Time Series Analysis

Yair Mau

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about

Welcome to **Time Series Analysis for Environmental Sciences** (71606) at the Hebrew University of Jerusalem. This is Yair Mau, your host for today. I am a senior lecturer at the Institute of Environmental Sciences, at the Faculty of Agriculture, Food and Environment, in Rehovot, Israel.

This website contains (almost) all the material you'll need for the course. If you find any mistakes, or have any comments, please email me.

disclaimer

The material here is not comprehensive and `does not` constitute a stand alone course in Time Series Analysis. This is only the support material for the actual presential course I give.

what, who, when and where?

Course number 71606, 3 academic points
Yair Mau (lecturer), Erez Feuer (TA)
Tuesdays, from 14:15 to 17:00
Computer classroom #18
Office hours: upon request

syllabus

course description

Data analysis of time series, with practical examples from environmental sciences.

course aims

This course aims at giving the students a broad overview of the main steps involved in the analysis of time series: data management, data wrangling, visualization, analysis, and forecast. The course will provide a hands-on approach, where students will actively engage with real-life datasets from the field of environmental science.

learning outcomes

On successful completion of this module,students should be able to:

- Explore a time-series dataset, while formulating interesting questions.
- Choose the appropriate tools to attack the problem and answer the questions.
- Communicate their findings and the methods they used to achieve them, using graphs, statistics, text, and a well-documented code.

course content

- **Data wrangling:** organization, cleaning, merging, filling gaps, excluding outliers, smoothing, resampling.
- **Visualization:** best practices for graph making using leading python libraries.
- **Analysis:** stationarity, seasonality, (auto)correlations, lags, derivatives, spectral analysis.
- **Forecast:** ARIMA
- **Data management:** how to plan ahead and best organize large quantities of data. If there is enough time, we will build a simple time-series database.

books and other sources

Click here.

course evaluation

There will be 2 projects during the semester (each worth 25% of the final grade), and one final project (50%).

weekly program

week 1

- **Lecture:** Course overview, setting of expectations. Introduction, basic concepts, continuous vs discrete time series, sampling, aliasing
- **Exercise:** Loading csv file into python, basic time series manipulation with pandas and plotting

week 2

- **Lecture:** Filling gaps, removing outliers
- **Exercise:** Practice the same topics learned during the lecture. Data: air temperature and relative humidity

week 3

- **Lecture:** Interpolation, resampling, binning statistics
- **Exercise:** Practice the same topics learned during the lecture. Data: air temperature and relative humidity, precipitation

week 4

- **Lecture:** Time series plotting: best practices. Dos and don'ts and maybes
- **Exercise:** Practice with Seaborn, Plotly, Pandas, Matplotlib

Project 1
Basic data wrangling, using real data (temperature, relative humidity, precipitation) downloaded from USGS. 25% of the final grade

week 5

- **Lecture:** Smoothing, running averages, convolution
- **Exercise:** Practice the same topics learned during the lecture. Data: sap flow, evapotranspiration

week 6

- **Lecture:** Strong and weak stationarity, stochastic processes, auto-correlation
- **Exercise:** Practice the same topics learned during the lecture. Data: temperature and wind speed

week 7

- **Lecture:** Correlation between signals. Pearson correlation, time-lagged cross-correlations, dynamic time warping
- **Exercise:** Practice the same topics learned during the lecture. Data: temperature, solar radiation, relative humidity, soil moisture, evapotranspiration

week 8

Same as lecture 7 above

week 9

- **Lecture:** Download data from repositories, using API, merging, documentation
- **Exercise:** Download data from USGS, NOAA, Fluxnet, Israel Meteorological Service

Project 2
Students will study a Fluxnet site of their choosing. How do gas fluxes (CO2, H2O) depend on environmental conditions? 25% of the final grade

week 10

- **Lecture:** Fourier decomposition, filtering, Nyquist–Shannon sampling theorem
- **Exercise:** Practice the same topics learned during the lecture. Data: dendrometer data

week 11

- **Lecture:** Seasonality, seasonal decomposition (trend, seasonal, residue), Hilbert transform
- **Exercise:** Practice the same topics learned during the lecture. Data: monthly atmospheric CO2 concentration, hourly air temperature

week 12

- **Lecture:** Derivatives, differencing
- **Exercise:** Practice the same topics learned during the lecture. Data: dendrometer data

week 13

- **Lecture:** Forecasting. ARIMA
- **Exercise:** Practice the same topics learned during the lecture. Data: vegetation variables (sap flow, ET, DBH, etc)

Final Project
In consultation with the lecturer, students will ask a specific scientific question about a site of their choosing (from NOAA, USGS, Fluxnet), and answer it using the tools learned during the semester. The report will be written in Jupyter Notebook,

combining in one document all the calculations, documentation, figures, analysis, and discussion. 50% of the final grade.

who cares?

why "Time Series Analysis?"

Time has two aspects. There is the arrow, the running river, without which there is no change, no progress, or direction, or creation. And there is the circle or the cycle, without which there is chaos, meaningless succession of instants, a world without clocks or seasons or promises.
URSULA K. LE GUIN

You are here because you are interested in how things change, evolve. In this course I want to discuss with you how to make sense of data whose temporal nature is in its very essence. We will talk about randomness, cycles, frequencies, correlations, and more.

why "Environmental Sciences"

This same time series analysis (TSA) course could be called instead "TSA for finance", "TSA for Biology", or any other application. The emphasis in this course is **not** Environmental Sciences, but the concepts and tools of TSA. Because my research is in Environmental Science, and many of the graduate students at HUJI-Rehovot research this, I chose to use examples "close to home". The same toolset should be useful for students of other disciplines.

what is it good for?

In many fields of science we are flooded by data, and it's hard to see the forest for the trees. I hope that the topics we'll

discuss in this course can help you find meaningful patterns in your data, formulate interesting hypotheses, and design better experiments.

do I need it?

Maybe. If you are a grad student and you have temporal data to analyze, then probably yes. However, I have very fond memories of courses that I took as a grad student that were completely unrelated to my research. Sometimes "because it's fun" is a perfectly good answer.

what will I actually gain from it?

By the end of this course you will have gained:

- a **hands-on** experience of fundamental time-series analysis tools
- an **intuition** regarding the basic concepts
- **technical** abilities
- a **springboard** for learning more about the subject by yourself

Part I

start here

1 the boring stuff you absolutely need to do

I assume everyone registered has taken a basic Python course. On your computer, do the following:

1.1 Anaconda

Install Anaconda's Python distribution. The Anaconda installation brings with it all the main python packages we will need to use. In order to install extra packages, refer to these two tutorials: tutorial 1, tutorial 2.

1.2 VSCode

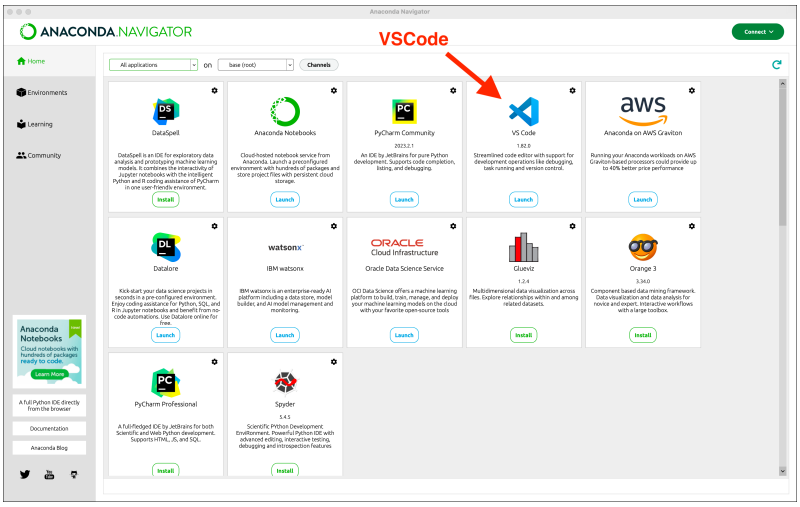
Install VSCode. Visual Studio Code is a very nice IDE (Integrated Development Environment) made by Microsoft, available to all operating systems. Contrary to the title of this page, it is not absolutely necessary to use it, but I like VSCode, and as my student, so do you .

1.3 jupyter notebooks

We will code exclusively in Jupyter Notebooks. Get acquainted with them. Make sure you can point VSCode to the Anaconda environment of your choice ("base" by default). Don't worry, this is easier than it sounds.

One failproof way of making sure VSCode uses the Anaconda installation is the following:

- Open Anaconda Navigator
- If you are using HUJI's computers, in "Environments", choose "asgard". If you are using your own computer, ignore this step.
- open VSCode from inside Anaconda Navigator (see image below).



Sometimes you will need to manualy install the Jupyter extension on VSCode. In this case follow this tutorial.

1.4 folder structure

You **NEED** to be confortable with you computer's folder (or directory) structure. Where are files located? How to navigate through different folders? How is my stuff organized? If you don't feel **absolutely** comfortable with this, then read this, Windows, MacOS. If you use Linux then you surely know this stuff. **Make yourself a "time-series" folder** wherever you want, and have it backed up regularly (use Google Drive, Dropbox, do it manually, etc). "My dog deleted my files" is not an excuse.

2 numpy, pandas, matplotlib

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

The three lines above are the most common way you will start every project in this course.

- **numpy** = numerical python. This library has a ton useful mathematical functions, and most importantly, it has an object called `numpy array`, which is one of the most useful data structures we have for time series analysis.
- **pandas** is built upon numpy, and allows us to easily manipulate data stored in `dataframes`, a fancy name for a table.
- **pyplot** is a submodule of `matplotlib`, and allows us to beautifully plot data.

The best resource I know to get acquainted with all three packages is Python Data Science Handbook, by Jake VanderPlas. This is a free online book, with excellent step by step examples.

2.1 pandas

We will primarily use the Pandas package to deal with data. Pandas has become the standard Python tool to manipulate time series, and you should get acquainted with its basic usage. This course will provide you the opportunity to learn by example, but I'm sure we will only scratch the surface, and you'll be left with lots of questions.

I provide below a (non-comprehensive) list of useful tutorials, they are a good reference for the beginner and for the experienced user.

- Python Data Science Handbook, by Jake VanderPlas
- Data Wrangling with pandas Cheat Sheet
- Working with Dates and Times in Python
- Cheat Sheet: The pandas DataFrame Object
- YouTube tutorials by Corey Schafer

2.2 pyplot

Matplotlib, and its submodule pyplot, are probably the most common Python plotting tool. Pyplot is both great and horrible:

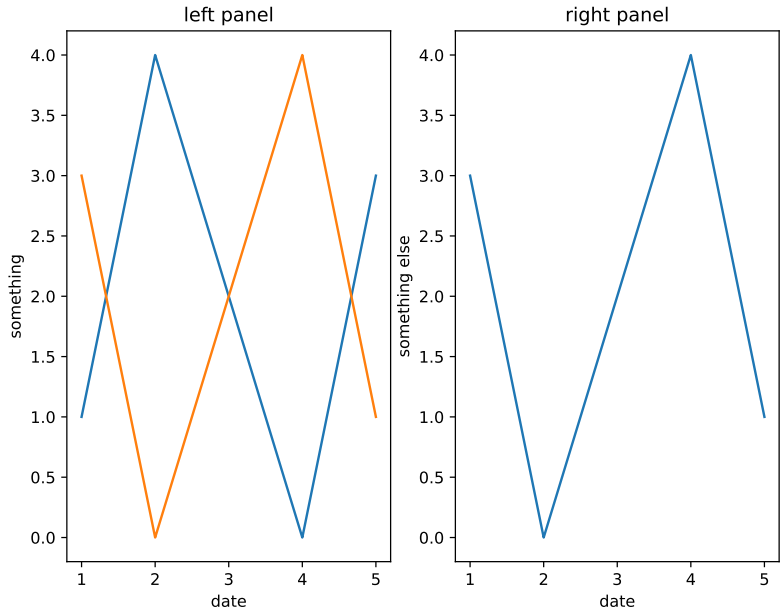
- Great: you'll have absolutely full control of everything you want to plot. The sky is the limit.
- Horrible: you'll cry as you do it, because there is so much to know, and it is not the most friendly plotting package.

Pyplot is *object oriented*, so you will usually manipulate the **axes** object like this.

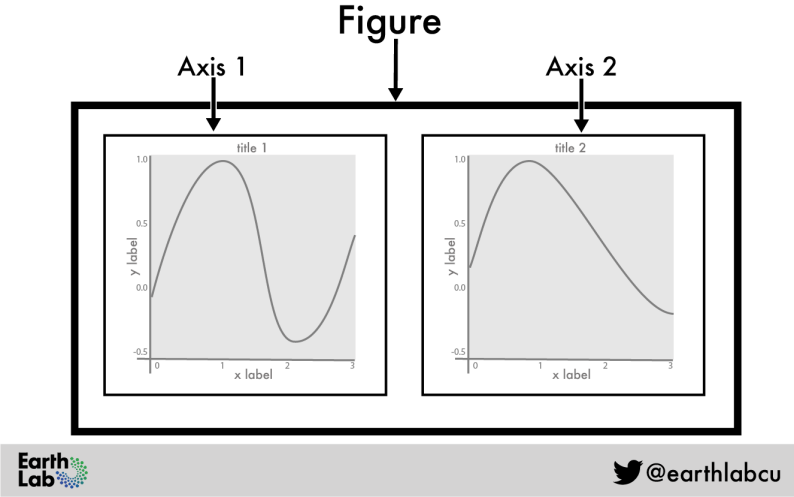
```
import matplotlib.pyplot as plt

x = [1, 2, 3, 4, 5]
y = [1, 4, 2, 0, 3]

# Figure with two plots
fig, (ax1, ax2) = plt.subplots(1, 2, figsize = (8, 6))
# plot on the left
ax1.plot(x, y, color="tab:blue")
ax1.plot(x, y[::-1], color="tab:orange")
ax1.set(xlabel="date",
        ylabel="something",
        title="left panel")
# plot on the right
ax2.plot(x, y[::-1])
ax2.set(xlabel="date",
```

```
        ylabel="something else",
        title="right panel")
```

```
[Text(0.5, 0, 'date'),
 Text(0, 0.5, 'something else'),
 Text(0.5, 1.0, 'right panel')]
```



For the very beginners, you need to know that `figure` refers to the whole white canvas, and `axes` means the rectangle inside which something will be plotted:



If you are new to all this, I recommend that you go to:

- Earth Lab's Introduction to Plotting in Python Using Matplotlib
- Jake VanderPlas's Python Data Science Handbook

3 learn by example

Now that everything is installed, try to run the code below *before* the first lecture. Don't worry if you don't understand everything.

- If you manage to run everything without errors, this means that your computer is good to go!
- You might encounter a few problems. That's ok. Make a note and we will solve everything in the first lecture.

Let's make a first plot of real data. We will use NOAA's Global Monitoring Laboratory data on Trends in Atmospheric Carbon Dioxide.

3.1 open a new Jupyter Notebook

1. On your computer, open the program `Anaconda Navigator` (it may take a while to load).
2. Find the white box called `VS Code` and click `Launch`.
3. Now go to `File` > `Open Folder`, and open the folder you created for this course. VS Code may ask you if you trust the authors, and the answer is "yes" (it's your computer).
4. `File` > `New File`, and call it `example.ipynb`
5. You can start copying and pasting code from this website to your Jupyter Notebook. To run a cell, press Shift+Enter.
6. You may be asked to choose to Select Kernel. This is VS Code wanting to know which python installation to use. Click on "Python Environments", and then choose the option with the word `anaconda` in it.
7. That's all! Congratulations!

3.2 import packages

First, import packages to be used. They should all be already included in the Anaconda distribution you installed.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)  # white graphs, with large and legible letters
```

3.3 load data

Load CO2 data into a Pandas dataframe. You can load it directly from the URL (option 1), or first download the CSV to your computer and then load it (option 2). The link to download the data directly form NOAA is this. If for some reason this doesn't work, download from this website's GitHub repository.

