CENG 384 - Signals and Systems for Computer Engineers
Spring 2022
Homework 1

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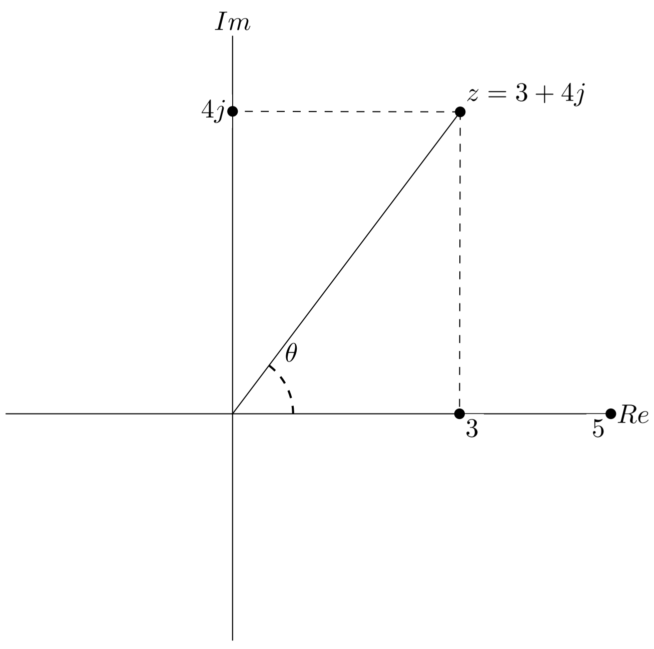
1. (a) Let say $z = x + yj$ where $z \in \mathbb{C}$ and $x, y \in \mathbb{R}$
$2z - 9 = 4j - \bar{z}$
$2x + 2yj - 9 = 4j - x + yj$
$3x + yj = 9 + 4j$
$\implies x = 3$ and $y = 4$
So, $|z| = \sqrt{x^2 + y^2} = 5$
$|z|^2 = 25$



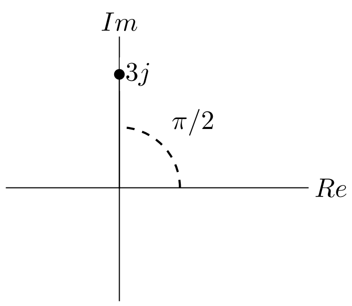
(b)

$$z^3 = -27j$$

$$z^3 = 27j^3$$

$$z = 3j$$

İf we write $3j$ in polar form



$$z = 3e^{\frac{\pi}{2}j}$$

(c)

$$\begin{aligned}
z &= \frac{(1+j)(\sqrt{3}-j)}{\sqrt{3}+j} \times \frac{\sqrt{3}-j}{\sqrt{3}-j} \\
&= \frac{(1+j)(\sqrt{3}-j)^2}{4} = \frac{(1+j)(2-2\sqrt{3}j)}{4} \\
&= \frac{1}{2}(1+j)(1-\sqrt{3}j) = \frac{1}{2} \times \sqrt{2}e^{\frac{\pi}{4}j} \times 2e^{\frac{-\pi}{3}j} \\
&= \sqrt{2}e^{\frac{-\pi}{12}j}
\end{aligned}$$

$\Longrightarrow$ Magnitude $= \sqrt{2}$ and Angle $= \frac{\pi}{12}$ (clockwise)

(d)

$$(1+j)^2 = 2j$$

$$j^4 = 1$$

$$(1+j)^8 = ((1+j)^2)^4 = (2j)^4 = 16(j^4) = 16$$

then :

$$z = -16e^{j\frac{\pi}{2}}$$

2. (a) .

