Time Series Analysis

Yair Mau

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about

Welcome to **Time Series Analysis for Environmental Sciences** (71606) at the Hebrew University of Jerusalem. This is Yair Mau, your host for today. I am a senior lecturer at the Institute of Environmental Sciences, at the Faculty of Agriculture, Food and Environment, in Rehovot, Israel.

This website contains (almost) all the material you'll need for the course. If you find any mistakes, or have any comments, please email me.

disclaimer

The material here is not comprehensive and `does not` constitute a stand alone course in Time Series Analysis. This is only the support material for the actual presential course I give.

what, who, when and where?

Course number 71606, 3 academic points
Yair Mau (lecturer), Erez Feuer (TA)
Tuesdays, from 14:15 to 17:00
Computer classroom #16
Office hours: upon request

syllabus

course description

Data analysis of time series, with practical examples from environmental sciences.

course aims

This course aims at giving the students a broad overview of the main steps involved in the analysis of time series: data management, data wrangling, visualization, analysis, and forecast. The course will provide a hands-on approach, where students will actively engage with real-life datasets from the field of environmental science.

learning outcomes

On successful completion of this module,students should be able to:

- Explore a time-series dataset, while formulating interesting questions.
- Choose the appropriate tools to attack the problem and answer the questions.
- Communicate their findings and the methods they used to achieve them, using graphs, statistics, text, and a well-documented code.

course content

- **Data wrangling:** organization, cleaning, merging, filling gaps, excluding outliers, smoothing, resampling.
- **Visualization:** best practices for graph making using leading python libraries.
- **Analysis:** stationarity, seasonality, (auto)correlations, lags, derivatives, spectral analysis.
- **Forecast:** ARIMA
- **Data management:** how to plan ahead and best organize large quantities of data. If there is enough time, we will build a simple time-series database.

books and other sources

Click here.

course evaluation

There will be 2 projects during the semester (each worth 25% of the final grade), and one final project (50%).

weekly program

week 1

- **Lecture:** Course overview, setting of expectations. Introduction, basic concepts, continuous vs discrete time series, sampling, aliasing
- **Exercise:** Loading csv file into python, basic time series manipulation with pandas and plotting

week 2

- **Lecture:** Filling gaps, removing outliers
- **Exercise:** Practice the same topics learned during the lecture. Data: air temperature and relative humidity

week 3

- **Lecture:** Interpolation, resampling, binning statistics
- **Exercise:** Practice the same topics learned during the lecture. Data: air temperature and relative humidity, precipitation

week 4

- **Lecture:** Time series plotting: best practices. Dos and don'ts and maybes
- **Exercise:** Practice with Seaborn, Plotly, Pandas, Matplotlib

Project 1
Basic data wrangling, using real data (temperature, relative humidity, precipitation) downloaded from USGS. 25% of the final grade

week 5

- **Lecture:** Smoothing, running averages, convolution
- **Exercise:** Practice the same topics learned during the lecture. Data: sap flow, evapotranspiration

week 6

- **Lecture:** Strong and weak stationarity, stochastic processes, auto-correlation
- **Exercise:** Practice the same topics learned during the lecture. Data: temperature and wind speed

week 7

- **Lecture:** Correlation between signals. Pearson correlation, time-lagged cross-correlations, dynamic time warping
- **Exercise:** Practice the same topics learned during the lecture. Data: temperature, solar radiation, relative humidity, soil moisture, evapotranspiration

week 8

Same as lecture 7 above

week 9

- **Lecture:** Download data from repositories, using API, merging, documentation
- **Exercise:** Download data from USGS, NOAA, Fluxnet, Israel Meteorological Service

Project 2
Students will study a Fluxnet site of their choosing. How do gas fluxes (CO2, H2O) depend on environmental conditions? 25% of the final grade

week 10

- **Lecture:** Fourier decomposition, filtering, Nyquist–Shannon sampling theorem
- **Exercise:** Practice the same topics learned during the lecture. Data: dendrometer data

week 11

- **Lecture:** Seasonality, seasonal decomposition (trend, seasonal, residue), Hilbert transform
- **Exercise:** Practice the same topics learned during the lecture. Data: monthly atmospheric CO2 concentration, hourly air temperature

week 12

- **Lecture:** Derivatives, differencing
- **Exercise:** Practice the same topics learned during the lecture. Data: dendrometer data

week 13

- **Lecture:** Forecasting. ARIMA
- **Exercise:** Practice the same topics learned during the lecture. Data: vegetation variables (sap flow, ET, DBH, etc)

Final Project
In consultation with the lecturer, students will ask a specific scientific question about a site of their choosing (from NOAA, USGS, Fluxnet), and answer it using the tools learned during the semester. The report will be written in Jupyter Notebook,

combining in one document all the calculations, documentation, figures, analysis, and discussion. 50% of the final grade.

who cares?

why "Time Series Analysis?"

Time has two aspects. There is the arrow, the running river, without which there is no change, no progress, or direction, or creation. And there is the circle or the cycle, without which there is chaos, meaningless succession of instants, a world without clocks or seasons or promises.
URSULA K. LE GUIN

You are here because you are interested in how things change, evolve. In this course I want to discuss with you how to make sense of data whose temporal nature is in its very essence. We will talk about randomness, cycles, frequencies, correlations, and more.

why "Environmental Sciences"

This same time series analysis (TSA) course could be called instead "TSA for finance", "TSA for Biology", or any other application. The emphasis in this course is **not** Environmental Sciences, but the concepts and tools of TSA. Because my research is in Environmental Science, and many of the graduate students at HUJI-Rehovot research this, I chose to use examples "close to home". The same toolset should be useful for students of other disciplines.

what is it good for?

In many fields of science we are flooded by data, and it's hard to see the forest for the trees. I hope that the topics we'll

discuss in this course can help you find meaningful patterns in your data, formulate interesting hypotheses, and design better experiments.

do I need it?

Maybe. If you are a grad student and you have temporal data to analyze, then probably yes. However, I have very fond memories of courses that I took as a grad student that were completely unrelated to my research. Sometimes "because it's fun" is a perfectly good answer.

what will I actually gain from it?

By the end of this course you will have gained:

- a **hands-on** experience of fundamental time-series analysis tools
- an **intuition** regarding the basic concepts
- **technical** abilities
- a **springboard** for learning more about the subject by yourself

Part I

start here

1 the boring stuff you absolutely need to do

I assume everyone registered has taken a basic Python course. On your computer, do the following:

1.1 Anaconda

Install Anaconda's Python distribution. The Anaconda installation brings with it all the main python packages we will need to use. In order to install extra packages, refer to these two tutorials: tutorial 1, tutorial 2

1.2 VSCode

Install VSCode. Visual Studio Code is a very nice IDE (Integrated Development Environment) made by Microsoft, available to all operating systems. Contrary to the title of this page, it is not absolutely necessary to use it, but I like VSCode, and as my student, so do you.

1.3 jupyter notebooks

We will code exclusively in Jupyter Notebooks. Get acquainted with them. Make sure you can point VSCode to the Anaconda environment of your choice ("base" by default). Don't worry, this is easier than it sounds.

1.4 HUJI's computers

If you are using the computers in the Hebrew University's computer lab, then:

- open Anaconda Navigator
- in "Environments", choose "asgard"
- open VSCode from inside Anaconda Navigator

1.5 folder structure

You **NEED** to be confortable with you computer's folder (or directory) structure. Where are files located? How to navigate through different folders? How is my stuff organized? If you don't feel **absolutely** comfortable with this, then read this, Windows, MacOS. If you use Linux then you surely know this stuff. **Make yourself a "time-series" folder** wherever you want, and have it backed up regularly (use Google Drive, Dropbox, do it manually, etc). "My dog deleted my files" is not an excuse.

2 pandas

3 pyplot

4 all together

5 old stuff

Import packages. If you don't have a certain package, e.g. 'new-package', just type
`pip install newpackage`

```
import urllib
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import os.path
import matplotlib.dates as mdates
import datetime as dt
import matplotlib as mpl
from pandas.tseries.frequencies import to_offset
from scipy.signal import savgol_filter
```

This is how you download data from Thingspeak

```
filename1 = "test_elad.csv"
# if file is not there, go fetch it from thingspeak
if not os.path.isfile(filename1):
    # define what to download
    channels = "1690490"
    fields = "1,2,3,4,6,7"
    minutes = "30"

    # https://www.mathworks.com/help/thingspeak/readdata.html
    # format YYYY-MM-DD%20HH:NN:SS
    start = "2022-05-01%2000:00:00"
    end = "2022-05-08%2000:00:00"

    # download using Thingspeak's API
    # url = f"https://api.thingspeak.com/channels/{channels}/fields/{fields}.csv?minutes={mi
```

```
    url = f"https://api.thingspeak.com/channels/{channels}/fields/{fields}.csv?start={start}
    data = urllib.request.urlopen(url)
    d = data.read()

    # save data to csv
    file = open(filename1, "w")
    file.write(d.decode('UTF-8'))
    file.close()
```

