3 + 0i$ ay u taagan yihiin tirooyinka maangalka ah ee $0 + 0i$ iyo $-3 + 0i$.

Qeexihii 2, 3, 5 iyo 6 waxaa loo qori karaa sida hoos ku taxan iyada oo la isticmaalayo $a + bi$.

Q e e x 10 :

Haddii $a, b \in R$ oo $i^2 = -1$, markaa :

1. $a + bi = c + di$ haddii iyo $a = c, b = d$
2. $(a + bi) + (c + di) = (a + c) + (b + d)i$
3. $(a + bi) - (c + di) = (a - c) + (b - d)i$
4. $(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$
5. $\frac{a + bi}{c + di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i$

F i i r o :

$$\begin{aligned}i^1 &= i \\i^2 &= -1 \\i^3 &= i^2 \cdot i = -1 \cdot i = -i \\i^4 &= i^2 \cdot i^2 = (-1)(-1) = 1 \\i^5 &= i^4 \cdot i = 1i = i \\i^6 &= i^4 \cdot i^2 = 1(-1) = -1 \\i^7 &= i^4 \cdot i^3 = 1(-i) = -i \\i^8 &= i^4 \cdot i^4 = 1 \cdot 1 = 1\end{aligned}$$

Waxaad aragtaa in marka la fududeeyo jibbaarrada isku xiga ee i, qii mayaashoodu ay yihiin afarta tiro ee i, -1 , $-i$, 1 , oo noqnoqoto ah.

F i i r o : $+ i(-1) = -1(i^2) = (-1)(-1) = 1$.

Waxaad halkan inooga cad in iyo $-i$ ay isu rogaal yihiin t.a $= \frac{1}{i} 1$ iyo $\frac{1}{-i} = i$

Sidaas awgeed fiirada 2 waa lagama maarmaan marka aynu fududaynayno jajabyada hooseeyaashoodu tirooyin maanagal soocan yihiin. Markaa :

$$\begin{aligned}\frac{9}{6i} &= \frac{9}{6} \cdot \frac{1}{i} = \frac{3}{2} \cdot (-1) = -\frac{3}{2}i \\ \frac{12}{i^3} &= \frac{12}{-i} = 12 \cdot \frac{i}{-1} = 12i\end{aligned}$$

Qeex 11 :

Xistiga kakan ama xistiga tirada kakan ee $a + bi$, waxa weeye $a - bi$. Had iyo jeer xistiaga tirada kakan ee Z waxaa loo qoraa \bar{Z} . Bal hadda fiiri waxa dhaca marka tiro kakan iyo xistigeeda la isku dhufto.

Ka soo qaad in $Z = a + bi$, markaa $Z^- = (a - bi)$

$$\begin{aligned}(a + bi) \cdot (a - bi) &= a^2 - abi + abi - b^2i^2 \\ &= a^2 - abi + abi + b^2 \\ &= a^2 + b^2.\end{aligned}$$

Tusaale :

U qor $\frac{3 - 2i}{-1 + i}$ saansaanka $a + bi$.

Furfuris :

$$\begin{aligned}\frac{3 - 2i}{-1 + i} &= \left[\frac{3 - 2i}{-1 + i} \right] \cdot \left[\frac{-1 - i}{-1 - i} \right] \\ &= \frac{-3 - 3i + 2i + 2i^2}{2} \\ &= \frac{-5 - i}{2} = \frac{-5}{2}, \frac{1}{2}i\end{aligned}$$

Dariiqo kale : Qeex ahaan, $\frac{3 - 2i}{-1 + i}$ waa tirada $a + bi$, ee a iyo b ay yihiin maangal ee

$$\frac{3 - 2i}{-1 + i} = a + bi$$

$$\begin{aligned}(1 + i)(a + bi) &= 3 - 2i \\ -a - b + (a - b)i &= 3 - 2i\end{aligned}$$

Markaa $-a - b = 3$; $a - b = -2$. Haddaba markaynu wadajir ahaan u furfurno waxaynu helnaa:

$$a = \frac{-5}{2}, \quad b = \frac{1}{2}, \quad \text{ama } a + b = \frac{-5}{2} - \frac{1}{2}i$$

Bal hadda u fiirso laylisyadan loo qoray sansaanka $a + bi$:

- b. $(3 + 2i) + (-7 - i) = (3 - 7) + (2 - 1)i = -4 + i$
- t. $(8 - 6i)(2i - 7) = (8 + 7) + (6 - 2)i = 15 - 8i$
- j. $(2 - 3i)(4 + 2i) = 2(4 + 2i) - 3i(4 + 2i)$
 $= 8 + 4i - 12i - 6i^2$
 $= 8 + 4i - 12i + 6$
 $= 14 - 8i$

$$\begin{aligned}
 x. \quad & \frac{5+5i}{3-4i} + \frac{20}{4+3i} = \left[\frac{5+5i}{3-4i} \right] \quad \left[\frac{3+4i}{3+4i} \right] \\
 & \quad + \left[\frac{20}{4+3i} \right] \quad \left[\frac{4-3i}{4-3i} \right] \\
 & = \frac{15+20i+15i+20i^2}{9-16i^2} + \frac{80-60i}{16-9i^2} \\
 & = \frac{-5+35i}{25} + \frac{80-60i}{25} \\
 & = 3-i.
 \end{aligned}$$

Si ay u fududaato ka shaqaynta dhammaan tibaaxaha sansaanka $\sqrt{-b}$ oo kale ah. Waxaa loo qoraa : $\sqrt{-b} = \sqrt{b}i$

Tusaale :

$$3 + \sqrt{-5} = 3 + \sqrt{5}i$$

$$(2 + \sqrt{-3})(2 - \sqrt{-3}) = (2 + \sqrt{3}i)(2 - \sqrt{3}i) \\ = 4 - 3i^2 = 7$$

Layli :

1. U qor laylisyadan sansaanka $a + bi$.

1. $[(5+3i) + (-1+2i)] + (7-5i) = 11$ Jawaab
2. $(2-i) + [(-3+2i) + (5-4i)] = 4-3i$ Jawaab
3. $(-1+2i)[(7-5i) + (-3+4i)] = -6-7i$ Jawaab
4. $\frac{3i^{30}-i^{19}}{2i-1} = \frac{1-3i}{5}$ Jawaab
5. $(2+4i) \cdot (3+i) = [2+14i]$
6. $(4-i) - (6-2i) = [-2-i]$
7. $(2-i) \cdot (3-2i) = [4-7i]$
8. $(2+i) - (4-2i) = [-2+3i]$
9. $3-4+2i = [-1-2i]$
10. $\frac{2}{1-i} = [1+i]$
11. $\frac{6}{3+2i} = [\frac{18}{13} - \frac{12}{13}i]$
12. $\frac{2+i}{1-3i} = [-\frac{2}{5} + \frac{4}{5}i]$
13. $\frac{3-i}{2i} = [-\frac{1}{2} - \frac{3}{2}i]$
14. $(1-3i)^2 = [-8-6i]$
15. $(2+i)^2 = [3+4i]$
16. $(3-4i)^2 - (1-i) = [-31-17i]$
17. $\frac{-3}{1-7i} = \frac{-3}{50} - \frac{21}{50}i$
18. $2 + \sqrt{(-1)} = [4/13 + 7/13i]$

2. U qor lammaane horsan oo kasta (tiro kakan ah) sansaanka a + bi :

- | | | | |
|-------------|------------|------------|-----------|
| 1. (2, 6) | 2. (-3, 4) | 3. (5, -2) | 4. (0, 6) |
| 5. (-7, -3) | 6. (-3, 2) | 7. (4, 0) | 8. (0, 0) |

3. Tiro kakan oo kasta u qor sida lammaane horsan :

- | | |
|--------------|----------|
| b. $2 + 3i$ | kh. $4i$ |
| t. $4 - 2i$ | d. 0 |
| j. $3 + i$ | r. 7 |
| x. $-6 - 3i$ | s. $-i$ |

4. Qor xistiga tiro kakan oo kasta oo ku jira su'aasha- saddexaad.

Tusaale 1:

Doon tirooyinka maangalka ah x iyo y si ay :

$$3x + 2yi - ix + 5y = 7 + 5i$$

Furfuris :

$$3x + 5y + i(2y - x) = 7 + 5i$$

Maangadyadana isle'egkeysii

$$3x + 5y = 7; \quad 2y - x = 5.$$

Marka aynnu wadajir u furfurno $x = -1, y = 2$.

Tusaale 2:

Raadi tirooyinka maangalka ah x iyo y ee run ka dhigaya isle'egta $(x - 2i)^2 = yi$.

Furfuris : U qor tiro kasta sansaanka a + bi

$$\begin{aligned} (x - 2i)^2 &= yi \\ (x - 2i)(x - 2i) &= yi \\ x^2 - 4xi + 4i^2 &= yi \\ x^2 - 4xi - 4 &= yi \\ (x^2 - 4) - 4xi &= yi \end{aligned}$$

Markaa innaga oo raacaynna Qeexda (2), $x^2 - 4 = 0, -4xi = yi$. Markaa $x = 2$ ama $x = -2$.

Haddii $x = 2$, markaa $y = -8$; haddii $x = -2$, markaa $y = 8$. Haddaba tirooyinka la doonayaa waa : $(2, -8)$ iyo $(-2, 8)$.

5. Doon tirooyinka maangalka ah x iyo y si ay :

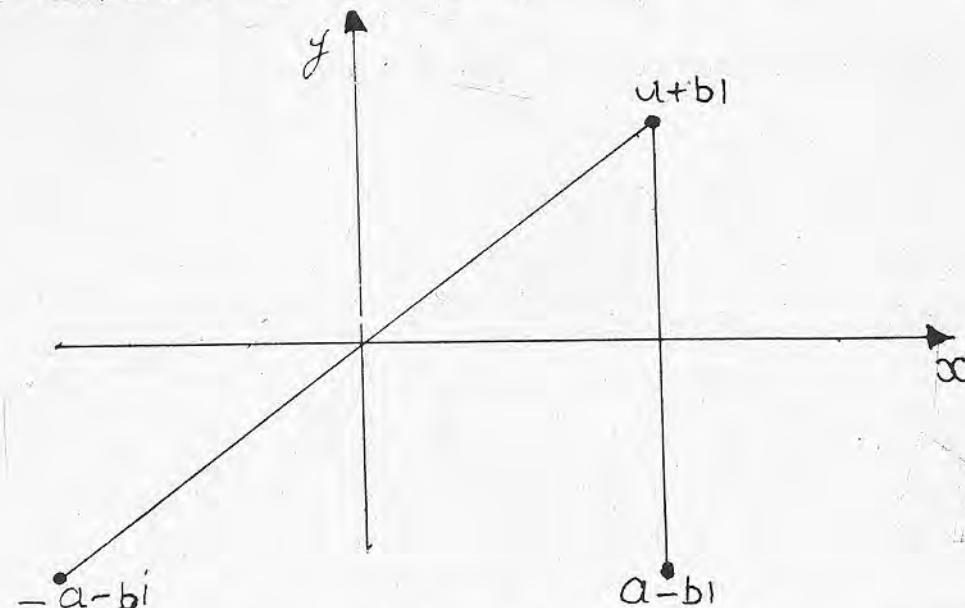
- 1) $2x - yi = 3 + 2i$
- 2) $-2i = 3x + yi$
- 3) $4 + xi = x^2 - yi$
- 4) $x + 9i = y + y^2i$
- 5) $(x + 3i)^2 = 2yi$
- 6) $(x - 2i)^2 = 3x + yi$

**TIROOYINKA KAKAN
OO GARAAF AHAAN LOO MUUJIYO**

Kulannada Kaartis isku aaddin 1 — 1 ah ayaa ka dhexaysaa ururka lammaanayaasha horsan (a, b) ee ku jira $R \times R$ (Ra lanqayr Ra) iyo ururka baraha p (a, b) ee ku jira sallaxa joometeriga ah. Isla markaa, haddii tiro kakan oo kasta loo qori karo sansaanka lammaane horsan (a, b) bar kasta oo ku jirta ama ku taal sallaxa waxaa loo arki karaa garaafka tiro kakan (a, b) ama a + bi sida shaxankan hoose ina tusayo.

Haddii qaybta maangalka ah ee a + bi taasoo ah, $(a = m_1(a + bi))$ loo qaato absisada ama kulanka —x ee dhobicda p, markaas ayaa dhidibka —x la yiraa **Dhidibka maangadka ah**. Sidoo kale haddii qaybta maangadka ah ee a + bi (t.a. $b = M_2(a + bi)$) loo qaato ordinayaalka barta p y waxaa la yiraa **Dhidibka maangadka ah**.

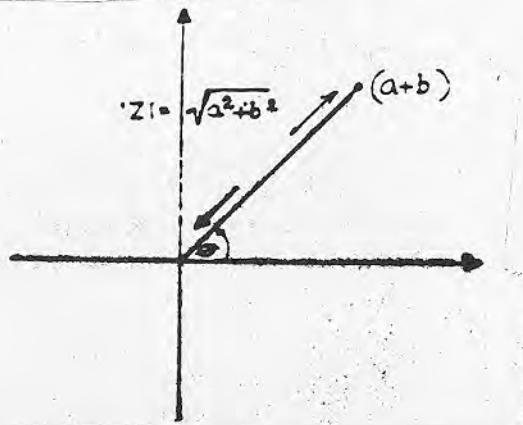
Sida aynu u niraa barta (2, 3) marka aynu u jeedno barta kulannadeedu yihin (2, 3); ayeynu iyana u niraa barta $2 + 3i$, marka aynu u jeedno barta u taagan tirada kakan ee $2 + 3i$. Sallaxa tirada kakan lagu muujiyo waxaa la yiraa **Sallax kakan**. Shaxankan hoose wuxuu muujinaya tirada kakan ee $Z = a + bi$, iyo xistigeeda $Z = a - bi$, iyo tabanaheeda $-Z = -a - bi$, iyo tabanaha xistigeeda $-Z = -a + bi$



Shaxanku wuxuu ina tusayaa in Z uu yahay noqodka \bar{Z} marka loo eego dhidibka maangalka ah $-Z$ waa noqodka Z marka loo eego unugga; $-Z$ waxay tahay noqodka \bar{Z} marka loo eego dhidibka maangadka ah.

Q e e x 1 2 :

Qiimaha sugar ee tirada kakan $Z = a + bi$, qormadeedu waa r, $|Z|$ ama $|a + bi|$, sida loo helaana waa $r = |Z| = |a + bi| = \sqrt{a^2 + b^2}$. Qiimaha sugar ee $|a + bi|$ wax kale maaha ee waa fogaanta u dhexeysa unugga iyo barta $a + bi$.



Q e e x 13 :

Baaxa tirada kakan ee $Z = a + bi$, loona qoro Baaxa ($a + bi$), waa xagasha θ ee bar bilowgeedu yahay dhidibka $-x$ ee tagan, bar dhammaadkeeduna yahay fallaarta ka tagta unugga ee ay ku jirto $a + bi$.

O G O O W. Haddii ay θ tahay Baaxa $a + bi$, markaas $(\theta + K \cdot 360)^\circ$ waa baaxa $a + bi$, marka K u taagan tahay abyone kasta. Haddii θ ay tahay baaxa $a + bi$, oo $a \neq 0$, markaas $\frac{b}{a} = \tan \theta$

L a y l i :

Samee garaafka, tiro kasta oo kakan, xistigeeda, tabanaheeda iyo tabanaha xistigeeda. Sawir xarriijimaha isku xiraya lammaane kasta oo ka mid ah afartaa dhibcood.

- | | |
|--------------|--------------|
| 1) $2 + 3i$ | 6) $-3i$ |
| 2) $-3 + 4i$ | 8) -5 |
| 3) $4 - i$ | 7) 6 |
| 4) $-i - 2$ | 9) $-4 - 3i$ |
| 5) $4i$ | |

2. Qor adoon isticmaalayn qormada qiimaha sugan.

T u s a a l e :

$$b. |2 + 5i| = \sqrt{2^2 + 5^2} = \sqrt{29}$$

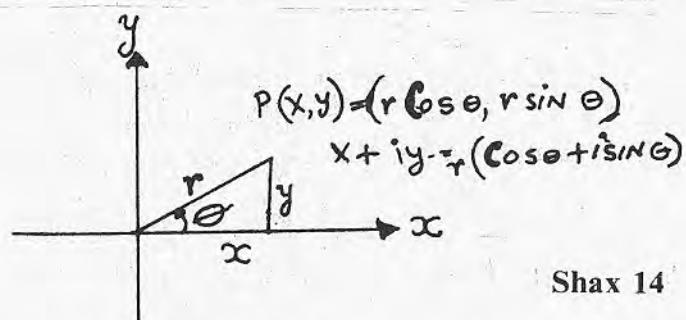
$$t. |-3| = \sqrt{(-3)^2} = 3$$

$$j. |-3,2| = \sqrt{(-3)^2 + (2)^2} = \sqrt{13}$$

- | | |
|---------------|------------------|
| 1) $ 4 $ | 7) $ (2,0) $ |
| 2) $ -2 $ | 8) $ (3, -5) $ |
| 3) $ 3 + 2i $ | 9) $ (-2,1) $ |
| 4) $ 4 - i $ | 10) $ (0,3) $ |
| 5) $ 5i $ | 11) $ (-2, -1) $ |
| 6) $ -3i $ | 12) $ (-7, -1) $ |

SANSAAN GEFTIMAADKA TIRO KAKAN

Haddii p tahay bar ku taal sallaxa kakan oo ku beegan tirada kakan ee (x, y) ama $x + iy$, markaa shaxanka waxaan ka aragnaa in $x = r \cos \theta$; $y = r \sin \theta$; $r = \sqrt{x^2 + y^2} = |x + iy|$.



Shax 14

Marka waa arrin cad in $Z = r(\cos \theta + i \sin \theta) \dots (I)$. Isle'egta (I) waxa la yiraa **Sansaan geftimaadka tiro** kakan, r iyo θ waxaa la yiraa **Kulanno geftimaadka**. Si sahan oo loo soo gaabiyo tibaaxda $\cos \theta + i \sin \theta$ waa **Cis** θ (oo loo akhriyo: kosayn θ oo loo geeyey i sayn θ) markaa waxaynu qorraa.

$$a + bi = r \operatorname{Cis} (\theta + 2\pi K), K \in \mathbb{Z}$$

O G O W : j waxay u taagan tahay ururka abyooneyaaasha. R waxay u taagan tahay Gacansiin.

Had iyo jeer θ waxaynu u qaadanaa cabbirka ugu yar ee aan tabnayn ee $a + bi = r \text{ Cis } \theta$.

Tusaale 1 :

Sansaan geftimaad u qor tiradan kakan $2 + 2\sqrt{3}i$

Furfuris :

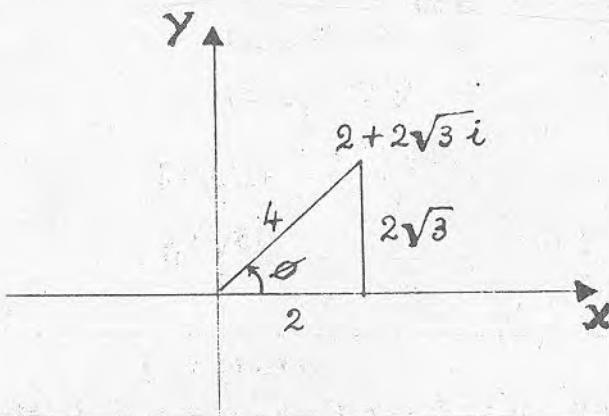
$$\begin{aligned} \text{Qiima sugar ee } r &= |2 + 2\sqrt{3}i| = \sqrt{4 + 12} = \\ &= \sqrt{16} = 4 \end{aligned}$$

$$\text{Baxaa } \theta = \sin^{-1} \frac{2\sqrt{3}}{4}$$

$$= \sin^{-1} \frac{\sqrt{3}}{2} = 60^\circ = \frac{\pi}{3}$$

$$\begin{aligned} \text{Markaa } 2 + 2\sqrt{3}i &= r(\cos \theta + i \sin \theta) \\ &= 4(\cos 60^\circ + i \sin 60^\circ) \\ &= 4 \left[\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right] \\ &= 4 \text{ Cis } \frac{\pi}{3} \end{aligned}$$

Shax 15



Tusaale 2 :

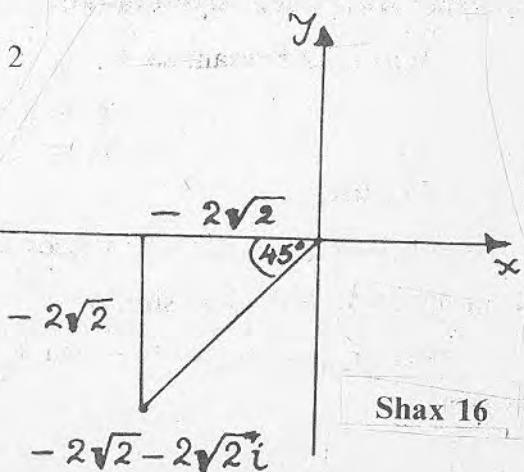
U muuji $4 \text{ Cis } 225^\circ$ garaaf ahaan, dabadeedna tirada sansaan laydi u qor.

Furfuris :

$$a = r \cos \theta = 4 \left[-\frac{\sqrt{2}}{2} \right] = -2\sqrt{2}$$

$$b = r \sin \theta = 4 \left[-\frac{\sqrt{2}}{2} \right] = -2\sqrt{2}$$

$$\text{Markaa } a + bi = -2\sqrt{2} - 2\sqrt{2}i$$



Layli :

1. Tirooyinka kakan u qor sansaanka $r \text{ Cis } \theta$

Jawaab.

- | | |
|-----------------------------------------|-------------------------------------------|
| 1) $-5 + 5i$ | $[5\sqrt{2} \text{ Cis } \frac{3\pi}{4}]$ |
| 2) $-\sqrt{6} - \sqrt{2}i$ | $[2\sqrt{2} \text{ Cis } \frac{7\pi}{6}]$ |
| 3) $-3i$ | $[3 \text{ Cis } \frac{3\pi}{2}]$ |
| 4) $2 - 2i$ | $(2\sqrt{2} \text{ Cis } 315^\circ)$ |
| 5) $-1 + \sqrt{3}i$ | $(2 \text{ Cis } 120^\circ)$ |
| 6) $2\sqrt{2} + 2\sqrt{2}i$ | $(4 \text{ Cis } 45^\circ)$ |
| 7) $-i$ | $(\text{Cis } 270^\circ)$ |
| 8) -4 | $(4 \text{ Cis } 180^\circ)$ |
| 9) $-2\sqrt{3} - 2i$ | $(4 \text{ Cis } 210^\circ)$ |
| 10) $2i$ | $(2 \text{ Cis } 90^\circ)$ |
| 11) $\frac{\sqrt{3}}{2} - \frac{3i}{2}$ | $(3 \text{ Cis } 300^\circ)$ |

2. Tiro kasta oo kakan u qor sansaanka $a + bi$

- | | |
|-------------------------------------|-------------------------------------------|
| b. $6 \text{ Cis } 135^\circ$ | $(-3\sqrt{2} + 3\sqrt{2}i)$ |
| t. $12 \text{ Cis } 90^\circ$ | $(12i)$ |
| j. $4 \text{ Cis } 315^\circ$ | $(2\sqrt{2} - 2\sqrt{2}i)$ |
| x. $2 \text{ Cis } \frac{5\pi}{4}$ | $(-\sqrt{2} - \sqrt{2}i)$ |
| kh. $3 \text{ Cis } \frac{7\pi}{6}$ | $[\frac{-3\sqrt{3}}{2} - (3/2)i]$ |
| d. $5 \text{ Cis } \frac{-2\pi}{3}$ | $[\frac{-5}{2} - \frac{(5\sqrt{3})i}{2}]$ |

Aragtiinka I

Haddii Z_1 iyo $Z_2 \in K$, oo ay $Z_1 = r_1 \text{ Cis } \theta_1$; $Z_2 = r_2 \text{ Cis } \theta_2$. Markaa :

- 1) $Z_1 \cdot Z_2 = r_1 \cdot r_2 \text{ Cis } (\theta_1 + \theta_2)$
- 2) $\frac{Z_1}{Z_2} = \frac{r_1}{r_2} \text{ Cis } (\theta_1 - \theta_2), \quad (Z_2 \neq 0)$

Caddeyntha Aragtiinka qaybtiisa hore.

Waxaynu haysannaan in

$$Z_1 = r_1 (\cos \theta_1 + i \sin \theta_1)$$

$$Z_2 = r_2 (\cos \theta_2 + i \sin \theta_2)$$

Markaa :

$$\begin{aligned} Z_1 \cdot Z_2 &= r_1 (\cos \theta_1 + i \sin \theta_1) \cdot r_2 (\cos \theta_2 + i \sin \theta_2) = r_1 \cdot r_2 (\cos \theta_1 \cos \theta_2 + i \cos \theta_1 \sin \theta_2 \\ &\quad + i \sin \theta_1 \cos \theta_2 + i^2 \sin \theta_1 \sin \theta_2) \\ &= r_1 \cdot r_2 [(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) + i (\cos \theta_1 \sin \theta_2 + \sin \theta_1 \cos \theta_2)] \end{aligned}$$

$$= r_1 \cdot r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

$$\therefore Z_1 \cdot Z_2 = r_1 \cdot r_2 \operatorname{Cis}(\theta_1 + \theta_2).$$

Aragtiinkan qaybtiiisa (2) laylis ahaan baa laguugu dhaafay.

Tusaale 1:

U qor taranka $3 \operatorname{Cis} 80^\circ \cdot 5 \operatorname{Cis} 40^\circ$ sansaanka $a + bi$.

Furfuris:

Raac aragtiinka I-1:

$$\begin{aligned} 3 \operatorname{Cis} 80^\circ \cdot 5 \operatorname{Cis} 40^\circ &= 3 \cdot 5 \operatorname{Cis}(80^\circ + 40^\circ) \\ &= 15 \operatorname{Cis} 120^\circ \\ &= 15 (\cos 120^\circ + i \sin 120^\circ) \\ \cos 120^\circ &= -\frac{1}{2} \\ \sin 120^\circ &= \frac{\sqrt{3}}{2} \end{aligned}$$

Markaa:

$$\begin{aligned} 15 (\cos 120^\circ + i \sin 120^\circ) &= 15 \left[-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right] \\ &= -\frac{15}{2} + \frac{15\sqrt{3}i}{2} \end{aligned}$$

Tusaale 2:

U qor $\frac{8 \operatorname{Cis} 540^\circ}{2 \operatorname{Cis} 225^\circ}$ sansaanka $a + bi$.

Furfuris:

Raac aragtiinka I-2:

$$\begin{aligned} \frac{8 \operatorname{Cis} 540^\circ}{2 \operatorname{Cis} 225^\circ} &= \frac{8}{2} \operatorname{Cis}(540^\circ - 225^\circ) = 4 \operatorname{Cis} 315^\circ \\ &= 4 (\cos 315^\circ + i \sin 315^\circ) \\ \cos 315^\circ &= \frac{1}{\sqrt{2}} \\ \sin 315^\circ &= -\frac{1}{\sqrt{2}} \end{aligned}$$

Marka

$$\begin{aligned} 4 (\cos 315^\circ + i \sin 315^\circ) &= 4 \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right] \\ &= 2\sqrt{2} - 2\sqrt{2}i \end{aligned}$$

Layli:

Haddii lammaanayaashan Z_1 iyo Z_2 ay tirooyin kakan yihii, raadi i) $Z_1 \cdot Z_2$; ii) $\frac{Z_1}{Z_2}$ isla markaana u qor jadeeyada sansaanka $a + bi$.

$$\begin{aligned} \text{i)} \quad Z_1 &= 3 \operatorname{Cis} 90^\circ \quad Z_2 = \sqrt{2} \operatorname{Cis} 45^\circ \\ [Z_1 \cdot Z_2 = -3 + 3i; \quad \frac{Z_1}{Z_2} = \frac{3}{2} + \frac{3i}{2}] \text{ Jawaab} \end{aligned}$$

2) $Z_1 = 6 \text{ Cis } 150^\circ \quad Z_2 = 18 \text{ Cis } 570^\circ$

$$[Z_1 \therefore Z_2 = +108 \quad \frac{Z_1}{Z_2} = \frac{1}{6} - \frac{\sqrt{3}i}{6}] \text{ Jawaab}$$

3) $|Z_1| = -3 + i \quad Z_2 = -2 - 4i$

$$[Z_1 \cdot Z_2 = 10 + 10i \quad \frac{Z_1}{Z_2} = \frac{1}{10} - \frac{7i}{10}] \text{ Jawaab}$$

4) U qor $[\frac{1}{2} - i\frac{\sqrt{3}}{2}]$ sansaanka r cis θ

[Cis 240°] Jawaab

5) Tus in xistiga tirada kakan ee r Cis θ , uu yahay r Cis $(-\theta)$.

Aragtiinka II (Demoivre)

Waxaynu ognahay in $a + bi = r \text{ Cis } \theta$, marka

$$(a + bi)^2 = (r \text{ Cis } \theta)(r \text{ Cis } \theta) = r^2 \text{ Cis } 2\theta$$

$$(a + bi)^3 = (a + bi)^2(a + bi) = (r^3 \text{ Cis } 2\theta)(r \text{ Cis } \theta) = r^3 \text{ Cis } 3\theta$$

$$(a + bi)^4 = (r^3 \text{ Cis } 3\theta)(r \text{ Cis } \theta) = r^4 \text{ Cis } 4\theta$$

Haddaba Aragtiinka Demoivre waxuu oranayaa: haddii $Z \in K$, $n \in N$, oo ay $Z = r \text{ Cis } \theta$
Markaa $Z^n = r^n \text{ Cis } n\theta$

Tusaale :

$$U \text{ qor } (\sqrt{3} + i)^7 \text{ sansaanka } a + bi.$$

Furfuris :

$$r = \sqrt{(3)^2 + (1)^2} = 2.$$

Si aynu u hello Baaxa $(a + bi)$, waxaynu ognahay in $\tan \theta = \frac{1}{\sqrt{3}}$, markaa $\theta = 30^\circ$

$$\text{Haddaba } (\sqrt{3} + i)^7 = (2 \text{ Cis } 30^\circ)^7 = 2^7 \text{ Cis } (7 \cdot 30^\circ)$$

$$= 128 \text{ Cis } 210^\circ = 128 (\cos 210^\circ + i \sin 210^\circ)$$

$$= 128 \left[-\frac{\sqrt{3}}{2} - \frac{1}{2}i \right] = -64\sqrt{3} - 64i$$

$$\text{Markaa } (\sqrt{3} + i)^7 = -64\sqrt{3} - 64i.$$

Qeex 14 :

Haddii aanay $Z \neq 0 + 0i$, markaa

1) $Z^0 = 1 + 0i$

2) $Z^{-n} = \frac{1}{Z^n}$, $n \in j$, (n waa kutirsane ururka abyoneyaalka togan).

Aragtiinka III

Haddii $Z \in K$, (oo $Z \neq 0 + 0i$), $n \in J$. oo $Z = r \text{ Cis } \theta$, markaa $Z^n = r^n \text{ Cis } n\theta$.

a d d e y n :

Ore waxaynu u aragnay marka $n \in N$. Mar haddii

$$Z^\circ = 1^\circ \text{ Cis } 0 \cdot 0 = 1 \text{ Cis } \theta = 1 (\cos \theta + i \sin \theta) = 1 + 0i = 1$$

Markaa aragtiinku waa dhab marka $n = 0$. Bal hadda aan eegno, marka aynu ha sanlo $-n$, (n waa tirsimo), taas oo ah in tusno in

$$Z^{-n} = r^{-n} \text{Cis } (-n \theta)$$

Haddaba haddii $Z \neq 0 + 0i$, oo $Z = r \text{ Cis } \theta$, markaa $Z^{-n} = \text{Cis } (-\theta)$. Haddaba innaga raad na Qeexda 14 - 2 :

IRO : Aragtiin III waa run marka n ay tahay tiro uun lakab ah.

$$Z^{-n} = \left(\frac{1}{Z^n}\right) = \left(\frac{1}{Z^n}\right)^n$$

$$\frac{1}{Z} = \frac{1}{Z} \cdot \frac{\bar{Z}}{\bar{Z}} = \frac{\bar{Z}}{|Z|^2}$$

Markaa

$$\begin{aligned} Z^{-n} &= \left(\frac{1}{Z}\right)^n = \left(\frac{Z^-}{|Z|^2}\right)^n \\ &= \frac{1}{|Z|^{2n}} (Z)^n = \frac{1}{|Z|^{2n}} [r \text{ Cis } (-\theta)]^n \\ &= \frac{1}{|Z|^{2n}} r^n \text{ Cis } (-n\theta) \end{aligned}$$

Markaa haddii $|Z| = r$, $Z^{-n} = \frac{1}{r^{2n}} r^n \text{Cis } (-n\theta) = r^{-n} \text{ cis } (-n\theta)$.