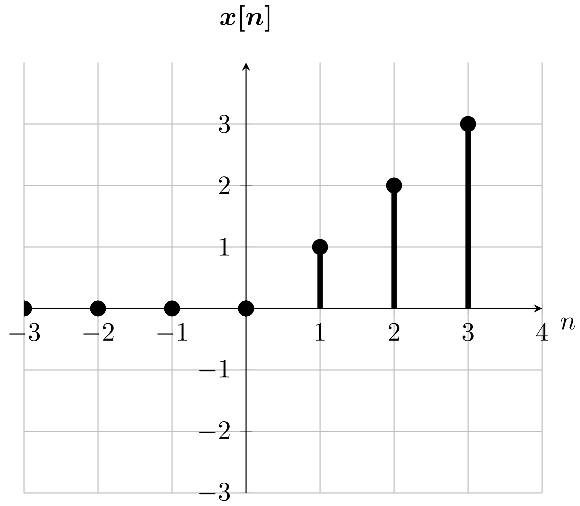


Figure 1: n vs. $x[n]$.

$$E_x = \sum_{-\infty}^{\infty} |x[n]|^2 = \infty$$

$\Longrightarrow$ This is not an energy signal.

$$P_x = \lim_{N\to\infty} \frac{E_x}{2N+1} = 0$$

$\Longrightarrow$ This is not a power signal since it does not satisfy the condition $P_x \neq 0$

(b)

$$
\begin{aligned}
E_x &= \lim_{T\to\infty}\int_{-T}^{T}|x(t)|\,dt = \lim_{T\to\infty}\int_{-T}^{T}e^{-2t}u(t)\,dt\\
&= \lim_{T\to\infty}\int_{-T}^{0}e^{-2t}u(t)\,dt + \lim_{T\to\infty}\int_{0}^{T}e^{-2t}u(t)\,dt\\
&= \lim_{T\to\infty}\int_{0}^{T}e^{-2t}u(t)\,dt = \lim_{T\to\infty}\int_{0}^{T}e^{-2t}\,dt\\
&= \lim_{T\to\infty}-\frac{1}{2}(e^{-2T}-e^{0}) = -\frac{1}{2}(0-1) = \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
P_x &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}e^{-2t}u(t)\,dt = \lim_{T\to\infty}\frac{1}{2T}\int_{0}^{T}e^{-2t}u(t)\,dt\\
&= \lim_{T\to\infty}\frac{1}{2T}(-\frac{1}{2})(e^{-2T}-e^{0}) = 0
\end{aligned}
$$

$\implies E_x < \infty \implies$ x(t) is an energy signal
$\implies P_x = 0 \implies$ x(t) is not a power signal

3. Signal can be expressed as a piecewise function:

$$
x(t) = \begin{cases} 0 & t \le -1\\ 2t+2 & -1 \le t \le 0\\ 2 & 0 \le t \le 1\\ -2t+4 & 1 \le t \le 2\\ 0 & 2 \le t \end{cases}
$$

$$
x(\frac{-1}{3}t+2) = \begin{cases} 0 & \frac{-1}{3}t+2 \le -1\\ 2(\frac{-1}{3}t+2)+2 & -1 \le \frac{-1}{3}t+2 \le 0\\ 2 & 0 \le \frac{-1}{3}t+2 \le 1\\ -2(\frac{-1}{3}t+2)+4 & 1 \le \frac{-1}{3}t+2 \le 2\\ 0 & 2 \le \frac{-1}{3}t+2 \end{cases}
$$

$$
x(\frac{-1}{3}t+2) = \begin{cases} 0 & 9 \le t\\ \frac{-2}{3}t+6 & 6 \le t \le 9\\ 2 & 3 \le t \le 6\\ \frac{2}{3}t & 0 \le t \le 3\\ 0 & t \le 0 \end{cases}
$$

$$
\frac{1}{2}x(\frac{-1}{3}t+2) = \begin{cases} 0 & 9 \le t\\ \frac{-t}{3}+3 & 6 \le t \le 9\\ 1 & 3 \le t \le 6\\ \frac{t}{3} & 0 \le t \le 3\\ 0 & t \le 0 \end{cases}
$$

