CENG 384 - Signals and Systems for Computer Engineers
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Homework 3

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1. (a)

$$sin(\frac{\pi}{5}t) = \frac{e^{j(\pi/5)t} - e^{-j(\pi/5)t}}{2j}$$

$$cos(\frac{\pi}{4}t) = \frac{e^{j(\pi/4)t} + e^{-j(\pi/4)t}}{2}$$

$$x(t) = \frac{1}{2j}e^{j(\pi/5)t} - \frac{1}{2j}e^{-j(\pi/5)t} + \frac{1}{2}e^{j(\pi/4)t} + \frac{1}{2}e^{-j(\pi/4)t}$$

Fundamental period of x(t) is 40. So,

$$a_4 = \frac{-j}{2} \quad , \quad a_{-4} = \frac{j}{2} \quad , \quad a_5 = \frac{1}{2} \quad , \quad a_{-5} = \frac{1}{2}$$

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(b) Since $sin(4\pi n) = 0$ and $cos(2\pi n) = 1$ for all n, and since fundamental period N = 2;

$$x[n] = \frac{1}{2} + e^{j\pi n} + 0 + 1$$

$$a_0 = \frac{3}{2} \quad , \quad a_1 = 1$$

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2.

$$\begin{aligned} x[n] &= a_1 e^{j(2\pi/N)n} + a_{-1}e^{-j(2\pi/N)n} + a_2 e^{2j(2\pi/N)n} + a_{-2}e^{-2j(2\pi/N)n} + a_3 e^{3j(2\pi/N)n} + a_{-3}e^{-3j(2\pi/N)n} \\ &= 2je^{j(2\pi/7)n} - 2je^{-j(2\pi/7)n} + 2e^{2j(2\pi/7)n} + 2e^{-2j(2\pi/7)n} + 2je^{3j(2\pi/7)n} - 2je^{-3j(2\pi/7)n} \\ &= -4sin(\frac{2\pi}{7}n) + 4cos(\frac{4\pi}{7}n) - 4sin(\frac{6\pi}{7}n) \\ &= -4sin(\frac{2\pi}{7}n) + 4sin(\frac{4\pi}{7}n + \frac{\pi}{2}) - 4sin(\frac{6\pi}{7}n) \end{aligned}$$

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3. (a)

$$sin(\frac{\pi}{8}t) = \frac{1}{2j}e^{j(\pi/8)t} - \frac{1}{2j}e^{-j(\pi/8)t}$$

$$\implies a_1 = \frac{1}{2j} \quad , \quad a_{-1} = -\frac{1}{2j}$$

.

(b)

$$cos(\frac{\pi}{8}t) = \frac{1}{2}e^{j(\pi/8)t} + \frac{1}{2}e^{-j(\pi/8)t}$$

$$\implies a_1 = \frac{1}{2} \quad , \quad a_{-1} = \frac{1}{2}$$

(c)

$$x(t) \leftrightarrow a_k \quad , \quad y(t) \leftrightarrow b_k \quad , \quad z(t) \leftrightarrow c_k$$

By multiplication property,

$$c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

Hence,

$$c_k = a_k * b_k = \frac{1}{4j}\delta[k-2] - \frac{1}{4j}\delta[k+2]$$
$$\implies c_2 = \frac{1}{4j} \quad , \quad c_{-2} = -\frac{1}{4j}$$

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4. From ii :

$$x(t) = a_0 + a_1 e^{jw_0 t} + a_{-1} e^{-jw_0 t} + a_2 e^{2jw_0 t} + a_{-2} e^{-2jw_0 t}$$

From iii :

$$x(t) = a_0 + a_1 e^{jw_0 t} + a_{-1} e^{-jw_0 t} + 3j(cos(2w_0 t) + jsin(2w_0 t)) + a_{-2}(cos(2w_0 t) - jsin(2w_0 t))$$

From i (since function is odd):

$$a_{-2} = -3j$$

From iv :

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{\forall k} |a_k|^2 = 18$$
$$|a_0|^2 + |a_1|^2 + |a_{-1}|^2 + 9 + 9 = 18$$
$$a_0 = a_1 = a_{-1} = 0$$

$$\implies x(t) = -6sin(2w_0 t)$$
$$w_0 = \frac{2\pi}{4}$$
$$\implies x(t) = -6sin(\pi t)$$

5. (a)

$$x[n] \leftrightarrow a_k$$
$$a_k = \frac{1}{N}\sum_{n=N} x[n] e^{-jk(\frac{2\pi}{9})n}$$
$$a_k = \frac{1}{9}(e^0 + e^{-jk\frac{2\pi}{9}} + e^{-jk\frac{4\pi}{9}} + e^{-jk\frac{6\pi}{9}} + e^{-jk\frac{8\pi}{9}})$$

(b)

$$y[n] \leftrightarrow b_k$$
$$b_k = \frac{1}{9}(e^0 + e^{-jk\frac{2\pi}{9}} + e^{-jk\frac{4\pi}{9}} + e^{-jk\frac{6\pi}{9}})$$

(c) For $k \neq 0, 9, 18...$

$$\frac{b_k}{a_k} = \frac{\frac{1}{9}(\frac{1-(e^{-jk\frac{2\pi}{9}})^4}{1-e^{-jk\frac{2\pi}{9}}})}{\frac{1}{9}(\frac{1-(e^{-jk\frac{2\pi}{9}})^5}{1-e^{-jk\frac{2\pi}{9}}})} = \frac{1-e^{-jk\frac{8\pi}{9}}}{1-e^{-jk\frac{10\pi}{9}}}$$

For $k = 0, 9, 18...$

$$a_0 = \frac{5}{9} \;, b_0 = \frac{4}{9}$$
$$\frac{b_0}{a_0} = \frac{4}{5}$$