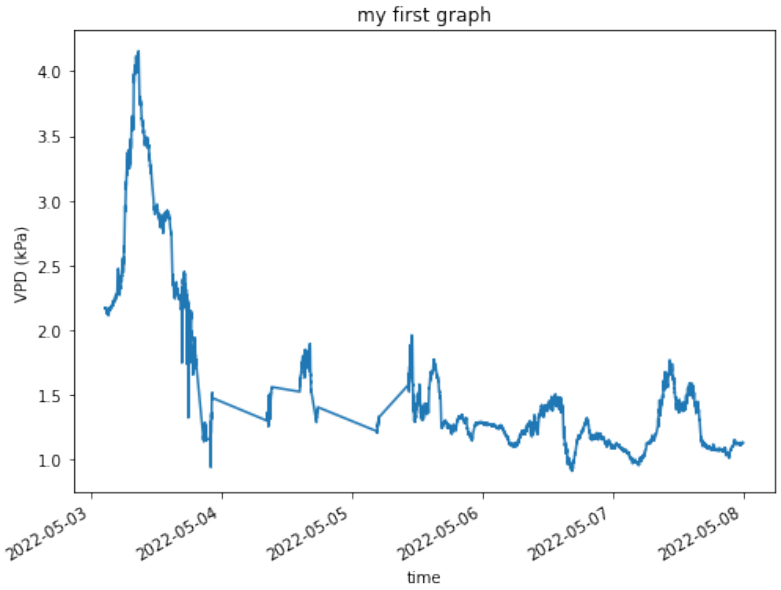
You can load the data using Pandas. Here we create a "dataframe", which is a fancy name for a table.

```
# load data
df = pd.read_csv(filename1)
# rename columns
df = df.rename(columns={"created_at": "timestamp",
                        "field1": "T1",
                        "field2": "RH",
                        "field3": "T2",
                        "field4": "motion_sensor",
                        "field6": "VWC",
                        "field7": "VPD",})
# set timestamp as index
df['timestamp'] = pd.to_datetime(df['timestamp'])
df = df.set_index('timestamp')
```

5.0.1 First graph

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,6))

ax.plot(df['VPD'])
# add labels and title
ax.set(xlabel = "time",
       ylabel = "VPD (kPa)",
       title = "my first graph")
# makes slanted dates
```

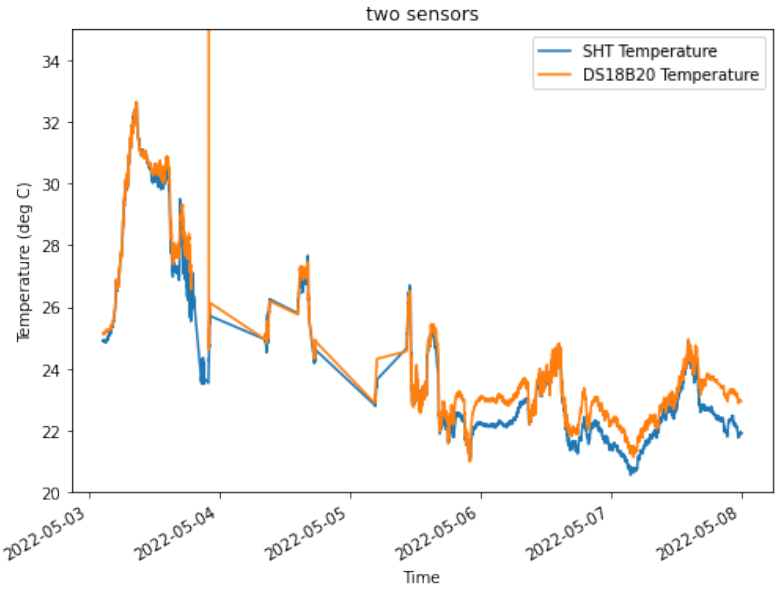
```
plt.gcf().autofmt_xdate()
```



5.0.2 Two columns in the same graph

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,6))

ax.plot(df['T1'], color="tab:blue", label="SHT Temperature")
ax.plot(df['T2'], color="tab:orange", label="DS18B20 Temperature")
# add labels and title
ax.set(xlabel = "Time",
       ylabel = "Temperature (deg C)",
       title = "two sensors",
       ylim=[20,35],
       )
# makes slanted dates
plt.gcf().autofmt_xdate()
ax.legend(loc="upper right")
```

```
<matplotlib.legend.Legend at 0x7fe6c9730610>
```



5.0.3 Calculate stuff

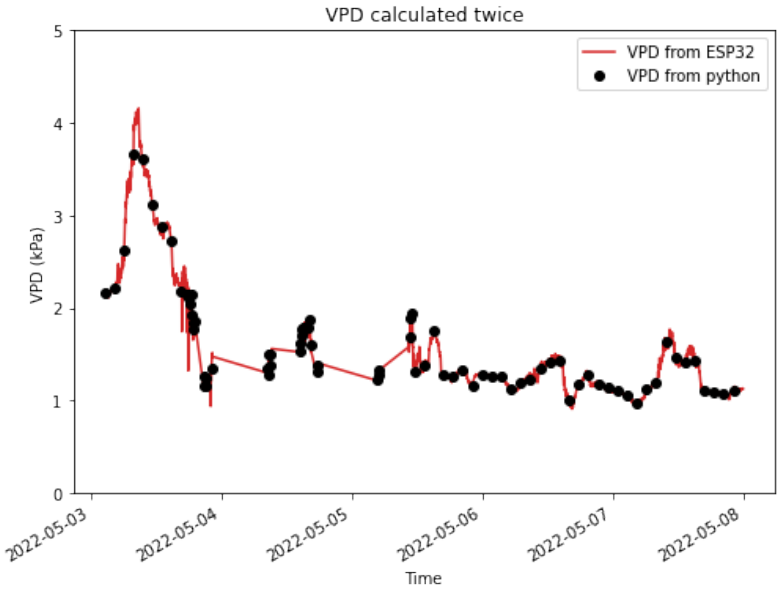
You can calculate new things and save them as new columns of your dataframe.

```
def calculate_es(T):
    es = np.exp((16.78 * T - 116.9) / (T + 237.3))
    return es

def calculate_ed(es, rh):
    return es * rh / 100.0

es = calculate_es(df['T1'])
ed = calculate_ed(es, df['RH'])
df['VPD2'] = es - ed
```

See if what you calculated makes sense.

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,6))

ax.plot(df['VPD'], color="tab:red", label="VPD from ESP32")
ax.plot(df['VPD2'][::100], "o", color="black", label="VPD from python")
# add labels and title
ax.set(xlabel = "Time",
       ylabel = "VPD (kPa)",
       title = "VPD calculated twice",
       ylim=[0,5],
       )
# makes slanted dates
plt.gcf().autofmt_xdate()
ax.legend(loc="upper right")
```

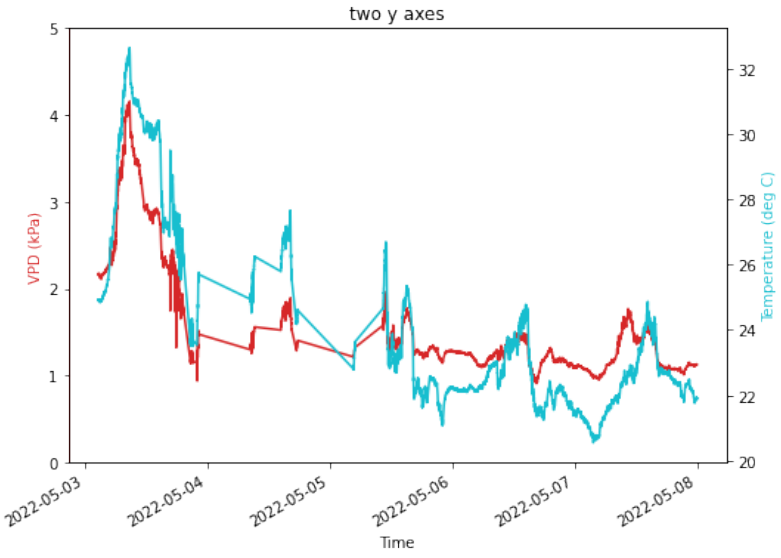
```
<matplotlib.legend.Legend at 0x7fe6989ca700>
```



5.0.4 Two y axes

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,6))

ax.plot(df['VPD'], color="tab:red", label="VPD")
plt.gcf().autofmt_xdate()
ax2 = ax.twinx()
ax2.plot(df['T1'], color="tab:cyan", label="Temperature")
ax.set(xlabel = "Time",
       title = "two y axes",
       ylim=[0,5],
       )
ax.set_ylabel('VPD (kPa)', color='tab:red')
ax.spines['left'].set_color('red')

ax2.set_ylabel('Temperature (deg C)', color='tab:cyan')
```

```
Text(0, 0.5, 'Temperature (deg C)')
```

