reactivity

a 1d linear dynamical system

$$\frac{dx}{dt} = ax$$

whose solution is

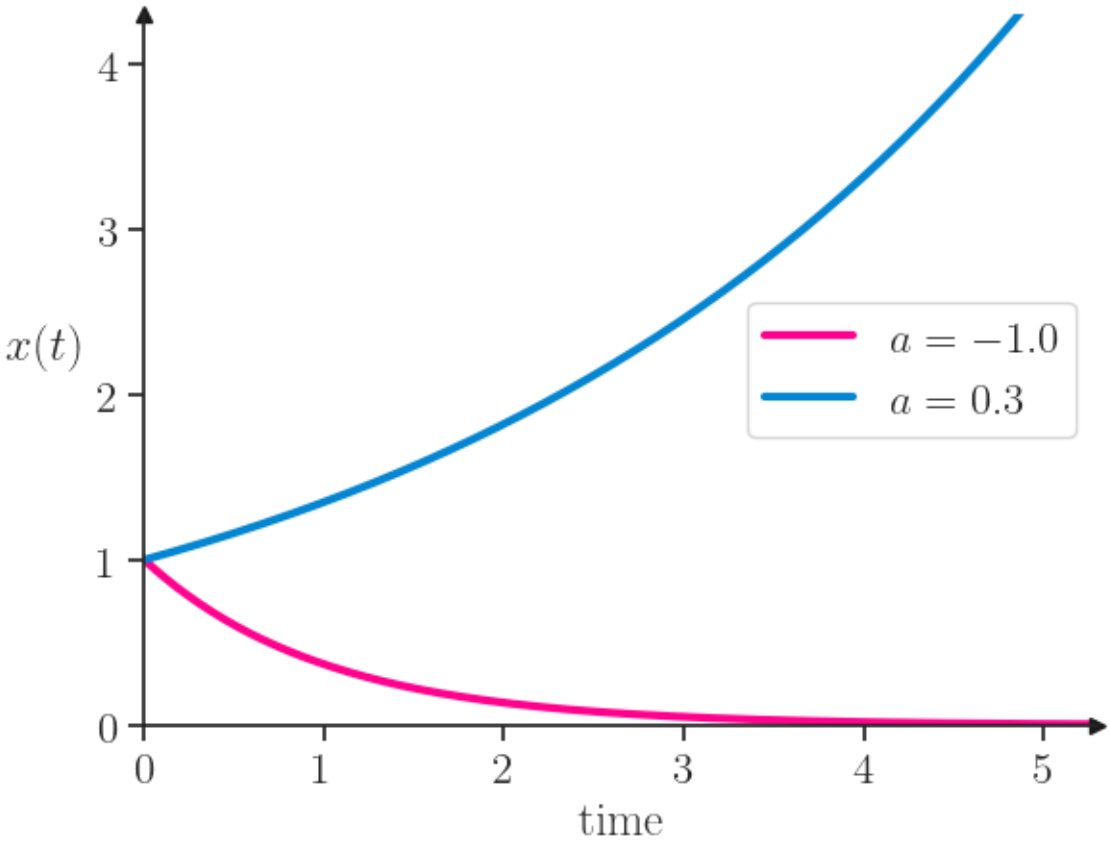
$$x(t) = x_0 e^{at}$$

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib
import matplotlib.gridspec as gridspec
from scipy.integrate import solve_ivp
import seaborn as sns
sns.set_theme(style="ticks", font_scale=1.5)  # white graphs, with large and legible letters
# %matplotlib widget
```

```
def equation_1d(a, x):
    return [a * x]

# parameters as a dictionary
a1 = -1.0
a2 = +0.3

tmax=6
dt=0.01
x0 = 1.0
t_eval = np.arange(0, tmax, dt)
```

```
# solve the system

sol1 = solve_ivp(lambda t, y: equation_1d(a1, y),
                 [0, tmax], [x0], t_eval=t_eval)
sol2 = solve_ivp(lambda t, y: equation_1d(a2, y),
                 [0, tmax], [x0], t_eval=t_eval)
```