```
# option 1: load data directly from URL
# url = "https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_weekly_mlo.csv"
# df = pd.read_csv(url,
#                  header=34,
#                  na_values=[-999.99]
#                  )

# option 2: download first (use the URL above and save it to your computer), then load csv
filename = "co2_weekly_mlo.csv"
df = pd.read_csv(filename,
                 comment='#',  # will ignore rows starting with #
                  na_values=[-999.99]  # substitute -999.99 for NaN (Not a Number), data not
                  )
# check how the dataframe (table) looks like
df
```

	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
0	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.39
1	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.05
2	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.59
3	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.64
4	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...	...	...	...	...	...	...	...	...	...
2566	2023	7	23	2023.5575	421.28	4	418.03	397.30	141.60
2567	2023	7	30	2023.5767	420.83	6	418.10	396.80	141.69
2568	2023	8	6	2023.5959	420.02	6	417.36	395.65	141.41
2569	2023	8	13	2023.6151	418.98	4	417.25	395.24	140.89
2570	2023	8	20	2023.6342	419.31	2	416.64	395.22	141.71

3.4 dealing with dates

Create a new column called `date`, that combines the information from three separate columns: `year`, `month`, `day`.

```
# function to_datetime translates the full date into a pandas datetime object,
# that is, pandas knows this is a date, it's not just a string
df['date'] = pd.to_datetime(df[['year', 'month', 'day']])
# make 'date' column the dataframe index
df = df.set_index('date')
# now see if everything is ok
df
```

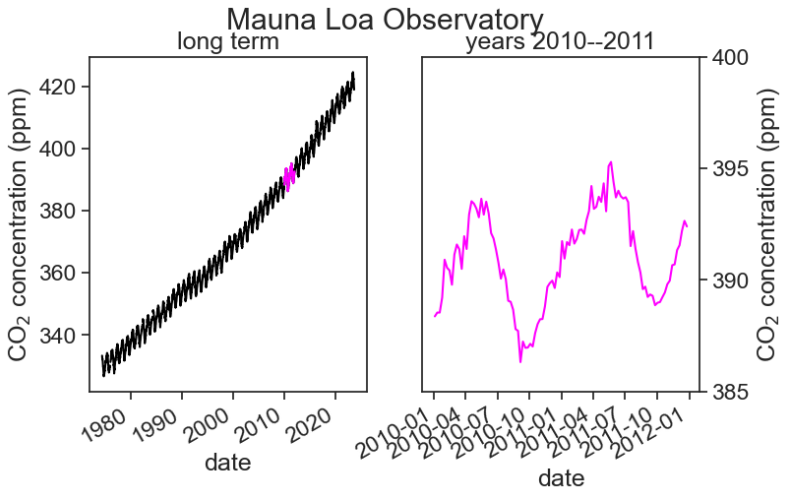
date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
1974-05-19	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.39
1974-05-26	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.05
1974-06-02	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.59
1974-06-09	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.64
1974-06-16	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...	...	...	...	...	...	...	...	...	...
2023-07-23	2023	7	23	2023.5575	421.28	4	418.03	397.30	141.60
2023-07-30	2023	7	30	2023.5767	420.83	6	418.10	396.80	141.69
2023-08-06	2023	8	6	2023.5959	420.02	6	417.36	395.65	141.41

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
2023-08-13	2023	8	13	2023.6151	418.98	4	417.25	395.24	140.89
2023-08-20	2023	8	20	2023.6342	419.31	2	416.64	395.22	141.71

3.5 first plot

We are now ready for our first plot! Let's see the weekly CO2 average.

```
# %matplotlib widget
# uncomment the above line if you want dynamic control of the figure when using VSCode
fig, (ax1, ax2) = plt.subplots(1, 2,  # 1 row, 2 columns
                               figsize=(8,5)  # width, height, in inches
                               )
# left panel
ax1.plot(df['average'], color="black")
ax1.plot(df.loc['2010-01-01':'2011-12-31','average'], color="magenta")
ax1.set(xlabel="date",
       ylabel=r"CO$_2$ concentration (ppm)",
       title="long term");
# right panel
ax2.plot(df.loc['2010-01-01':'2011-12-31','average'], color="magenta")
ax2.set(xlabel="date",
        ylabel=r"CO$_2$ concentration (ppm)",
        ylim=[385, 400],  # choose y limits
        yticks=np.arange(385, 401, 5),  # choose ticks
        title="years 2010--2011");
# put ticks and label on the right for ax2
ax2.yaxis.tick_right()
ax2.yaxis.set_label_position("right")
# title above both panels
fig.suptitle("Mauna Loa Observatory")
# makes slanted dates
plt.gcf().autofmt_xdate()
```



3.6 first plot, v2.0

The dates in the x-label are not great. Let's try to make them prettier.

We need to import a few more packages first.

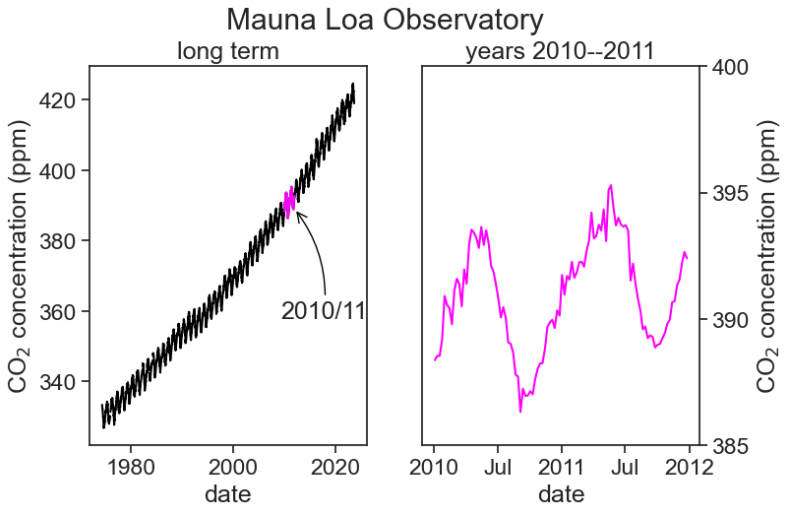
```
import matplotlib.dates as mdates
from matplotlib.dates import DateFormatter
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()  # datetime converter for a matplotlib
```

Now let's replot.

```
# %matplotlib widget
# uncomment the above line if you want dynamic control of the figure when using VSCode
fig, (ax1, ax2) = plt.subplots(1, 2,  # 1 row, 2 columns
                               figsize=(8,5)  # width, height, in inches
                               )
# left panel
ax1.plot(df['average'], color="black")
ax1.plot(df.loc['2010-01-01':'2011-12-31','average'], color="magenta")
ax1.set(xlabel="date",
       ylabel=r"CO$_2$ concentration (ppm)",
```

```
        title="long term");
# right panel
ax2.plot(df.loc['2010-01-01':'2011-12-31','average'], color="magenta")
ax2.set(xlabel="date",
        ylabel=r"CO$_2$ concentration (ppm)",
        ylim=[385, 400],  # choose y limits
        yticks=np.arange(385, 401, 5),  # choose ticks
        title="years 2010--2011");
# put ticks and label on the right for ax2
ax2.yaxis.tick_right()
ax2.yaxis.set_label_position("right")
# title above both panels
fig.suptitle("Mauna Loa Observatory", y=1.00)

locator = mdates.AutoDateLocator(minticks=3, maxticks=5)
formatter = mdates.ConciseDateFormatter(locator)
ax1.xaxis.set_major_locator(locator)
ax1.xaxis.set_major_formatter(formatter)

locator = mdates.AutoDateLocator(minticks=4, maxticks=5)
formatter = mdates.ConciseDateFormatter(locator)
ax2.xaxis.set_major_locator(locator)
ax2.xaxis.set_major_formatter(formatter)

ax1.annotate(
    "2010/11",
    xy=('2011-12-25', 389),  xycoords='data',
    xytext=(-10, -80), textcoords='offset points',
    arrowprops=dict(arrowstyle="->",
                    color="black",
                    connectionstyle="arc3,rad=0.2"))
fig.savefig("CO2-graph.png", dpi=300)
```

```
/var/folders/hc/jhnmlst937d27zzq9fhfks780000gn/T/ipykernel_10652/850389963.py:42: UserWarning:
  fig.savefig("CO2-graph.png", dpi=300)
/opt/anaconda3/lib/python3.9/site-packages/IPython/core/pylabtools.py:151: UserWarning: AutoDat
  fig.canvas.print_figure(bytes_io, **kw)
```



The dates on the horizontal axis are determined thus:

1. `locator = mdates.AutoDateLocator(minticks=3, maxticks=5)`
   This deremines the location of the ticks (between 3 and 5 ticks, whatever "works best")
2. `ax1.xaxis.set_major_locator(locator)`
   This actually puts the ticks in the positions determined above
3. `formatter = mdates.ConciseDateFormatter(locator)`
   This says that the labels will be placed at the locations determined in 1.
4. `ax1.xaxis.set_major_formatter(formatter)`
   Finally, labels are written down

The arrow is placed in the graph using `annotate`. It has a tricky syntax and a million options. Read Jake VanderPlas's excellent examples to learn more.

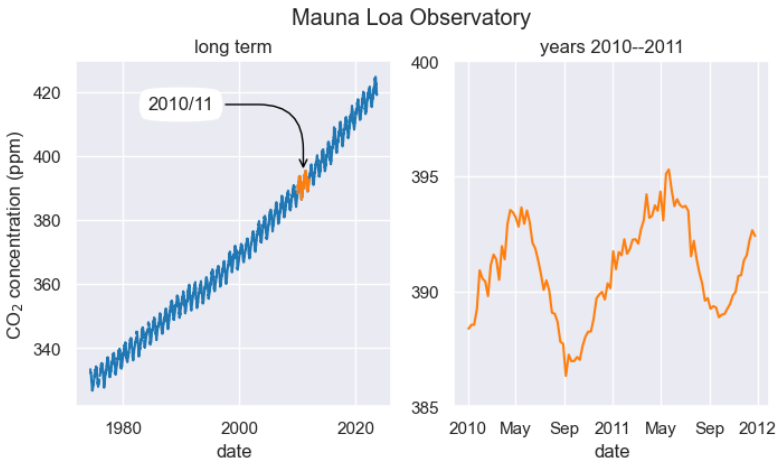
3.7 modifications

Let's change a lot of plotting options to see how things could be different.

```
sns.set(style="darkgrid")
sns.set_context("notebook")

# %matplotlib widget
# uncomment the above line if you want dynamic control of the figure when using VSCode
fig, (ax1, ax2) = plt.subplots(1, 2,  # 1 row, 2 columns
                               figsize=(8,4)  # width, height, in inches
                               )
# left panel
ax1.plot(df['average'], color="tab:blue")
ax1.plot(df.loc['2010-01-01':'2011-12-31','average'], color="tab:orange")
ax1.set(xlabel="date",
       ylabel=r"CO$_2$ concentration (ppm)",
       title="long term");
# right panel
ax2.plot(df.loc['2010-01-01':'2011-12-31','average'], color="tab:orange")
ax2.set(xlabel="date",
        ylim=[385, 400],  # choose y limits
        yticks=np.arange(385, 401, 5),  # choose ticks
        title="years 2010--2011");
# title above both panels
fig.suptitle("Mauna Loa Observatory", y=1.00)

locator = mdates.AutoDateLocator(minticks=3, maxticks=5)
formatter = mdates.ConciseDateFormatter(locator)
ax1.xaxis.set_major_locator(locator)
ax1.xaxis.set_major_formatter(formatter)

locator = mdates.AutoDateLocator(minticks=5, maxticks=8)
formatter = mdates.ConciseDateFormatter(locator)
ax2.xaxis.set_major_locator(locator)
ax2.xaxis.set_major_formatter(formatter)

ax1.annotate(
    "2010/11",
    xy=('2010-12-25', 395),  xycoords='data',
    xytext=(-100, 40), textcoords='offset points',
    bbox=dict(boxstyle="round4,pad=.5", fc="white"),
    arrowprops=dict(arrowstyle="->",
```

```
                    color="black",
                    connectionstyle="angle,angleA=0,angleB=-90,rad=40"))
```

```
Text(-100, 40, '2010/11')
```



The main changes were:

1. Using the Seaborn package, we changed the fontsize and the overall plot style. Read more.
   `sns.set(style="darkgrid")`
   `sns.set_context("notebook")`
2. We changed the colors of the lineplots. To know what colors exist, click here.
3. The arrow annotation has a different style. Read more.

3.8 playing with the code

I encourage you to play with the code you just ran. An easy way of learning what each line does is to comment something out and see what changes in the output you see. If you feel brave, try to modify the code a little bit.

4 AI policy

The guidelines below are an adaptation of Ethan Mollick's extremely useful ideas on AI as an assistant tool for teaching.

I EXPECT YOU to use LLMs (large language models) such as ChatGPT, Bing AI, Google Bard, Lex, or whatever else springs up since the time of this writing. You should familiarize yourself with the AI's capabilities and limitations.

Use LLMs to help you learn, chat with them about what you want to accomplish and learn from them how to do it. **Ask** your LLM what each part of the code means, copy and pasting blindly is unacceptable. You are here to learn.

Consider the following important points:

- Ultimately, you, the student, are responsible for the assignment.
- Acknowledge the use of AI in your assignment. Be transparent about your use of the tool and the extent of assistance it provided.

Part II

resampling

5 motivation

5.1 Jerusalem, 2019

Data from the Israel Meteorological Service, IMS.

See the temperature at a weather station in Jerusalem, for the whole 2019 year. This is an interactive graph: to zoom in, play with the bottom panel.

```
alt.VConcatChart(...)
```

5.1.0.0.1 * discussion

The temperature fluctuates on various time scales, from daily to yearly. Let's think together a few questions we'd like to ask about the data above.

Now let's see precipitation data:

```
alt.VConcatChart(...)
```

5.1.0.0.2 * discussion

What would be interesting to know about precipitation?

We have not talked about what kind of data we have in our hands here. The csv file provided by the IMS looks like this:

	Station	Date & Time (Winter)	Diffused radiation (W/m^2)	Global radiation (W/m^
0	Jerusalem Givat Ram	01/01/2019 00:00	0.0	0.0
1	Jerusalem Givat Ram	01/01/2019 00:10	0.0	0.0
2	Jerusalem Givat Ram	01/01/2019 00:20	0.0	0.0
3	Jerusalem Givat Ram	01/01/2019 00:30	0.0	0.0

	Station	Date & Time (Winter)	Diffused radiation (W/m^2)	Global radiation (W/m^
4	Jerusalem Givat Ram	01/01/2019 00:40	0.0	0.0
...	...	...	...	...
52549	Jerusalem Givat Ram	31/12/2019 22:20	0.0	0.0
52550	Jerusalem Givat Ram	31/12/2019 22:30	0.0	0.0
52551	Jerusalem Givat Ram	31/12/2019 22:40	0.0	0.0
52552	Jerusalem Givat Ram	31/12/2019 22:50	0.0	0.0
52553	Jerusalem Givat Ram	31/12/2019 23:00	0.0	0.0

We see that we have data points spaced out evenly every 10 minutes.

5.2 Challenges

Let's try to answer the following questions:

What is the mean temperature for each month?

First we have to divide temperature data by month, and then take the average for each month.
a possible solution

```
df_month = df['temperature'].resample('M').mean()
```

For each month, what is the mean of the daily maximum temperature? What about the minimun?

This is a bit trickier.

1. We need to find the maximum/minimum temperature for each day.
2. Only then we split the daily data by month and take the average.

a possible solution

```
df_day['max temp'] = df['temperature'].resample('D').max()
df_month['max temp'] = df_day['max temp'].resample('MS').mean()
```

What is the average night temperature for every season? What about the day temperature?

1. We need to filter our data to contain only night times.
2. We need to divide rain data by seasons (3 months), and then take the mean for each season.

a possible solution

```
solution
```

What is the daily precipitation?

First we have to divide rain data by day, and then take the sum for each day.

How much rain was there every month?

We have to divide rain data by month, and then sum the totals of each month.

How many rainy days were there each month?

1. We need to sum rain by day.
2. We need to count how many days are there each month where `rain > 0`.

How many days, hours, and minutes were between the last rain of the season (Malkosh) to the first (Yore'h)?

1. We need to divide our data into two: `rainy_season_1` and `rainy_season_2`.
2. We need to find the time of the last rain in `rainy_season_1`.
3. We need to find the time of the first rain in `rainy_season_2`.
4. We need to compute the time difference between the two dates.

What was the rainiest morning (6am-12pm) of the year? Bonus, what about the rainiest night (6pm-6am)?

1. We need to filter our data to contain only morning times.
2. We need to sum rain by day.
3. We need to find the day with the maximum value.

Note: this whole webpage is actually a Jupyter Notebook rendered as html. If you want to know how to make interactive graphs, go to the top of the page and click on " Code"

Useful functions compatible with `pandas.resample()` can be found here. The full list of resampling frequencies can be found here.

6 resampling

We can only really understand how to calculate monthly means if we do it ourselves.

First, let's import a bunch of packages we need to use.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import warnings
# Suppress FutureWarnings
warnings.simplefilter(action='ignore', category=FutureWarning)
warnings.simplefilter(action='ignore', category=UserWarning)
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)  # white graphs, with large and legible letters
```

Now we load the csv file for Jerusalem (2019), provided by the IMS.