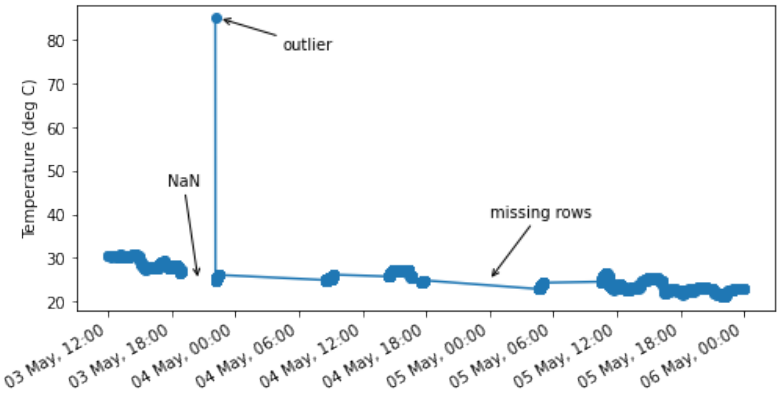


5.1 NaN, Missing data, Outliers

```
# %matplotlib widget

start = "2022-05-03 12:00:00"
end = "2022-05-06 00:00:00"

fig, ax = plt.subplots(1, figsize=(8,4))

# plot using pandas' plot method
df.loc[start:end, 'T2'].plot(ax=ax,
                             linestyle='-',
                             marker='o',
                             color="tab:blue",
                             label="data")

# annotate examples here:
# https://jakevdp.github.io/PythonDataScienceHandbook/04.09-text-and-annotation.html
ax.annotate("NaN",                              # text to write, if nothing, then ""
            xy=('2022-05-03 20:30:00', 25),     # (x,y coordinates for the tip of the arrow)
            xycoords='data',                    # xy as 'data' coordinates
            xytext=(-20, 60),                   # xy coordinates for the text
            textcoords='offset points',         # xytext relative to xy
            arrowprops=dict(arrowstyle="->")    # pretty arrow
           )
ax.annotate("outlier",
            xy=('2022-05-03 22:30:00', 85),
            xycoords='data',
            xytext=(40, -20),
            textcoords='offset points',
            arrowprops=dict(arrowstyle="->")
           )
ax.annotate("missing rows",
            xy=('2022-05-05 00:00:00', 25),
            xycoords='data',
            xytext=(0, 40),
            textcoords='offset points',
            arrowprops=dict(arrowstyle="->")
           )
```

```
ax.xaxis.set_major_formatter(mdates.DateFormatter('%d %b, %H:00'))
plt.gcf().autofmt_xdate()
ax.set(xlabel="",
       ylabel="Temperature (deg C)")
```

```
[Text(0.5, 0, ''), Text(0, 0.5, 'Temperature (deg C)')]
```



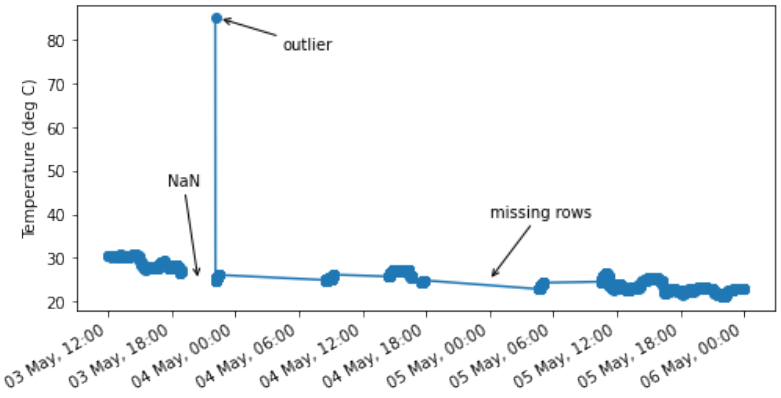
The arrows (annotate) work because the plot was
`df['column'].plot()`

If you use the usual
`ax.plot(df['column'])`
then you matplotlib will not understand timestamps as x-positions. In this case follow the instructions below.

```
# %matplotlib widget

start = "2022-05-03 12:00:00"
end = "2022-05-06 00:00:00"

fig, ax = plt.subplots(1, figsize=(8,4))

ax.plot(df.loc[start:end, 'T2'], linestyle='-', marker='o', color="tab:blue", label="data")

t_nan = '2022-05-03 20:30:00'
x_nan = mdates.date2num(dt.datetime.strptime(t_nan, "%Y-%m-%d %H:%M:%S"))
```

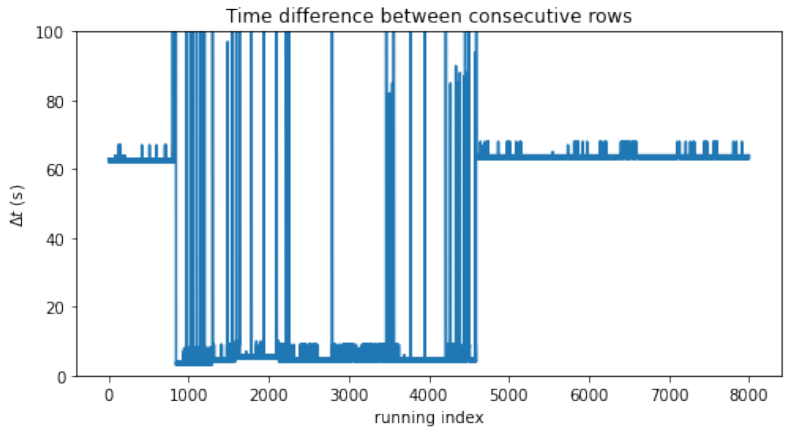
```
ax.annotate("NaN",
            xy=(x_nan, 25),
            xycoords='data',
            xytext=(-20, 60),
            textcoords='offset points',
            arrowprops=dict(arrowstyle="->")
           )
t_outlier = '2022-05-03 22:30:00'
x_outlier = mdates.date2num(dt.datetime.strptime(t_outlier, "%Y-%m-%d %H:%M:%S"))
ax.annotate("outlier",
            xy=(x_outlier, 85),
            xycoords='data',
            xytext=(40, -20),
            textcoords='offset points',
            arrowprops=dict(arrowstyle="->")
           )
t_missing = '2022-05-05 00:00:00'
x_missing = mdates.date2num(dt.datetime.strptime(t_missing, "%Y-%m-%d %H:%M:%S"))
ax.annotate("missing rows",
            xy=(x_missing, 25),
            xycoords='data',
            xytext=(0, 40),
            textcoords='offset points',
            arrowprops=dict(arrowstyle="->")
           )
# code for hours, days, etc
# https://docs.python.org/3/library/datetime.html#strftime-and-strptime-format-codes
ax.xaxis.set_major_formatter(mdates.DateFormatter('%d %b, %H:00'))
plt.gcf().autofmt_xdate()
ax.set(xlabel="",
       ylabel="Temperature (deg C)")
```

```
[Text(0.5, 0, ''), Text(0, 0.5, 'Temperature (deg C)')]
```




```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,4))

delta_index = (df.index.to_series().diff() / pd.Timedelta('1 sec') ).values
ax.plot(delta_index)
ax.set(ylim=[0, 100],
       xlabel="running index",
       ylabel=r"$\Delta t$ (s)",
       title="Time difference between consecutive rows")
```

```
[(0.0, 100.0),
 Text(0.5, 0, 'running index'),
 Text(0, 0.5, '$\\Delta t$ (s)'),
 Text(0.5, 1.0, 'Time difference between consecutive rows')]
```

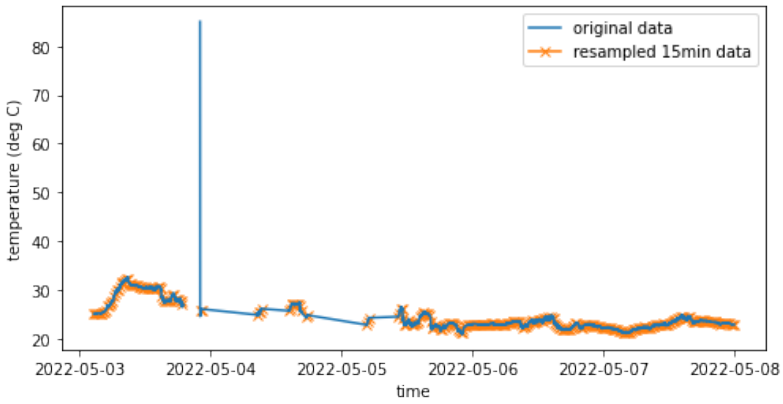


5.2 Resample

5.2.1 Downsampling

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,4))

# Downsample to spaced out data points. Change the number below, see what happens.
window_size = '15min'
df_resampled = (df['T2'].resample(window_size)  # resample doesn't do anything yet, just div
                        .mean()                 # this is where stuff happens. you can also
               )
# optional, add half a window size to timestamp
df_resampled.index = df_resampled.index + to_offset(window_size) / 2

ax.plot(df['T2'], color="tab:blue", label="original data")
ax.plot(df_resampled, marker='x', color="tab:orange", zorder=-1,
        label=f"resampled {window_size} data")
ax.legend()

ax.set(xlabel="time",
       ylabel="temperature (deg C)")
```

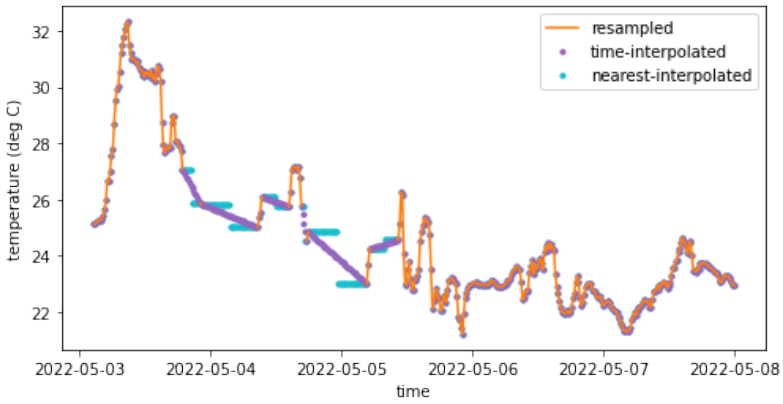
```
[Text(0.5, 0, 'time'), Text(0, 0.5, 'temperature (deg C)')]
```



5.2.2 Filling missing data

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,4))

# see options for interpolation methods here:
# https://pandas.pydata.org/docs/reference/api/pandas.DataFrame.interpolate.html
df_interpolated1 = df_resampled.interpolate(method='time')
df_interpolated2 = df_resampled.interpolate(method='nearest')

ax.plot(df_resampled, color="tab:orange", label="resampled")
ax.plot(df_interpolated1, '.', color="tab:purple", zorder=-1,
        label=f"time-interpolated")
ax.plot(df_interpolated2, '.', color="tab:cyan", zorder=-2,
        label=f"nearest-interpolated")
ax.legend()

ax.set(xlabel="time",
       ylabel="temperature (deg C)")
```

```
[Text(0.5, 0, 'time'), Text(0, 0.5, 'temperature (deg C)')]
```