T u s a a l e :

$$\text{U qor } (1 + i)^{-6} \text{ sansaanka } a + bi.$$

F u r f u r i s :

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\text{oo markaa } \tan \theta = 1 \text{ ama } \theta = 45^\circ$$

Markaa waxaynu haysannaa

$$\begin{aligned} (1 + i)^{-6} &= (\sqrt{2} \text{ Cis } 45^\circ)^{-6} \\ &= (\sqrt{2})^{-6} \text{ Cis } (-6 \cdot 45^\circ) \\ &= \frac{1}{8} \text{ Cis } (-270^\circ) \\ &= \frac{1}{8} [\cos (-270^\circ) + i \sin (-270^\circ)] \end{aligned}$$

$$\cos (-270^\circ) = 0, \sin (-270^\circ) = 1 \text{ waxayna helaynaa}$$

$$\therefore (1 + i)^{-6} = \frac{1}{8} (0 + i) = 0 + \frac{1}{8} i = \frac{1}{8} i$$

L a y l i :

- U qor tibaaxahana sansaanka $a + bi$
- 1) $[2 \text{ Cis } (-30^\circ)]^7$ $(-64\sqrt{3} + 64i)$
 - 2) $[\text{Cis } 360^\circ]^5$ $(1 + 0i)$
 - 3) $[-\frac{1}{2} + \frac{1}{2}\sqrt{3}i]^3$ $(1 + 0i)$
 - 4) $\left[\frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \right]^{10}$ $\left[-1 + \frac{\sqrt{3}i}{2} \right]$
 - 5) $\frac{(2 \text{ Cis } 150^\circ)^7}{(4 \text{ Cis } 45^\circ)^3}$ $(\sqrt{3})^{-1}$
 - 6) $(\sqrt{3} \text{ Cis } 50^\circ)^{-12}$ $\frac{1}{729} \left[\frac{1}{2} - \frac{3}{2}i \right]$
 - 7) $(\sqrt{3} - i)^{-5}$ $\frac{1}{64} (-\sqrt{3} + i)$
 - 8) $\frac{(1 - i)^5}{(1 + i)^6}$ $-\frac{1}{2} - \frac{1}{2}i$

XIDIDDO NAADADA TIRO KAKAN

Sida ugu fudud ee lagu fahmo xididdo naadada tiro kakan, waa innagoo ku bilowna usaale :

T u s a a l e 1 :

Raadi xididdo saddexjibbaarka $4(\sqrt{3} + i)$

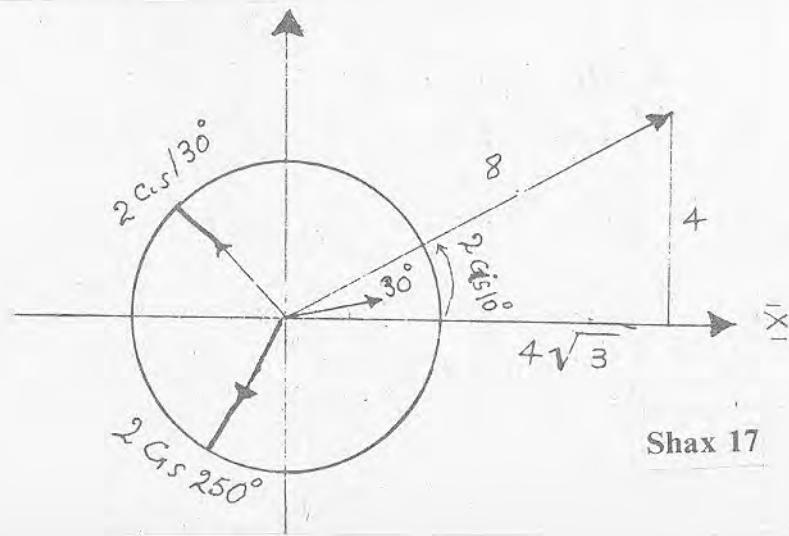
F u r f u r i s :

Xisaabta macnaheedu waxaa weeye raadi qiimayaasha $[4(\sqrt{3} + i)]^{1/3}$ Tirada iyada ah marka sansaan geftimeed ahaan loo dhigo waxay noqonaysaa

$$(8 \text{ Cis } 30^\circ)^{1/3} = 8^{1/3} (\text{Cis } 30^\circ)^{1/3}$$

Marka waxaad mooddaa in marka aynu adeegsanno aragtiiinka DEMOIVRE, $8(\text{Cis } 30^\circ)^{1/3}$ jawaabteedu tahay $2 \text{ Cis } 10^\circ$. Run ahaantiise warku intaa kuma eka. Baaxa tirada kakan ee ugu horaantii laynä siiyey waa 30° , waxaynuse ognahay in Baaxa tiradaasi aanu 30° qudha ahayn ee uu noqon karo $390^\circ, 750^\circ, 1110^\circ$, iwm. ama $-330^\circ, -690^\circ$ iwm. Run ahaantii $8 \text{ Cis } 30^\circ = 8 \text{ Cis } 390^\circ = 8 \text{ Cis } 750^\circ = 8 \text{ Cis } 1110^\circ$ Markaa $(8 \text{ Cis } 30^\circ)^{1/3} = 2 \text{ Cis } 130^\circ$ ama $2 \text{ Cis } 250^\circ$; ama $2 \text{ Cis } 370^\circ$

Waxaad aragtaa in qiimayaasha tiradooda aan cidina soo koobi karin, laakiin sida shaxanku ina tusayo, qiimayaashu waa saddex uun, inta kalena waa saddexdii qiime ee soo noqnoqday, waayo $2 \text{ Cis } 10^\circ = 2 \text{ Cis } 370^\circ$.



Shax 17

Guulahaan xididdo Naadada tiro kakan tiradoodu waxay le'eg tahay n, waxayna sameeyaan geesoole hufan oo n.gees leh.

Hadalkii oo yar xididdo Naadada tirada kakan ee $Z = r \text{ Cis } \theta$ waxaa weeye :

$$\sqrt[n]{r \text{ Cis } \theta} = \sqrt[n]{r \text{ Cis}} \left[-\frac{\theta}{n} + \frac{2\pi k}{n} \right]$$

$$K = 0, +1, +2, \dots$$

f u s a a l e 2 :

$$Z = -16, r = +16, \theta = \pi.$$

$$\sqrt[4]{16 \text{ Cis } \pi} = \sqrt[4]{16 \text{ Cis}} \left[\frac{\pi}{4} + \frac{2\pi K}{4} \right]$$

$$\sqrt[4]{16 \text{ Cis } \pi} = 2 \text{ Cis} \left[\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right]$$

Afartii Xididna waxay noqonayaan :

$$w_0 = 2 \left[\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right] = \sqrt{2}(1+i)$$

$$w_1 = 2 \left[\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right] = \sqrt{2}(-1+i)$$

$$w_2 = 2 \left[\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right] = \sqrt{2}(-1-i)$$

$$w_3 = 2 \left[\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right] = \sqrt{2}(1-i)$$

L a y l i :

Raadi xididdada xusan ee laylisyadan :

I Raadi xididdada isle'egtan :

- a) $Z^5 = -32$
- b) $Z^6 + 1 = 0$

II Raadi xididdada xusan :

- | | |
|----------------------------|---------------------------|
| 1. $(-1+i)^{1/3}$ | 6. $(2+2\sqrt{3}i)^{1/3}$ |
| 2. $(-2\sqrt{3}-2i)^{1/4}$ | 7. $(-16i)^{1/4}$ |
| 3. $(-15-8i)^{1/2}$ | 8. $(64)^{1/6}$ |
| 4. $(-4+4i)^{1/5}$ | 9. $(i)^{2/3}$ |
| 5. $(2\sqrt{3}-2i)^{1/2}$ | |

C u t u b H I L O O J I G

A R A R :

Waxaa laga yaabaa in aad in badan ka maqasha sheekooinka caadiga ah ama wadaha-dallada rasmiga dad isku eedeynaya loojiga la'aan ama isku ammaanaya loojighayina cutubkan gaaban waxaan ka baranaynaa saldhigga maadada loojingga rasmiga ah, siduu u yahay qayb abyaa oo ka mid ah barashada xisaabta maanta ee ballaaran.

Haddana cutubkani, haba gaabnaadeen wuxuu inoo awood siinayaa in aynu qof ku eedeynno loojig la'aan qofna ku ammaanno loojig hayin dhexdhexaadil la'aan ama sidii aynnu fikradaheenna si habboon isugu duwan lahayn.

Loojig haddaba, waa qayb horay u marsan oo ka mid ah xisaabta shaqa-galka ah ee ku saabsan saafiid kasta. Loojiggoo waa barasho habdhisan oo xaaladaha guud ee dhuuxis dhisan ku saabsan.

Dhuuxiddii waa habka lagu gaarayo xukun ama hawraar go'aan ah. Go'aankaasoo hawraar kale oo la ogyahay malagelinayaan. Sida laan kasta ee waxbarasho loojiggoo wuxu leeyahay tabo iyo xeerar lagama maarmaan u ah saafidda wixii sax ah iyo wixii gef ah garaakasta. Hawsha caynkaas ahi ma sahlana. Hase ahaatee, haddii aynnu akhrinno ama dhegeysanno qof dooddii, waynu ku dhici karnaa in aynu dooddaas u qiiimaynno mid «dhisan» ama mid jaban, hadba siday u caddahay.

Sidaa awgeed waxaa caadi ah in si la isla yaqaan oo summado ah loo tarjumo macnaha dood kasta inta aan la sugin inay jaban tahay ama inay dhisan tahay.

Sida ay run ku tahay af kasta fikradi waxay macna buuxa ku sameysaa saddex fansaar oo waaweyn (af kastaa leeyahay) mid uun.

- b. Fansaar warbixin
- t. Fansaar sifeeyn
- j. Fansaar amar bixin.

Hase ahaatee mid kasta oo ka mid ah saddexda qaybood waxaa loo tibaaxi karaa sansaan weedheed; weedhahaasoo u qaybsami kara kuwa fudud, kuwa tudcan ama kakan. Weedhuhu misna waxay u qaybsamaan :

- 1) Amarro
- 2) Hawraaro
- 3) Su'aalo
- 4) Yaabyo

Mar haddii loojig ku saabsan yahay dhuuxid waxaan ku koobeynaa falamqeynteenha Hawraaro oo keliya.

Q e e x :

Hawraari waa weedh run ama been mid uun ah.

T u s a a l e :

- 1) Aamina waa ardayad fahmo leh.
- 2) Qamaan wuxuu ahaa gabayaa weyn.
- 3) Loojig waa xisaab waxtar leh.

Saddexda weedhood ee kor ku qorani dhammiaan waa fudud yihiin, waynu isku raaci karnaa tii run ah iyo tii been ah.

Si sumado loogu qoro weedha, waxaan u tarjumnaa sida soo socota: Ka dhig in qay u taagan tahay «Aamina waa ardayad fahmo leh». Tarjumadaa daraadeed, markasta oo aynu la kulanno «Aamina waa ardayad fahmo leh». Waxaynnu u joojin q.

q: Aamina waa ardayad fahmo leh.

q

R

B

B iyo R waxay u taagan yihiin been iyo run. Labadaba waynu qornay waayo waa marjiime saafidda u suuroobi karta hawraar fudud oo kasta, t.a., q run bay noqon ama been, abadabase ma wada noqon karto.

Tusaalooyin Hawraaro :

- 1) 4 waa tiro kisi ah.
- 2) 3 way qaybisaa 21.
- 3) Xamar waxay u dhexeysaa Hargeysa iyo Jowhar.
- 4) Caasha waa guulwade firfircooni.

Tusaalooyin weedho aan ahayn hawraaro.

- b. Fariiso
- t. Hawraartan aad akhrineysaa waa been.
- j. Alla ! qurux badnaa !
- x. Qodh iyo xero waa Maxamed !
- kh. Magacaa ?

Q e e x :

Hawraar mutuxan ama atam ah waa weedh aanay ku jirin weedh dabarro. Dhammaan hawraaraha ilaa iyo iminka innaga hor yimid waa hawraaro mutuxan.

L a y l i :

- I) Sheeg in weedhahani yihiin hawraaro iyo in kale.
 - b. 3 waa u isir 7.
 - t. 12 waa tiro dhaban ah.
 - j. Ii sheeg da'daada.
 - x. Xamar weligeedba roob ayey heshaa 4ta Abril
 - kh. Maroodi waa ka dheereeyaa deero.
 - d. Siduu yahay macallinkiina Xisaabtu ?
 - s. Iska warran.
 - sh. Xamar waa magaalo weyn.
 - dh. Waa wareey.
 - c. Inanta kursiga uga kac.
 - g. Ma ka qayb gashay Iskaa-wax-u-qabsadii shalay ?
- II) U tarjum hawraaraha soo socda sumado. Kala sheeg hawraar kasta run ama been kolba kay tahay.
 - I) Qof kastoo Soomaaliya joogaa Af-Soomaaliga si wanaagsan ayuu ugu hadlaa.
 - 2) Roodiisiya caddaan yar baa si sharcidarro ah u xukuma.
 - 3) $2 + 3$ waa ka weyn tahay 2×3 .
 - 4) $1 + 1 > 1 \times 1$

- 5) Ma tahay taasu dood dhisan ?
- 6) Badweynta Hindiya waa deggan tahay sannadka oo idil.

Q e e x :

Haddii Hawraari leedahay hal weedh-dabar ama in ka badan, hawraartaas waxaa la yiraa **Hawraar tudcan** ama **Molikiyool**.

T u s a a l e :

- Hawraaro tudcan : . . .
- b. Maanta ma aha 1dii Maajo.
 - t. Cali iyo Nuur waa arday.
 - j. 3 waa tiro kisi ama tiro farcan.
 - x. Haddii aan baska haleeli waayo, markaa shaqada waan ka daahi doonaa.
 - 4) $3 \cdot 6 = 18$ weliba $3 + 6 = 9$

Weedh-Dabarro	Dabarro
ma (~)	Diidmo
yo (Λ)	Isku xire
ama (V)	Kala xire
haddii . . . markaa (→)	Shardiile
Haddii iyo haddi qudha (↔)	Labashardiile

OGOW : dabarrada qaarkood sida isku xiraha iyo Shardiilaha, sansaanno dhawr ah bay qaataan.

T u s a a l e :

- Hawraaro isku-xir ah.
- 1) Muuse waa arday waana shaqaale.
 - 2) $3 \cdot 6 = 18$ isla markaa $3 + 6 = 9$
 - 3) $3 \cdot 6 = 18$ haddana $3 + 6 = 9$

T u s a a l e :

Sansaanno kale oo «haddii . . . markaa ay qaadato waa :

- 1) Haddii aan baska haleeli waayo, shaqada waan ka daahi doonaa.
- 2) Haddii aan baska haleeli waayo, kolkaa shaqada waan ka daahi doonaa.

Mar haddii aynu aragnay sida loo tarjumo hawraar mutuxan (atam) cabsi kama qabno sidaynu u tarjumi lahayn hawraar tuducan, maxaa yeelay waxay ka kooban tahay laba atam ama in ka badan oo ay la jiraan hal weedh dabar ama in ka badan.

Bal aynu dayno in aynu tarjumno «Maanta ma aha 1dii Maajo.» Waxaa jirta tab ah «mar walba u tarjumo hawaarahaga sansaankooda aqbalka ah».

Markan aynu ka dhigno q in ay u taagan tahay «Maanta waa 1dii Maajo.» Haddaba hawraarta ah «Maanta ma aha 1dii Maajo.» Waxay u tarjumantaa ~ q.

$$\begin{array}{c} q \sim q \\ \hline R & B \\ B & R \end{array}$$

Qodob kale oo aynnu aragnaa waa : Haddii q been tahay ~ q waa run, haddii ~ q been tahay, q waa run. Tani isla markaa waxay dhammeynaysaa mar-qiime saafidda u suuroobi karta hawraar iyo diidmadeed.

Hadda, aynu tarjumno « $3 \times 6 = 18$, $3 + 6 = 9$ »; hawraartu waxay u qormi kartaa «3 lagu dhuftay 6 waa 18, 6 loo geeyay 3na waa 9». Na-da 3da ka dambeynsaa waxay la shaqa tahay iyo.

Ku dhig q: $3 \times 6 = 18$, k: $3 + 6 = 9$. Hawraarta tuducdani waxay sumad ahaan u qormi sidan : q \wedge k. Si aynu u helno qiimaha marar saafidda q \wedge k waxaynu baareynaa inta raacayn kala jaad ah u suuroobi karta qiimaha q \wedge k.

- 1) q iyo k labaduba run bay noqon karaan.
- 2) q run bay noqon kartaa k-na been.
- 3) q been bay noqon kartaa k-na run.
- 4) q iyo k labaduba been bay noqon karaan.

Sidaa awgeed isku xirka hawraarta tuducani waxay leedahay 4 racayn oo qiime rumeedka ama beeneedka q iyo k isku raaci karaan sida **tuse rumeedkanu** bandhigayo.

	q	k	q \wedge k
1	R	R	R
2	R	B	B
3	B	R	B
4	B	B	B

Waa ka caddahay tusaha in isku-xirka hawraar tudcani yahay run marka hawraaraha mutuxan ee ay ka koobanto hawraarta tuducani wada yihin run, haddii kalese waa been.

Inta mar ee mar-qiime saafid ee hawraartuducani, oo ka koobantaa 2 atam, yeelan karto waa 4.

Marka kala xirka waxaa la heli :

	q	k	q V k
	R	R	R
	R	B	R
	B	R	R
	B	B	B

Markan, hawraar tudcan oo kala xir ahi waa been haddii atamyada ay ka koobantaa mid walba been tahay, haddii kale waa run.

Tusaale :

- b. Waan farxi lahaa haddaan gudbo Fisikiska ama Kimistiriga.
- t. Caawa filim baan aadayaa ama aqalkaan joogayaa.

«Ama» waxaa loo isticmaalaa si reeban iyo si kujirid ah labadaba. Tusaalahaa kore, «ama»-da (b) waa kujirid, taasoo ah, waa suuroobi kartaa inaad gudubto Kimistiri iyo Fisikis labadaba markaasna aad faraxdo. Laakiin «ama»-da (t) waxaa loo isticmaalay macna reeba taasoo ah, ma dhici karto in labada atamba run noqdaan sidaa daraadeed, si aynu u fogeynno madmadow, «ama»-da loojig waxaa mar walba loo isticmaalaa macna kujirid.

Marka shardiga, bal aynu tixgelino hawraartan oo nin ooridiisii ku yiri «Haddii aan roob di'in filimkaan caawa ku geyn».

Ka dhig q : roob baa di'i.

k : filimkaan caawa ku geyn,

Sumad ahaan hawraartan tuducan waxay u qormi ~ q \longrightarrow k

- 1) Ka soo aad in aanu roob di'in isna uu haweentiisi filim geeyay — markan ninku ma ballan furin.
- 2) Ka soo qaad in aanu roob di'in isna aanu iyada geeyn filim — markan ninku waa ballan furay.
- 3) Ka soo qaad in roob da'ay isna aanu haweentiisa geeyn filim — markan ninku ma ballan furin.
- 4) Ka soo qaad in roob da'ay isna uu haweentiisa geeyay filim — markan ninku ma ballan furin.

Haddaba, hawraar shardiile kasta waxaa u jira 4 xaaladood oo suuragal ah sida : tuse rumeedka hoose muujinayo.

q	\sim q	k	\sim q	\longrightarrow k
R	B	R	R	
R	B	B	R	
B	R	R	R	
B	R	B	B	

Haddii ay hawraartu tahay labashardiile, markaa furfuristeedu waxay noqon sida hoos ku tusan :

Hawraar labashardiile kasta $q \leftrightarrow k$, waxaa loo qori karaa sidan ($q \longrightarrow k$) \wedge ($k \longrightarrow q$), taasoo ah isku xirka laba hawraar shardiile.

$$(q \leftrightarrow k) = (q \longrightarrow k) \wedge (k \longrightarrow q).$$

Sidaa awgeed labashardiile wuxuu leeyahay tuse rumeedka soo socda.

q \wedge k	$(q \longrightarrow k)$	\wedge	$(k \longrightarrow q)$	$(q \longrightarrow k)$	\wedge	$(k \longrightarrow q)$
R R	R		R	R		
R B	B		R	B		
B R	R		B	B		
B B	R		R	R		

Haddii la soo gaabiyo

q	k	$q \leftrightarrow k$
R	R	R
R	B	B
B	R	B
B	B	R

Labashardiile wuxuu run yahay marka labada hawraarood ee uu ka koobmaa yihii wada run iyo markay yihii wada been haddii kale waa been.

Layli :

- 1) U kala bixi shardileyaalkan run iyo been sababtana sheeg.
- b. Haddii $9 + 4 = 17$ markaa $8 + 7 = 15$.
- t. Haddii $9 + 4 = 17$ haddaba $8 + 7 = 15$.
- j. Haddii b. t. $\in \mathbb{R}$ markaa $\frac{b}{t}$ waa qeexan tahay.
- kh. Haddii dad si mukhlisnimo ah u wada shaqeeyo markaa nolol wanaagsan ba wada noolaadaan.
- x. Haddii dayaxu cas yahay markaa biyaha baddu waa buluug.
2. U kala sheeg run ama been.
- 1) $9 + 5 = 15$ ama $15 \times 5 = 75$.
- 2) 2 waa tiro dhaban iyo tiro mutuxan labaduba.
- 3) 15 waa tiro mutuxan ama waa tiro farcan.
- 4) $12 + 5 = 14 \longrightarrow 21 - 20 = 0$
- 5) Haddii 13 tiro kisi tahay haddana $0 < 1$ markaa Hargeysa Bari bay ka xigtaa Afgooye ama $4^3 = 64$.
3. Buuxi meeshii bannaan ee tuseyaalka soo socda haddii ay suuroowdo.

(B)	q	k	$q \wedge k$	q	k	$q \longrightarrow k$
R		R	R	B		B
		R	B		B	R
				R		R
(T)	q	k	$q \vee k$			
B		B				
B		R				
B		B				
(X)	q	k	$q \wedge \sim k$	$\sim (q \vee k)$	$q \longrightarrow k$	$q \longrightarrow k$
R		R				
R		B				
B		R				
B		B				

DHISNAANTA IYO JABNAANTA DOODO

Doodi waa urur hawraarro ah oo loo yaqaan afeefiyo iyo hawaar lagu soo gooyay oo loo yaqaan go'aan.

Tusaale 1 :

- 1) Sugaal waa arday fasalka IV ama waa horjooge — afeef.

- 2) Sugaal waa horjooge — afeef. Sidaa awgeed, Sugaal ma aha arday fasalka IV — go'aan.

T u s a a l e 2 :

- 1) Haddii Cali gudbo imtixaanka maamulaha dugsigu wuxuu siin doonaa abaalgud — afeef.
- 2) Cali waa gudbay imtixaankii — afeef. Markaa maamuluuhu wuxuu siin doonaa abaalgud — go'aan.

DHISNAANTA DOOD IYO ISTICMAALKA TUSE RUMEED

Habkani wuxuu isticmaalayaa tuse rumeed si loo kala ogaado dhisnaanta ama jabaanta dood la soo jeediyay. Si aad u aragto in dooddi dhisan tahay ama jaban tahay, marka hore u tarjim dooddaas summado, marka xiga afeefaha iyo go'aanka dhinactax u qor adoo u dhxeysiinaya afeefaha haya « / », laba hayfana « / / » ka horreeysiinaya go'aanka. Úgu dambeyn ka dhis tuse rumeed ururka hawraaraha oo idil. Baar dhinactax kasta. Haddii uu jiro dhinactax ay afeefihiisa oo idil run yihiin go'aankuna been yahay, marka dooddaasi waa jaban tahay; haddii kale waa dhisan tahay.

XUSUUSIN : U fiirso tilmaan iyo dood ekaanshahooda.

T u s a a l e 1 :

Sug dhisnaanta doodda soo socota.

- 1) Axmed waa arday sannadka IV ama waa horjooge
- 2) Axmed waa horjooge.
- 3) Haddaba, Axmed ma aha arday fasalka IV.

Tarjimo q: Axmed waa arday sannadka IV ah.
k: Axmed waa horjooge.

q	V	k	/	k	/	/	~	q
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1.	R	R	R	R			B
2.	R	R	B	B			B
3.	B	R	R	R			R
4.	B	B	B	B			R

Doodani waa jaban tahay maxaa yeelay afeefaha dhinactax oo idil waa wada run, go'aankuna waa been. Sidaa awged waxaa dhisan in Axmed noqon karo arday fasalka IV iyo horjooge labadaba

T u s a a l e 2 :

- 1) Haddii Xasan laba buug akhriyo bishii, markaa Xasan waa nin wax garad ah.
- 2) Xasan laba buug ayuu akhriyaa bishii. Haddaba Xasan waa nin wax garad ah.

T a r j u m o :

- q: Xasan laba buug ayuu akhriya bishii.
k: Xasan waa nin wax-garad ah.

q	\longrightarrow	k/ q / / k
R R	R	R R R
R B	B	R B B
B R	R	B R R
B R	B	B B B

Dooddani waa dhisan tahay waayo ma jiro dhinactax afeefihiis oo idili run yihiin go'aankuna been yahay.

Layli :

Sug dhisnaanta doodaha soo socda :

- b. 1) Faarax waa arday fasalka IV ama fasalka III.
2) Faarax ma aha arday fasalka IV.
Haddaba Faarax waa arday fasalka III ah.
- t. 1) Haddii x tiro dhaban tahay markaa x² waa tiro dhaban.
2) x² waa tiro dhaban.
 \therefore x waa tiro dhaban.
- j. 1) Haddii m gudbo imtixaanka xisaabta, macallinku wuxuu siin doonaa buug.
2) Macallinku m wuxuu siiyay buug.
 \therefore m waa gudbay imtixaankii xisaabta.
- x. 1) Haddii roob da'o koodhkaygaan xiran.
2) Roob baa da'ay.
 \therefore Koodhkaygaan xiran.
- kh. 1) Haddii dawadii wax tartay, Cali waa ladan yahay.
2) Cali ma ladna.
 \therefore Daawadii wax ma tarin.
- d. 1) Haddii aan mushaharadeyda soo qaato, filim baan ku geeyn.
2) Ma soo qaadan mushahaaradii.
 \therefore Ku geeyn maayo filim.
- r. 1) Haddii aan mushahaaradeyda soo qaato, filim baan ku geyn.
2) Ma soo qaadan mushaharadeydi.
 \therefore Filimkaan ku geeyn doonaa.
- s. 1) Milanku waa dhexdhexaad ama midabkiisu waa casaan.
2) Milanku ma aha dhexdhexaad.
 \therefore Midabkiisu waa casaan.
- sh. 1) Kansarka waxaa u sabab ah sigaarka ama qiiqa baabuurta.
2) Kansarka waxaa u sabab ah sigaarka.
 \therefore Qiiqa baabuurtu sabab uma aha kansarka.
- dh. 1) Run ma aha in Cali arday fasalka III yahay isla markaasna horjooge yahay.

- 2) Cali waa arday fasalka IV.
 \therefore Cali ma aha horjooge.
- c. 1) Run ma aha in Xaliimo tahay ardayad fasalka IV ama horjooge.
 2) Xaliimo ma aha horjooge.
 \therefore Xaliimo ma aha ardayda fasalka IV.
- g. 1) Haddii Cumar yahay arday fasalka IV ama horjooge, galabtuu soo noqon doonaa.
 2) Cumar waa horjooge.
 \therefore Galabtu soo noqon doonaa.
- f. 1) Haddii x tiro dhaban tahay, markaa 3x waa tiro dhaban.
 \therefore Run ma aha in x tiro dhaban tahay 3x-na aynan ahayn tiro dhaban.
 \therefore 3x waa tiro dhaban.
- q. 1) Haddii webigu biyo keeno, beeraha waa la abuuri.
 Haddii beeraha la abuuro, galley badan ayaa la goosan.
 \therefore Haddii webigu biyo keeno, galley badan ayaa la goosan.
 1) Haddii aan run sheego, Eebaan raalli gelin.
 2) Sheegi maayo run.
 \therefore Raalli gelin maayo Eebbe.

DHAB-OODYO

Qeex :

Hawraari waa dhab-ood haddii iyo haddii qudha ay tahay run mar kasta oo ka mid ah mar-qiime saafiddeeda.

Qeex kale :

Dhab-ood waa hawraar tudcan oo had iyo jeer run ah iyadoon loo eegin runnimada iyo beennimada xubnaheeda.

Tusaale 1 :

Bal aynu tixgelinno hawraarta ah «jaamac waa arday ama ma aha.»

Ka dhig q: Jaamac waa arday.

Haddaba hawraarta tudcani waxay summad ahaan u tarjumaysaa q V ~ q.

q ~ q q V ~ q

R	B	R
B	R	R

Mar haddii q V ~ q ay mar walba run tahay, q V ~ q waa dhab-ood. Dhab-oodkan waxaa loo yaqaan sharciga reebnaan u dhaxaynta.

Tusaale 2 :

Haddii Ibraahim arday fasalka IV yahay isla markaana horjooge yahay, markaa Ibraahim waa arday fasalka IV ah.

Ka dhig q: Ibraahim waa arday fasalka IV ah.
k: Ibraahim waa horjooge.

Hawraarta tudcani waxay u qormaysaa
 $(q \wedge k) \longrightarrow q$

Tuse rumeedka hawraartu wuxuu noqon.

q	k	$q \wedge k$	$(q \wedge k) \longrightarrow q$
R	R	R	R
R	B	B	R
B	R	B	R
B	B	B	R

Mar haddii $(q \wedge k) \longrightarrow q$ ay run tahay mar kasta oo ka mid ah mar-qiime saafideedda haddaba $(q \wedge k) \longrightarrow q$ waa dhab-ood.

Layli :

Adoo la kaashanaya tuse rumeed sug inay kuwa soo socdaa yihiin dhab-oodyo.

- 1) $\sim (q \wedge \sim q)$
- 2) $(q \wedge k) \longrightarrow q$
- 3) $q \longrightarrow (q \vee k)$
- 4) $[(q \vee k) \wedge \sim q] \longrightarrow k$
- 5) $q \leftrightarrow \sim (\sim q)$
- 6) $(q \longrightarrow k) \leftrightarrow (\sim k \longrightarrow \sim q)$
- 7) $[(q \longrightarrow k) \wedge q] \longrightarrow k$
- 8) $[(q \longrightarrow k) \wedge \sim k] \longrightarrow \sim q$
- 9) $[(q \longrightarrow k) \wedge (k \longrightarrow I)] \longrightarrow (q \longrightarrow I)$
- 10) $(q \wedge q) \longrightarrow q$
- 11) U tarjim (1 — 10) Hawraaro.

HAWRAARO ISUDHIGMA

Qeexid :

Laba hawrarood dhab-ood ahaan bay isugu dhigmaan haddii iyo haddii qudha uu labashardiilaha ka sameysma labadoodu u yahay dhab-ood.

Tusaale :

Bal aynu tixgelinno labadan hawrarood :

- 1) Haddii Cali arday fasalka IV yahay, markaa wuxuu u fariisan doonaa imtixaanka ugu dambeeyaa.
- 2) Cali ma aha arday fasalka IV ama wuxuu u fariisan doonaa imtixaanka ugu dambeeyaa.

Hawraarta koowaad waxay u tarjumeysaa $q \longrightarrow k$ tan labaadna waxay u tarjum-neysaa $\sim q \vee k$.

Haddana aynu tixgelinno labashardiilaha ay sameenayaan labada hawraaro
 $(q \longrightarrow k) \leftrightarrow (q \sim \vee k)$.

q	k	$q \rightarrow k$	$\sim q$	$(\sim q \vee k)$	$(q \rightarrow k) \leftrightarrow (\sim q \vee k)$
R	R	R	B	R	R
R	B	B	B	B	R
B	R	R	R	R	R
B	B	R	R	R	R

Mar kasta oo ka mid ah mar-qiime saafidda ($q \rightarrow k$) iyo ($\sim q \vee k$) waa isku qiime, sidaa awgeed labashardiilaha ay sameynayaan waa dhab ood. Haddaba ($q \rightarrow k$) iyo ($\sim q \vee k$) dhab-ood ahaan waa hawraaro isu dhigma. Taasi waa ($q \rightarrow k$) \equiv ($\sim q \vee k$).

OGOW : in \equiv ay u taagan tahay isudhignaan dhab-oodeed.

Tusaale 2 :

Aynu tixgelinno hawraaraha :

- 1) Waa been in Cali yahay arday fasalka IV ah isla markaana yahay horjooge.
- 2) Cali ma aha arday fasalka IV ama ma aha horjooge.

Ka dhig q: cali waa arday fasalka IV ah.

k: Cali waa horjooge.

