

Time Series Analysis

Yair Mau

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about

Welcome to **Time Series Analysis for Environmental Sciences** (71606) at the Hebrew University of Jerusalem. This is Yair Mau, your host for today. I am a senior lecturer at the Institute of Environmental Sciences, at the Faculty of Agriculture, Food and Environment, in Rehovot, Israel.

This website contains (almost) all the material you'll need for the course. If you find any mistakes, or have any comments, please email me.

disclaimer

The material here is not comprehensive and **does not** constitute a stand alone course in Time Series Analysis. This is only the support material for the actual presential course I give.

what, who, when and where?

Course number 71606, 3 academic points
Yair Mau (lecturer), Erez Feuer (TA)
Tuesdays, from 14:15 to 17:00
Computer [classroom #18](#)
Office hours: upon request

syllabus

course description

Data analysis of time series, with practical examples from environmental sciences.

course aims

This course aims at giving the students a broad overview of the main steps involved in the analysis of time series: data management, data wrangling, visualization, analysis, and forecast. The course will provide a hands-on approach, where students will actively engage with real-life datasets from the field of environmental science.

learning outcomes

On successful completion of this module, students should be able to:

- Explore a time-series dataset, while formulating interesting questions.
- Choose the appropriate tools to attack the problem and answer the questions.
- Communicate their findings and the methods they used to achieve them, using graphs, statistics, text, and a well-documented code.

course content

- **Data wrangling:** organization, cleaning, merging, filling gaps, excluding outliers, smoothing, resampling.
- **Visualization:** best practices for graph making using leading python libraries.
- **Analysis:** stationarity, seasonality, (auto)correlations, lags, derivatives, spectral analysis.
- **Forecast:** ARIMA
- **Data management:** how to plan ahead and best organize large quantities of data. If there is enough time, we will build a simple time-series database.

books and other sources

[Click here.](#)

course evaluation

There will be assignments during the semester (totaling 50% of the final grade), and one final project (50%).

Evaluation policy

- **Individual Work:** While we support helping your peers, it's important to remember that all assignments must be completed individually. This means that your submissions should be your own unique work and not contain code or text that is identical to someone else's.
- **Zero Plagiarism:** Do not copy text verbatim from any source. Always express ideas in your own words.
- **On-Time Submission:** Assignments must be turned in by the specified deadline. Late submissions will receive a grade of 0. If you require an extension, requests will only be considered if made at least 24 hours before the due date.
- **Non-Compliance Consequence:** Assignments that do not adhere to these guidelines will automatically receive a grade of 0.

weekly program

This year's course will be a bit different than planned due to the shortening of the academic semester. The information below is NOT up to date. Ask Yair what is relevant this year.

week 1

- **Lecture:** Course overview, setting of expectations. Introduction, basic concepts, continuous vs discrete time series, sampling, aliasing
- **Exercise:** Loading csv file into python, basic time series manipulation with pandas and plotting

week 2

- **Lecture:** Filling gaps, removing outliers
- **Exercise:** Practice the same topics learned during the lecture. Data: air temperature and relative humidity

week 3

- **Lecture:** Interpolation, resampling, binning statistics
- **Exercise:** Practice the same topics learned during the lecture. Data: air temperature and relative humidity, precipitation

week 4

- **Lecture:** Time series plotting: best practices. Dos and don'ts and maybes
- **Exercise:** Practice with Seaborn, Plotly, Pandas, Matplotlib

Project 1

Basic data wrangling, using real data (temperature, relative humidity, precipitation) downloaded from USGS. 25% of the final grade

week 5

- **Lecture:** Smoothing, running averages, convolution
- **Exercise:** Practice the same topics learned during the lecture. Data: sap flow, evapotranspiration

week 6

- **Lecture:** Strong and weak stationarity, stochastic processes, auto-correlation
- **Exercise:** Practice the same topics learned during the lecture. Data: temperature and wind speed

week 7

- **Lecture:** Correlation between signals. Pearson correlation, time-lagged cross-correlations, dynamic time warping
- **Exercise:** Practice the same topics learned during the lecture. Data: temperature, solar radiation, relative humidity, soil moisture, evapotranspiration

week 8

Same as lecture 7 above

week 9

- **Lecture:** Download data from repositories, using API, merging, documentation
- **Exercise:** Download data from USGS, NOAA, Fluxnet, Israel Meteorological Service

Project 2

Students will study a Fluxnet site of their choosing. How do gas fluxes (CO₂, H₂O) depend on environmental conditions? 25% of the final grade

week 10

- **Lecture:** Fourier decomposition, filtering, Nyquist–Shannon sampling theorem
- **Exercise:** Practice the same topics learned during the lecture. Data: dendrometer data

week 11

- **Lecture:** Seasonality, seasonal decomposition (trend, seasonal, residue), Hilbert transform
- **Exercise:** Practice the same topics learned during the lecture. Data: monthly atmospheric CO₂ concentration, hourly air temperature

week 12

- **Lecture:** Derivatives, differencing
- **Exercise:** Practice the same topics learned during the lecture. Data: dendrometer data

week 13

- **Lecture:** Forecasting. ARIMA
- **Exercise:** Practice the same topics learned during the lecture. Data: vegetation variables (sap flow, ET, DBH, etc)

Final Project

In consultation with the lecturer, students will ask a specific scientific question about a site of their choosing (from NOAA, USGS, Fluxnet), and answer it using the tools learned during the semester. The report will be written in Jupyter Notebook, combining in one document all the calculations, documentation, figures, analysis, and discussion. 50% of the final grade.

who cares?

why “Time Series Analysis?”

Time has two aspects. There is the arrow, the running river, without which there is no change, no progress, or direction, or creation. And there is the circle or the cycle, without which there is chaos, meaningless succession of instants, a world without clocks or seasons or promises.

URSULA K. LE GUIN

You are here because you are interested in how things change, evolve. In this course I want to discuss with you how to make sense of data whose temporal nature is in its very essence. We will talk about randomness, cycles, frequencies, correlations, and more.

why “Environmental Sciences”

This same time series analysis (TSA) course could be called instead “TSA for finance”, “TSA for Biology”, or any other application. The emphasis in this course is **not** Environmental Sciences, but the concepts and tools of TSA. Because my research is in Environmental Science, and many of the graduate students at HUJI-Rehovot research this, I chose to use examples “close to home”. The same toolset should be useful for students of other disciplines.

what is it good for?

In many fields of science we are flooded by data, and it’s hard to see the forest for the trees. I hope that the topics we’ll

discuss in this course can help you find meaningful patterns in your data, formulate interesting hypotheses, and design better experiments.

do I need it?

Maybe. If you are a grad student and you have temporal data to analyze, then probably yes. However, I have very fond memories of courses that I took as a grad student that were completely unrelated to my research. Sometimes “because it’s fun” is a perfectly good answer.

what will I actually gain from it?

By the end of this course you will have gained:

- a **hands-on** experience of fundamental time-series analysis tools
- an **intuition** regarding the basic concepts
- **technical** abilities
- a **springboard** for learning more about the subject by yourself

Part I

start here

1 the boring stuff you absolutely need to do

I assume everyone registered has taken a basic Python course.
On your computer, do the following:

1.1 Anaconda

Install [Anaconda's Python distribution](#). The Anaconda installation brings with it all the main python packages we will need to use. In order to install extra packages, refer to these two tutorials: [tutorial 1](#), [tutorial 2](#).

1.2 VSCode

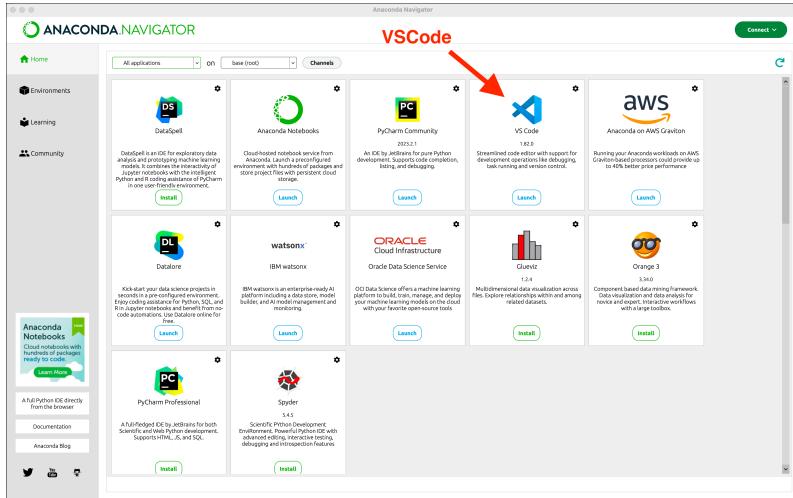
Install [VSCode](#). Visual Studio Code is a very nice IDE (Integrated Development Environment) made by Microsoft, available to all operating systems. Contrary to the title of this page, it is not absolutely necessary to use it, but I like VSCode, and as my student, so do you .

1.3 jupyter notebooks

We will code exclusively in Jupyter Notebooks. [Get acquainted with them](#). Make sure you can [point VSCode](#) to the Anaconda environment of your choice (“base” by default). Don’t worry, this is easier than it sounds.

One failproof way of making sure VSCode uses the Anaconda installation is the following:

- Open Anaconda Navigator
- If you are using HUJI's computers, in “Environments”, choose “asgard”. If you are using your own computer, ignore this step.
- open VSCode from inside Anaconda Navigator (see image below).



Sometimes you will need to manually install the Jupyter extension on VSCode. In this case follow [this tutorial](#).

1.4 folder structure

You **NEED** to be comfortable with your computer's folder (or directory) structure. Where are files located? How to navigate through different folders? How is my stuff organized? If you don't feel **absolutely** comfortable with this, then read this, [Windows](#), [MacOS](#). If you use Linux then you surely know this stuff. **Make yourself a “time-series” folder** wherever you want, and have it backed up regularly (use Google Drive, Dropbox, do it manually, etc). “My dog deleted my files” is not an excuse.

2 numpy, pandas, matplotlib

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

The three lines above are the most common way you will start every project in this course.

- **numpy** = numerical python. This library has a ton useful mathematical functions, and most importantly, it has an object called **numpy array**, which is one of the most useful data structures we have for time series analysis.
- **pandas** is built upon numpy, and allows us to easily manipulate data stored in **dataframes**, a fancy name for a table.
- **pyplot** is a submodule of **matplotlib**, and allows us to beautifully plot data.

The best resource I know to get acquainted with all three packages is [Python Data Science Handbook](#), by Jake VanderPlas. This is a free online book, with excellent step by step examples.

We will primarily use the Pandas package to deal with data. Pandas has become the standard Python tool to manipulate time series, and you should get acquainted with its basic usage. This course will provide you the opportunity to learn by example, but I'm sure we will only scratch the surface, and you'll be left with lots of questions.

I provide below a (non-comprehensive) list of useful tutorials, they are a good reference for the beginner and for the experienced user.

- [Python Data Science Handbook](#), by Jake VanderPlas

- Data Wrangling with pandas Cheat Sheet
- Working with Dates and Times in Python
- Cheat Sheet: The pandas DataFrame Object
- YouTube tutorials by Corey Schafer

2.1 pyplot

Matplotlib, and its submodule pyplot, are probably the most common Python plotting tool. Pyplot is both great and horrible:

- Great: you'll have absolutely full control of everything you want to plot. The sky is the limit.
- Horrible: you'll cry as you do it, because there is so much to know, and it is not the most friendly plotting package.

Pyplot is *object oriented*, so you will usually manipulate the `axes` object like this.

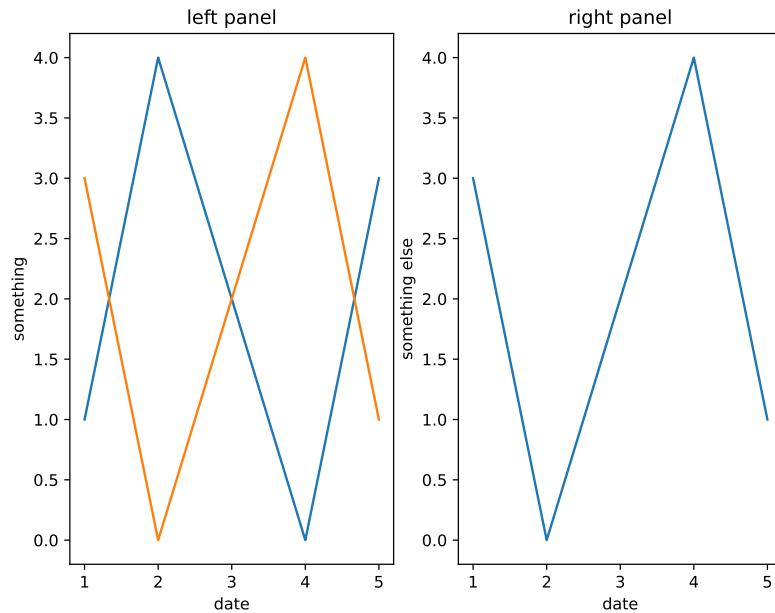
```
import matplotlib.pyplot as plt

x = [1, 2, 3, 4, 5]
y = [1, 4, 2, 0, 3]

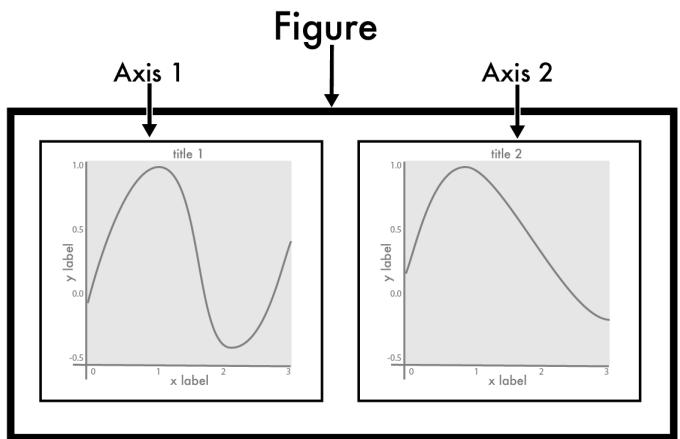
# Figure with two plots
fig, (ax1, ax2) = plt.subplots(1, 2, figsize = (8, 6))
# plot on the left
ax1.plot(x, y, color="tab:blue")
ax1.plot(x, y[::-1], color="tab:orange")
ax1.set(xlabel="date",
         ylabel="something",
         title="left panel")
# plot on the right
ax2.plot(x, y[::-1])
ax2.set(xlabel="date",
         ylabel="something else",
         title="right panel")

[Text(0.5, 0, 'date'),
```

```
Text(0, 0.5, 'something else'),
Text(0.5, 1.0, 'right panel')]
```

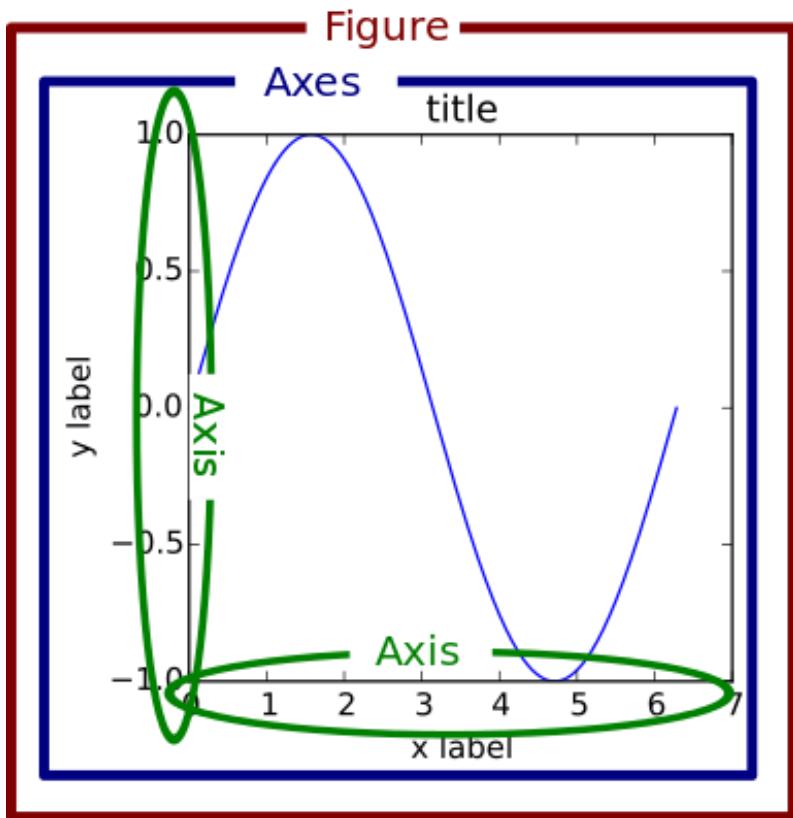


For the very beginners, you need to know that `figure` refers to the whole white canvas, and `axes` means the rectangle inside which something will be plotted:



The image above is good because it has 2 panels, and it's easy to understand what going on. Sadly, they mixed the two terms, axis and axes.

- **axes** is where the whole plot will be drawn. In the figure above it is the same as each panel.
- **axis** is each of the vertical and horizontal lines, where you have ticks and numbers.



If you are new to all this, I recommend that you go to:

- [Earth Lab's Introduction to Plotting in Python Using Matplotlib](#)
- [Jake VanderPlas's Python Data Science Handbook](#)

3 learn by example

Now that everything is installed, try to run the code below *before* the first lecture. Don't worry if you don't understand everything.

- If you manage to run everything without errors, this means that your computer is good to go!
- You might encounter a few problems. That's ok. Make a note and we will solve everything in the first lecture.

Let's make a first plot of real data. We will use NOAA's Global Monitoring Laboratory data on [Trends in Atmospheric Carbon Dioxide](#).

1. On your computer, open the program **Anaconda Navigator** (it may take a while to load).
2. Find the white box called **VS Code** and click **Launch**.
3. Now go to **File > Open Folder**, and open the folder you created for this course. VS Code may ask you if you trust the authors, and the answer is "yes" (it's your computer).
4. **File > New File**, and call it `example.ipynb`
5. You can start copying and pasting code from this website to your Jupyter Notebook. To run a cell, press Shift+Enter.
6. You may be asked to choose to Select Kernel. This is VS Code wanting to know which python installation to use. Click on "Python Environments", and then choose the option with the word `anaconda` in it.
7. That's all! Congratulations!

3.1 import packages

First, import packages to be used. They should all be already included in the Anaconda distribution you installed.

```

import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters

```

3.2 load data

Load CO₂ data into a Pandas dataframe. You can load it directly from the URL (option 1), or first download the CSV to your computer and then load it (option 2). The link to download the data directly from NOAA is [this](#). If for some reason this doesn't work, download [here](#).

```

# option 1: load data directly from URL
# url = "https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_weekly_mlo.csv"
# df = pd.read_csv(url,
#                   header=34,
#                   na_values=[-999.99]
#                   )

# option 2: download first (use the URL above and save it to your computer), then load csv
filename = "co2_weekly_mlo.csv"
df = pd.read_csv(filename,
                  comment='#', # will ignore rows starting with #
                  na_values=[-999.99] # substitute -999.99 for NaN (Not a Number), data not
                  )
# check how the dataframe (table) looks like
df

```

	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
0	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.39
1	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.05
2	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.59
3	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.64
4	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...
2566	2023	7	23	2023.5575	421.28	4	418.03	397.30	141.60

	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
2567	2023	7	30	2023.5767	420.83	6	418.10	396.80	141.69
2568	2023	8	6	2023.5959	420.02	6	417.36	395.65	141.41
2569	2023	8	13	2023.6151	418.98	4	417.25	395.24	140.89
2570	2023	8	20	2023.6342	419.31	2	416.64	395.22	141.71

3.3 dealing with dates

Create a new column called `date`, that combines the information from three separate columns: `year`, `month`, `day`.

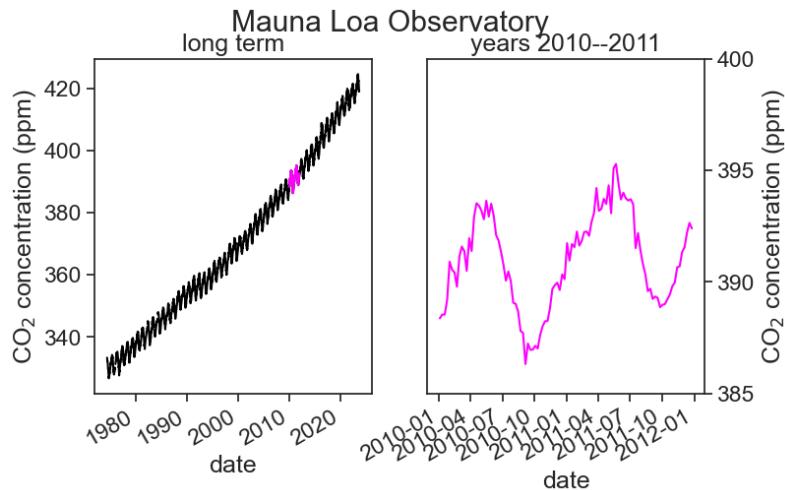
```
# function to_datetime translates the full date into a pandas datetime object,
# that is, pandas knows this is a date, it's not just a string
df['date'] = pd.to_datetime(df[['year', 'month', 'day']])
# make 'date' column the dataframe index
df = df.set_index('date')
# now see if everything is ok
df
```

	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
date									
1974-05-19	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.39
1974-05-26	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.05
1974-06-02	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.59
1974-06-09	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.64
1974-06-16	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...
2023-07-23	2023	7	23	2023.5575	421.28	4	418.03	397.30	141.60
2023-07-30	2023	7	30	2023.5767	420.83	6	418.10	396.80	141.69
2023-08-06	2023	8	6	2023.5959	420.02	6	417.36	395.65	141.41
2023-08-13	2023	8	13	2023.6151	418.98	4	417.25	395.24	140.89
2023-08-20	2023	8	20	2023.6342	419.31	2	416.64	395.22	141.71

3.4 first plot

We are now ready for our first plot! Let's see the weekly CO₂ average.

```
# %matplotlib widget
# uncomment the above line if you want dynamic control of the figure when using VSCode
fig, (ax1, ax2) = plt.subplots(1, 2, # 1 row, 2 columns
                               figsize=(8,5) # width, height, in inches
)
# left panel
ax1.plot(df['average'], color="black")
ax1.plot(df.loc['2010-01-01':'2011-12-31','average'], color="magenta")
ax1.set(xlabel="date",
        ylabel=r"CO$_2$ concentration (ppm)",
        title="long term");
# right panel
ax2.plot(df.loc['2010-01-01':'2011-12-31','average'], color="magenta")
ax2.set(xlabel="date",
        ylabel=r"CO$_2$ concentration (ppm)",
        ylim=[385, 400], # choose y limits
        yticks=np.arange(385, 401, 5), # choose ticks
        title="years 2010--2011");
# put ticks and label on the right for ax2
ax2.yaxis.tick_right()
ax2.yaxis.set_label_position("right")
# title above both panels
fig.suptitle("Mauna Loa Observatory")
# makes slanted dates
plt.gcf().autofmt_xdate()
```



3.5 first plot, v2.0

The dates in the x-label are not great. Let's try to make them prettier.