5.3 Smoothing noisy data

Let's first download data from a different project.

```
filename2 = "test_peleg.csv"
# if file is not there, go fetch it from thingspeak
if not os.path.isfile(filename2):
    # define what to download
    channels = "1708067"
    fields = "1,2,3,4,5"
    minutes = "30"

    # https://www.mathworks.com/help/thingspeak/readdata.html
    # format YYYY-MM-DD%20HH:NN:SS
    start = "2022-05-15%2000:00:00"
    end = "2022-05-25%2000:00:00"

    # download using Thingspeak's API
    # url = f"https://api.thingspeak.com/channels/{channels}/fields/{fields}.csv?minutes={mi
    url = f"https://api.thingspeak.com/channels/{channels}/fields/{fields}.csv?start={start}
    data = urllib.request.urlopen(url)
    d = data.read()

    # save data to csv
    file = open(filename2, "w")
    file.write(d.decode('UTF-8'))
```

```
    file.close()
```

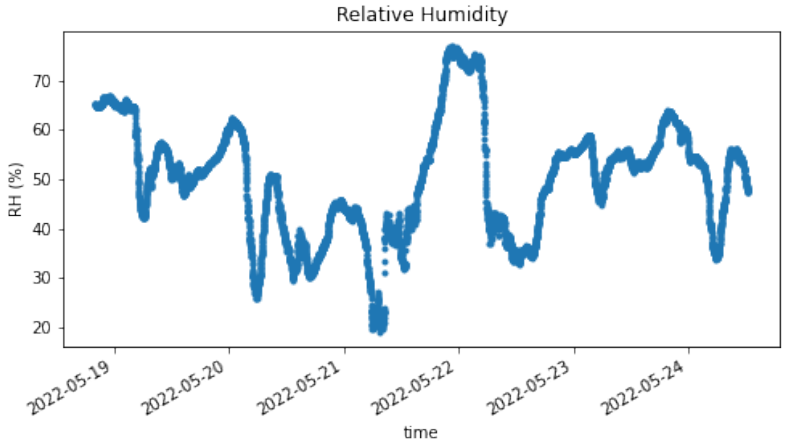
```
# load data
df = pd.read_csv(filename2)
# rename columns
df = df.rename(columns={"created_at": "timestamp",
                        "field1": "T",
                        "field2": "Tw",
                        "field3": "RH",
                        "field4": "VPD",
                        "field5": "dist",
                        })
# set timestamp as index
df['timestamp'] = pd.to_datetime(df['timestamp'])
df = df.set_index('timestamp')
```

```
df
```

timestamp	entry_id	T	Tw	RH	VPD	dist
2022-05-18 20:09:31+00:00	24716	23.85	23.3125	65.32	1.02532	7.208
2022-05-18 20:10:32+00:00	24717	23.88	23.2500	65.32	1.02717	7.208
2022-05-18 20:11:33+00:00	24718	23.90	23.2500	65.23	1.03107	7.276
2022-05-18 20:12:33+00:00	24719	23.90	23.2500	65.19	1.03226	7.208
2022-05-18 20:13:34+00:00	24720	23.89	23.2500	65.15	1.03282	7.633
...	...	...	...	...	...	...
2022-05-24 12:18:35+00:00	32711	27.47	26.1250	47.49	1.92397	8.925
2022-05-24 12:19:36+00:00	32712	27.47	26.1250	47.62	1.91921	8.925
2022-05-24 12:20:39+00:00	32713	27.47	26.1250	47.96	1.90675	8.925
2022-05-24 12:21:40+00:00	32714	27.47	26.1875	47.75	1.91444	8.925
2022-05-24 12:22:41+00:00	32715	27.49	26.1875	47.94	1.90971	8.925

5.4 Smoothing noisy data

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,4))

ax.plot(df['RH'], '.')
# add labels and title
ax.set(xlabel = "time",
       ylabel = "RH (%)",
       title = "Relative Humidity")
# makes slanted dates
plt.gcf().autofmt_xdate()
```

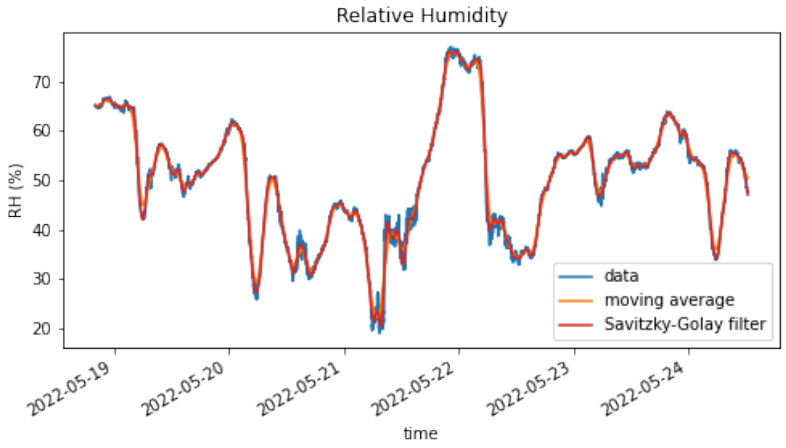


5.4.1 Moving average and SavGol

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,4))

# apply a rolling average of size "window_size",
# it can be either by number of points, or by window time
# window_size = 30  # number of measurements
window_size = '120min'  # minutes
```

```
RH_smooth = df['RH'].rolling(window_size, center=True).mean().to_frame()
RH_smooth.rename(columns={'RH': 'rolling_avg'}, inplace=True)

RH_smooth['SG'] = savgol_filter(df['RH'], window_length=121, polyorder=2)

ax.plot(df['RH'], color="tab:blue", label="data")
ax.plot(RH_smooth['rolling_avg'], color="tab:orange", label="moving average")
ax.plot(RH_smooth['SG'], color="tab:red", label="Savitzky-Golay filter")
# add labels and title
ax.set(xlabel = "time",
       ylabel = "RH (%)",
       title = "Relative Humidity")
# makes slanted dates
plt.gcf().autofmt_xdate()
ax.legend()
```

```
<matplotlib.legend.Legend at 0x7fe6a0525730>
```

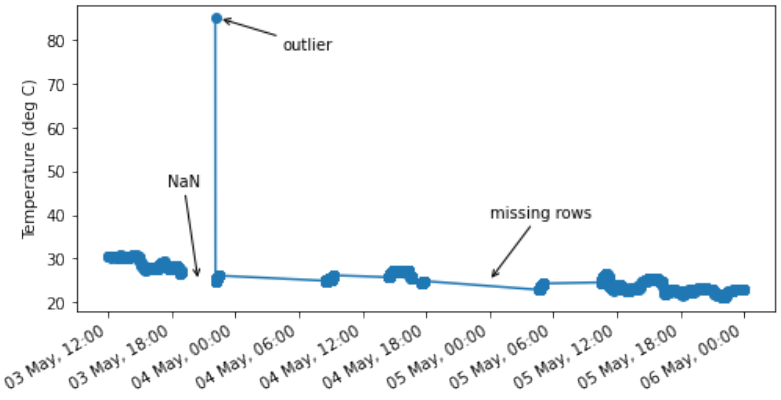


Part II

outliers and gaps

6 motivation

Outliers are observations significantly different from all other observations. Consider, for example, this temperature graph:



While most measured points are between 20 and 30 °C, there is obviously something very wrong with the one data point above 80 °C.

How could such a thing come about? This could be the result of non-natural causes, such as measurement errors, wrong data collection, or wrong data entry. On the other hand, this point could have natural sources, such as a very hot spark flying next to the temperature sensor.

Identifying outliers is important, because they might greatly impact measures like mean and standard deviation. When left untouched, outliers might make us reach wrong conclusions about our data. See what happens to the slope of this linear regression with and without the outliers.