Hawraarta laad waxay u qormi $\sim (q \rightarrow k)$, tan 2aad-na $\sim q \vee \sim k$.

q	k	$(q \wedge k)$	$\sim (q \wedge k)$	$\sim q \vee \sim k$	$\sim (q \wedge k) \leftrightarrow (\sim q \vee \sim k)$
R	R	R	B	B	R
R	B	B	R	R	R
B	R	B	R	R	R
B	B	B	R	R	R

U fiirso tuse rumeedka, $\sim (q \rightarrow k)$ iyo $\sim q \vee \sim k$ mar kasta waa isku qiime. Sidaa daraadeed labashardiilaha ay sameynayaan waa dhab-ood. Haddaba waxaan aragnaa in $\sim (q \wedge k)$ iyo ($\sim q \vee \sim k$) ay yihiin dhab-ood ahaan hawraaro isudhigma. Taasi waa

$$\sim (q \wedge k) \equiv (\sim q \vee \sim k)$$

Layli :

Caddee in

- 1) $q \vee k \equiv k \vee q$
- 2) $q \wedge k \equiv k \wedge q$
- 3) $q \vee (k \vee L) \equiv (q \vee k) \vee L$
- 4) $q \wedge (k \vee L) \equiv (q \wedge k) \vee (q \wedge L)$
- 5) $q \vee (k \wedge L) \equiv (q \vee k) \wedge (q \vee L)$
- 6) $(q \vee q) \equiv q$
- 7) $(q \wedge q) \equiv q$
- 8) $q \vee (q \wedge k) \equiv q$
- 9) $q \wedge (q \vee k) \equiv q$
- 10) $q \vee \sim q \equiv R$
- 11) $q \wedge \sim q \equiv B$

- 12) $\sim(\sim q) \equiv q$
 13) $\sim(q \vee k) \equiv (\sim q \wedge \sim k)$
 14) $\sim(q \wedge k) \equiv \sim q \vee \sim k$
 15) $R \vee q \equiv B$
 16) $B \wedge q \equiv B$
 17) $B \vee q \equiv q$
 18) $R \wedge q \equiv q$
 19) $\sim R \equiv B$
 20) $q \longrightarrow k \equiv \sim q \vee k$
 21) $q \longrightarrow k \equiv \sim(q \wedge \sim k)$
 22) $q \vee k \equiv \sim q \longrightarrow k$
 23) $[(q \longrightarrow k) \wedge q] \longrightarrow k \equiv R$
 24) $[(q \vee k) \wedge (\sim q \longrightarrow k)] \equiv R$
 25) $(q \longrightarrow k) \equiv (\sim k \longrightarrow \sim q)$

OGOW : R macnaheedu waa hawraar had iyo jeer run ah. B waxay u taagan tahay hawraar had iyo jeer been ah ama isburis ah.

ALJEBRADA HAWRAARO

Haddii aynu u qaadanno E in ay u taagan tahay ururka hawraarrada oo idil, (q, k, l, \dots), aynu u tixgelinana weedh-dabarrada V iyo \wedge in ay yihiin xisaabfallo ururka E, kuwaasoo lammane kastoo horsan oo hawraarro ah ($q, k \in E$, ku aadiya hawraar dhab-ood ahaan ugu dhiganta, markaa waxaan dajin karnaa dhardhaarada loojig ama midaallada hawraareed ee soo socda :

- 1) KALA HORMARINTA
 a. $V q, k \in E$
 b. $q \vee k \equiv k \vee q$
 t. $q \wedge k \equiv k \wedge q$
- 2) HORMOGELINTA :
 a. $V q, k \text{ iyo } L \in E$
 b. $q \vee (k \vee L) \equiv (q \vee k) \vee L$
 t. $q \wedge (k \wedge L) \equiv (q \wedge k) \wedge L$
- 3) SHARCIYADA KALA DHIGGA :
 a. $V q, k \text{ iyo } l \in E$
 b. $q \vee (k \wedge L) \equiv (q \vee k) \wedge (q \vee L)$
 t. $q \wedge (k \vee L) \equiv (q \wedge k) \vee (q \wedge L)$
- 4) TAAG-MADOORSHE :
 a. $V q \in E$
 b. $q \vee q \equiv q$
 t. $q \wedge q \equiv q$
- 5) JIRITAANKA ASAL MADOORSHE :
 a. $V q \in E$
 b. $q \vee B \equiv q$
 t. $q \wedge R \equiv q$
- 6) SHARCIGA DULEEDINTA :
 a. $V q \in E$
 b. $q \vee \sim q \equiv R$
 t. $q \wedge q \equiv B$

7) SHARCIGA NUUGIDDA

- a. $\forall q \in E$
- b. $q \vee (\neg q \wedge k) \equiv q$
- c. $\neg(\neg q \wedge k) \equiv q$

8) SHARCIGA LABALAABKA DIIDMO :

- a. $\forall q \in E$
- b. $\sim(\sim q) \equiv q$

9) SHARCIGA DEMORGAN :

- b. $\sim(q \vee k) \equiv \neg q \wedge \neg k$
- c. $\sim(q \wedge k) \equiv \neg q \vee \neg k$

10) XISAABFALLO AY LA JIRAAN R IYO B :

- b. $\sim B \equiv R$
- c. $\sim R \equiv B$

11) b. $R \vee q \equiv R$
c. $B \wedge q \equiv B$

Haddii aynu baarno sharciyada kor ku qoran, waxaan aragnaa in ay u dhacayaan lammaanayaal laba-eef ah, t.a. Sharci kasta ee Aljebrada hawraaro haddii kala xir kasta lagu beddelo isku xir, isku xir kastana lagu beddelo kala xir, B lagu beddelo R, R-na B. Markaa hawraarta soo baxdaa waxay noqon sharci ku jira Aljebrada hawraaro. Sharci kastoo habkaa ku yimaada waxaa la yiraa **waa kala guradka ka kale**. Intaa waxaa soo raacaya haddii aynu qaadanno midaal kasta oo Aljebrada hawraaro oo ka soo diirmay sharciyaasha kor ku qoran, haddana aynu sameynno isku beddelka V iyo **A**, iyo B iyo R, markaa hawraarta soo baxaysaa iyana waa midaal. Taasna waxaa loo yaqaan **sharciga kala gurashada**.

FUDUDEYNTA HAWRAARO

Tusaale 1 :

$$\text{Caddee in } q \vee (\neg q \wedge k) \equiv q \vee k$$

Caddeeyn :

$$\begin{aligned} q \vee (\neg q \wedge k) &\equiv (q \vee \neg q) \wedge (q \vee k) \text{ Sharciaga 3b} \\ &\equiv R \quad (q \vee k) \text{ Sharciga 6b} \\ &\equiv (q \vee k) \text{ Sharciga 5b} \\ \therefore q \vee (\neg q \wedge k) &\equiv q \vee k \end{aligned}$$

Layli :

Caddee in :

- 1) $q \wedge (\neg q \vee k) \equiv q \wedge k$
- 2) $(q \wedge k) \longrightarrow q \equiv R$
- 3) $q \longrightarrow (q \vee k) \equiv R$
- 4) $\sim(q \wedge k \wedge l) \equiv \sim q \vee \sim k \vee \sim l$

LOOJIGGA URURRO

Bilaw ahaan, bal aynu tixgelinno weedhaha ama tibaaxaha soo socda oo ku saabsan ururro.

- b. Dadka 70 ka weyn oo idil waa gabyaan.
- t. Ardayda qaarkood waa cayaaryahanno.
- j. Tirooyinka maangal ah oo dhan.
- x. Geela 8 lugood leh oo dhan.
- kh. Dhibcaha Sallax ku yaal oo dhan.
- d. Dhammaan kutirsanyaasha Golaha Sare ee Kacaanka.

Waxaan aragnaa in ururrada kor ku qoran qaarkood yihiin kuwa la tirin karo, qaarkoodna kuwa aan la tirin karin, ama tirada ku beegmaysa aanay koobnayn. Masalan kutirsanyaasha G.S.K. waa 20 sannadkan 74; kutirsanyaasha ururka tirooyinka maangalku ma koobna; kutirsanyaalka ururka geela 8da lugood lehi ma jiro. Haddii dhammaan kutirsanyaasha urur yihiin kutirsanyaasha urur kale, markaa ururka hore waa u hormo ururka dambe, sida uu tuaaleynayo xiriirka ka dhexecyaa ururrada (b) ku qoran ama kuwa (t) ku qoran.

Fikradda loojig dheegidda qaarkeed waxaa loo geli karaa ama loo hollan karaa si aan rasmi ahayn iyadoo lala kaashanayo xiriirka ka dhexecyaa ururro. Bal hadda aynu tixgelinno. Tusaalooyinka soo socda.

Tusaale 1 :

- 1) Laydiyo oo idil waa barbarrooleyaal.
- 2) Barbarrooleyaal oo idil waa geesooleyaal.
- 3) Laydiyo oo idil waa geesooleyaal.

Weedh kasta oo tusaalahaa laad waxay ku saabsan tahay laba urur, waxayna qeexeysa xiriirka kutirsanyaashooda ka dhexecyaa. Hadda bal aynu isku qaadno saddexda weedhood oo dhan, kana fikirno waxay tilmaamayaan. Qof kastaa waa garan karaa in weedha 3aad tahay go'aan aan laga fursaneyn oo weedha laad iyo tan 2aad oo afeefo loo qaataa malagelinayaan.

Weedha 2aad ma tahay go'aan lagama fursadaan ah oo weedha laad iyo tan 3aad malagelinayaan ? Weedhaha tusaalahaa laad oo idil joometeri ahaan waa dhab laakiinse markan, runnimada iyo beennimada weedhaha, waxaa ka muhiimsan sida ay isku raacsan tahay garaadinta ina geyneysa go'aan aan laga fursaneyn.

Israacsanaanta garaadinta aynu ka hadleeynaa waxay caddaaneysaa marka magacyada lagu beddelo summado sida :

Tusaale 2 :

- 1) X oo idil waa Y.
- 2) Y oo idil waa W.
- 3) X oo idil waa W.

Tusaale 3 :

- b. Bash oo idil waa Nuf.
- t. Nuf oo idil waa ran.
- j. Bash oo idil waa ran.

Sidii tusaalahaa laad oo kale weedha 3aad ee tusaalahaa 2aad iyo ka 3aad waa go'aan aan laga fursaneyn oo ka soo arooraya weedha laad iyo tan 2aad tusaale walba. Inkasta oo tusaalooyinka aynu ku isticmaalnay ereyo aan qeexnayn, haddana

habka garadiintu u dhacaysaa wuxuu daadixin jiday saldhig u ah xeerka loojigga, t.a, dood kasta oo xisaab ahi waxay ku bilaabataa tibxo aan qeexnayn oo la isticmaalo iyo xeer garadiin. Mar haddii la oggolaado taas, habka loogu gaarayo afeefaha doodda lagu bilaabay, go'aan lagama fursadaan ah waxaa la yiraa **Dheegid garaadin**.

Tusaalaha soo socda wuxuu inoo muujinayaa in aanay dhib ridda go'aan lagama fursadaan ah, xataa haddii la isticmaalo afeefo fudud.

Tusaale 4 :

- 1) cayaaryahannada qaarkood waa arday Jaammacadda ku jirta.
- 2) Ma jiro arday Jaamicadda ku jira oo makaanig ahi.
- 3) Darawal oo idil waa makaanig.

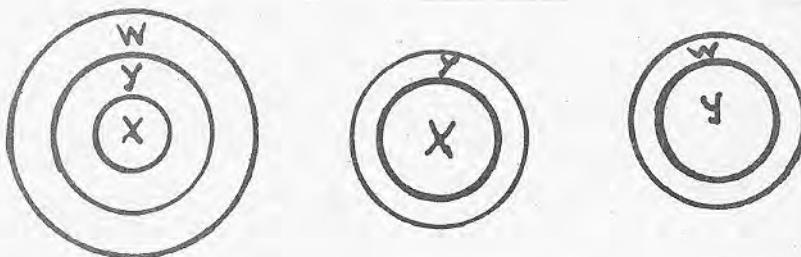
Go'aanno suuroobi karaa waa :

- b. ma jiro cayaaryahaan makaanig ah.
- t. Ardayda jaamiciyiinta ah qaarkood ma aha cayaaryahanno.
- j. Ma jiro arday Jaamici ah oo darawal ahi.
- x. Cayaaryahannada qaarkood ma aha darawallo.

Haddii weedhaha (1), (2) iyo (3) afeefo loo qaato, weedhaha (b), (t) (j) iyo (x) teebaa go'aan lagama fursadaan ah, haddiba go'aan jiri karo? Intaanad sii socon su'aalahaas uga jawaab si aad ku kalsoon tahay.

Qormo ahaan, urur loojig waxaynu u taagnaa xaraf weyn sida X, kutirsanaha ururkana waxaynu u taagnaa xarafkaas oo yar x.

Jaantusyo u taagan ururrada loojig iyo xiriirka ka dhexeeya waxay inoo sahli karaan in aynu fiiro dheer u yeelanno sida garaadinta ururrada ku saabsan u dhacayso. Ka dhig in gudaha xood oodani u taagan yahay ururka X, baraha oodanta ku jiraana u taagan yihiin kutirsanayaasha ururka X, haddaba ururro kasta oo sidaa u sawirmaya xiriirka ka dhexeeya waxaa muujin karaa xiriirka oodannadooda ka dhexeeya. Tusaalaha 2aad waxay jaantus muujintiisu noqon sida soo socota.



Tusaal 2 :

- 1) X oo idil waa Y.
- 2) Y oo idil waa W.
- 3) X oo idil waa W.

Mar haddii X oodonkeedu ku dhacayo gudaha oodanka W, haddaba bar kasta oo X ku jirtaa waa bar W ku jirta, waana taa awgeed waxa weedha 3aad u tahay go'aan aan laga fursaneyn.

Jaantuska caynkaas ah waxaa la yiraa **Jaantuska «Ven»** ama **Jaantuska «Euler»**. Marka waxaa isweydiin leh, waa maxay qaybaha loo kala bixin karaa jaantuska «Ven» marka lagu isticmaalayo loojig dheegidda ?

Fiiradeena xagga shaxannada joometeri waxaynu ka heleynaa heshiisyo. Ugu horreyn, weedhaha ku saabsan ururro intooda badani waxay u tibaaxan yihiin ama u muuqdaan sansaannada soo socda mid uun.

- B. X oo idil waa Y.
- T. Ma jirto X ah Y.
- J. X qaarkeed waa Y.
- D. X qaarkeed ma aha Y.

Marka ayllu ka hadleyno urur, waxaan ka fikirnaa in ururkaasi hormo u yahay urur weyn oo la yiraa **Urur guud G**

Tusaalaha laad, laydiyo, barbarrooleyaal iyo geesooleyaa waxaynu uga fekeri karnaa in ay hormooyin u yihiin ururka Shaxannada Joometeri ee Sallax. Kutirsanyaal kasta oo ku jira G aan ku jirinse urur la magacaabay waxy ku jiraan urur duleedkiisa. Haddii X urur yahay, markaa kutirsanyaalka G ee aan ku jirin X waxaa loo taagaan \bar{X} .

Baraha jaantuska oodanta X

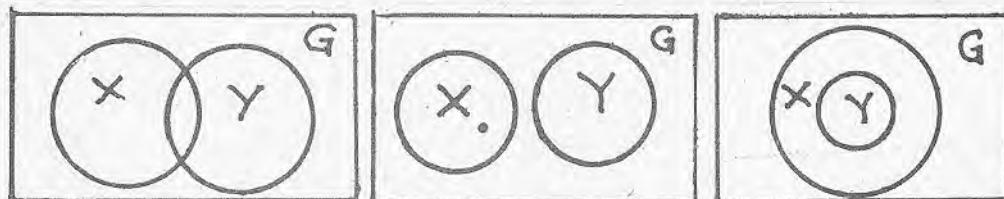
ka baxsan laakiinse yaal gudaha oodanka G waxay sameeyaan ururka \bar{X} t.a. \bar{X} ee X.

Haddii la doonayo in lagu heshiyo jaantusyada u suuroobi kara weedhaha (B). (T), (J) iyo (D), waa in si cad loo kasaa ereyada weedhuhu ka kooban yihiin, «Idil» waxay la macna tahay «kasta», «walba», «dhammaan» ama «kulli»; haddii ereyadaa la isku beddelo macnaha weedhahaa ama hawraarahaa isbeddelku kuma dhaco inkastoo laga yaabo in dhismahooda isbeddel ku dhacay. «Qaar» waxay mar-mar la shaqa tahay «in» macnaheeduna waa «ugu yaraan mid lagana yaabee dhammaan». «Qaar»-tani waa ka jaad midda sida caadiga ah loo isticmaalo taasoo aan wadan macnaha ah «laga yaabee dhammaan».

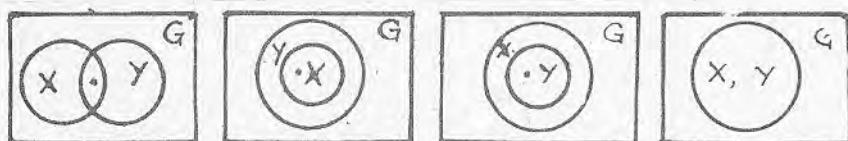
OGOW : in hawraarta (T) u qormaysa sansaankan :

Dhammaan Xi ma aha yama X oo idil ma aha Y: u fiirso in labadan sansanood iyo kuwa kale oo aad sameyn kartaa aanay malagelinayn in X qaarkeed Y yahay sida la is-moodsiin karo.

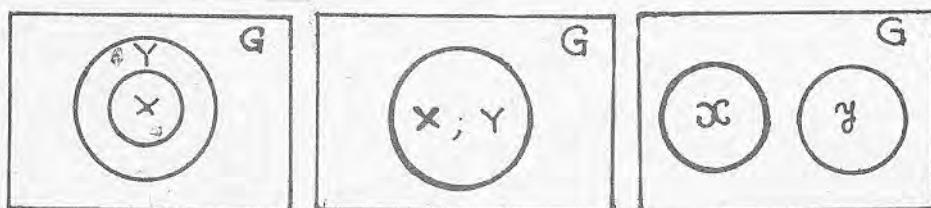
Jaantusyada u suuroobi kara hawraaraha (B), (T), (J) iyo (D).



(B) X oo idil waa Y (T) Ma jirto X ah Y.



(J) X qaarkeed waa Y.



(D) X qaarkeed ma aha Y.

U fiirso in laydi kastaa ee jaantusku u taagan yahay urur guud barahooduna u taagan yihii kutirsanaha ugu yaraaneed ee raalligelinaya ereyga «qaar».

Garaadin dhisan oo xagga ururradu, waa habka loo helayo go'aannada, gaar ahaan aan laga fursaneyn ee ka soo arooraya jaantus kasta oo afeefuhu wadajir si sax ah u sawirayaan. Go'aankaas isaga ah waxaa la yiraa **Go'aan dhisan**.

Haddii la helo jaantus sax ah oo qudha oo go'aan la isa siiyay ka soo horjeeda, markaa waxaynu niraahnaa go'aankaasi **ma dhisna** ama waa **jaban yahay**.

F a a h f a a h i n :

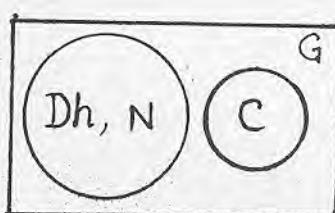
Habka loo sawirayaan jaantusyada Venn mararka qaarkood ma ballan qaadayo in aynnu helno go'aan dhisan, sababta oo ah ma hubin karno in aynu dhammaynay inta jaantus ee suuragal u ah xiriirrada ka dhexeeyaa aseefaha. Hase ahaatee, inta jaantusyo ee aynu karno oo aynnu sameynaa waxy inaga caawin kartaa in aynnu duuxno go'aannada inoogu muuqda dhisnaan. Waynu hubi karnaa in go'aan jaban yahay, haddii aynu helno jaantuska sax ah oo burinaya.

T u s a a l e 5 :

- Afeefo : 1) Dhaqtar oo idil. «Dh» waa naxariis badan yahay «N».
2) Ma jiro qof naxariis badan oo dhabcaal ah «C».

Go'aan ma jiro dhaqtar dhabcaal ahi.

Labada jaantus ee hoose oo qudha bay afeefuhu u sawirmi karaan. Jaantus walba N iyo C oodannadoodu isma taabanayaan. Sidaa awgeed go'aanku wuxuu u muuqdaa mid dhisan.



Shax 20

Suuroowga 1

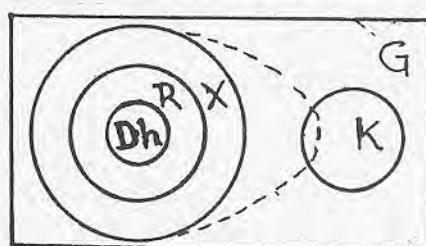
Suuroowga 2

T u s a a l e 6 :

- Afeefo : 1) Xoola-dhaqato oo idil «Dh» waa reer guuraa «R».
2) Ma jiro reer guuraa Kalluunsata ahi «K».
3) Reer xeebeedka qaarkood, «X» waa kalluunsato.

Go'aan : Reer xeebeedka qaarkood ma aha xoolo-dhaqato.

Jaantusyo dhawr ah ayaa loo sawiri karaa dooddan, laakiinse mar walba ugu yaraan hal kutirsane ayaa ku jira dhexyaalka oodanta X iyo K. Kutirsanahaasu mar hadduu ku jiro K kuna jiri karo Dh (Eeg jaantuska hoose). Sidaa awgeed garaadintu waa dhisan, go'aankuna waa dhisan. sawir jaantusyada kale, caddeena natijjada mar kasta.



Shax 21

T u s a a l e 7 : Hal ka mid ah jaantusyada u suuragala tusaale 6.

Afeefo : Sida tusaalah 6.

Go'aan : Xoola-dhaqatada qaarkood ma aha kalluunsato. Shaxanka tusaale 6 wuxuu ina leeyahay go'aanku waa dhisan yahay. Laakiin haddii Dhiyo Ray ururro madhan yihiin markaa go'aanku waa jabnaan lahaa. Marka lagu kordhiyo afeefaha in aanu ururna madhnayn (in xoolo dhaqato jirto, in-reer guuraa jiro) markaa go'aanku waa dhisan yahay.

Layli :

Dood kasta oo socota u tax afeefo iyo go'aan, caddeena dhisnaanta ama jabnaanta garaadinta adoo isticmaalaya jaantuska «Euler».

- 1) Dadka Caqliga badan oo idili waa hubqaad wanaagsan yihiin.

Ardayda Fasalka afraad oo idil waa dad caqli badan.

Haddaba, ardayda Fasalka afraad oo idil waa hubqaad wanaagsan yihiin.

- 2) Cayaaryahanno oo idil waa xoog badan yihiin.

Darawallada xamuulqaadyada oo idil waa xoog badan yihiin.

Haddaba darawallada xamuulqaadyada oo idil waa cayaaryahanno.

- 3) Saddexagallada qaarkood waa shaxanno siman. Ma jiro saddexagal afar gees ahi.

Haddaba ma jiraan afar geesyo shaxanno siman ahi.

- 4) Ma jiraan barayaal jaajaalayaal ahi.

Dadka caqliga badan qaarkood ma aha jaajaalayaa. Sidaa darteed barayaalka qaarkood waa dad caqli badan.

- 5) Naasleyda qaarkeed biyo dhexdooda bay ku nooshahay.

Nafleeyda biyaha ku dhex nool oo idil waa kalluun. Haddaba, naasleeyda qaarkeed waa Kalluun.

- 6) Dadka cilmiga bulshada barta oo idili waxay aaminsan yihiin in dadku aqallo wada helo.

Dadka kumuniistaha oo idil waxay aaminsan yihiin in dadku aqallo wada helo.

Haddaba, dadka cilmiga bulshada bartaa qaarkood waa kumuniiste.

- 7) Ma jiro dhismo loox ka samaysan oo aan dab qabsan karin.

Dhismaha dugsiga oo idil wuxuu ka samaysan yahay dhagax.

Haddaba dhismaha dugsiga oo idil dab ma qabsan karo.

- 8) Dhalo khad oo kastaa waxay ka sameysan tahay quraarad.

Wax kasta oo quraarad ka sameysani waa jabid sahlan yihiin.

Wixii jabid sahlan ee kastaa waa khatar.

Haddaba dhalo khad oo idil waa khatar.

- 9) Qof kasta oo buurani waa cunta badan yahay.

Ma jiro bare cunta badani.

Haddaba, ma jiro bare buurani.

- 10) Koor kastaa waa afar geesle.

barbarroole kastaa waa afar geesle.

Haddaba barbarroole kastaa waa koor..

ALJEBRADA BOOLE

A R A R :

Markii sannadku ahayaa 1840kii ayaa George Boole oo Xisaab-yahan iyo Loojig-yahan Ingiriis ahayaa, dejiyey habdhis ah loojigga suntan. Habdhiskaasoo hawaarihi loojigga Aristotel u soo celiyey isle'egyo iyo tibaaxo la sansaana aljebrada fudud. Habdhiska loojigga suntan waxa lagu magacaabay **Aljebrada Boole**, iyadoo lagu sharfayo George Boole. Aljebrada Boole, waxay waafaqsan tahay xeerarka artimetigga caadiga ah intooda badan oo ay ka mid yihiin kala hormarinta, hormogelinta iyo kala dhigga. Aljebrada Boole waxay waxtar weyn u leedahay injiyeerinta danab mareegga. Aynu ku bilownee Aljebrada Boole waa habdhis xisaabeed oo ah (E , \cup , \cap , $-$). E waxay u taagan tahay urur ku-tirsaneyaalkiisu yihiin dhammaan hormooyinka urur la og yahay, ururkaa E waxa la yiraa **Urur jibbaar**. Habdhis xisaabeed waxa loola jeedaa waa hawaaro ay la jiraan xisaab-fallo qeexan; sida ururka tirooyinka maangalka ah oo ay la jiraan xisaab-fallada isugeynta (+) iyo iskudhufashada (\times). Aljebrada Boole waa habdhiska leh astaamaha soo socda:

1) Oodnaan.

Haddii B iyo T ay yihiin ku-tirsaneyaal urur la og yahay E , markaa $B \cup T$, $B \cap T$, B iyo T waa ku-tirsaneyaalka ururka E .

2) Hormogelinta.

- a. $\forall B, T \text{ iyo } J \in E \quad (\forall - \text{ dhammaan})$
- b. $(B \cup T) \cup J = B \cup (T \cup J)$
- c. $(B \cap T) \cap J = B \cap (T \cap J)$

3) Kala hormarinta.

- a. $\forall B, T \in E$
- b. $B \cup T = T \cup B$
- c. $B \cap T = T \cap B$

4) Jiritaanka asal madoorshe.

Waxaa jira ku-tirsaneyaal gaar ah oo E ku jira, kuwaasoo ah ϕ iyo G oo haddii $B \subset G$ markaa

- b. $G \cap B = B \cap G = B$
- c. $\phi \cup B = B \cup \phi = B$

Haddaba, ururka guud G waxaynu niraan asal madoorshaha dhexyaal, ururka madhanna waxaynu niraan asal madoorshaha isutagga.

5) Jiritaanka duleedyo.

B kasta oo E waxa u jira urur maddi ah \bar{B} oo la yiraa **Duleedka B**, B waa in ay ku run noqotaa.

- b. $B \cap \bar{B} = \phi$
- c. $B \cup \bar{B} = G$

6) Kala dhigidda.

- a. $\forall B, T \text{ iyo } J \in E$
- b. $B \cap (T \cup J) = (B \cap T) \cup (B \cap J)$
- c. $B \cup (T \cap J) = (B \cup T) \cap (B \cup J)$

U fiirso in marka (b) aynu falka dhexyaalka ku kala dhigno isutagga. Astaanta waxaa loo yaqaan **Kala dhigidda dhexyaal**. Markaa (t-na) falka isutag baynu ku kala dhignay falka dhexyaal, sidaa awgeedna waxaa loo yaqaan **Kala dhigidda isutag**.

Aljebrada ururro waxaa la yiraa **Aljebrada Boole**, haddii ay waafaqdo lixda dhardhaar ee kor ku qoran. Dhardhaarradaa waxaan ku dhalin karnaa astaamo kale oo la yiraa aragtiinyo.

Aragtiin 1

Haddii B tahay ku-tirsane ururkaa E, markaa $B \cap \phi = \phi$.

Caddeyn

$$\begin{aligned}
 \phi &= B \cap B && \text{Astaanta } 5 \text{ (b)} \\
 &= B \cap (\overline{B}) \cup \phi && " \quad 4 \text{ (t)} \\
 &= (B \cap \overline{B}) \cup (B \cap \phi) && " \quad 6 \text{ (b)} \\
 &= \phi \cup (B \cap \phi) && " \quad 5 \text{ (b)} \\
 &= B \cap \phi && " \quad 4 \text{ (t)}
 \end{aligned}$$

Aragtiin 2

Haddii B tahay ku-tirsane kasta oo ururka E, markaa $B \cup G = G$.

Caddeyn:

$$\begin{aligned}
 B \cup G &= B \cup (B \cup \overline{B}) && \text{Astaanta } 5 \text{ (t)} \\
 &= (B \cup B) \cup \overline{B} && " \quad 2 \text{ (b)} \\
 &= B \cup \overline{B} && " \quad 5 \text{ (t)} \\
 &= G
 \end{aligned}$$

Aragtiin 3

$$\overline{G} = \phi$$

Caddeyn :

Waxaan caddeyn rabnaa in duleedka urur guud yahay urur madhan; haddaba, astaamaha 5 (b) iyo 5 (t) waxaan ka aragnaa in $G \cap \overline{G} = \phi$, $G \cup \overline{G} = G$. Sidaa awgeed, $G = \phi$.

Layli:

- 1) Caddee in $B \cup G = G$.
- 2) Magacaw astaamaha lagu isticmaalay tallaabo kastaa.

Caddeyn in

$$\begin{aligned}
 B \cap B &= B \\
 B &= B \cap B \\
 &= B \cap (B \cup \overline{B}) \\
 &= (B \cap B) \cup (B \cap \overline{B}) \\
 &= (B \cap B) \cup \phi \\
 &= B \cap B
 \end{aligned}$$

- 3) Caddee in $B \cup B = B$.

Aragtiin 4 (Aragtiinka De Morgan)

Haddii B iyo T ay ku-tirsaneyaal E yihin, markaa

b. $(\overline{B \cap T}) = \overline{B} \cup \overline{T}$

t. $(\overline{B \cup T}) = \overline{B} \cap \overline{T}$

C a d d e y n :

Si loo caddeeyo qaybta (b) waa in aan tusnaa in $(\overline{B \cap T}) \cap (B \cap T) = \phi$, isla markaana $(\overline{B} \cup \overline{T}) \cap (B \cap T) = \phi$

$$\begin{aligned} (B \cap T) \cap (\overline{B} \cap \overline{T}) &= \phi \quad \text{astaanta 5 (b)} \quad 1 \\ (B \cup T) \cap (B \cap T) &= [(B \cup T) \cap B] \cap [(B \cup T) \cap T] \\ &\quad \text{kala dhigidda dhexyaal} \\ &= [B \cap (B \cup T)] \cap [T \cap (B \cup T)] \\ &\quad \text{astaanta kala hormarinta} \\ &= [(B \cap B) \cup (B \cap T)] \cap [(T \cap B) \cup (T \cap T)] \\ &\quad \text{kala dhigidda dhexyaal} \\ &= [\phi \cup (B \cap T)] \cap [(T \cap B) \cup \phi] \\ &\quad \text{astaanta 5 (b)} \\ &= (B \cap T) \cap (T \cap B) \\ &\quad \text{astaanta 4 (t)} \\ &= [(B \cap T) \cap T] \cap [(B \cap T) \cap B] \\ &\quad \text{kala dhigidda dhexyaal} \\ &= [B \cap (T \cap T)] \cap [(B \cap T) \cap B] \\ &= (B \cap \phi) \cap [(B \cap T) \cap B] \\ &\quad \text{astaanta 5 (b)} \\ &= \phi \cap [(B \cap T) \cap B] \\ &\quad \text{aragtiin 1} \\ &= \phi \quad \dots\dots\dots 2 \end{aligned}$$

Isle'egyada 1 iyo 2 waxaan ka helnaa:

$$\begin{aligned} (B \cap T) \cap (B \cap T) &= (B \cup T) \cap (B \cap T) \\ \therefore (B \cap T) &= B \cup T \end{aligned}$$

Mar haddii duleedka $(B \cap T)$ madi yahay markaa

$$(\overline{B \cap T}) = (\overline{B} \cup \overline{T}) \text{ waa dhisan tahay.}$$

L a y l i 2:

b. Tus (caddee) in $(\overline{B \cup T}) = \overline{B} \cap \overline{T}$.

t. Sheeg dhardhaarta Boole ee aynu u cuskannay caddeyn ta aragtiina De Morgan qaybta (b) (ii).

j. Qor been ama run:

1) $(B \cap T) \cap J = B \cap (T \cap J)$

2) $(B \cup T) \cap J = B \cup (T \cap J)$

3) $B \cup B = B$

4) $\phi = B \cap \overline{B}$

—

5) $\overline{(\overline{B})} = B$

- 6) $B \cap (B \cup T) = (B \cup \phi) \cap (T \cup B)$
 - 7) $B \cap (B \cup T) = (\phi \cap T) \cup B$
 - 8) $B \cap (B \cup T) = B$
 - 9) $\phi = \phi$
 - 10) $B \cup (T \cup J) = (B \cup T) \cap (B \cup J)$
- X) Caddee in:

- 1) $\overline{\phi} = G$
- 2) $\overline{G} = \phi$
- 3) Haddii $B, T \in E, B \cup (B \cap T) = B$
- 4) Haddii $B, T \in E, B \cap (B \cup T) = B$
- 5) $G \cup B = G$
- 6) $\phi \cup B = B$

- 7) $\overline{(B)} = B$
- 8) $B \cup (\overline{B} \cap T) = B \cup T$
- 9) $(B \overline{\cup T \cup J}) = \overline{B} \cap \overline{T} \cap \overline{J}$
- 10) $(B \cap T \cap J) \cup (\overline{B} \cap J) \cup (\overline{T} \cap J) = J$

HABDHIS HAWAARO

Qaybtani waxay ku saabsan tahay xiriirka Boole iyo loojigga.

Tusahan soo socda wuxuu muujinayaan isku beegnaanta u degsami karta aljebrada Boole iyo loojigga.