We need to import a few more packages first.

```
import matplotlib.dates as mdates
from matplotlib.dates import DateFormatter
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters() # datetime converter for a matplotlib
```

Now let's replot.

```
# %matplotlib widget
# uncomment the above line if you want dynamic control of the figure when using VSCode
fig, (ax1, ax2) = plt.subplots(1, 2, # 1 row, 2 columns
                               figsize=(8,5) # width, height, in inches
)
# left panel
ax1.plot(df['average'], color="black")
ax1.plot(df.loc['2010-01-01':'2011-12-31','average'], color="magenta")
ax1.set(xlabel="date",
        ylabel=r"CO$ _2$ concentration (ppm)",
```

```

        title="long term");
# right panel
ax2.plot(df.loc['2010-01-01':'2011-12-31','average'], color="magenta")
ax2.set(xlabel="date",
        ylabel=r"CO2 concentration (ppm)",
        ylim=[385, 400], # choose y limits
        yticks=np.arange(385, 401, 5), # choose ticks
        title="years 2010--2011");
# put ticks and label on the right for ax2
ax2.yaxis.tick_right()
ax2.yaxis.set_label_position("right")
# title above both panels
fig.suptitle("Mauna Loa Observatory", y=1.00)

locator = mdates.AutoDateLocator(minticks=3, maxticks=5)
formatter = mdates.ConciseDateFormatter(locator)
ax1.xaxis.set_major_locator(locator)
ax1.xaxis.set_major_formatter(formatter)

locator = mdates.AutoDateLocator(minticks=4, maxticks=5)
formatter = mdates.ConciseDateFormatter(locator)
ax2.xaxis.set_major_locator(locator)
ax2.xaxis.set_major_formatter(formatter)

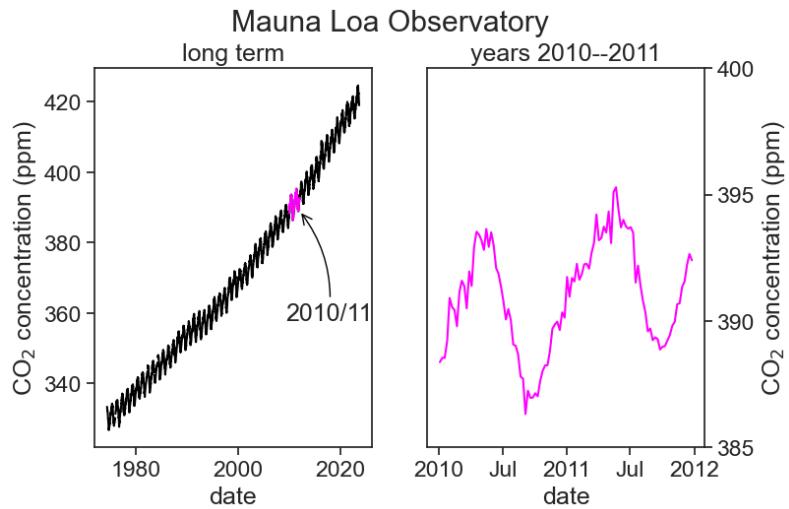
ax1.annotate(
    "2010/11",
    xy=('2011-12-25', 389), xycoords='data',
    xytext=(-10, -80), textcoords='offset points',
    arrowprops=dict(arrowstyle="->",
                    color="black",
                    connectionstyle="arc3,rad=0.2"))
fig.savefig("CO2-graph.png", dpi=300)

```

```

/var/folders/hc/jhnmlst937d27zzq9fhfks780000gn/T/ipykernel_10652/850389963.py:42: UserWarning:
  fig.savefig("CO2-graph.png", dpi=300)
/opt/anaconda3/lib/python3.9/site-packages/IPython/core/pylabtools.py:151: UserWarning: AutoDat
  fig.canvas.print_figure(bytes_io, **kw)

```



The dates on the horizontal axis are determined thus:

1. `locator = mdates.AutoDateLocator(minticks=3, maxticks=5)`
This determines the location of the ticks (between 3 and 5 ticks, whatever “works best”)
2. `ax1.xaxis.set_major_locator(locator)`
This actually puts the ticks in the positions determined above
3. `formatter = mdates.ConciseDateFormatter(locator)`
This says that the labels will be placed at the locations determined in 1.
4. `ax1.xaxis.set_major_formatter(formatter)`
Finally, labels are written down

The arrow is placed in the graph using `annotate`. It has a tricky syntax and a million options. Read [Jake VanderPlas’s excellent examples](#) to learn more.

3.6 modifications

Let’s change a lot of plotting options to see how things could be different.

```

sns.set(style="darkgrid")
sns.set_context("notebook")

# %matplotlib widget
# uncomment the above line if you want dynamic control of the figure when using VSCode
fig, (ax1, ax2) = plt.subplots(1, 2, # 1 row, 2 columns
                               figsize=(8,4) # width, height, in inches
)
# left panel
ax1.plot(df['average'], color="tab:blue")
ax1.plot(df.loc['2010-01-01':'2011-12-31','average'], color="tab:orange")
ax1.set(xlabel="date",
        ylabel=r"CO$_2$ concentration (ppm)",
        title="long term");
# right panel
ax2.plot(df.loc['2010-01-01':'2011-12-31','average'], color="tab:orange")
ax2.set(xlabel="date",
        ylim=[385, 400], # choose y limits
        yticks=np.arange(385, 401, 5), # choose ticks
        title="years 2010--2011");
# title above both panels
fig.suptitle("Mauna Loa Observatory", y=1.00)

locator = mdates.AutoDateLocator(minticks=3, maxticks=5)
formatter = mdates.ConciseDateFormatter(locator)
ax1.xaxis.set_major_locator(locator)
ax1.xaxis.set_major_formatter(formatter)

locator = mdates.AutoDateLocator(minticks=5, maxticks=8)
formatter = mdates.ConciseDateFormatter(locator)
ax2.xaxis.set_major_locator(locator)
ax2.xaxis.set_major_formatter(formatter)

ax1.annotate(
    "2010/11",
    xy=('2010-12-25', 395), xycoords='data',
    xytext=(-100, 40), textcoords='offset points',
    bbox=dict(boxstyle="round4,pad=.5", fc="white"),
    arrowprops=dict(arrowstyle="->",

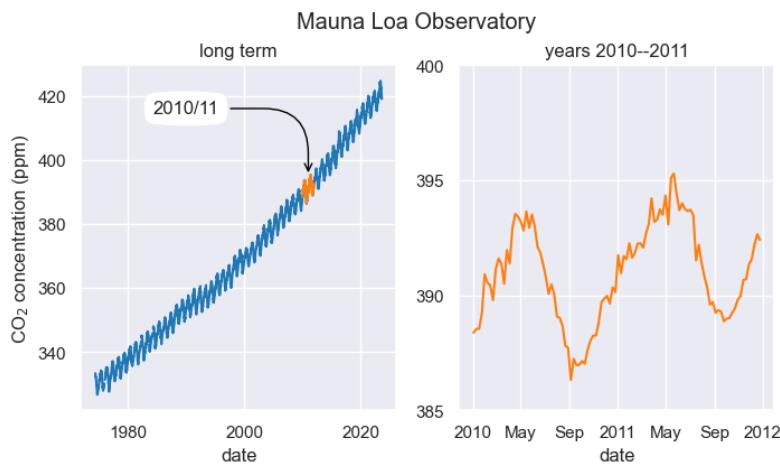
```

```

    color="black",
    connectionstyle="angle,angleA=0,angleB=-90,rad=40"))

Text(-100, 40, '2010/11')

```



The main changes were:

1. Using the Seaborn package, we changed the fontsize and the overall plot style. [Read more](#).

```

sns.set(style="darkgrid")
sns.set_context("notebook")

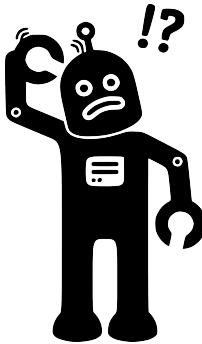
```

2. We changed the colors of the lineplots. To know what colors exist, [click here](#).
3. The arrow annotation has a different style. [Read more](#).

3.7 playing with the code

I encourage you to play with the code you just ran. An easy way of learning what each line does is to comment something out and see what changes in the output you see. If you feel brave, try to modify the code a little bit.

4 AI policy



The guidelines below are an adaptation of [Ethan Mollick's extremely useful ideas on AI](#) as an assistant tool for teaching.

I EXPECT YOU to use LLMs (large language models) such as ChatGPT, Bing AI, Google Bard, or whatever else springs up since the time of this writing. You should familiarize yourself with the AI's capabilities and limitations.

Use LLMs to help you learn, chat with them about what you want to accomplish and learn from them how to do it. **Ask** your LLM what each part of the code means, copy and pasting blindly is unacceptable. You are here to learn.

Consider the following important points:

- Ultimately, you, the student, are responsible for the assignment.
- Acknowledge the use of AI in your assignment. Be transparent about your use of the tool and the extent of assistance it provided.



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Learning
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Part II

resampling

5 motivation

Data from the [Israel Meteorological Service](#), IMS.

See the temperature at a weather station in Jerusalem, for the whole 2019 year. This is an interactive graph: to zoom in, play with the bottom panel.

```
alt.VConcatChart(...)
```

5.0.0.1 * discussion

The temperature fluctuates on various time scales, from daily to yearly. Let's think together a few questions we'd like to ask about the data above.

Now let's see precipitation data:

```
alt.VConcatChart(...)
```

5.0.0.2 * discussion

What would be interesting to know about precipitation?

We have not talked about what kind of data we have in our hands here. The csv file provided by the IMS looks like this:

	Station	Date & Time (Winter)	Diffused radiation (W/m ²)	Global radiation (W/m ²)
0	Jerusalem Givat Ram	01/01/2019 00:00	0.0	0.0
1	Jerusalem Givat Ram	01/01/2019 00:10	0.0	0.0
2	Jerusalem Givat Ram	01/01/2019 00:20	0.0	0.0
3	Jerusalem Givat Ram	01/01/2019 00:30	0.0	0.0
4	Jerusalem Givat Ram	01/01/2019 00:40	0.0	0.0
...
52549	Jerusalem Givat Ram	31/12/2019 22:20	0.0	0.0

	Station	Date & Time (Winter)	Diffused radiation (W/m ²)	Global radiation (W/m ²)
52550	Jerusalem Givat Ram	31/12/2019 22:30	0.0	0.0
52551	Jerusalem Givat Ram	31/12/2019 22:40	0.0	0.0
52552	Jerusalem Givat Ram	31/12/2019 22:50	0.0	0.0
52553	Jerusalem Givat Ram	31/12/2019 23:00	0.0	0.0

We see that we have data points spaced out evenly every 10 minutes.

5.1 Challenges

Let's try to answer the following questions:

What is the mean temperature for each month?

First we have to divide temperature data by month, and then take the average for each month.
a possible solution

```
df_month = df['temperature'].resample('M').mean()
```

For each month, what is the mean of the daily maximum temperature? What about the minimum?

This is a bit trickier.

1. We need to find the maximum/minimum temperature for each day.
2. Only then we split the daily data by month and take the average.

a possible solution

```
df_day['max temp'] = df['temperature'].resample('D').max()
df_month['max temp'] = df_day['max temp'].resample('MS').mean()
```

What is the average night temperature for every season?
What about the day temperature?

1. We need to filter our data to contain only night times.
2. We need to divide rain data by seasons (3 months), and then take the mean for each season.

a possible solution

```
# filter only night data
df_night = df.loc[((df.index.hour < 6) | (df.index.hour >= 18))]
season_average_night_temp = df_night['temperature'].resample('Q').mean()
```

another possible solution

```
# filter using between_time
df_night = df.between_time('18:00', '06:00', inclusive='left')
season_average_night_temp = df_night['temperature'].resample('Q').mean()
```

What is the daily precipitation?

First we have to divide rain data by day, and then take the sum for each day.

a possible solution

```
daily_precipitation = df['rain'].resample('D').sum()
```

How much rain was there every month?

We have to divide rain data by month, and then sum the totals of each month.

a possible solution

```
monthly_precipitation = df['rain'].resample('M').sum()
```

How many rainy days were there each month?

1. We need to sum rain by day.
2. We need to count how many days are there each month where `rain > 0`.

a possible solution

```
daily_precipitation = df['rain'].resample('D').sum()  
only_rainy_days = daily_precipitation.loc[daily_precipitation > 0]  
rain_days_per_month = only_rainy_days.resample('M').count()
```

How many days, hours, and minutes were between the last rain of the season (Malkosh) to the first (Yoreh)?

1. We need to divide our data into two: `rainy_season_1` and `rainy_season_2`.
2. We need to find the time of the last rain in `rainy_season_1`.
3. We need to find the time of the first rain in `rainy_season_2`.
4. We need to compute the time difference between the two dates.

a possible solution

```
split_date = '2019-08-01'  
rainy_season_1 = df[:split_date] # everything before split date  
rainy_season_2 = df[split_date:] # everything after split date  
malkosh = rainy_season_1['rain'].loc[rainy_season_1['rain'] > 0].last_valid_index()  
yoreh = rainy_season_2['rain'].loc[rainy_season_2['rain'] > 0].first_valid_index()  
dry_period = yoreh - malkosh  
# extracting days, hours, and minutes  
days = dry_period.days  
hours = dry_period.components.hours  
minutes = dry_period.components.minutes  
print(f'The dry period of 2019 was {days} days, {hours} hours and {minutes} minutes.')
```

i What was the rainiest morning (6am-12pm) of the year?
Bonus, what about the rainiest night (6pm-6am)?

1. We need to filter our data to contain only morning times.
2. We need to sum rain by day.
3. We need to find the day with the maximum value.

a possible solution

```
# filter to only day data
morning_df = df.loc[((df.index.hour >= 6) & (df.index.hour < 18))]
morning_rain = morning_df['rain'].resample('D').sum()
rainiest_morning = morning_rain.idxmax()
# plot
morning_rain.plot()
plt.axvline(rainiest_morning, c='r', alpha=0.5, linestyle='--')
```

bonus solution

```
# filter to only night data
df_night = df.loc[((df.index.hour < 6) | (df.index.hour >= 18))]
# resampling night for each day is tricky because the date changes at 12:00. We can do this
# we shift the time back by 6 hours so all the data for the same night will have the same index
df_shifted = df_night.tshift(-6, freq='H')
night_rain = df_shifted['rain'].resample('D').sum()
rainiest_night = night_rain.idxmax()
# plot
night_rain.plot()
plt.axvline(rainiest_night, c='r', alpha=0.5, linestyle='--')
```

Note: this whole webpage is actually a Jupyter Notebook rendered as html. If you want to know how to make interactive graphs, go to the top of the page and click on “Code”

Useful functions compatible with `pandas.resample()` can be found [here](#). The full list of resampling frequencies can be found [here](#).

6 resampling

We can only really understand how to calculate monthly means if we do it ourselves.

First, let's import a bunch of packages we need to use.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
warnings.simplefilter(action='ignore', category=UserWarning)
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters
```

Now we load the csv file for Jerusalem (2019), provided by the [IMS](#).

6.0.0.1 * discussion

We will go to the IMS website together and see what are the options available and how to download. If you just need the csv right away, download it [here](#).

- We substitute every occurrence of - for NaN (not a number, that is, the data is missing).
- We call the columns Temperature ($^{\circ}\text{C}$) and Rainfall (mm) with more convenient names, since we will be using them a lot.
- We interpret the column Date & Time (Winter) as a date, saying to python that day comes first.
- We make date the index of the dataframe.

```

filename = "../archive/data/jerusalem2019.csv"
df = pd.read_csv(filename, na_values=['-'])
df.rename(columns={'Temperature (°C)': 'temperature',
                   'Rainfall (mm)': 'rain'}, inplace=True)
df['date'] = pd.to_datetime(df['Date & Time (Winter)'], dayfirst=True)
df = df.set_index('date')
df

```

	Station	Date & Time (Winter)	Diffused radiation (W/m^2)	Global rad
date				
2019-01-01 00:00:00	Jerusalem Givat Ram	01/01/2019 00:00	0.0	0.0
2019-01-01 00:10:00	Jerusalem Givat Ram	01/01/2019 00:10	0.0	0.0
2019-01-01 00:20:00	Jerusalem Givat Ram	01/01/2019 00:20	0.0	0.0
2019-01-01 00:30:00	Jerusalem Givat Ram	01/01/2019 00:30	0.0	0.0
2019-01-01 00:40:00	Jerusalem Givat Ram	01/01/2019 00:40	0.0	0.0
...
2019-12-31 22:20:00	Jerusalem Givat Ram	31/12/2019 22:20	0.0	0.0
2019-12-31 22:30:00	Jerusalem Givat Ram	31/12/2019 22:30	0.0	0.0
2019-12-31 22:40:00	Jerusalem Givat Ram	31/12/2019 22:40	0.0	0.0
2019-12-31 22:50:00	Jerusalem Givat Ram	31/12/2019 22:50	0.0	0.0
2019-12-31 23:00:00	Jerusalem Givat Ram	31/12/2019 23:00	0.0	0.0

With `resample` it's easy to compute monthly averages. Resample by itself only divides the data into buckets (in this case monthly buckets), and waits for a further instruction. Here, the next instruction is `mean`.

```

df_month = df['temperature'].resample('M').mean()
df_month

```

date	temperature
2019-01-31	9.119937
2019-02-28	9.629812
2019-03-31	10.731571
2019-04-30	14.514329
2019-05-31	22.916894
2019-06-30	23.587361
2019-07-31	24.019403

```

2019-08-31    24.050822
2019-09-30    22.313287
2019-10-31    20.641868
2019-11-30    17.257153
2019-12-31    11.224131
Freq: M, Name: temperature, dtype: float64

```

Instead of M for month, which other options do I have? The full list can be [found here](#), but the most commonly used are:

M	month end frequency
MS	month start frequency
A	year end frequency
AS, YS	year start frequency
D	calendar day frequency
H	hourly frequency
T, min	minutely frequency
S	secondly frequency

The results we got for the monthly means were given as a pandas series, not dataframe. Let's correct this:

```

df_month = (df['temperature'].resample('M')           # resample by month
            .mean()                  # take the mean
            .to_frame('mean temp')) # make output a dafaframe
)
df_month

```

mean temp	
date	
2019-01-31	9.119937
2019-02-28	9.629812
2019-03-31	10.731571
2019-04-30	14.514329
2019-05-31	22.916894
2019-06-30	23.587361
2019-07-31	24.019403
2019-08-31	24.050822

	mean temp
date	
2019-09-30	22.313287
2019-10-31	20.641868
2019-11-30	17.257153
2019-12-31	11.224131

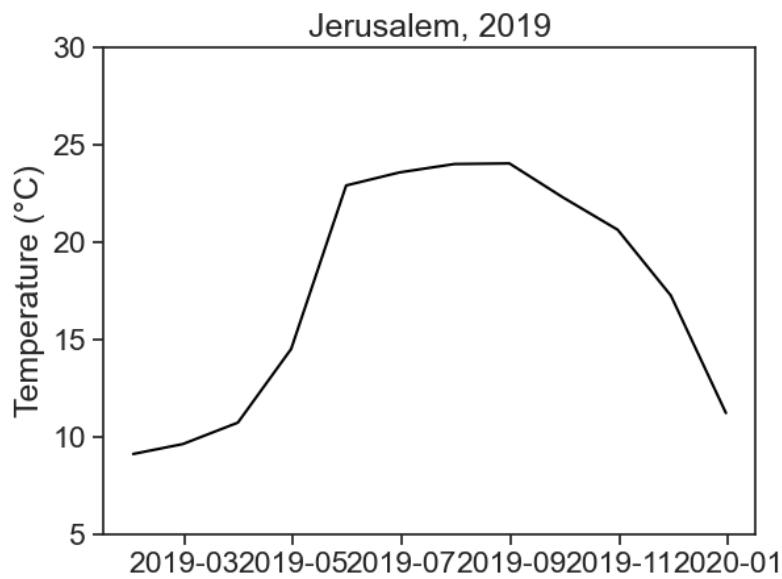
6.0.0.2 * hot tip

Sometimes, a line of code can get too long and messy. In the code above, we broke line for every step, which makes the process so much cleaner. We **highly** advise you to do the same. **Attention:** This trick works as long as all the elements are inside the same parenthesis.

Now it's time to plot!

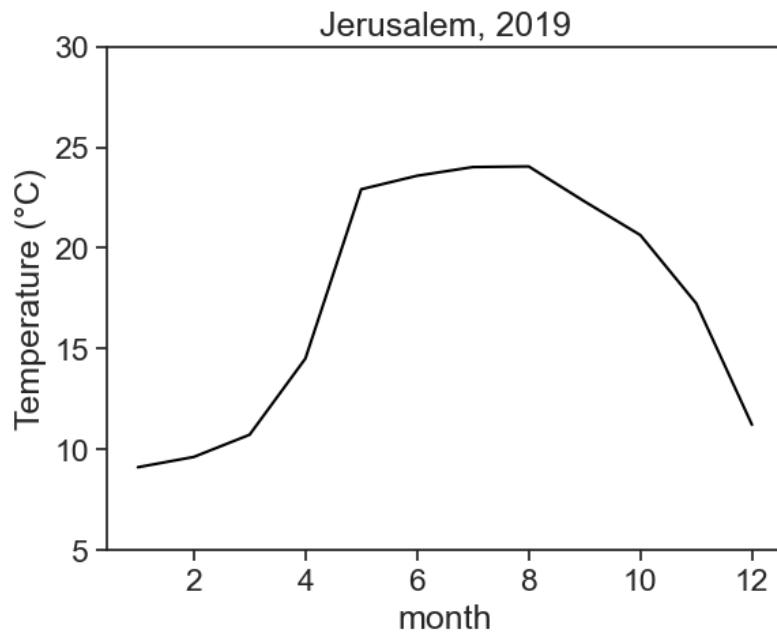
```
fig, ax = plt.subplots()
ax.plot(df_month['mean temp'], color='black')
ax.set(ylabel='Temperature (°C)',
       yticks=np.arange(5,35,5),
       title="Jerusalem, 2019")

[Text(0, 0.5, 'Temperature (°C)'),
 [matplotlib.axis.YTick at 0x7faf784c6d60>,
 <matplotlib.axis.YTick at 0x7faf7843a220>,
 <matplotlib.axis.YTick at 0x7faf784c62b0>,
 <matplotlib.axis.YTick at 0x7faf784f3400>,
 <matplotlib.axis.YTick at 0x7faf784f3760>,
 <matplotlib.axis.YTick at 0x7faf784fa5b0>],
Text(0.5, 1.0, 'Jerusalem, 2019')]
```



The dates in the horizontal axis are not great. An easy fix is to use the month numbers instead of dates.

```
fig, ax = plt.subplots()
ax.plot(df_month.index.month, df_month['mean temp'], color='black')
ax.set(xlabel="month",
       ylabel='Temperature (°C)',
       yticks=np.arange(5,35,5),
       title="Jerusalem, 2019");
```



6.0.0.3 * discussion

When you have datetime as the dataframe index, you don't need to give the function `plot` two arguments, date and values. You can just tell `plot` to use the column you want, the function will take the dates by itself.

What does this line mean?

```
df_month['mean temp'].index.month
```

Print on the screen the following, and see yourself what each thing is:

- `df_month`
- `df_month.index`
- `df_month.index.month`
- `df_month.index.day`

We're done! Congratulations :)

Now we need to calculate the average minimum/maximum daily temperatures. We start by creating an empty dataframe.

```
df_day = pd.DataFrame()
```

Now resample data by day (D), and take the min/max of each day.

```
df_day['min temp'] = df['temperature'].resample('D').min()
df_day['max temp'] = df['temperature'].resample('D').max()
df_day
```

	min temp	max temp
date		
2019-01-01	7.5	14.1
2019-01-02	6.6	11.5
2019-01-03	6.3	10.7
2019-01-04	6.6	14.6
2019-01-05	7.0	11.4
...
2019-12-27	4.4	7.4
2019-12-28	6.6	10.3
2019-12-29	8.1	12.5
2019-12-30	6.9	13.0
2019-12-31	5.2	13.3

The next step is to calculate the average minimum/maximum for each month. This is similar to what we did above.

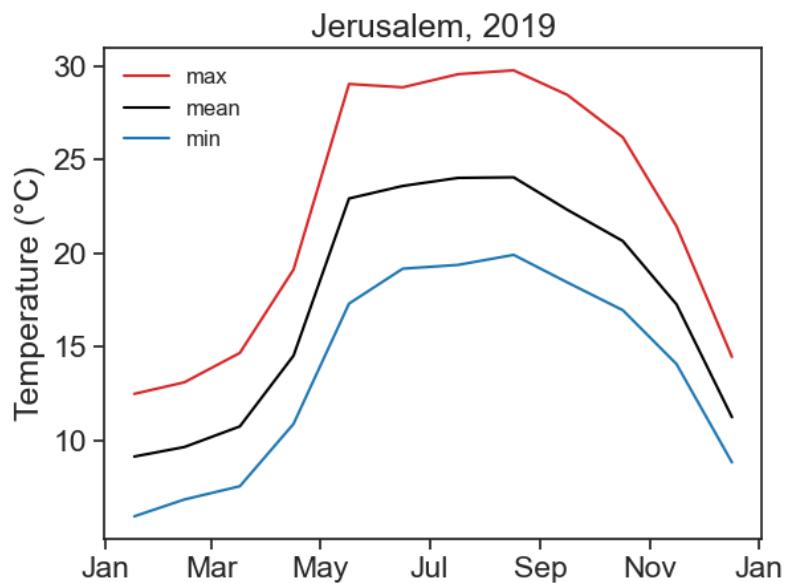
```
df_month['min temp'] = df_day['min temp'].resample('M').mean()
df_month['max temp'] = df_day['max temp'].resample('M').mean()
df_month
```

	mean temp	min temp	max temp
date			
2019-01-31	9.119937	5.922581	12.470968
2019-02-28	9.629812	6.825000	13.089286
2019-03-31	10.731571	7.532258	14.661290
2019-04-30	14.514329	10.866667	19.113333
2019-05-31	22.916894	17.296774	29.038710

	mean temp	min temp	max temp
date			
2019-06-30	23.587361	19.163333	28.860000
2019-07-31	24.019403	19.367742	29.564516
2019-08-31	24.050822	19.903226	29.767742
2019-09-30	22.313287	18.430000	28.456667
2019-10-31	20.641868	16.945161	26.190323
2019-11-30	17.257153	14.066667	21.436667
2019-12-31	11.224131	8.806452	14.448387

Let's plot...