```
# learn how to configure:
# http://matplotlib.sourceforge.net/users/customizing.html
params = {
          'font.family': 'serif',
          'ps.usedistiller': 'xpdf',
          'text.usetex': True,
          # include here any neede package for latex
          'text.latex.preamble': r'\usepackage{amsmath}',
          }
plt.rcParams.update(params)
# matplotlib.rcParams['text.latex.preamble'] = [
#     r'\usepackage{amsmath}',
#     r'\usepackage{mathtools}']

fig, ax = plt.subplots()

bright_color1 = "xkcd:hot pink"
bright_color2 = "xkcd:cerulean"

ax.plot(sol1.t, sol1.y[0], color=bright_color1, lw=3, label=f'$a={a1}$')
ax.plot(sol2.t, sol2.y[0], color=bright_color2, lw=3, label=f'$a={a2}$')


ax.legend(loc='center right')
ax.set(xlim=[0,5.3],
       ylim=[0,4.3],
       xlabel='time',)
ax.set_ylabel(r'$x(t)$', labelpad=20, rotation=0)
# only left and bottom spines
ax.spines['right'].set_color('none')
ax.spines['top'].set_color('none')

ax.plot(1, 0, ">k", transform=ax.get_yaxis_transform(), clip_on=False)
ax.plot(0, 1, "^k", transform=ax.get_xaxis_transform(), clip_on=False)
```



the simplest 2d dynamical system

$$\frac{dx_1}{dt} = ax_1 + bx_2$$
$$\frac{dx_2}{dt} = cx_1 + dx_2$$

...or in matrix form:

$$\frac{d\mathbf{x}}{dt} = M\mathbf{x},$$

where $\mathbf{x} = (x_1, x_2)$ and $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

```
def system_equations_2d(p, x, y):
    return [p['a'] * x + p['b'] * y,
            p['c'] * x + p['d'] * y,
           ]

# parameters as a dictionary
A0 = {'a': -1.0, 'b': +0.0,
      'c': +0.0, 'd': -2.0}
A1 = {'a': -1.0, 'b': +1.0,
      'c': +0.0, 'd': -2.0}
A2 = {'a': -1.0, 'b': +10,
      'c': +0.0, 'd': -2.0}
```

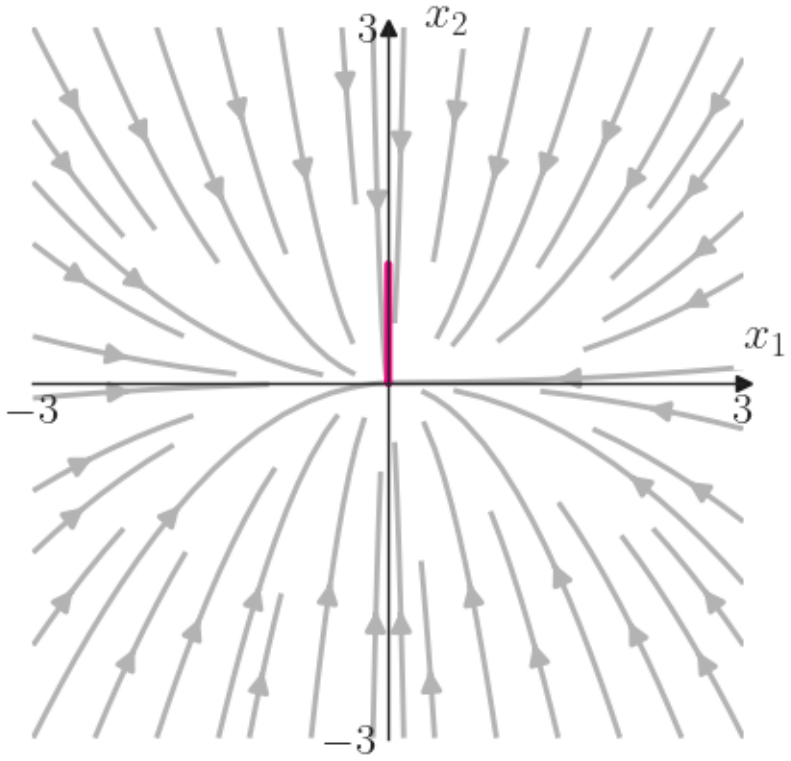
```
min_x, max_x = [-3, 3]
min_y, max_y = [-3, 3]
div = 50
X, Y = np.meshgrid(np.linspace(min_x, max_x, div),
                   np.linspace(min_y, max_y, div))