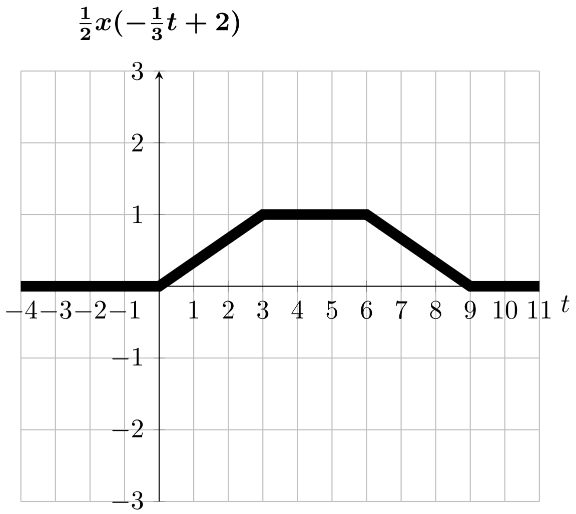


Figure 2: t vs. $\frac{1}{2}x(-\frac{1}{3}t+2)$.

4. (a) .

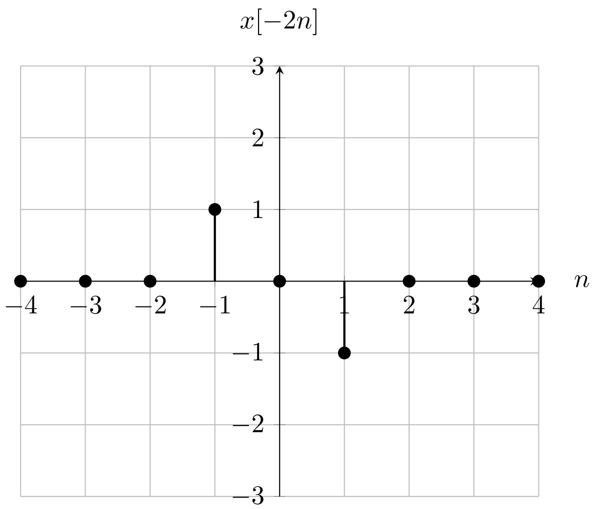


Figure 3: x vs. $x[-2n]$.

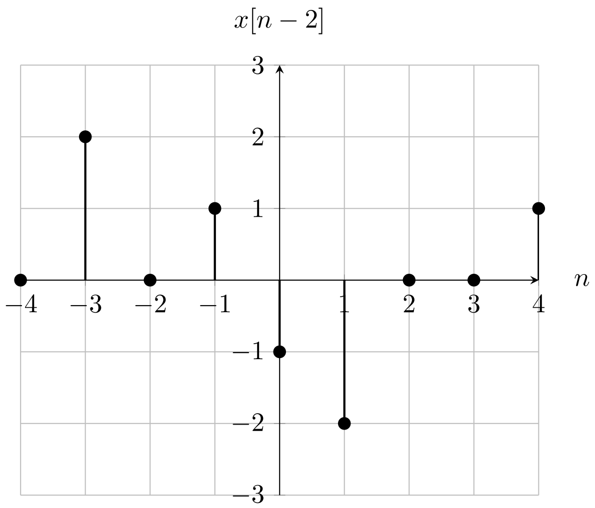


Figure 4: x vs. $x[n-2]$.