```
fig, ax = plt.subplots()
ax.plot(df_month['max temp'], color='tab:red', label='max')
ax.plot(df_month['mean temp'], color='black', label='mean')
ax.plot(df_month['min temp'], color='tab:blue', label='min')
ax.set(ylabel='Temperature (°C)',
       yticks=np.arange(10,35,5),
       title="Jerusalem, 2019")
ax.xaxis.set_major_locator(mdates.MonthLocator(range(1, 13, 2), bymonthday=15))
date_form = DateFormatter("%b")
ax.xaxis.set_major_formatter(date_form)
ax.legend(fontsize=12, frameon=False);
```



Voilà! You made a beautiful graph!

6.0.0.4 * discussion

This time we did not put month numbers in the horizontal axis, we now have month names. How did we do this black magic, you ask? See lines 8–10 above. Matplotlib gives you absolute power over what to put in the axis, if you can only know how to tell it to... Wanna know more? [Click here.](#)

7 upsampling

In the previous chapter, we resampled from fine temporal resolution to a coarser one. This is also called **downsampling**. We will learn the **upsampling** now: how to go from coarse data to a finer scale.

Sadly, there is no free lunch, and we just can't get data that was not measured. What to do then?

It's best to consider a practical example.

We want to calculate the daily potential evapotranspiration using [Penman's equation](#). Part of the calculation involves characterizing the energy budget on soil surface. When direct solar radiation measurements are not available, we can estimate the energy balance by knowing the “cloudless skies mean solar radiation”, R_{so} . This is the amount of energy ($\text{MJ/m}^2/\text{d}$) that hits the surface, assuming no clouds. This radiation depends on the season and on the latitude you are. For Israel, located at latitude 32° N , we can use the following data for 30° :

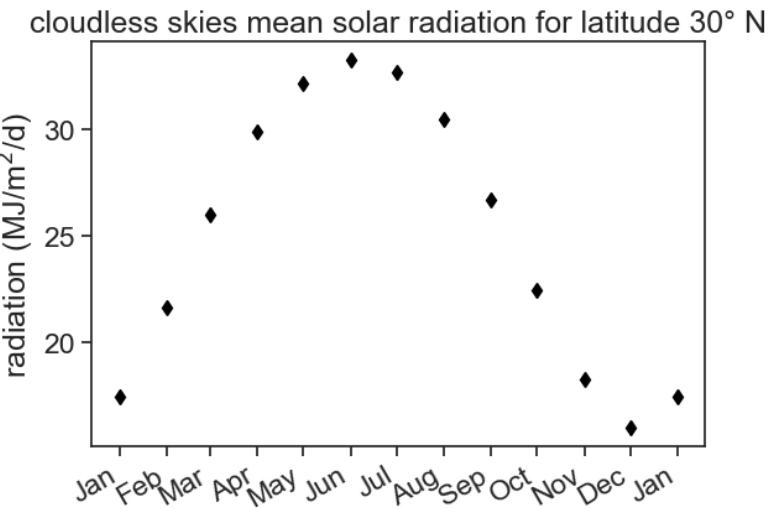
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters

dates = pd.date_range(start='2021-01-01', periods=13, freq='MS')
values = [17.46, 21.65, 25.96, 29.85, 32.11, 33.20, 32.66, 30.44, 26.67, 22.48, 18.30, 16.04
df = pd.DataFrame({'date': dates, 'radiation': values})
df = df.set_index('date')
```

```
df
```

date	radiation
2021-01-01	17.46
2021-02-01	21.65
2021-03-01	25.96
2021-04-01	29.85
2021-05-01	32.11
2021-06-01	33.20
2021-07-01	32.66
2021-08-01	30.44
2021-09-01	26.67
2021-10-01	22.48
2021-11-01	18.30
2021-12-01	16.04
2022-01-01	17.46

```
fig, ax = plt.subplots()
ax.plot(df['radiation'], color='black', marker='d', linestyle='None')
ax.set(ylabel=r'radiation (MJ/m$^2$/d)',
       title="cloudless skies mean solar radiation for latitude 30° N")
ax.xaxis.set_major_locator(mdates.MonthLocator())
date_form = DateFormatter("%b")
ax.xaxis.set_major_formatter(date_form)
plt.gcf().autofmt_xdate() # makes slanted dates
```



We only have 12 values for the whole year, and we can't use this dataframe to compute daily ET. We need to upsample!

In the example below, we resample the monthly data into daily data, and do nothing else. Pandas doesn't know what to do with the new points, so it fills them with NaN.

```
df_nan = df[['radiation']].resample('D').asfreq().to_frame()
df_nan.head(33)
```

radiation	
date	
2021-01-01	17.46
2021-01-02	NaN
2021-01-03	NaN
2021-01-04	NaN
2021-01-05	NaN
2021-01-06	NaN
2021-01-07	NaN
2021-01-08	NaN
2021-01-09	NaN
2021-01-10	NaN
2021-01-11	NaN
2021-01-12	NaN

radiation	
date	
2021-01-13	NaN
2021-01-14	NaN
2021-01-15	NaN
2021-01-16	NaN
2021-01-17	NaN
2021-01-18	NaN
2021-01-19	NaN
2021-01-20	NaN
2021-01-21	NaN
2021-01-22	NaN
2021-01-23	NaN
2021-01-24	NaN
2021-01-25	NaN
2021-01-26	NaN
2021-01-27	NaN
2021-01-28	NaN
2021-01-29	NaN
2021-01-30	NaN
2021-01-31	NaN
2021-02-01	21.65
2021-02-02	NaN

7.1 Forward/Backward fill

We can forward/backward fill these NaNs:

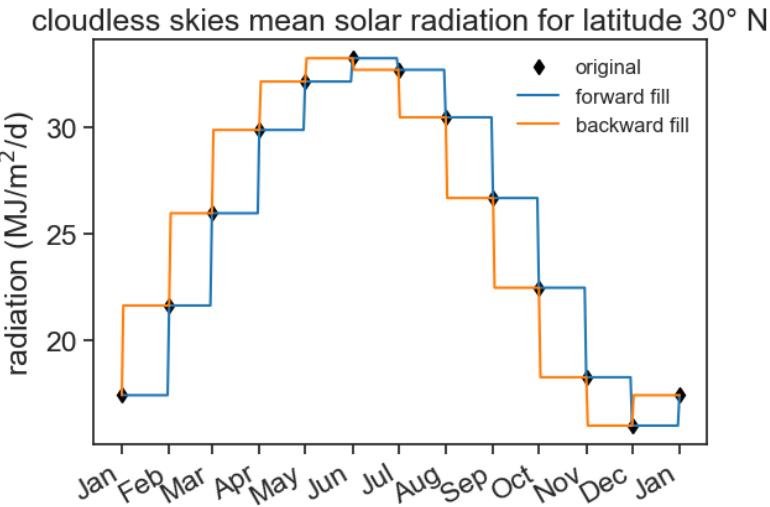
```
df_forw = df['radiation'].resample('D').ffill().to_frame()
df_back = df['radiation'].resample('D').bfill().to_frame()

fig, ax = plt.subplots()
ax.plot(df['radiation'], color='black', marker='d', linestyle='None', label="original")
ax.plot(df_forw['radiation'], color='tab:blue', label="forward fill")
ax.plot(df_back['radiation'], color='tab:orange', label="backward fill")
ax.set(ylabel=r'radiation (MJ/m$^2$/d)',
       title="cloudless skies mean solar radiation for latitude 30° N")
```

```

ax.legend(frameon=False, fontsize=12)
ax.xaxis.set_major_locator(mdates.MonthLocator())
date_form = DateFormatter("%b")
ax.xaxis.set_major_formatter(date_form)
plt.gcf().autofmt_xdate() # makes slanted dates

```



This does the job, but I want something better, not step functions. The radiation should vary smoothly from day to day. Let's use interpolation.

7.2 Interpolation

```

df_linear = df['radiation'].resample('D').interpolate(method='time').to_frame()
df_cubic = df['radiation'].resample('D').interpolate(method='cubic').to_frame()

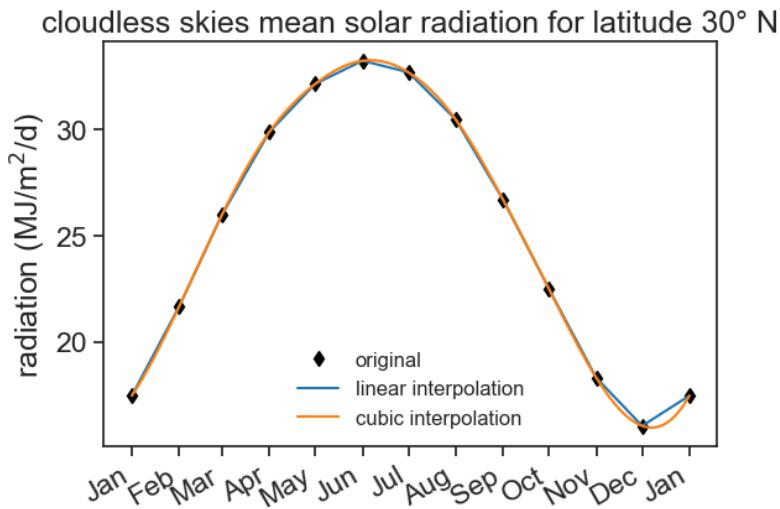
fig, ax = plt.subplots()
ax.plot(df['radiation'], color='black', marker='d', linestyle='None', label="original")
ax.plot(df_linear['radiation'], color='tab:blue', label="linear interpolation")
ax.plot(df_cubic['radiation'], color='tab:orange', label="cubic interpolation")
ax.set(ylabel=r'radiation (MJ/m$^2$/d)',
       title="cloudless skies mean solar radiation for latitude 30° N")

```

```

ax.legend(frameon=False, fontsize=12)
ax.xaxis.set_major_locator(mdates.MonthLocator())
date_form = DateFormatter("%b")
ax.xaxis.set_major_formatter(date_form)
plt.gcf().autofmt_xdate() # makes slanted dates

```



There are many ways to fill NaNs and to interpolate. A nice detailed guide can be [found here](#).

8 interpolation

Interpolation is the act of getting data you don't have from data you already have. We used some interpolation when upsampling, and now it is time to talk about it a little bit more in depth.

There is no one correct way of interpolating, the method you use depends in the end on what you want to accomplish, what are your (hidden or explicit) assumptions, etc. Let's see a few examples.

9 FAQ

This is called [anchored offset](#). One possible use to it is to calculate statistics according to the hydrological year that, for example, ends in September.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters

filename = "../archive/data/Kinneret_Kvuza_daily_rainfall.csv"
df = pd.read_csv(filename, na_values=['-'])
df.rename(columns={'Date': 'date',
                   'Daily Rainfall (mm)': 'rain'}, inplace=True)
df['date'] = pd.to_datetime(df['date'], dayfirst=True)
df = df.set_index('date')
df = df.resample('D').asfreq().fillna(0) # asfreq = replace
df
```

	Station	rain
date		
1980-01-02	Kinneret Kvuza 09/1977-08/2023	0.0
1980-01-03	0	0.0
1980-01-04	0	0.0
1980-01-05	Kinneret Kvuza 09/1977-08/2023	35.5
1980-01-06	Kinneret Kvuza 09/1977-08/2023	2.2
...
2019-12-26	Kinneret Kvuza 09/1977-08/2023	39.4

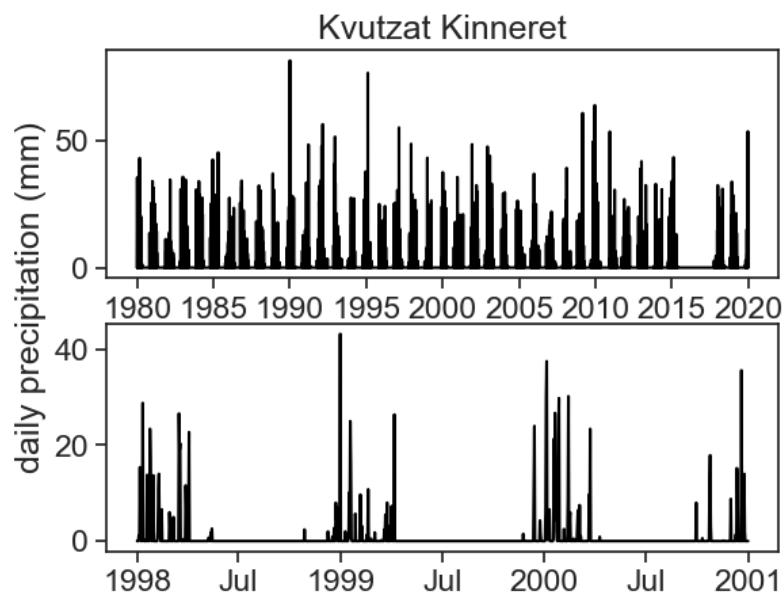
	Station	rain
date		
2019-12-27	Kinneret Kvaza 09/1977-08/2023	5.2
2019-12-28	Kinneret Kvaza 09/1977-08/2023	1.6
2019-12-29	0	0.0
2019-12-30	Kinneret Kvaza 09/1977-08/2023	0.1

```

fig, ax = plt.subplots(2,1)
ax[0].plot(df['rain'], color='black')
ax[1].plot(df.loc['1998':'2000', 'rain'], color='black')
locator = mdates.AutoDateLocator(minticks=4, maxticks=8)
formatter = mdates.ConciseDateFormatter(locator)
ax[1].xaxis.set_major_locator(locator)
ax[1].xaxis.set_major_formatter(formatter)
fig.text(0.02, 0.5, 'daily precipitation (mm)', va='center', rotation='vertical')
ax[0].set_title("Kvutzat Kinneret")

Text(0.5, 1.0, 'Kvutzat Kinneret')

```



We see a marked dry season during the summer, so let's assume the Hydrological Year ends in September.

```
df_year = df.resample('A-SEP').sum()  
df_year = df_year.loc['1980':'2003']  
df_year
```

```
/var/folders/c3/7hp0d36n6vv8jc9hm2440__00000gn/T/ipykernel_94063/2047090134.py:1: FutureWarning:  
df_year = df.resample('A-SEP').sum()
```

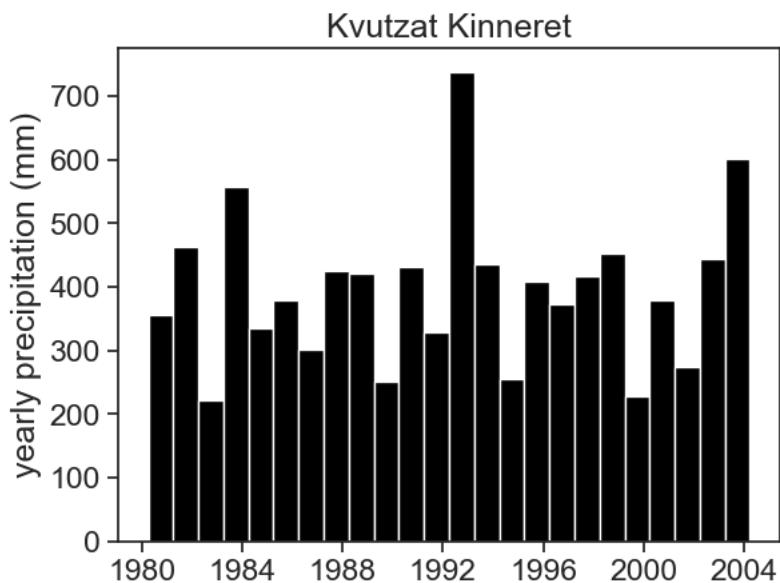
date	rain
1980-09-30	355.5
1981-09-30	463.1
1982-09-30	221.7
1983-09-30	557.1
1984-09-30	335.3
1985-09-30	379.8
1986-09-30	300.7
1987-09-30	424.7
1988-09-30	421.6
1989-09-30	251.6
1990-09-30	432.5
1991-09-30	328.3
1992-09-30	738.4
1993-09-30	434.9
1994-09-30	255.4
1995-09-30	408.6
1996-09-30	373.0
1997-09-30	416.2
1998-09-30	451.9
1999-09-30	227.8
2000-09-30	378.9
2001-09-30	273.9
2002-09-30	445.2
2003-09-30	602.4

```

fig, ax = plt.subplots()
ax.bar(df_year.index, df_year['rain'], color='black',
       width=365)
ax.set_ylabel("yearly precipitation (mm)")
ax.set_title("Kvutzat Kinneret")

Text(0.5, 1.0, 'Kvutzat Kinneret')

```



9.1 When upsampling, how to fill missing values with zero?

We did that in the example above, like this:

```
df = df.resample('D').asfreq().fillna(0) # asfreq = replace
```

Part III

smoothing

10 motivation

This is the temperature for the Yatir Forest (Shani station, see [map](#)), between 2 and 5 of January 2022. Data is in intervals of 10 minutes, and was downloaded from the Israel Meteorological Service.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import datetime as dt
import matplotlib.ticker as ticker
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters
import requests
import json
import os
# %matplotlib widget

# read token from file
with open('../archive/IMS-token.txt', 'r') as file:
    TOKEN = file.readline()
# 28 = SHANI station
STATION_NUM = 28
start = "2022/01/01"
end = "2022/01/07"
filename = 'shani_2022_january.json'

# check if the JSON file already exists
# if so, then load file
```

```

if os.path.exists(filename):
    with open(filename, 'r') as json_file:
        data = json.load(json_file)
else:
    # make the API request if the file doesn't exist
    url = f"https://api.ims.gov.il/v1/envista/stations/{STATION_NUM}/data/?from={start}&to={end}"
    headers = {'Authorization': f'ApiToken {TOKEN}'}
    response = requests.get(url, headers=headers)
    data = json.loads(response.text.encode('utf8'))

    # save the JSON data to a file
    with open(filename, 'w') as json_file:
        json.dump(data, json_file)
# show data to see if it's alright
# data

df = pd.json_normalize(data['data'], record_path=['channels'], meta=['datetime'])
df['date'] = (pd.to_datetime(df['datetime'])
              .dt.tz_localize(None) # ignores time zone information
              )
df = df.pivot(index='date', columns='name', values='value')
# df

# dirty trick to have dates in the middle of the 24-hour period
# make minor ticks in the middle, put the labels there!
# from https://matplotlib.org/stable/gallery/ticks/centered_ticklabels.html

def centered_dates(ax):
    date_form = DateFormatter("%d %b") # %d 3-letter-Month
    # major ticks at midnight, every day
    ax.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax.xaxis.set_major_formatter(date_form)
    # minor ticks at noon, every day
    ax.xaxis.set_minor_locator(mdates.HourLocator(byhour=[12]))
    # erase major tick labels
    ax.xaxis.set_major_formatter(ticker.NullFormatter())
    # set minor tick labels as define above
    ax.xaxis.set_minor_formatter(date_form)
    # completely erase minor ticks, center tick labels

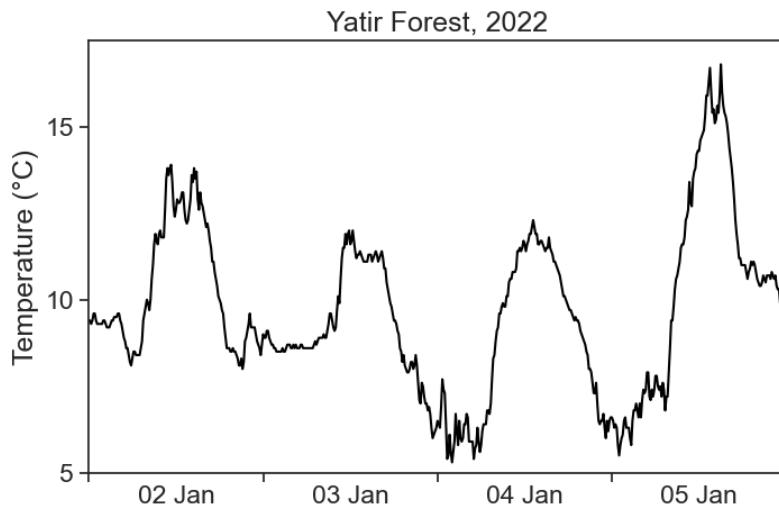
```

```

for tick in ax.xaxis.get_minor_ticks():
    tick.tick1line.set_markersize(0)
    tick.tick2line.set_markersize(0)
    tick.label1.set_horizontalalignment('center')

fig, ax = plt.subplots(figsize=(8,5))
start = "2022-01-02"
end = "2022-01-05"
df = df.loc[start:end]
ax.plot(df['TD'], color='black')
ax.set(ylim=[5, 17.5],
       xlim=[df.index[0], df.index[-1]],
       ylabel="Temperature (°C)",
       title="Yatir Forest, 2022",
       yticks=[5,10,15])
centered_dates(ax)
fig.savefig("YF-temperature_2022_jan.png", dpi=300)

```



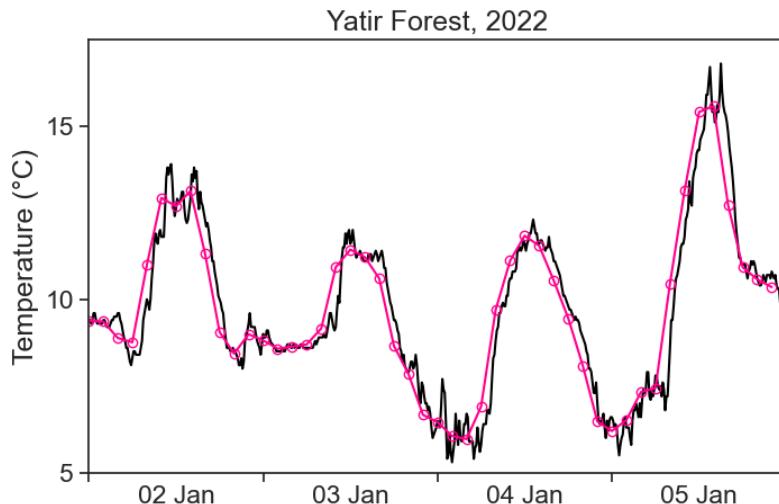
We see that the temperature curve has a rough profile. Can we find ways of getting smoother curves?

We learned how to average over a window with `resample`. Let's try that for a 2-hour window:

```

fig, ax = plt.subplots(figsize=(8,5))
ax.plot(df['TD'], color='black')
ax.plot(df['TD'].resample('2H').mean(),
        color='xkcd:hot pink', ls='-' ,
        marker="o", mfc="None")
ax.set(ylim=[5, 17.5],
       xlim=[df.index[0], df.index[-1]],
       ylabel="Temperature (°C)",
       title="Yatir Forest, 2022",
       yticks=[5,10,15])
centered_dates(ax)

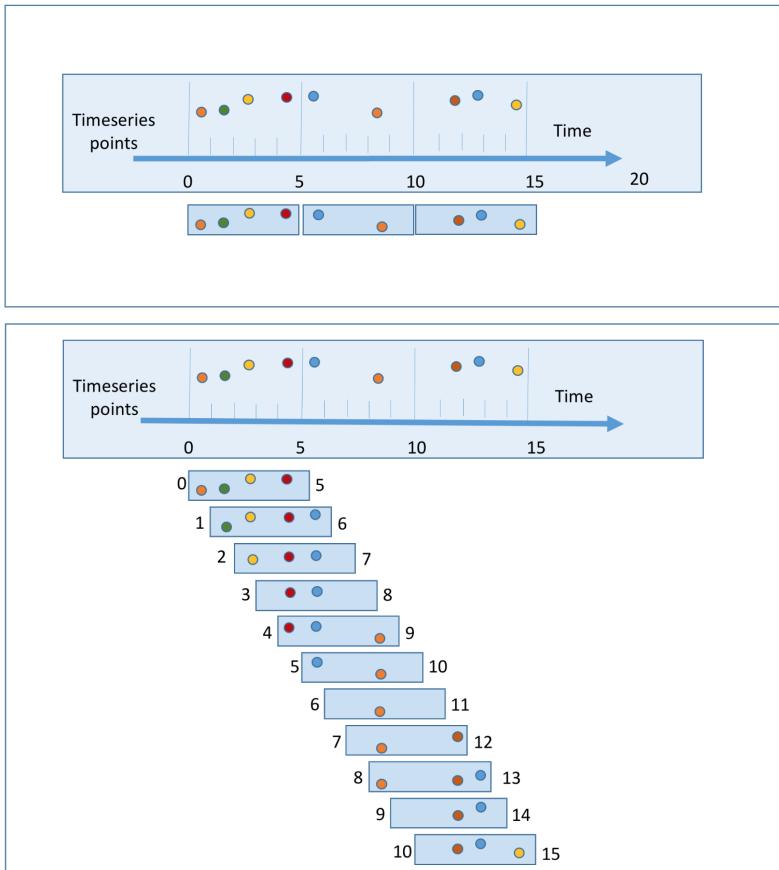
```



The temperature profile now is much smoother, but when using `resample`, we lost temporal resolution. Our original data had 10-minute frequency, and now we have a 2-hour frequency.

How can we get a smoother curve without losing resolution?