6.0.0.0.1 * discussion

We will go to the IMS website together and see what are the options available and how to download. If you just need the csv right away, download it here.

- We substitute every occurence of `-` for NaN (not a number, that is, the data is missing).
- We call the columns `Temperature (°C)` and `Rainfall (mm)` with more convenient names, since we will be using them a lot.
- We interpret the column `Date & Time (Winter)` as a date, saying to python that day comes first.

- We make `date` the index of the dataframe.

```
filename = "../archive/data/jerusalem2019.csv"
df = pd.read_csv(filename, na_values=['-'])
df.rename(columns={'Temperature (°C)': 'temperature',
                   'Rainfall (mm)': 'rain'}, inplace=True)
df['date'] = pd.to_datetime(df['Date & Time (Winter)'], dayfirst=True)
df = df.set_index('date')
df
```

date	Station	Date & Time (Winter)	Diffused radiation (W/m^2)	Global rad
2019-01-01 00:00:00	Jerusalem Givat Ram	01/01/2019 00:00	0.0	0.0
2019-01-01 00:10:00	Jerusalem Givat Ram	01/01/2019 00:10	0.0	0.0
2019-01-01 00:20:00	Jerusalem Givat Ram	01/01/2019 00:20	0.0	0.0
2019-01-01 00:30:00	Jerusalem Givat Ram	01/01/2019 00:30	0.0	0.0
2019-01-01 00:40:00	Jerusalem Givat Ram	01/01/2019 00:40	0.0	0.0
...	...	...	...	...
2019-12-31 22:20:00	Jerusalem Givat Ram	31/12/2019 22:20	0.0	0.0
2019-12-31 22:30:00	Jerusalem Givat Ram	31/12/2019 22:30	0.0	0.0
2019-12-31 22:40:00	Jerusalem Givat Ram	31/12/2019 22:40	0.0	0.0
2019-12-31 22:50:00	Jerusalem Givat Ram	31/12/2019 22:50	0.0	0.0
2019-12-31 23:00:00	Jerusalem Givat Ram	31/12/2019 23:00	0.0	0.0

With `resample` it's easy to compute monthly averages. Resample by itself only divides the data into buckets (in this case monthly buckets), and waits for a further instruction. Here, the next instruction is `mean`.

```
df_month = df['temperature'].resample('M').mean()
df_month
```

```
date
2019-01-31     9.119937
2019-02-28     9.629812
2019-03-31    10.731571
2019-04-30    14.514329
2019-05-31    22.916894
```

```
2019-06-30    23.587361
2019-07-31    24.019403
2019-08-31    24.050822
2019-09-30    22.313287
2019-10-31    20.641868
2019-11-30    17.257153
2019-12-31    11.224131
Freq: M, Name: temperature, dtype: float64
```

Instead of `M` for month, which other options do I have? The full list can be found here, but the most commonly used are:

```
M          month end frequency
MS         month start frequency
A          year end frequency
AS, YS     year start frequency
D          calendar day frequency
H          hourly frequency
T, min     minutely frequency
S          secondly frequency
```

The results we got for the monthly means were given as a pandas series, not dataframe. Let's correct this:

```
df_month = (df['temperature'].resample('M')          # resample by month
                             .mean()                 # take the mean
                             .to_frame('mean temp') # make output a dafaframe
            )
df_month
```

date	mean temp
2019-01-31	9.119937
2019-02-28	9.629812
2019-03-31	10.731571
2019-04-30	14.514329
2019-05-31	22.916894
2019-06-30	23.587361

date	mean temp
2019-07-31	24.019403
2019-08-31	24.050822
2019-09-30	22.313287
2019-10-31	20.641868
2019-11-30	17.257153
2019-12-31	11.224131

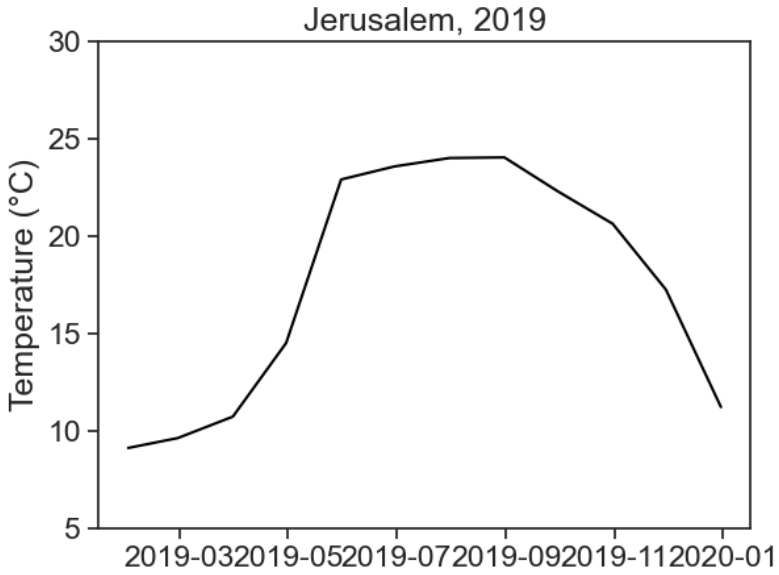
6.0.0.0.2 * hot tip

Sometimes, a line of code can get too long and messy. In the code above, we broke line for every step, which makes the process so much cleaner. We **highly** advise you to do the same. **Attention:** This trick works as long as all the elements are inside the same parenthesis.

Now it's time to plot!

```
fig, ax = plt.subplots()
ax.plot(df_month['mean temp'], color='black')
ax.set(ylabel='Temperature (°C)',
       yticks=np.arange(5,35,5),
       title="Jerusalem, 2019")
```

```
[Text(0, 0.5, 'Temperature (°C)'),
 [<matplotlib.axis.YTick at 0x7fa6a2f412b0>,
  <matplotlib.axis.YTick at 0x7fa6a2f3ec10>,
  <matplotlib.axis.YTick at 0x7fa685442f40>,
  <matplotlib.axis.YTick at 0x7fa6853d7e80>,
  <matplotlib.axis.YTick at 0x7fa6853f9850>,
  <matplotlib.axis.YTick at 0x7fa685401070>],
 Text(0.5, 1.0, 'Jerusalem, 2019')]
```



Although all the calculations are correct, the graph is not great.

- Each monthly average was assigned to the last day of the month.
- The ticks on the x-axis are on the first of the month, so there is a mismatch between data and labels.

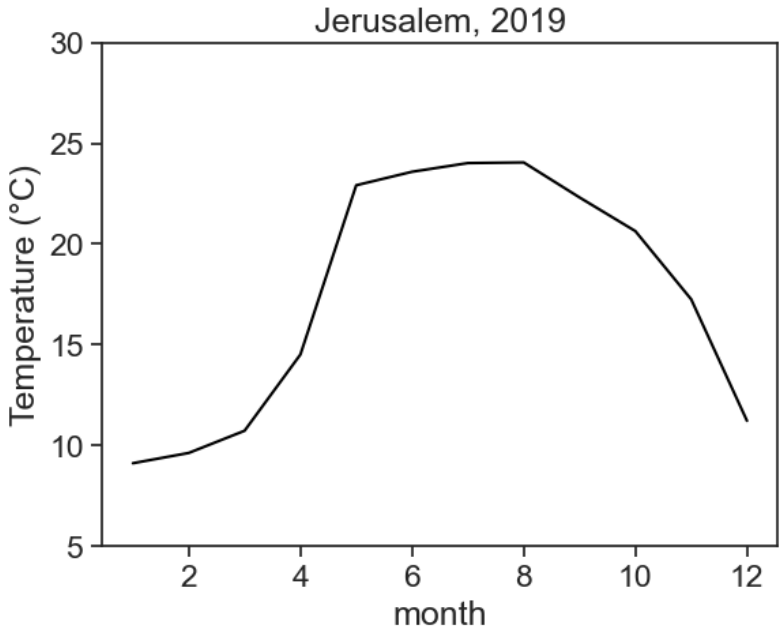
We will resample now using `MS`, which assigns the result to the first of the month (Month Start), and we will then add 14 more days to it (left offset), so that every point is assigned to the middle (15th) of the month.

```
df_month = (df['temperature'].resample('MS',
                                       loffset=pd.Timedelta(14, 'd'))
                             .mean()
                             .to_frame('mean temp')
            )
df_month
```

date	mean temp
2019-01-15	9.119937
2019-02-15	9.629812
2019-03-15	10.731571
2019-04-15	14.514329
2019-05-15	22.916894
2019-06-15	23.587361
2019-07-15	24.019403
2019-08-15	24.050822
2019-09-15	22.313287
2019-10-15	20.641868
2019-11-15	17.257153
2019-12-15	11.224131

Now let's plot again, and we'll choose a cleaner x-axis labeling.

```
fig, ax = plt.subplots()
ax.plot(df_month.index.month, df_month['mean temp'], color='black')
ax.set(xlabel="month",
       ylabel='Temperature (°C)',
       yticks=np.arange(5,35,5),
       title="Jerusalem, 2019",);
```



6.0.0.0.3 * discussion

What does this line mean?
`df_month['mean temp'].index.month`

Print on the screen the following, and see yourself what each thing is:

- `df_month`
- `df_month.index`
- `df_month.index.month`
- `df_month.index.day`

We're done! Congratulations :)

Now we need to calculate the average minimum/maximum daily temperatures. We start by creating an empty dataframe.

```
df_day = pd.DataFrame()
```

Now resample data by day (`D`), and take the min/max of each day.

```
df_day['min temp'] = df['temperature'].resample('D').min()
df_day['max temp'] = df['temperature'].resample('D').max()
df_day
```

date	min temp	max temp
2019-01-01	7.5	14.1
2019-01-02	6.6	11.5
2019-01-03	6.3	10.7
2019-01-04	6.6	14.6
2019-01-05	7.0	11.4
...	...	...
2019-12-27	4.4	7.4
2019-12-28	6.6	10.3
2019-12-29	8.1	12.5
2019-12-30	6.9	13.0
2019-12-31	5.2	13.3

The next step is to calculate the average minimum/maximum for each month. This is similar to what we did above.

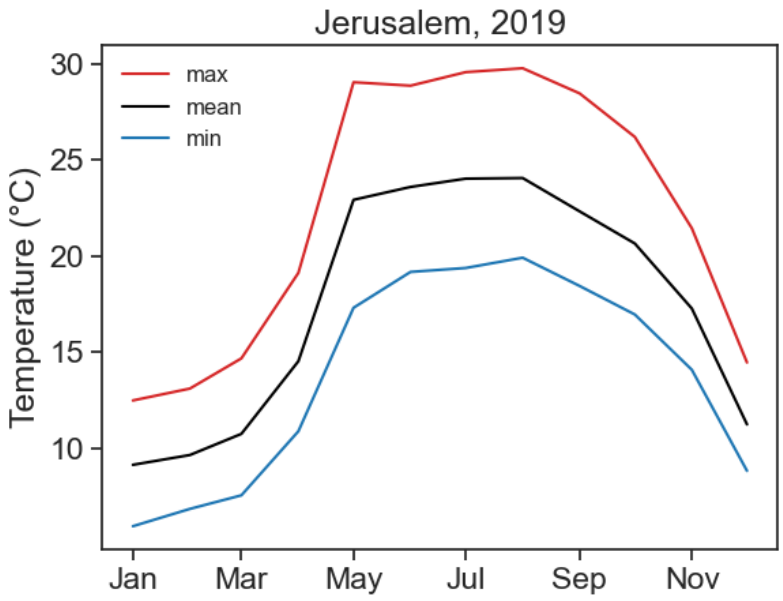
```
df_month['min temp'] = df_day['min temp'].resample('MS',loffset=pd.Timedelta(14, 'd')).mean(
df_month['max temp'] = df_day['max temp'].resample('MS',loffset=pd.Timedelta(14, 'd')).mean(
df_month
```

date	mean temp	min temp	max temp
2019-01-15	9.119937	5.922581	12.470968
2019-02-15	9.629812	6.825000	13.089286
2019-03-15	10.731571	7.532258	14.661290
2019-04-15	14.514329	10.866667	19.113333
2019-05-15	22.916894	17.296774	29.038710
2019-06-15	23.587361	19.163333	28.860000
2019-07-15	24.019403	19.367742	29.564516
2019-08-15	24.050822	19.903226	29.767742
2019-09-15	22.313287	18.430000	28.456667
2019-10-15	20.641868	16.945161	26.190323

date	mean temp	min temp	max temp
2019-11-15	17.257153	14.066667	21.436667
2019-12-15	11.224131	8.806452	14.448387

Let's plot...

```
fig, ax = plt.subplots()
ax.plot(df_month['max temp'], color='tab:red', label='max')
ax.plot(df_month['mean temp'], color='black', label='mean')
ax.plot(df_month['min temp'], color='tab:blue', label='min')
ax.set(ylabel='Temperature (°C)',
       yticks=np.arange(10,35,5),
       title="Jerusalem, 2019")
ax.xaxis.set_major_locator(mdates.MonthLocator(range(1, 13, 2), bymonthday=15))
date_form = DateFormatter("%b")
ax.xaxis.set_major_formatter(date_form)
ax.legend(fontsize=12, frameon=False);
```



Voilà! You made a beautiful graph!

6.0.0.0.4 * discussion

When you have datetime as the dataframe index, you don't need to give the function `plot` two arguments, date and values. You can just tell `plot` to use the column you want, the function will take the dates by itself.

Another comment:
This time we did not put month numbers in the horizontal axis, we now have month names. How did we do this black magic, you ask? See lines 8–10 above. Matplotlib gives you absolute power over what to put in the axis, if you can only know how to tell it to… Wanna know more? Click here.

7 upsampling

In the previous chapter, we resampled from fine temporal resolution to a coarser one. This is also called **downsampling**. We will learn the **upsampling** now: how to go from coarse data to a finer scale.

Sadly, there is no free lunch, and we just can't get data that was not measured. What to do then?

It's best to consider a practical example.

7.1 Potential Evapotranspiration using Penman's equation

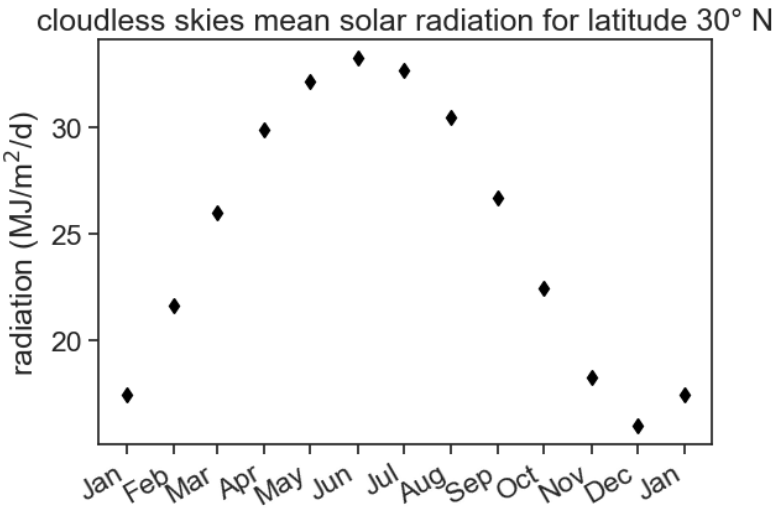
We want to calculate the daily potential evapotranspiration using Penman's equation. Part of the calculation involves characterizing the energy budget on soil surface. When direct solar radiation measurements are not available, we can estimate the energy balance by knowing the "cloudless skies mean solar radiation", R_{so}. This is the amount of energy ($MJ/m^2/d$) that hits the surface, assuming no clouds. This radiation depends on the season and on the latitude you are. For Israel, located at latitude 32° N, we can use the following data for 30°:

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)  # white graphs, with large and legible letters
```

```
dates = pd.date_range(start='2021-01-01', periods=13, freq='MS')
values = [17.46, 21.65, 25.96, 29.85, 32.11, 33.20, 32.66, 30.44, 26.67, 22.48, 18.30, 16.04
df = pd.DataFrame({'date': dates, 'radiation': values})
df = df.set_index('date')
df
```

date	radiation
2021-01-01	17.46
2021-02-01	21.65
2021-03-01	25.96
2021-04-01	29.85
2021-05-01	32.11
2021-06-01	33.20
2021-07-01	32.66
2021-08-01	30.44
2021-09-01	26.67
2021-10-01	22.48
2021-11-01	18.30
2021-12-01	16.04
2022-01-01	17.46

```
fig, ax = plt.subplots()
ax.plot(df['radiation'], color='black', marker='d', linestyle='None')
ax.set(ylabel=r'radiation (MJ/m$^2$/d)',
       title="cloudless skies mean solar radiation for latitude 30° N")
ax.xaxis.set_major_locator(mdates.MonthLocator())
date_form = DateFormatter("%b")
ax.xaxis.set_major_formatter(date_form)
plt.gcf().autofmt_xdate()  # makes slanted dates
```



We only have 12 values for the whole year, and we can't use this dataframe to compute daily ET. We need to upsample!

In the example below, we resample the monthly data into daily data, and do nothing else. Pandas doesn't know what to do with the new points, so it fills them with NaN.