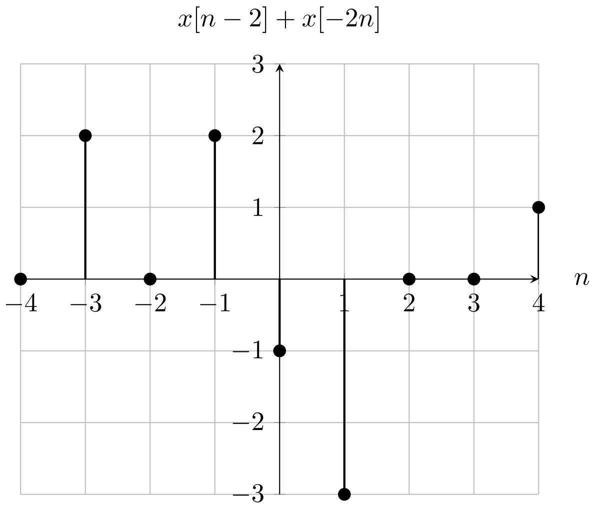


Figure 5: x vs. $x[n-2]+x[-2n]$.

(b)

$$\boldsymbol{x[-2n]+x[n2]} = \delta[n-4]+2\delta[n-2.5]+\delta[n-1.5]-3\delta[n-1]-2\delta[n-0.5]-\delta[n]+2\delta[n+1]+2\delta[n+3]$$

5. (a) Assume $x(t)$ is periodic
then $x(t) = x(t + T_0)$ $\ ,T > 0$

$$\frac{e^{j3t}}{-j} = \frac{e^{j3(t+T_0)}}{-j}$$
$$\frac{e^{j3t}}{-j} = \frac{e^{j3t}}{-j}e^{j3T_0}$$
$$1 = e^{j3T_0} = \cos(3T_0) + j\sin(3T_0)$$

let

$$T_0 = \frac{2\pi}{3}$$

then

$$\cos(2\pi) + j\sin(2\pi) = 1$$

$\Longrightarrow$ x(t) is periodic, and one period of x(t) is $\frac{2\pi}{3}$

(b) To be periodic, $x[n] = x[n + N_0]$

$$\frac{1}{2}\sin\left[\frac{7\pi}{8}n\right] + 4\cos\left[\frac{3\pi}{4}n - \frac{\pi}{2}\right] = \frac{1}{2}\sin\left[\frac{7\pi}{8}(n + N_0)\right] + 4\cos\left[\frac{3\pi}{4}(n + N_0) - \frac{\pi}{2}\right]$$

$$\Longrightarrow \quad \frac{7\pi}{8}(n + N_0) = \frac{7\pi}{8}n + 2k_1\pi \Longrightarrow N_0 = \frac{16k_1}{7}$$
$$\Longrightarrow \quad \frac{3\pi}{4}(n + N_0) - \frac{\pi}{2} = \frac{3\pi}{4}n - \frac{\pi}{2} + 2k_2\pi \Longrightarrow N_0 = \frac{8k_2}{3}$$

So for $k_1 = 7$ and $k_2 = 3$

$$N_{0_1} = 16 \quad N_{0_2} = 8$$

$$lcm(8, 16) = 16$$

So, the signal is periodic and 16 is fundamental period of the signal.

6. (a) Assume $x(t)$ is odd,
then, for every t in the domain of x,

$$x(t) = -x(-t)$$

for $t = -1$:

$$x(-1) \neq -x(1)$$

CONTRADICTION !

Assume $x(t)$ is even,
then, for every t in the domain of x,

$$x(t) = x(-t)$$

for $t = -1$:

$$x(-1) \neq x(1)$$

CONTRADICTION !

$\Longrightarrow$ Signal is neither even nor odd.

(b) We found formula for signal in figure 1 as :

$$x(t) = \begin{cases} 0, & \text{if } t \leq -1 \\ 2t+2, & \text{if } -1 < t \leq 0 \\ 2, & \text{if } 0 < t \leq 1 \\ -2t+4, & \text{if } 1 < t \leq 2 \\ 0, & \text{if } 2 < t \end{cases}$$