10.1 Tumbling vs Sliding



11 sliding window

This is the temperature for the Yatir Forest, between 2 and 5 of January 2022. Data (download .csv here) is in intervals of 10 minutes, and was downloaded from the Israel Meteorological Service.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import datetime as dt
import matplotlib.ticker as ticker
import os
import warnings
import scipy
warnings.simplefilter(action='ignore', category=FutureWarning)
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters
# %matplotlib widget

# dirty trick to have dates in the middle of the 24-hour period
# make minor ticks in the middle, put the labels there!
# from https://matplotlib.org/stable/gallery/ticks/centered_ticklabels.html

def centered_dates(ax):
    date_form = DateFormatter("%d %b") # %d 3-letter-Month
    # major ticks at midnight, every day
    ax.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax.xaxis.set_major_formatter(date_form)
    # minor ticks at noon, every day
    ax.xaxis.set_minor_locator(mdates.HourLocator(byhour=[12]))
    # erase major tick labels
```

```

ax.xaxis.set_major_formatter(ticker.NullFormatter())
# set minor tick labels as define above
ax.xaxis.set_minor_formatter(date_form)
# completely erase minor ticks, center tick labels
for tick in ax.xaxis.get_minor_ticks():
    tick.tick1line.set_markersize(0)
    tick.tick2line.set_markersize(0)
    tick.label1.set_horizontalalignment('center')

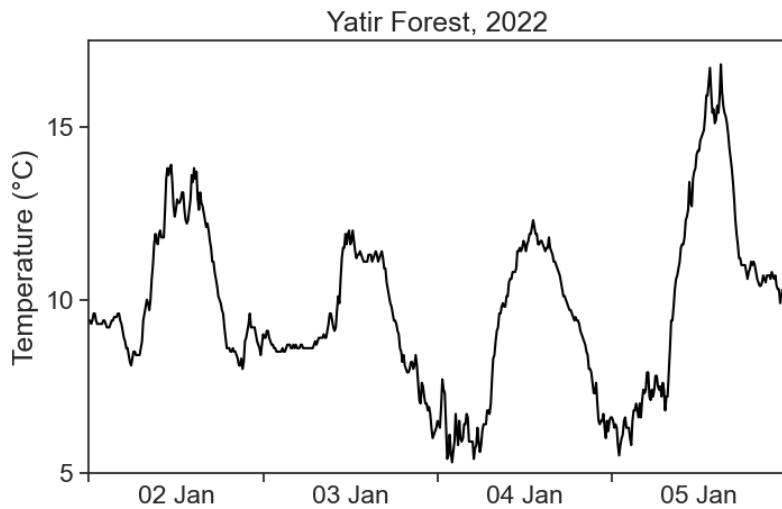
df = pd.read_csv('shani_2022_january.csv', parse_dates=['date'], index_col='date')

fig, ax = plt.subplots(figsize=(8,5))
start = "2022-01-02"
end = "2022-01-05"
df = df.loc[start:end]
ax.plot(df['TD'], color='black')

plot_settings = {
    'ylim': [5, 17.5],
    'xlim': [df.index[0], df.index[-1]],
    'ylabel': "Temperature (°C)",
    'title': "Yatir Forest, 2022",
    'yticks': [5, 10, 15]
}

ax.set(**plot_settings)
centered_dates(ax)

```



We see that the temperature curve has a rough profile. Can we find ways of getting smoother curves?

11.1 convolution

Convolution is a fancy word for averaging a time series using a sliding window. We will use the terms **convolution**, **running average**, and **rolling average** interchangeably. See the animation below. We take all temperature values inside a window of width 500 minutes (51 points), and average them with equal weights. The weights profile is called **kernel**.

The pink curve is much smoother than the original! However, the running average cannot describe sharp temperature changes. If we decrease the window width to 200 minutes (21 points), we get the following result.

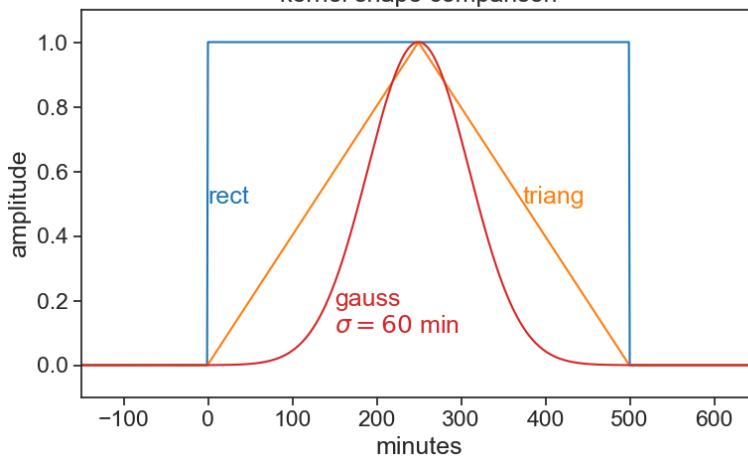
There is a tradeoff between the smoothness of a curve, and its ability to describe sharp temporal changes.

11.2 kernels

We can modify our running average, so that values closer to the center of the window have higher weights, and those further away count less. This is achieved by changing the weight profile, or the shape of the kernel. We see below the result of a running average using a triangular window of base 500 minutes (51 points).

Things can get as fancy as we want. Instead of a triangular kernel, which has sharp edges, we can choose a smoother gaussian kernel, see the difference below. We used a gaussian kernel with 60-minute standard deviation.

See how the three kernel shapes compare. There are [many kernels to chose from](#).



[many kernels to chose from](#).

11.3 math

The definition of a convolution between signal $f(t)$ and kernel $k(t)$ is

$$(f * k)(t) = \int f(\tau)k(t - \tau)d\tau.$$

The expression $f * k$ denotes the convolution of these two functions. The argument of k is $t - \tau$, meaning that the kernel runs

from left to right (as t does), and at every point the two signals (f and k) are multiplied together. It is the product of the signal with the weight function k that gives us an average. Because of $-\tau$, the kernel is flipped backwards, but this has no effect to symmetric kernels, like to ones in the examples above. Finally, the actual running average is not the convolution, but

$$\frac{(f * k)(t)}{\int k(t)dt}.$$

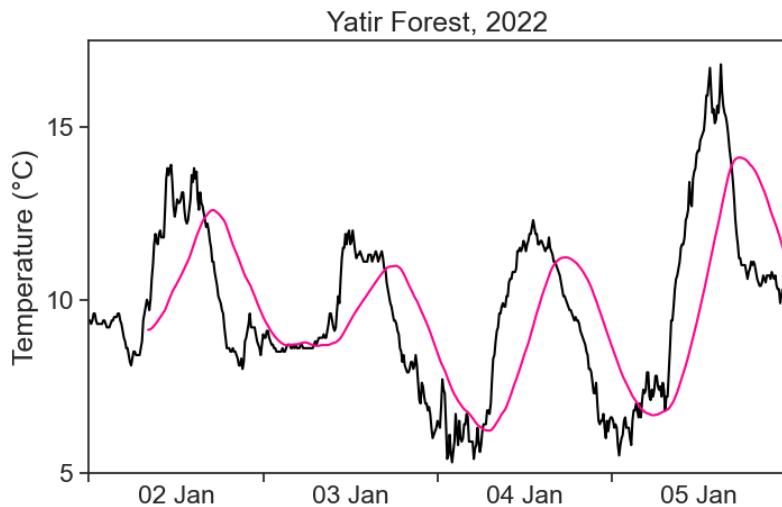
Whenever the integral of the kernel is 1, then the convolution will be identical with the running average.

11.4 numerics

Running averages are very common tools in time-series analysis. The `pandas` package makes life quite simple. For example, in order to calculate the running average of temperature using a rectangular kernel, one writes:

```
df['temp_smoothed'] = (
    df['TD'].rolling(window='500min',
                     min_periods=50      # comment this to see what happens
                     )
    .mean()
)

fig, ax = plt.subplots(figsize=(8,5))
ax.plot(df['TD'], color='black')
ax.plot(df['temp_smoothed'], color='xkcd:hot pink')
ax.set(**plot_settings)
centered_dates(ax)
```



The pink curve looks smooth, but why does it lag behind the data?! What's going on?

11.4.1 7-day average of COVID-19 infections

During the COVID-19 pandemic, we would see graphs like this all the time in the news:

```
# data from https://health.google.com/covid-19/open-data/raw-data?loc=IL
# define the local file path
local_file_path = 'COVID_19_israel.csv'
# check if the local file exists
if os.path.exists(local_file_path):
    # if the local file exists, load it
    covid_IL = pd.read_csv(local_file_path, parse_dates=['date'], index_col='date')
else:
    # if the local file doesn't exist, download from the URL
    url = "https://storage.googleapis.com/covid19-open-data/v3/location/IL.csv"
    covid_IL = pd.read_csv(url, parse_dates=['date'], index_col='date')
    # save the downloaded data to the local file for future use
    covid_IL.to_csv(local_file_path)

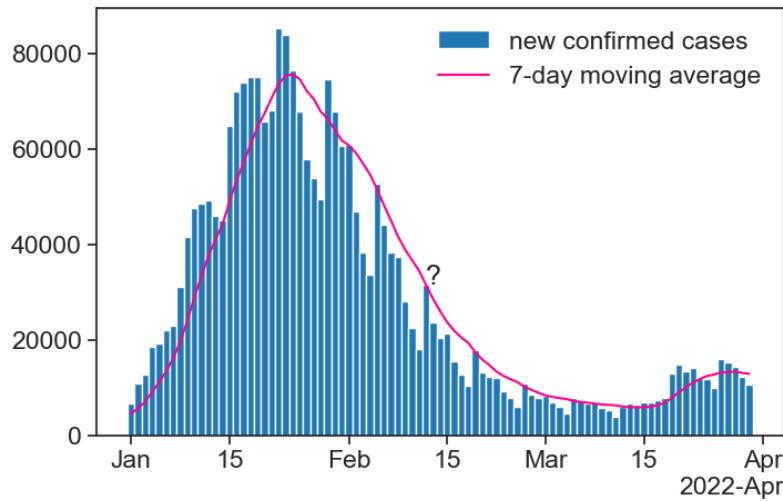
df_covid = covid_IL['new_confirmed'].to_frame()
```

```

df_covid['7d_avg'] = df_covid['new_confirmed'].rolling(window='7D').mean()

fig, ax = plt.subplots(figsize=(8,5))
st = '2022-01-01'
en = '2022-03-30'
new_cases = ax.bar(df_covid[st:en].index, df_covid.loc[st:en,'new_confirmed'],
                    color="tab:blue", width=1)
mov_avg, = ax.plot(df_covid.loc[st:en,'7d_avg'],
                    color='xkcd:hot pink')
ax.legend(handles=[new_cases, mov_avg],
           labels=['new confirmed cases', '7-day moving average'],
           frameon=False)
weird_day = "2022-02-12"
weird_day_x = mdates.date2num(dt.datetime.strptime(weird_day, "%Y-%m-%d"))
ax.text(weird_day_x, df_covid.loc[weird_day,'new_confirmed'], "?")
# formating dates on x axis
locator = mdates.AutoDateLocator(minticks=7, maxticks=11)
formatter = mdates.ConciseDateFormatter(locator)
ax.xaxis.set_major_locator(locator)
ax.xaxis.set_major_formatter(formatter)

```



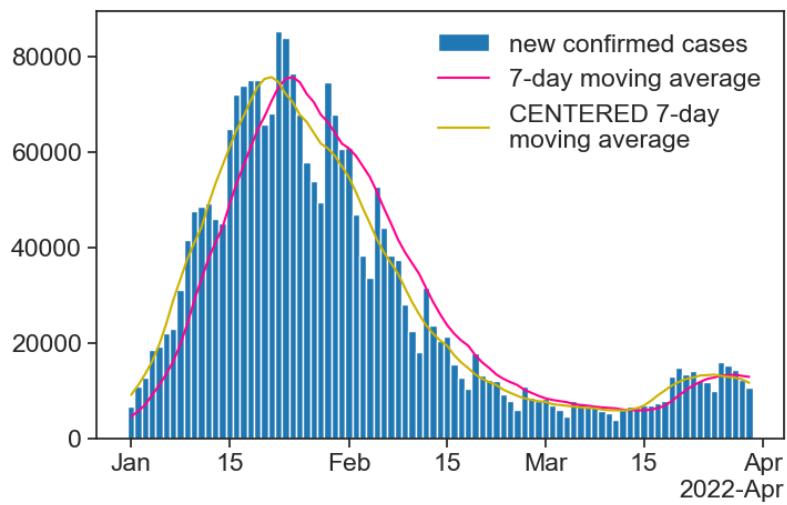
Take a look at the moving average next to the question mark. How can it be that high, when all the bars around that date are lower? Is the calculation right?

The answer is that the result of the moving average is assigned to the right-most date in the running window. This is reasonable for COVID-19 cases: for a given day, I can only calculate a 7-day average based on **past** values, I don't know what the future will be.

There is a simple way of assigning the result to the center of the window:

```
df_covid['7d_avg_center'] = (
    df_covid['new_confirmed']
        .rolling(window='7D',
                center=True) # THIS
        .mean()
)

fig, ax = plt.subplots(figsize=(8,5))
st = '2022-01-01'
en = '2022-03-30'
new_cases = ax.bar(df_covid[st:en].index, df_covid.loc[st:en,'new_confirmed'],
                    color="tab:blue", width=1)
mov_avg, = ax.plot(df_covid.loc[st:en,'7d_avg'],
                    color='xkcd:hot pink')
mov_avg_center, = ax.plot(df_covid.loc[st:en,'7d_avg_center'],
                           color='xkcd:mustard')
ax.legend(handles=[new_cases, mov_avg, mov_avg_center],
           labels=['new confirmed cases',
                   '7-day moving average',
                   'CENTERED 7-day\nmoving average'],
           frameon=False)
# formating dates on x axis
locator = mdates.AutoDateLocator(minticks=7, maxticks=11)
formatter = mdates.ConciseDateFormatter(locator)
ax.xaxis.set_major_locator(locator)
ax.xaxis.set_major_formatter(formatter)
```



As a rule, we will used a **centered** moving average (`center=True`), unless stated otherwise. Also, only use `min_periods` if you know what you are doing.

11.4.2 gaussian

You can easily change the kernel shape by using the `win_type` argument. See how to perform a rolling mean with a gaussian kernel:

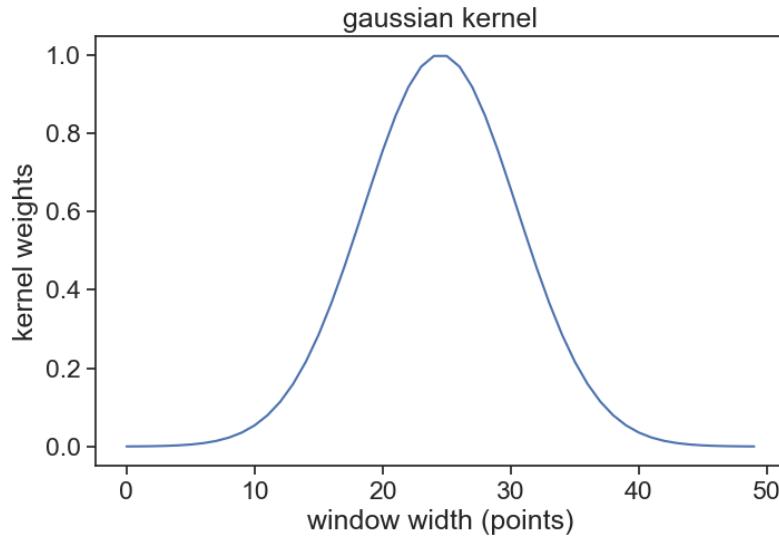
```
(  
    df['temperature'].rolling(window=window_width,  
                            center=True,  
                            win_type="gaussian")  
    .mean(std=std_gaussian)  
)
```

where

- `window_width` is an integer, number of points in your window
- `std_gaussian` is the standard deviation of your gaussian, **measured in sample points, not time!**

For instance, if we have measurements every 10 minutes, and our window width is 500 minutes, then `window_width = 500/10 + 1` (first and last included). If we want a standard deviation of 60 minutes, then `std_gaussian = 6`. The gaussian kernel will look like this:

```
window_width = 50 # in points = 500 min
std = 6 # in points = 60 min
fig, ax = plt.subplots(figsize=(8,5))
g = scipy.signal.gaussian(window_width, std)
ax.plot(g)
ax.set(xlabel="window width (points)",
       ylabel="kernel weights",
       title="gaussian kernel");
```



You can take a look at various options for kernel shapes [here](#), provided by the `scipy` package.

11.4.3 triangular

Same idea as gaussian, but simpler, because we don't need to think about standard deviation.

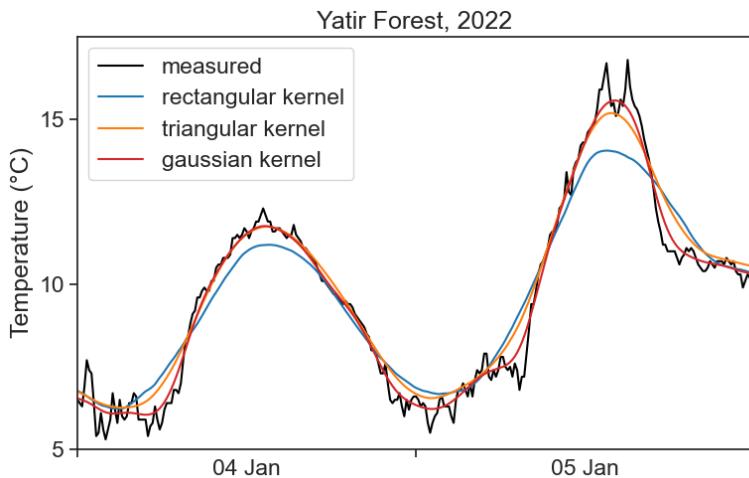
```

(
df['temperature'].rolling(window=window_width,
    center=True,
    win_type="triang")
.mean()
)

```

11.5 which window shape and width to choose?

Sorry, there is not definite answer here... It really depends on your data and what you need to do with it. See below a comparison of all examples in the videos above.



One important question you need to ask is: what are the time scales associated with the processes I'm interested in? For example, if I'm interested in the daily temperature pattern, getting rid of 1-minute-long fluctuations would probably be ok. On the other hand, if we were to smooth the signal so much that all that can be seen are the temperature changes between summer and winter, then my smoothing got out of hand, and I threw away the very process I wanted to study.

All this is to say that you need to know in advance a few things about the system you are studying, otherwise you can't know what is "noise" that can be smoothed away.

12 not only averages

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import datetime as dt
import matplotlib.ticker as ticker
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters
import requests
import json
import os
# %matplotlib widget

# read token from file
with open('../archive/IMS-token.txt', 'r') as file:
    TOKEN = file.readline()
# 28 = SHANI station
STATION_NUM = 28
start = "2022/01/01"
end = "2022/01/07"
filename = 'shani_2022_january.json'

# check if the JSON file already exists
# if so, then load file
if os.path.exists(filename):
    with open(filename, 'r') as json_file:
        data = json.load(json_file)
else:
```

```

# make the API request if the file doesn't exist
url = f"https://api.ims.gov.il/v1/envista/stations/{STATION_NUM}/data/?from={start}&to={end}"
headers = {'Authorization': f'ApiToken {TOKEN}'}
response = requests.get(url, headers=headers)
data = json.loads(response.text.encode('utf8'))

# save the JSON data to a file
with open(filename, 'w') as json_file:
    json.dump(data, json_file)
# show data to see if it's alright
# data

df = pd.json_normalize(data['data'], record_path=['channels'], meta=['datetime'])
df['date'] = (pd.to_datetime(df['datetime'])
              .dt.tz_localize(None) # ignores time zone information
              )
df = df.pivot(index='date', columns='name', values='value')
# let's work only with a few days, and only temperature
start = "2022-01-02"
end = "2022-01-05"
df = df.loc[start:end, 'TD'].to_frame()
df.rename(columns={"TD": "temp"}, inplace=True)
# df

# dirty trick to have dates in the middle of the 24-hour period
# make minor ticks in the middle, put the labels there!
# from https://matplotlib.org/stable/gallery/ticks/centered_ticklabels.html

def centered_dates(ax):
    date_form = DateFormatter("%d %b") # %d 3-letter-Month
    # major ticks at midnight, every day
    ax.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax.xaxis.set_major_formatter(date_form)
    # minor ticks at noon, every day
    ax.xaxis.set_minor_locator(mdates.HourLocator(byhour=[12]))
    # erase major tick labels
    ax.xaxis.set_major_formatter(ticker.NullFormatter())
    # set minor tick labels as define above
    ax.xaxis.set_minor_formatter(date_form)

```

```

# completely erase minor ticks, center tick labels
for tick in ax.xaxis.get_minor_ticks():
    tick.tick1line.set_markersize(0)
    tick.tick2line.set_markersize(0)
    tick.label1.set_horizontalalignment('center')

# creating the dictionary with the desired settings
plot_settings = {
    'ylim': [5, 17.5],
    'xlim': [df.index[0], df.index[-1]],
    'ylabel': 'Temperature (°C)',
    'title': 'Yatir Forest, 2022',
    'yticks': [5, 10, 15]
}

```

Let's see on a graph the average temperature, with an envelope of 1 standard deviation around it:

```

df['mean'] = df['temp'].rolling('3H', center=True).mean()
df['std'] = df['temp'].rolling('3H', center=True).std()

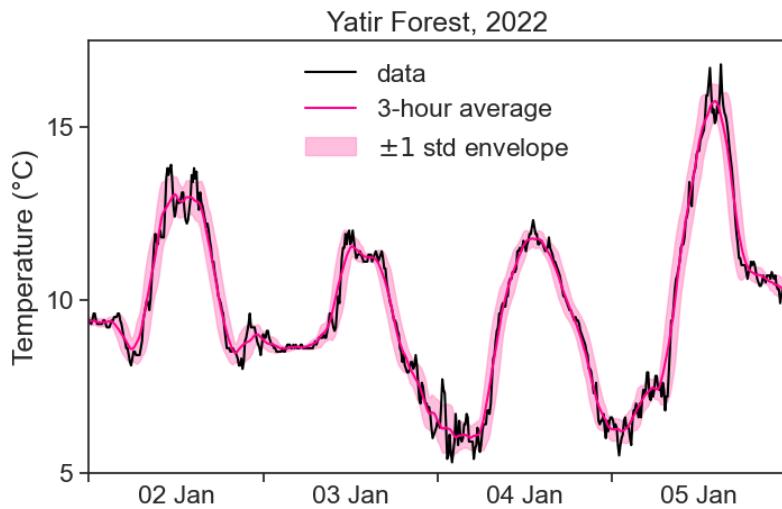
fig, ax = plt.subplots(figsize=(8,5))

plot_std = ax.fill_between(df.index,
                           df['mean'] + df['std'],
                           df['mean'] - df['std'],
                           color="xkcd:pink", alpha=0.5)
plot_data, = ax.plot(df['temp'], color='black')
plot_mean, =ax.plot(df['mean'], color='xkcd:hot pink')

ax.legend([plot_data, plot_mean, plot_std],
          ['data', '3-hour average', r"$\pm$1 std envelope"],
          frameon=False)

# applying the settings to the ax object
ax.set(**plot_settings)
centered_dates(ax)
# fig.savefig("YF-temperature_2022_jan.png", dpi=300)

```



12.1 Confidence Interval

We can calculate anything we want inside the sliding window. One good example is the **Confidence Interval of the Mean**, given by:

$$CI(\alpha) = Z(\alpha) \cdot SE.$$

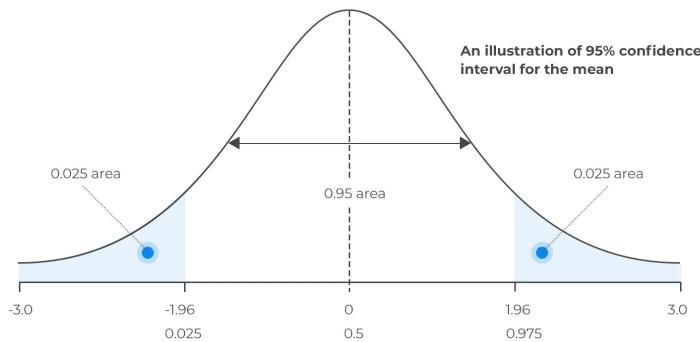
$Z(\alpha)$ is the Z-score corresponding to the chosen confidence level α . The most commonly used confidence level is 95%, which corresponds to a Z-score of 1.96. What does this mean? This means that we expect to find 95% of the points within ± 1.96 standard deviations away from the mean.

This is called “ ” - in hebrew.

- $Z(\alpha)$ = Z-score.
- SE = standard error.



95% Interval



An illustration of 95% confidence interval for the mean

You can find the Z-score using the following python code:

```
from scipy.stats import norm

confidence_level = 0.95
# 5% outside
out = 1 - confidence_level
# 0.975 of points to the left of right boundary
p = 1 - out/2
# inverse of cdf: 0.975 of the points will be smaller than what distance (in sigma units)?
z_score = norm.ppf(p)
print(f"z-score = {z_score}")
```

Source: Dhaval Raval's Medium article

z-score = 1.959963984540054

If you are still not convinced why we need 0.975 instead of 0.95, read this [excellent response on stackoverflow](#).