```
df_nan = df['radiation'].resample('D').asfreq().to_frame()
df_nan.head(33)
```

date	radiation
2021-01-01	17.46
2021-01-02	NaN
2021-01-03	NaN
2021-01-04	NaN
2021-01-05	NaN
2021-01-06	NaN
2021-01-07	NaN
2021-01-08	NaN
2021-01-09	NaN
2021-01-10	NaN
2021-01-11	NaN
2021-01-12	NaN

date	radiation
2021-01-13	NaN
2021-01-14	NaN
2021-01-15	NaN
2021-01-16	NaN
2021-01-17	NaN
2021-01-18	NaN
2021-01-19	NaN
2021-01-20	NaN
2021-01-21	NaN
2021-01-22	NaN
2021-01-23	NaN
2021-01-24	NaN
2021-01-25	NaN
2021-01-26	NaN
2021-01-27	NaN
2021-01-28	NaN
2021-01-29	NaN
2021-01-30	NaN
2021-01-31	NaN
2021-02-01	21.65
2021-02-02	NaN

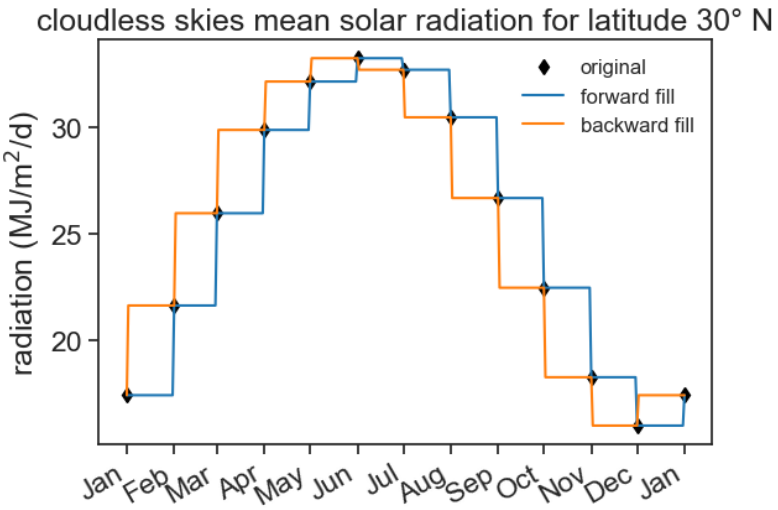
7.2 Forward/Backward fill

We can forward/backward fill these NaNs:

```
df_forw = df['radiation'].resample('D').ffill().to_frame()
df_back = df['radiation'].resample('D').bfill().to_frame()
```

```
fig, ax = plt.subplots()
ax.plot(df['radiation'], color='black', marker='d', linestyle='None', label="original")
ax.plot(df_forw['radiation'], color='tab:blue', label="forward fill")
ax.plot(df_back['radiation'], color='tab:orange', label="backward fill")
ax.set(ylabel=r'radiation (MJ/m$^2$/d)',
       title="cloudless skies mean solar radiation for latitude 30° N")
```

```
ax.legend(frameon=False, fontsize=12)
ax.xaxis.set_major_locator(mdates.MonthLocator())
date_form = DateFormatter("%b")
ax.xaxis.set_major_formatter(date_form)
plt.gcf().autofmt_xdate()  # makes slanted dates
```



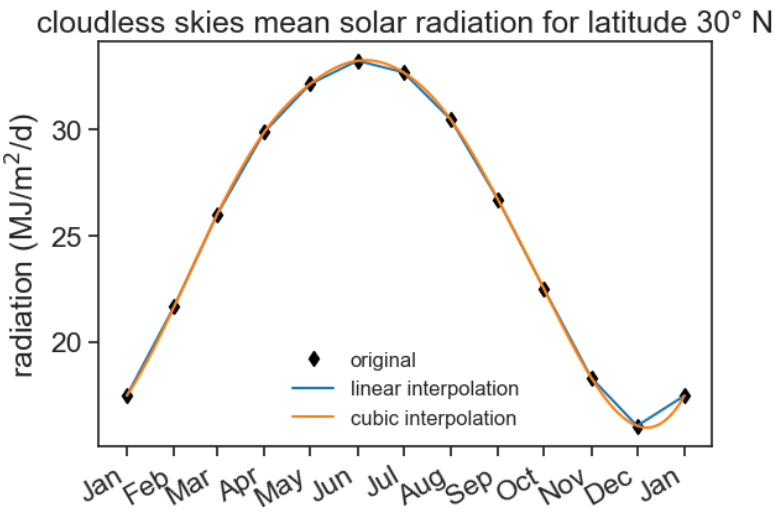
This does the job, but I want something better, not step functions. The radiation should vary smoothly from day to day. Let's use interpolation.

7.3 Interpolation

```
df_linear = df['radiation'].resample('D').interpolate(method='time').to_frame()
df_cubic = df['radiation'].resample('D').interpolate(method='cubic').to_frame()
```

```
fig, ax = plt.subplots()
ax.plot(df['radiation'], color='black', marker='d', linestyle='None', label="original")
ax.plot(df_linear['radiation'], color='tab:blue', label="linear interpolation")
ax.plot(df_cubic['radiation'], color='tab:orange', label="cubic interpolation")
ax.set(ylabel=r'radiation (MJ/m$^2$/d)',
       title="cloudless skies mean solar radiation for latitude 30° N")
```

```
ax.legend(frameon=False, fontsize=12)
ax.xaxis.set_major_locator(mdates.MonthLocator())
date_form = DateFormatter("%b")
ax.xaxis.set_major_formatter(date_form)
plt.gcf().autofmt_xdate()  # makes slanted dates
```



There are many ways to fill NaNs and to interpolate. A nice detailed guide can be found here.

8 interpolation

Interpolation is the act of getting data you don't have from data you alreay have. We used some interpolation when upsampling, and now it is time to talk about it a little bit more in depth.

There is no one correct way of interpolating, the method you use depends in the end on what you want to accomplish, what are your (hidden or explicit) assumptions, etc. Let's see a few examples.

9 FAQ

9.1 How to resample by year, but have it end in September?

This is called anchored offset. One possible use to it is to calculate statistics according to the hydrological year that, for example, ends in September.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)  # white graphs, with large and legible letters
```

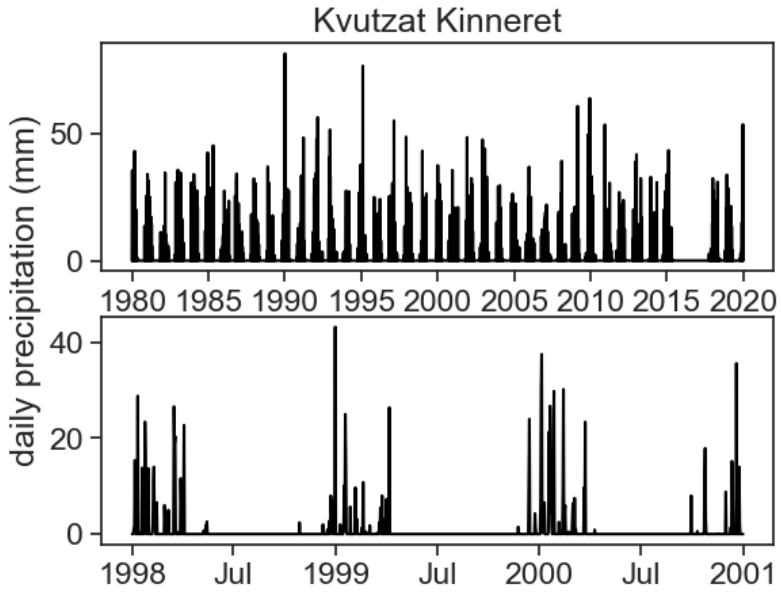
```
filename = "../archive/data/Kinneret_Kvuza_daily_rainfall.csv"
df = pd.read_csv(filename, na_values=['-'])
df.rename(columns={'Date': 'date',
                   'Daily Rainfall (mm)': 'rain'}, inplace=True)
df['date'] = pd.to_datetime(df['date'], dayfirst=True)
df = df.set_index('date')
df = df.resample('D').asfreq().fillna(0)  # asfreq = replace
df
```

date	Station	rain
1980-01-02	Kinneret Kvuza 09/1977-08/2023	0.0
1980-01-03	0	0.0
1980-01-04	0	0.0

date	Station	rain
1980-01-05	Kinneret Kvuza 09/1977-08/2023	35.5
1980-01-06	Kinneret Kvuza 09/1977-08/2023	2.2
...	...	...
2019-12-26	Kinneret Kvuza 09/1977-08/2023	39.4
2019-12-27	Kinneret Kvuza 09/1977-08/2023	5.2
2019-12-28	Kinneret Kvuza 09/1977-08/2023	1.6
2019-12-29	0	0.0
2019-12-30	Kinneret Kvuza 09/1977-08/2023	0.1

```
fig, ax = plt.subplots(2,1)
ax[0].plot(df['rain'], color='black')
ax[1].plot(df.loc['1998':'2000', 'rain'], color='black')
locator = mdates.AutoDateLocator(minticks=4, maxticks=8)
formatter = mdates.ConciseDateFormatter(locator)
ax[1].xaxis.set_major_locator(locator)
ax[1].xaxis.set_major_formatter(formatter)
fig.text(0.02, 0.5, 'daily precipitation (mm)', va='center', rotation='vertical')
ax[0].set_title("Kvutzat Kinneret")
```

```
Text(0.5, 1.0, 'Kvutzat Kinneret')
```



We see a marked dry season during the summer, so let's assume the Hydrological Year ends in September.

```
df_year = df.resample('A-SEP').sum()
df_year = df_year.loc['1980':'2003']
df_year
```

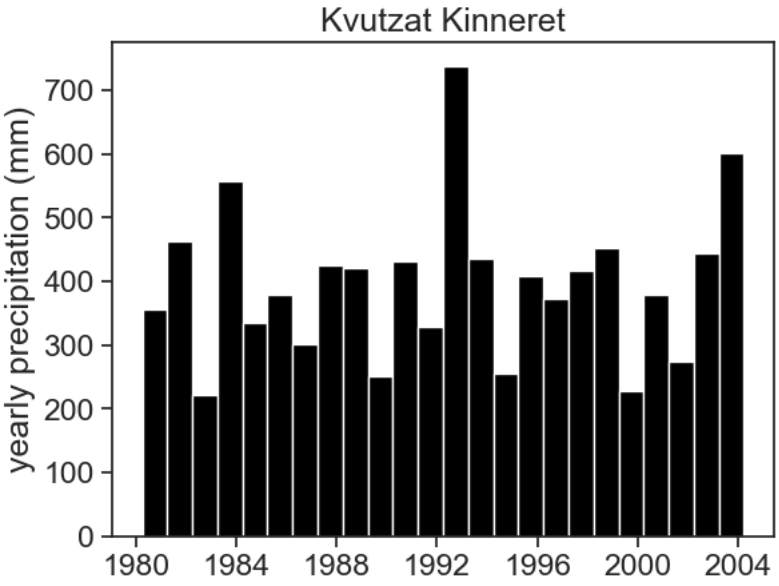
```
/var/folders/c3/7hp0d36n6vv8jc9hm2440__00000gn/T/ipykernel_94063/2047090134.py:1: FutureWarning
  df_year = df.resample('A-SEP').sum()
```

date	rain
1980-09-30	355.5
1981-09-30	463.1
1982-09-30	221.7
1983-09-30	557.1
1984-09-30	335.3
1985-09-30	379.8
1986-09-30	300.7
1987-09-30	424.7

date	rain
1988-09-30	421.6
1989-09-30	251.6
1990-09-30	432.5
1991-09-30	328.3
1992-09-30	738.4
1993-09-30	434.9
1994-09-30	255.4
1995-09-30	408.6
1996-09-30	373.0
1997-09-30	416.2
1998-09-30	451.9
1999-09-30	227.8
2000-09-30	378.9
2001-09-30	273.9
2002-09-30	445.2
2003-09-30	602.4

```
fig, ax = plt.subplots()
ax.bar(df_year.index, df_year['rain'], color='black',
       width=365)
ax.set_ylabel("yearly precipitation (mm)")
ax.set_title("Kvutzat Kinneret")
```

```
Text(0.5, 1.0, 'Kvutzat Kinneret')
```



9.2 When upsampling, how to fill missing values with zero?

We did that in the example above, like this:

```
df = df.resample('D').asfreq().fillna(0)  # asfreq = replace
```

Part III

smoothing

10 motivation

See below a graph of tree transpiration as a function of time. The dots represent actual measurements from several transpiration chambers placed on individual tree branches. It might be hard to find a pattern amidst all these scattered points, so we can ask "what is the average behavior of these points across time?".

When we learned resampling, we did exactly that, and we saw examples of how to take daily or monthly temperature averages. Downsampling, however, decreases the temporal frequency of our data (e.g. from daily to monthly). How can we take averages of our data without decreasing its frequency?

10.1 moving window

One common solution is to analyze our data in small windows, and slide this window from left to right in order to get tbe aggregate behavior of our data in a small time interval. For daily data, if we have have a 1-month long window sliding from January to December in 1-month steps, then we exactly resampled our data into monthly bins, and also decreased its frequency from 1 day to 1 month. If the window slides by only 1 day, however, then we get a (slightly smaller) array also with a 1-day frequency. This is the central idea of a moving window.

10.2 example

Temperature in jerusalem 2019

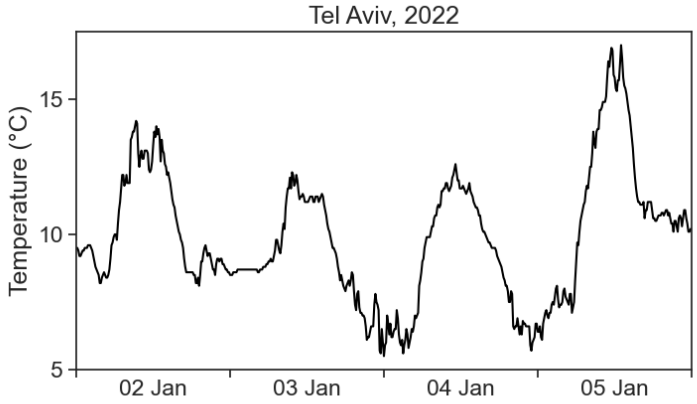
10.3 we don't need to take averages

There are many ways of aggregating your data inside a window, and we will see some of them in the following sections.

11 convolution

Running windows of different shapes (kernels)

This is the temperature for Tel Aviv, between 2 and 5 of January 2022. Data is in intervals of 10 minutes, and was downloaded from the Israel Meteorological Service.



We see that the temperature curve has a rough profile. Can we find ways of getting smoother curves?