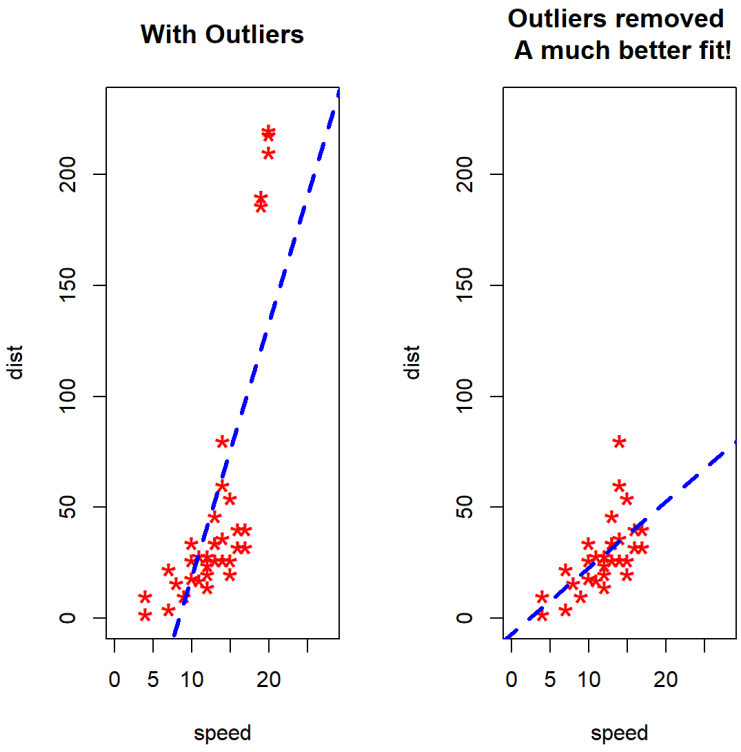


Figure 6.1: Source: Zhang (2020)

7 Z-score

$$z = \frac{x - \mu}{\sigma},$$

where

- x = data point,
- μ = time series mean
- σ = time series standard deviation.

Let's write a function that identifies outliers according to the Z-score.

```
def zscore(df, degree=3):
    data = df.copy()
    data['zscore'] = (data - data.mean())/data.std()
    outliers = data[(data['zscore'] <= -degree) | (data['zscore'] >= degree)]
    return outliers['value'], data
```

Now we can simply use this function:

```
threshold = 2.5
outliers, transformed = zscore(tx, threshold)
```

Source: Atwan (2022)

8 IQR (Inter Quartile Range)

The IQR (Inter Quartile Range) is the distance between the 25th percentile (Q1) and the 75th percentile (Q3). In a box plot, the whiskers usually extend 1.5×IQR beyond Q1 and Q3, see below.

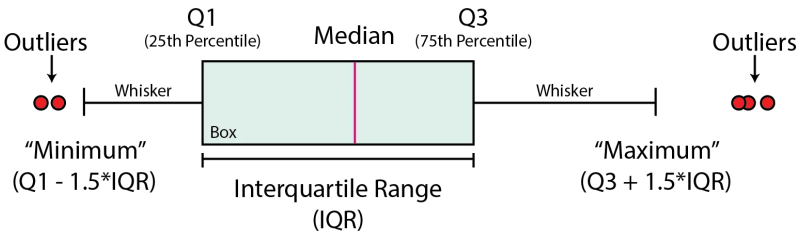


Figure 8.1: Source: McDonald (2022)

A common outlier detection method is to consider whatever points outside the whisker range as outliers.

Part III

resampling

9 motivation

10 upsampling, interpolation

11 downsampling

Part IV

best practices

12 motivation

13 dates

Part V

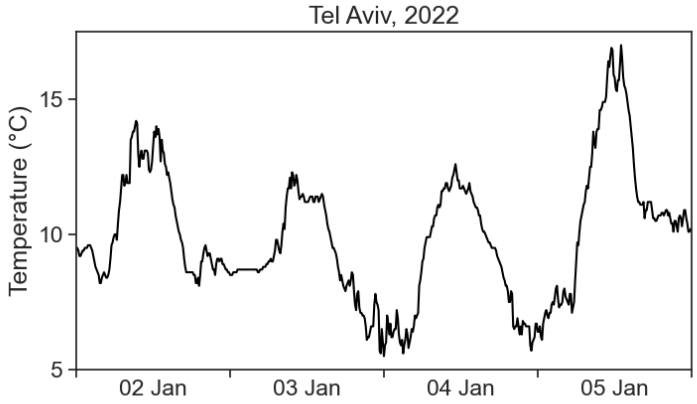
smoothing

14 motivation

15 convolution

Running windows of different shapes (kernels)

This is the temperature for Tel Aviv, between 2 and 5 of January 2022. Data is in intervals of 10 minutes, and was downloaded from the Israel Meteorological Service.



We see that the temperature curve has a rough profile. Can we find ways of getting smoother curves?

Convolution is a fancy word for averaging a time series using a running window. We will use the terms **convolution, running average, and rolling average** interchangeably. See the animation below. We take all temperature values inside a window of width 500 minutes (51 points), and average them with equal weights. The weights profile is called `kernel`.

The pink curve is much smoother than the original! However, the running average cannot describe sharp temperature

changes. If we decrease the window width to 200 minutes (21 points), we get the following result.

There is a tradeoff between the smoothness of a curve, and its ability to describe sharp temporal changes.

15.1 kernels

We can modify our running average, so that values closer to the center of the window have higher weights, and those further away count less. This is achieved by changing the weight profile, or the shape of the kernel. We see below the result of a running average using a triangular window of base 500 minutes (51 points).

Things can get as fancy as we want. Instead of a triangular kernel, which has sharp edges, we can choose a smoother gaussian kernel, see the difference below. We used a gaussian kernel with 60-minute standard deviation (the window in the animation is 4 standard deviations wide).

15.2 math

The definition of a convolution between signal $f(t)$ and kernel $k(t)$ is

$$(f * k)(t) = \int f(\tau)k(t-\tau)d\tau.$$

The expression $f * k$ denotes the convolution of these two functions. The argument of k is $t-\tau$, meaning that the kernel runs from left to right (as t does), and at every point the two signals (f and k) are multiplied together. It is the product of the signal with the weight function k that gives us an average. Because of $-\tau$, the kernel is flipped backwards, but this has no effect to symmetric kernels, like to ones in the examples above. Finally, the actual running average is not the convolution, but

$$\frac{(f * k)(t)}{\int k(t)dt}.$$

Whenever the integral of the kernel is 1, then the convolution will be identical with the running average.

15.3 numerics

Running averages are very common tools in time-series analysis. The `pandas` package makes life quite simple. For example, in order to calculate the running average of temperature using a rectangular kernel, one writes

```
df['temperature'].rolling(window='20', center=True).mean()
```

- `window=20` means that the width of the window is 20 points. Pandas lets us define a window width in time units, for example, `window='120min'`.
- `center=True` is needed in order to assign the result of averaging to the center of the window. Make it `False` and see what happens.
- `mean()` is the actual calculation, the average of temperature over the window. The `rolling` part does not compute anything, it just creates a moving window, and we are free to calculate whatever we want. Try to calculate the standard deviation or the maximum, for example.

It is implicit in the command above a "rectangular" kernel. What if we want other shapes?

15.3.1 gaussian

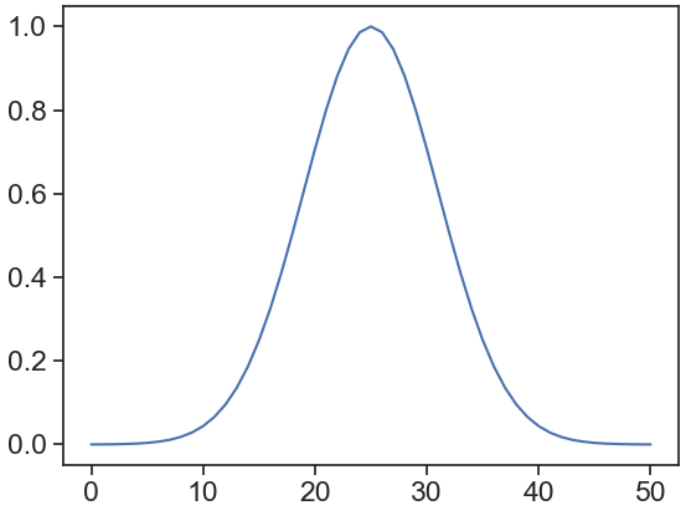
```
(
df['temperature'].rolling(window=window_width,
                          center=True,
                          win_type="gaussian")
```

```
                    .mean(std=std_gaussian)
)
```

where

- `window_width` is an integer, number of points in your window
- `std_gaussian` is the standard deviation of your gaussian, measured in sample points, not time!

For instance, if we have measurements every 10 minutes, and our window width is 500 minutes, then `window_width = 500/10 + 1` (first and last included). If we want a standard deviation of 60 minutes, then `std_gaussian = 6`. The gaussian kernel will look like this:



You can take a look at various options for kernel shapes here, provided by the `scipy` package. The graph above was achieved by running:

```
g = scipy.signal.gaussian(window_width, std)
plt.plot(g)
```

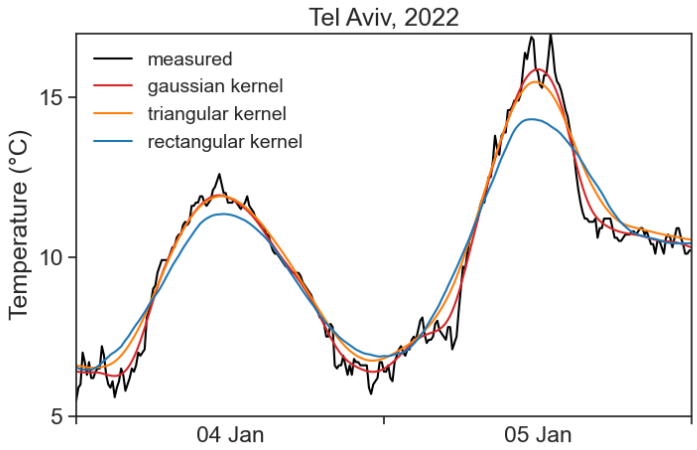
15.3.2 triangular

Same idea as gaussian, but simpler, because we don't need to think about standard deviation.

```
(
df['temperature'].rolling(window=window_width,
                          center=True,
                          win_type="triang")
                 .mean()
)
```

15.4 which window shape and width to choose?