SE is the standard error:

$$SE = \frac{\sigma}{\sqrt{N}}.$$

We can write a function to calculate the confidence interval of the mean, and use it with the sliding window:

- σ = standard deviation.
- N = number of points.

```

def std_error_of_the_mean(window):
    return window.std() / np.sqrt(window.count())

def confidence_interval(window):
    return z_score * std_error_of_the_mean(window)

df['std_error'] = (
    df['temp'].rolling('3H',
                       center=True)
    .apply(std_error_of_the_mean)
)
df['confidence_int'] = (
    df['temp'].rolling('3H',
                       center=True)
    .apply(confidence_interval)
)

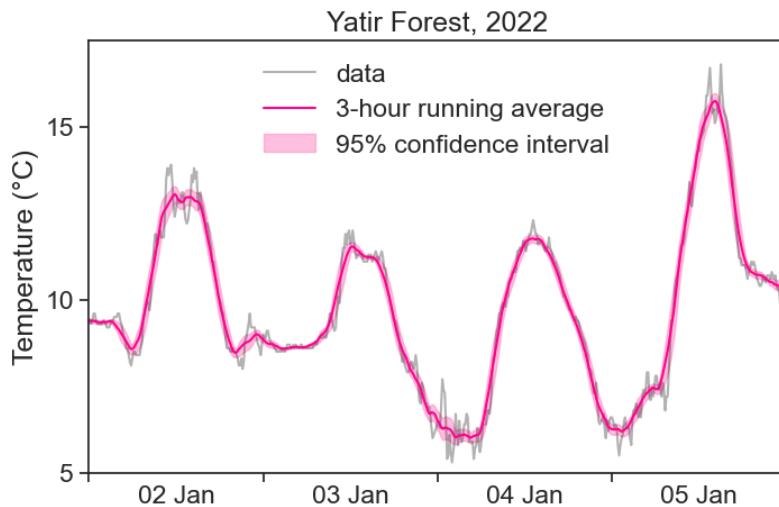
fig, ax = plt.subplots(figsize=(8,5))

plot_std = ax.fill_between(df.index,
                           df['mean'] + df['confidence_int'],
                           df['mean'] - df['confidence_int'],
                           color="xkcd:pink", alpha=0.5)
plot_data, = ax.plot(df['temp'], color='black', alpha=0.3)
plot_mean, =ax.plot(df['mean'], color='xkcd:hot pink')

ax.legend([plot_data, plot_mean, plot_std],
          ['data', '3-hour running average', r"95% confidence interval"],
          frameon=False)

# applying the settings to the ax object
ax.set(**plot_settings)
centered_dates(ax)
# fig.savefig("YF-temperature_2022_jan.png", dpi=300)

```



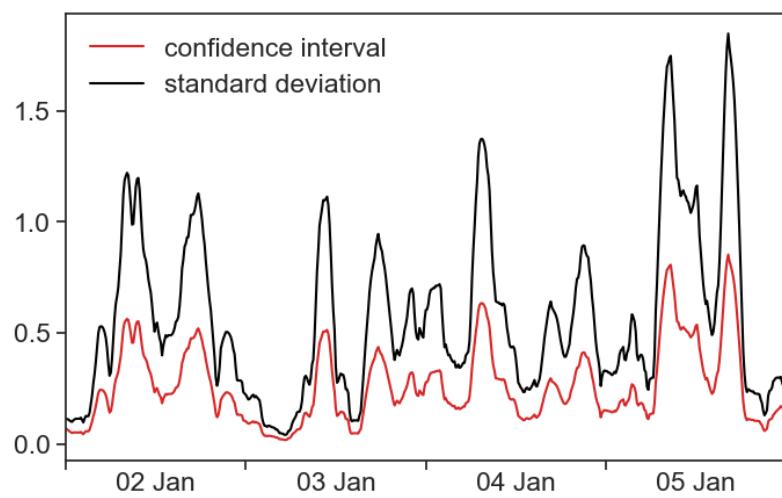
When the time series has a regular sampling frequency, all positions of the running window will have the same number of data points in them. Because the Confidence Interval is proportional to the Standard Error, and the SE is proportional to the Standard Deviation (\sqrt{N} is constant), then the envelope created by the CI is identical to the envelope created by the standard deviation, up to a multiplying constant. Nice.

```

fig, ax = plt.subplots(figsize=(8,5))
plot_ci, = ax.plot(df['confidence_int'], color='tab:red')
plot_std, = ax.plot(df['std'], color="black")
ax.legend([plot_ci, plot_std],
          ['confidence interval', 'standard deviation'],
          frameon=False)

# applying the settings to the ax object
# ax.set(**plot_settings)
ax.set(xlim=[df.index[0], df.index[-1]])
centered_dates(ax)

```



13 fit

We will make a little parenthesis to talk about a very important topic: **fitting**.

See below temperature data inside and outside a greenhouse, for a period of about 2 weeks.



```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import altair as alt
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
from scipy.optimize import curve_fit
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters
pd.options.mode.chained_assignment = None # default='warn'
# %matplotlib widget

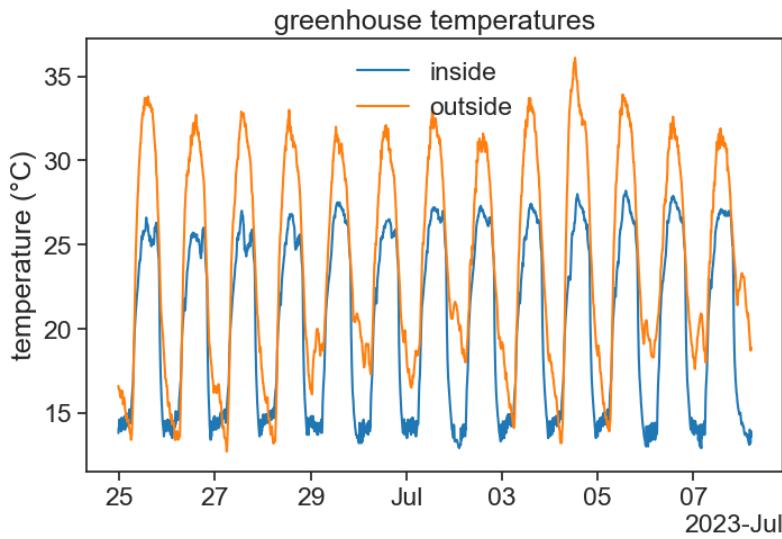
df = pd.read_csv('greenhouse_cooling.csv', index_col='time', parse_dates=True)
# df

fig, ax = plt.subplots(figsize=(8,5))

ax.plot(df['T_in'], c='tab:blue', label='inside')
ax.plot(df['T_out'], c='tab:orange', label='outside')
ax.set(ylabel='temperature (°C)',
       title="greenhouse temperatures")

# formating dates on x axis
locator = mdates.AutoDateLocator(minticks=7, maxticks=11)
formatter = mdates.ConciseDateFormatter(locator)
ax.xaxis.set_major_locator(locator)
ax.xaxis.set_major_formatter(formatter)

ax.legend(frameon=False);
```



Every evening, at 20:00, the air conditioning turns on, and we see a fast decrease in temperature:

```

df_fit = df['2023-06-29 20:10:00':'2023-06-29 22:00:00']

fig, ax = plt.subplots(2, 1, figsize=(8,6))
fig.subplots_adjust(hspace=0.4) # Adjust the vertical space between subplots

ax[0].plot(df.loc['2023-06-29 16:10:00':'2023-06-30 16:00:00', 'T_in'], color='tab:green')
ax[0].set(ylabel='temperature (°C)',
           title="temperature inside the greenhouse")

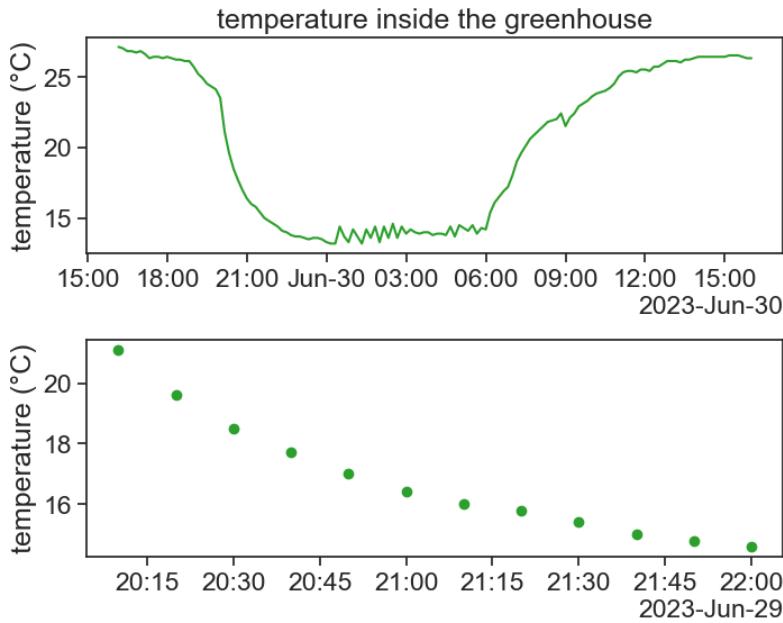
ax[1].scatter(df_fit['T_in'].index, df_fit['T_in'], color='tab:green')
ax[1].set(ylabel='temperature (°C)',)

# formating dates on x axis
locator = mdates.AutoDateLocator(minticks=7, maxticks=11)
formatter = mdates.ConciseDateFormatter(locator)
ax[0].xaxis.set_major_locator(locator)
ax[0].xaxis.set_major_formatter(formatter)

locator = mdates.AutoDateLocator(minticks=7, maxticks=11)
formatter = mdates.ConciseDateFormatter(locator)

```

```
ax[1].xaxis.set_major_locator(locator)
ax[1].xaxis.set_major_formatter(formatter)
```



The AC is able to bring the temperature down, but up to a limit. The AC can work at a maximum given power, and the cooler it is outside, the more effectively the AC will be able to bring down the temperature inside the greenhouse. We can imagine that the AC behaves as an effective external environment to the greenhouse, and the greenhouse cools down according to [Newton's law of cooling](#):

$$\frac{dT}{dt} = r \cdot (T_{\text{env}} - T)$$

The cooling rate is proportional to the difference in temperature between the inside and outside. Assuming T_{env} and r to be constant, the solution of this differential equation is:

$$T(t) = T_{\text{env}} + (T_0 - T_{\text{env}}) e^{-rt}.$$

- T = the greenhouse temperature
- T_{env} = the outside environment temperature
- r = coefficient of heat transfer.
- T_0 = the initial greenhouse temperature

We want to check if the temperature measured inside the greenhouse behaves like Newton's law of cooling, and if so, what can we say about the cooling coefficient r and about T_{env} .

13.1 linear fit

The following is a **very short** introduction to curve fitting.
The natural place to start is with a linear fit.

```
# the "fit" process can't deal with datetimes
# we therefore make a new column 'minutes', that will be used here
df_fit['minutes'] = (df_fit.index - df_fit.index[0]).total_seconds() / 60
# linear Fit (degree 1)
degree = 1
coeffs = np.polyfit(df_fit['minutes'], df_fit['T_in'], degree)
# linear Function
linear_function = np.poly1d(coeffs)

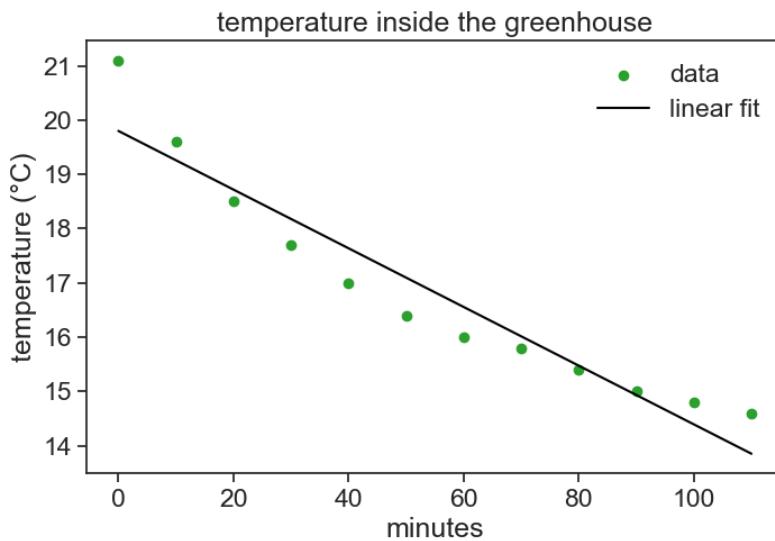
fig, ax = plt.subplots(figsize=(8,5))

ax.scatter(df_fit['minutes'], df_fit['T_in'],
           color='tab:green', label='data')
ax.plot(df_fit['minutes'], linear_function(df_fit['minutes']),
        color='black', label='linear fit')

ax.set(xlabel='minutes',
       ylabel='temperature (°C)',
       title="temperature inside the greenhouse")

ax.legend(frameon=False)
print(f"starting at {coeffs[1]:.2f} degrees,\nthe temperature decreases by {-coeffs[0]:.2f}")

starting at 19.80 degrees,
the temperature decreases by 0.05 degrees every minute.
```



The line above is the “best” straight line that describes our data. Defining the residual as the difference between our data and our model (straight line),

$$e = T_{\text{data}} - T_{\text{model}},$$

the straight line above is the one that **minimizes** the sum of the squares of residuals. For this reason, the method used above to fit a curve to the data is called “least-squares method”.

Can we do better than a straight line?

it minimizes the sum

13.2 polynomial fit

$$S = \sum_i e_i^2$$

```
# polynomial fit (degree 2)
degree = 2
coeffs2 = np.polyfit(df_fit['minutes'], df_fit['T_in'], degree)
quad_function = np.poly1d(coeffs2)

# polynomial fit (degree 2)
degree = 3
coeffs3 = np.polyfit(df_fit['minutes'], df_fit['T_in'], degree)
```

```

cubic_function = np.poly1d(coeffs3)

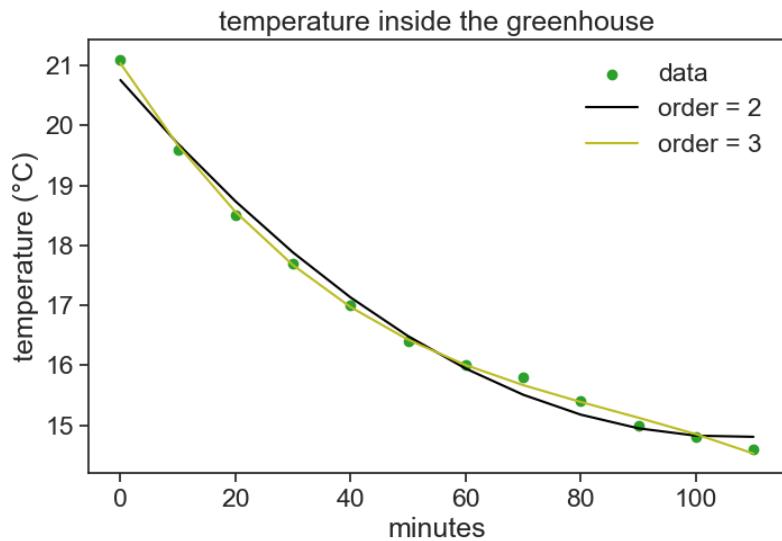
fig, ax = plt.subplots(figsize=(8,5))

ax.scatter(df_fit['minutes'], df_fit['T_in'],
           color='tab:green', label='data')
ax.plot(df_fit['minutes'], quad_function(df_fit['minutes']),
        color='black', label='order = 2')
ax.plot(df_fit['minutes'], cubic_function(df_fit['minutes']),
        color='tab:olive', label='order = 3')

ax.set(xlabel='minutes',
       ylabel='temperature (°C)',
       title="temperature inside the greenhouse")
ax.legend(frameon=False)

```

<matplotlib.legend.Legend at 0x7fd0a0833eb0>



13.3 any function you want

Now let's get back to our original assumption, that the green-house cools according to Newton's cooling law. We can still use the least-squares method for any function we want!

```
def cooling(t, T_env, T0, r):
    """
    t = time
    other stuff = parameters to be fitted
    """
    return T_env + (T0 - T_env)*np.exp(-r*t)

t = df_fit['minutes'].values
y = df_fit['T_in'].values

T_init = df_fit['T_in'][0]

popt, pcov = curve_fit(f=cooling,
                       xdata=t,
                       ydata=y,
                       p0=(2, T_init, 0.5),
)
print(f"the optimal parameters are {popt}")
```

```
the optimal parameters are [14.01663586 21.0074623  0.02121802]
```

```
fig, ax = plt.subplots(sharex=True)

ax.scatter(df_fit['minutes'], df_fit['T_in'],
           color='tab:green', label='data')
ax.plot(t, cooling(t, *popt),
         color='black', label='exponential fit')

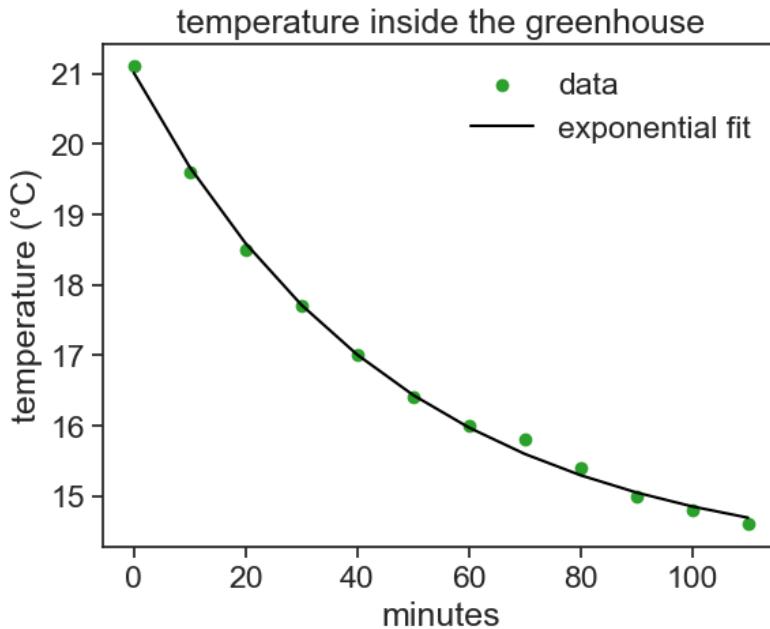
ax.set(xlabel='minutes',
       ylabel='temperature (°C)',
       title="temperature inside the greenhouse")
```

```

ax.legend(frameon=False)

<matplotlib.legend.Legend at 0x7fd0b0140850>

```



That looks really good :)

We can use curve fitting to retrieve important parameters from our data. Let's write a function that executes the fit and returns two of the fitted parameters: `T_env` and `r`.

```

def run_fit(data):
    data['minutes'] = (data.index - data.index[0]).total_seconds() / 60
    t = data['minutes'].values
    y = data['T_in'].values
    T_init = data['T_in'][0]
    popt, pcov = curve_fit(f=cooling,                  # model function
                           xdata=t,                      # x data
                           ydata=y,                      # y data
                           p0=(2, T_init, 0.5),          # initial guess of the parameters
                           )

```

```
    return popt[0],popt[2]
```

We now apply this function to several consecutive evenings, and we keep the results in a new dataframe.

```
df_night = df.between_time('20:01', '22:01', inclusive='left')

# group by day and apply the function
# this is where the magic happens.
# if you are not familiar with "groupby", this will be hard to understand
result_series = df_night.groupby(df_night.index.date).apply(run_fit)

# convert the series to a dataframe
result_df = pd.DataFrame(result_series.tolist(), index=result_series.index, columns=['T_env'])
result_df.index = pd.to_datetime(result_df.index)
result_df
```

	T_env	r
2023-06-25	13.275540	0.019354
2023-06-26	13.331949	0.027034
2023-06-27	13.254827	0.018753
2023-06-28	13.392919	0.020449
2023-06-29	14.016636	0.021218
2023-06-30	13.807517	0.021749
2023-07-01	14.994207	0.023504
2023-07-02	14.314220	0.023705
2023-07-03	14.585848	0.019438
2023-07-04	14.377220	0.019504
2023-07-05	14.814939	0.021202
2023-07-06	14.667792	0.022264
2023-07-07	15.535115	0.024421

```
fig, ax = plt.subplots(3,1,sharex=True, figsize=(8,8))

ax[0].plot(df['T_in'], c='tab:blue', label='inside')
ax[0].plot(df['T_out'], c='tab:orange', label='outside')
ax[0].set(ylabel='temperature (°C)',
           title="actual temperatures",
```

```

    ylim=[10,45])

# formating dates on x axis
locator = mdates.AutoDateLocator(minticks=7, maxticks=11)
formatter = mdates.ConciseDateFormatter(locator)
ax[0].xaxis.set_major_locator(locator)
ax[0].xaxis.set_major_formatter(formatter)

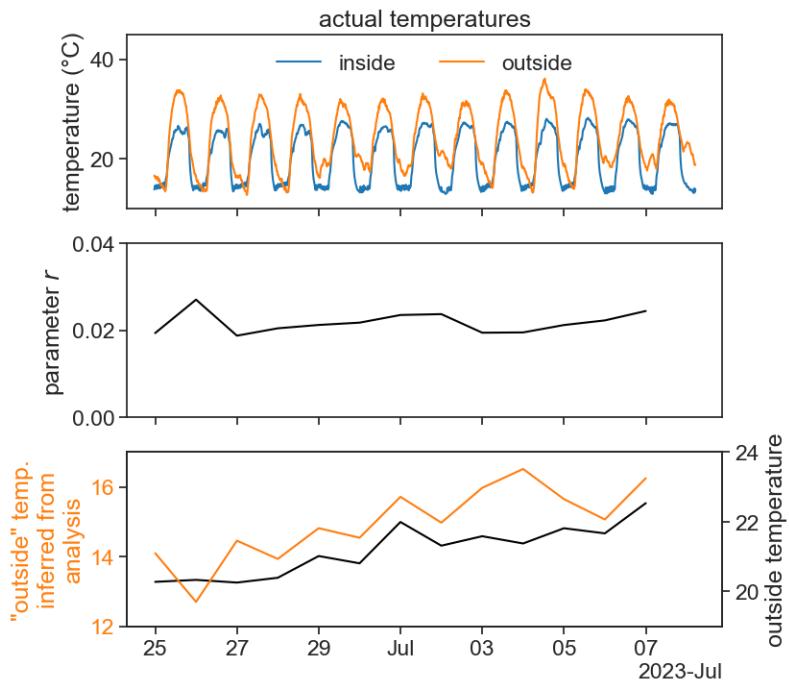
ax[0].legend(ncol=2, loc='upper center', frameon=False)

ax[1].plot(result_df['r'], color='black')
ax[1].set(ylabel=r"parameter $r$",
          ylim=[0, 0.04])

ax[2].plot(result_df['T_env'], color='black')
ax2b = ax[2].twinx()
ax2b.plot(df_night['T_out'].resample('D').mean(), color='tab:orange')
ax[2].set(ylim=[12, 17])
ax2b.set(ylim=[19, 24],
          ylabel='outside temperature')
# color the xticks
for tick in ax[2].get_yticklabels():
    tick.set_color('tab:orange')
# color the xlabel
ax[2].set_ylabel(r'"outside" temp.'+'\n"inferred from\nanalysis', color='tab:orange')

Text(0, 0.5, '"outside" temp.\n"inferred from\nanalysis")

```



Conclusions:

1. The cooling coefficient r seems quite stable throughout the two weeks of measurements. This probably says that the greenhouse and AC properties did not change much. For instance, the greenhouse thermal insulation stayed constant, and the AC power output stayed constant.
2. The AC tracks very well the outside temperature! This is to say: the AC works better (more easily) when temperatures outside are low, and vice-versa.

14 Savitzky–Golay

The Savitzky-Golay filter, also known as LOESS, smoothes a noisy signal by performing a polynomial fit over a sliding window.

