CENG 384 - Signals and Systems for Computer Engineers
Spring 2022
Homework 2

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1. (a) Since the sum of the items entering the adder is equal to the sum of the items leaving the adder we can write the equation as,

$$x(t) + 2\frac{dx(t)}{dt} + 3y(t) - 2\int_{-\infty}^{t} y(\sigma)\, d\sigma = \frac{dy}{dt}$$

İf we take the derivative of both sides,

$$x^{'}(t) - 2x^{''}(t) + 3y^{'}(t) - 2y(t) = y^{''}(t)$$

$$y^{''}(t) - 3y^{'}(t) + 2y(t) = -2x^{''}(t) + x^{'}(t)$$

(b) Assume

$$y_h = Ce^{st}$$

and

$$y_p = Kx(t) = K(e^{-t} + e^{-2t})u(t)$$

Homogeneous solution:

$$y^{''}(t) - 3y^{'}(t) + 2y(t) = 0$$

$$Cs^2e^{st} - 3Cse^{st} + 2Ce^{st} = 0$$

$$Ce^{st}(s^2 - 3s + 2) = 0$$

$$Ce^{st}(s-2)(s-1) = 0 \implies s_1 = 1, s_2 = 2$$

$$\implies y_h = C_1e^t + C_2e^{2t}$$

Particular solution:

$$K(e^{-t} + 4e^{-2t})u(t) - 3K(-e^{-t} - 2e^{-2t})u(t) + 2K(e^{-t} + e^{-2t})u(t) = -2(e^{-t} + 4e^{-2t})u(t) + (-e^{-t} - 2e^{-2t})u(t)$$

$$(K + 3K + 2K)e^{-t} + (4K + 6K + 2K)e^{-2t} = -3e^{-t} - 10e^{-2t}$$

$$6K = -3 \implies K_1 = -1/2$$

$$12K = -10 \implies K_2 = -5/6$$

$$\implies y_p(t) = (-\frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t})u(t)$$

General solution:

$$y(t) = y_h + y_p = (C_1e^t + C_2e^{2t} - \frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t})u(t)$$

Since the system is initially at rest,

$$y(0) = C_1 + C_2 - 1/2 - 5/6 = 0 \implies C_1 + C_2 = 8/6$$

$$y^{'}(0) = C_1 + 2C_2 + 1/2 + 10/6 = 0 \implies C_1 + 2C_2 = -13/6$$

$$\implies C_1 = \frac{29}{6} \quad C_2 = \frac{-7}{2}$$

$$\implies y(t) = (\frac{29}{6}e^t - \frac{7}{2}e^{2t} - \frac{1}{2}e^{-t} - \frac{5}{6}e^{-2t})u(t)$$

2. (a) since system is linear, we can solve in 2 parts :
for $\delta[n-1]$;
since system is time invariant, we can say :

$$\delta[n-1] * h[n] = 2\delta[n+1] - \delta[n]$$

for $3\delta[n+2]$;
since system is time invariant and linear, we can say :

$$\delta[n-1] * h[n] = 3(2\delta[n+4] - \delta[n+3]) = 6\delta[n+1] - 3\delta[n]$$

by adding two solutions :

$$x[n] * h[n] = 6\delta[n+4] - 3\delta[n+3] + 2\delta[n+1] - \delta[n]$$

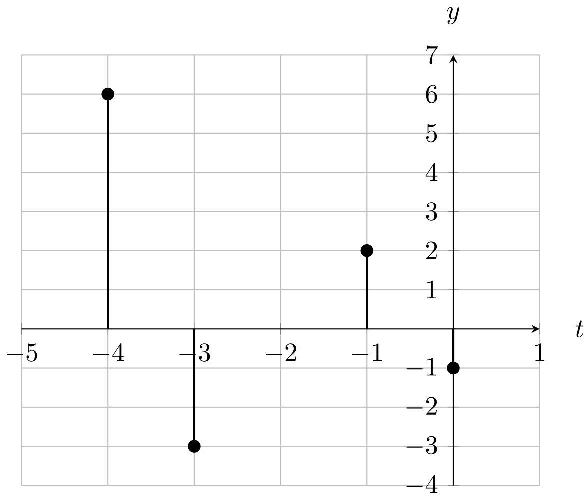


Figure 1: t vs. y.