We know $Odd\{x(t)\} = \frac{1}{2}(x(t) - x(-t))$
then, for $t \leq -2$:

$$\frac{1}{2}(0-0) = 0$$

for $-2 < t \leq -1$:

$$\frac{1}{2}(0-(2t+4)) = -t-2$$

for $-1 < t \leq 0$:

$$\frac{1}{2}((2t+2)-(2)) = t$$

since it is odd, we can directly write remaining part without calculation:
for $0 < t \leq 1$:

$$= t$$

for $1 < t \leq 2$:

$$= -t+2$$

for $2 < t$:

$$= 0$$

Then,

$$Odd\{x(t)\} = \begin{cases} 0, & \text{if } t \leq -2 \\ -t-2, & \text{if } -2 < t \leq -1 \\ t, & \text{if } -1 < t \leq 0 \\ t, & \text{if } 0 < t \leq 1 \\ -t+2, & \text{if } 1 < t \leq 2 \\ 0, & \text{if } 2 < t \end{cases}$$

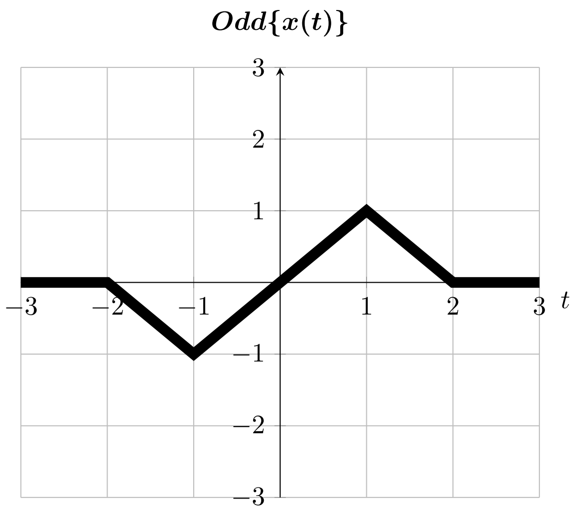


Figure 6: t vs. $\boldsymbol{Odd\{x(t)\}}$.

We know $Even\{x(t)\} = \frac{1}{2}(x(t) + x(-t))$

then, for $t \leq -2$:

$$\frac{1}{2}(0+0) = 0$$

for $-2 < t \leq -1$:

$$\frac{1}{2}(0+(2t+4)) = 2+t$$

for $-1 < t \leq 0$:

$$\frac{1}{2}((2t+2)+(2)) = 2+t$$

since it is even, we can directly write remaining part without calculation:
for $0 < t \leq 1$:

$$= -t+2$$

for $1 < t \leq 2$:

$$= -t+2$$

for $2 < t$:

$$= 0$$

Then,

$$Odd\{x(t)\} = \begin{cases} 0, & \text{if } t \leq -2 \\ t+2, & \text{if } -2 < t \leq -1 \\ t+2, & \text{if } -1 < t \leq 0 \\ -t+2, & \text{if } 0 < t \leq 1 \\ -t+2, & \text{if } 1 < t \leq 2 \\ 0, & \text{if } 2 < t \end{cases}$$

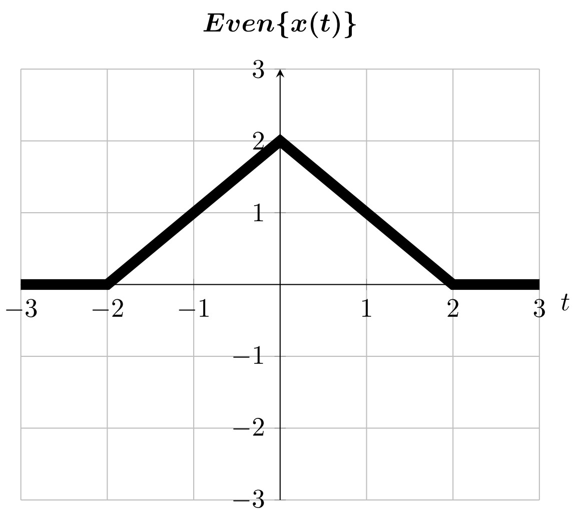


Figure 7: t vs. $\boldsymbol{Even\{x(t)\}}$.