Polynomial fit of order 3, window size = 51 pts

Polynomial fit of order 2, window size = 51 pts

The simulations look different because the order of the polynomial makes a very different impression on us, but in reality the outcome of the two filtering is almost identical:

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import datetime as dt
import matplotlib.ticker as ticker
from scipy.signal import savgol_filter
import os
import warnings
import scipy
warnings.simplefilter(action='ignore', category=FutureWarning)
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters
# %matplotlib widget

# dirty trick to have dates in the middle of the 24-hour period
# make minor ticks in the middle, put the labels there!
# from https://matplotlib.org/stable/gallery/ticks/centered_ticklabels.html

def centered_dates(ax):
```

```

date_form = DateFormatter("%d %b") # %d 3-letter-Month
# major ticks at midnight, every day
ax.xaxis.set_major_locator(mdates.DayLocator(interval=1))
ax.xaxis.set_major_formatter(date_form)
# minor ticks at noon, every day
ax.xaxis.set_minor_locator(mdates.HourLocator(byhour=[12]))
# erase major tick labels
ax.xaxis.set_major_formatter(ticker.NullFormatter())
# set minor tick labels as define above
ax.xaxis.set_minor_formatter(date_form)
# completely erase minor ticks, center tick labels
for tick in ax.xaxis.get_minor_ticks():
    tick.tick1line.set_markersize(0)
    tick.tick2line.set_markersize(0)
    tick.label1.set_horizontalalignment('center')

df = pd.read_csv('shani_2022_january.csv', parse_dates=['date'], index_col='date')
start = "2022-01-02"
end = "2022-01-05"
df = df.loc[start:end]

df['sg_3_51'] = savgol_filter(df['TD'], window_length=51, polyorder=3)
df['sg_2_51'] = savgol_filter(df['TD'], window_length=51, polyorder=2)

fig, ax = plt.subplots(figsize=(8,5))

plot_data, = ax.plot(df['TD'], color='black')
plot_sg2, = ax.plot(df['sg_2_51'], color='xkcd:hot pink')
plot_sg3, = ax.plot(df['sg_3_51'], color='xkcd:mustard')

ax.legend(handles=[plot_data, plot_sg2, plot_sg3],
           labels=['data', 'sg order 2', 'sg order 3'],
           frameon=False)

plot_settings = {
    'ylim': [5, 17.5],
    'xlim': [df.index[0], df.index[-1]],
    'ylabel': "Temperature (°C)",
}

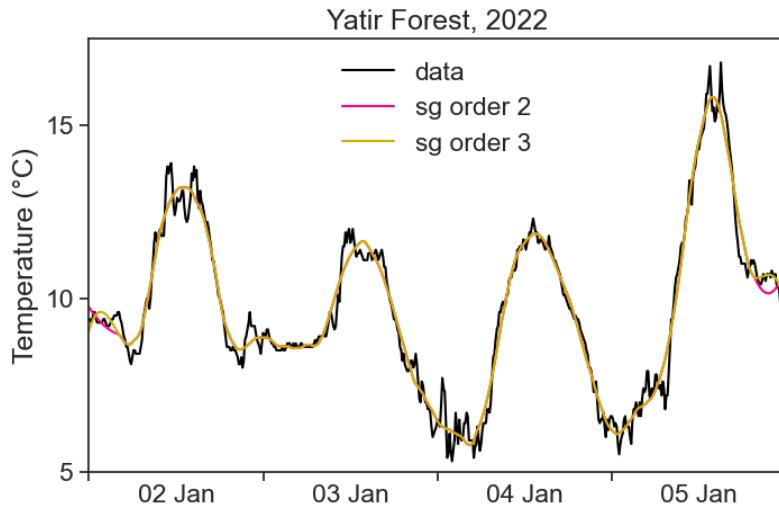
```

```

        'title': "Yatir Forest, 2022",
        'yticks': [5, 10, 15]
    }

ax.set(**plot_settings)
centered_dates(ax)

```



To really see the difference between window width and polynomial order, we need to play with their ratio,

$$\text{ratio} = \frac{w}{p} = \frac{\text{window width}}{\text{polynomial order}}$$

```

start = "2022-01-02 00:00:00"
end = "2022-01-02 23:50:00"
df = df.loc[start:end]

# window_length, polyorder
df['sg_1'] = savgol_filter(df['TD'], 5, 3)
df['sg_2'] = savgol_filter(df['TD'], 11, 2)
df['sg_3'] = savgol_filter(df['TD'], 25, 3)

```

```

fig, ax = plt.subplots(figsize=(8,5))

plot_data, = ax.plot(df['TD'], color='black')
plot_sg1, = ax.plot(df['sg_1'], color='xkcd:hot pink')
plot_sg2, = ax.plot(df['sg_2'], color='xkcd:mustard')
plot_sg3, = ax.plot(df['sg_3'], color='xkcd:royal blue')

ax.legend(handles=[plot_data, plot_sg1, plot_sg2, plot_sg3],
           labels=['data', r'$w/p=1.5$', r'$w/p=5.5$', r'$w/p=8.3$'],
           frameon=False)

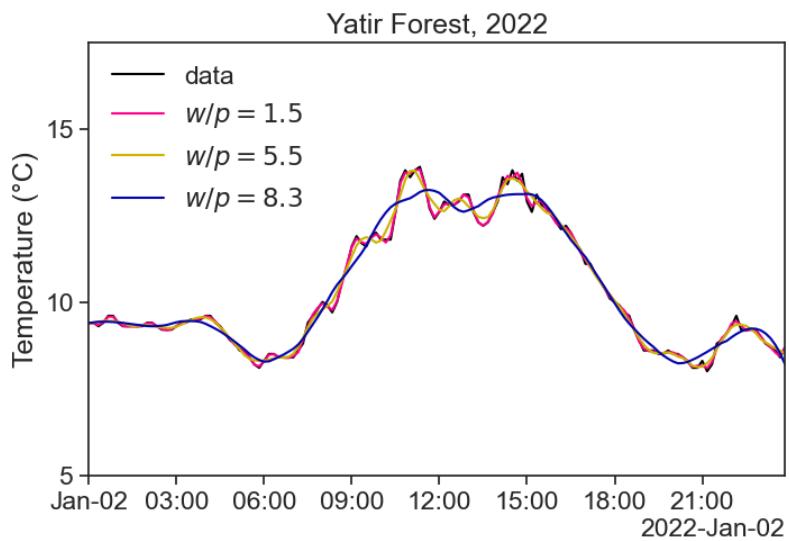
plot_settings = {
    'ylim': [5, 17.5],
    'xlim': [df.index[0], df.index[-1]],
    'ylabel': "Temperature (°C)",
    'title': "Yatir Forest, 2022",
    'yticks': [5, 10, 15]
}

ax.set(**plot_settings)

locator = mdates.AutoDateLocator(minticks=7, maxticks=11)
formatter = mdates.ConciseDateFormatter(locator)

ax.xaxis.set_major_locator(locator)
ax.xaxis.set_major_formatter(formatter)

```



The higher the ratio, the more aggressive the smoothing.

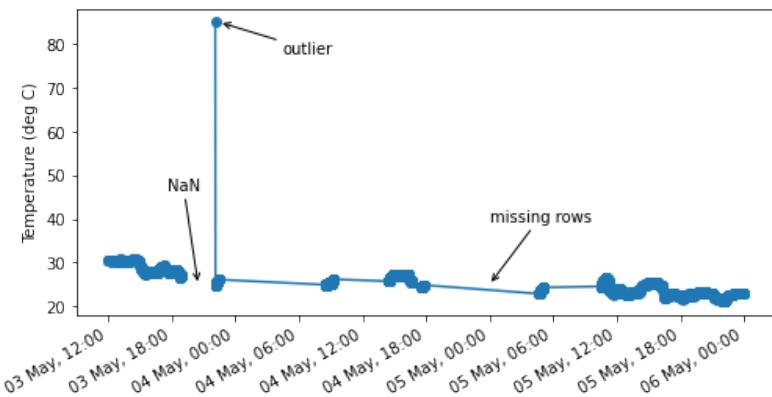
There is **a lot** more about the Savitzky-Golay filter, but for our purposes this is enough. If you want some more discussion about how to choose the parameters of the filter, [read this](#).

Part IV

outliers and gaps

15 motivation

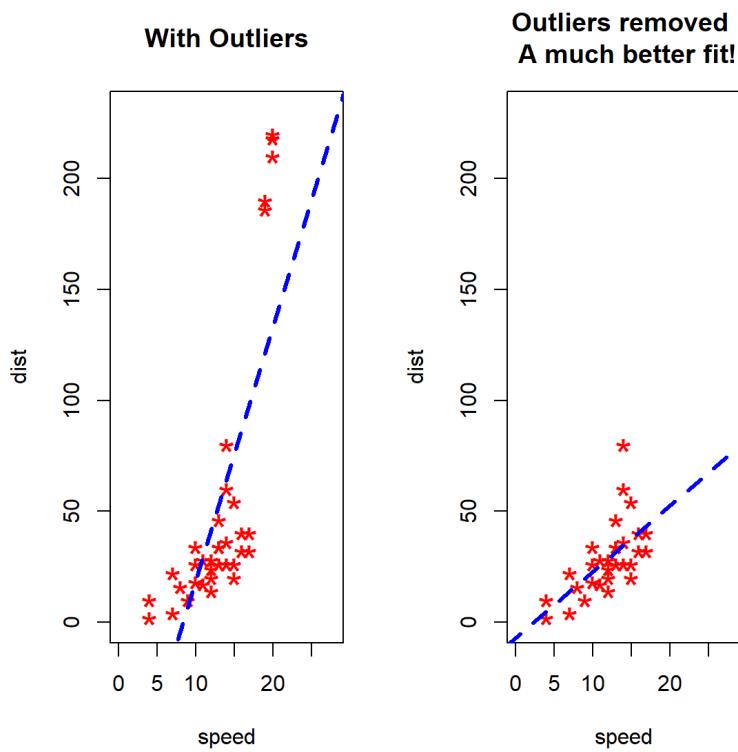
Outliers are observations significantly different from all other observations. Consider, for example, this temperature graph:



While most measured points are between 20 and 30 °C, there is obviously something very wrong with the one data point above 80 °C.

How could such a thing come about? This could be the result of **non-natural causes**, such as measurement errors, wrong data collection, or wrong data entry. On the other hand, this point could have **natural** sources, such as a very hot spark flying next to the temperature sensor.

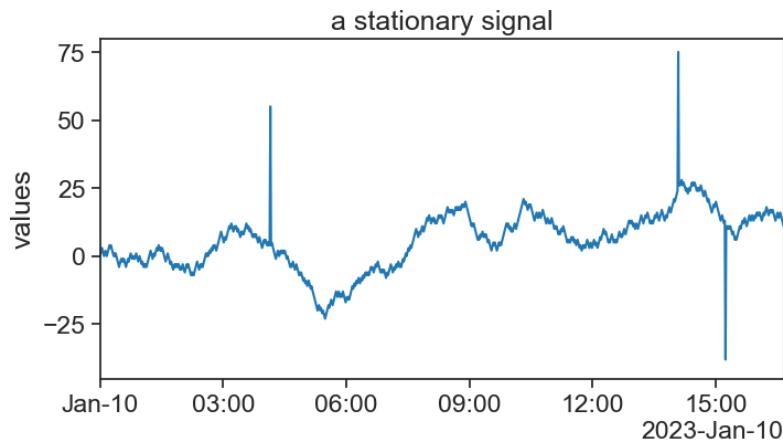
Identifying outliers is important, because they might greatly impact measures like mean and standard deviation. When left untouched, outliers might make us reach wrong conclusions about our data. See what happens to the slope of this linear regression with and without the outliers.



Source: Zhang (2020)

16 outlier identification

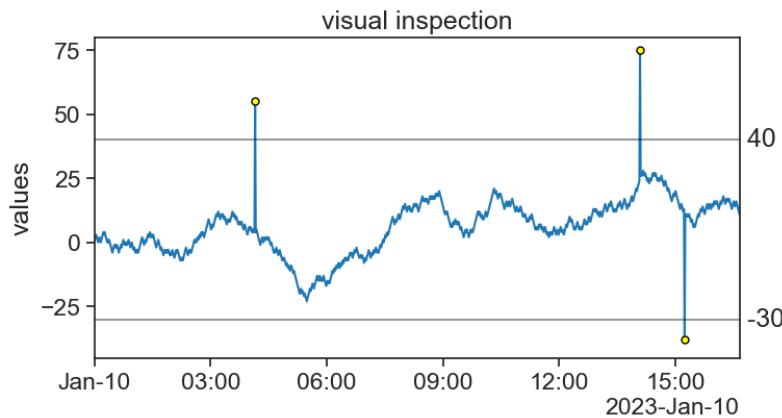
I produced a stationary signal and added to it a few outliers. Can you tell where just by looking at the graph?



The easiest way of identifying the outliers is:

- First plot the time series.
- Choose upper and lower boundaries. Whatever falls outside these boundaries is an outlier.

Easy.



If all you have is this one time series, you're done, congratulations. However, it is often the case that one has very long time series, or a great number of time series to analyze. In this case it is impractical to use the visual inspection method. We would like to devise an algorithm to automate this task.

16.1 Z-score

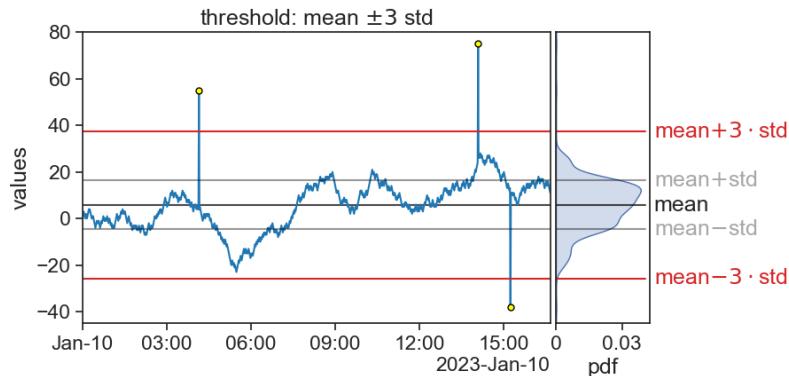
The Z-score is the distance, in units of 1 standard deviation, of a point in the series with respect to the mean:

$$z = \frac{x - \mu}{\sigma},$$

A common choice is to consider an outlier a point whose Z-score is greater than 3, in absolute value. In other words: If a point is more than 3 standard deviations away from the mean, then we call it an outlier.

where

- x = data point,
- μ = time series mean
- σ = time series standard deviation.

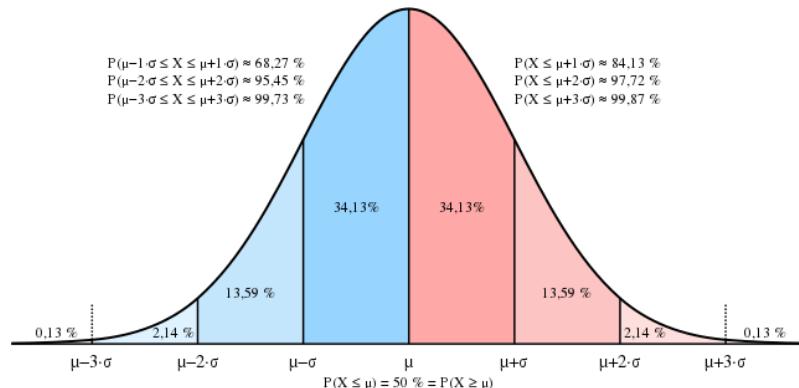


You can now use this algorithm to any number of time series, let the computer do the hard work.

Of course, there is nothing sacred about the number 3. You can choose any Z-score you want to perform an analysis on your own data, depending on your needs.

16.1.1 ATTENTION!

For data that is gaussianly distributed, we expect that 99.73% of data to fall within 3 standard deviations from the mean. In other words, 0.27% of points would be considered as *outliers* according to the Z-score method.

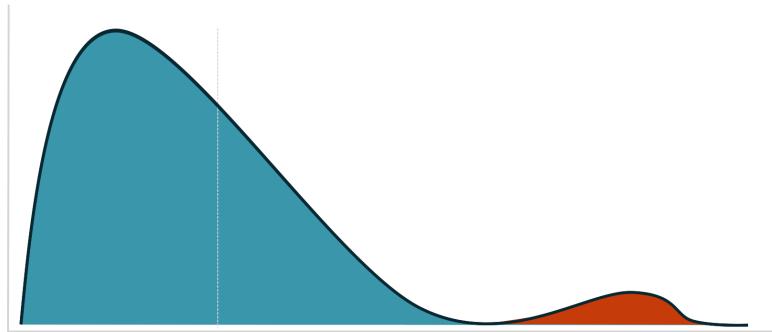


Assume you have a time series gaussianly distributed, with 10k measurements. We would expect to find about 27 outliers in this time series.

Source: [Wikimedia Commons](#)

So what is the problem?!

The thing is, outliers are not supposed to be only data points far from the other points. That's not enough. A better way of understanding outliers is to imagine that our expected measurements are sampled from a given distribution, and every now and then we have measurements that are sampled from **another** distribution.



We should have this in mind always. We wouldn't want to single out good data as something weird. Our true task is to identify which points in our time series were sampled from a different distribution. This can be a very challenging task.

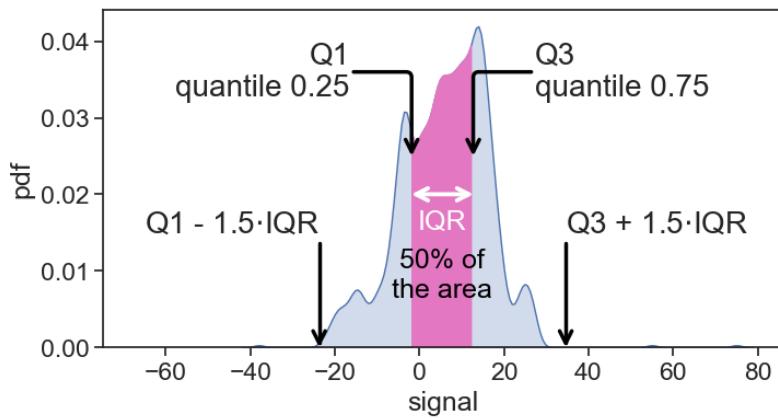
Source: Taylor Wilson's "Dealing with Outliers (Part 1): Ignore Them at Your Peril"

16.2 IQR

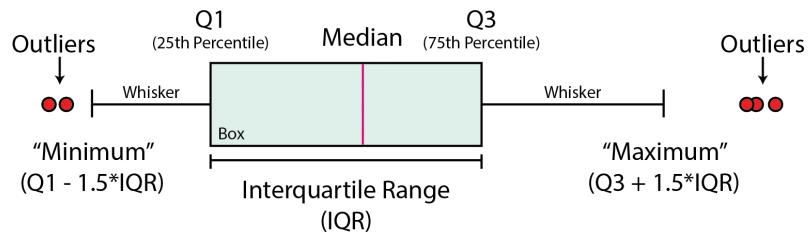
Another super common criterion for identifying outliers is the IQR, or InterQuartile Range.

Take a look at the statistics below of the time series we have been working with so far. The IQR is the distance between the first quartile (Q1) and the third quartile (Q3), where exactly 50% of the data is.

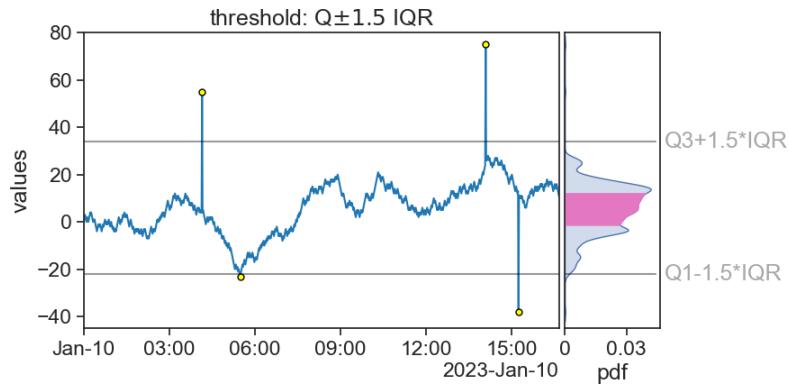
The algorithm here is to determine two thresholds, whose distance is 1.5 times the IQR from Q1 and Q3. Whatever falls outside these two thresholds is an outlier.



We are used to see this in box plots:



Again, the distance 1.5 is not sacred, it's only the most common.
You might want to choose other values depending on your needs.
Let's now apply the IQR method to our time series.

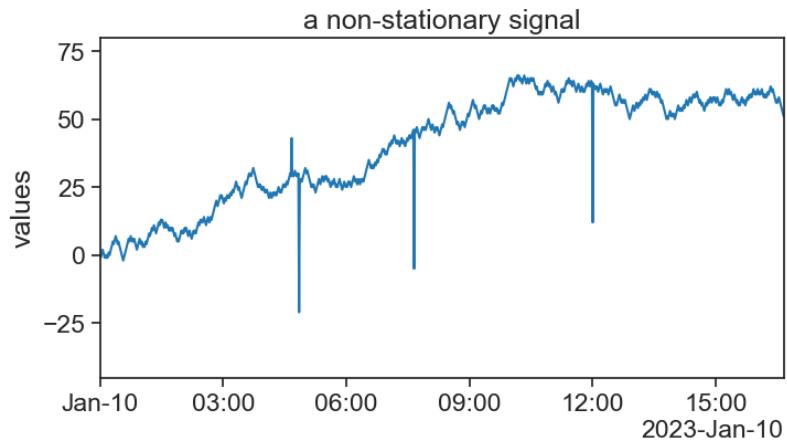


It works pretty well! Notice that now we have an additional outlier (a bit before 06:00). What do we do with that?

Source: McDonald (2022)

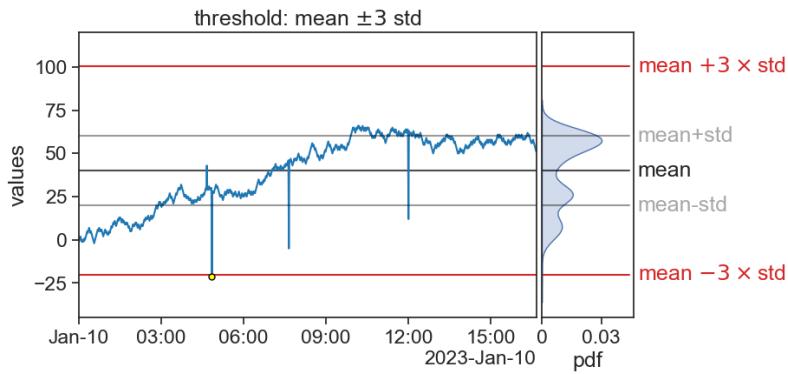
16.3 non-stationary time series

I have produced a new time series, one that on average goes up with time. Can you point in the graph where are the outliers?

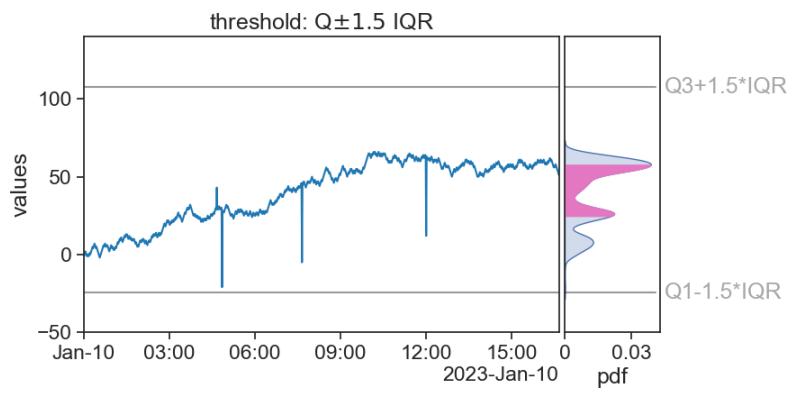


Now, see what happens when we apply the previous two methods to this time series.

Z-score



IQR



What happened? Do you have ideas how to solve this?

16.4 Sources

17 robust analysis

A tool is said to be **robust** if outliers don't influence (much) its results.

The average and standard deviation are **not** robust.