(b)

$$x[n] = u[n+1] - u[n-2] = \delta[n+1] + \delta[n] + \delta[n-1]$$

using linearity and time invariance :

$$\delta[n] * h[n] = u[n-4] - u[n-6] = \delta[n-4] + \delta[n-5]$$
$$\delta[n+1] * h[n] = \delta[n-3] + \delta[n-4]$$
$$\delta[n-1] * h[n] = \delta[n-5] + \delta[n-6]$$

by adding ;

$$x[n] * h[n] = \delta[n-3] + 2\delta[n-4] + 2\delta[n-5] + \delta[n-6]$$

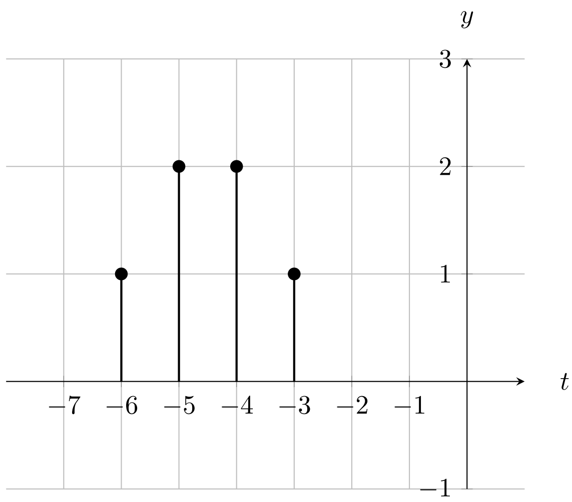


Figure 2: t vs. y.

3. (a)

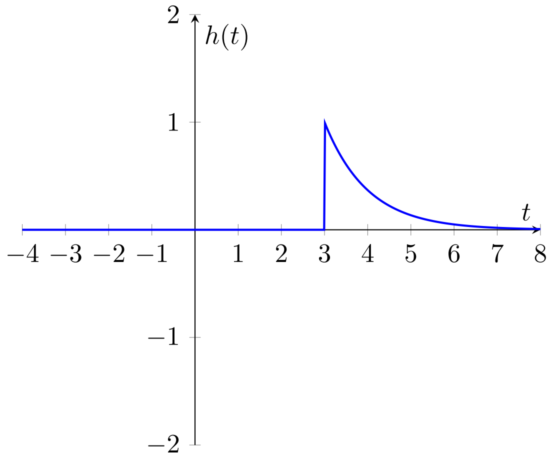
$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(\tau)d\tau = \int_{-\infty}^{\infty} e^{\tau}u(\tau)e^{\frac{-1}{2}(t-\tau)}u(t-\tau)d\tau = \int_{0}^{t} e^{-\frac{1}{2}t}e^{-\frac{1}{2}\tau}d\tau = -2e^{\frac{-1}{2}t}(e^{\frac{-1}{2}t} - 1)$$

(b) using distributive property;

$$x(t)*h(t) = e^{-3t}u(t)*u(t) - e^{-3t}u(t)*u(t-4) = u(t)\int_{0}^{t} e^{-3\tau}d\tau - u(t-4)\int_{0}^{t-4} e^{-3\tau}d\tau = u(t)\frac{e^{-3t}-1}{-3} - u(t-4)\frac{e^{-3t+12}-1}{-3}$$

4. (a)

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)}\delta(\tau - 3)d\tau = e^{t-3}u(t-3)$$