Sorry, there is not definite answer here... It really depends on your data and what you need to do with it. See below a comparison of all examples in the videos above.



One important question you need to ask is: what are the time scales associated with the processes I'm interested in? For example, if I'm interested in the daily temperature pattern, getting rid of 1-minute-long fluctuations would probably be ok.

On the other hand, if we were to smooth the signal so much that all that can be seen are the temperature changes between summer and winter, then my smoothing got out of hand, and I threw away the very process I wanted to study.

All this is to say that you need to know in advance a few things about the system you are studying, otherwise you can't know what is "noise" that can be smoothed away.

16 LOESS

17 a perfect smoother

Source: Eilers (2003)

GitHub repository

Noisy series y of length m.

The smoothed series is called z.

We have conflicting interests:

- we want a z series "as smooth as possible".
- however, the smoother z is, the farthest from y it will be (low fidelity).

Roughness:

$$R = \sum_i (z_i - z_{i-1})^2$$

Fit to data:

$$S = \sum_i (y_i - z_i)^2$$

Cost functional to be minimized:

$$Q = S + \lambda R$$

Part VI

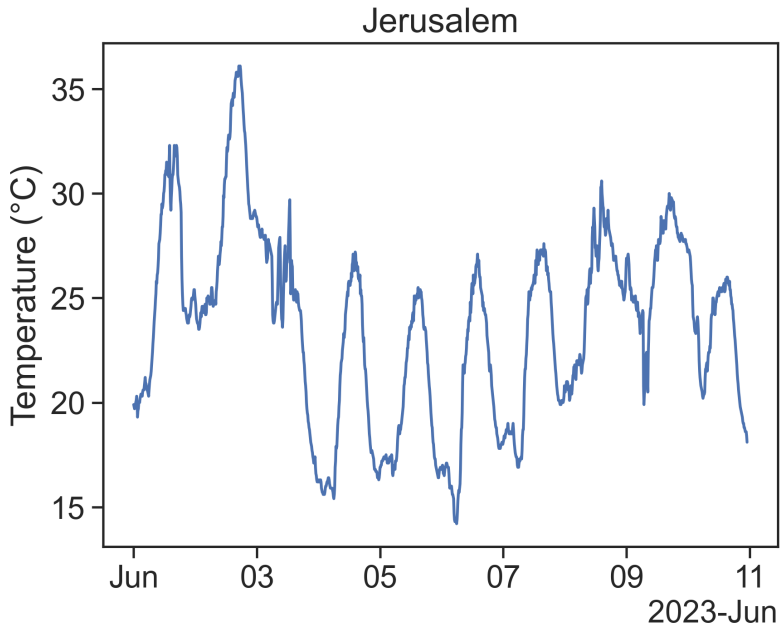
stationarity

18 motivation

19 stochastic processes

20 autocorrelation

See the temperatures for Jerusalem in a 4-day interval:



20.1 question

If I know the temperature right now, what does that tell me about the temperature 10 minutes from now? How about 100 minutes? 1000 minutes?

To answer this, we need to talk about **autocorrelation**. Let's start by introducing the necessary concepts.

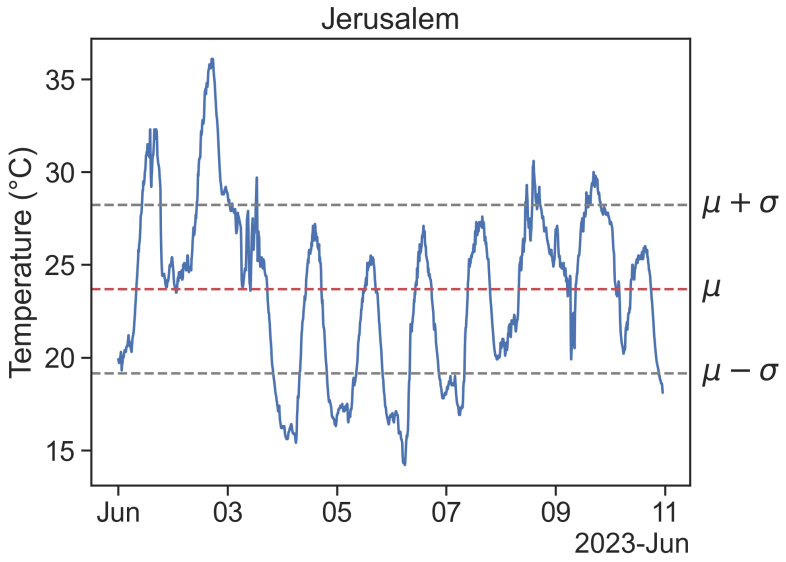
20.2 mean and standard deviation

Let's call our time series from above X, and its length N. Then:

$$\text{mean} \quad \mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

$$\text{standard deviation} \quad \sigma = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}}$$

The mean and standard deviation can be visualized thus:



One last basic concept we need is the expected value:

$$E[X] = \sum_{i=1}^{N} X_i p_i$$

For our time series, the probability p_i that a given point X_i is in the dataset is simply $1/N$, therefore the expectation becomes

$$E[X] = \frac{\sum_{i=1}^{N} X_i}{N}$$

20.3 autocorrelation

The autocorrelation of a time series X is the answer to the following question:

> if we shift X by τ units, how similar will this be to the original signal?

In other words:

> how correlated are $X(t)$ and $X(t+\tau)$?

Using the Pearson correlation coefficient

we get

$$\rho_{XX}(\tau) = \frac{E\left[(X_t - \mu)(X_{t+\tau} - \mu)\right]}{\sigma^2}$$

Pearson correlation coefficient between X and Y:

$$\rho_{X,Y} = \frac{E\left[(X - \mu_X)(X_Y - \mu_Y)\right]}{\sigma_X \sigma_Y}$$

A video is worth a billion words, so let's see the autocorrelation in action:

https://youtu.be/tpf-tuYHR5w

A few comments:

- The autocorrelation for $\tau = 0$ (zero shift) is always 1. [Can you prove this? All the necessary equations are above!]

Part VII

time lags

21 motivation

22 cross-correlation

```
import numpy as np
```

```
print('dfvdfv')
```

```
dfvdfv
```

23 dynamic time warp

24 LDTW

according to this paper

Part VIII

frequency

25 Motivation

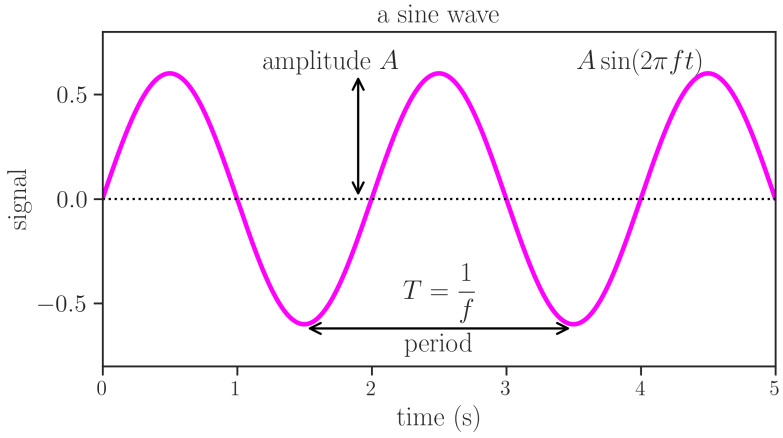
26 Fourier transform

26.1 basic wave concepts

The function

$$f(t) = B\sin(2\pi f t) \tag{26.1}$$

has two basic characteristics, its amplitude B and frequency f.

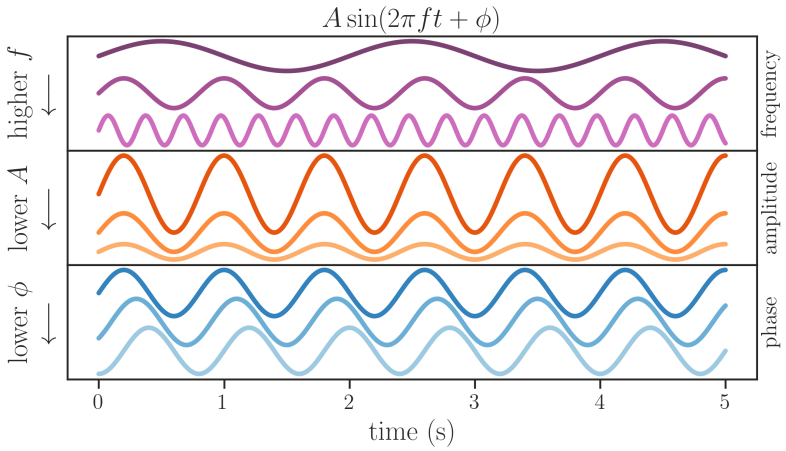


In the figure above, the amplitude $B = 0.6$ and we see that the distance between two peaks is called period, $T = 2$ s. The frequency is defined as the inverse of the period:

$$f = \frac{1}{T}. \tag{26.2}$$

When time is in seconds, then the frequency is measured in Hertz (Hz). For the graph above, therefore, we see a wave whose frequency is $f = 1/(2\text{ s}) = 0.5$ Hz.