11.1 convolution

Convolution is a fancy word for averaging a time series using a running window. We will use the terms **convolution, running average, and rolling average** interchangeably. See the

animation below. We take all temperature values inside a window of width 500 minutes (51 points), and average them with equal weights. The weights profile is called `kernel`.

The pink curve is much smoother than the original! However, the running average cannot describe sharp temperature changes. If we decrease the window width to 200 minutes (21 points), we get the following result.

There is a tradeoff between the smoothness of a curve, and its ability to describe sharp temporal changes.

11.2 kernels

We can modify our running average, so that values closer to the center of the window have higher weights, and those further away count less. This is achieved by changing the weight profile, or the shape of the kernel. We see below the result of a running average using a triangular window of base 500 minutes (51 points).

Things can get as fancy as we want. Instead of a triangular kernel, which has sharp edges, we can choose a smoother gaussian kernel, see the difference below. We used a gaussian kernel with 60-minute standard deviation (the window in the animation is 4 standard deviations wide).

11.3 math

The definition of a convolution between signal $f(t)$ and kernel $k(t)$ is

$$(f * k)(t) = \int f(\tau)k(t-\tau)d\tau.$$

The expression $f * k$ denotes the convolution of these two functions. The argument of k is $t-\tau$, meaning that the kernel runs from left to right (as t does), and at every point the two signals (f and k) are multiplied together. It is the product of the signal with the weight function k that gives us an average. Because

of $-\tau$, the kernel is flipped backwards, but this has no effect to symmetric kernels, like to ones in the examples above. Finally, the actual running average is not the convolution, but

$$\frac{(f * k)(t)}{\int k(t)dt}.$$

Whenever the integral of the kernel is 1, then the convolution will be identical with the running average.

11.4 numerics

Running averages are very common tools in time-series analysis. The `pandas` package makes life quite simple. For example, in order to calculate the running average of temperature using a rectangular kernel, one writes

```
df['temperature'].rolling(window='20', center=True).mean()
```

- `window=20` means that the width of the window is 20 points. Pandas lets us define a window width in time units, for example, `window='120min'`.
- `center=True` is needed in order to assign the result of averaging to the center of the window. Make it `False` and see what happens.
- `mean()` is the actual calculation, the average of temperature over the window. The `rolling` part does not compute anything, it just creates a moving window, and we are free to calculate whatever we want. Try to calculate the standard deviation or the maximum, for example.

It is implicit in the command above a "rectangular" kernel. What if we want other shapes?

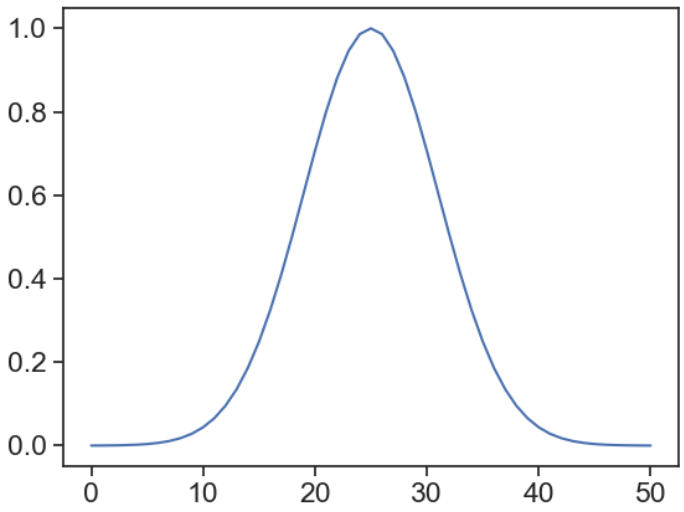
11.4.1 gaussian

```
(
df['temperature'].rolling(window=window_width,
                          center=True,
                          win_type="gaussian")
                 .mean(std=std_gaussian)
)
```

where

- `window_width` is an integer, number of points in your window
- `std_gaussian` is the standard deviation of your gaussian, measured in sample points, not time!

For instance, if we have measurements every 10 minutes, and our window width is 500 minutes, then `window_width = 500/10 + 1` (first and last included). If we want a standard deviation of 60 minutes, then `std_gaussian = 6`. The gaussian kernel will look like this:



You can take a look at various options for kernel shapes here, provided by the `scipy` package. The graph above was achieved

by running:

```
g = scipy.signal.gaussian(window_width, std)
plt.plot(g)
```

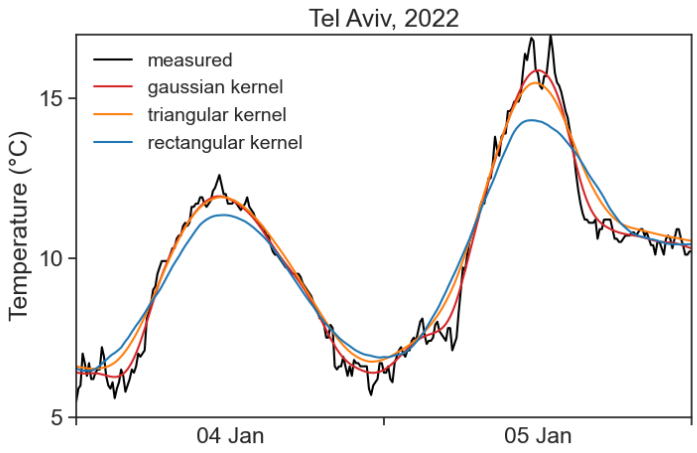
11.4.2 triangular

Same idea as gaussian, but simpler, because we don't need to think about standard deviation.

```
(
df['temperature'].rolling(window=window_width,
                          center=True,
                          win_type="triang")
                 .mean()
)
```

11.5 which window shape and width to choose?

Sorry, there is not definite answer here… It really depends on your data and what you need to do with it. See below a comparison of all examples in the videos above.



One important question you need to ask is: what are the time scales associated with the processes I'm interested in? For example, if I'm interested in the daily temperature pattern, getting rid of 1-minute-long fluctuations would probably be ok. On the other hand, if we were to smooth the signal so much that all that can be seen are the temperature changes between summer and winter, then my smoothing got out of hand, and I threw away the very process I wanted to study.

All this is to say that you need to know in advance a few things about the system you are studying, otherwise you can't know what is "noise" that can be smoothed away.

12 LOESS

13 a perfect smoother

Source: Eilers (2003)

GitHub repository

Noisy series y of length m.

The smoothed series is called z.

We have conflicting interests:

- we want a z series "as smooth as possible".
- however, the smoother z is, the farthest from y it will be (low fidelity).

Roughness:

$$R = \sum_i (z_i - z_{i-1})^2$$

Fit to data:

$$S = \sum_i (y_i - z_i)^2$$

Cost functional to be minimized:

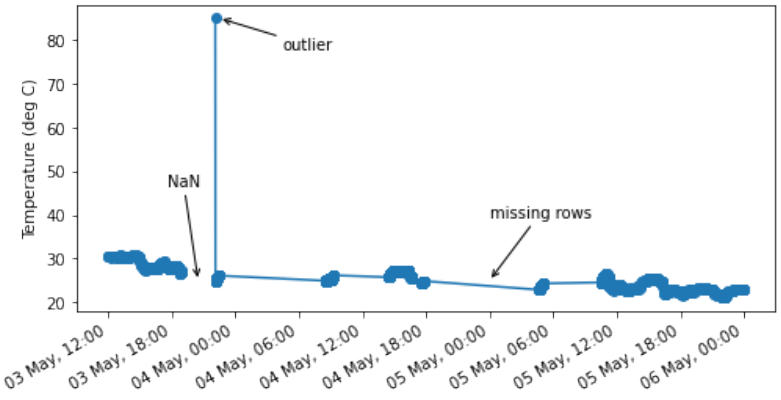
$$Q = S + \lambda R$$

Part IV

outliers and gaps

14 motivation

Outliers are observations significantly different from all other observations. Consider, for example, this temperature graph:



While most measured points are between 20 and 30 °C, there is obviously something very wrong with the one data point above 80 °C.

How could such a thing come about? This could be the result of non-natural causes, such as measurement errors, wrong data collection, or wrong data entry. On the other hand, this point could have natural sources, such as a very hot spark flying next to the temperature sensor.

Identifying outliers is important, because they might greatly impact measures like mean and standard deviation. When left untouched, outliers might make us reach wrong conclusions about our data. See what happens to the slope of this linear regression with and without the outliers.

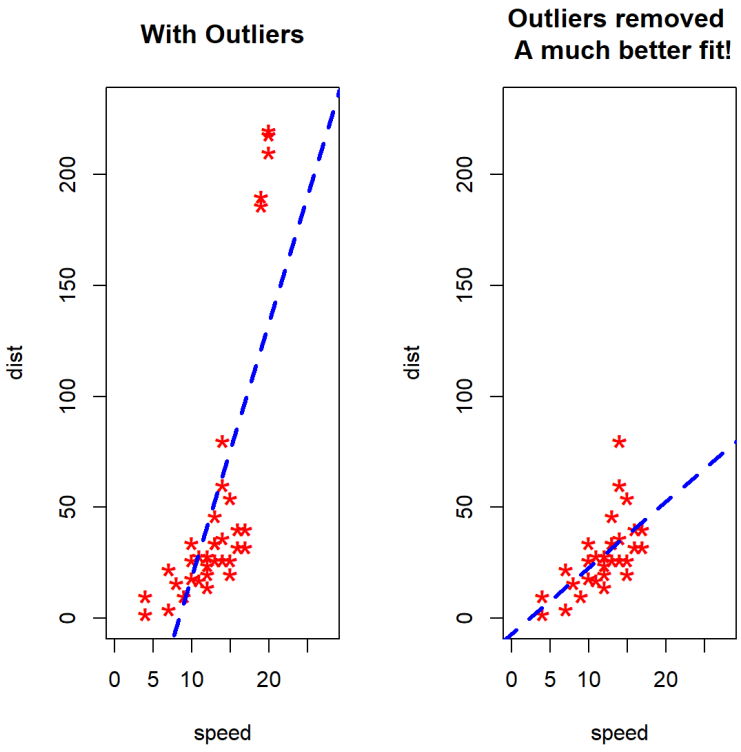


Figure 14.1: Source: Zhang (2020)

15 Z-score

$$z = \frac{x - \mu}{\sigma},$$

where

- x = data point,
- μ = time series mean
- σ = time series standard deviation.

Let's write a function that identifies outliers according to the Z-score.

```
def zscore(df, degree=3):
    data = df.copy()
    data['zscore'] = (data - data.mean())/data.std()
    outliers = data[(data['zscore'] <= -degree) | (data['zscore'] >= degree)]
    return outliers['value'], data
```

Now we can simply use this function:

```
threshold = 2.5
outliers, transformed = zscore(tx, threshold)
```

Source: Atwan (2022)

16 IQR (Inter Quartile Range)

The IQR (Inter Quartile Range) is the distance between the 25th percentile (Q1) and the 75th percentile (Q3). In a box plot, the whiskers usually extend 1.5×IQR beyond Q1 and Q3, see below.

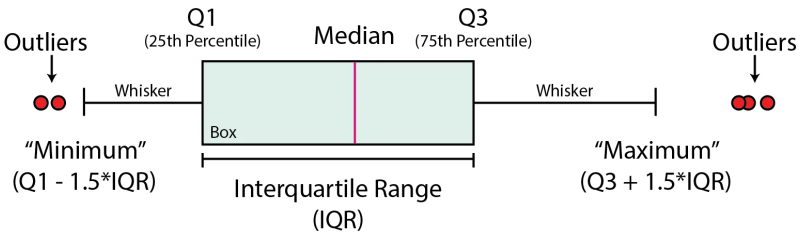


Figure 16.1: Source: McDonald (2022)

A common outlier detection method is to consider whatever points outside the whisker range as outliers.

Part V

best practices

17 motivation

18 date formatting

Here you will find several examples of how to format dates in your plots. Not many explanations are provided.

How to use this page? Find first an example of a plot you like, only then go to the code and see how it's done.

```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import datetime
from datetime import timedelta
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)
import matplotlib.gridspec as gridspec
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
```

```
import pandas as pd

start_date = '2018-01-01'
end_date = '2018-04-30'

# create date range with 1-hour intervals
dates = pd.date_range(start_date, end_date, freq='1H')
# create a random variable to plot
var = np.random.rand(len(dates)) - 0.51
var = var.cumsum()
var = var - var.min()
# create dataframe, make "date" the index
df = pd.DataFrame({'date': dates, 'variable': var})
df.set_index(df['date'], inplace=True)
```

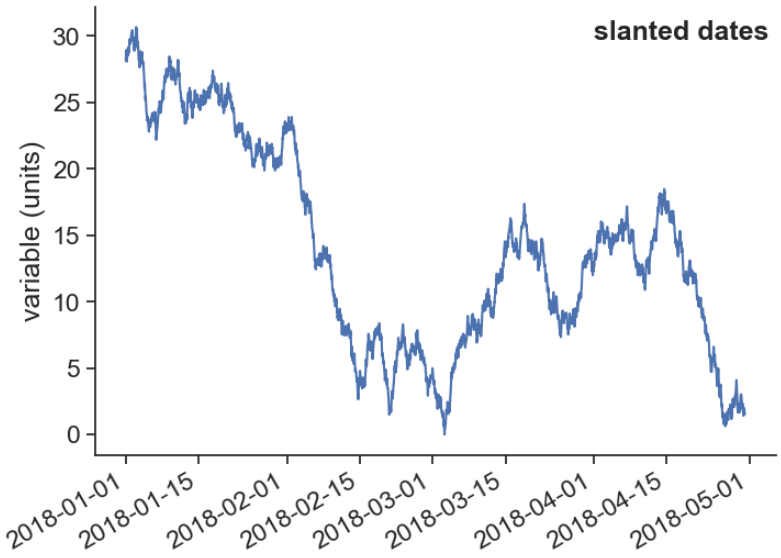
```
df
```

	date	variable
date		
2018-01-01 00:00:00	2018-01-01 00:00:00	28.317035
2018-01-01 01:00:00	2018-01-01 01:00:00	28.120523
2018-01-01 02:00:00	2018-01-01 02:00:00	28.596894
2018-01-01 03:00:00	2018-01-01 03:00:00	28.931941
2018-01-01 04:00:00	2018-01-01 04:00:00	28.561778
...	...	...
2018-04-29 20:00:00	2018-04-29 20:00:00	1.914343
2018-04-29 21:00:00	2018-04-29 21:00:00	1.648757
2018-04-29 22:00:00	2018-04-29 22:00:00	1.992956
2018-04-29 23:00:00	2018-04-29 23:00:00	1.500860
2018-04-30 00:00:00	2018-04-30 00:00:00	1.650439

define a useful function to plot the graphs below

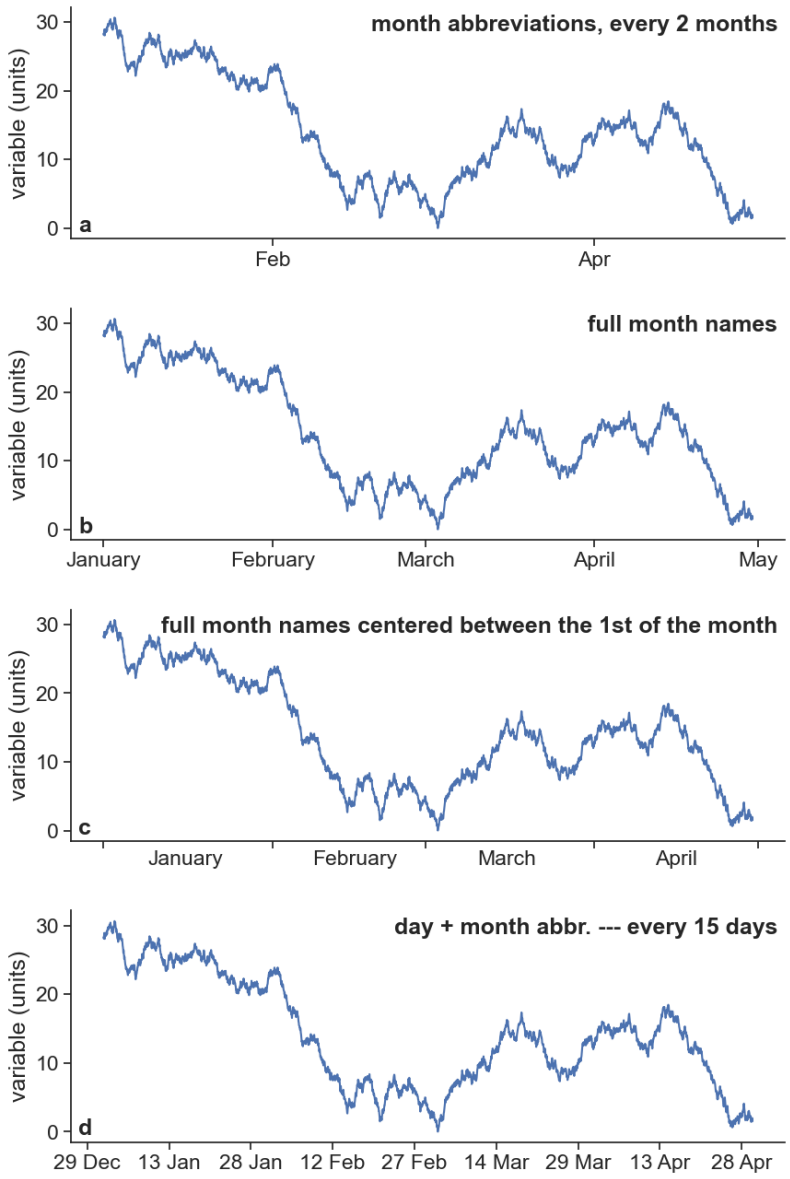
```
def explanation(ax, text, letter):
    ax.text(0.99, 0.97, text,
            transform=ax.transAxes,
            horizontalalignment='right', verticalalignment='top',
            fontweight="bold")
    ax.text(0.01, 0.01, letter,
            transform=ax.transAxes,
            horizontalalignment='left', verticalalignment='bottom',
            fontweight="bold")
    ax.set(ylabel="variable (units)")
    ax.spines['top'].set_visible(False)
    ax.spines['right'].set_visible(False)
```

```
fig, ax = plt.subplots(1, 1, figsize=(8, 6))
ax.plot(df['variable'])
plt.gcf().autofmt_xdate()  # makes slated dates
explanation(ax, "slanted dates", "")
fig.savefig("dates1.png")
```



