The Discrete Update Algorithm

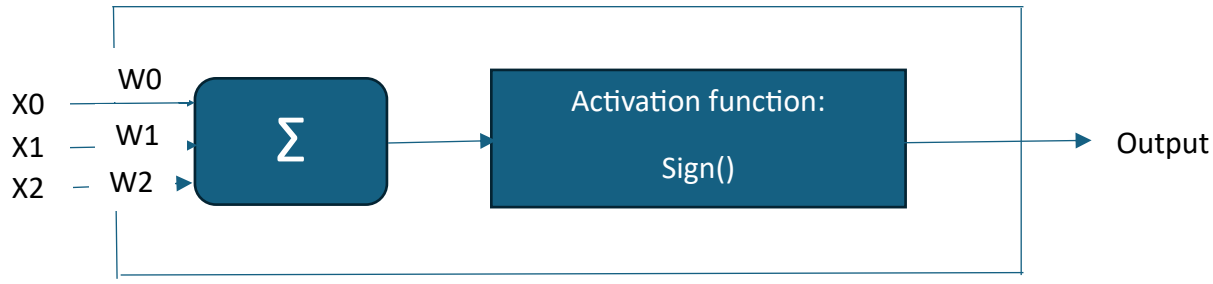
Solution of AND Gate by Perceptron

By Yair Levi

AND Gate:

X0	X1	X2	Output
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

The Perceptron:



Output = W0*X0 + W1*X1 + W2*X2

Since always X0=1 then:

Output = W0 + W1*X1 + W2*X2

Let's choose a random W vector:

W = [3, -2, 5]

Moving on the AND Gate rows staring from the first.

First epoch:

1. Y = 3 + 0 + 0 = 3
   Need to correct W since output is 0:
   W = [3-1, 12-0, 5-0] = [2, -2, 5]

2. Y = 2 + 0 + 5 = 7
   Need to correct W since output is 0:
   W = [2-1, -2-0, 5-1] = [1, -2, 4]
3. Y = 1 + (-2) + 0 = -1 Correct.
4. Y = 1 +(-2) + 4 = 3 > 0 Correct.

Second epoch:

5. Y = 1 + 0 + 0 = 1
   Need to correct W since output is 0:
   W = [1-1, -2, 4] = [0, -2, 4]
6. Y = 0 + 0 + 4 = 4
   Need to correct W since output is 0:
   W = [0-1, -2-0, 4-1] = [-1, -2, 3]
7. Y = -1 +(-2) + 0 = -3 Correct.
8. Y = -1 +(-2) + 3 = 0
   Need to correct W since output is 1:
   W = [-1+1, -2+1, 4] = [0, -1, 4]

Third epoch:

9. Y = 0 + 0 + 0 = 0
   Need to correct W since output is 0 and we need the result to be below the line and not on the line:
   W = [0-1, -1, 4] = [-1, -1, 4]
10. Y = -1 + 0 + 4 = 3
    Need to correct W since output is 0:
    W = [-2, -1, 3]
11. Y = -2 -1 + 0 = -3 Correct.
12. Y = -2 + (-1) + 3 = 0
    Need to correct W since output is 1:
    W = [-2+1, -1+1, 3+1] = [-1, 0, 4]

Forth epoch:

13. Y = -1 +0 +0 = -1 Correct.
14. Y = -1 +0 + 4 = 3
    Need to correct W since output is 0:
    W = [-1-1, -1-0, 3-1] = [-2, 0, 3]
15. Y = -2 +0 +0 = -2 Correct.
16. Y = -2 +0 + 3 = 1

Fifth Epoch:

17. Y = -2 +0 + 0 = -2 Correct.
18. Y = -2 +0 +3 = 1
    Need to correct W since output is 0:
    W = [-2-1 , 0, 3-1] = [-3, 0, 2]
19. Y = -3 +0 +0 = -3 Correct.
20. Y = -3 +0 +2 = -1
    Need to correct W since output is 1:
    W = [-2, 1, 3]

Sixth epoch:

21. Y = -2 +0 +0 = -2 Correct.
22. Y = -2 +3 = 1
    Need to correct W since output is 0:
    W = [-3, 1, 2]
23. Y = -3 +1 +0 = -2 Correct.
24. Y = -3 + 1 +2 =0
    Need to correct W since output is 1:
    W = [-2, 2, 3]

Seventh epoch:

25. Y = -2 +0 +0 = -2 Correct.
26. Y = -2 +0 +3 = 1
    Need to correct W since output is 0:
    W = [-3, 2, 2]
27. Y = -3 +2 +0 = -1 Correct.
28. Y = -3 +2 +2 = 1 Correct.

Eighth epoch:

29. Y = -3 Correct.
30. Y = -3 +2 = -1 Correct.
31. Y = -3 + 2 = -1 Correct.
32. Y = -3 +2 +2 =1 Correct.

So:

W = [-3, 2, 2]
W0 = -3

W1 = 2

W2 = 2

Since:

Output = W0 + W1*X1 + W2*X2

We want to find the separation line:

0 = W0 + W1*X1 + W2*X2

W2*X2 = -W1*X1 -W0

X2 = -(W1/W2)*X1 –(W0/W2)

X2 = -X1 +1.5

Points of this line are (0,1.5) and (1.5,0).

The graph is shown below.

All points below the graph the output is 0 and all points above the graph the output is 1.