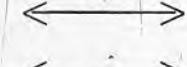
LOOJIG

R (Run)

B (Been)

A

V



Hawaaro (q, k, L,...)

Diidmo (\sim)

ALJEBRADA BOOLE

G

ϕ

\cap

\cup

=

\subset

Ururro (B, T, J,...)

Duleed (—)

Waxaynu urur jibbaarka E u macneynaa ururka dhammaan hawaaraha (q, k, L,...). B $\cup T$ macnaheedu waa kala-xirkalabha hawaarood q V k, $B \cap T$ waa isku xirkalabha q A k, run (R) waxaa loo macneeyaa G, beenna (B) ϕ . Tibxo kale ee aljebrada Boole waa $B \subset T$ iyo $B = T$ oo loojig ahaan loo tarjumo $q \rightarrow k$ iyo $q \leftrightarrow k$ say isugu xigaan. Urur duleedka, B wuxuu xagga loojigga macnahiisu noqonayaa diidmo hawaar « \sim ».

QARAABANNIMADA DHARDHAARRADA ALJEBRADA BOOLE IYO LOOJIG

DHARDHAARRADA BOOLE

- 1) **Hormogelinta**

$V B, T$ iyo $J \in E$

b. $(B \cup T) \cup J = B \cup (T \cup J)$

t. $(B \cap T) \cap J = B \cap (T \cap J)$

DHARDHAARRADA LOOJIGA

- 1) **Hormogelinta**

$V q, k$ iyo $L \in E$

b. $(q V k) V L = q V (k V L)$

t. $(q A k) A L = q A (k A L)$

2) **Kala hormaynta**

- a. $\forall B, T \in E$
- b. $B \cup T = T \cup B$
- c. $B \cap T = T \cap B$

3) **Jiritaanka asal madoorshe**

- a. $\forall B \in E$
- b. $B \cup \phi = B$
- c. $B \cap G = B$

4) **Jiritaanka duleed madi**

- a. $\forall B \in E$
- b. $B \cup B = G$
- c. $B \cap B = \phi$

5) Ugu dambeyn astaanta kala dhigga ee dhardhaarka Boole iyo astaanta kala dhigga ee hawraar loojig waa is-oggol yihiin. Sidoo kale aragtiin kasta oo Aljebra Boole, waxaa loo macnayn karaa tibaax loojig.

T u s a a l e 1:

$\overline{(B \cup T)} = \overline{B} \cap \overline{T}$, isle'egtan Boole waxay loojig ahaan u tarjimeysaa
 $\sim (q \vee k) \leftrightarrow \sim q \wedge \sim k$.

T u s a a l e 2:

Haddii B, T iyo $J \in E$ markaa

$$(B \cup T) \cap (B \cup J) = [B \cup (B \cap J)] \cup [(B \cap J) \cup (J \cap T)]$$

isle'egtan Boole waxay noqonaysaa dhab-oodka soo socda, haddii loo tarjumo hawraar loojigeed $V q, k$ iyo $j \in E$, $(q \vee k) A (q \vee j) \leftrightarrow [q \vee (q \wedge k)] V [(q \wedge j) V (k \wedge j)]$ sidoo kale $\sim q \vee (k \wedge \sim q) \leftrightarrow \sim q$ waxay u macnaysamaysaa

$$\overline{B} \cap (\overline{T} \cap \overline{B}) = \overline{B}.$$

Si loo caddeeyo isudhiganta loojigeed:

$$\sim q \vee (k \wedge \sim q) \leftrightarrow \sim q$$

aynu isticmaalno tuse rumeed

$q \sim q \wedge k$	$(k \wedge \sim q)$	$\sim q \vee (k \wedge \sim q)$	$\sim q \vee (k \wedge \sim q) \leftrightarrow q$
R	B	B	R
R	B	B	R
B	R	R	R
B	B	R	R

Haddii aynu caddeyno in $\overline{B} \cup (\overline{T} \cap \overline{B}) = \overline{B}$ waxaa la innaga doonayaa inaynu tusno in isle'egtu jirto iyo in kale.

$$\begin{aligned}
 \overline{B} \cup (\overline{T} \cap \overline{B}) &= (\overline{B} \cap G) \cup (\overline{T} \cap \overline{B}) \\
 &= \overline{B} \cap (G \cup \overline{T}) \\
 &= \overline{B} \cap G \\
 &= \overline{B}
 \end{aligned}$$

L a y l i 3:

I) Tibaaxahan Boole ee soo socda u tarjum tibaaxo ah habdhiska hawaaro.

- | | |
|----------------------------------|-------------------------------------------|
| 1. $B \cap T = B$ | 6. $A \cap \phi =$ |
| 2. $B = B \cap G$ | 7. $B \subset G$ |
| 3. $\phi = \bar{B} \cap \bar{B}$ | 8. $[(A \cap B) \cup J] \subset A \cup J$ |
| 4. $\bar{G} = \phi$ | 9. $\phi \subset G$ |
| 5. $\bar{A} = A$ | 10. $(B \cap B) \subset J$ |

2) U tarjum hawaaraha soo socda tibaaxo Aljebrada Boole ah.

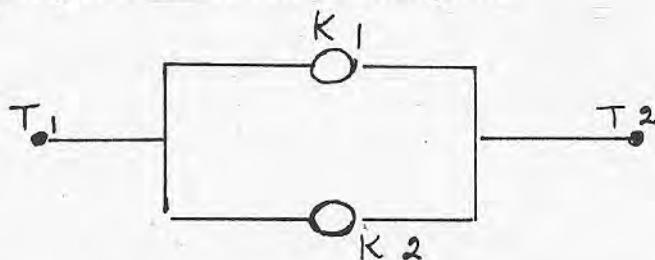
- b. $\sim q \vee (q \wedge k) \leftrightarrow R$
- t. $\sim (q \wedge k) \leftrightarrow \sim q \vee \sim k$
- j. $(\sim q \vee k) \leftrightarrow (q \rightarrow k)$
- x. $\sim (q \vee \sim k) \leftrightarrow \sim q \wedge k$
- kh. $(q \vee k) \wedge (q \wedge k) \leftrightarrow (q \wedge k)$
- d. $(q \wedge \sim k) \vee (k \wedge \sim q) \leftrightarrow (q \vee k) \wedge \sim (q \wedge k)$

3) Adoo isticmaalaya tuse rumeed tus dhisnaanta hawaaraha layliska 2aad laga bilaabo (b) ilaa (d).

MAREEGO DANAB

Aljebrada Boole, waxaa si la yaab leh loogu adeegsadaa dhismaha mareegyo danab, sidaa aweed ayey lagama maarmaan ugu tahay injiyeerinta danabeed ee maanta.

Laba fure waxaa la yiraa waxay u xiran yihiin barbarro haddii xiis marayo mareegta marka fure xiran yahay ama labada fureba xiran yihiin.



K_1 waxay ka joogtaa furaha # 1. K_2 waxay ka joogtaa furaha # 2. T_1 iyo T_2 waxay u taagan yihiin cirifyada danabka.

Xiisku waa isu maraya T_1 iyo T_2 haddii K_1 ama K_2 xiran tahay. Marka K_1 iyo K_2 ay isdabayaal u xiran yihiin xiisku wuxuu isu maraya (iskaga socdaa) T_1 iyo T_2 haddii iyo haddii oo qudha ay K_1 iyo K_2 labaduba xiran yihiin.



Waxaan fure kasta siinnaa qiime la iska doortay. Haddii furuhu xiran yahay waxaan siinnaa qiimaha G , hadduu furan yahayna ϕ . Fududeyn daraaddeed ϕ waxaan ka tagnaa 0. G -na 1, \cup -na +, \cap -na waxaa laga joojiyaa X. Waxaan qornaa $K_1 + K_2$ haddii K_1 iyo K_2 ay u xiran yihiin barbarro, haddii kale oo ay u xiran yihiin isdabayaal waxaan qornaa $K_1 \times K_2$ ama $K_1 K_2$.

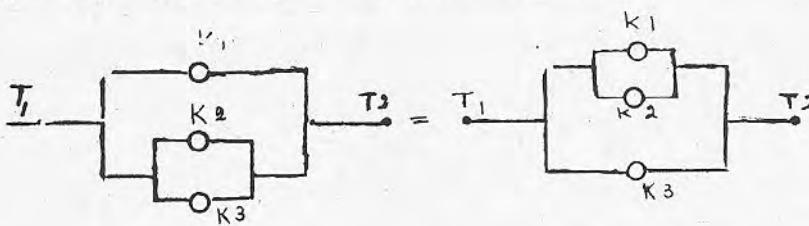
M u u j i n 1:

Ku tus jaantus in mareegyada $K_1 + (K_2 + K_3)$ iyo $(K_1 + K_2) + K_3$ ay isu dhigmaan.

Fur furiis:

Waa in la tuso in:

$$K_1 + (K_2 + K_3) = (K_1 + K_2) + K_3 \text{ (hormogelinta)}$$

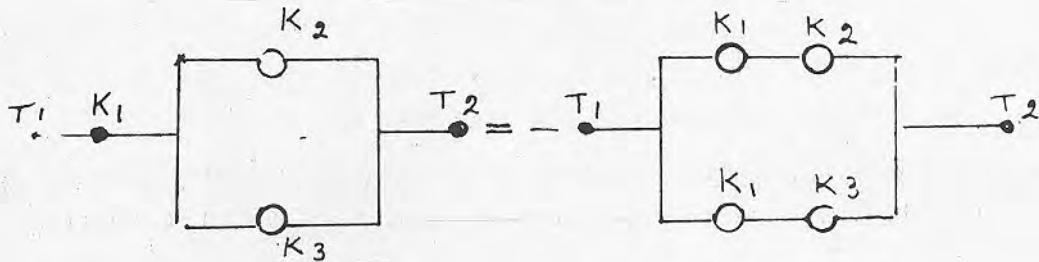


Ogow in laba mareeg isu dhigmaan haddii xaalad kasta oo ay ku sugan yihiin furayaashoodu, xiisku labadoodaba wada marayo ama aannu midnaba mareyn.

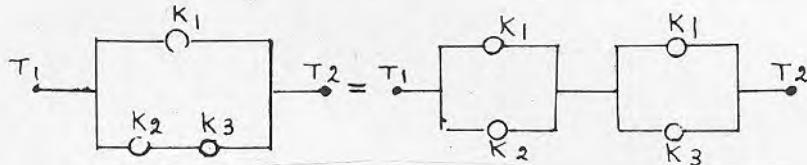
Muujin 2:

Astaanta kala dhigga oo lagu muujinayo mareeg:

$$1. K_1 \times (k_2 + k_3) = (k_1 \times k_2) + (k_1 \times k_3)$$



$$2. K_1 + (k_2 \times k_3) = (k_1 + k_2) \times (k_1 + k_3)$$



Ogow in + iyo \times ayan la shaqa ahayn summdaha isugeynta iyo isku dhufashada.

Layli:

1) Sawir mareegta ku beegan isle'egyada soo socda:

- b. $k_1 \times k_2 = k_2 \times k_1$
- t. $(k_1 \times k_2) \times k_3 = k_1 \times (k_2 \times k_3)$
- j. $k_1 + k_2 = k_2 + k_1$
- x. $1 \times k = k$

$$kh \quad k + \bar{k} = 1$$

$$d. \quad k \times 0 = 0$$

$$r. \quad k \times k = k$$

$$s. \quad \overline{(k_1 + k_2)} = \bar{k}_1 \times \bar{k}_2$$

$$sh. \quad \overline{(k_1 \times k_2)} = \bar{k}_1 + \bar{k}_2$$

$$dh. \quad \bar{k}_1 \times (\bar{k}_2 \times \bar{k}_1) = \bar{k}_1$$

2) Sawir mareegta ku beegan tibaax kasta ee soo socota, raadina mareegta ay isu dhigmaan ee u fudud

- | | |
|----------------------------------|-----------------------------------|
| 1) $k_1 \times (k_2 \times k_3)$ | 6) $\bar{k}_1 + (k_1 \times k_2)$ |
| 2) $k_1 + (\bar{k}_1 + k_3)$ | 7) $k \times k$ |
| 3) $k \times 0$ | 8) $(\bar{k}_1 \times \bar{k}_2)$ |
| 4) $k \times 1$ | 9) $k \times \bar{k}$ |
| 5) $k + \bar{k}$ | |

Cutub V
I T I M A A L

RAABAQAAD

Ku-tirsaneyaasha ururka {a, b, c} waxaad u ratibi kartaa lix siyaalood oo kala duwan:

$$\begin{array}{l} a \ b \ c \\ \quad a \ c \ b \end{array}$$

$$\begin{array}{l} b \ a \ c \\ \quad b \ c \ a \end{array}$$

$$\begin{array}{l} c \ a \ b \\ \quad c \ b \ a \end{array}$$

Ratibaad kasta waxay kaga duwan tahay tan kale horsiimada ku-tirsaneyaasha.

Q e e x :

Raabaqaad waa ratibaad kasta oo ku-tirsenayaasha urur la siinayo horsiimo huban.

A B N A Q A N

Abnaqaka n oo loo qoro n! waa taranka tirsiiimada laga bilaabo 1 ilaa n, taasoo ah:

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \dots (n-3) (n-2) (n-1) n.$$

$$\text{Matalan } 5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$$

$$\text{Ogow: } 0! = 1, \text{ sidoo kale } 1! = 1.$$

Sumadda ${}_n p_r$ waxay u taagan tahay tirada Raabaqad n shey oo markiiba la qaato r shey sida, ${}_8 p_3$ waa tirada Raabaqaad 8 shey oo markiiba la qaad qaatay 3 shey.

Tirada Raabaqaad n oo kala duwan oo markiiba r shey la israacsho waxay la mid tahay:

$${}_n p_r = n (n - 1) (n - 2) \dots (n - r + 1)$$

$$= \frac{n!}{(n - r)!}$$

$$\text{Haddii } r = n$$

$${}_n p_r = {}_n p_n = n (n - 1) (n - 2) \dots 3 \cdot 2 \cdot 1 = n!$$

T u s a a l e :

- b) ${}_5 p_1 = 5; {}_5 p_2 = 5 \cdot 4 = 20; {}_5 p_3 = 5 \cdot 4 \cdot 3 = 60$
 ${}_5 p_4 = 5 \cdot 4 \cdot 3 \cdot 2 = 120;$
 ${}_5 p_5 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$

- t) Tirada siyaalah 4 qof ugu fariisan karta 6 kursi waa:
 ${}_6 p_4 = 6 \cdot 5 \cdot 4 \cdot 3 = 360.$

- j) Immisa siyaabood ayaa loo safi karaa 5 buug oo kala duwan? Taasi waxay ka dhigan tahay tirada raabaqaadka 5 buug oo la qaato markiiba 5, ama ${}_5 p_5$ waa immisa?

$${}_5 p_5 = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$$

L a y l i :

- 1) Qiimee: ${}_{20} p_2; {}_{8} p_5; {}_7 p_5; {}_7 p_7.$
2) Hel n haddii:
b) ${}_7 {}_n p_3 = 6(n + 1) p_3$
t) $3 {}_n p_4 = (n - 1) p_5$
3) Raadi inta siyaalood ee 5 shaql loo ratibi karo haddii markiiba 3 la qaato. Caddeena in tiradu tahay 60.

- 4) Baakad turub ah ka soo saar yekaha, baashaha, quleenka, raanida iyo tobantaha oo kulligood wada isbig yihii. Immisa siyaalood ayaa 3 ka mid ah 5-taa warqadood loogu dhigi karaa dhinactax min bidix ilaa midig ah?

R A C A Y N :

Q e e x :

Racayntu waa kooxeyn ama xulid tiro waxyaaleed iyadoo aan loo eegin horsanaan.

T u s a a l e :

Racaynta 3da xaraf, a, b, c, oo la qaato markiiba 2 waa ab, ac, bc.

Ogow: ab iyo ba waa 1 racayn, laakiin, waa 2 raabaqaad. Summadda ${}_nC_r$ waxay u taagan tahay racaynta n shey oo la qaatay markiiba r, sida ${}_9C_4$ oo ah tirada racaynta 9 shey oo markiiba la qaatay 4.

Racaynta n shey oo kala duwan oo markiiba la qaatay r shey waxay la mid tahay:

$${}_nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!(n-r)!}$$

T u s a a l e :

Tirada gacan qaadka ka dhex dhici kara 12 qof haddii mid kasta uu salaamo kan kale waa:

$${}_{12}C_2 = \frac{12!}{2!(12-2)!} = \frac{12 \cdot 11}{2} = 66.$$

Jidka hoos ku qoran wuxuu kuu fududeynaya xisaabinta qaarkeed:

$${}_nC_r = {}_nC_{n-r}.$$

$$\text{Sida, } {}_9C_7 = {}_9C_2 = \frac{9 \cdot 8}{2} = 36$$

Ogow:

a. ${}_nC^0 = {}_nC_n = 1.$

b. ${}_nC_r$ waxaa loo qori karaa $[r]$.

L a y l i :

- 1) Hel n haddii:
 - b. ${}_nC_{n-2} = 10$
 - t. ${}_nC_{15} = {}_nC_{11}$
 - j. ${}_nP_4 = 30 {}_nP_5$
- 2) Immisa saddex-xagal ayaa lagu sameyn karaa 6da gees ee lix geesle?
- 3) Immisa xagla-gooye ayuu leeyahay siddeed geesluhu.

Aragtiinka Labatibaaxlaha

Ka soo qaad kala bixinta soo socota:

$$\begin{aligned}(a + b)^3 &= (a + b)(a + b)(a + b) \\ &= aaaa + baab + aabb + abba + bab + abb + bbb\end{aligned}$$

Tartan kasta oo ka mid ah kala bixintaa waxaad ku heli kartaa adoo isku dhufta 3 doorsoome oo mid kastaana ka mid yahay isirrada labatibaaxlahan saddexjibbaaran, $(a + b)^3$. Tibixda **baa**, matalan, waa taranka la helo marka laga doorto **b** isirka koowaad ee isirrada labatibaaxlaha saddexjibbaaran, **a** kan labaad, iyo kan saddexaad, kaddibna isku wada dhufatid. Eeg tarannada **baa**, **aab** iyo **aba** waxaad heshaa markaad ka xulato isirka koowaad **b**, **a-ka labaad**, **a-da kalena kan saddexaad**. Markaa, haddaad sidaa oo kale u raacayso tibaaxaha ku jira kala bixinta waxaad heleysaa:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Haddaba, 3, weheliyaha a^2b , waa tirada siyaalaha looga xulan karo **hal b saddexda** isir; taasi waxay ka dhigan tahay ${}_3C_1$. Sidoo kale 1, weheliyaha a^3 , waa adoo aan ka xulan b **saddexda** isir, taasoo ka dhigan ${}_3C_0$. Sidaas oo kale haddaad uga fekerto kala bixintii waxaa loo qori karaa:

$$(a + b)^3 = {}_3C_0 a^3 + {}_3C_1 a^2b + {}_3C_2 ab^2 + {}_3C_3 b^3$$

Sidoo kale waxaa loo heli karaa weheliyaasha ku jira kala bixinta $(a + b)^n$.

$$\begin{aligned}(a + b)^n &= {}_nC_0 a^n + {}_nC_1 a^{n-1} b + {}_nC_2 a^{n-2} b^2 + {}_nC_{n-1} ab^{n-1} + {}_nC_n b^n \\ &= \sum_{k=0}^n {}_nC_k a^{n-k} b^k\end{aligned}$$

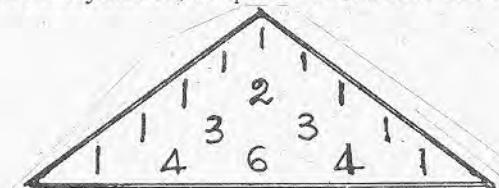
Tibixda r-aad waxay la mid tahay:

$${}_nC_{r-1} ab^{n-(r-1)} b^{r-1}$$

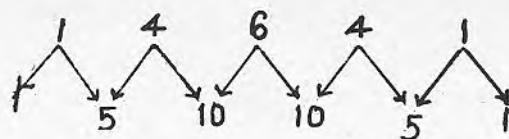
Haddii aad qorto kala bixinta $(a + b)^n$ adoo n siiyey qiimooyin u socda sida: 0, 1, 2, 3, 4, ... waxaad heleysaa:

$$\begin{aligned}(a + b)^0 &= 1 \\ (a + b)^1 &= a + b \\ (a + b)^2 &= a^2 + 2ab + b^2 \\ (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\ (a + b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\end{aligned}$$

Hadda-haddii la qoro weheliyaasha oo qura waxaa kuu muuqanaya saddexagalaha hoos ku muujisan



Eeg dhinactax kasta laga bilaabo kan saddexaad, tibxihiisa waxaa loo heli karaa adoo isku dara tibixda midig ka xigta iyo tan bidixda oo ku jira dhinactaxa ka sarreeya, sida:



Tirooyinkan u sameysan sida saddexagal waxaa la yiraa **saddexagalkii Baskal** (Pascal). Guud ahaan tibixda raad ee ku jirta dhinactaxa n-aad ee saddexaglahaa waxaad u heli kartaa:

$${}^n C_r = n - 1 \cdot r - 1 + n - 1 \cdot r (n \cdot r + 1)$$

T u s a a l e 1:

$$\text{Kala bixi: } (x + 2y)^4$$

F u r f u r i s :

$$\begin{aligned} (x + 2y)^4 &= {}^4 C^0 x^4 + {}^4 C_1 x^3 (2y) + {}^4 C_2 x^2 (2y)^2 + {}^4 C_3 x (2y)^3 + {}^4 C_4 (2y)^4 \\ &= 1 \cdot x^4 + 4x^3 (2y) + 6x^2 (4y^2) + + 4x (8y^3) + 1 \cdot (16y^4) \\ &= x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4 \end{aligned}$$

T u s a a l e 2 :

Hel tibixda lixaad ee ku jirta kala bixinta $(a + 3b)^9$.

F u r f u r i s :

Tibixda r-aad waxaa lagu helaa:

$${}^n C_{r-1} a^{n-(r-1)} 3b^{r-1}$$

$$\text{Markaa ta 6aad} = {}^9 C_{6-1} a^{9-(6-1)} (3b)^{6-1}$$

$$\text{Ogow: } {}^9 C_5 = \frac{9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4} = 126$$

$$\begin{aligned} \text{Tibixda 6aad} &= 126 a^4 (3b)^5 \\ &= 126 a^4 (243 b^5) \\ &= 30,618 a^4 b^5. \end{aligned}$$

L a y l i :

B. Kala bixi laba tibaaxleyaasha soo socda:

$$1. (1 + 3y)^4 \quad 4. \left(\frac{x}{3} + 3\right)^6$$

$$2. (2x + 1)^5 \quad 5. \left(2 - \frac{y}{3}\right)^6$$

$$3. (2 + \frac{y}{2})^6 \quad 6. -\left(\frac{x}{3} - 1\right)^6$$

T. Raadi:

- 1) Tibixda 5-aad ee ku jirta $(x + y)^{10}$
- 2) Tibixda 8-aad ee ku jirta $\left(2 - \frac{y}{2}\right)^{10}$
- 3) Tibixda 6-aad ee ku jirta $(a + b)^{11}$
- 4) Tibixda 14-aad ee ku jirta $(5 + a)^{15}$

J. Caddee :

- 1) $nC_2 = n_1C_1 + n_2C_2$, haddii $n \geq 3$;
- 2) $nCr = n_1Cr_1 + n_2Cr_2$, haddii $n \geq r + 1$.

MUNDALEEL

Haddii aad wax tijaabiso waxaad dhowrtaaba waa urur ka kooban jadeeyada suuragalka ah. Matalan, haddii aad tuurto laf-laadhuu waxaad dhowrtaaba waa in uu soo ko maro hal weji oo ay ku dul qoran yihin tirooyinka 1, 2, 3, 4, 5 ama 6 miduun. Taasi waxay koobtay tirooyinkii suuragalka ahaa oo idil.

Qeex :

Ururka jadeeyada suuragalka ah oo idil ee tijaabo waxaa la yiraa **Mundaleelkiitijaabada**.

Tusaale :

Tuur kumi, Mundaleelku waa (H, X). H waxay u taagan tahay madax, X waxay u taagan tahay xarar.

Tusaale :

Markan, tuur laf-laadhuu. Haddii aan doonayno tiro ku dul qoran wejiga u sara mara, mundaleelku waa $M_1 = \{1, 2, 3, 4, 5\}$ iyo $6\}$. Haddii aan doonayno tirada ku dul qoran wejiga u sara mara inuu noqdo kisi ama dhaban mundaleelku waa $M_2 = \{kisi, dhaban\}$.

Taasi waxay na tusaysaa in la isticmaali karo mundaleel ka badan mid, si loo tilmaamo waxyaabaha ka soo fulaya tijaabo la sameeyey. Wuxuu inoo muuqata in M_1 uu ina siiyay warbixin ka badan M_2 . Haddii aannu ognahay ku-tirsanaha M_1 , waxaan sheegi karnaa M_2 waxyaabaha ka soo fuli kara; laakiin, aqoonta waxyaabaha ka soo fula M_2 inoo sheegi mayso ku-tirsanihi kuu jirey M_1 . Markaa, waxa habboon in la isticmaalo mundaleel ku siinaya warbixin buuxda marka aad tijaabo samaysaba.

Tusaale :

Dambiil bay ku jiraan kubbado yaryari, oo qaarkood ka samaysan yihin quraarad, qaarkoodna cinjir. Labada noocba qaar baa casaan ah, qaarna waa buluug. Kaddib haddii aan sameyno tijaabo ku saabsan ka soo saaridda hal kubbud dambiisha, waxa dhici kara in ay soo baxdo kubbud:

- b. Ka sameysan quraarad ama cinjir. Haddii C tahay soo saaridda kubbud cinjir ah, Q-na mid quraarad ah, mundaleelkeeduna waa {C, Q}.
- t. Buluug ama casaan. Haddii B tahay soo saaridda kubbud buluug ah, G-na tahay mid casaan ah, mundaleelkeennu waa {B, G}.
- j. Ka sameysan quraarad ama cinjir mid ahna leh. Ka dhig Q, C, B, G. Kuwaan horey u soo qeexinay, mundaleelkeennu waa {(Q, B), (Q, G), (C, G)}.

Markan ku-tirsanyaasha mundaleelkeenna waa lammaaneyaa hoorsan. Suuragal kale oo badan ayaa jira. Arrintu markaa waxay tahay, marka la dhisayo mundaleelka waa in aad maskaxda ku haysaa jadaayo khaas ah.

WAQDHAC

Qeex :

Horma kasta oo mundaleel waxa la yiraa **Waqdhac**, waxaana loo soo gaabiyaa W.

Tusaale :

Haddii tuuridda laf-laadhuga mundaleelku yahay {1, 2, 3, 4, 5, 6}, marka waqdhac jadeeyadu tahay abyoone dhaban ah waa {2, 4, 6} oo horma u ah mundaleelkeennii.

Waqdhacaha marka jadeeyadu noqoto abyone kisi ah waa {1, 3, 5}. Tirada waqdhacaha suuragalka ah ee mundaleel t ku-tirsane leh waxay la mid tahay tirada hormooyinka suuragal ka ah ee urur kasta, oo t ku-tirsane leh, taasoo ah 2¹, iyagoo ay ku wada jiraan, ururka madhan waqdhac aan suuragal ahayn iyo mundaleelkoo idil (waqdhaca xaqiiq ah).

L a y l i :

- 1) Laf-laadhuu baa la tuuray. Tax mundaleelka. Tax waqdhacaha, marka tirada ku dul qoran wejiga u sare mara ay tahay mid ka weyn 2.
- 2) Kumi baa la tuuray. Tax mundaleelka. Tax waqdhacaha marka madax soo baxo.
- 3) Tuur laba kuumi. Tax mundaleelka. Tax waqdhaca marka ay u soo baxaan laba madax ama laba xarar.
- 4) Laf-laadhuu baa la tuuray. Tax mudaleelka. Haddii x tahay ururka jadeeyada suuragalka ah, oo laf-laadhuuga y-na tahay ururka jadeeyada suuragalka ah ee Kumiga, hel n (x) iyo n (y). Xisaabi tirada ku-tirsaneyasha taranka Kaartis ee ururrada x iyo y.
- 5) Immisa warqadood ayaa ku jira mundaleelka {1, 2, 3}, haddii Q (urur madhan) loo qaato waqdhac?
- 6) Immisa waqdhacood ayaa ku jira mudaleelka {1, 2, 3, 4, 5, 6}, haddii waqdhac kastaa ay ku jirto ugu yaraan hal jadeeyo.

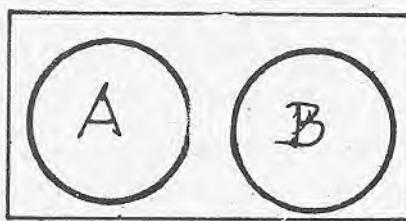
WAQDHACYO KALA REEBAN

Tuur Kumi. Wuxuu u dhici madax ama xarar, laakiin islama soo bixi karaan labadooduba mar. Kuwaas oo kale waxaa la yiraa **waqdhaco kala reeban**. Markan, tuur laf-laadhuu. Waqdhacaha in uu laf-laadhuugu soo saaro tirada 4 iyo isla markaa in uu soo saaro tiro dhufsane 3 ah ta {3, 6} ma wada dhici karaan, sababtoo ah waa waqdhaco kala reeban.

Laakiin, waqdhacaha inaan ka soo saarno baakad turub oo meel taal, hal warqad oo ah yeke, isla markaasna ah isbig, ma aha waqdhaco kala reeban.

Q e e x :

Laba waqdhacood A iyo B waxa la yiraa **waqdhaco kala reeban**, haddii $A \cap B = \emptyset$.



Shaxda 23

Waqdhaco kala reeban

L a y l i :

Lammaanayaasha soo socda kuwee baa ah waqdhaco kala reeban?

1. Magaalo ku taal (a) Afrika, (b) Soomaaliyaa.
2. Magaalo ku taal (a) Suudaan, (b) Aasiya.
3. Tiro (a) dhaban, (b) mid mutuxan.
4. Tiro (a) labajibbaaran, (b) saddexjibbaaran.
5. $A = \{1, 2, 5, -1, 4\}$ $B = \{a, x, y, 1\}$
6. $A = \{a, b, c, d\}$ $B = \{16, 41, 7, 8, -1\}$
7. $A = \{\square, \Delta, +\}$ $B = \{\div, -, x\}$
8. Tiro ah (b) kisi.

ITIMAAL WAQDHACEED

Markan, waxaa noo suurtowday inaan garanno waxa looga jeedo **Itimaalka jadeeyo tijaabo**. Ka dhig inaan haysanno kumi u dhaca madax ama xarar, laakiin, ayna ku suurtoobin inuu girtigira isku taago. Waxa la fili karaa in 50 jeer uu madax u dhaco, 50-na xarar marka boqol jeer la tuuro. Haddaba, waxaynu oran karnaa in uu madax u dhaco waa 50 — 50. Taasi ma aha haddii 100 jeer la tuuro kuumiga inaan heleyno 50 madax iyo 50 xarar, ee waxa loola jeedaa oo keli ah marar aad u fara badan haddii la tuuro, celceliska inta jeer ee la tuuray barkeed ayaa noqon madaxyo. Wuxaan sii afeefanaynaa in waqdhaca «madax» iyo «xarar» ay isku itimaal yihii. Itimaalka waqdhaca kastana innagoo sameyna tijaabo kasta, laakiin, waxaan qaadannaa ama ku fekernaa hindiso, markaasna waxaan fiirinnaa in tijaabadeennu waafaqsan tahay iyo in kale. Matalan, markaan sawireyno saddexagalka ma cabbirno in wadarta xaglihiisu tahay 18° , laakiin, waxaan isku qaadannaa in wadartoodu tahay 180° .

Q e e x :

Itimaalka waqdhaca W waa tiro, loo qoro $I(w)$ oo ay $0 \leq I(w) \leq 1$. Haddii M tahay mundaleelka markaa $I(M) = 1$. Haddii $w_1, w_2, w_3, \dots, w_n$ ay waqdhacyo kala reebban oo fudud yihii, markaa $I(M) = I(W_1) + I(W_2) + \dots + I(W_n) = 1$.

T u s a a l e :

Tuur laba kuumi, markaa $M = [HH, HX, XH, XX]$. Waqdhacyada fudud waa $w_1 = HH$, $w_2 = HX$, $w_3 = XH$, $w_4 = XX$. Waqdhacyada soo socdaa waa isku itimaal:

$$I(w_1) = I(w_2) = I(w_3) = I(w_4).$$

Markaa

$$\begin{aligned} I(M) &= I(w_1) + I(w_2) + I(w_3) + I(w_4) \\ &= 1/4 + 1/4 + 1/4 + 1/4 = 1. \end{aligned}$$

Baakad turub ah oo 52 warqadood ah haddaad jeceshahay in aad ka soo saarto hal warqad oo ah yake, waa immisa itimaalku? Wuxaan ognahay in baakadda ay ku jiraan 4 yake, markaa itimaalka hal yake kuu soo baxaa waa $4/52$. Guud ahaan, haddii aynu haysanno waqdhaco isku itimaal ah, markaa itimaalka $I(w)$ ee waqdhaci waa

$$I(w) = \frac{\text{Tirada suurogalka ah ee waqdhacdu soo baxdo}}{\text{tirada guud ee suuragalka ah}}$$

Markaa, haddii aan tuurno kuumi, itimaalka in uu u dhaco madax waa $1/2$, in uu u dhaco xararna waa $1/2$. Sidaan aragno labada waqdhacood waa isku itimaal, laakiin, haddii aan doonno in aan sameyno tijaabo waxa dhici kara in tirada soo bixidda madaxo ka badato tan xararka. Waxase jirta, haddii tijaabada aan sameyno marar fara badan in inta jeer ee uu soo baxo madaxu ay ku dhowaato inta jeer ee uu soo baxo xararku.