# given initial conditions (x0,y0), simulate the trajectory of the system as ivp
def simulate_trajectory(p, x0, y0, tmax=10, dt=0.01):
    t_eval = np.arange(0, tmax, dt)
    sol = solve_ivp(lambda t, y: system_equations_2d(p, y[0], y[1]),
                    [0, tmax], [x0, y0], t_eval=t_eval)
    return sol

t0 = simulate_trajectory(A0, 0, 1, 100)
t1 = simulate_trajectory(A1, 0, 1, 100)
t2 = simulate_trajectory(A2, 0, 1, 100)
```

```
fig, ax = plt.subplots()

density = 2 * [0.80]
minlength = 0.2
arrow_color = 3 * [0.7]
bright_color1 = "xkcd:hot pink"
bright_color2 = "xkcd:cerulean"

# make sure that each axes is square
ax.set_aspect('equal', 'box')
ax.streamplot(X, Y, system_equations_2d(A0, X, Y)[0], system_equations_2d(A0, X, Y)[1],
              density=density, color=arrow_color, arrowsize=1.5,
```

```
                linewidth=2,
                minlength=minlength,
                zorder=-10
                )
ax.plot(t0.y[0], t0.y[1], color=bright_color1, lw=3)

# make spines at the origin, put arrow at the end of the axis
ax_list = [ax]
for axx in ax_list:
    axx.spines['left'].set_position('zero')
    axx.spines['bottom'].set_position('zero')
    axx.spines['right'].set_color('none')
    axx.spines['top'].set_color('none')
    axx.spines['left'].set_linewidth(1.0)
    axx.spines['bottom'].set_linewidth(1.0)
    axx.xaxis.set_ticks_position('bottom')
    axx.yaxis.set_ticks_position('left')
    axx.xaxis.set_tick_params(width=0.5)
    axx.yaxis.set_tick_params(width=0.5)
    # put arrow at the end of the axis
    axx.plot(1, 0, ">k", transform=axx.get_yaxis_transform(), clip_on=False)
    axx.plot(0, 1, "^k", transform=axx.get_xaxis_transform(), clip_on=False)
    axx.text(1, 0.55, r"$x_1$", transform=axx.transAxes, clip_on=False, bbox=dict(facecolor=
    axx.text(0.55, 1, r"$x_2$", transform=axx.transAxes, clip_on=False, bbox=dict(facecolor=
    # set limits
    axx.set(xticks=[-3,3],
                    yticks=[-3,3],
                    xlim=[-3, 3],
                    ylim=[-3, 3],)
    # remove ticks from both axes
    axx.tick_params(axis='both', which='both', length=0)

# put on title the respective parameters as matrix, use latex equation
# add pad to title to avoid overlap with x-axis
ax.set_title(r'$M_1=\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$', pad=40)
```

