CENG 384 - Signals and Systems for Computer Engineers
Spring 2022
Homework 4

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June 8, 2022

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1. (a)

$$x(t) + \frac{dx(t)}{dt} - \int_{-\infty}^{t} x(\tau)\,d\tau - \int_{-\infty}^{t} y(\tau)\,d\tau - 2y(t) = \frac{dy(t)}{dt}$$

(b) Let's take the derivative of both sides,

$$\dot{x} + \ddot{x} - x - y - 2\dot{y} = \ddot{y}$$

let,

$$x = e^{jwt} \implies y = H(jw).e^{jwt}$$

$$\dot{x} + \ddot{x} - x = y + 2\dot{y} + \ddot{y}$$

$$\implies jw + (jw)^2 - 1 = H + 2Hjw + H(jw)^2$$

$$\implies jw + (jw)^2 - 1 = H + H(1 + jw)^2$$

$$\implies H(jw) = \frac{(jw)^2 + jw - 1}{(1 + jw)^2} \quad , (w = k.w_0)$$

(c)

$$H(jw) = \frac{(jw)^2 + jw - 1}{(1 + jw)^2} = 1 - \frac{1}{1 + jw} - \frac{1}{(1 + jw)^2}$$

Inverse fourier table from lecture note 10;

$$H(jw) \longleftrightarrow \delta(t) - e^{-t}u(t) - te^{-t}u(t) = h(t)$$

(d)

$$Y(jw) = H(jw).X(jw) = H(jw)(\frac{1}{1 + jw}) = \frac{1}{1 + jw} - \frac{1}{(1 + jw)^2} - \frac{1}{(1 + jw)^3}$$

Inverse fourier;

$$Y(jw) \longleftrightarrow e^{-t}u(t) - te^{-t}u(t) - \frac{t^2}{2}e^{-t}u(t) = y(t)$$

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2. (a)

$$\int_{-\infty}^{t} \dot{h}(\tau)\,d\tau = \int_{-\infty}^{t} \delta(\tau + 1)\,d\tau - \int_{-\infty}^{t} \delta(\tau - 1)\,d\tau$$

$$h(t) = u(t + 1) - u(t - 1)$$

(b)

$$H(jw) = \int_{-\infty}^{\infty} h(t)e^{-jwt}\,dt = \int_{-\infty}^{\infty} u(t + 1)e^{-jwt}\,dt - \int_{-\infty}^{\infty} u(t - 1)e^{-jwt}\,dt$$

$$= e^{jw}.Fourier(u(t)) - e^{-jw}.Fourier(u(t))$$

$$= \frac{e^{jw}}{jw} - \frac{e^{-jw}}{jw}$$

3. (a)

$$h[n] = h_1[n] * h_2[n] \longleftrightarrow H(e^{jw}) = H_1(e^{jw})H_2(e^{jw})$$
$$H(e^{jw}) = (\frac{1}{1 - \frac{e^{-jw}}{2}})^2$$

(b)

$$x[n] = sin(\frac{\pi}{3}(n + \frac{3\pi}{4}))$$
$$sin(\frac{\pi}{3}n) \longleftrightarrow \frac{\pi}{j} \sum_{n=-\infty}^{\infty} [\delta(w - \frac{\pi}{3} - 2\pi k) - \delta(w + \frac{\pi}{3} - 2\pi k)]$$
$$x[n] = sin(\frac{\pi}{3}(n + \frac{3\pi}{4})) \longleftrightarrow \frac{e^{jw\frac{3\pi}{4}}\pi}{j} \sum_{n=-\infty}^{\infty} [\delta(w - \frac{\pi}{3} - 2\pi k) - \delta(w + \frac{\pi}{3} - 2\pi k)]$$

(c)

$$Y(e^{jw}) = H(e^{jw})X(e^{jw})$$
$$= (\frac{1}{1 - \frac{e^{-jw}}{2}})^2 \quad .\frac{e^{jw\frac{3\pi}{4}}\pi}{j} \sum_{n=-\infty}^{\infty} [\delta(w - \frac{\pi}{3} - 2\pi k) - \delta(w + \frac{\pi}{3} - 2\pi k)]$$

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4. (a)

$$h[n] = 2\delta[n] + (\frac{1}{2})^n u[n] \longleftrightarrow 2 + \frac{1}{1 - \frac{1}{2}e^{-jw}} = H(jw)$$

(b)

$$2 + \frac{2}{2 - e^{-jw}} = \frac{6 - 2e^{-jw}}{2 - e^{-jw}} = \frac{Y(jw)}{X(jw)}$$
$$2Y(e^{jw}) - e^{-jw}Y(e^{jw}) = 6X(e^{jw}) - 2e^{-jw}X(e^{jw})$$

Take the inverse fourier transform;

$$2y[n] - y[n-1] = 6x[n] - 2x[n-1]$$

(c)

$$x[n] = (-1)^n = (e^{j\pi})^n = e^{-jn\pi}$$
$$\implies X(e^{jw}) = 2\pi \sum_{n=-\infty}^{\infty} \delta(w - \pi - 2\pi k)$$
$$Y(e^{jw}) = H(e^{jw})X(e^{jw}) = (2 + \frac{1}{1 - \frac{1}{2}e^{-jw}}).2\pi \sum_{n=-\infty}^{\infty} \delta(w - \pi - 2\pi k)$$