T u s a a l e :

Hel itimaala in laba laf-laadhuu isla baxaan wadar 9 ah.

F u r f u r i s :

Laf-laadhuuda hore waxay u dhici kartaa 6 siyaabood, mid kasta oo ka mid ah siyaabahaas, kan labaadna wuxuu u dhici karaa si la mid ah. Haddaba, waxa jira $6 \times 6 = 36$ siyaalood oo ay laba laf-laadhuu u dhici karaan: wadarta 9 waxaa loo heli karaa:

Laf-laadhuu hore	6 5 4 3
Laf-laadhuu dambe	3 4 5 6

Marka waxa jirta 4 siyaalood oo loo heli karo 9. Itimaalka in la helo tirooyin wadartoodu tahay 9 waa $\frac{4}{36} = \frac{1}{9}$.

Waxaan tusaaleynay in wadarta 9 loo heli karo 4 siyaalood, iyaguna u wada dhici karaan 36 siyaalood. Marka waxa jira 32 siyaalood oon 9 lagu heli karin.

Itimaalka in la helo wadarta 9 waa $I(W) = \frac{4}{36} = \frac{1}{9}$. Itimaal duleedka ama itimaalka in la waayo wadarta 9 waa $I(\bar{W}) = \frac{32}{36} = \frac{8}{9}$.

Eeg $I(W) + I(\bar{W}) = 1$. Guud ahaan haddii $I(W)$ yahay itimaalka waqdhaci soo bixi karto oo tirooyinka wada suuragal ah, markaa:

$$I(w) = \frac{\text{Tirada suurogalka waqdhacdu soo baxdo}}{\text{tirada guud ee suuragalka ah}}$$

$$I(\bar{w}) = \frac{\text{Tirada suurogalka waqdhacdu ayna soo bixin}}{\text{tirada guud ee suuragalka ah}}$$

Haddaba, waqdhacdu waa in ay soo baxdaa ama waa in äyna soo bixin; markaa wadarta sarreeyaasha ee dhinaca midigta xigtaa waxay la mid tahay tirada guud ee suuragal ah, taasoo keentay $I(W) + I(\bar{W}) = 1$.

Guud ahaan, haddii w_1, w_2, \dots, w_i yihiin waqdhacyo kala reebban oo mandaleel khaas ah, markaa

$$I(w_1) + I(w_2), \dots + I(w_i) = 1.$$

Ogow. Haddii $I(w) = 0$, waqdhacu weligii ma dhaco (ma suuragalo). Haddii $I(w) = 1$, waqdhacdu mar walba waa soo baxdaa (xaqiiqo), waana mundaleelka. Itimaalka $I(w)$, had iyo jeer, wuxuu u dhexeeyaa 0 iyo 1. Taasoo ah $0 \leq I(w) \leq 1$.

Aragtiin 1

Haddii w iyo \bar{w} ay yihiin waqdhaco isu duleed ah, markaa

$$I(\bar{w}) = 1 - I(w).$$

Caddeyn :

Waxaan haysannaa $w \cup \bar{w} = M$ markaa qeexda itimaalka waxaan ka heleynaa

$$I(w \cup \bar{w}) = I(w) + I(\bar{w})$$

Laakiin

$$I(w \cup \bar{w}) = I(M) = 1$$

ama

$$I(w) + I(\bar{w}) = 1$$

Markaa

$$I(\bar{w}) = 1 - I(w).$$

Aragtiin 2

$$I(\phi) = 0$$

C a d d e y n :

Innagoo og in Q iyo M ay isu yihiin duleed, Aragtiinkii 1 waxaan ka haysannaan:

$$\begin{aligned} I(Q) &= 1 - I(M) \\ I(Q) &= 1 - 1 \\ I(Q) &= 0 \end{aligned}$$

L a y l i :

- 1) Hel itimaalka:
 - b. Soo saaridda hal isbig oo ku jira baakad turub ah.
 - t. Soo saarid la'aanta hal isbig isla baakadda.
- 2) Hel itimaalka soo saaridda:
 - b. Tirooyinka wadartoodu ka yar tahay 18, tirooyin wadartoodu ka yar tahay 17, markii isku mar la tuuro 3 laf-laadhuu.
- 3) Tuur 2 laf-laadhuu ka dhig w waqdhaca labada laf-laadhuuyadu ay soo saaraan tirooyin isku mid ah. Ka dhig F waqdhaca tirooyinka wadartoodu ka weyn tahay 8. Hel:
 - b. $I(\overline{w})$, t. $I(f)$, j. $I(\overline{w})$, x. $I(F)$.

Xarfaha qaarkood (a, e, i o, u) waa shaqallo gaaban, qaarkoodna waa shaqallo dhaadheer. Haddii aan doonno inaan ka soo saarno xarfa alif-Soomaaliga, itimaalka soo saaridda hal shaqal oo gaaban waa $\frac{5}{32}$, itimaalka soo saaridda shaqal gaaban ama shaqal dheer, ama shaqallo waa $\frac{10}{32} = \frac{5}{16}$.

Guud ahaan, haddii ay jiraan t suuragal, t_a ay siiso waqdhaca A, t_b ay siiso waqdhaca B, itimaalka waqdhaca A ama B waa:

$$I(A \text{ ama } B) = \frac{t_b + t_b}{t}$$

Haddii A iyo B yihiin waqdhac kala reebban, marka

$$I(A \text{ ama } B) = -\frac{t_a}{t} + \frac{t_b}{t}$$

$$\text{ama } I(A \text{ am } B) = I(A) + I(B).$$

$$\begin{aligned} I(A) \text{ iyo } I(B) \text{ waa itimaallada waqdhacyada } A \text{ iyo } B \\ \text{ama } I(A \cup B) = I(A) + I(B). \end{aligned}$$

L a y l i :

- 1) Tiraan laga soo saari 1, 2, ..., 9, 10. Hel itimaalka si tirada soo baxdaa ay u noqoto:
 - b. Dhufsane 3;
 - t. Dhufsane 7;
 - j. Dhufsane 3 ama dhufsane 7.
- 2) Tiraan laga soo saari 1, 2, ..., 9, 10. Hel itimaalka:
 - b. Dhufsane 2;
 - t. Dhufsane 5. Maxaa reebay in la isku daro itimaallada si loo helo itimaalka dhufsanaha 2 ama dhufsanaha 5?

- 3) Sanduuq bay ku jiraan 40 kubbadoo oo 6 caddaan tahay, 8-na casaan tahay iyo 2-na cagaar tahay inta kalena midab kale yihii. Hel itimaalka si ay u soo baxdo kubbad caddaan ah ama casaan ah ama cagaar ah.

Waqdhaco Madaxbannaan

Q e e x :

Laba waqdhacood ama in ka badan waxa la yiraa **Way madaxbannaan yihii**, haddii soo bixidda ama soo bixid la 'aanta mid kasta oo ka mid ahi ayna wax ka beddeleyn soo bixidda mid kasta oo ka mid ah kuwa kale.

Haddii aan 4 jeer tuurro kumi uuna u wada dhaco madax, marka 5aad wuxuu u dhici karaa madax ama xarar, taasoo aan waxba ka beddelayn kuwii hore.

Itimaalka 2 waqdhac oo madaxbannaani ama in ka badan waa taranka itimaalkooda.

Summad ahaan:

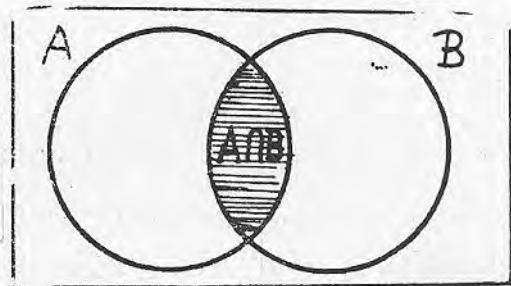
$$I(A \text{ iyo } B) = I(A) \cdot I(B)$$

ama

$$I(A \cap B) = I(A) \cdot I(B)$$

$I(A \cap B)$ waa itimaalka in ay mar dhacaan laba waqdhacood oo madaxbannaani A iyo

shax 24



T u s a a l e 1:

Kumi tuur, itimaalka in tuuryada 5aad iyo tan 6aad ay labaduba u wada dhacaan madax waa $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

T u s a a l e 2:

Laf-laadhuu baa 3 jeer la tuuray. Hel itimaalka in ay soo baxdo 6.

- 1) Mar kasta;
- 2) Marka saddexaad oo keliya.

F u r f u r i s :

- 1) Itimaalka in la helo 6 waa $\frac{1}{6}$; 3-da jeerna waa $I = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$.

- 2) Itimaalka in la helo 6 waa $\frac{1}{6}$. Itimaalka in aan 6 la helin waa $\frac{5}{6}$. Labada jeer ee hore waa kala madaxbannaan yihii marka itimaalkoodu waa $\frac{5}{6} \times \frac{5}{6}$. Tuuryada saddexaad way ka madaxbannaan tahay labadii ka horreeyey; markaa, itimaalka in la helo 6 tuuryada 3aad oo keliya waa $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$.

T u s a a l e 3:

- 84 Hel itimaalka si ay u soo baxdo ugu yaraan 3, marka laba jeer la tuuro laf-laadhuu.

Furfuriis:

Itimaalka in aan la helin 3 markii hal jeer la tuuro waa $1 - \frac{1}{6} = \frac{5}{6}$. Itimaalka in aan la helin 3 markii laba jeer la tuuro waa $\frac{5}{6} \times \frac{5}{6}$. Markaa itimaalka in la helo ugu yaraan 3 marka laba jeer la tuuro waa $1 - \frac{5}{6} \times \frac{5}{6} = 1 - \frac{25}{36} = \frac{11}{36}$.

Layli:

- 1) Warqad baa laga soo saaray baakad turub ah, kaddibna waa lagu ceshay; haddana mid kale ayaa laga soo saaray. Hel itimaalkaa in la soo saaro (b) laba yake, (t) laba isbig.
- 2) 3 laf-laadhuu baa la wada tuuray. Hel itimaalka ay soo baxaan (b) 3 lixaad; (t) laba lixaad; (j) hal lix ugu yaraa.
- 3) Itimaalka in Cali noolaado 25 sano oo dambe waa $\frac{3}{7}$; itimaalka in xaaskiisu noolaado 25 sano oo dambe waa $\frac{4}{5}$. Raadi itimaalka in 25ka sannadood ee soo socda ay:
 - b. Labadooduba noolaadaan;
 - t. Ugu yaraan midkood noolaado;
 - j. Cali kelidii noolaado.
- 4) Itimaalka in A furfuro masalo waa $\frac{4}{5}$, in B furfurana waa $\frac{2}{3}$, in C furfurana waa $\frac{3}{7}$. Haddii saddexdooduba isku dayaan, raadi itimaalka masalada lagu furfurayo.
- 5) Tuur 3 kumi. Raadi itimaalka in la helo laba madax iyo hal xarar.
- 6) Sanduuq baa waxa ku jira 3 kubbadood oo casaan ah iyo 8 cadaan ah. Haddii kubbad laga soo saaro sanduuq kasta hel itimaalka in ay soo baxaan:
 - b. 2 casaan ah;
 - t. 2 caddaan ah;
 - j. Mid casaan ah iyo mid caddaan ah.

Waqdhaco isku xiran

Haddii warqad aan ka soo saarno baakad turub ah, kaddibna aan lagu celin, itimaalka in la soo saaro mid labaad way isbeddeleysaa. Waqdhacahaas oo kale waxaa la yiraa **Waqdhaco isku xiran**.

Tusaaile:

Warqad baa laga soo saaray baakad turub ah. Mid kale ayaa la soo saaray, iyadoo aan tii hore lagu celin. Hel itimaalka si ay u soo baxaan:

- 1) Laba yake;
- 2) Laba isbig.

Furfuriis:

- 1) Itimaalka in warqadda koowaad ay noqoto yeke waa $\frac{4}{52}$. Markii laga saaray hal yake waxa baakadda ku haray 3 yake oo 51 warqadood ku dhix jira; itimaalka in warqadda

2aad ay noqoto yake waa $\frac{3}{51}$. Haddaba, si ay u soo wada baxaan 2 yake itimaalka waa in

$$\text{uu noqdaa } \frac{4}{52} \times \frac{3}{51} = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}.$$

- 2) Itimaalka in warqadda koowaad ay u noqoto isbig waa $\frac{13}{52}$. Itimaalka in warqadda

2aad ay u noqoto isbig waa $\frac{12}{51}$. Markaa itimaalka in labadooduba ay noqdaan isbig waa $\frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$.

L a y l i :

- 1) 9 kubblood oo 4 madow tahay, 5na caddaan tahay aaya ku jira sanduuq. Haddii la iska soo saaro labo, hel itimaalka in labadooduba noqdaan: (b) caddaan, (t) madow.
- 2) Sanduuq baa waxaa ku jira kubblood 5 cagaar tahay, 5 buluug tahay, 5 casaan tahay, 5 caddaan tahay, 5na madow tahay. Raadi itimaalka in ay soo baxaan:
 - a. 2 cagaar ahi;
 - b. 2 isku midab ah.
- 3) Dambiil baa waxaa ku jira 3 caddaan tahay, 2na madow tahay. Laba kubblood ayaa laga soo saaray. Raadi itimaalka in ay soo wada baxaan:
 - b. Laba madow ah;
 - t. Mid caddaan ah iyo mid madow ah.
- 4) Haddii 2 warqadood laga soo saaro baakad turub ah oo aan lagu celin, raadi itimaalka in ay soo baxaan:
 - i) 2 hadhiin ah;
 - ii) Mid madow ah iyo mid casaan ah.

TIJAABOOYIN BADAN

Haddii I tahay itimaalka in waqdhaci dhacdo tijaabo kasta oo la sameeyaba, $L = I - I$, $I - n$ tahay itimaalka in waqdhacdaasi ay dhici weydo tijaabo kasta, markaa itimaalka in ay jeer dhacdo markii la sameeyey n tijaabo waa:

$$_n C_r I^r L^{n-r}$$

Itimaalka in waqdhacdaasi ay dhacdo ugu yaraan r jeer markii la sameeyey n tijaabo waa: $I^n + {}_n C_1 I^{n-1} L + {}_n C_2 I^{n-2} L^2 + \dots + {}_n C_r I^{n-r} L^r$ (1)

Tibaaxda (1) waa wadarta $n - r + 1$ tibxood ee ugu horreeya kala bixinta laba tibaaxlahaa.

$$(I + L)^n.$$

T u s a a l e 1:

Xisaabi itimaalka haddii 9 laf-laadhuu mar la wada tuuro, ay soo baxaan 2 weji oo ay ku dul qoran yihiiin 1.

F u r f u r i s :

Itimaalka laba kasta oo ka mid ah 9ka laf-laadhuu in ay soo saaraan 2 weji ay ku dul yaallaan 1 waa $\frac{1}{6} \times (\frac{1}{6}) = (\frac{1}{6})^2$.

Itimaalka in 7da kale soo saarin 2 weji ay ku dul yaallaan 1 waa

$$(1 - \frac{1}{6})^7 = (\frac{5}{6})^7$$

Mar haddii laga soo saari karo ${}_9C_2$ lammaaneyaa kala duwan, 9 laf-laadhuu, itimaalka hal lammaane oo 1 ahi waa

$${}_9C_2 \cdot \{\frac{1}{6}\}^2 \cdot \{\frac{5}{6}\}^7 = \frac{78,125}{279,936}$$

ama jidka guud

$${}_{10}C_1 \cdot I \cdot L = {}_9C_2 \cdot (\frac{1}{6})^2 \cdot (\frac{5}{6})^7 = \frac{78,125}{279,936}$$

T u s a a l e 2:

Haddii itimaalka in celceliska ardada cusubi ayan ku dhammayn Dugsiga Sare 4 sano uu yahay $\frac{1}{3}$, hel itimaalka in 4tii ardayba ugu yaraan 3 ku dhammayso 4 sano.

F u r f u r i s :

Itimaalka in 3 dhammayso halna uusan dhammayn waa

$${}_4C_3 \cdot \{\frac{2}{3}\}^3 \cdot \{\frac{1}{3}\} = {}_4C_1 \cdot \{\frac{2}{3}\}^3 \cdot \{\frac{1}{3}\},$$

Itimaalka in 4 dhammaysana waa $\{\frac{2}{3}\}^4$

Itimaalka in ugu yaraan 4-tiiba 3 dhammayso waa

$$\{\frac{2}{3}\}^4 + {}_4C_1 \cdot \{\frac{2}{3}\}^3 \cdot \{\frac{1}{3}\} = \frac{16}{27}.$$

ama jidka guud. Itimaal labada tibxood ee ugu horreysa ($n - r + 1 = 4 - 3 + 1$) ee kala bixinta $\{\frac{2}{3} + \frac{1}{3}\}^4$

waa $\{\frac{2}{3}\}^4 + {}_4C_1 \cdot \{\frac{2}{3}\}^3 \cdot \{\frac{1}{3}\}$

ama

$$\frac{16}{81} + \frac{32}{81} = \frac{16}{27}.$$

L a y l i :

- 1) Hel itimaalka in ay soo baxdo wadarta 9 markii 3 jeer la tuuro laba laf-laadhuu.
- 2) Tuur kumi 6 jeer. Hel itimaalka in ay soo baxaan ugu yaraan 3 madax.
- 3) Tuur kumi 5 jeer. Hel itimaalka in ay soo baxaan ugu yaraan 4 xarar.
- 4) Hel itimaalka in qoys leh 5 carruur ah ay ku jiraan ugu yaraan 2 wiil iyo gabar. U qaado in itimaalka dhalashada 1 wiil uu yahay $\frac{1}{2}$.
- 5) Xisaabi itimaalka marka la isla tuuro 6 laf-laadhuu ay soo baxaan 3 weji oo tirada 2 leh.

- 6) Haddii itimaalka in celceliska dumarku ay k dhalaan wiil uu yahay $\frac{3}{5}$, hel itimaalka in 4-tiiba 3 ay dhalaan wiilal.

FILAAN XISAABEED

Q e e x :

Taranka itimaalka Loo ah in la helo hanti lacag ah H, iyo hantida qudheeda, waxaa la yiraa **Filaanka xisaabeeed** F ee helidda H.

Summad ahaan F = I. H.

T u s a a l e 1 :

Ninka cayaaraya bakhtiyaanasiib oo haysta hal warqad oo laga soo saaray 100 warqadood, itimaalkiisa inuu reeyo waa 0.01. Wuxaan u qaadanaynaa in nin kasta oo sita hal warqad in itimaalkisu yahay sidaas oo kale. Haddii ninkii reeyaa uu helayo Sh. So. 250, falaan xisaabeedkiisu waa

$$\begin{aligned} F &= (0.01)(250) \\ &= \text{Sh. So. } 2.50. \end{aligned}$$

T u s a a l e 2 :

Kaddib markii la eegay tusaha dhimashada, Cabdi itimaalkiisa in uu noolaado hal sano waa 0.98. Markaa wuxuu iibsaday warqadda badbaadada iyadoo uu ka faa'iideysan doono Sh. So. 4,000 haddii uu dhinto sannadka gudihiisa.

Hel filaan xisaabeedka in uu Cabdi lacagtaa helo.

F u r f u r i s :

Wakaaladda Badbaadadu, waxay lacagtaa bixinaysaa haddii uu Cabdi ku dhinto muddadaa. Haddaba itimaalka in uu dhinto sannadkaa gudihiisa waa $1 - 0.98 = 0.02$.

Markaa

$$\begin{aligned} \text{F i I. H} &= (0.02)(40,000) \\ &= \text{Sh. So. } 800. \end{aligned}$$

L a y l i :

- 1) 1000 warqadood ayaa loo sameeyey bakhtiyaanasiib. Ninkii reeyaa wuxuu helaya Sh. So. 300. Hel filaan xisaabeedka ninka warqad sita.
- 2) Itimaalka in waqdhaci ay dhacdo waa 0.23. Haddii waqdhacdii ay dhacdo, Faarax wuxuu helaya Sh. So. 500. Hel filaan xisaabkiisa.
- 3) Hel filaan xisaabeedka si loo helo Sh. So. 35.75, haddii itimaalka in la helo lacagtaa uu yahay $\frac{1}{25}$.
- 4) Haddii itimaalka in la helo Sh. So. 4,000 uu yahay 25%, hel filaan xisaabeedka in lagu guuleysto.

K A L K U L A S

Kalkulasku wuxuu ka mid yahay laamaha xisaabta, kuwa ugu muhiimsan. Haddii la raad raacana, Kalkulkasku wuxuu dabada ku hayaa Fisikis iyo Joometeri. Laan la mid ah Fisikiska oo la yiraahdo **Makaanikis** ayaa ku saabsan walxo socodkooda. Socodna waxaa badanaaba la doonaya in la ogaado, ammin go'an halka ay joogto walax soconaysa. Markaa, waxa loo baahday habdhis xisaabeed oo innaga kaalmeeya dersidda socodkaa in la soo saaro. Habdhiskaasu wuxuu noqday laanta ka mid ah Kalkulaska ee loo yaqaan **Xigta Kalkulaska**. Xigta Kalkulaska waxay ku saabsan tahay, inta isbeddel ah ee xaddi ku dhaca marka loo eego xaddi kale.

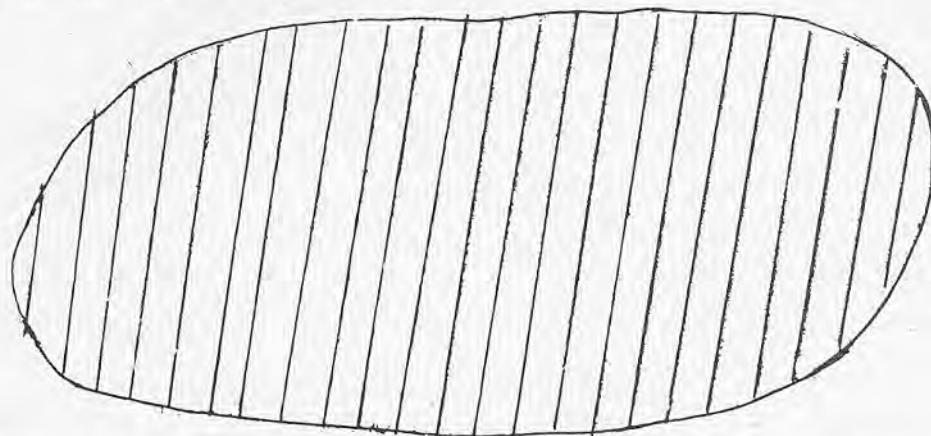
Laanta labaad ee Kalkulasku waa ta la yiraa **Kalkulaska Abyane**. Kalkulaska Abyane wuxuu asal ahaan ka soo jeedaa Joometeri. Run ahaantii, Kalkulkaska Abyane wuxuu ku saabsan yahay raadinta fansaar marka inta isbeddelkiisu oggol tahay. Wuxuu kala oo la oran karaa Kalkulaska Abyane, saldhiggeedu waa raadinta bededka xoodadka oo lagu doonayo Seebid. Arrintaasu waxay markaa inoo geyneysaa fikradda soohdinta, oo ah fikradda aasaaska u ah Kalkulaska.

FIKRADDA XADKA U AASAASKA AH

Waxaynu hore u soo sheegnay, in Kalkulasku ku dhisan yahay fikradda xadka. Si aynu u garanno waxa looga jeedo xad fansaar, bal aan ugu horreyn ku bilawno tusaaloooyin.

T u s a a l e 1:

Ka soo qaad in la rabo in la helo bedka xood Sallaax. Si aynu u hello bedkaas, waxaynu xoodka ku dhexmaraynaa laydiyo yaryar.



Halkan waxa innooga muuqata in bedka sallaxu in yar ka weyn yahay wadarta bededka laydiyada xoodku ku meeran yahay. Haddii laydiyada meeran la sii yareeyo, bedadkoodu waxay seebayaan bedka xoodka. Taas macnahedu waxa weeye, wadarta bededka laydiyadu waxay ku siqaysaa bedka xoodka.

T u s a a l e 2:

Malee arday ka lugeynaya gurigoodii oo ku socda dugsigiisii oo u jira $2\frac{1}{2}$ Km. Ka soo qaad in ardaygaa laga doonayo in uu joogo dugsiga marka saacaddu tahay 7:30 subaxnimo. Markii ay saacaddu ahayd 7:05 ardaygii wuxuu socday fogaan $\frac{5}{12}$ Km. ah, 7:10 markii ay ahaydna wuxuu socday fogaan $\frac{5}{6}$ Km, ah 7:15 markii ay ahaydna ardaygii wuxuu socday fogaan $\frac{5}{6}$ Km, ah, 7:20 wuxuu socday fogaan $1\frac{1}{4}$ Km. ah, 7:25 wuxuu socday fogaan $\frac{21}{12}$ Km. ah, 7:30 wuxuu socday fogaan $2\frac{1}{2}$ Km. ah.

Waxaynu aragnaa in fogaanta ardaygu u jiro dugsiga ay ku siqayso eber marka ammintu ku siqayso 7:30. Halkaa waxaynu uga jeednaa hab xadysan. Markaas oo kale waxaynu niraahnaa «Dugsiga fogaantiisu waxay ku siqaysaa eber marka ammintu ku siqdo 7:30».

T u s a a l e 3:

Waxaad ka soo qaaddaa in sallaan ku tiirsan yahay derbi taagan oo uu la sameynayo xagal le'eg Q^0 jiiftada. Haddii aynu sallaanka u dhaqaajinno xagga derbiga, xagasha sallaanku la sameynayo derbiga waxay fansaar u tahay fogaanta d ee sallaanku u jiro derbiga (t.a. $Q = f(d)$). Marka sallaanku derbiga ku sii dhowaadoba, Q waxay ku sii dhowaaneysaa 90^0 , inkasta oo aanay marnaba le'ekaanayn 90^0 ; waayo, ma suurtoobi karto in sallaanku ku yaallo derbiga. Markaa, waxaynu niraahnaa « Q waxay ku siqaysaa 90^0 marka ay d ku siqdo eber».

T u s a a l e 4:

Tusaha hoose adoo la kaashanaya tixgeli saamiga $\frac{\sin x}{x}$, oo ay x tahay gacansinyo.

x	sin x	$\frac{\sin x}{x}$
0.00	0.00000	—
0.01	0.01000	1
0.02	0.02000	1
0.03	0.03000	1
0.04	0.03999	0.9998
0.05	0.04998	0.9993
0.06	0.05996	0.9993
0.07	0.09694	0.9991
0.08	0.07991	0.9989
0.09	0.08988	0.9987
0.1	0.09983	0.9983

Joog-taxa midigta ugu xiga markaad u fiirsato, waxaad aragtaa in saamiga $\frac{\sin x}{x}$ uu ka sii dhawaanayo 1 marka x ku dhawaato 0. Markaas oo kale waxaynu niraahnaa: xadka $\frac{\sin x}{x}$ waxay ku siqdaa 1 marka x ku siqdo 0.

Tusaalooyinka kore waxay ina tusayaan wixa looga jeedo Xad. Giddigoodna biyadhacoodu wixa weeye, in xaddi go'ani ku sii dhawaado qiime go'an marka, xaddi kale oo ogaal ahi ku sii dhawaado qiime kale oo ogaal ah.

Q e e x 1:

Ka soo qaad in $f(x)$ (loo akhriyo f -da x) uu yahay fansaar maangal ah, kana soo qaad in L tahay tiro maangal ah. Markaas, xadka $f(x)$ waa L marka x ku siqdo tiro maangal oo « a » ah, waxay la macna tahay: Qiimaha $f(x)$ wuxuu aad ugu sii dhowaanayaa L marka x ku sii dhowaato a .

Summad ahaan qoraalkeedu waa: $\underset{x \rightarrow a}{\text{Xad}} f(x) = L$ (marmarna waxaan qoraa $f(x) \rightarrow L$) marka $x \rightarrow a$. Tusaalooyinkii aynu hore u soo sheegnay iyo Qeexda 1, ayanu ka heleynaa fekradda xad saldhingga u ah. Imminkara aynu hal sheegno caddeyn la'aan, aragtiiyada aasaaska u ah Xad.

Aragtiin 1-b

Haddii $f(x) = C$ oo C tahay madoorsoome, markaas $\lim_{x \rightarrow a} f(x) = C$, hadal ahaan waxa weeye madoorsoome xadkiisu waa madoorsoome.

Aragtiin 1-t

Madoorsoome kasta C , iyo fansaar kasta $f(x)$.
 $\lim_{x \rightarrow a} Cf(x) = C \lim_{x \rightarrow a} f(x)$, hadal ahaan xadka

fansaar lagu dhuftay madoorsoome, waxay le'eg tahay madoorsoome lagu dhuftay xadka fansaarka.

Aragtiin 2

Fansaar kasta f iyo g , haddii
 $\lim_{x \rightarrow a} f(x) = A$, $\lim_{x \rightarrow a} g(x) = B$
markaas, $\lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x) = A + B$.

Taa macnaheedu waxa weeye: xadka wadari waxay le'eg tahay wadarta xadadka.

Aragtiin 3

Haddii $\lim_{x \rightarrow a} f(x) = A$, $\lim_{x \rightarrow a} g(x) = B$,

markaa:

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) \\ = A \cdot B.$$

Hadal ahaan, xadka taraneed waxay le'eg tahay taranka xadadka.

Aragtiin 4

Haddii

$$\lim_{x \rightarrow a} f(x) = A, \lim_{x \rightarrow a} g(x) = B,$$

markaas

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \\ = \frac{A}{B}, \quad B \neq 0.$$

Taas macnaheedu waxa weeye, Xadadka qaybeed waxay le'eg tahay Qaybta Xadadka, iyada oo shardi ay tahay inaan hooseeyuhu eber noqon.

T u s a a l e 1:

Aragtiinka (1) wuxuu ina oranayaa, Xadka fansaar madoorsoome waa madoorsoomaha. Markaas, haddii $f(x) = 8$, Xad $f(x) = \lim_{x \rightarrow a} f(x) = 8$.