```
import numpy as np
series1 = np.array([0, 1, 2, 3, 4, 5, 6])
series2 = np.array([0, 1, 2, 3, 4, 5, 60])
print(f"series 1: mean={series1.mean():.2f}, std={series1.std():.2f}")
print(f"series 2: mean={series2.mean():.2f}, std={series2.std():.2f}")

series 1: mean=3.00, std=2.00
series 2: mean=10.71, std=20.18
```

On the other hand, the median and IQR are robust:

```
from scipy.stats import iqr
print(f"series 1: median={np.median(series1):.2f}, IQR={iqr(series1):.2f}")
print(f"series 2: median={np.median(series2):.2f}, IQR={iqr(series2):.2f}")

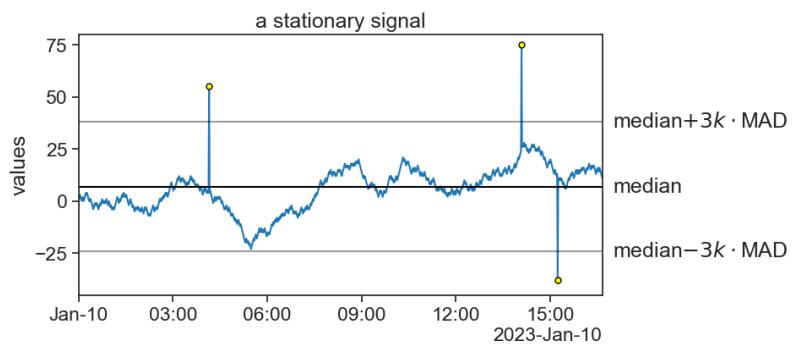
series 1: median=3.00, IQR=3.00
series 2: median=3.00, IQR=3.00
```

Another robust method is MAD, the Median Absolute Deviation, given by

$$\text{MAD} = \text{median}(|x_i - \text{median}(x)|),$$

where $|\cdot|$ is the absolute value.

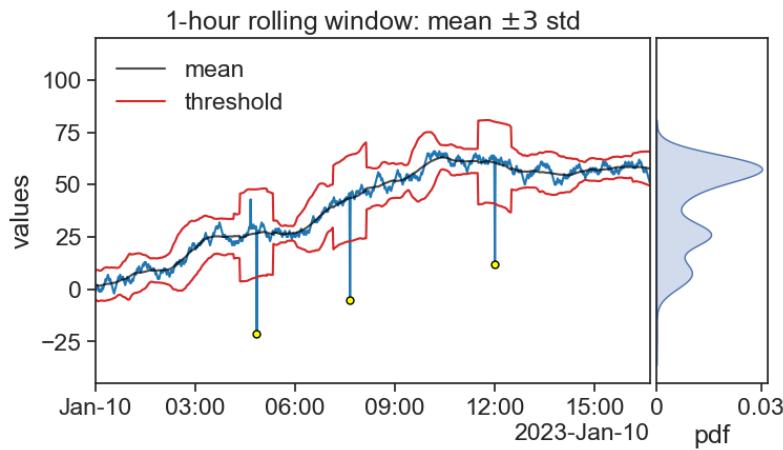
Applying MAD to the stationary time series from before, yields



Here, the threshold is the median $\pm 3k \cdot \text{MAD}$, where the value $k = 1.4826$ scales MAD so that when the data is gaussianly distributed, $3k$ equals 1 standard deviation.

18 sliding algorithms

None of the methods learned before seem appropriate to deal with non-stationary data. A simple solution is to apply those methods for sliding windows of “appropriate” widths.

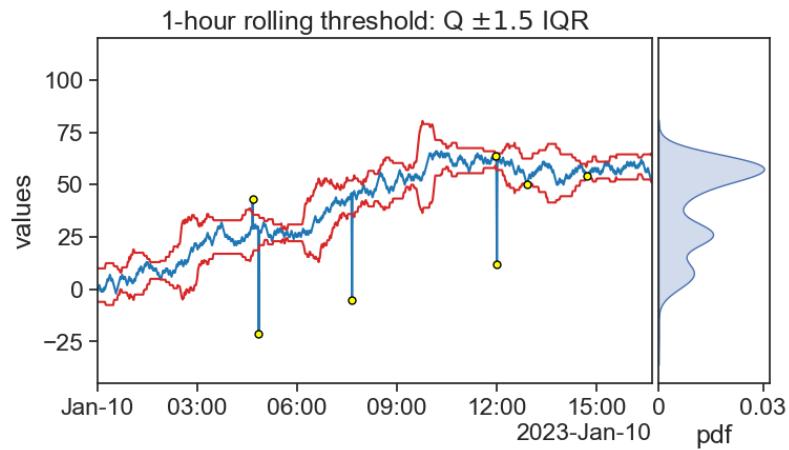


Now the Z-score seems to give really nice results (but not perfect). Maybe playing with the window width and Z-score threshold would give better results?

In any case, we clearly see why the Z-score is not a robust algorithm. See how the standard deviation is sensitive to outliers?

18.1 Sliding IQR

Let's see how well the sliding IQR method fares.



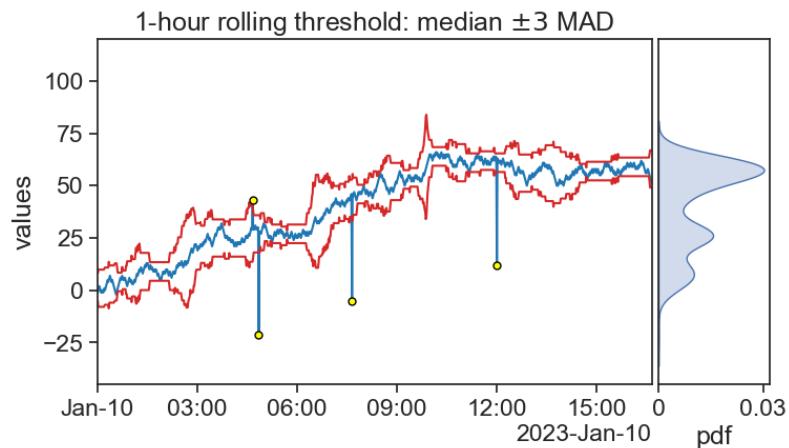
It identified all the outliers, but also found that a few *other* points should be considered outliers? What do you think of that?

See that the threshold does not jump abruptly when the sliding window includes an outlier. In fact, the threshold doesn't even care! This is what it means to be robust.

However, we do see large fluctuations in the threshold. When does this happen? Why?

18.2 Sliding MAD

Now it's MAD's time to shine.



Compare this result to the previous two. Which yields best results?

MAD is robust to outliers, and again we see that the threshold envelope widens when there is a rising or falling trend in the data.

18.3 Challenges

Now it's your turn to work, I'm tired! Write algorithms for the following outlier identification methods:

1. visual inspection
2. Z-score
3. IQR
4. MAD

Excluding the visual inspection method, write first an algorithm that operates on a full time series, and then write a new version that can work with sliding windows.

19 substituting outliers

Ok. We found the outliers. Now what?!

As usual, it depends.

Assuming the outlier indeed happened in real life, and is not the result of faulty data transmission or bad data recording, then excluding an outlier might be the last thing you want to do. Sometimes extreme events do happen, such as a one-in-a-hundred-year storm, and they have a disproportionate weight on the system you are studying. The outliers might actually be the most interesting points in your data for all you know!

In case the outliers are not of interest to you, if you are using **robust** methods to analyze your data, you don't necessarily need to do anything either. For instance, let's say that you want to smooth your time series. If instead of taking the `mean` inside a sliding window you choose to calculate the `median`, then outliers shouldn't be a problem. Test it and see if it's true. Go on.

For many things you need to do (not only smoothing), you might be able to find robust methods. What do you do if you **have** to use a non-robust method? Well, then you can substitute the outlier for two things: NaN or imputed values.

19.1 NaN

Substitute outliers for NaN.

NaN means “Not a Number”, and is what you get when you try to perform a mathematical operation like $0/0$. It is common to see NaN in dataset rows when data was not collected for some reason.

This might seem like a neutral solution, but it actually can generate problems down the line. See this example:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters
from scipy.signal import savgol_filter

# example using numpy
series = np.array([2, 4, 5, np.nan, 8, 15])
mean = np.mean(series)
print(f"the series average is {mean}")

the series average is nan
```

A single NaN in your time series ruins the whole calculation! There is a workaround though:

```
mean = np.nanmean(series)
print(f"the series average is {mean}")

the series average is 6.8
```

You have to make sure what is the behavior of each function you use with respect to NaNs, and if possible, use a suitable substitute.

The same example in `pandas` would not fail:

```
date_range = pd.date_range(start='2024-01-01', periods=len(series), freq='1D')
df = pd.DataFrame({'series': series}, index=date_range)
mean = df['series'].mean()
print(f"the series average is {mean}")

the series average is 6.8
```

19.2 impute values

To “impute values” means to fill in the missing value with a guess, an estimation of what this data point “should have been” if it were measured in the first place. Why should we bother to do so? Because many tools that we know and love don’t do well with missing values.

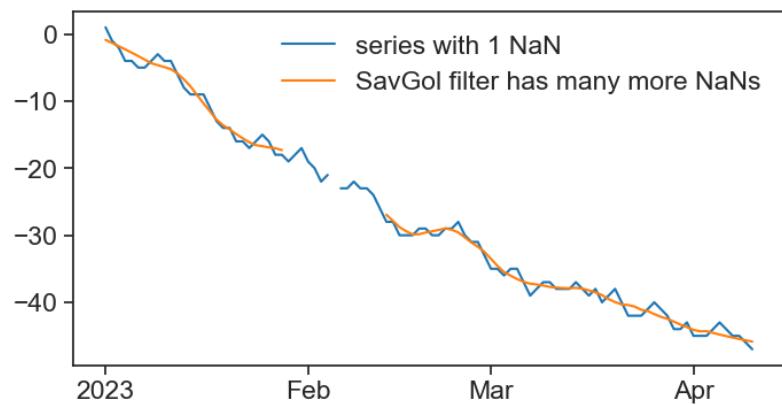
We learned about the Savitzky-Golay filter for smoothing data. See what happens when there is a single NaN in the series:

```
steps = np.random.randint(low=-2, high=2, size=100)
data = steps.cumsum()
date_range = pd.date_range(start='2023-01-01', periods=len(data), freq='1D')
df = pd.DataFrame({'series': data}, index=date_range)
df.loc['2023-02-05', 'series'] = np.nan

df['sg'] = savgol_filter(df['series'], window_length=15, polyorder=2)

def concise(ax):
    locator = mdates.AutoDateLocator(minticks=3, maxticks=7)
    formatter = mdates.ConciseDateFormatter(locator)
    ax.xaxis.set_major_locator(locator)
    ax.xaxis.set_major_formatter(formatter)

fig, ax = plt.subplots(figsize=(8,4))
ax.plot(df['series'], color="tab:blue", label="series with 1 NaN")
ax.plot(df['sg'], color="tab:orange", label="SavGol filter has many more NaNs")
concise(ax)
ax.legend(frameon=False);
```



We will deal with this topic in the next chapter, “interpolation”. There, we will learn a few methods to fill in missing data, and basic NaN operations you should be acquainted with.

20 interpolation

Interpolation is the act of getting data you don't have from data you already have.

Part V

best practices

21 motivation

22 date formatting

Here you will find several examples of how to format dates in your plots. Not many explanations are provided.

How to use this page? Find first an example of a plot you like, only then go to the code and see how it's done.

```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import datetime
from datetime import timedelta
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)
import matplotlib.gridspec as gridspec
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker

import pandas as pd

start_date = '2018-01-01'
end_date = '2018-04-30'

dates = pd.date_range(start_date, end_date, freq='1H')
# create a random variable to plot
var = np.random.rand(len(dates)) - 0.51
var = var.cumsum()
var = var - var.min()
# create dataframe, make "date" the index
df = pd.DataFrame({'date': dates, 'variable': var})
df.set_index(df['date'], inplace=True)
df
```

	date	variable
date		
2018-01-01 00:00:00	2018-01-01 00:00:00	28.317035
2018-01-01 01:00:00	2018-01-01 01:00:00	28.120523
2018-01-01 02:00:00	2018-01-01 02:00:00	28.596894
2018-01-01 03:00:00	2018-01-01 03:00:00	28.931941
2018-01-01 04:00:00	2018-01-01 04:00:00	28.561778
...
2018-04-29 20:00:00	2018-04-29 20:00:00	1.914343
2018-04-29 21:00:00	2018-04-29 21:00:00	1.648757
2018-04-29 22:00:00	2018-04-29 22:00:00	1.992956
2018-04-29 23:00:00	2018-04-29 23:00:00	1.500860
2018-04-30 00:00:00	2018-04-30 00:00:00	1.650439

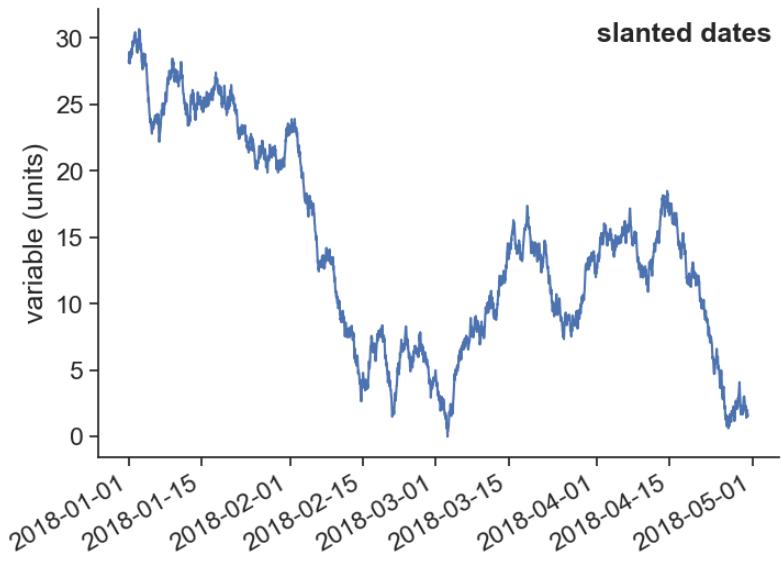
define a useful function to plot the graphs below

```

def explanation(ax, text, letter):
    ax.text(0.99, 0.97, text,
            transform=ax.transAxes,
            horizontalalignment='right', verticalalignment='top',
            fontweight="bold")
    ax.text(0.01, 0.01, letter,
            transform=ax.transAxes,
            horizontalalignment='left', verticalalignment='bottom',
            fontweight="bold")
    ax.set(ylabel="variable (units)")
    ax.spines['top'].set_visible(False)
    ax.spines['right'].set_visible(False)

fig, ax = plt.subplots(1, 1, figsize=(8, 6))
ax.plot(df['variable'])
plt.gcf().autofmt_xdate() # makes slanted dates
explanation(ax, "slanted dates", "")
fig.savefig("dates1.png")

```



```

fig, ax = plt.subplots(4, 1, figsize=(10, 16),
                      gridspec_kw={'hspace': 0.3})

### plot a ####
ax[0].plot(df['variable'])
date_form = DateFormatter("%b")
ax[0].xaxis.set_major_locator(mdates.MonthLocator(interval=2))
ax[0].xaxis.set_major_formatter(date_form)

### plot b ####
ax[1].plot(df['variable'])
date_form = DateFormatter("%B")
ax[1].xaxis.set_major_locator(mdates.MonthLocator(interval=1))
ax[1].xaxis.set_major_formatter(date_form)

### plot c ####
ax[2].plot(df['variable'])
ax[2].xaxis.set_major_locator(mdates.MonthLocator())
# 16 is a slight approximation for the center, since months differ in number of days.
ax[2].xaxis.set_minor_locator(mdates.MonthLocator(bymonthday=16))
ax[2].xaxis.set_major_formatter(ticker.NullFormatter())
ax[2].xaxis.set_minor_formatter(DateFormatter('%B'))

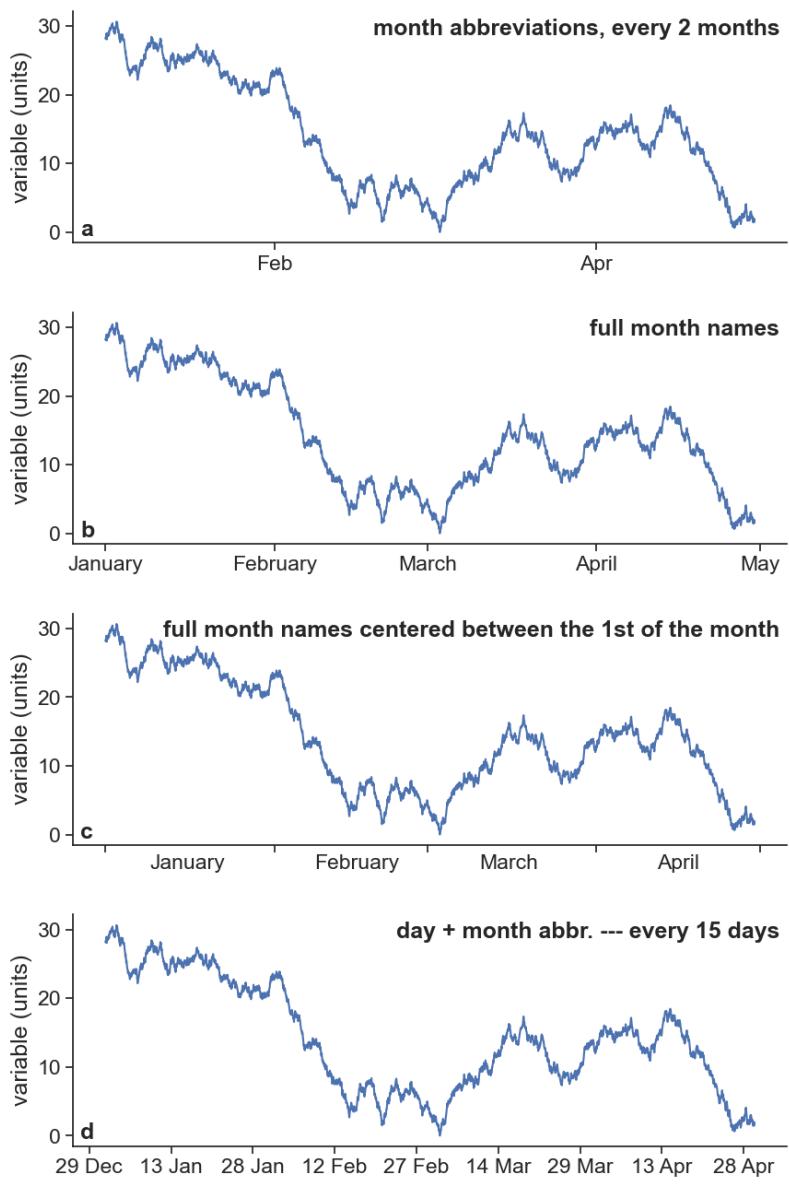
```

```
for tick in ax[2].xaxis.get_minor_ticks():
    tick.tick1line.set_markersize(0)
    tick.tick2line.set_markersize(0)
    tick.label1.set_horizontalalignment('center')

### plot d ####
ax[3].plot(df['variable'])
date_form = DateFormatter("%d %b")
ax[3].xaxis.set_major_locator(mdates.DayLocator(interval=15))
ax[3].xaxis.set_major_formatter(date_form)

explanation(ax[0], "month abbreviations, every 2 months", "a")
explanation(ax[1], "full month names", "b")
explanation(ax[2], "full month names centered between the 1st of the month", "c")
explanation(ax[3], "day + month abbr. --- every 15 days", "d")

fig.savefig("dates2.png")
```



```

fig, ax = plt.subplots(4, 1, figsize=(10, 16),
                      gridspec_kw={'hspace': 0.3})

### plot e ####
ax[0].plot(df['variable'])

```

```

date_form = DateFormatter("%d/%m")
ax[0].xaxis.set_major_locator(mdates.DayLocator(bymonthday=[5, 20]))
ax[0].xaxis.set_major_formatter(date_form)

### plot f ####
ax[1].plot(df['variable'])
locator = mdates.AutoDateLocator(minticks=11, maxticks=17)
formatter = mdates.ConciseDateFormatter(locator)
ax[1].xaxis.set_major_locator(locator)
ax[1].xaxis.set_major_formatter(formatter)

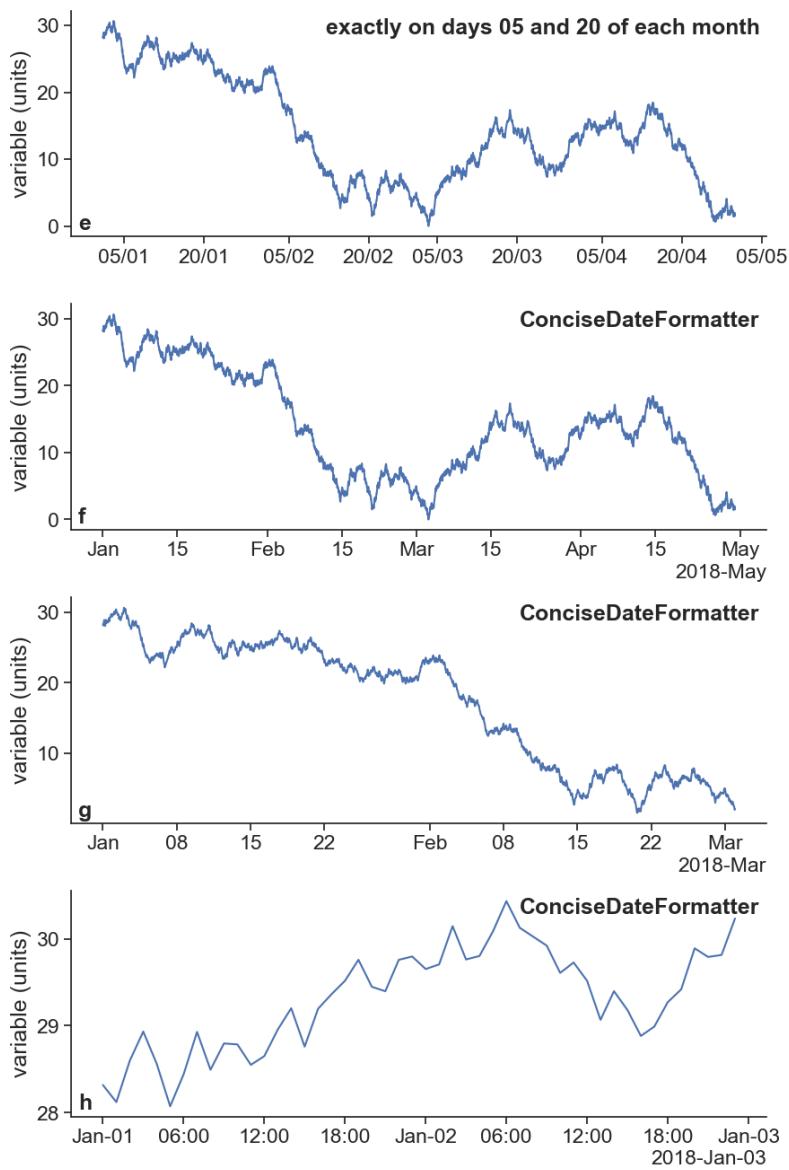
### plot g ####
ax[2].plot(df.loc['2018-01-01':'2018-03-01', 'variable'])
locator = mdates.AutoDateLocator(minticks=6, maxticks=14)
formatter = mdates.ConciseDateFormatter(locator)
ax[2].xaxis.set_major_locator(locator)
ax[2].xaxis.set_major_formatter(formatter)

### plot h ####
ax[3].plot(df.loc['2018-01-01':'2018-01-02', 'variable'])
locator = mdates.AutoDateLocator(minticks=6, maxticks=10)
formatter = mdates.ConciseDateFormatter(locator)
ax[3].xaxis.set_major_locator(locator)
ax[3].xaxis.set_major_formatter(formatter)

explanation(ax[0], "exactly on days 05 and 20 of each month", "e")
explanation(ax[1], "ConciseDateFormatter", "f")
explanation(ax[2], "ConciseDateFormatter", "g")
explanation(ax[3], "ConciseDateFormatter", "h")

fig.savefig("dates3.png")

```



```

fig, ax = plt.subplots(1, 1, figsize=(10, 4),
                      gridspec_kw={'hspace': 0.3})

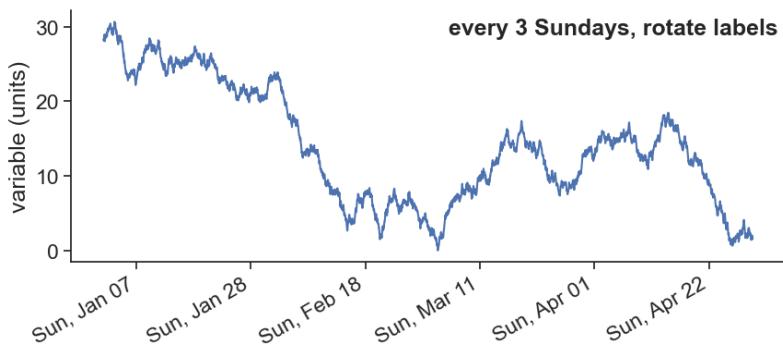
# import constants for the days of the week
from matplotlib.dates import MO, TU, WE, TH, FR, SA, SU
ax.plot(df['variable'])

```

```

# tick on sundays every third week
loc = mdates.WeekdayLocator(byweekday=SU, interval=3)
ax.xaxis.set_major_locator(loc)
date_form = DateFormatter("%a, %b %d")
ax.xaxis.set_major_formatter(date_form)
fig.autofmt_xdate(bottom=0.2, rotation=30, ha='right')
explanation(ax, "every 3 Sundays, rotate labels", "")

```



Code	Explanation
%Y	4-digit year (e.g., 2022)
%y	2-digit year (e.g., 22)
%m	2-digit month (e.g., 12)
%B	Full month name (e.g., December)
%b	Abbreviated month name (e.g., Dec)
%d	2-digit day of the month (e.g., 09)
%A	Full weekday name (e.g., Tuesday)
%a	Abbreviated weekday name (e.g., Tue)
%H	24-hour clock hour (e.g., 23)
%I	12-hour clock hour (e.g., 11)
%M	2-digit minute (e.g., 59)
%S	2-digit second (e.g., 59)
%p	"AM" or "PM"
%Z	Time zone name
%z	Time zone offset from UTC (e.g., -0500)

Part VI

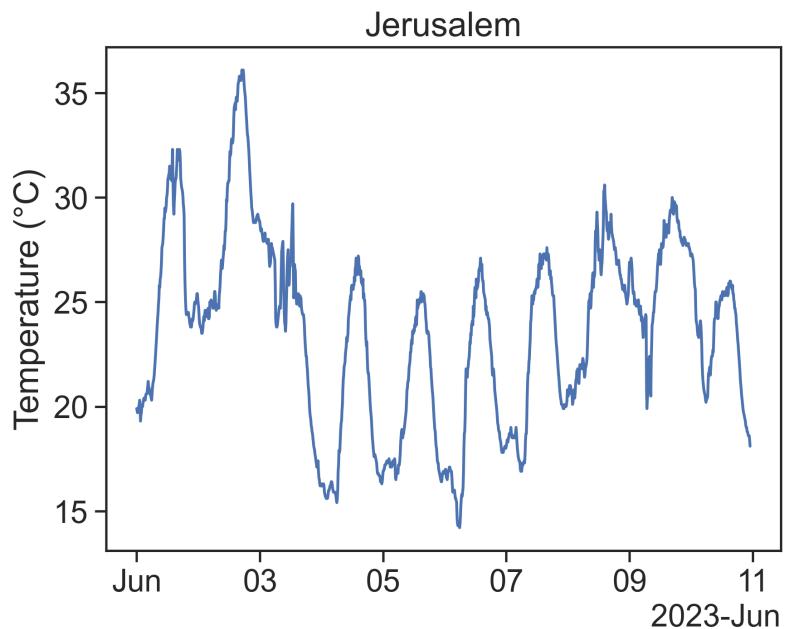
stationarity

23 motivation

24 stochastic processes

25 autocorrelation

See the temperatures for Jerusalem in a 4-day interval:



25.1 question

If I know the temperature right now, what does that tell me about the temperature 10 minutes from now? How about 100 minutes? 1000 minutes?