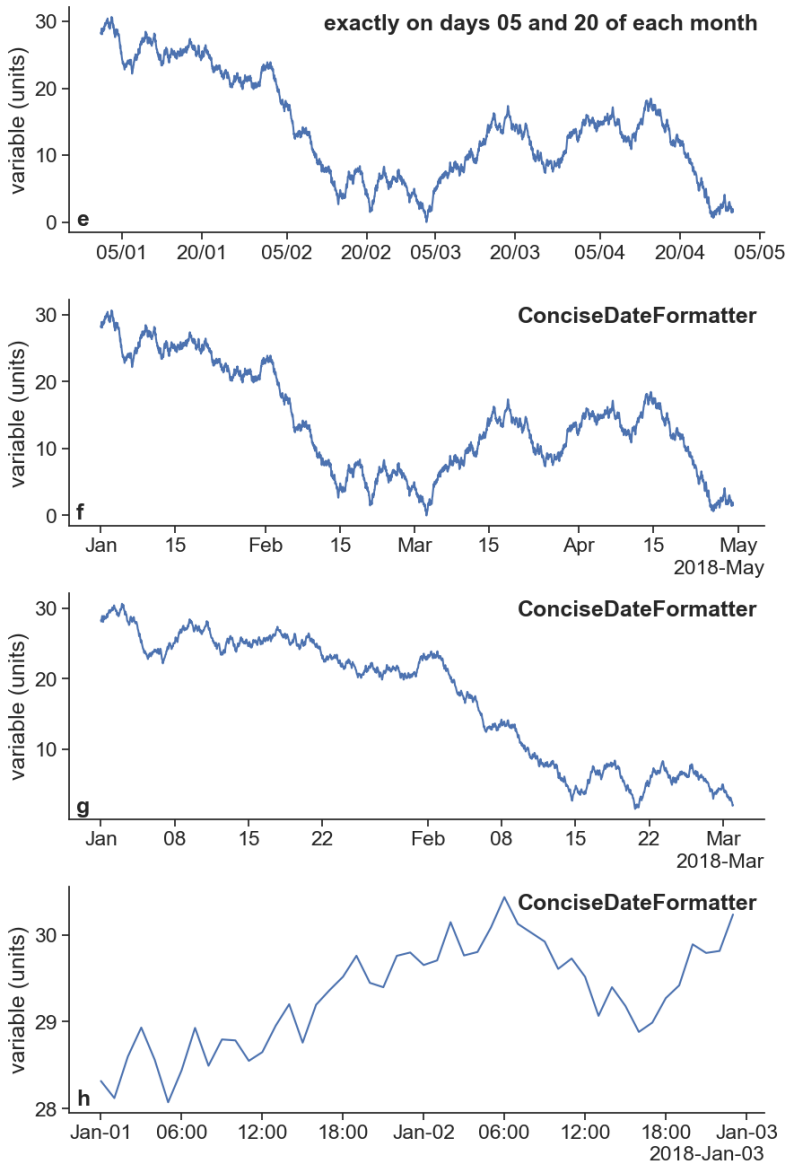

```
fig, ax = plt.subplots(4, 1, figsize=(10, 16),
                       gridspec_kw={'hspace': 0.3})

### plot a ###
ax[0].plot(df['variable'])
date_form = DateFormatter("%b")
ax[0].xaxis.set_major_locator(mdates.MonthLocator(interval=2))
ax[0].xaxis.set_major_formatter(date_form)

### plot b ###
ax[1].plot(df['variable'])
date_form = DateFormatter("%B")
ax[1].xaxis.set_major_locator(mdates.MonthLocator(interval=1))
ax[1].xaxis.set_major_formatter(date_form)

### plot c ###
ax[2].plot(df['variable'])
ax[2].xaxis.set_major_locator(mdates.MonthLocator())
# 16 is a slight approximation for the center, since months differ in number of days.
ax[2].xaxis.set_minor_locator(mdates.MonthLocator(bymonthday=16))
ax[2].xaxis.set_major_formatter(ticker.NullFormatter())
ax[2].xaxis.set_minor_formatter(DateFormatter('%B'))
```

```
for tick in ax[2].xaxis.get_minor_ticks():
    tick.tick1line.set_markersize(0)
    tick.tick2line.set_markersize(0)
    tick.label1.set_horizontalalignment('center')

### plot d ###
ax[3].plot(df['variable'])
date_form = DateFormatter("%d %b")
ax[3].xaxis.set_major_locator(mdates.DayLocator(interval=15))
ax[3].xaxis.set_major_formatter(date_form)

explanation(ax[0], "month abbreviations, every 2 months", "a")
explanation(ax[1], "full month names", "b")
explanation(ax[2], "full month names centered between the 1st of the month", "c")
explanation(ax[3], "day + month abbr. --- every 15 days", "d")

fig.savefig("dates2.png")
```



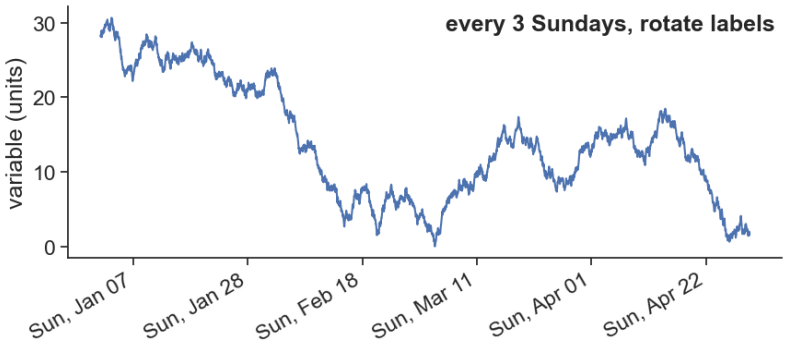

```
fig, ax = plt.subplots(4, 1, figsize=(10, 16),
                       gridspec_kw={'hspace': 0.3})

### plot e ###
ax[0].plot(df['variable'])
```

```
date_form = DateFormatter("%d/%m")
ax[0].xaxis.set_major_locator(mdates.DayLocator(bymonthday=[5, 20]))
ax[0].xaxis.set_major_formatter(date_form)

### plot f ###
ax[1].plot(df['variable'])
locator = mdates.AutoDateLocator(minticks=11, maxticks=17)
formatter = mdates.ConciseDateFormatter(locator)
ax[1].xaxis.set_major_locator(locator)
ax[1].xaxis.set_major_formatter(formatter)

### plot g ###
ax[2].plot(df.loc['2018-01-01':'2018-03-01', 'variable'])
locator = mdates.AutoDateLocator(minticks=6, maxticks=14)
formatter = mdates.ConciseDateFormatter(locator)
ax[2].xaxis.set_major_locator(locator)
ax[2].xaxis.set_major_formatter(formatter)

### plot h ###
ax[3].plot(df.loc['2018-01-01':'2018-01-02', 'variable'])
locator = mdates.AutoDateLocator(minticks=6, maxticks=10)
formatter = mdates.ConciseDateFormatter(locator)
ax[3].xaxis.set_major_locator(locator)
ax[3].xaxis.set_major_formatter(formatter)

explanation(ax[0], "exactly on days 05 and 20 of each month", "e")
explanation(ax[1], "ConciseDateFormatter", "f")
explanation(ax[2], "ConciseDateFormatter", "g")
explanation(ax[3], "ConciseDateFormatter", "h")

fig.savefig("dates3.png")
```




```
fig, ax = plt.subplots(1, 1, figsize=(10, 4),
                       gridspec_kw={'hspace': 0.3})

# import constants for the days of the week
from matplotlib.dates import MO, TU, WE, TH, FR, SA, SU
ax.plot(df['variable'])
```

```
# tick on sundays every third week
loc = mdates.WeekdayLocator(byweekday=SU, interval=3)
ax.xaxis.set_major_locator(loc)
date_form = DateFormatter("%a, %b %d")
ax.xaxis.set_major_formatter(date_form)
fig.autofmt_xdate(bottom=0.2, rotation=30, ha='right')
explanation(ax, "every 3 Sundays, rotate labels", "")
```



Code	Explanation
%Y	4-digit year (e.g., 2022)
%y	2-digit year (e.g., 22)
%m	2-digit month (e.g., 12)
%B	Full month name (e.g., December)
%b	Abbreviated month name (e.g., Dec)
%d	2-digit day of the month (e.g., 09)
%A	Full weekday name (e.g., Tuesday)
%a	Abbreviated weekday name (e.g., Tue)
%H	24-hour clock hour (e.g., 23)
%I	12-hour clock hour (e.g., 11)
%M	2-digit minute (e.g., 59)
%S	2-digit second (e.g., 59)
%p	“AM” or “PM”
%Z	Time zone name
%z	Time zone offset from UTC (e.g., -0500)

Part VI

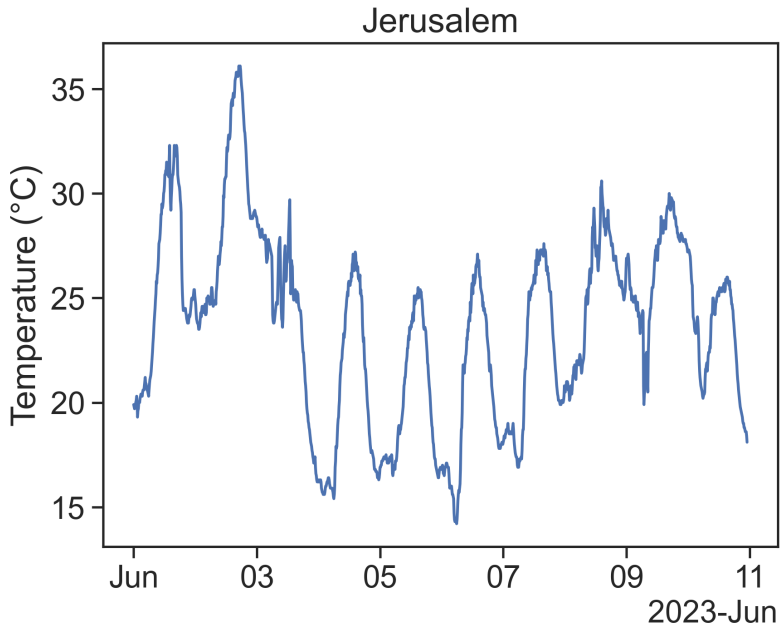
stationarity

19 motivation

20 stochastic processes

21 autocorrelation

See the temperatures for Jerusalem in a 4-day interval:



21.1 question

If I know the temperature right now, what does that tell me about the temperature 10 minutes from now? How about 100 minutes? 1000 minutes?

To answer this, we need to talk about **autocorrelation**. Let's start by introducing the necessary concepts.

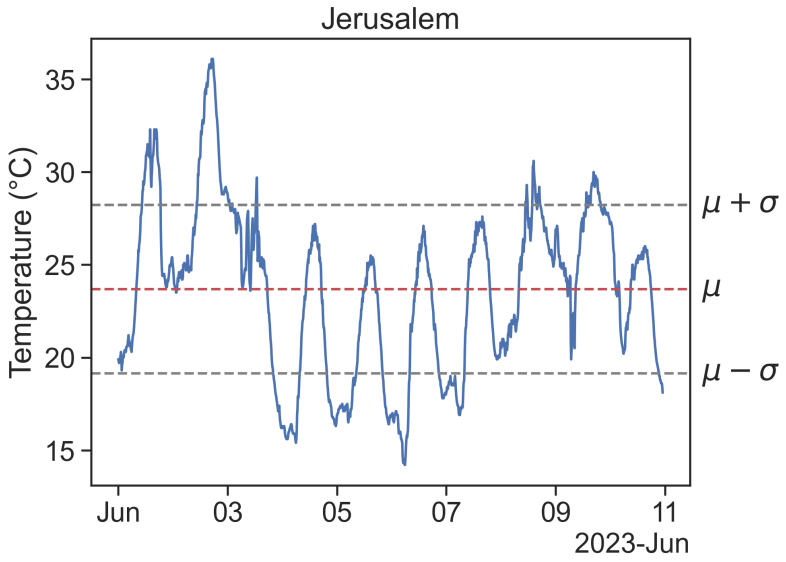
21.2 mean and standard deviation

Let's call our time series from above X, and its length N. Then:

$$\text{mean} \quad \mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

$$\text{standard deviation} \quad \sigma = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}}$$

The mean and standard deviation can be visualized thus:



One last basic concept we need is the expected value:

$$E[X] = \sum_{i=1}^{N} X_i p_i$$

For our time series, the probability p_i that a given point X_i is in the dataset is simply $1/N$, therefore the expectation becomes

$$E[X] = \frac{\sum_{i=1}^{N} X_i}{N}$$

21.3 autocorrelation

The autocorrelation of a time series X is the answer to the following question:

> if we shift X by τ units, how similar will this be to the original signal?

In other words:

> how correlated are $X(t)$ and $X(t+\tau)$?

Using the Pearson correlation coefficient

we get

$$\rho_{XX}(\tau) = \frac{E\left[(X_t - \mu)(X_{t+\tau} - \mu)\right]}{\sigma^2}$$

Pearson correlation coefficient between X and Y:

$$\rho_{X,Y} = \frac{E\left[(X - \mu_X)(X_Y - \mu_Y)\right]}{\sigma_X \sigma_Y}$$

A video is worth a billion words, so let's see the autocorrelation in action:

https://youtu.be/tpf-tuYHR5w

A few comments:

- The autocorrelation for $\tau = 0$ (zero shift) is always 1. [Can you prove this? All the necessary equations are above!]

Part VII

time lags

22 motivation

23 cross-correlation

```
import numpy as np
```

```
print('dfvdfv')
```

```
dfvdfv
```

24 dynamic time warp

25 LDTW

according to this paper

Part VIII

frequency

26 motivation

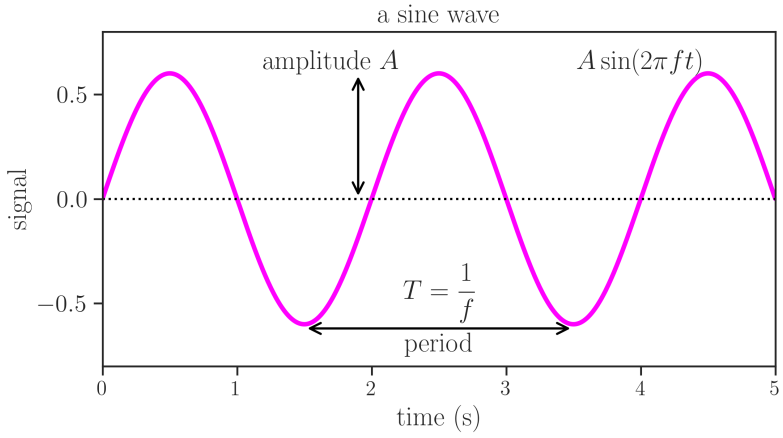
27 Fourier transform

27.1 basic wave concepts

The function

$$f(t) = B\sin(2\pi f t) \tag{27.1}$$

has two basic characteristics, its amplitude B and frequency f.