7. (a) x(t) can be expressed as,

$$x(t) = 3u(t+3) - 3u(t+1) + 2u(t-2) - 4u(t-4) + 3u(t-6)$$

(b) Derivative of step function is impulse function. And since x(t) has spikes at -3 with magnitude 3, at -1 with magnitude -3, at -2 with magnitude 2, at 4 with magnitude -4, at 6 with magnitude 3; graph of x(t) is:

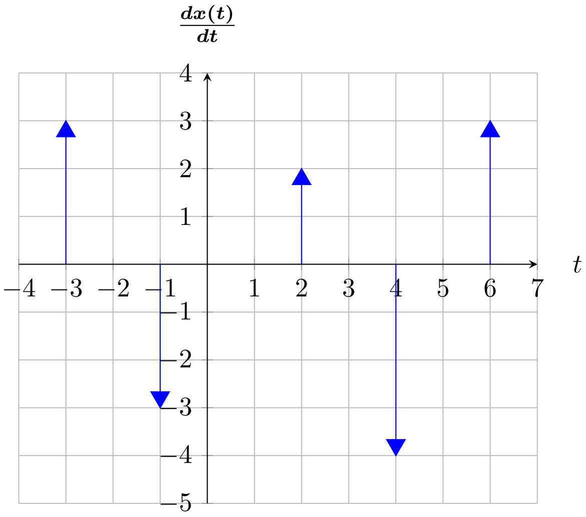


Figure 8: t vs. $\frac{dx(t)}{dt}$.

8. (a) **Memory:** It has memory. For example, $n = 1, y[1] = x[0]$.
Causality: It is not causal. For, $n > 2 \implies 2n - 2 > n$. Since output signal value depends on future values for $n > 2$, it is not causal.
Invertibility: h[n] = x[$\frac{n+2}{2}$] so it is invertible.
Stability: It is stable. $\implies$ For any bounded input, y(t) will be bounded and also if we change $x[2n-2]$ with a constant, output also constant.
Linearity: It is linear.

$$y_1 = x_1[2n-2],\ y_2 = x_2[2n-2],\ y_{eq} = x_1(n) + x_2(n)$$
$$y_1 + y_2 = x_1[2n-2] + x_2[2n-2] = x_{eq}[2n-2] = y_{eq}(n)$$

Time-Invariance: It is time-invariant. Lets shift input as t_0, for input $x[n+t_0]$

$$y^{'} = x[2(n-t_0)-2] = x[2n-2t_0-2] = y[n+t_0]$$

Input is shifted t_0, while output is shifted t_0, it is time-invariant.

(b) **Memory:** It has memory. For example, for $t = 1, y(1) = x(-\frac{1}{2})$ it depends on past values.
Causality: For $t = -3,\ \frac{t}{2} - 1 = -\frac{5}{2} > -3 = t$. Since it depends on future values, it is not causal.
Invertibility: It is not invertable. For $t = 0$, since $h(t) = \frac{x(2t+2)}{t}$, inverse is not defined.
Stability: It is not stable. For bounded input, output will not be bounded and also if we change input with a constant, output is still vary with t.
Linearity: It is linear. For

$$x_1(t) \implies y_1 = tx_1(\frac{t}{2} - 1)$$

and for

$$x_2(t) \implies y_2 = tx_2(\frac{t}{2} - 1)$$

Let $x_{eq} = a_1x_1 + a_2x_2$

$$\implies y_{eq} = t(a_1x_1(\frac{t}{2} - 1) + a_2x_2(\frac{t}{2} - 1)) = a_1y_1 + a_2y_2$$

Time-Invariance: It is not time-invariant. Let's shift the input:

$$x(t) \implies x(t-t_0)$$

$$x(t-t_0) \implies y(t) = tx(\frac{t-t_0}{2} - 1) \neq y(t-t_0) = (t-t_0)x(\frac{t-t_0}{2} - 1)$$