In the figure below, we see what happens when we vary the values of the frequency and amplitude.



The graph above introduces two new characteristics of a wave, its phase ϕ, and its offset B. A more general description of a sine wave is

$$f(t) = B\sin(2\pi ft + \phi) + B_0. \tag{26.3}$$

The offset B_0 moves the wave up and down, while changing the value of ϕ makes the sine wave move left and right. When the phase $\phi = 2\pi$, the sine wave will have shifted a full period, and the resulting wave is identical to the original:

$$B\sin(2\pi ft) = B\sin(2\pi ft + 2\pi). \tag{26.4}$$

All the above can also be said about a cosine, whose general for can be given as

$$A\cos(2\pi ft + \phi) + A_0 \tag{26.5}$$

One final point before we jump into the deep waters is that the sine and cosine functions are related through a simple phase shift:

$$\cos\left(2\pi ft + \frac{\pi}{2}\right) = \sin\left(2\pi ft\right)$$

26.2 Fourier's theorem

Fourier's theorem states that

> Any periodic signal is composed of a superposition of pure sine waves, with suitably chosen amplitudes and phases, whose frequencies are harmonics of the fundamental frequency of the signal.

See the following animations to visualize the theorem in action.

Source: https://en.wikipedia.org/wiki/File:Fourier_series_and_transform.gif

Source: https://commons.wikimedia.org/wiki/File:Fourier_synthesis_square_wave_animated.gif

Source: https://commons.wikimedia.org/wiki/File:Sawtooth_Fourier_Animation.gif

Source: https://commons.wikimedia.org/wiki/File:Continuous_Fourier_transform_of_rect_and_sinc_functions.gif

26.3 Fourier series

> a periodic function can be described as a sum of sines and cosines.

The classic examples are usually the square function and the sawtooth function:

[Source: https://www.geogebra.org/m/tkajbzmg]

https://www.geogebra.org/m/k4eq4fkr

Not any function, but certainly most functions we will deal with in this course. The function has to fullful the Dirichlet conditions

$$F[x(t)] = F(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt$$

$$f(t) = \int_{-\infty}^{\infty} F(f) e^{2\pi i f t} df$$

https://dibsmethodsmeetings.github.io/fourier-transforms/

https://www.jezzamon.com/fourier/index.html

27 filtering

28 Nyquist-Shannon sampling theorem

Part IX

seasonality

29 motivation

30 seasonal decomposition

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()  # datetime converter for a matplotlib
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)
from statsmodels.tsa.seasonal import seasonal_decompose
import matplotlib.dates as mdates
from matplotlib.dates import DateFormatter
```

30.1 trends in atmospheric carbon dioxide

Mauna Loa CO2 concentration.
data from NOAA

```
url = "https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_weekly_mlo.csv"
df = pd.read_csv(url, header=47, na_values=[-999.99])

# you can first download, and then read the csv
# filename = "co2_weekly_mlo.csv"
# df = pd.read_csv(filename, header=47, na_values=[-999.99])

df
```

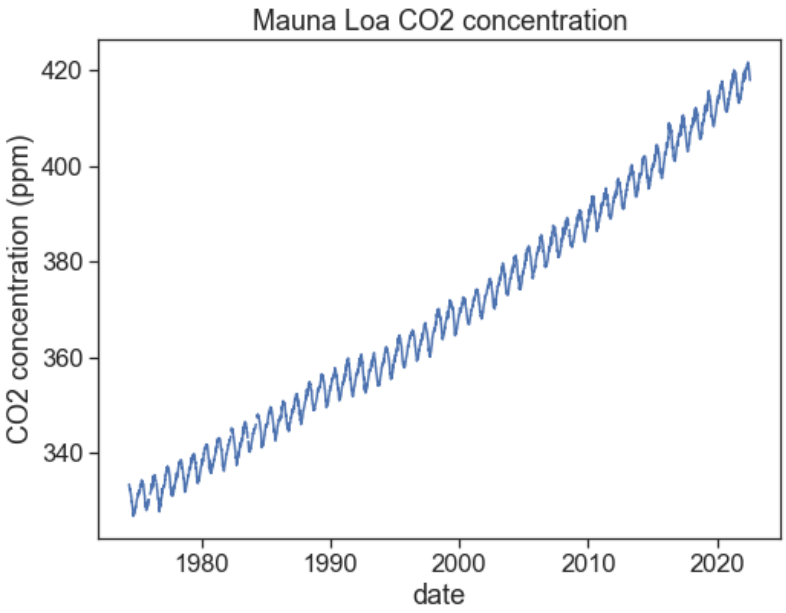
	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
0	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.40
1	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.06
2	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.60

	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
3	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.65
4	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...	...	...	...	...	...	...	...	...	...
2510	2022	6	26	2022.4836	420.31	7	418.14	395.36	138.71
2511	2022	7	3	2022.5027	419.73	6	417.49	395.15	138.64
2512	2022	7	10	2022.5219	419.08	6	417.25	394.59	138.52
2513	2022	7	17	2022.5411	418.43	6	417.14	394.64	138.41
2514	2022	7	24	2022.5603	417.84	6	415.68	394.11	138.36

```
df['date'] = pd.to_datetime(df[['year', 'month', 'day']])
df = df.set_index('date')
df
```

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
1974-05-19	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.40
1974-05-26	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.06
1974-06-02	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.60
1974-06-09	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.65
1974-06-16	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...	...	...	...	...	...	...	...	...	...
2022-06-26	2022	6	26	2022.4836	420.31	7	418.14	395.36	138.71
2022-07-03	2022	7	3	2022.5027	419.73	6	417.49	395.15	138.64
2022-07-10	2022	7	10	2022.5219	419.08	6	417.25	394.59	138.52
2022-07-17	2022	7	17	2022.5411	418.43	6	417.14	394.64	138.41
2022-07-24	2022	7	24	2022.5603	417.84	6	415.68	394.11	138.36

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,6))
ax.plot(df['average'])
ax.set(xlabel="date",
       ylabel="CO2 concentration (ppm)",
       # ylim=[0, 430],
       title="Mauna Loa CO2 concentration");
```



fill missing data. interpolate method: 'time'
interpolation methods visualized

```
df['co2'] = (df['average'].resample("D") #resample daily
                          .interpolate(method='time') #interpolate by time
             )
df
```

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
1974-05-19	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.40
1974-05-26	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.06
1974-06-02	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.60
1974-06-09	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.65
1974-06-16	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...	...	...	...	...	...	...	...	...	...
2022-06-26	2022	6	26	2022.4836	420.31	7	418.14	395.36	138.71
2022-07-03	2022	7	3	2022.5027	419.73	6	417.49	395.15	138.64
2022-07-10	2022	7	10	2022.5219	419.08	6	417.25	394.59	138.52
2022-07-17	2022	7	17	2022.5411	418.43	6	417.14	394.64	138.41

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
2022-07-24	2022	7	24	2022.5603	417.84	6	415.68	394.11	138.36

30.2 decompose data

`seasonal_decompose` returns an object with four components:

- observed: $Y(t)$
- trend: $T(t)$
- seasonal: $S(t)$
- resid: $e(t)$

Additive model:

$$Y(t) = T(t) + S(t) + e(t)$$

Multiplicative model:

$$Y(t) = T(t) \times S(t) \times e(t)$$

30.2.0.1 Interlude

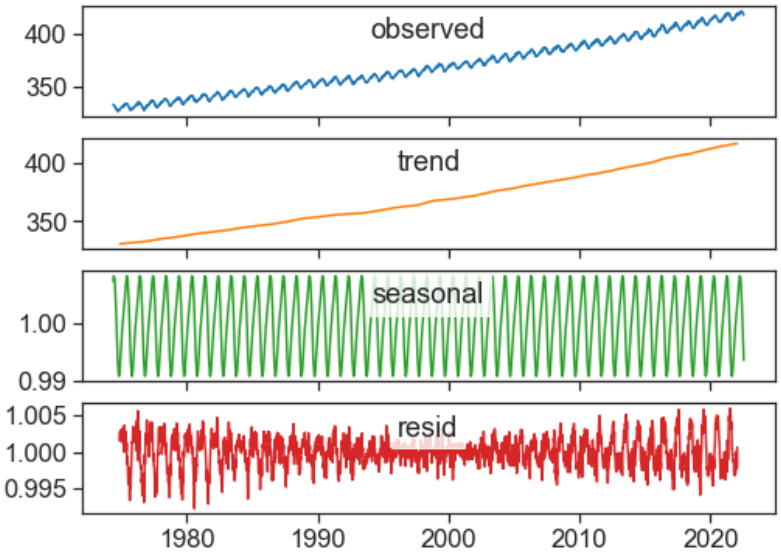
learn how to use `zip` in a loop

```
letters = ['a', 'b', 'c', 'd', 'e']
numbers = [1, 2, 3, 4, 5]
# zip let's us iterate over to lists at the same time
for l, n in zip(letters, numbers):
    print(f"{l} = {n}")
```

```
a = 1
b = 2
c = 3
d = 4
e = 5
```