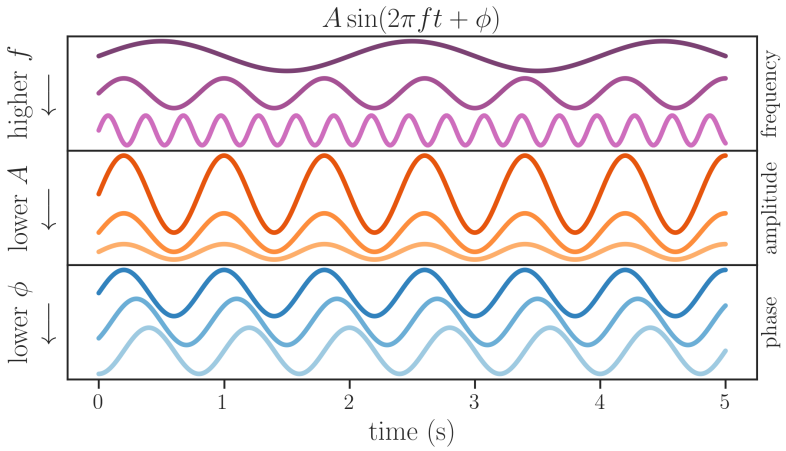


In the figure above, the amplitude $B = 0.6$ and we see that the distance between two peaks is called period, $T = 2$ s. The frequency is defined as the inverse of the period:

$$f = \frac{1}{T}. \tag{27.2}$$

When time is in seconds, then the frequency is measured in Hertz (Hz). For the graph above, therefore, we see a wave whose frequency is $f = 1/(2\text{ s}) = 0.5$ Hz.

In the figure below, we see what happens when we vary the values of the frequency and amplitude.



The graph above introduces two new characteristics of a wave, its phase ϕ, and its offset B. A more general description of a sine wave is

$$f(t) = B\sin(2\pi ft+\phi) + B_0. \tag{27.3}$$

The offset B_0 moves the wave up and down, while changing the value of ϕ makes the sine wave move left and right. When the phase $\phi = 2\pi$, the sine wave will have shifted a full period, and the resulting wave is identical to the original:

$$B\sin(2\pi ft) = B\sin(2\pi ft+2\pi). \tag{27.4}$$

All the above can also be said about a cosine, whose general for can be given as

$$A\cos(2\pi ft+\phi) + A_0 \tag{27.5}$$

One final point before we jump into the deep waters is that the sine and cosine functions are related through a simple phase shift:

$$\cos\left(2\pi ft+\frac{\pi}{2}\right) = \sin\left(2\pi ft\right)$$

27.2 Fourier's theorem

Fourier's theorem states that

> Any periodic signal is composed of a superposition of pure sine waves, with suitably chosen amplitudes and phases, whose frequencies are harmonics of the fundamental frequency of the signal.

See the following animations to visualize the theorem in action.

Source: https://en.wikipedia.org/wiki/File:Fourier_series_and_transform.gif

Source: https://commons.wikimedia.org/wiki/File:Fourier_synthesis_square_wave_animated.gif

Source: https://commons.wikimedia.org/wiki/File:Sawtooth_Fourier_Animation.gif

Source: https://commons.wikimedia.org/wiki/File:Continuous_Fourier_transform_of_rect_and_sinc_functions.gif

27.3 Fourier series

> a periodic function can be described as a sum of sines and cosines.

The classic examples are usually the square function and the sawtooth function:

[Source: https://www.geogebra.org/m/tkajbzmg]

https://www.geogebra.org/m/k4eq4fkr

Not any function, but certainly most functions we will deal with in this course. The function has to fullful the Dirichlet conditions

$$F[x(t)] = F(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt$$

$$f(t) = \int_{-\infty}^{\infty} F(f) e^{2\pi i f t} df$$

https://dibsmethodsmeetings.github.io/fourier-transforms/

https://www.jezzamon.com/fourier/index.html

28 filtering

29 Nyquist-Shannon sampling theorem

Part IX

seasonality

30 motivation

31 seasonal decomposition

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()  # datetime converter for a matplotlib
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)
from statsmodels.tsa.seasonal import seasonal_decompose
import matplotlib.dates as mdates
from matplotlib.dates import DateFormatter
```

31.1 trends in atmospheric carbon dioxide

Mauna Loa CO2 concentration.
data from NOAA

```
url = "https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_weekly_mlo.csv"
# df = pd.read_csv(url, header=47, na_values=[-999.99])

# you can first download, and then read the csv
filename = "co2_weekly_mlo.csv"
df = pd.read_csv(filename, header=35, na_values=[-999.99])

df
```

	1974	5	19	1974.3795	333.37	5.1	-999.99	-999.99.1	50.39
0	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.05
1	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.59
2	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.64

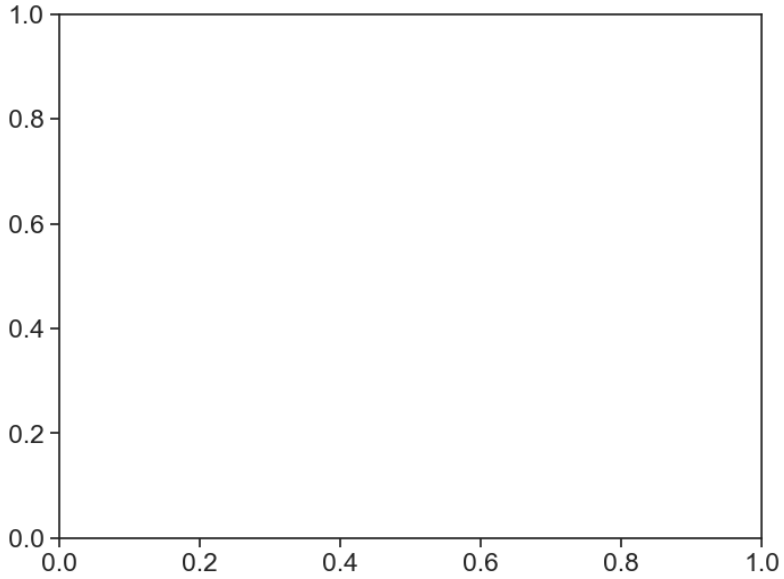
	1974	5	19	1974.3795	333.37	5.1	-999.99	-999.99.1	50.39
3	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
4	1974	6	23	1974.4753	331.73	5	NaN	NaN	49.72
...	...	...	...	...	...	...	...	...	...
2565	2023	7	23	2023.5575	421.28	4	418.03	397.30	141.60
2566	2023	7	30	2023.5767	420.83	6	418.10	396.80	141.69
2567	2023	8	6	2023.5959	420.02	6	417.36	395.65	141.41
2568	2023	8	13	2023.6151	418.98	4	417.25	395.24	140.89
2569	2023	8	20	2023.6342	419.31	2	416.64	395.22	141.71

```
df['date'] = pd.to_datetime(df[['year', 'month', 'day']])
df = df.set_index('date')
df
```

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
1974-05-19	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.40
1974-05-26	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.06
1974-06-02	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.60
1974-06-09	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.65
1974-06-16	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...	...	...	...	...	...	...	...	...	...
2022-06-26	2022	6	26	2022.4836	420.31	7	418.14	395.36	138.71
2022-07-03	2022	7	3	2022.5027	419.73	6	417.49	395.15	138.64
2022-07-10	2022	7	10	2022.5219	419.08	6	417.25	394.59	138.52
2022-07-17	2022	7	17	2022.5411	418.43	6	417.14	394.64	138.41
2022-07-24	2022	7	24	2022.5603	417.84	6	415.68	394.11	138.36

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,6))
ax.plot(df['average'])
ax.set(xlabel="date",
       ylabel="CO2 concentration (ppm)",
       # ylim=[0, 430],
       title="Mauna Loa CO2 concentration");
```

```
KeyError: 'average'
```



fill missing data. interpolate method: 'time'
interpolation methods visualized

```
df['co2'] = (df['average'].resample("D") #resample daily
                          .interpolate(method='time') #interpolate by time
             )
df
```

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
1974-05-19	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.40
1974-05-26	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.06
1974-06-02	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.60
1974-06-09	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.65
1974-06-16	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...	...	...	...	...	...	...	...	...	...
2022-06-26	2022	6	26	2022.4836	420.31	7	418.14	395.36	138.71
2022-07-03	2022	7	3	2022.5027	419.73	6	417.49	395.15	138.64
2022-07-10	2022	7	10	2022.5219	419.08	6	417.25	394.59	138.52

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
2022-07-17	2022	7	17	2022.5411	418.43	6	417.14	394.64	138.41
2022-07-24	2022	7	24	2022.5603	417.84	6	415.68	394.11	138.36

31.2 decompose data

`seasonal_decompose` returns an object with four components:

- observed: $Y(t)$
- trend: $T(t)$
- seasonal: $S(t)$
- resid: $e(t)$

Additive model:

$$Y(t) = T(t) + S(t) + e(t)$$

Multiplicative model:

$$Y(t) = T(t) \times S(t) \times e(t)$$

31.2.0.1 Interlude

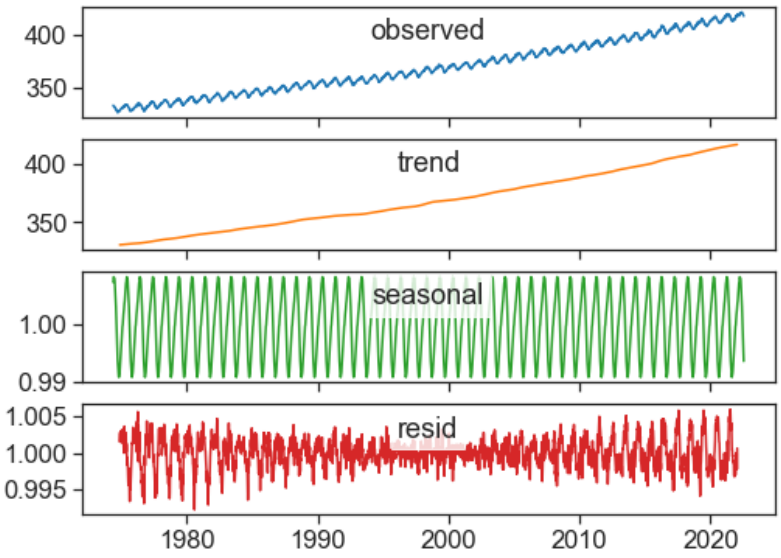
learn how to use `zip` in a loop

```
letters = ['a', 'b', 'c', 'd', 'e']
numbers = [1, 2, 3, 4, 5]
# zip let's us iterate over to lists at the same time
for l, n in zip(letters, numbers):
    print(f"{l} = {n}")
```

```
a = 1
b = 2
c = 3
d = 4
e = 5
```

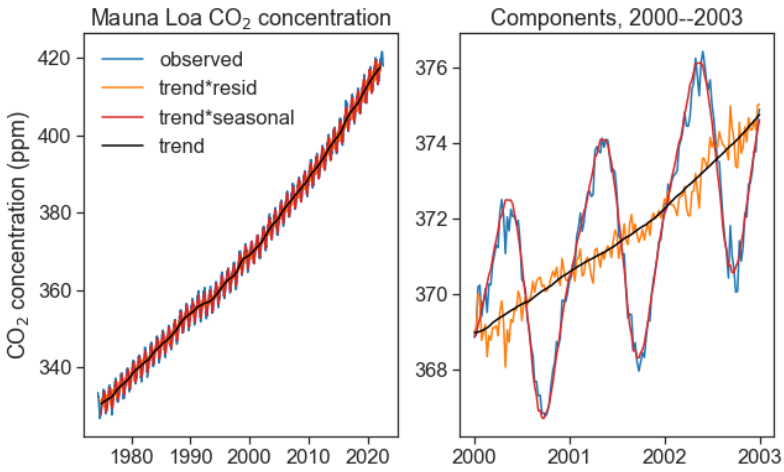
Plot each component separately.

```
# %matplotlib widget

fig, ax = plt.subplots(4, 1, figsize=(8,6), sharex=True)
decomposed_m = seasonal_decompose(df['co2'], model='multiplicative')
decomposed_a = seasonal_decompose(df['co2'], model='additive')
decomposed = decomposed_m
pos = (0.5, 0.9)
components =["observed", "trend", "seasonal", "resid"]
colors = ["tab:blue", "tab:orange", "tab:green", "tab:red"]
for axx, component, color in zip(ax, components, colors):
    data = getattr(decomposed, component)
    axx.plot(data, color=color)
    axx.text(*pos, component, bbox=dict(facecolor='white', alpha=0.8),
           transform=axx.transAxes, ha='center', va='top')
```




```
# %matplotlib widget

decomposed = decomposed_m

fig, ax = plt.subplots(1, 2, figsize=(10,6))
```

```
ax[0].plot(df['co2'], color="tab:blue", label="observed")
ax[0].plot(decomposed.trend * decomposed.resid, color="tab:orange", label="trend*resid")
ax[0].plot(decomposed.trend * decomposed.seasonal, color="tab:red", label="trend*seasonal")
ax[0].plot(decomposed.trend, color="black", label="trend")
ax[0].set(ylabel="CO$_2$ concentration (ppm)",
          title="Mauna Loa CO$_2$ concentration")
ax[0].legend(frameon=False)

start = "2000-01-01"
end = "2003-01-01"
zoom = slice(start, end)
ax[1].plot(df.loc[zoom, 'co2'], color="tab:blue", label="observed")
ax[1].plot((decomposed.trend * decomposed.resid)[zoom], color="tab:orange", label="trend*res
ax[1].plot((decomposed.trend * decomposed.seasonal)[zoom], color="tab:red", label="trend*sea
ax[1].plot(decomposed.trend[zoom], color="black", label="trend")
date_form = DateFormatter("%Y")
ax[1].xaxis.set_major_formatter(date_form)
ax[1].xaxis.set_major_locator(mdates.YearLocator(1))
ax[1].set_title("Components, 2000--2003");
```



32 Hilbert transform

Part X

rates of change

33 motivation

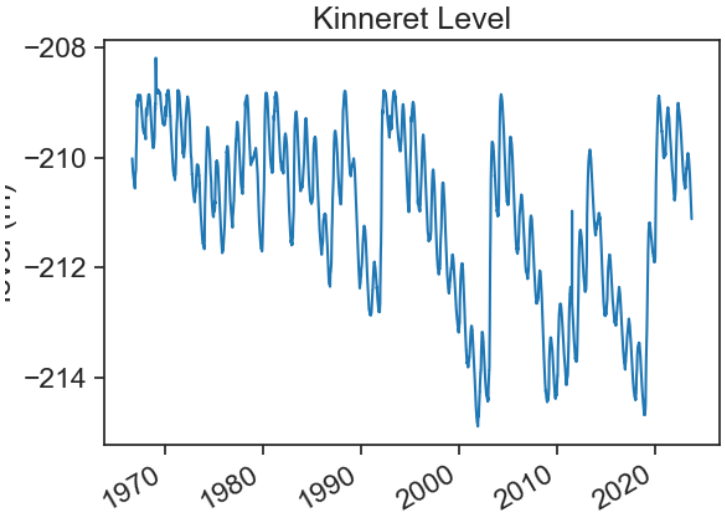
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)  # white graphs, with large and legible letters
%matplotlib widget
```

```
filename = "../archive/data/kinneret_cleaned.csv"
df = pd.read_csv(filename)
df['date'] = pd.to_datetime(df['date'], dayfirst=True)
df = df.set_index('date')
df
```

date	level
2023-09-12	-211.115
2023-09-11	-211.105
2023-09-10	-211.095
2023-09-09	-211.085
2023-09-08	-211.070
...	...
1966-11-01	-210.390
1966-10-15	-210.320
1966-10-01	-210.270
1966-09-15	-210.130
1966-09-01	-210.020

```
fig, ax = plt.subplots()
ax.plot(df['level'], color="tab:blue")
ax.set(title="Kinneret Level",
```

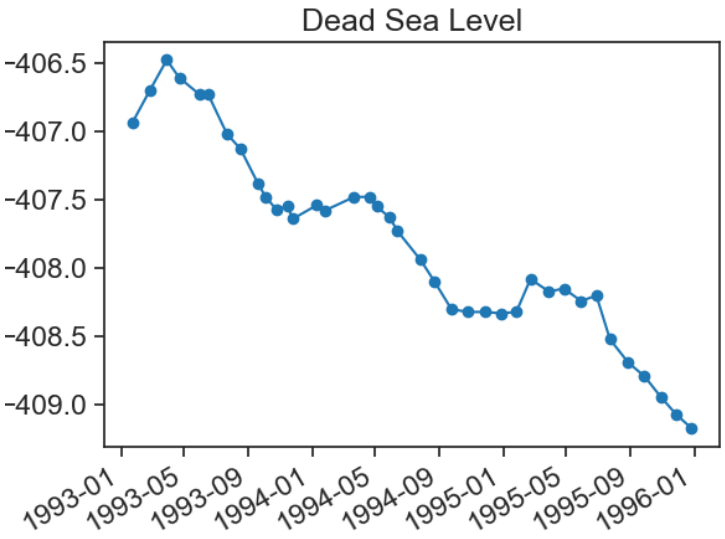
```
       ylabel="level (m)")
plt.gcf().autofmt_xdate()  # makes slanted dates
```



The data seems ok, until we take a closer look. Data points are not evenly spaced in time.