```
Text(0.5, 1.0, '$M_1=\\begin{bmatrix} -1 & 0 \\\\ 0 & -2 \\end{bmatrix}$')
```

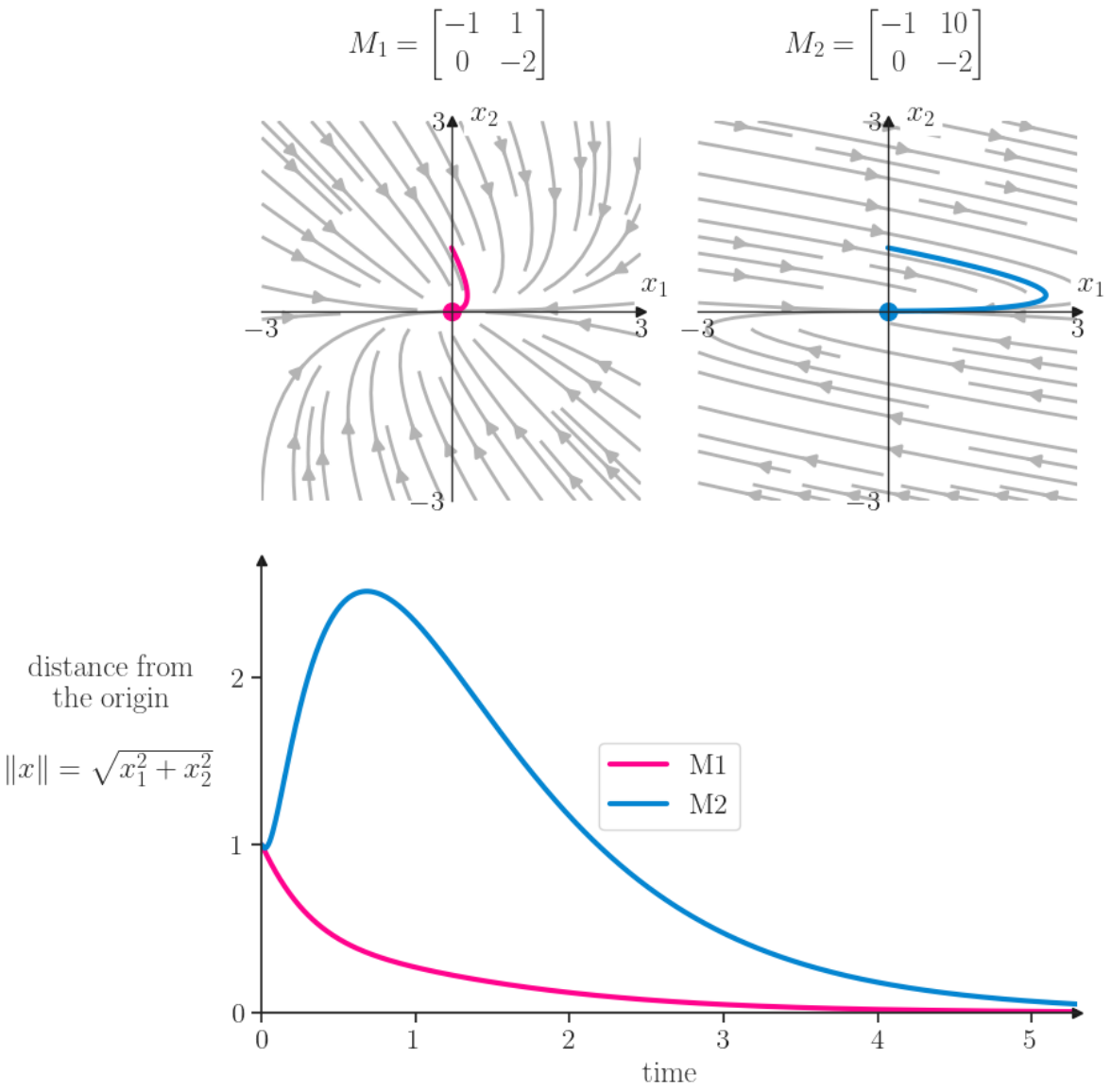
$$M_1 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$




```
# learn how to configure:
# http://matplotlib.sourceforge.net/users/customizing.html
params = {
          'font.family': 'serif',
          'ps.usedistiller': 'xpdf',
          'text.usetex': True,
          # include here any neede package for latex
          'text.latex.preamble': r'\usepackage{amsmath}',
          }
plt.rcParams.update(params)
# matplotlib.rcParams['text.latex.preamble'] = [
#     r'\usepackage{amsmath}',
#     r'\usepackage{mathtools}']

fig = plt.figure(figsize=(10, 10))
```

```
gs = gridspec.GridSpec(2, 2, width_ratios=[1,1], height_ratios=[1,1])
gs.update(left=0.20, right=0.86,top=0.88, bottom=0.13, hspace=0.05, wspace=0.15)

ax0 = plt.subplot(gs[0, 0])
ax1 = plt.subplot(gs[0, 1])
ax2 = plt.subplot(gs[1, :])

density = 2 * [0.80]
minlength = 0.2
arrow_color = 3 * [0.7]
bright_color1 = "xkcd:hot pink"
bright_color2 = "xkcd:cerulean"