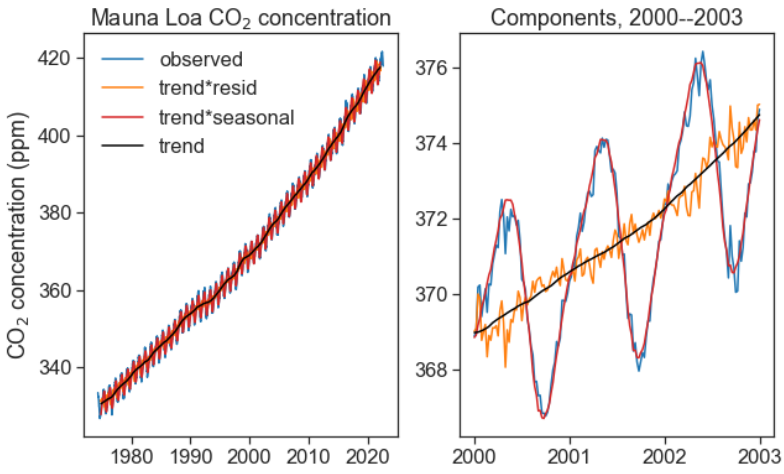
Plot each component separately.

```
# %matplotlib widget

fig, ax = plt.subplots(4, 1, figsize=(8,6), sharex=True)
decomposed_m = seasonal_decompose(df['co2'], model='multiplicative')
decomposed_a = seasonal_decompose(df['co2'], model='additive')
decomposed = decomposed_m
pos = (0.5, 0.9)
components =["observed", "trend", "seasonal", "resid"]
colors = ["tab:blue", "tab:orange", "tab:green", "tab:red"]
for axx, component, color in zip(ax, components, colors):
    data = getattr(decomposed, component)
    axx.plot(data, color=color)
    axx.text(*pos, component, bbox=dict(facecolor='white', alpha=0.8),
           transform=axx.transAxes, ha='center', va='top')
```




```
# %matplotlib widget

decomposed = decomposed_m

fig, ax = plt.subplots(1, 2, figsize=(10,6))
ax[0].plot(df['co2'], color="tab:blue", label="observed")
ax[0].plot(decomposed.trend * decomposed.resid, color="tab:orange", label="trend*resid")
```

```
ax[0].plot(decomposed.trend * decomposed.seasonal, color="tab:red", label="trend*seasonal")
ax[0].plot(decomposed.trend, color="black", label="trend")
ax[0].set(ylabel="CO$_2$ concentration (ppm)",
          title="Mauna Loa CO$_2$ concentration")
ax[0].legend(frameon=False)

start = "2000-01-01"
end = "2003-01-01"
zoom = slice(start, end)
ax[1].plot(df.loc[zoom, 'co2'], color="tab:blue", label="observed")
ax[1].plot((decomposed.trend * decomposed.resid)[zoom], color="tab:orange", label="trend*res
ax[1].plot((decomposed.trend * decomposed.seasonal)[zoom], color="tab:red", label="trend*sea
ax[1].plot(decomposed.trend[zoom], color="black", label="trend")
date_form = DateFormatter("%Y")
ax[1].xaxis.set_major_formatter(date_form)
ax[1].xaxis.set_major_locator(mdates.YearLocator(1))
ax[1].set_title("Components, 2000--2003");
```



31 Hilbert transform

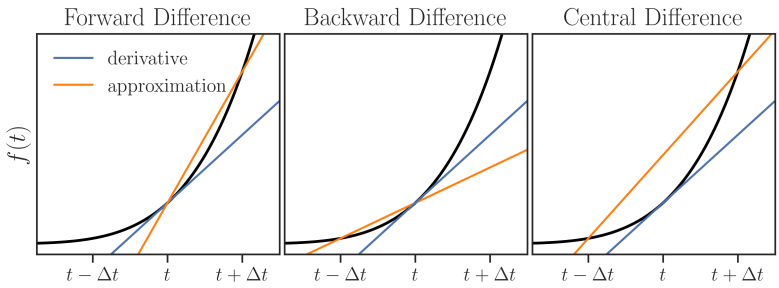
Part X

rates of change

32 motivation

33 derivatives

34 finite differences



Definition of a **derivative**:

$$\underbrace{\dot{f} = f'(t) = \frac{df(t)}{dt}}_{\text{same thing}} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}.$$

Numerically, we can approximate the derivative $f'(t)$ of a time series $f(t)$ as

$$\frac{df(t)}{dt} = \frac{f(t+\Delta t) - f(t)}{\Delta t} + \mathcal{O}(\Delta t). \tag{34.1}$$

The expression above is called the *two-point forward difference formula*. Likewise, we can define the *two-point backward difference formula*:

The expression $\mathcal{O}(\Delta t)$ means that the error associated with the approximation is proportional to Δt. This is called "Big O notation".

$$\frac{df(t)}{dt} = \frac{f(t) - f(t-\Delta t)}{\Delta t} + \mathcal{O}(\Delta t). \tag{34.2}$$

If we sum together Equation 34.1 and Equation 34.2 we get:

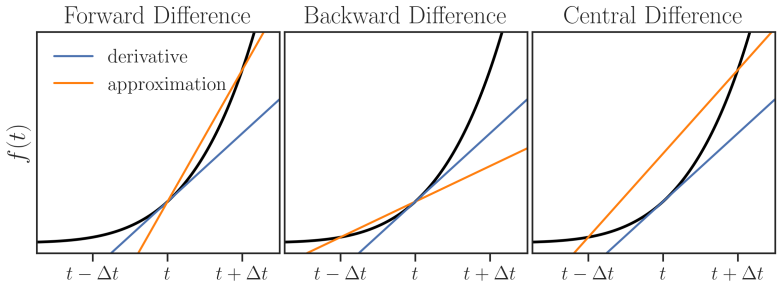
$$\begin{aligned} 2\frac{df(t)}{dt} &= \frac{f(t+\Delta t) - \cancel{f(t)}}{\Delta t} + \frac{\cancel{f(t)} - f(t-\Delta t)}{\Delta t} \\ &= \frac{f(t+\Delta t) - f(t-\Delta t)}{\Delta t}. \end{aligned} \tag{34.3}$$

Dividing both sides by 2 gives the *two-point central difference formula*:

$$\frac{df(t)}{dt} = \frac{f(t+\Delta t) - f(t-\Delta t)}{2\Delta t} + \mathcal{O}(\Delta t^2). \tag{34.4}$$

Two things are worth mentioning about the approximation above:

1. it is balanced, that is, there is no preference of the future over the past.
2. its error is proportional to Δt^2, it is a lot more precise than the unbalanced approximations :)



To understand why the error is proportional to Δt^2, one can subtract the Taylor expansion of $f(t-\Delta t)$ from the Taylor expansion of $f(t+\Delta t)$. See this, pages 3 and 4.

The function `np.gradient` calculates the derivative using the central difference for points in the interior of the array, and uses the forward (backward) difference for the derivative at the beginning (end) of the array.

Check out this nice example.

The "gradient" usually refers to a first derivative with respect to space, and it is denoted as $\nabla f(x) = \frac{df(x)}{dx}$. However, it doesn't really matter if we call the independent variable x or t, the derivative operator is exactly the same.

35 Fourier-based derivatives

This tutorial is based on Pelliccia (2019).

nice trick: https://math.stackexchange.com/questions/430858/fourier-transform-of-derivative

36 LOESS-based derivatives

Part XI

forecasting

37 motivation

38 ARIMA

technical stuff

operating systems

I recommend working with UNIX-based operating systems (MacOS or Linux). Everything is easier.

If you use Windows, consider installing Linux on Windows with WSL.

software

Anaconda's Python distribution

VSCode

python packages

Kats — a one-stop shop for time series analysis
Developed by Meta

statsmodels statsmodels is a Python package that provides a complement to scipy for statistical computations including descriptive statistics and estimation and inference for statistical models.

ydata-profiling
Quick Exploratory Data Analysis on time-series data. Read also this.

sources

books

from Data to Viz

Fundamentals of Data Visualization, by Claus O. Wilke

PyNotes in Agriscience

Forecasting: Principles and Practice (3rd ed), by Rob J Hyndman and George Athanasopoulos

Python for Finance Cookbook 2nd Edition - Code Repository

Practical time series analysis,: prediction with statistics and machine learning, by Aileen Nielsen
The online edition of this book is available for Hebrew University staff and students.

Time series analysis with Python cookbook : practical recipes for exploratory data analysis, data preparation, forecasting, and model evaluation, by Tarek A. Atwan
The online edition of this book is available for Hebrew University staff and students.

Hands-on Time Series Analysis with Python: From Basics to Bleeding Edge Techniques, by B V Vishwas, Ashish Patel
The online edition of this book is available for Hebrew University staff and students.

videos

Times Series Analysis for Everyone, by Bruno Goncalves
This series is available for Hebrew University staff and students.

Time Series Analysis with Pandas, by Joshua Malina This video is available for Hebrew University staff and students.

references

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