T u s a a l e 2:

Marka la raaco Aragtiinka (3):

$$\begin{aligned} \text{Xad } (x+7)(x^2 - 1) &= \text{Xad } (x+7) \cdot \text{Xad } (x^2 - 1) \\ x \rightarrow 1 &\quad x \rightarrow 1 \quad x \rightarrow 1 \\ &= \text{Xad } (x+7) \cdot \text{Xad } [(x-1)(x+1)] \\ x \rightarrow 1 &\quad x \rightarrow 1 \\ &= \text{Xad } (x+7) \cdot \text{Xad } (x-1) \cdot \text{Xad } (x+1) \\ x \rightarrow 1 &\quad x \rightarrow 1 \quad x \rightarrow 1 \\ &= 8 \cdot 0 \cdot 2 = 0. \end{aligned}$$

T u s a a l e 3:

Marka la raaco aragtiinka (2)

$$\begin{aligned} \text{Xad } (x^2 + 2x - 5) &= \text{Xad } x^2 + \text{Xad } 2x - \text{Xad } 5 \\ x \rightarrow 3 &\quad x \rightarrow 3 \quad x \rightarrow 3 \quad x \rightarrow 3 \\ &= 9 + 6 - 5 = 10. \end{aligned}$$

T u s a a l e 4:

Marka la raaco aragtiinka (4):

$$\begin{aligned} \text{Xad } \frac{x^2 - 5x + 6}{x \rightarrow 2} &= \text{Xad } \frac{(x-2)(x-3)}{x \rightarrow 2} \\ &= \text{Xad } (x-3) \\ x \rightarrow 2 & \\ &= -1. \end{aligned}$$

T u s a a l e 5:

Raadi

$$\text{Xad } \frac{x^4 - 16}{x \rightarrow 2} \frac{x^3 - 8}{}$$

Waxaynu haysannaa

$$\begin{aligned} \frac{x^4 - 16}{x^3 - 8} &= \frac{x^4 - 2^4}{x^3 - 2^3} = \frac{(x^2 - 4)(x^2 + 4)}{(x-2)(x^2 + 2x + 4)} \\ &= \frac{(x-2)(x+2)(x^2 + 4)}{(x-2)(x^2 + 2x + 4)} \\ &= \frac{(x+2)(x^2 + 4)}{x^2 + 2x + 4} \end{aligned}$$

$$\begin{aligned} \text{Xad } \frac{x^4 - 16}{x \rightarrow 2} \frac{x^3 - 8}{} &= \text{Xad } \frac{(x+2)(x^2 + 4)}{x \rightarrow 2} \frac{x^2 + 2x + 4}{} \\ &= \frac{4 \cdot 8}{4 + 4 + 4} = \frac{32}{12} = \frac{8}{3}. \end{aligned}$$

T u s a a l e 6:

Raadi

$$\underset{\theta \rightarrow 0}{\text{Xad}} \frac{\sin^2 \theta}{\theta^2}$$

Waxaynu haysannaa

$$\frac{\sin^2 \theta}{\theta^2} = \frac{(\sin \theta)(\sin \theta)}{(\theta)(\theta)}$$

$$\begin{aligned} \underset{\theta \rightarrow 0}{\text{Xad}} \frac{\sin^2 \theta}{\theta^2} &= \underset{\theta \rightarrow 0}{\text{Xad}} \frac{\sin \theta}{\theta} \cdot \underset{\theta \rightarrow 0}{\text{Xad}} \frac{\sin \theta}{\theta} \\ &= 1 \cdot 1 = 1. \end{aligned}$$

T u s a a l e 7:

Raadi

$$\underset{\theta \rightarrow 2}{\text{Xad}} \frac{\sin 2\theta}{\theta}$$

Mar haddii

$$\underset{\theta \rightarrow 0}{\text{Xad}} \frac{\sin 2\theta}{\theta} = \underset{2\theta \rightarrow 0}{\text{Xad}} \frac{2 \sin 2\theta}{2\theta}$$

$$\underset{\theta \rightarrow 0}{\text{Xad}} \frac{\sin 2\theta}{\theta} = 2 \underset{2\theta \rightarrow 0}{\text{Xad}} \frac{\sin 2\theta}{2\theta} = 2 \cdot 1 = 2.$$

L a y l i :

1) Raadi xadadka soo socda:

- b. Xad $(2x^2 + x - 3)$
 $x \rightarrow 3$
- t. Xad $(x^3 + 3x^2 - x + 1)$
 $x \rightarrow 1$
- j. Xad $(2x^2 + 4x - 6)$
 $x \rightarrow 4$
- x. Xad $(2x^2 - x + 18)$
 $x \rightarrow 1$
- kh. Xad $(x^3 + 8)$
 $x \rightarrow 2$

2) Raadi xadadka soo socda:

- 1) Xad $(x - 2)(x + 1)$
 $x \rightarrow 3$
- 2) Xad $(x + 1)(x^2 + 1)(x + 2)$
 $x \rightarrow 1$

3) Xad $3(x^3 + 1)(x^2 - 1)$
 $x \rightarrow 2$

4) Xad $\frac{1}{2} \frac{1}{x^3}$
 $x \rightarrow 2$

5) Xad $\sqrt{x^2 + 4}$
 $x \rightarrow 0$

6) Xad $x \sqrt{x^2 + 4}$
 $x \rightarrow 2$

7) Xad $3(x^4 - 4)$
 $x \rightarrow 3$

8) Xad $(1 - x)(x^2 - 1)$
 $x \rightarrow 2$

3) Xisaabi xadadka:

1) Xad $\frac{1}{2} \frac{1}{x^3}$

2) Xad $4 \frac{1}{x - 4}$

3) Xad $3 \frac{x - 3}{x^2 + x - 12}$

4) Xad $1 \frac{x + 1}{x(x^2 - 1)}$

5) Xad $2 \frac{x^2 + 8x - 16}{x^2 - 4x + 4}$

6) Xad $5 \frac{x^2 - 9x + 20}{(x-1)(x-3)x-5}$

7) Xad $3 \frac{3}{x^2} - \frac{1}{3}$
 $x \rightarrow 3$

8) Xad $a \frac{x^2 - 2x - 35}{x(x^2 + 3x - 10)}$

4) Doon xadadka soo socda. Ka soo qaad in $a \neq 0$.

1) Xad $a \frac{x^2 + a^2}{x^4 - a^4}$
 $x \rightarrow a$

2) Xad $a \frac{x^3 - a^3}{x^2 - a^2}$
 $x \rightarrow a$

$$3) \lim_{x \rightarrow 2} \frac{(3x+1)(x^2+2)^2}{(x^2-4)(x^2+3x+2)}$$

$$4) \lim_{x \rightarrow a} \frac{x^3 + a^3}{x^2 - a^2}$$

$$\frac{3}{x^2} - \frac{3}{a^2}$$

$$5) \lim_{x \rightarrow a} \frac{\dots}{x-a}$$

$$6) \lim_{x \rightarrow 1} \frac{x^2 - 8x + 7}{x^3 - 4x^2 - x + 4}$$

$$7) \lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$$

$$8) \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x} - \sqrt{2}} \text{ ku dhufu } \frac{\sqrt{x} + \sqrt{2}}{\sqrt{x} + \sqrt{2}}$$

$$9) \lim_{x \rightarrow 2} \frac{3x^2 - 12x - 36}{4x^2 + 16x + 16}$$

$$10) \lim_{x \rightarrow 2} \frac{x^2 - 4x - 21}{x^2 + 2x - 63}$$

$$11) \lim_{\phi \rightarrow 0} \frac{\sin 4\phi}{\phi}$$

$$12) \lim_{\phi \rightarrow 0} \frac{\sin 2\phi}{\phi}$$

$$13) \lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{\theta}$$

$$\sin \frac{\theta}{2}$$

$$14) \lim_{\theta \rightarrow 0} \frac{\dots}{\theta}$$

$$15) \lim_{\theta \rightarrow 0} \frac{\sin(3\theta - \theta)}{\theta}$$

5) Layliyada soo socda:

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Tusaale:

$$f(x) = x^2 - 3x$$

Furfuris:

$$f(x) = x^2 - 3x; f(x+h) = (x+h)^2 - 3(x+h)$$

markaa

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{x^2 + 2hx + h^2 - 3h - x^2 + 3x - 3x}{h} \\ &= \frac{2hx + h^2 - 3h}{h} \\ &= \frac{x \rightarrow 0}{x \rightarrow 0} (2x + h - 3) = 2x - 3 \\ &= \frac{x \rightarrow 0}{h \rightarrow 0} (2x + h - 3) = 2x - 3. \end{aligned}$$

- | | |
|---------------------------|--------------------------|
| 1. $f(x) = x^3$ | 5. $f(x) = x^2 + 3x + 5$ |
| 2. $f(x) = 2x^2 + 3x$ | 6. $f(x) = x^2$ |
| 3. $f(x) = \frac{1}{x}$ | 7. $f(x) = x^4$ |
| 4. $f(x) = \frac{1}{x^2}$ | 8. $f(x) = C$ |
| | 9. $f(x) = \sqrt{x}$ |

XADAD JAAD KALE AH

Tiro Beel.

Bal fiiri fansaarka $\frac{1}{x}$, $x \neq 0$ waxa halkaa ka cad:

- b. Marka x tahay tiro yar oo toogan, $\frac{1}{x}$ waa tiro weyn oo toogan. Tusaalin $\frac{1}{0.001} = 1000$
- t. Marka x tahay tiro weyn ama badan, $\frac{1}{x}$ waa yar tahay. Tusaalin, $\frac{1}{10,000} = 0.0001$.
- j. Marka x taban tahay, $\frac{1}{x}$ waa taban tahay. Tusaalin:

$$\frac{1}{(-0.001)} = -1000 \text{ iyo } \frac{1}{(-10,000)} = -0.0001.$$

Hubaalladaa sidan ayaan loo soo gaabiyaa:

- b. $\frac{1}{x}$ Waxay ku siqdaa 0, markaa x ku siqdo $+\infty$ ama $-\infty$.
- t. $\frac{1}{x}$ Waxay ku siqdaa $+\infty$, marka x xagga midig kaga siqdo 0.
- j. $\frac{1}{x}$ waxay ku siqdaa $-\infty$, marka x xagga bidix kaga siqdo 0.

Astada ∞ , waxa loo akhriyaa **tiro-dhaaf**, amase taagna wax tiro ah oo maangal ah.

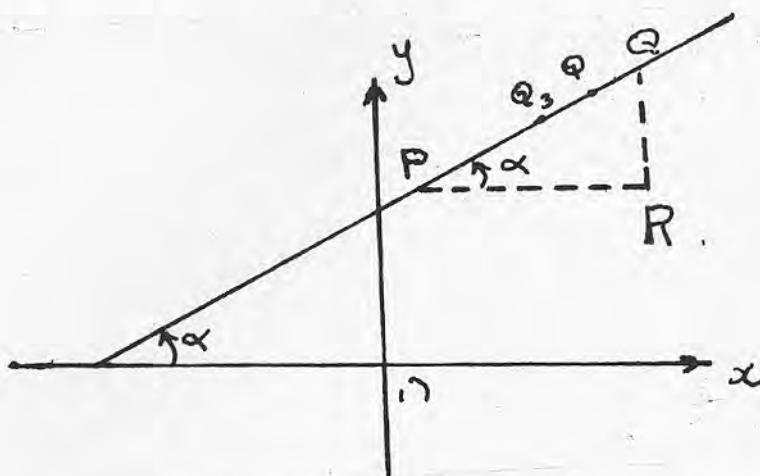
Ogow:

$$\begin{aligned}
 a + (+\infty) &= \infty \quad a \in \mathbb{R} \\
 a - (+\infty) &= -\infty \\
 a \cdot (+\infty) &= +\infty \quad \text{haddii } a > 0 \\
 a \cdot (+\infty) &= -\infty \quad \text{haddii } a < 0 \\
 (+\infty) \cdot (+\infty) &= +\infty \\
 (+\infty) + (+\infty) &= +\infty
 \end{aligned}$$

X I G I D D A

Tiirada Xood.

Waynu naqaan sida loo raadiyo xarriiq toosan tiirateeda. Haddii L tahay xarriiq toosan oo la sameynayo Dhidibka $\rightarrow x$ ee togan xagal cabbirkeedu yahay α , P iyo Q-in a yihiin laba barood oo ku yaal xarriiqda L (Eeg shaxanka 25). Markaas $m = \tan \alpha = \frac{RQ}{PR}$.



Shax 25

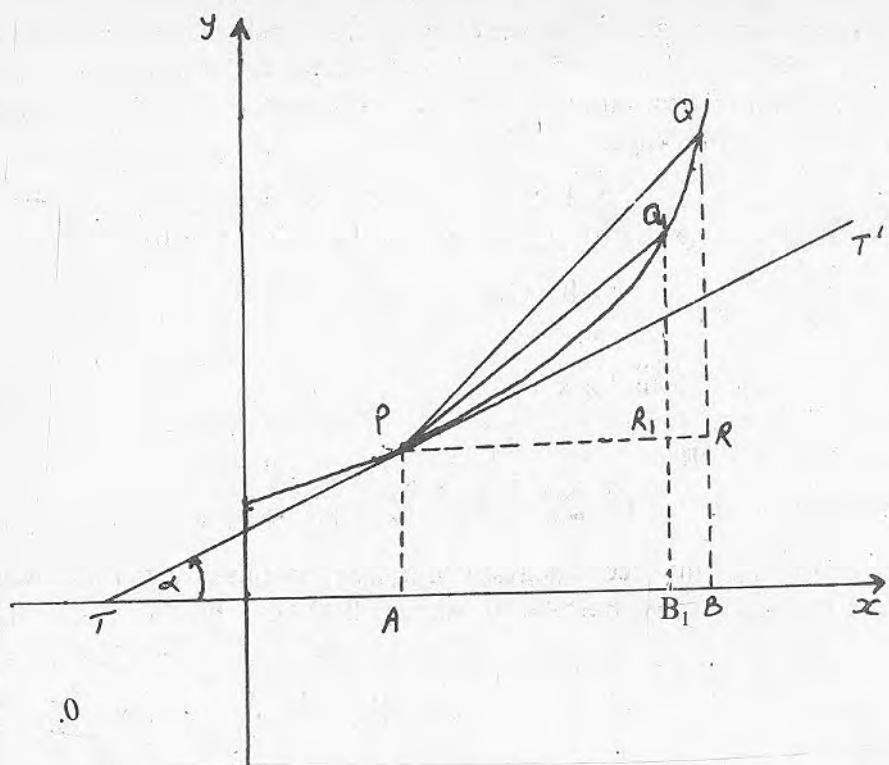
Bal hadda, ka dhig inaynu doonayno inaynu hello tiiro xood. Ka soo qaad P iyo Q in ay yihiin laba barood oo ku yaal xoodka, soona jeex ordineytyada PA iyo QB. PR-in a ha noqoto xarriiq la barbarro ah dhidibka — x.

Markaas saamiga $\frac{RQ}{PR}$ wuxuu u taagan yahay tiiro boqonka PQ. Haddaba si loo helo tiirada xoodka halka barta P tahay, waxaynu u baahannahay in ayu derisno siyaabuhuu yeelo boqonka PQ marka barta Q u dhaqdhaqaaqdo xagga P. Waynu aragnaa haddii Q u dhaqdhaqaaqdo Q_1 . in tirada boqonka PQ noqonayso $\frac{R_1 Q_1}{PR_1}$.

Sidaas oo kale haddii Q tagto rugaha cusub ee Q_2, Q_3, \dots waxa hubaal ah in Q ku sii durkayso P. Fogaanta PR-in a ay aad iyo aad u sii yaraanayso. Markaa, saamiga $\frac{RQ}{PR}$ wuxuu aad iyo aad ugu dhowaanayaa tiirada xoodka. Boqonka PQ wuxuu aad ugu sii dhowaanayaa tiirada xoodka. Boqonka PQ wuxuu aad ugu sii dhowaanayaa taabtaha TT¹.

(Eeg shax. 26. Haddaba, sidan hoos ku qoran ayeynu u qeexnaa tiirada xoodka halka barta. P.

$$\begin{aligned}
 \text{Tiirada xoodka halka barta P} &= \frac{\text{Xad}}{Q \rightarrow P} \frac{RQ}{PR} \\
 &= \text{tiirada taabtaha barta P.}
 \end{aligned}$$



Shax 26

Tusaale ahaan, ka soo qaad in xoodka shaxanka sare uu leeyahay isle'egta $y = x^2 + 5$.
Marka $OA = 1$, $AP = 1^2 + 5 = 6$. Marka $OB = 2$, $BQ = 2^2 + 5 = 9$.

Haddaba,

$$PR = AP = OB - OA = 1, RQ = BQ - AP = 3.$$

$$\text{Markaas tiirada } PQ = \frac{RQ}{PR} = \frac{3}{1} = 3.$$

Imminka, haddii Q u dhaqaaqdo Q_1 oo $OB = 1.5$, markaas,

$$PR_1 = 1.5 - 1 = 0.5.$$

$$R_1Q_1 = B_1Q_1 - AP = (1.5) + 5 - 6 = 1.25.$$

$$\therefore \frac{R_1Q_1}{PR_1} = \frac{1.25}{0.5} = 2.5.$$

$$\text{Haddii } OB_2 = 1.2; B_2Q_2 = (1.2)^2 + 5 = 6.44$$

$$\therefore R_2Q_2 = 6.44 - 6 = 0.44.$$

$$\therefore \frac{R_2Q_2}{PR_2} = \frac{0.44}{0.2} = 2.2$$

$$\text{Haddii } OB_3 = 1.1, B_3Q_3 = (1.1)^2 + 5 = 6.21$$

$$\therefore R_3Q_3 = 6.21 - 6 = 0.21$$

$$\therefore \frac{R_3Q_3}{PR_3} = \frac{0.21}{0.1} = 2.1$$

$$\text{Haddii } OB_4 = 1.05, B_4Q_4 = (1.05)^2 + 5 = 6.1025$$

$$\therefore R_4Q_4$$

$$\therefore R_4Q_4 = 6.1025 - 6 = 0.1025$$

$$\therefore \frac{R_4Q_4}{PR_4} = \frac{0.1025}{0.05} = 2.05$$

Haddaba, waxaa muuqata oon mar kasta oo Q ku sii dhowaanaysop. Markaa, waxaynu niraahnaa tiirada xoodka $y = x^2 + 5$ waa 2 halka x tahay 1 (halka x = 1). Haddii ay P tusaalaheenna hore tahay barta, kulan $-x$ -du (kulan x-deedu) = x_1 , Q tahay bar u dhow oo kulan $-x$ -du = $x_1 + h$; markaas:

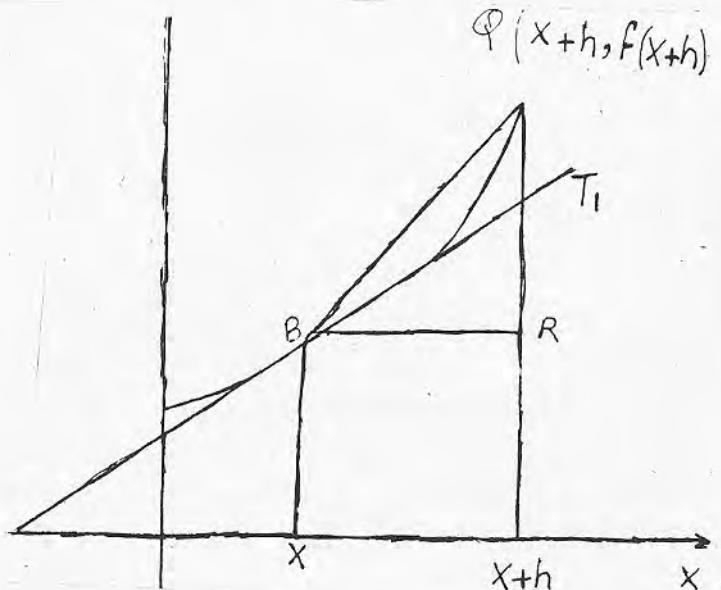
$$\begin{aligned} AP &= x_1^2 + 5, PR = (x_1 + h) - x_1 = h \\ RQ &= (x_1 + h)^2 + 5 - (x_1^2 + 5) = 2x_1 h + h^2 \\ \therefore \frac{RQ}{PR} &= \frac{2x_1 h + h^2}{h} = 2x_1 + h \end{aligned}$$

Markaas, tiirada xoodka halka x_1 :

$$\frac{\underset{Q \rightarrow P}{\text{Xad}}}{\underset{R \rightarrow 0}{\text{RQ}}} \underset{h \rightarrow 0}{\text{PR}} = \underset{Q \rightarrow P}{\text{Xad}} (2x_1 + h) = 2x_1$$

Guud ahaan, haddii isle'egta xoodka aynu

tiiradiisa doonayno marka uu meel ogaal ah maraayo, ay tahay $y = f(x)$; waxaynu tixgelinnaa barta P(x), f(x) iyo barta u dhow oo ah Q(x+h), f(x+h), oo h aad u yar tahay.



Shax 27

Sidii hore oo kale, tiirada boqonka

$$PQ = \frac{RQ}{PR} = \frac{f(x+h) - f(x)}{h}$$

Marka, tiirada xoodka halka barta x_1 :

$$\frac{\underset{Q \rightarrow P}{\text{Xad}}}{\underset{R \rightarrow 0}{\text{RQ}}} = \underset{h \rightarrow 0}{\text{Xad}} \frac{f(x+h) - f(x)}{h}$$

Waxaa halkaa ka muuqata, in mar kasta oo Q ku sii dhowaado P in boqonka PQ ku dhawaato taabtaha TT₁, markaa tiirada xoodka halka barta P:

$$P = \underset{h \rightarrow 0}{\text{Xad}} \frac{f(x+h) - f(x)}{h}$$

Tusaale :

Raadi tiirada xoodka $y = f(x) = 2x^2 - x + 3$

- i) Halka bartii aad doonto ee x ah.
- ii) Halka $x = C$.
- iii) Halka $x = 3$.

Fururis :

- i) Innaga oo jadeeyada sare ka ogaanay in tiirada bar kasta Xad $\frac{f(x+h) - f(x)}{h - 0}$ waxaan aragnaa in:

$$\begin{aligned} \text{Xad}_{\frac{h}{0}} &= \frac{2(x+h)^2 - (x+h) + 3 - (2x^2 - x + 3)}{h} \\ &= \frac{x \text{ ad}_{\frac{h}{0}}}{h} \frac{(4xh - h + 2h^2)}{h} \\ &= \frac{x \text{ ad}}{h} \frac{4x - 1 - 2h}{h} = 4x - 1 \end{aligned}$$

- ii) Aan x ku beddello C jadeeyada (i), waxaan heleynaa $4C - 1$.
- iii) Haddii $x = 3$. waxaynu heleynaa
 $4 \cdot 3 - 1 = 11$.

LAYLI:

b. Sawir garaafka $y = x^2$, ka soo qaad in P tahay barta $x = 2$, Q tahay barta $x = 2.5$.

- 1) Xisaabi tiirada xoodka halka $x = 2$; tahay barta P (2,4).

- 2) Sawir taabtaha P, tiiradiisana soo saar.

- 3) Labada jadeeyo is-garab dhig, waa sidee?

t. Sida layliga kore ka yeel marka $y = 2x^2 + 5$ labada barood yihin P (1, 3) iyo Q (1.4, 1.08).

- j) Sawir garaafka $y = \frac{1}{4x^2}$; adoo qiimayaasha x ka qaadanaya gaaliska oodan ee (-4,4). Soo saar tiirada xoodka halka i) $x = -3$; ii) $x = 2 \cdot 4$.

- x. Haddii isle'egta socodka kubbad ,la cirbixiyey uu yahay $S = 128t - 16t^2$, xisaabi tiirada xoodka marka $t = 4$ iyo marka $t = 5$.

- kh. Adoo tixgelinaya fansaarka $f(x) = 96x - \frac{1}{2}x^3$.

Xisaabi tiirada xoodka halka $x = C$, kaddibna halka $x = 2$ iyo halka $x = 8$.

XIGSIINTA FANSAAR

Si aynu u garanno waxa looga jeedo xigsiiin fansaar. Bal aan tixgelinno tusaalooyinka soo socda:

Tusaale 1:

Lammaanaha kul-fidka

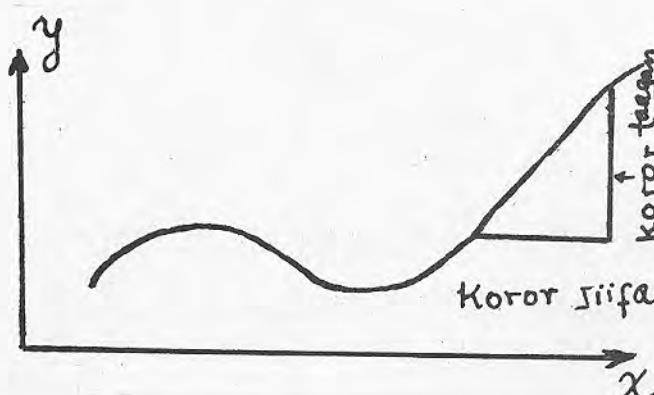
Marka bir la kululeeyo way fiddaa, laakiin, fididdaasi isku wada mid maaha. Qeexda meheliyaha kul-fidka, waxa weeye, isbeddelka dhererka ku dhaca, marka hal derejo uu kor u kaco heerkulmitirku. Taas oo ah:

$$\text{Celceliska weheliyaha kul-fidka toosan} = \frac{\text{Isbeddelka ku dhacay dhererka}}{\text{Isbeddelka Uumibeeegga}}$$

Tusaale 2:

TIIRADA WADDO

Cabbirka tiirada ay waddo leedahay waxa ku siinaya; isbeddelka fogaanta taagan oo loo qaybsho isbeddelka fogaanta jiifta.



Tusaale 3:

KAYNAAN

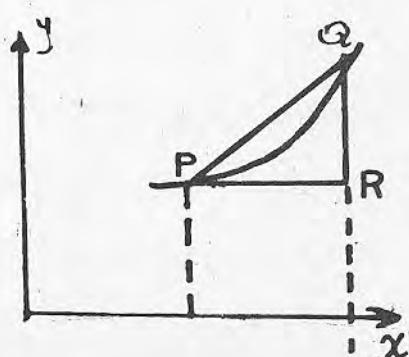
Haddii gaari u kala socdo 3 saac laba magaalo A iyo B, oo isu jira 240 km. waxaynu niraahnnaa celceliska kaynaanka gaarigu waa 80 km./sac.

$$\text{Celceliska Kaynaanku} = \frac{\text{Isbeddelka Barabixidda}}{\text{Isbeddelka amminta}}$$

Tusaale 4:

TIIRADA XOOD

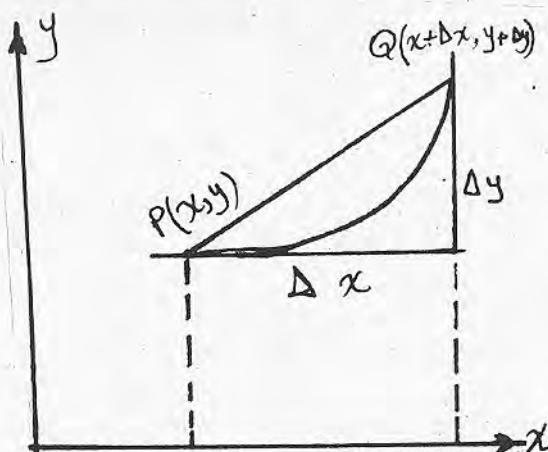
Haddii aynu xood haysanno (sida shaxanka 29aad tusayo) waxaynu aragnaa in tiirada boqonka $PQ = \frac{RQ}{PR}$.



Tusaalooyinkaas oo dhan biyo-dhacoodu waxa weeye Celceliska inta isbeddelka ee

fansaar = Isbeddelka ku dhacay doorsoomaha siyaaban
Isbeddelka ku dhacay doorsoomaha madaxbannaan

Guud ahaan, ka soo qaad in $y = f(x)$ ay tahay xood, kana dhig in $P(x, y)$ ay tahay bar, $Q(x + \Delta x, y + \Delta y)$ tahay bar kale oo u dhow. Δx (delta x) waa inta ku korodhay jihada x, Δy -na waa inta ku korodhay jihada y ee ku beegan.



Shax 30

Tiirada boqonka PQ, ama celceliska tiirada waa $\frac{\Delta y}{\Delta x}$.

Mar haddii fansaarku yahay $y = f(x)$, waxaynu haysannaa:

$$\begin{aligned} y + \Delta y &= f(x + \Delta x) \\ \Delta y &= f(x + \Delta x) - f(x) \\ \frac{\Delta y}{\Delta x} &= \frac{f(x + \Delta x) - f(x)}{\Delta x} \end{aligned}$$

ama

$$\text{Celceliska tiirada} = \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

= Celceliska inta isbeddelka y marka loo eego x.

Fiiro 1. Δx waa xaddi qura, oo u taagan koror aad u yar oo ku dhacay jihada x, ee ma aha taranta Δ iyo x.

Fiiro 2. Δy ma aha hor bannaane, ee waxay ku xiran tahay Δx , (eeg shaxanka 30). Mar haddii kororka jiiftada la ogaado, kororka taagan ee Δy , waa lala soo bixi karayaa.

Fiiro 3. Waxaa dhici karta in Δx iyo Δy labaduba yar yihiiin, oo haddana Saamiga $\frac{\Delta y}{\Delta x}$ uu weyn yahay.

T u s a a l e :

Haddii $\Delta x = 0.00002$, $\Delta y = 0.004$; markaas $\frac{\Delta y}{\Delta x} = \frac{0.004}{0.00002} = 200$. Waxaynu ognahay celceliska isbeddelka fansaar in uu yahay $\frac{\Delta y}{\Delta x}$. Haddii aynu imminka doonayno inaan ogaanno biriqsiga inta isbeddelka, waynu aragnaa inay tahay, xadaynta qiimaha celceliska, t.a.

Biriqsiga inta isbeddelka \equiv Xad celceliska inta isbeddelka

$$= \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x} \dots (1)$$

Tibaaxda (1). Waxa la yiraa **Diiridda fansaarka**.

X I G S I N

Q e e x :

Haddii $y = f(x)$ ay tahay fansaar, markaas xigsinta fansaarka $f(x)$, oo ay u taagan tahay $f'(x)$ (loo akhriyo f-da kor dhaban ee x), waxaa weeye:

$$f'(x) = \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x}$$

Habka raadinta xigsinta waxa la yiraa **xigid**.

Ogow: Mar haddii $y = f(x)$, $y + \Delta y = f(x + \Delta x)$, markaas $\Delta y = f(x + \Delta x) - f(x)$.
Sidaa awgeed

$$\begin{aligned} \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x} &= \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ f'(x) &= \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{f(x + \Delta x) - f(x)}{\Delta x} \end{aligned}$$

Xigsinta $y = f(x)$, marka loo eego x, waxa lagu muujin karaa qormooyinkan soo socda tii la doono:

$$f'(x), \frac{dy}{dx}, Dxy, y' \text{ ama } \frac{d}{dx} f(x).$$

Ogow: $\frac{dy}{dx}$ waxa loo akhriyaa «xigsinta y marka loo eego x».

T u s a a l e 1:

Haddii $y = f(x) = x^2$; raadi celceliska isbeddelka y marka loo eego x.

F u r f u r i s :

$$\Delta y = (x + \Delta x)^2 - x^2 = x^2 + 2x\Delta x + (\Delta x)^2 - x^2 = 2x\Delta x + (\Delta x)^2.$$

$$\begin{aligned} \text{Celceliska isbeddelka } y &= \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x} = \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\ &= 2x + \Delta x \end{aligned}$$

T u s a a l e 2:

Raadi xigsinta tusaalaha 1aad:

F u r f u r i s :

$$\begin{aligned} -\frac{dy}{dx} \text{ ama } f'(x) &= \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x} \\ &= 2x. \end{aligned}$$

Tusaaile 3:

Raadi xigsinta $y = x^2 + 3x - 2$.

Furfuris:

$$\begin{aligned}y + \Delta y &= (x + \Delta x)^2 + 3(x + \Delta x) - 2 \\&= x^2 + 2x\Delta x + \Delta x^2 + 3x + 3\Delta x - 2 \\ \therefore \Delta y &= y + \Delta y - y \\&= x^2 + 2x\Delta x + \Delta x^2 + 3x + 3\Delta x - 2 - (x^2 + 3x - 2)\end{aligned}$$

$$\begin{aligned}\therefore \frac{\Delta y}{\Delta x} &= 2x + \Delta x + 3 \\ \therefore \frac{\Delta y}{\Delta x} &= \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x} \\&= \underset{\Delta x \rightarrow 0}{\text{Xad}} (2x + \Delta x + 3) = 2x + 3\end{aligned}$$

Tusaaile 4:

$$\begin{aligned}\text{Xig } y &= x^3 \\ \Delta y &= (x + \Delta x)^3 - x^3 \\&= x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - x^3 \\&= 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 \\ \therefore \frac{\Delta y}{\Delta x} &= 3x^2 + 3x\Delta x + \Delta x^2 \\ \therefore \frac{\Delta y}{\Delta x} &= \underset{\Delta x \rightarrow 0}{\text{Xad}} (3x^2 + 3x\Delta x + \Delta x^2) \\&= 3x^2\end{aligned}$$

Layli:

- 1) Socodka walax iskeed u soo dhacaysa ayaa lagu siiyey in isle'egta socodkeedu yahay $S = 16t^2$. Raadi celceliska kaynaanka ee gaaliska Ammineed, min: i) 3 ilaa 3.1 ii) 3 ilaa 3.05 iii) t_1 ilaa t_2 iv) t^0 ilaa $t^0 + h$.
- 2) Saxar jid toosan haysta, ayaa t sekenba socda fogaan S mitir ah, iyadoo $S = t^2 + t - 6$.
 - i) Fogaan intee le'eg ayuu jaraa 2da seken ee u horreeya? 3da seken ee u horreeya?
 - ii) Waa intee celceliska kaynaanka sekenba saddexaad?
 - iii) Waa intee celceliska kaynaanka gaaliska ammineed, min: $t = t_0$ ilaa $t = t_0 + h$?
 - iv) Waa intee biriqsiga kaynaanku marka $t = t_0$.
- 3) Fansaarrada soo socda, mid walba raadi celceliska inta isbeddelka doorsoomaha hore marka loo eego doorsoomaha labaad.
 - b. $y = 2x^2 + 2$, min:
 - i) $x = 2$ ilaa $x = 2.2$
 - ii) $x = 2$ ilaa $x = 2.1$
 - iii) $x = 2$ ilaa $x = 2.01$.

- t. $y = x^3$, min:
 i) $x = 1.9$ ilaa $x = 2$
 ii) $x = 2$ ilaa $x = 2.1$.
- j. $S = 2 + t' t^2$, min:
 i) $t = 1.1$ ilaa $t = 1.2$
 ii) $t = 2$ ilaa $t = 2.2$
 iii) $t = t_2$ ilaa $t = t_1$

Haddii $y = f(x)$, waxaynu ognahay in Qeexda $f'(x)$ ay noqonayso

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

b. Haddaynu Δx ku beddelno h, sansaanka ay qeexdii $f'(x)$ yeelanaysaa waa kan:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

Adoo sansaankan laftiisa raacaya, raadi $f'(x)$ haddii $f(x) = 3x^2 + 2x - 1$.

t. Haddaynu x ku beddelo C, Δx -na h, markaas sansaanka ay qeexdi $f'(x)$ yeelanaysaa waa kan:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(C + h) - f(C)}{h}$$

Adoo sansaankan raacaya, qiimee $f'(2)$ haddii $f(x) = 3x^2 + 2x - 1$.

j. Haddaynu $x + \Delta x$ ku beddelo x_2 , x ku beddelo x_1 , sansaanka ay qeexdii $f'(x)$ yeelanaysaa waa kan:

$$f'(x_1) = \lim_{x_2 - x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Adoo sansaankan raacaya, qiimee $f'(3)$ haddii $f(x) = x^3 - x^2 - 4$.