# make sure that each axes is square
ax0.set_aspect('equal', 'box')
ax1.set_aspect('equal', 'box')

ax0.streamplot(X, Y, system_equations_2d(A1, X, Y)[0], system_equations_2d(A1, X, Y)[1],
               density=density, color=arrow_color, arrowsize=1.5,
               linewidth=2,
               minlength=minlength,
               zorder=-10
               )
ax1.streamplot(X, Y, system_equations_2d(A2, X, Y)[0], system_equations_2d(A2, X, Y)[1],
               density=density, color=arrow_color, arrowsize=1.5,
               linewidth=2,
               minlength=minlength,
               zorder=-10
               )
ax0.plot(t1.y[0], t1.y[1], color=bright_color1, lw=3)
ax1.plot(t2.y[0], t2.y[1], color=bright_color2, lw=3)
ax0.plot(t1.y[0][-1], t1.y[1][-1], 'o', color=bright_color1, markersize=10)
ax1.plot(t2.y[0][-1], t2.y[1][-1], 'o', color=bright_color2, markersize=10)

# make spines at the origin, put arrow at the end of the axis
ax_list = [ax0, ax1]
for axx in ax_list:
    axx.spines['left'].set_position('zero')
    axx.spines['bottom'].set_position('zero')
    axx.spines['right'].set_color('none')
    axx.spines['top'].set_color('none')
    axx.spines['left'].set_linewidth(1.0)
```

```
    axx.spines['bottom'].set_linewidth(1.0)
    axx.xaxis.set_ticks_position('bottom')
    axx.yaxis.set_ticks_position('left')
    axx.xaxis.set_tick_params(width=0.5)
    axx.yaxis.set_tick_params(width=0.5)
    # put arrow at the end of the axis
    axx.plot(1, 0, ">k", transform=axx.get_yaxis_transform(), clip_on=False)
    axx.plot(0, 1, "^k", transform=axx.get_xaxis_transform(), clip_on=False)
    axx.text(1, 0.55, r"$x_1$", transform=axx.transAxes, clip_on=False, bbox=dict(facecolor=
    axx.text(0.55, 1, r"$x_2$", transform=axx.transAxes, clip_on=False, bbox=dict(facecolor=
    # set limits
    axx.set(xticks=[-3,3],
                  yticks=[-3,3],
                  xlim=[-3, 3],
                  ylim=[-3, 3],)
    # remove ticks from both axes
    axx.tick_params(axis='both', which='both', length=0)

# put on title the respective parameters as matrix, use latex equation
# add pad to title to avoid overlap with x-axis
ax0.set_title(r'$M_1=\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$', pad=40)
ax1.set_title(r'$M_2=\begin{bmatrix} -1 & 10 \\ 0 & -2 \end{bmatrix}$', pad=40)

L2_one = np.sqrt(t1.y[0]**2 + t1.y[1]**2)
L2_two = np.sqrt(t2.y[0]**2 + t2.y[1]**2)


# bottom plot
ax2.plot(t1.t, L2_one, color=bright_color1, lw=3, label='M1')
ax2.plot(t2.t, L2_two, color=bright_color2, lw=3, label='M2')
ax2.legend(loc='center')
ax2.set(xlim=[0,5.3],
        ylim=[0,2.7],
        yticks=[0,1,2],
        xlabel='time',)
ax2.set_ylabel('distance from\nthe origin\n\n' + r"$\lVert x\rVert =\sqrt{x_1^2+x_2^2}$", lab
# only left and bottom spines
ax2.spines['right'].set_color('none')
ax2.spines['top'].set_color('none')

ax2.plot(1, 0, ">k", transform=ax2.get_yaxis_transform(), clip_on=False)
ax2.plot(0, 1, "^k", transform=ax2.get_xaxis_transform(), clip_on=False)
```



The question arises whether asymptotic behavior adequately characterizes the response to perturbations. Because of the short duration of many ecological experiments, transients may dominate the observed responses to perturbations. In addition, transient responses may be at least as important as asymptotic responses. Managers charged with ecosystem restoration, for example, are likely to be interested in both the short-term and long-term effects of their manipulations, particularly if the short-term effects can be large.

Source: Neubert & Caswell, 1997, *Ecology*