```
fig, ax = plt.subplots()
ax.plot(df.loc["1993":"1995", 'level'], color="tab:blue", marker="o")
ax.set(title="Dead Sea Level",
       ylabel="level (m)")
plt.gcf().autofmt_xdate()  # makes slanted dates
```

```
/var/folders/c3/7hp0d36n6vv8jc9hm2440__00000gn/T/ipykernel_3777/934261896.py:2: FutureWarning:
  ax.plot(df.loc["1993":"1995", 'level'], color="tab:blue", marker="o")
```



We can resample by day (a much higher rate than the original),
and linearly interpolate:

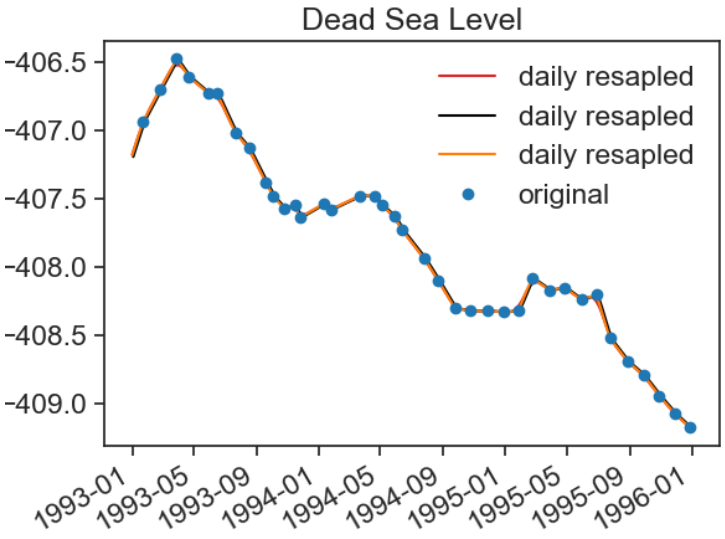
```
df2 = df['level'].resample('D').interpolate('time').to_frame()
df2['level_sm'] = df2['level'].rolling('30D', center=True).mean()
df3 = df2['level'].resample('W').mean().to_frame()
```

```
fig, ax = plt.subplots()
ax.plot(df2.loc["1993":"1995", 'level_sm'],
        color="tab:red",
        label="daily resapled")
ax.plot(df3.loc["1993":"1995", 'level'],
        color="black",
        label="daily resapled")
ax.plot(df2.loc["1993":"1995", 'level'],
        color="tab:orange",
        label="daily resapled")
ax.plot(df.loc["1993":"1995", 'level'],
        color="tab:blue",
        marker="o",
        linestyle="None",
```

```
        label="original")
ax.set(title="Dead Sea Level",
       ylabel="level (m)")
plt.gcf().autofmt_xdate()  # makes slanted dates
ax.legend(frameon=False)
```

```
/var/folders/c3/7hp0d36n6vv8jc9hm2440__00000gn/T/ipykernel_3777/2583247388.py:11: FutureWarning
  ax.plot(df.loc["1993":"1995", 'level'],
```

```
<matplotlib.legend.Legend at 0x7fa71e8b0bb0>
```

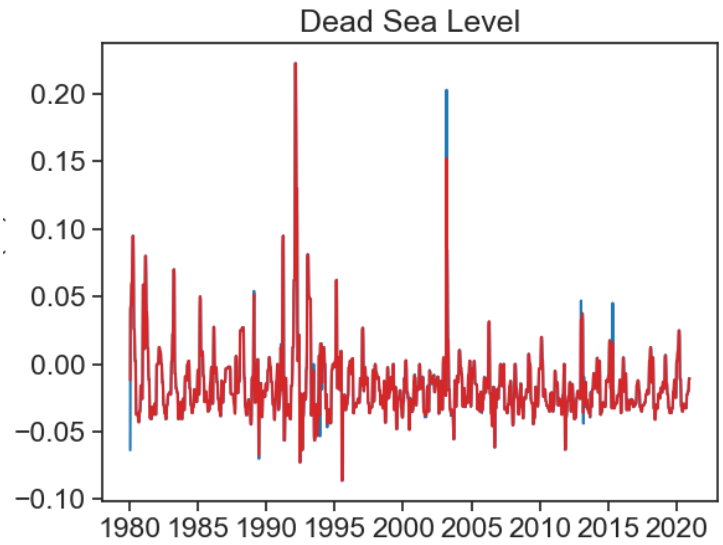



```
df2['naive'] = df2['level'].diff()
df2['gradient'] = np.gradient(df2['level'])

df3['naive'] = df3['level'].diff()
df3['gradient'] = np.gradient(df3['level'])
```

```
fig, ax = plt.subplots()
ax.plot(df3.loc["1980":"2020", 'naive'], color="tab:blue")
ax.plot(df3.loc["1980":"2020", 'gradient'], color="tab:red")
```

```
ax.set(title="Dead Sea Level",
       ylabel="level (m)")
```

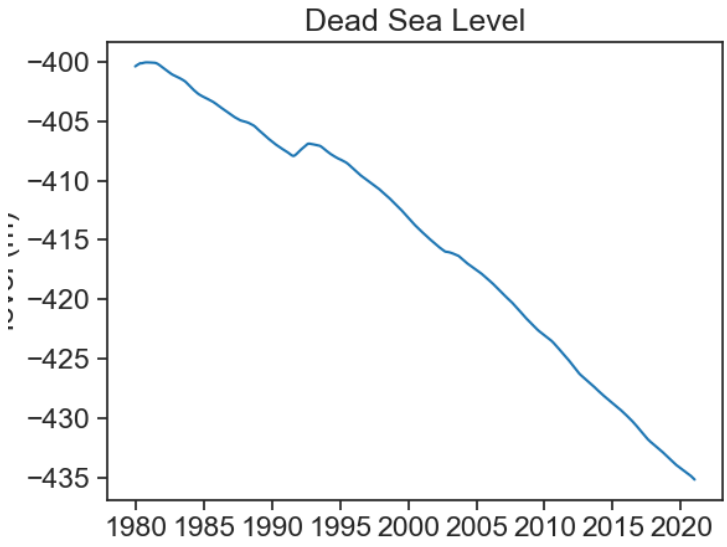
```
[Text(0.5, 1.0, 'Dead Sea Level'), Text(0, 0.5, 'level (m)')]
```




```
df3 = df2["level"].rolling('365.24D', center=True).mean().to_frame()
```

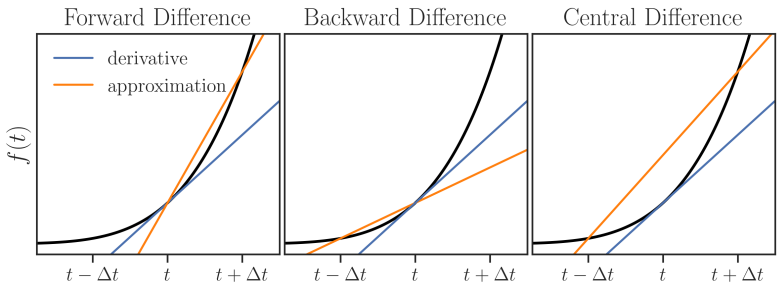
```
fig, ax = plt.subplots()
ax.plot(df3.loc["1980":"2020", 'level'], color="tab:blue")
ax.set(title="Dead Sea Level",
       ylabel="level (m)")
```

```
[Text(0.5, 1.0, 'Dead Sea Level'), Text(0, 0.5, 'level (m)')]
```


Dead Sea Level
−400
−405
−410
−415
−420
−425
−430
−435
1980 1985 1990 1995 2000 2005 2010 2015 2020

34 derivatives

35 finite differences



Definition of a **derivative**:

$$\underbrace{\dot{f} = f'(t) = \frac{df(t)}{dt}}_{\text{same thing}} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}.$$

Numerically, we can approximate the derivative $f'(t)$ of a time series $f(t)$ as

$$\frac{df(t)}{dt} = \frac{f(t+\Delta t) - f(t)}{\Delta t} + \mathcal{O}(\Delta t). \tag{35.1}$$

The expression above is called the *two-point forward difference formula*. Likewise, we can define the *two-point backward difference formula*:

The expression $\mathcal{O}(\Delta t)$ means that the error associated with the approximation is proportional to Δt. This is called "Big O notation".

$$\frac{df(t)}{dt} = \frac{f(t) - f(t-\Delta t)}{\Delta t} + \mathcal{O}(\Delta t). \tag{35.2}$$

If we sum together Equation 35.1 and Equation 35.2 we get:

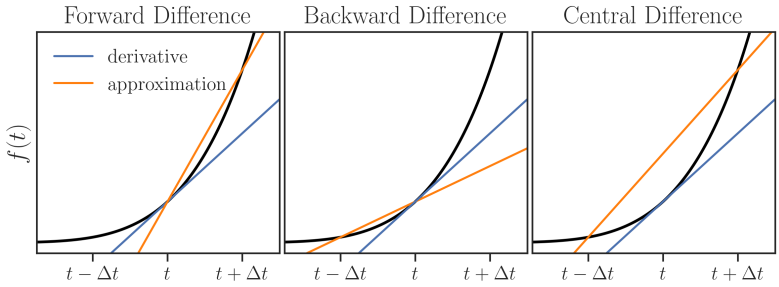
$$\begin{aligned} 2\frac{df(t)}{dt} &= \frac{f(t+\Delta t) - \cancel{f(t)}}{\Delta t} + \frac{\cancel{f(t)} - f(t-\Delta t)}{\Delta t} \\ &= \frac{f(t+\Delta t) - f(t-\Delta t)}{\Delta t}. \end{aligned} \tag{35.3}$$

Dividing both sides by 2 gives the *two-point central difference formula*:

$$\frac{df(t)}{dt} = \frac{f(t+\Delta t) - f(t-\Delta t)}{2\Delta t} + \mathcal{O}(\Delta t^2). \tag{35.4}$$

Two things are worth mentioning about the approximation above:

1. it is balanced, that is, there is no preference of the future over the past.
2. its error is proportional to Δt^2, it is a lot more precise than the unbalanced approximations :)



To understand why the error is proportional to Δt^2, one can subtract the Taylor expansion of $f(t-\Delta t)$ from the Taylor expansion of $f(t+\Delta t)$. See this, pages 3 and 4.

The function `np.gradient` calculates the derivative using the central difference for points in the interior of the array, and uses the forward (backward) difference for the derivative at the beginning (end) of the array.

Check out this nice example.

The "gradient" usually refers to a first derivative with respect to space, and it is denoted as $\nabla f(x) = \frac{df(x)}{dx}$. However, it doesn't really matter if we call the independent variable x or t, the derivative operator is exactly the same.

36 Fourier-based derivatives

This tutorial is based on Pelliccia (2019).

nice trick: https://math.stackexchange.com/questions/430858/fourier-transform-of-derivative

37 LOESS-based derivatives

Part XI

forecasting

38 motivation

39 ARIMA

Part XII

assignments

40 assignment 1

This assignment comes right after the first session, where we discussed resampling.

40.1 Task

Go to the IMS website, and choose another weather station we have not worked with yet. Download 10-minute data for a full year, any year.

Make 3 graphs:

1. Monthly temperature averages
2. Daily maximum temperature
3. For each day of the year, show the number of hours when global solar radiation was above, on average, the threshold 10 W/m^2. Now add another line, for the threshold 500 W/m^2.

Make 7 more graphs (total of 10 graphs) of whatever you find interesting. Explore the dataset you downloaded. Use the tools we learned during class. Be creative.

You must download this Jupyter Notebook template. Create a zip file with your Jupyter notebook and with the csv you used. Upload this csv to the moodle task we created.

40.2 Evaluation

All your assignments will be evaluated according to the following criteria:

- 40% Presentation. How the graphs look, labels, general organization, markdown, clean code.
- 30% Discussion. This is where you explain what you did, what you found out, etc.
- 15% Depth of analysis. You can analyze/explore the data with different levels of complexity, this is where we take that into consideration.
- 10% Replicability: Your code runs flawlessly.
- 5%: Code commenting. Explain in your code what you are doing, this is good for everyone, especially for yourself!
- Bonus: 10% for originality, creative problem solving, or notable analysis.

41 assignment 2

42 final assignment

technical stuff

operating systems

I recommend working with UNIX-based operating systems (MacOS or Linux). Everything is easier.

If you use Windows, consider installing Linux on Windows with WSL.

software

Anaconda's Python distribution

VSCode

python packages

Kats — a one-stop shop for time series analysis
Developed by Meta

statsmodels statsmodels is a Python package that provides a complement to scipy for statistical computations including descriptive statistics and estimation and inference for statistical models.

ydata-profiling
Quick Exploratory Data Analysis on time-series data. Read also this.

sources

books

from Data to Viz

Fundamentals of Data Visualization, by Claus O. Wilke

PyNotes in Agriscience

Forecasting: Principles and Practice (3rd ed), by Rob J Hyndman and George Athanasopoulos

Python for Finance Cookbook 2nd Edition - Code Repository

Practical time series analysis,: prediction with statistics and machine learning, by Aileen Nielsen
The online edition of this book is available for Hebrew University staff and students.

Time series analysis with Python cookbook : practical recipes for exploratory data analysis, data preparation, forecasting, and model evaluation, by Tarek A. Atwan
The online edition of this book is available for Hebrew University staff and students.

Hands-on Time Series Analysis with Python: From Basics to Bleeding Edge Techniques, by B V Vishwas, Ashish Patel
The online edition of this book is available for Hebrew University staff and students.

videos

Times Series Analysis for Everyone, by Bruno Goncalves
This series is available for Hebrew University staff and students.

Time Series Analysis with Pandas, by Joshua Malina This video is available for Hebrew University staff and students.

references

Atwan, Tarek A. 2022. *Time Series Analysis with Python Cookbook: Practical Recipes for Exploratory Data Analysis, Data Preparation, Forecasting, and Model Evaluation*. Packt.

Eilers, Paul HC. 2003. "A Perfect Smoother." *Analytical Chemistry* 75 (14): 3631–36. https://doi.org/10.1021/ac034173t.

McDonald, Andy. 2022. "Creating Boxplots with the Seaborn Python Library." *Medium*. Towards Data Science. https://towardsdatascience.com/creating-boxplots-with-the-seaborn-python-library-f0c20f09bd57.

Pelliccia, Daniel. 2019. "Fourier Spectral Smoothing Method." 2019. https://nirpyresearch.com/fourier-spectral-smoothing-method/.

Zhang, Ou. 2020. "Outliers-Part 3:outliers in Regression." ouzhang.me. https://ouzhang.me/blog/outlier-series/outliers-part3/.