5) Adoo isticmaalaya qeexdii xigsinta, raadi $\frac{dy}{dx}$

Haddii:

- b. $y = 2x + 3$
 t. $y = 3x^2 - 4x + 2$
 j. $y = 2x^3 - x$
 x. $y = 3x^2 + 7x - 2$
 kh. $x^2 - x - 1$.

XEERARKA XIGIDDA EE AASAASKA AH

Ilaa iyo imminka, marka aynu xigeyno fansaar, waxaynu toos u raacaynay Qeexdii. Waxase jirta, inaan Qeex raacu ahayn sida ugu fudud ee loo xigo fansaar. Sidaa awgeed, waxaynu halkan ku sheegi doonaa, xeerarka xigidda jideynaya ee aasaaska ah qaarkood. Xeerarkaa badidooda waynu caddaynaynaa, qaarkoodse waynu tibaaxaynaa caddeyn la'aan. Ka soo qaad in f iyo g yihin laba fansaar oo xigma, kana dhig in C tahay madoorsoome, n tahay tiro maangal ah. Markaas:

Xeer 1. $(f \pm g)' = f' \pm g'$. Hadal ahaan waxa weeye: Xigsinta wadartu ama faraqu waxay le'eg tahay wadar ama faraqa xigsimaha.

Xeer 2. Madoorsoome xigsiiantu waa eber.

Xeer 3. $(cf') = Cf'$, t.a., xigsinta madoorsoome lagu dhuftay fansaar, waxay le'eg tahay madoorsoomaha oo lagu dhuftay xigsinta fansaarka.

Xeer 4. $(fg)' = f \cdot g' + f' \cdot g$.

Xeer 5. $(x^n)' = n \cdot x^{n-1}$.

Xeerarkan markaynu caddeynayno waxaynu adeegsanaynaa Qeexdii xigsinta iyo arag tiinyadii Xadka.

X e e r 1:

Ka soo qaad in $y = f(x) + g(x)$, markaas $y + \Delta y = f(x + \Delta x) + g(x + \Delta x)$

$$\therefore \Delta y = f(x + \Delta x) - f(x) + g(x + \Delta x) - g(x)$$

$$\therefore y' = \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x}$$

$$= \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{f(x + \Delta x) - f(x) + g(x + \Delta x) - g(x)}{\Delta x}$$

$$= \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{f(x + \Delta x) - f(x)}{\Delta x} +$$

$$+ \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$

$$\therefore (f + g)' = f' + g'$$

X e e r 2:

- Ka soo qaad in $y = f(x) = C$, markaa
 $y + \Delta y = C$

$$\therefore \Delta y = C - C = 0$$

$$\therefore y' = \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x} = \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{C - C}{\Delta x} = 0.$$

X e e r 3:

Ka soo qaad in $y = C f(x)$, markaas

$$\begin{aligned}\Delta y &= C f(x + \Delta x) - C f(x) \\ &= C [f(x + \Delta x) - f(x)]\end{aligned}$$

$$\therefore y' = \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{\Delta y}{\Delta x}$$

$$= \underset{\Delta x \rightarrow 0}{\text{Xad}} C \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= C \underset{\Delta x \rightarrow 0}{\text{Xad}} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= C f'.$$

X e e r 5:

Ka soo qaad in $y = x^n$, n ay tahay abyooone togan markaa:

$$y' = \frac{\Delta y}{\Delta x} \rightarrow 0 \quad \frac{(x + \Delta x)^n - x^n}{\Delta x}$$

$= \frac{\Delta y}{\Delta x} \rightarrow 0$

$$(x^n + n x^{n-1} \Delta x + \frac{n(n-1)}{1 \cdot 2} x^{n-2} (\Delta x)^2 + \dots + (\Delta x)^n - x^n)$$

$$\frac{\Delta y}{\Delta x}$$

$$= \frac{\Delta y}{\Delta x} \rightarrow 0 \quad (nx^{n-1} + \frac{n(n-1)}{1 \cdot 2} x^{n-2} \Delta x + \dots + (\Delta x)^{n-1})$$

$$= n x^{n-1}.$$

T u s a a l e 1

Haddii $y = x^4$, raadi y' .

F u r f u r i s :

$$\begin{aligned}\Delta y &= (x + \Delta x)^4 - x^4 \\ &= x^4 + 4 x^3 \Delta x + 6 x^2 (\Delta x)^2 + 4 x (\Delta x)^3 + (\Delta x)^4 - x^4 \\ &= 4 x^3 \Delta x + 6 x^2 (\Delta x)^2 + 4 x (\Delta x)^3 + (\Delta x)^4.\end{aligned}$$

$$\therefore \frac{\Delta y}{\Delta x} = 4 x^3 + 6 x^2 \Delta x + 4 x (\Delta x)^2 + (\Delta x)^3$$

$$\therefore \frac{\Delta y}{\Delta x} \rightarrow 0 \quad 4 x^3$$

$$\text{Markaa } \frac{d}{dx}(x^4) = 4 x^3$$

$$\text{Sidoo kale } \frac{d}{dx}(x^5) = 5 x^4, \frac{d}{dx}(x^6) = 6 x^5, \text{iwm.}$$

T u s a a l e 2:

$$\text{Raadi } -\frac{d}{dx} \left\{ \frac{1}{x} \right\}$$

F u r f u r i s :

$$\text{Haddii } y = \left\{ \frac{1}{x} \right\}, \text{ markaas}$$

$$\Delta y = \frac{1}{x + \Delta x} - \frac{1}{x} = \frac{x - x - \Delta x}{x(x + \Delta x)}$$

$$= \frac{-\Delta x}{x(x + \Delta x)}$$

$$\therefore \frac{\Delta y}{\Delta x} = -\frac{1}{x(x + \Delta x)}$$

$$\therefore \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = - \lim_{\Delta x \rightarrow 0} \frac{1}{x(x + \Delta x)} = - \frac{1}{x^2}$$

Tusale 3:

$$\text{Raadi y' haddii } y = \frac{1}{x^2}$$

Fururis:

$$\begin{aligned}\Delta y &= \frac{1}{(x + \Delta x)^2} - \frac{1}{x^2} \\ &= \frac{x^2 - (x + \Delta x)^2}{x^2(x + \Delta x)^2} \\ &= \frac{-2x\Delta x - (\Delta x)^2}{x^2[x^2 + 2x\Delta x + (\Delta x)^2]} \\ \therefore \frac{\Delta y}{\Delta x} &= \frac{-2x - \Delta x}{x^2[x^2 + 2x\Delta x + (\Delta x)^2]} \\ \therefore \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} &= - \lim_{\Delta x \rightarrow 0} \frac{2x - \Delta x}{x^2[x^2 + 2x\Delta x + (\Delta x)^2]} \\ &= - \frac{2x}{x^4} = - \frac{2}{x^3} = - 2x^{-3}\end{aligned}$$

$$\text{Sidaas oo kale aaya } \frac{d}{dx}(x^{-3}) = -3x^{-4}, \text{ iyo}$$

$$\frac{d}{dx}(x^{-4}) = -4x^{-5}; \text{ iwm.}$$

Tusale 4:

$$\text{Raadi } \frac{d}{dx}(\sqrt{x}).$$

Fururis:

$$\text{Ka soo qaad } f(x) = \sqrt{x} \text{ markaa}$$

$$\begin{aligned}f(x + h) - f(x) &= \sqrt{x+h} - \sqrt{x} \\ &= \frac{(x+h) - x}{\sqrt{x+h} + \sqrt{x}} \\ &= \frac{h}{\sqrt{x+h} + \sqrt{x}}\end{aligned}$$

$$\therefore \text{Xad } \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \text{Xad } \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{d}{dx} (\sqrt{x}) = \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

Guud ahaan, waad aragtaa in $\frac{d}{dx} (x^n) = n x^{n-1}$.

Xeerka Isasuran. (Xigidda fansaarka fansaar). Haddii $y = f(x)$, $u = g(x)$, markaas $y = f[g(x)]$ waa fansaar x .

Haddii y tahay fansaar xigma oo u ay tahay fansaar xigma oo x , markaas $y = f[g(x)]$ waa fansaar xigma oo x , xigsinta $\frac{dy}{dx}$ waxa lagu heli karaa xeerkan oo loo yaqaan **Xeerkan Isasuran.**

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

T u s a a l e 1:

$$\text{Haddii } y = u^2 + 3, u = 2x + 1, \text{ markaas } \frac{dy}{du} = 2u, \frac{du}{dx} = 2.$$

$$\text{Markaas } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot 2 = 4u$$

$$\text{laakiinse } u = 2x + 1$$

$$\therefore \frac{dy}{dx} = 4u = 4(2x + 1) = 8x + 4.$$

T u s a a l e 2:

$$\text{Raadi } \frac{dy}{dx} \text{ haddii } y = \sqrt[3]{x^2 + 3}.$$

F u r f u r i s :

$$U \text{ qaado in } x^2 + 3 = u.$$

$$\therefore y = \sqrt[3]{u} = u^{1/3}$$

$$\frac{dy}{du} = \frac{1}{3} u^{-2/3} = \frac{1}{3u^{2/3}}$$

$$\frac{du}{dx} = 2x$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{3u^{2/3}} \cdot 2x$$

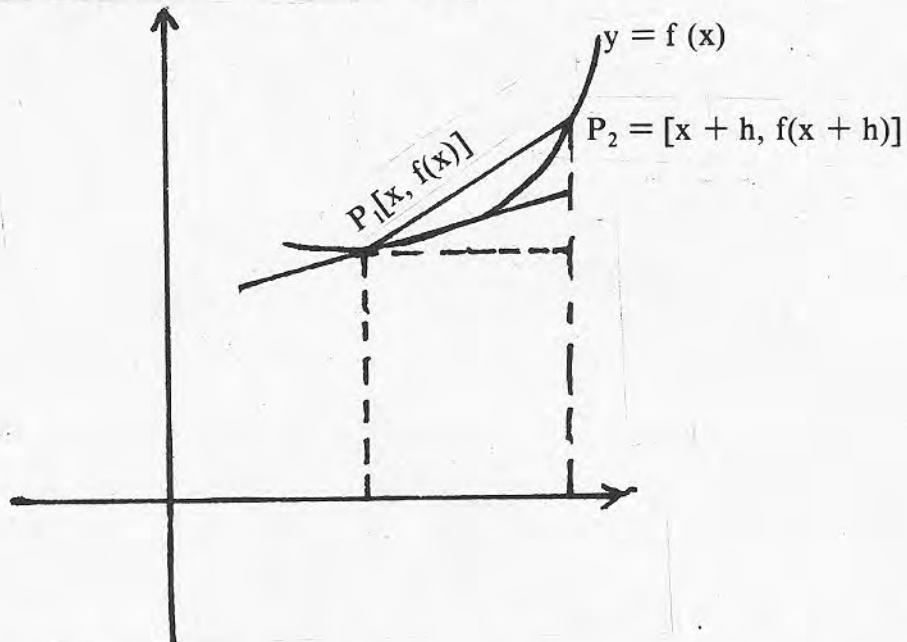
$$= \frac{1}{3\sqrt[3]{(x^2 + 3)^2}} \cdot 2x$$

$$= \frac{1}{\sqrt[3]{x^4 + 6x^2 + 9}} \cdot 2x$$

$$= \frac{2x}{\sqrt[3]{x^4 + 6x^2 + 9}}$$

MACNAHA JOOMETERIGA AH EE XIGSINTA

Raadi y' adoo tixgelinaya fansaarka $y = f(x)$.



Ka soo qaad in $P_1 [x, f(x)]$ ay tahay bar ku taal xoodka $y = f(x)$, tixgelina bar kale oo u dhow oo ah $P_2 [x+h, f(x+h)]$. Markaa waynu aragnaa in tiirada boqonka $P_1 P_2$

$$= \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$\frac{\Delta x}{\Delta x} \underset{0}{\overset{\text{Xad.}}{\longrightarrow}} \frac{\Delta y}{\Delta x} = \underset{0}{\overset{\text{Xad.}}{\longrightarrow}} \frac{f(x+h) - f(x)}{h} = f'(x) \dots (1)$$

Laakiin. . . (1) waa tiirada taabtaha ee barta P_1 . Sidaas darteed, xigsinta fansaar ee barta kasta waa tiirada xoodka ee bartaa.

T u s a a l e 1:

Raadi tiirada xoodka $y = 3x^3 - 4x^2 + 2x - 1$ halka barta $(1, 11)$.

F u r f u r i s :

$$y' = 9x^2 - 8x + 2$$

Markaa halka barta $(1, 11)$

$$y' = 9(1)^2 - 8(1) + 2 = 3.$$

Sidaas awgeed tiirada halka barta $(1, 11) = 3$.

Tusaale 2:

Raadi kulammada barta ku taal garaafka $y = 5x^2 + 8x - 1$, halka tiirada tahay — 12.

Furfuriis:

$$y' = 10x + 8.$$

Markaynu qaadanno in $\dot{y} = -12$, waxaynu heleyntaa $-12 = 10x + 8$.

$$\therefore x = -2$$

$$\therefore y = 5(-2)^2 + 8(-2) - 1 = 3.$$

\therefore Barta la doonayaan waa $(-2, 3)$.

Layli:

Xig fansaarradan.

1) $y = 3x^2 + 4x - 6$

2) $y = 4x^4 + 3x^3 - 6x^2 - 8$

3) $y = \frac{1}{x^3}$

4) $y = x^2 + 2x - \frac{1}{x}$

5) $y = x^7 + 8x^6 + x^5 + 3x^3 + 15$

6) $y = \frac{1}{\sqrt{x}}$

7) $y = (x+2)(3x^3 - 2x + 7)$

8) $y = 6 - \frac{1}{x^4}$

9) $y = 2x^2 + \frac{1}{x}\sqrt{x}$

10) $y = x^{3/2}$

11) Mid kastaba fansaarradan, raadi xigsinta halka baraha xusan:

b. $y = 3x^3 - 6x + 2$; halka $x = \frac{1}{2}$; $x = -1$

t. $y = x^2 - \frac{1}{x} + 6x$, $x = 2$, $x = -\frac{1}{2}$

j. $y = x^3 + 5x^2 - 6x + 3$; $x = 2$; $x = -\frac{1}{2}$

kh. $y = 3x - \frac{3}{x^4}$; $x = -1$; $x = \frac{1}{2}$

d. $y = (x-2)(x^2 - 4)$; $x = 0$

r. $y = 2x^3 - 4x^2 - 7x + 2$, $x = -1$, $x = 0$.

12) Raadi kulammada barta ku taal xoodka, $y = 3x^2 - 2x + 4$, halka tiirada tahay.

13) Xisaabi tiirada xoodka $y = x^3 + 4x - 6$, halka baraha $(1, -1)$, $(-2, -22)$, $(-1, -3)$.

14) Raadi kulammada baraha ku yaal xoodka $y = x^3 - 2x^2 +$ halka taabtuuhu, dhidibka — x kala sameeyo xagal 45° ah.

15) Raadi kulammada baraha ku yaal garaafka $y = 2x^2 + 6x - 8$, halka tiiradu le'eg tahay 8.

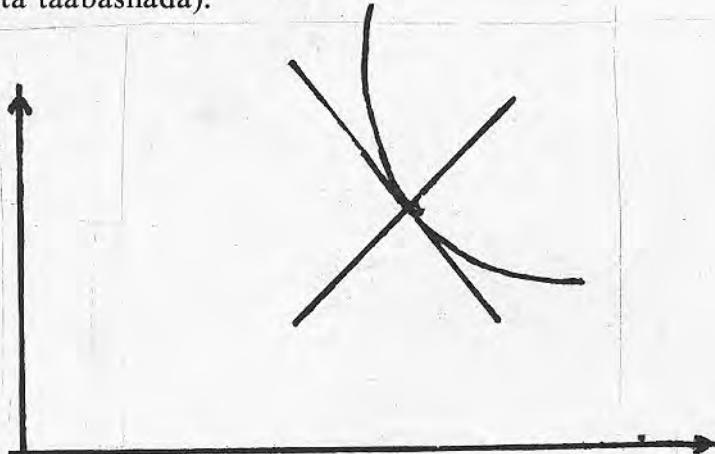
- 16) Raadi kulammada barta ku yaal garaafka $y = 3x^3 - 6x^2 + x - 2$, halka tiiradu tahay — 2.

M I D I I D S I

Taabte iyo Liganeyaal

Q e e x o :

Liganaha xood waa xarriiqada toosan ee kaga qotonta taabtaha xoodka halka barta P (barta P-na waa barta taabashada).



Shax 32

Xigsinta ayaa si fudud loo adeegsan karaa marka la raadinaayo isle'egta Ligane xoog. Waxaynu ognahay, haddii $f(x)$ ay tahay garaafka xood, in markaas $f'(x)$ tahay tiirada taabtaha. Marse haddii liganuhu ku qotomo taabtaha, tiiradiisu waa rogaalka taban ee taabtaha. Markaa tiirada liganuhu waa $-\frac{1}{f'(x)}$.

T u s a a l e 1:

Raadi isle'egyada taabtaha iyo liganaha ee xoodka $y = 3x^2 - 5x + 7$, halka (2, 9).

F u r f u r i s :

$$y' = 6x - 5.$$

$$\text{Halka } (2, 9) \quad y' = 6(2) - 5 = 7.$$

$$\text{Tiirada taabte} = 7.$$

\therefore Isle'egta taabtuuhu waa

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 9 &= 7(x - 2) \\ y &= 7x - 5 \end{aligned}$$

$$\text{Tiirada liganuhu} = -\frac{1}{7}$$

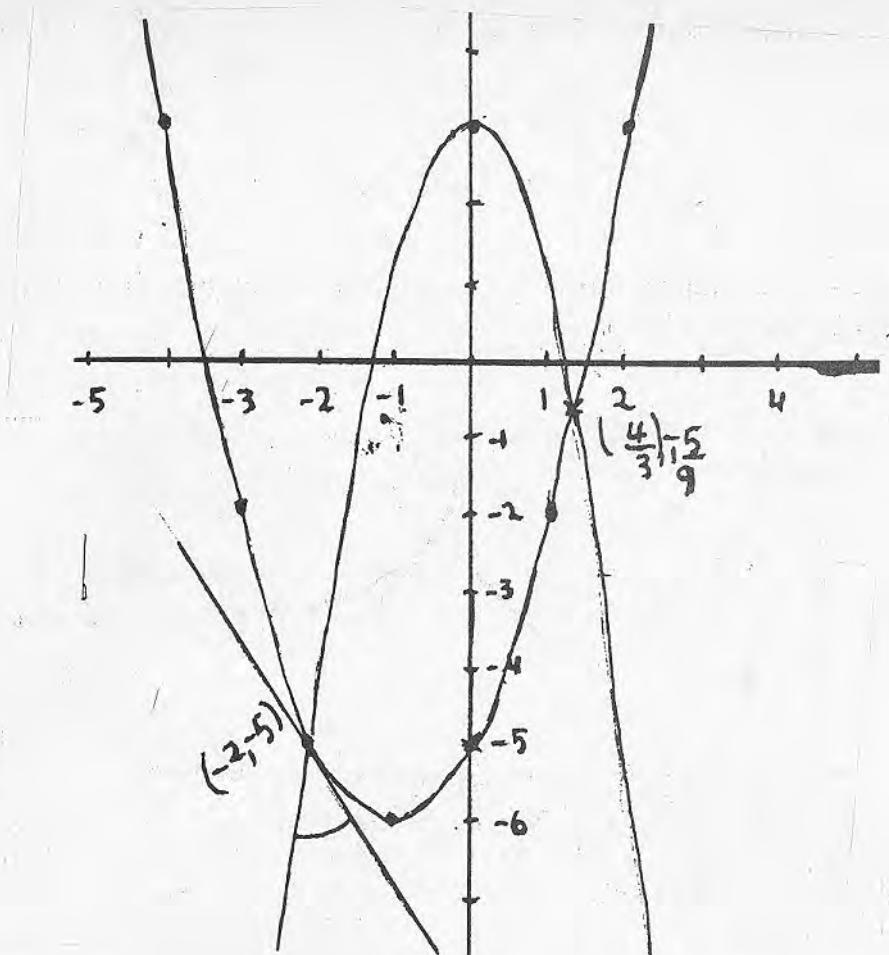
\therefore Isle'egta liganuhu waa

$$\begin{aligned} y - y_0 &= -\frac{1}{7}(x - x_0) \\ y - 9 &= -\frac{1}{7}(x - 2) \end{aligned}$$

$$\therefore 7y + x - 65 = 0.$$

T u s a a l e 2:

Xoodadka $y = 3 - 2x^2$ iyo $y = x^2 + 2x - 5$ ayaa iska gooya halka $(-2, -5)$, iyo $\left\{\frac{4}{3}, -\frac{5}{9}\right\}$. Raadi isle'gyada taabtayaasha halka $(-2, -5)$. Intay bay le'eg tahay xagasha u dhexeysaan taabtayaasha?



Shax | 33

Fur furiis:

Tiirada xoodka $y = 3 - 2x^2$ waa $y' = -4x = 8$ halka $x = 2$. Tiirada xoodka $y = x^2 + 2x - 5$ waa $y' = 2x + 2 = -2$ halka $x = -2$.

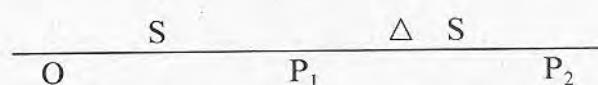
Taabtaha T_1 ee xoodka $y = 3 - x^2$ halka $(-2, -5)$ isle'egtiisu waa $y + 5 = 8(x + 2)$.

Taabtaha T_2 ee xoodka $y = x^2 + 2x - 5$ isle'egtiisu waa $y + 5 = -2(x - 2)$.

$$\begin{aligned} \tan \phi &= \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{-2 - 8}{1 + (-2)(8)} = \frac{-10}{-15} \\ &= 0.6666 \\ \therefore \phi &= \tan^{-1} 0.6666 = 33^\circ 50' \end{aligned}$$

KAYNAAN IYO KARAAR

Ka soo qaad in saxar soconayaa jid toosan haysto. Markii saxarku uu soo dhaafay barta 0 hadday ka dambeysay t seken wuxuu joogaa barta P_1 hadday ka dambeysay Δ t sekenna wuxuu joogaa barta P_2 .



Socodka dhammaantii waxaa lagu tibaaxi karaa foganta la jaray iyo amminta ka soo wareegtay. Ka soo qaad in foganta S ay tahay fansaar ammineed, markaa $S = f(t)$.

$$\therefore \Delta s = f(t + \Delta t) - f(t)$$

$$\therefore \text{Celceliska kaynaan } \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

Kaynaanka ammin kasta (t) waxaa laga helaa:

$$V = \frac{X_{ad}}{\Delta t} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$$
$$\therefore V = \frac{ds}{dt} = f'(t).$$

Sidaas oo kale, waxaynu oran karnaa, Karaarka saxar waa: $a = \frac{dv}{dt}$.

Tusaale 3:

Socodka kubbad sare loo tuuray, oo kaynaan bilowgeedu yahay 128 m/sek. ayaa lagu siiyay in fogaanta ay jarayso, S lagu helo $S = 128t - 16t^2$.

- 1) Goormaa kaynaanku yahay eber?
- 2) Waa intee kaynaanku 2 seken kaddib?
- 3) Waa intee karaarku 2 seken kaddib?

Fururis:

Kaynaanku ammin kasta (t) waa:

$$1) V = \frac{ds}{dt} = 128 - 32t.$$

Haddii $V = 0$, markaas $128 - 32t = 0$.

$$\therefore t = \frac{128}{32} = 4 \text{ seken.}$$

Markaas kaynaanku waa eber 4 seken ka bacdi.

- 1) Adigoo isle'egta $V = \frac{ds}{dt} = 128 - 32t$, t ku beddelaya 2 kaynaanku waa $128 - 64 = 64 \text{ m/sek.}$
- 3) Waxaynu haysanna $V = 128 - 32t$. Markaa karaarku waa $a = \frac{dv}{dt} = -32$. Mar haddii karaarku ammin kasta yahay -32 m/sek.^2 seken ka bacdina waa -32 m/sek.^2

Layli:

- 1) Raadi isle'egta taabtaha xoodka kasta halka barta xusan.
 - b. $y = x^3$, halka $x = 2$
 - t. $y = 3x^2 - 2x - 8$; $x = -1$
 - j. $y = \frac{4x^3}{3}$; halka $x = \frac{3}{2}$.
 - x. $y = 7x^2 + 6x - 2$; $x = -\frac{1}{2}$
 - kh. $128x - 16x^2$; $x = 6$
 - d. $y = 6x^2 - 2x^2$; $x = 1/2, 0, 1$.
- 2) Raadi isle'egta liganaha xood kasta halka barta xusan.

b. $y = 2x - \frac{x}{2}; x = 1$

t. $y = -\frac{x^3}{3}; x = 2$

j. $y = \frac{9x^4}{16}; x = \frac{4}{3}$

kh. $2x^3 - 9x^2 + 12x - 3; x = \frac{3}{2}$

d. $y = \frac{x^4}{3}; x = \frac{3}{2}$

r. $y = 3x^5 - 20x^2 + 4; x = 2$

- 3) Xoodkan hoos ku xusan, labadii isku lammaanba raadi is-goyskooda; ka bacdina raadi in xoodadku iskaga qotomaan halka barta is-goynta. (**Ogow:** laba xood waxay bar iskaga qotomaan, haddii taabtayaashoodu halka bartaa iskaga qotomaan).

b. $y = x^2; y = x^2 - 2x + 1$

t. $y = 3x^2 + 2x - 1; y = 3x^2 - 4x + 2$

j. $y = \frac{1}{3}x^2; y = \frac{3}{2} - x - x^2$

kh. $y = x^2; y = 2 - x^4$

d. $y = x^2; y = 2x^4$

r. $y = \frac{1}{4}x^2; y = 3 - \frac{x^2}{8}$

- 4) Saxar jid toosan haysta, ayaa lagu siiyey in fogaantiisa $S = t^2 - 2t + 5$. Samee garaafka fogaanta amminta min marka $t = 0$ ilaa marka $t = 4$; dabadeedna adoo cabbiraya tiirada, raadi kaynaanka:

i) Hal seken ka bacdi;

ii) 2 seken ka bacdi. Jadeeyada ku sii hubi xigid.

- 5) Saxar jid toosan haysta, ayaa lagu siiyey in fogaantiisu $S = 6t - 3$. Samee garaafka fogaan ammin, tusna in kaynaanku madoorsoome yahay. Jadeeyada ku sii hubi xigid.

- 6) Kaynaanka saxar jid toosan haysta waa: $V = 4t - 12$. Samee garaafka kaynaan, ammin tusna in karaarku madoorsoome yahay.

- 7) Kaynaanka saxar jid toosan haysta ayaa lagu siiyey in uu yahay $V = 4 - t^2$. Samee garaaf kaynaan-ammin, min marka $t = 0$ ilaa $t = 4$, adigoo sawiraya taabtaha raadi karaarka:

i) Hal seken ka bacdi;

ii) 2 seken ka bacdi.

- 8) Iyadoo $S = 128 - 16t^2$, ayaa saxar t sekenba socdaa "S" mitir.

i) Waa intee kaynaan bilowgu

ii) Goormuu nasasho biriweed ku jiraa?

iii) Waa intee karaarku 3 seken ka bacdi?

(S)

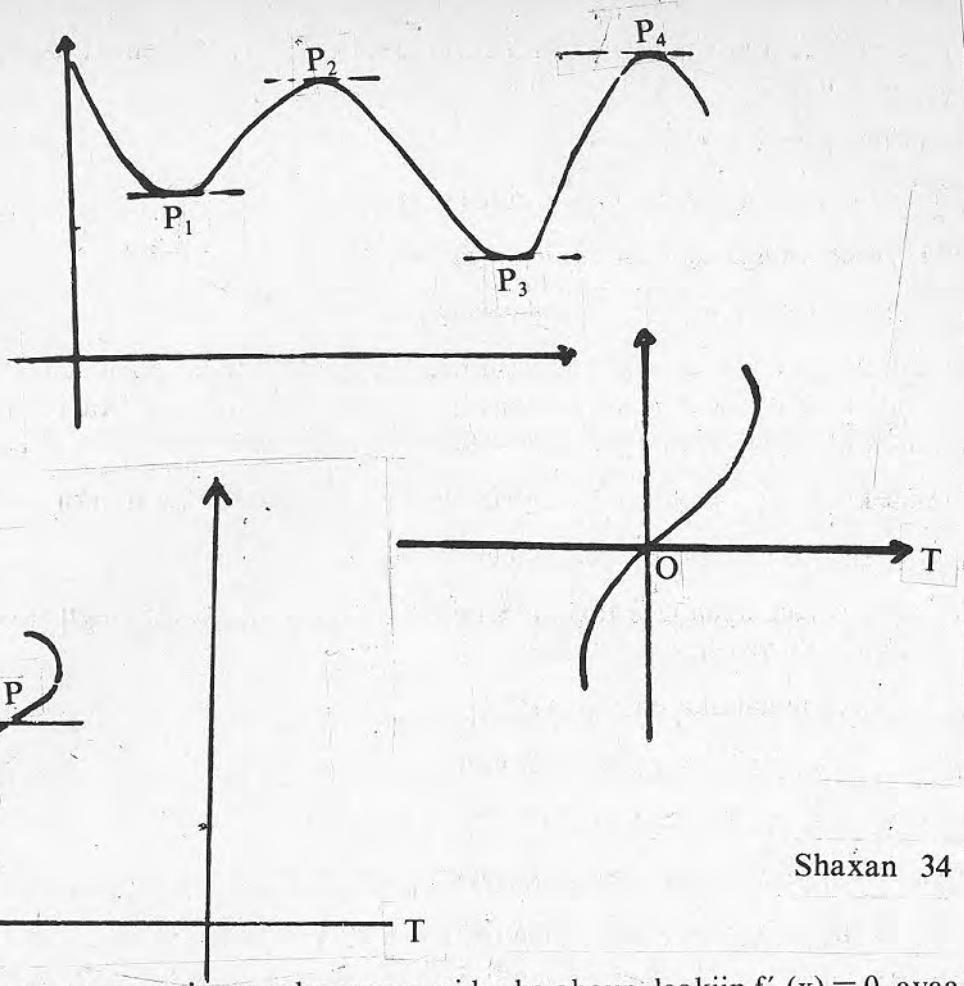
- 9) Kubbud baa la cirbixiyey. Fogaanta S ee ay sare u baxday t seken ka bacdi, ayaa lagu siiyey in ay tahay $S = -4.9t^2 + 196t$.
- Waa intee kaynaan bilowgu?
 - Goormay kubbaddu nasasho biriweed ku jirtaa?
 - Waa intee joogga ugu dheer ee ay gaartay?
 - Goormay dhulka ku soo noqonaysaa?
- 10) Saqaf cirifkii, ayaa kubbud sare looga tuuray, si ay hadhow ugu soo dhacdo dhulka, oo u jira 112 m. Hadday socoto, oo markaas inta ay ka fog tahay saqafka ay tahay $S = 96t - 16t^2$ raadi:
- Halka kubbaddu joogto kaynaankeeda iyo jihada socodka marka $t = 2$.
 - Kaynaankeeda marka ay ku dhacdo dhulka.
- 11) Saxar toos u socda, ayaa jara fogaan S mitir ah t sekenba. Haddii lagu siiyey in $S = t^3 - 9t^2 + 24t - 18$, raadi:
- Goormaa kaynaanka eber yahay?
 - Fogaanta ay jartay waa intee amminta ku xusan (i)?
 - Goormaa karaarku eber yahay?
 - Waa intee kaynaanku marka karaarku eber yahay?
- 12) T seken kaddib ayaa saxar socday S mitir iyadoo $S = t^3 - 6t^2 + 9t + 5$. Immisa seken ka dib ayaa kaynaanku eber yahay? Raadi kaynaanka hanhaankiisa iyo karaarkiisa:
- Afar (4) seken ka bacdi.
 - Marka bilowga.
- 13) Socda lagu siiyey in $S = 64t^2 - 16t^4$, raadi:
- Goorta kaynaanku eber yahay?
 - Fogaan intee le'eg bay socotay marka (i)?
 - Waa intee karaarka marka $t = 0$, marka $t = \Delta 2$?

BARTA UGU SARREYSA IYO TA UGU HOOSEYSA

Ka soo qaad in $y = f(x)$ ay tahay fansaarka xoodka ka muuqda shaxanka. Waxaynu aragnaa in taabitayaasha barta P_1, P_2, P_3 iyo P_4 ay giddigood yihiin xarriiqyo jiifa. Sidaa awgeed, tiirooyinka barahaasi waa eber. Barahaas oo kale waxaa la yiraa **Baro qiiraqiir**. Barta P_3 waa ta ugu hooseysa waxana loogu yeeraa ugu hooseyso. Barta P_4 waa ta ugu sarreysa iyadana waxa loogu yeeraa ugu sarreysa.