(b)

$$x(t) = \begin{cases} 1 & -2 \leq x \leq 1 \\ 0 & else \end{cases}$$

Then, for $t < 1$;

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau - 3)d\tau = 0$$

Then, for $1 \leq t \leq 4$;

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau - 3)d\tau = \int_{1}^{t} e^{-(t-\tau)}d\tau = 1 - e^{1-t}$$

Then, for $4 < t$;

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 3) d\tau = \int_{1}^{4} e^{-(t-\tau)} d\tau = e^{-t}(e^4 - e)$$

$$\implies y(t) = \begin{cases} e^{-t}(e^4 - e) & t > 41 - e^{1-t} \\ 1 \leq x \leq 4 & \\ 0 & t < 1 \end{cases}$$

5. (a) Assume $\exists A$ s.t.

$$h_1^{-1}[n] - Ah_1^{-1}[n-1] = \delta[n]$$
$$n = 1 \implies h_1^{-1}[1] - Ah_1^{-1}[0] = 0$$
$$\implies A = \frac{1}{2}$$

So there exists such A; and since,

$$(h * h^{-1})[n] = \delta[n] \quad x[n] * \delta[n] = x[n]$$
$$h_1^{-1}[n] * (\delta[n] - \frac{1}{2}\delta[n-1]) = \delta[n]$$
$$\implies h_1[n] = \delta[n] - \frac{1}{2}\delta[n-1]$$
$$h_1[n] * h_1[n] = \delta[n] - \delta[n-1] + \frac{1}{4}\delta[n-2]$$

(b)

$$h = h_0 * h_1 * h_1$$
$$h = h_0 * (\delta[n] - \delta[n-1] + \frac{1}{4}\delta[n-2])$$

By the shifting property,

$$h = h_0[n] - h_0[n-1] + \frac{1}{4}h_0[n-2]$$

Support of $h[n]$ is 0 to 4 so $h_0[n]$ must have support from 0 to some value,

$$h_0[n] = 0 \quad for \quad n < 0$$

$$h[0] = 4 = h_0[0] - h_0[-1] + \frac{1}{4}h_0[-2] \implies h_0[0] = 4$$
$$h[1] = 0 = h_0[1] - h_0[0] + \frac{1}{4}h_0[-1] \implies h_0[1] = 4$$
$$h[2] = 1 = h_0[2] - h_0[1] + \frac{1}{4}h_0[0] \implies h_0[2] = 4$$
$$h[3] = 8 = h_0[3] - h_0[2] + \frac{1}{4}h_0[1] \implies h_0[3] = 11$$
$$h[4] = 1 = h_0[4] - h_0[3] + \frac{1}{4}h_0[2] \implies h_0[4] = 11$$
$$h[5] = 0 = h_0[5] - h_0[4] + \frac{1}{4}h_0[3] \implies h_0[5] = \frac{33}{4}$$
$$h[6] = 0 = h_0[6] - h_0[5] + \frac{1}{4}h_0[4] \implies h_0[6] = \frac{22}{4}$$
$$h[7] = 0 = h_0[7] - h_0[6] + \frac{1}{4}h_0[5] \implies h_0[7] = \frac{55}{16}$$

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(c)

$$y[n] = h_0[n] * x[n]$$

$$y[n] = h_0[n] * (\delta[n] + \delta[n-2])$$

By shifting property,

$$y[n] = h_0[n] + h_0[n-2]$$

So,

$$y[n] = 0 \quad for \quad n < 0$$

$$y[0] = 4$$

$$y[1] = 4$$

$$y[2] = 4 + 4 = 8$$

$$y[3] = 4 + 11 = 15$$

$$y[4] = 4 + 11 = 15$$

$$y[5] = 11 + \frac{33}{4} = \frac{77}{4}$$

$$y[6] = 11 + \frac{22}{4} = \frac{66}{4}$$

$$y[7] = \frac{33}{4} + \frac{55}{16} = \frac{187}{16}$$

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