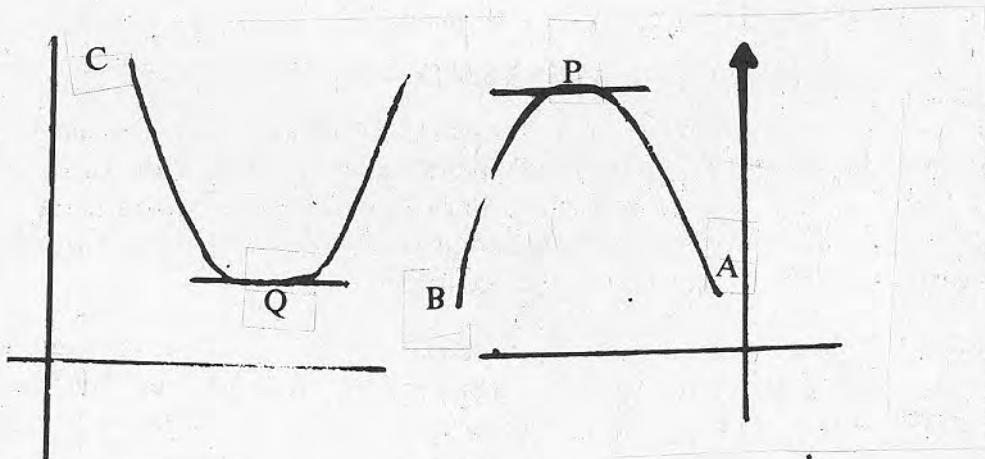
Haddaba, bar waxa ay ugu sarreysa tahay marka qimaha ugu sarreysa (ugu weyn) ee $f(x)$ laga helo ay tahay marka loo eego baraha agagaarkeeda ah. Sidaas oo kale bar waxa ay ugu hooseyso tahay haddii qimahaedu ugu hooseyso (ugu yar tahay) iyadoo loo eegayo baraha agagaarkeeda ah.

Waxa aynu aragnay, haddii fansaar ugu sarreysa ama ugu hooseyso leeyahay in markaa tiirada xoodku halka barahaasi yihiin uu eber yahay, t.a., $f'(x) = 0$; markaas halka ugu sarreysada ama ugu hoosayso $f'(x) = 0$. Ugama jeedno haddii $f'(x) = 0$. Isla halkaas ayay $f(x)$ ku leedahay ugu sarreysa ama ugu hooseyso. (Fiiri shaxanka 34).



Shaxan 34

Bar aan ugu sarreysyo iyo ugu hooseyso midnaba ahayn, laakiin $f'(x) = 0$, ayaa la yiraa **Baraha Dalac-rogga**. (Tusaalin: baraha 0 iyo P ee shaxan 34 . Si loo ogaado in bar qiiraqir ($f'(x) = 0$) u taagan tahay ugu sarreysyo ama ugu hoosayso, waxaanu derisnaa **dhaqmaadka** tiirada ee agagaarka bartaa.



Shax 35

Markaynu shaxanka 35 fiirinno, waxaynu aragnaa in tiirada xoodka togan yahay inta u dhexeyso A iyo P. Laakiin, ay tartiib u soo yaraanayso ilaa ay noqoto eber halka barta P. Markaas haddana way sii yaraanaysaa oo tiiradu tabnaaneysaa inta u dhexeyso P iyo B. Ugu sarrayso ayaa ka sameynta halka P. Sidaas oo kale, inta u dhexeyso C iyo Q, tiirada way taban tahay, laakiin, tartiib-tartiib ayay u kordhaysaa ilaa ay noqoto eber halka barta Q. Inta ka dambeyso Q, way togan tahay oo ugu hooseyso ayaa ka sameysanta. Habkaas, sida soo socota ayeynu ku soo gabagabaynaynaa. Haddii lagu siiyo fansarka $y = f(x)$; barta C waxay ugu sarreysuo u taagan tahay haddii:

- i) $f'(C) = 0$
- ii) $f'(x)$ Summaddiisu isu beddesho min tognaan ilaa tabnaan t.a., $f'(x) > 0$ dhanka bidix ee C; dhanka midig ee C-na $f'(x) < 0$. Sidaas oo kale C waxay ugu hooseyso u taagan tahay haddii:
- b. $f'(C) = 0$
- t. $f'(x)$ Summaddiisu isu beddesho min tabnaan ilaa tognaan t.a., $f'(x) < 0$; dhanka bidix ee C; dhanka midig ee C-na $f'(x) > 0$.

Arrintaasu waxay isugu biyo-dhuranaysaa:

Xood $y = f'(x)$ wuxuu barahiisa mid ka ah $x = x_0$ ku leeyahay bar dalac-roog:

- i) Haddii $f'(x) = 0$ ama aanay qeexnayn.
- ii) Haddii $f'(x)$ summaddiisu beddelanto, marka korodh x ku dhaco ilaa ay $x = x_0$. Run ahaantii shardiga (ii) wuxuu la macna yahay $f'(x) \neq 0$.

T u s a a l e 1:

Raadi qiimaha ugu sarreysada ama ugu hooseysada $y = x^2 - x - 6$.

F u r f u r i s :

$$F'(x) = 2x - 1.$$

$$F'(x) = 2x - 1. Lekaysii f'(x) eber t.a., f'(x) = 0. Markaas 2x - 1 = 0. x = \frac{1}{2}.$$

Waxaa muuqata in hal bar qiirqiir jirto. Taasoo ah $x = \frac{1}{2}$.

Bidixda marka $x = \frac{1}{2}$ (t.a., $x < \frac{1}{2}$), $f'(x) < 0$.

Tusaalin: ka dhig in $x = \frac{1}{4}$. Midigta marka $x = \frac{1}{2}$ (t.a., $x > \frac{1}{2}$), $f'(x) > 0$.

Tusaalin: ka dhig in $x = 1$.) Markaas $f'(x)$ summaddiisu waxay u beddelmeysaa min tabnaan ilaa tognaan.

$$\therefore x = \frac{1}{2}, \text{ waxay u taagan tahay ugu hooseyso, taasina waa}$$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 6 = -\frac{25}{4} = -6\frac{1}{4}$$

T u s a a l e 2:

Raadi qiimaha ugu sarreysada iyo ugu hooseysada ee $f(x) = x^3 - 3x^2 - 9x + 6$.

F u r f u r i s :

$$F'(x) = 3x^2 - 6x - 9, \text{ marka } f'(x) = 0 \text{ waxaynu heleyntaa:}$$

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3, -1.$$

Markaa, baaro qiiroo kii waxay noqonaysaa kuwan.

Haddii $x < 3$, markaa $(x - 3)(x + 1) < 0$.

Haddii $x > 3$, markaa $(x - 3)(x + 1) > 0$.

∴ $F'(x)$ summadiisu waxay u beddelmaysaa min tabnaan ilaa tognaan.

∴ $x = 3$ waa ugu hooseyso, taasina waa

$$f(3) = (3)^2 - 3(3)^2 - 9(3) + 6 = -21(3, -21).$$

Haddii $x < -1$, markaa $(x - 3)(x + 1) > 0$

Haddii $x > -1$, markaa $(x - 3)(x + 1) < 0$.

∴ $F'(x)$ summadiisu waxay u beddelmaysaa min tognaan ilaa tabnaan. Marka $x = -1$ waxay u taagan tahay ugu sarreysyo, taasina waa

$$f(-1) = (-1)^2 - 3(-1)^2 - 9(-1) + 6 = 11, (-1, 11).$$

Tusaale 3:

Tijaabi in $y = x^5$ ay ugu sarreysyo leedahay iyo in ay ugu hooseyso leedahay.

Furfuris:

$y' = 5x^4$; waxaanay eber tahay haddii iyo haddii qura oo $x = 0$; markaas waxaynu aragnaa:

i) Haddii $x > 0, y' > 0$.

ii) Haddii $x < 0, y' > 0$.

Sidaas darteed, xoodka summadiisu isma beddesho. Markaas $x = 0$ ugu sarreysana ma aha ugu hooseysana ma aha; waxayse tahay bar dalac-rog.

Tusaale 4:

Dhululubo labada afba ka xiran ama ka oodan, ayaa muggeedu yahay 539 sm^3 . Raadi gacanka salka, marka dhammaan bed dibadeedkeeda dul uu ugu yar yahay.

Furfuris:

$$\text{Muugga, } V = \pi r^2 h = 539.$$

$$\text{Bed duleedka, } S = 2\pi rh + 2\pi r^2.$$

$$\therefore h = \frac{639}{\pi r^2}$$

$$\therefore S = 2\pi r \left(\frac{539}{\pi r^2} \right) + 2\pi r^2 = \frac{539}{r} + 2\pi r^2$$

$$S' = -\frac{539}{r^2} + 4\pi r = \frac{4\pi r^3 - 539}{r^2}$$

$$\therefore \frac{4\pi r^3 - 539}{r^2} = 0$$

$$\therefore r^3 = \frac{539}{4\pi}$$

$$\therefore r = \sqrt[3]{\frac{539}{4\pi}} = \sqrt[3]{\frac{539 \times 7}{4 \times 22}} = \sqrt[3]{\frac{49 \times 7}{8}}$$

$$= \sqrt[3]{\frac{7^3}{2^3}} = \frac{7}{2}.$$

Tusale 5:

Dhammaan tirooyinka lammaanayaal ah, ee faraqoodu yahay 64, raadi lammaanaha tarankiisu u yar yahay.

Furfuris:

Ka soo qaad in tirooyinku yihin x iyo y .
 Markaa $x - y = 64$, ama $x = 64 + y$.
 Taranka, $P = xy = y(64 + y) = y^2 + 64y$.

$$\therefore P' = 2y + 64 = 0$$

$$y = -32$$

$$x = 64 - 32 = 32.$$

\therefore Markaas lammaanuhu waa $(32, -32)$.

Layli:

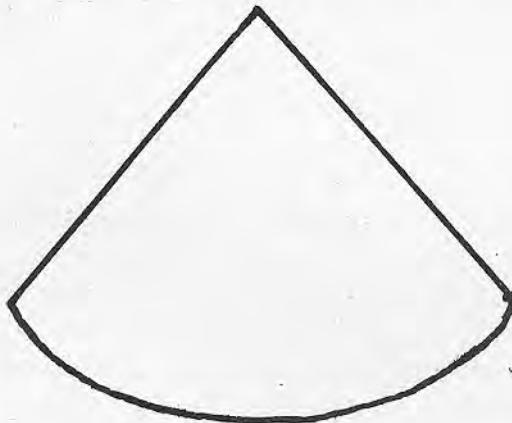
Raadi qiimaha ugu sarreysada iyo qiimaha ugu hooseysada ee fansaarkan:

- 1) $y = 2x^2 + 2x - 12$
- 2) $y = x^2 + \frac{2}{x}$
- 3) $y = 4 - 12x - 32x^2$
- 4) $y = x^3 - 6x^2 + 9x - 6$
- 5) $y = x^4 - 4x^4$
- 6) $y = \frac{x^2 - 128}{x}$
- 7) $y = 2x^3 + 3x^2 - 36x - 6$
- 8) $y = x^3 + x^2 - 4x - 4$.

Raadi baro qirkiirka xoodkan, xisaabina tiirada barahaasi.

- b. $y = x^3 - x$
- t. $y = \frac{x^3}{3} + x^2 - 15x + 16$
- j. $y = x^2 - \frac{x^4}{100}$
- x. $y = x^2 - 3x + 15$.
- 10) Dhammaan tirooyinka lammaaneyaa ah, ee wadartoodu tahay 64, raadi lammaanaha tarankiisu ugu weyn yahay.

- 11) Waqarta labo tiro oo togan waa 21. Raadi labadaa tiro, haddii midkooda iyo ka kale labajibbaarkii, tarankoodu intii la doono weynaan karo ama badan karo.
- 12) Raadi tiro togan, oo faraqa laba-jibbaarkeeda iyo afar laabka laba-jibbaarka rogaal-keedu intii la doono yar yahay.
- 13) Laydi baa bedkiisu yahay 46 sm^2 . Raadi addimihiisa haddii fogaanta u dhexeysa geesaha midkood, iyo bartamaha dhinaca aan deriskiisa ahayn la doonayo in ay ugu yaraato.
- 14) Raadi gacan salka dhululubo-goobo malaasan. Haddii dhammaan bed-duleedkeedu yahay $24\pi \text{ sm}^2$, mugguna ugu weyn yahay (Fiiro: dhammaan bed dulku = $2\pi rh + 2\pi r^2$). Waa intee mugga ugu badani (ha ka saarin jawaabta π).
- 15) Fogaanta kubbad laga cirbixiyey meel u jirta dhulka 64 m. oo kaynaan bilowgeedu ahaa 128 seken, ayaa waxa ina siyya $S = -16t^2 + 128t + 64$
 - i) Raadi joogga ugu dheer ee ay gaartay.
 - ii) Goorma kubbaddu dhulka ku soo noqonaysaa?
- 16) Taar dhererkiiisu yahay 2 mitir ayaa loo qalqalloociyey si uu u sameeyo faquuq goobo (Eeg shaxanka 51). Raadi xagasha u dhexeysa gacmo soohdimayaasha marka bedka faquuq ugu weyn yahay.



Shax 36

- 17) Nin beer leh ayaa doonay in uu lo'diisa u seero meel doog leh, oo sansaan laydi ah. Saddex dhinac ayuu kaga wareejiyey taar, dhinacii afraadna gidaarkii ayuu ugu qabtay. Haddii uu haysto taar dhererkiiisu yahay 2400 m., waa intee bedka ugu weyn ee uu oodi karaa?
- 18) Weel dhuuban oo sal goobo ah leh, ayaa la doonayaa in la sameeyo, si uu u qaado 64 m^3 . Raadi addimaha si ay birta loo baahan yahay (bed dhulka), ugu yaraato marka:
 - i) Weelku yahay dhululubo dabool la'.
 - ii) Weelku yahay oodan.
- 19) Salka saddexxagal labaale waa 20 m., jooggiisuna waa 40 m. Raadi addimaha laydigaa ugu weyn ee ku dhexmaraan, haddii geedihiisa laba ahaani ku yaallin saddexxagalka.

A B Y A N

Lidxigsin: Abyanaha aan hubnayn.

Haddii y tahay fansaar x, guud ahaan waxaynu aragnay, in aynu heli karno fansaar kale Z oo ah xigsinta y.

T u s a a l e :

$$\text{Haddii } y = x^3, \text{ markaas } Z = \frac{dy}{dx} = 3x^2$$

Imminka, waa inaynu tixgelinnaa arrintii aynu hore uga hadallay taas oo ah: haddii lagu siiyey xigsinta fansaar, raadi fansaarka.

T u s a a l e :

Waynu ognahay in keynaanka V, ee saxar yahay $V = \frac{ds}{dt}$, waxaana layna su'aali karaa fogaanta. Waxaa layna siin karaa karaarka, oo kaynaanka doona layna oran karaa.

Guud ahaan, haddii $f(x)$ iyo $g(x)$ ay yihiin laba fansaar, oo $f(x) = g(x)$ markaas $f(x)$ waxa la yiraa **Lidxigsinta** $g(x)$.

T u s a a l o o y i n :

$$\text{i) } \frac{dy}{dx} = 5x^2, \quad y = \frac{5}{3} x^3$$

$$\text{ii) } \frac{dy}{dx} = 7x^4, \quad y = \frac{7}{5} x^5$$

$$\text{iii) } \frac{dy}{dx} = 3x^{-3}, \quad y = -\frac{3}{2} x^{-2}$$

$$\text{iv) } \frac{dy}{dx} = 4x^{3/2}, \quad y = \frac{8}{5} x^{5/2}.$$

Tusaalooyinkaas oo dhan, xigan baa layna siiyey mar kastana waxaa la helay fansaarkii asalka ama unugta. Fansaarrada la helay ayaa la yiraa **lidxigsin**. Wuxuu qofaa jirta in fansaarrada: x^3 , $x^3 + 5$, $x^3 + 7$, $x^3 - 3$ ay dhammaantood xigsintoodu tahay $3x^2$. Marka, halkaa waxaa ka muuqata, haddii aynu ognahay fansaar xigsintiisa, hubaal ahaan ma sheegi karno fansaarka laf ahaantiisa. Waxaynuse ognahay, in uu yahay fansaar x , oo lagu daray madoorsoome. Madoorsoomahaana ma sheegi karro wuxuu yahay, ilaa war dheeraad ah la innaga siiyo fansaarka. Markaas, afarta tusaale ee kor ku qoran, furfuristoodu waa in ay u qorantaa sidan:

$$\text{i) } y = \frac{5}{3} x^3 + C$$

$$\text{ii) } y = \frac{7}{5} x^5 + C$$

$$\text{iii) } y = -\frac{3}{2} x^{-2} + C$$

$$\text{iv) } y = \frac{8}{5} x^{5/2} + C$$

C waxaa la yiraa **Madoorsoome aan hubnay**. Madoorsoomaha aan hubnayn, waynu ogaan karraa wuxuu yahay, haddii war dheeraad ah aynu ka ognahay fansaarka.

T u s a a l e :

gii) Haddii $\frac{dy}{dx} = 2x^3$; $y = 7$ marka $x = 2$

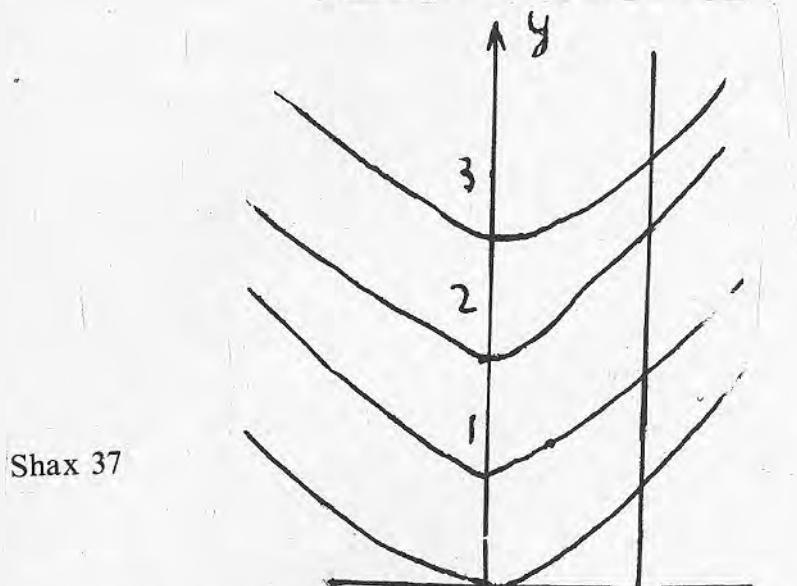
$$\text{Marka } y = \frac{1}{2} x^4 + C$$

$$\therefore 7 = 8 + C$$

$$\therefore C = -1$$

$$\text{t.a. } y = \frac{1}{2} x^4 - 1$$

Tusaale Joometeri



Shaxanku wuxuu muujinayaa garaafyada:

$$y = x^2, \quad y = x^2 + 1, \quad y = x^2 + 2, \quad y = x^2 + 3.$$

Garaafyadaa mid kastaba tiiradiisu waa $2x$. Marka, haddii layna yiraa tiiradu waa $2x$, waxaynu ogaan karraa oo keliya waa sansaanka guud ee xoodka, laakiinse kooxda xood-adka ah ee kor ku sawirami mid uu yahay ma garan karro. Laakiin, haddii iyana aynu ognahay in $y = 3$ marka $x = 0$; markiiba waynu sheegi karaa in xoodka gaarka ah ee loo jeedaa uu leeyahay isle'egta $y = x^2 + 3$.

Q O R M O

Habka dib-u-celinta xigidda waxaa la yiraa **Abyan**, fansaarka cusub ee ka soo baxana waxaa la yiraa **Abyanaha fansaarkii hore**. Marka, abyanaaha $3x^2$ waa $x^3 + C$.

Haddii $\frac{dy}{dx} = 3x^2$; marka tallaaboo yinkii xigidda dib loo celiyo waxaynu heleynaa

in $\frac{dy}{dx} = 3x^2$, (dy iyo dx waa xaddiyo yaryar), t.a. $dy = 3x^2 dx$. Taas ayaa inoo geyneysa

Qormo cusub oo loo qoro:

i) $dy = \int 3x^2 dx$, loona akhriyo: «y waa abyanaaha $3x^2$ marka loo eego x».

Ogow: Qormada (i) waa si kale oo loo qoray $\frac{dy}{dx} = 3x^2$.

Astada « \int » waxaa la yiraa **Summadda abyanaaha**.

$\int f(x) dx$, waxaa la yiraa **abyanaaha aan hubnayn**, waayo, waxaa furfurista ku jira madoorsoome aan hubnayn.

T u s a a l e v:

$$\text{Haddii } \frac{dy}{dx} = x^2 \text{ markaas}$$

$$y = \int x^2 dx = \frac{x^3}{3} + C.$$

T u s a a l e vi:

$$\text{Haddii } \frac{dy}{dx} = x^3 \text{ markaas}$$

$$y = \int x^3 dx = \frac{x^4}{4} + C$$

T u s a a l e vii:

$$\text{Haddii } dy = x^5, \text{ markaas}$$

$$y = \int x^5 dx = \frac{x^6}{6} + C$$

$$\text{Guud ahaan, haddii } \frac{dy}{dx} = x^n, \text{ markaas}$$

$$y = \int x^n dx = \frac{x^{n+1}}{n+1} + C.$$

T u s a a l e viii:

$$\int 4x^5 dx = 4 \int x^5 dx = -\frac{4x^6}{6} + C = \frac{2}{3} x^6 + C.$$

$$\text{Ogow: } dx = \int 1. dx = x + C.$$

Mar haddii aynu ognahay in xigsinta wadari le'eg tahay wadarta xigsinta, isla markaas, waxa run ah in Abyanaha wadari le'eg yahay wadarta abyanyaasha; waayo, abyantu waa rogga xigid.

Sidaas oo kale, abyanaaha, madoorsoome lagu dhuftay fansaar, wuxuu la mid yahay taranka madoorsoomaha iyo abyanaaha fansaarka. Sidaas awgeed, jidatkayanu isticmaalla marka laga shaqeynayo abyinta.

$$1) \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1.$$

$$2) \int cf(x) dx = c \int f(x) dx$$

$$3) \int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

T u s a a l e ix:

$$\text{Qiimee } \int (4x^7 - 3x^5 + 2x^3 - 6) dx.$$

$$\int (4x^7 - 3x^5 + 2x^3 - 6) dx = 4 \int x^7 dx - 3 \int x^5 dx + 2 \int x^3 dx - 6 \int 1 dx$$

$$= \frac{4x^8}{8} - \frac{3x^6}{6} + \frac{2x^4}{4} - \boxed{6x} + C.$$

$$= \frac{x^8}{2} - \frac{x^6}{2} + \frac{x^4}{2} - 6x + C.$$

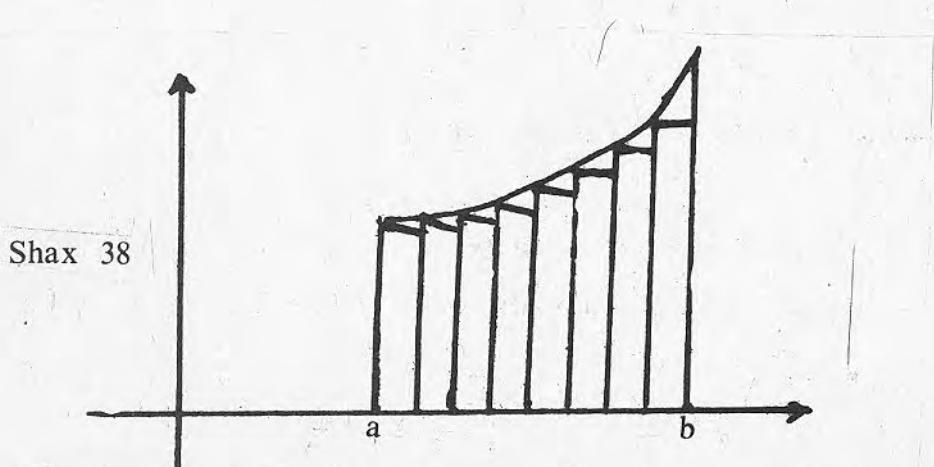
Q i i m e e :

- 1) $\int x^5 dx$
- 2) $\int 4x^3 dx$
- 3) $\int 3x dx$
- 4) $\int \frac{dx}{x^3}$
- 5) $\int \sqrt{x'} dx$
- 6) $\int (5x + 5) dx$
- 7) $\int (80x^{19} - 32x^{15} - 12x^{-3}) dx$
- 8) $\int (5x^6 + 2x^3 - 4x + 3) dx$
- 9) $\int 5(x + 3)^2 dx$
- 10) $\int (x - 1)(x + 2) dx$
- 11) $\int \frac{1}{2}x^9 dx$
- 12) $\int \frac{dx}{\sqrt[3]{x^2}}$
- 13) $\int (1 - x)\sqrt{x} dx$
- 14) $\int (3s + 4)^2 ds$
- 15) $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

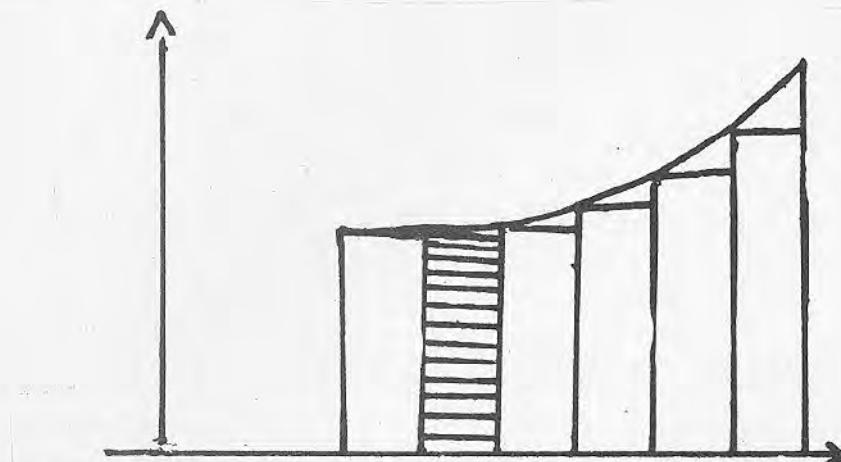
BEDKA KA HOOSEEEYA XOOD IYO ABYANAHAA HUBAN

Q e e x :

Bal ka soo qaad in layna siiyey fansaarka $y = f(x)$, tixgelina garaafka $y = f(x)$ min $x = a$ ilaa $x = b$ (eeg shaxanka 38). Bedka ka hooseeya xoodka $y = f(x)$ ayaa loo qeexaa, in uu yahay bedka ay soo lingaxeen xoodka laftiisa iyo xarriiqda $x = a$ iyo xarriiqda $x = b$.



Bal imminka aan isku dayno, inaynu raadinno bedka ay soo lingaxeen xoodka guud ee $y = f(x)$, xarriiqda $x = a$ iyo $x = b$ (eeg shaxanka 39). Haddaynu ku dhex-sameyno laydiyo is-ballac le'eg, waxaynu aragnaa in bedka ka hooseeyaa $y = f(x)$ uu ka weyn yahay wadarta bededka laydiyada .



Shax 39

Hase yeeshiee, haddii ballaca laydiyadaa, aad iyo aad loo yareeyo, bedka xoodka ka hooseeya, waxa lagu seebi karaa wadarta bededka xoodka $y = f(x)$, dhidibka — x , iyo xarriiqyada $x = a$ iyo $x = b$, ay u qaybsan tahay n-qaybood oo isle'eg ballacuna yahay Δx , iyadoo baraha $a = x_0, x_1, x_2, \dots, x_n = b$, lagu taxayo dhidibka — x , (eeg shaxankan 40).

Markaynnu laydiyo ku dhex-sameyno, kuwaas oo uu ka mid yahay laydi PQRS.

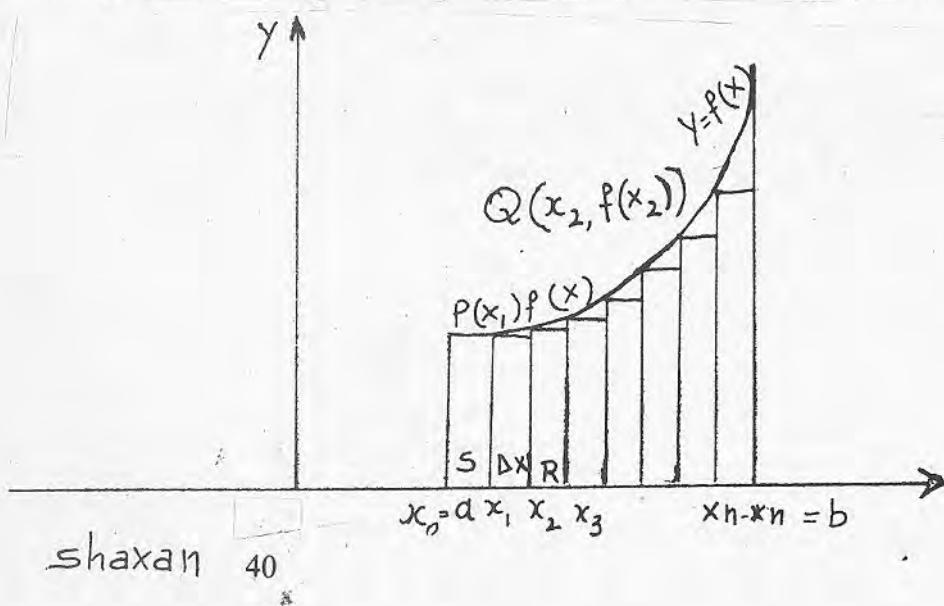
Bedka laydiga $PQRS = f(x_1) \Delta x$.

Wadarta S, ee bedka laydiyadaa waa:

$$S = f(x_0) \Delta x + f(x_1) \Delta x + \dots + f(x_n) \Delta x$$

$$= \sum_{i=0}^n f(x_i) \Delta x.$$

Wadarta bededka laydiyadaasi, way ka yar tahay bedka xoodka ka hooseeya. Hadday-nuse, ballaca laydiyada aad u yarayno (t.a. $\Delta x = 0$), markaas bedka xoodka ka hooseeya,



Shaxan 40

waxaa lagu seebin karaa, wadarta bededka laydiyada. Sidaas awgeed, bedkaa waxaynu u qeexnaa sidan:

$$S = \frac{1}{\Delta x} \sum_{i=0}^n f(x_i) \Delta x \dots \dots (1)$$

Waxaa loo qoraa:

$S = f(x) dx \dots \dots (2)$ (loo akhriyo: Abyanaha huban ee $f(x)$, ee u dhexeeyaa $x = a$ iyo $x = b$).

Tibaaxda (2) waxa la yiraa **Abyanaha Huban**, a waxa la yiraa **Xoodka Hoose**, b-na waxa la yiraa **Xadka Sare**.

T u s a a l e 1:

Raadi bedka ay soo lingaxeen xoodka $y = x^2$, xarriiqyada $x = 0$, iyo $x = 4$.

F u r f u r i s :

Bedka $A = \int_0^4 x^2 dx$. Ugu horreyn waxaynu raadinaynaa abyanaa aan hubnayn. Markaas

$$A = \frac{x^3}{3} + C$$

Laakiin

$$x = 0, A = 0$$

$$\therefore C = 0$$

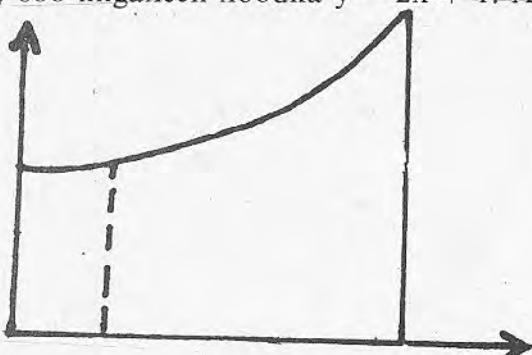
$$A = \frac{x^3}{3}$$

Innaga oo x ku beddeleyna 4, waxaynu heleynta:

$$A = -\frac{64}{3} = 21 \frac{1}{3}$$

T u s a a l e 2:

Raadi bedka ay soo lingaxeen xoodka $y = 2x + 1$. Xarriiqyada $x = 1$ iyo $x = 5$.



Shax 41

F u r f u r i s :

$$A = \int_1^5 (2x + 1) dx$$

Abyanaha aan hubnayn wuxuu noqonayaa:

$$A = x^2 + x + C$$

$$\text{Marka } x = 1, A = 0$$

$$\therefore C = -2$$

$$\text{Markaas } A = x^2 + x - 2$$

Innagoo x ku beddeleyna 5, $A = 28$. Bal hadda arag jadeeyada waynu heli karraa, in madoorsoomaha aan xaalayn, x mar ku beddel 5, marna 1, dabadeedna kala goo, t.a., $(5^2 + 5) - (1^2 + 1) = 30 - 2 = 28$. Tusaalihii horana waa sidoo kale.

G a b a g a b a y n :

$$A = \int_a^b f(x) dx = [f(x)]_a^b = f(b) - f(a).$$

Markaas habka lagu helaa bed iyadoo la isticmaalayo Abyan waa kan:

- 1) Abyi fansaarka, hana xaalayn madoorsoomaha.
- 2) Qiimayaasha x ku beddel xadka sare iyo xadka hoose ee Abyanaha.
- 3) Kala jar qiimayaasha.

T u s a a l e 3:

$$\text{Qiimee } \int_2^4 (x^2 + 2x - 4) dx$$

F u r f u r i s :

$$\begin{aligned} \int_2^4 (x^2 + 2x - 4) dx &= \left[\frac{x^3}{3} + x^2 - 4x \right]_2^4 \\ &= \left\{ \frac{64}{3} + 16 - 16 \right\} - \left[\frac{8}{3} + 4 - 8 \right] \\ &= \frac{64}{3} + \frac{4}{3} = \frac{68}{3} \end{aligned}$$

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