To answer this, we need to talk about **autocorrelation**. Let's start by introducing the necessary concepts.

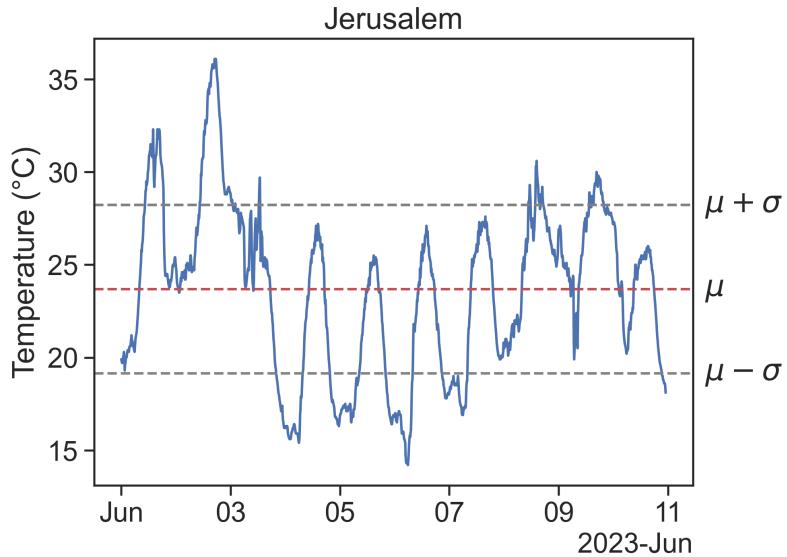
25.2 mean and standard deviation

Let's call our time series from above X , and its length N . Then:

$$\text{mean } \mu = \frac{\sum_{i=1}^N X_i}{N}$$

$$\text{standard deviation } \sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}}$$

The mean and standard deviation can be visualized thus:



One last basic concept we need is the expected value:

$$E[X] = \sum_{i=1}^N X_i p_i$$

For our time series, the probability p_i that a given point X_i is in the dataset is simply $1/N$, therefore the expectation becomes

$$E[X] = \frac{\sum_{i=1}^N X_i}{N}$$

25.3 autocorrelation

The autocorrelation of a time series X is the answer to the following question:

if we shift X by τ units, how similar will this be to the original signal?

In other words:

how correlated are $X(t)$ and $X(t + \tau)$?

Using the Pearson correlation coefficient

we get

$$\rho_{XX}(\tau) = \frac{E[(X_t - \mu)(X_{t+\tau} - \mu)]}{\sigma^2}$$

A video is worth a billion words, so let's see the autocorrelation in action:

<https://youtu.be/tpf-tuYHR5w>

A few comments:

- The autocorrelation for $\tau = 0$ (zero shift) is always 1.
[Can you prove this? All the necessary equations are above!]

Pearson correlation coefficient between X and Y :

$$\rho_{X,Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

Part VII

time lags

26 motivation

27 cross-correlation

```
import numpy as np  
  
print('dfvdfv')  
  
dfvdfv
```

28 dynamic time warp

29 LDTW

according to this paper

Part VIII

frequency

30 motivation

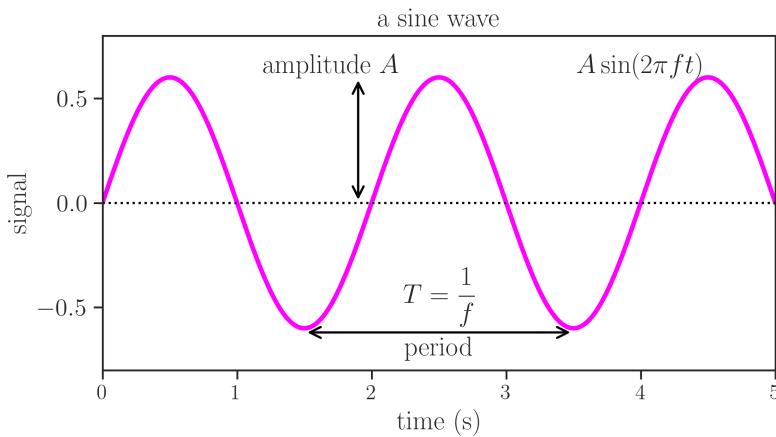
31 Fourier transform

31.1 basic wave concepts

The function

$$f(t) = B \sin(2\pi ft) \quad (31.1)$$

has two basic characteristics, its amplitude B and frequency f .

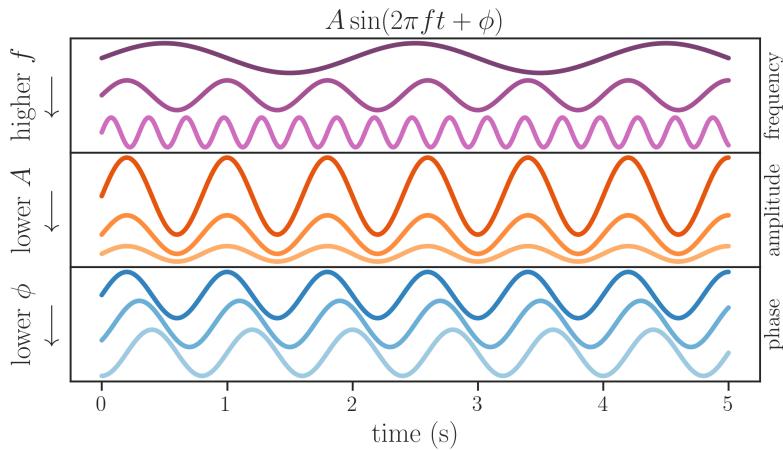


In the figure above, the amplitude $B = 0.6$ and we see that the distance between two peaks is called period, $T = 2$ s. The frequency is defined as the inverse of the period:

$$f = \frac{1}{T}. \quad (31.2)$$

When time is in seconds, then the frequency is measured in Hertz (Hz). For the graph above, therefore, we see a wave whose frequency is $f = 1/(2 \text{ s}) = 0.5 \text{ Hz}$.

In the figure below, we see what happens when we vary the values of the frequency and amplitude.



The graph above introduces two new characteristics of a wave, its phase ϕ , and its offset B . A more general description of a sine wave is

$$f(t) = B \sin(2\pi ft + \phi) + B_0. \quad (31.3)$$

The offset B_0 moves the wave up and down, while changing the value of ϕ makes the sine wave move left and right. When the phase $\phi = 2\pi$, the sine wave will have shifted a full period, and the resulting wave is identical to the original:

$$B \sin(2\pi ft) = B \sin(2\pi ft + 2\pi). \quad (31.4)$$

All the above can also be said about a cosine, whose general form can be given as

$$A \cos(2\pi ft + \phi) + A_0 \quad (31.5)$$

One final point before we jump into the deep waters is that the sine and cosine functions are related through a simple phase shift:

$$\cos\left(2\pi ft + \frac{\pi}{2}\right) = \sin(2\pi ft)$$

31.2 Fourier's theorem

Fourier's theorem states that

Any periodic signal is composed of a superposition of pure sine waves, with suitably chosen amplitudes and phases, whose frequencies are harmonics of the fundamental frequency of the signal.

See the following animations to visualize the theorem in action.

Source: https://en.wikipedia.org/wiki/File:Fourier_series_and_transform.gif

Source: https://commons.wikimedia.org/wiki/File:Fourier_synthesis_square_wave_animated.gif

Source: https://commons.wikimedia.org/wiki/File:Sawtooth_Fourier_Animation.gif

Source: https://commons.wikimedia.org/wiki/File:Continuous_Fourier_transform_of_rect_and_sinc_functions.gif

31.3 Fourier series

a periodic function can be described as a sum of sines and cosines.

The classic examples are usually the square function and the sawtooth function:

[Source: <https://www.geogebra.org/m/tkajbzmg>]

<https://www.geogebra.org/m/k4eq4fkr>

Not any function, but certainly most functions we will deal with in this course. The function has to fulfill the [Dirichlet conditions](#)

$$F[x(t)] = F(f) = \int_{-\infty}^{\infty} x(t)e^{-2\pi i ft} dt$$

$$f(t) = \int_{-\infty}^{\infty} F(f) e^{2\pi i f t} df$$

<https://dibsmethodsmeetings.github.io/fourier-transforms/>

<https://www.jezzamon.com/fourier/index.html>

32 filtering

33 Nyquist-Shannon sampling theorem

Part IX

seasonality

34 motivation

35 seasonal decomposition

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters() # datetime converter for a matplotlib
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)
from statsmodels.tsa.seasonal import seasonal_decompose
import matplotlib.dates as mdates
from matplotlib.dates import DateFormatter
```

35.1 trends in atmospheric carbon dioxide

Mauna Loa CO₂ concentration.

data from [NOAA](#)

```
url = "https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_weekly_mlo.csv"
# df = pd.read_csv(url, header=47, na_values=[-999.99])

# you can first download, and then read the csv
filename = "co2_weekly_mlo.csv"
df = pd.read_csv(filename, header=35, na_values=[-999.99])

df
```

	1974	5	19	1974.3795	333.37	5.1	-999.99	-999.99.1	50.39
0	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.05
1	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.59
2	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.64

	1974	5	19	1974.3795	333.37	5.1	-999.99	-999.99.1	50.39
3	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
4	1974	6	23	1974.4753	331.73	5	NaN	NaN	49.72
...
2565	2023	7	23	2023.5575	421.28	4	418.03	397.30	141.60
2566	2023	7	30	2023.5767	420.83	6	418.10	396.80	141.69
2567	2023	8	6	2023.5959	420.02	6	417.36	395.65	141.41
2568	2023	8	13	2023.6151	418.98	4	417.25	395.24	140.89
2569	2023	8	20	2023.6342	419.31	2	416.64	395.22	141.71

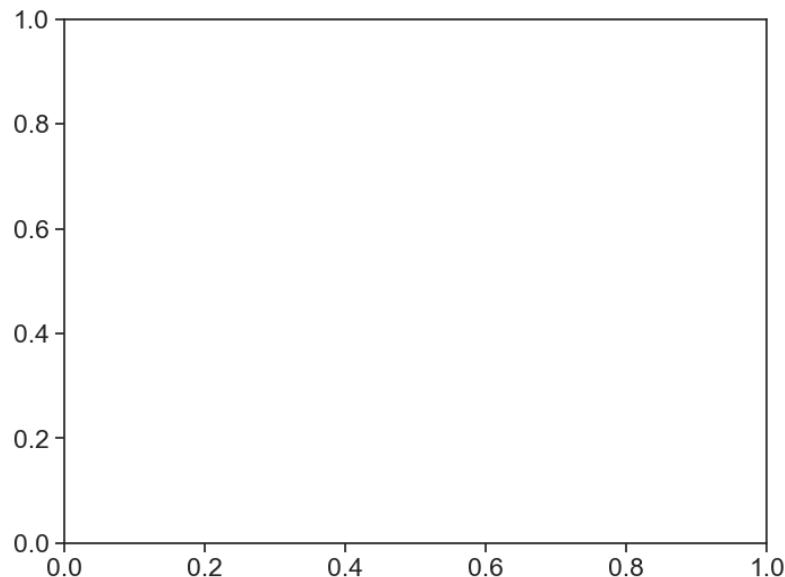
```
df['date'] = pd.to_datetime(df[['year', 'month', 'day']])
df = df.set_index('date')
df
```

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
1974-05-19	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.40
1974-05-26	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.06
1974-06-02	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.60
1974-06-09	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.65
1974-06-16	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...
2022-06-26	2022	6	26	2022.4836	420.31	7	418.14	395.36	138.71
2022-07-03	2022	7	3	2022.5027	419.73	6	417.49	395.15	138.64
2022-07-10	2022	7	10	2022.5219	419.08	6	417.25	394.59	138.52
2022-07-17	2022	7	17	2022.5411	418.43	6	417.14	394.64	138.41
2022-07-24	2022	7	24	2022.5603	417.84	6	415.68	394.11	138.36

```
# %matplotlib widget

fig, ax = plt.subplots(1, figsize=(8,6))
ax.plot(df['average'])
ax.set(xlabel="date",
       ylabel="CO2 concentration (ppm)",
       # ylim=[0, 430],
       title="Mauna Loa CO2 concentration");
```

KeyError: 'average'



fill missing data. interpolate method: 'time'

interpolation methods visualized

```
df['co2'] = (df['average'].resample("D") #resample daily  
              .interpolate(method='time') #interpolate by time  
            )  
df
```

date	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
1974-05-19	1974	5	19	1974.3795	333.37	5	NaN	NaN	50.40
1974-05-26	1974	5	26	1974.3986	332.95	6	NaN	NaN	50.06
1974-06-02	1974	6	2	1974.4178	332.35	5	NaN	NaN	49.60
1974-06-09	1974	6	9	1974.4370	332.20	7	NaN	NaN	49.65
1974-06-16	1974	6	16	1974.4562	332.37	7	NaN	NaN	50.06
...
2022-06-26	2022	6	26	2022.4836	420.31	7	418.14	395.36	138.71
2022-07-03	2022	7	3	2022.5027	419.73	6	417.49	395.15	138.64
2022-07-10	2022	7	10	2022.5219	419.08	6	417.25	394.59	138.52

	year	month	day	decimal	average	ndays	1 year ago	10 years ago	increase since 1800
date									
2022-07-17	2022	7	17	2022.5411	418.43	6	417.14	394.64	138.41
2022-07-24	2022	7	24	2022.5603	417.84	6	415.68	394.11	138.36

35.2 decompose data

`seasonal_decompose` returns an object with four components:

- observed: $Y(t)$
- trend: $T(t)$
- seasonal: $S(t)$
- resid: $e(t)$

Additive model:

$$Y(t) = T(t) + S(t) + e(t)$$

Multiplicative model:

$$Y(t) = T(t) \times S(t) \times e(t)$$

35.2.0.1 Interlude

learn how to use `zip` in a loop

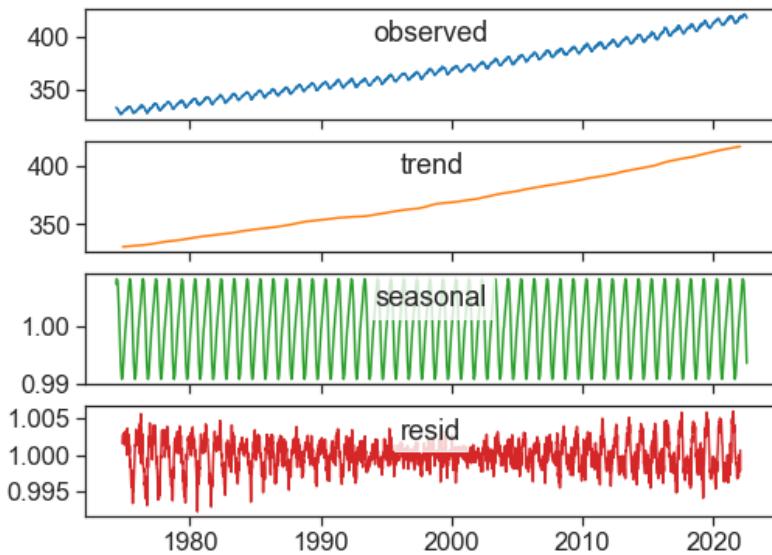
```
letters = ['a', 'b', 'c', 'd', 'e']
numbers = [1, 2, 3, 4, 5]
# zip let's us iterate over lists at the same time
for l, n in zip(letters, numbers):
    print(f"{l} = {n}")
```

```
a = 1
b = 2
c = 3
d = 4
e = 5
```

Plot each component separately.

```
# %matplotlib widget

fig, ax = plt.subplots(4, 1, figsize=(8,6), sharex=True)
decomposed_m = seasonal_decompose(df['co2'], model='multiplicative')
decomposed_a = seasonal_decompose(df['co2'], model='additive')
decomposed = decomposed_m
pos = (0.5, 0.9)
components = ["observed", "trend", "seasonal", "resid"]
colors = ["tab:blue", "tab:orange", "tab:green", "tab:red"]
for axx, component, color in zip(ax, components, colors):
    data = getattr(decomposed, component)
    axx.plot(data, color=color)
    axx.text(*pos, component, bbox=dict(facecolor='white', alpha=0.8),
              transform=axx.transAxes, ha='center', va='top')
```



```
# %matplotlib widget

decomposed = decomposed_m

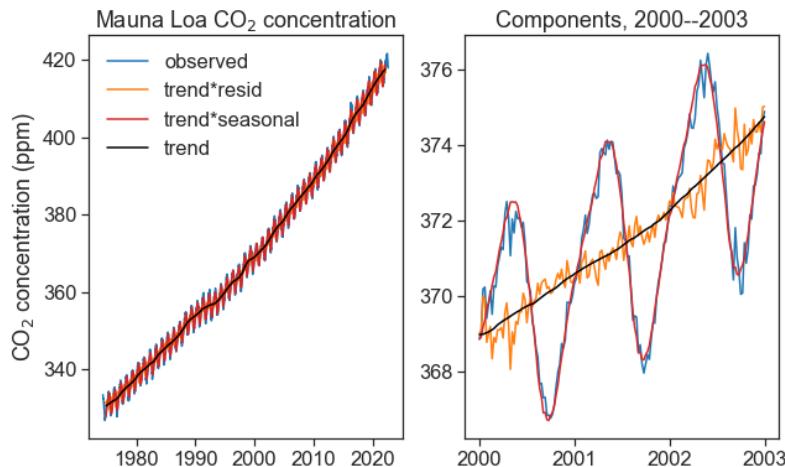
fig, ax = plt.subplots(1, 2, figsize=(10,6))
```

```

ax[0].plot(df['co2'], color="tab:blue", label="observed")
ax[0].plot(decomposed.trend * decomposed.resid, color="tab:orange", label="trend*resid")
ax[0].plot(decomposed.trend * decomposed.seasonal, color="tab:red", label="trend*seasonal")
ax[0].plot(decomposed.trend, color="black", label="trend")
ax[0].set(ylabel="CO2 concentration (ppm)",
           title="Mauna Loa CO2 concentration")
ax[0].legend(frameon=False)

start = "2000-01-01"
end = "2003-01-01"
zoom = slice(start, end)
ax[1].plot(df.loc[zoom, 'co2'], color="tab:blue", label="observed")
ax[1].plot((decomposed.trend * decomposed.resid)[zoom], color="tab:orange", label="trend*resid")
ax[1].plot((decomposed.trend * decomposed.seasonal)[zoom], color="tab:red", label="trend*seasonal")
ax[1].plot(decomposed.trend[zoom], color="black", label="trend")
date_form = DateFormatter("%Y")
ax[1].xaxis.set_major_formatter(date_form)
ax[1].xaxis.set_major_locator(mdates.YearLocator(1))
ax[1].set_title("Components, 2000--2003");

```



36 Hilbert transform

Part X

rates of change

37 motivation

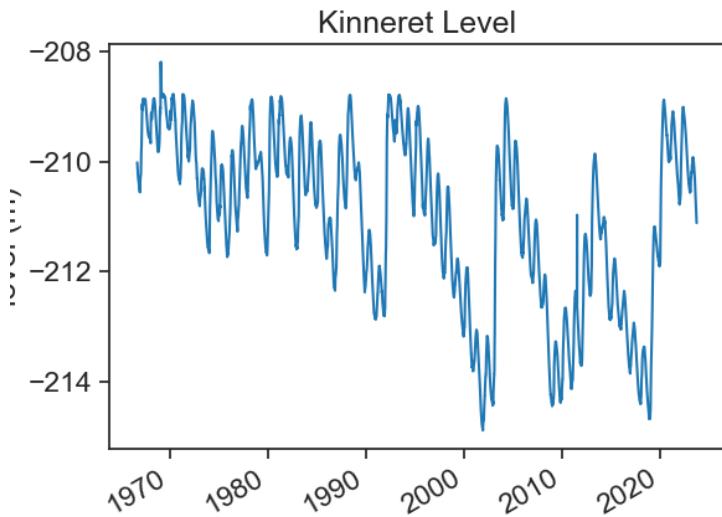
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters
%matplotlib widget
```

```
filename = "../archive/data/kinneret_cleaned.csv"
df = pd.read_csv(filename)
df['date'] = pd.to_datetime(df['date'], dayfirst=True)
df = df.set_index('date')
df
```

	level
date	
2023-09-12	-211.115
2023-09-11	-211.105
2023-09-10	-211.095
2023-09-09	-211.085
2023-09-08	-211.070
...	...
1966-11-01	-210.390
1966-10-15	-210.320
1966-10-01	-210.270
1966-09-15	-210.130
1966-09-01	-210.020

```
fig, ax = plt.subplots()
ax.plot(df['level'], color="tab:blue")
ax.set(title="Kinneret Level",
```

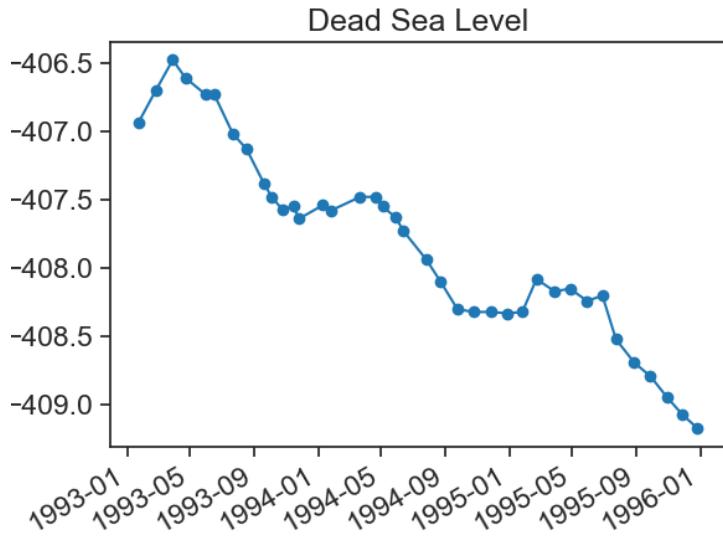
```
        ylabel="level (m)")
plt.gcf().autofmt_xdate() # makes slanted dates
```



The data seems ok, until we take a closer look. Data points are not evenly spaced in time.

```
fig, ax = plt.subplots()
ax.plot(df.loc["1993":"1995", 'level'], color="tab:blue", marker="o")
ax.set(title="Dead Sea Level",
       ylabel="level (m)")
plt.gcf().autofmt_xdate() # makes slanted dates
```

```
/var/folders/c3/7hp0d36n6vv8jc9hm2440__00000gn/T/ipykernel_3777/934261896.py:2: FutureWarning:
  ax.plot(df.loc["1993":"1995", 'level'], color="tab:blue", marker="o")
```



We can resample by day (a much higher rate than the original), and linearly interpolate:

```
df2 = df['level'].resample('D').interpolate('time').to_frame()
df2['level_sm'] = df2['level'].rolling('30D', center=True).mean()
df3 = df2['level'].resample('W').mean().to_frame()

fig, ax = plt.subplots()
ax.plot(df2.loc["1993":"1995", 'level_sm'],
        color="tab:red",
        label="daily resampled")
ax.plot(df3.loc["1993":"1995", 'level'],
        color="black",
        label="daily resampled")
ax.plot(df2.loc["1993":"1995", 'level'],
        color="tab:orange",
        label="daily resampled")
ax.plot(df.loc["1993":"1995", 'level'],
        color="tab:blue",
        marker="o",
        linestyle="None",
```

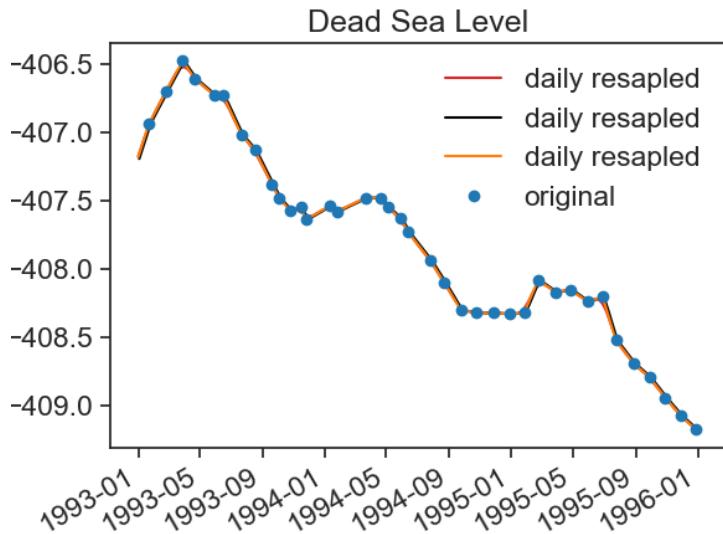
```

        label="original")
ax.set(title="Dead Sea Level",
       ylabel="level (m)")
plt.gcf().autofmt_xdate() # makes slanted dates
ax.legend(frameon=False)

/var/folders/c3/7hp0d36n6vv8jc9hm2440__00000gn/T/ipykernel_3777/2583247388.py:11: FutureWarning:
ax.plot(df.loc["1993":"1995", 'level'],

<matplotlib.legend.Legend at 0x7fa71e8b0bb0>

```



```

df2['naive'] = df2['level'].diff()
df2['gradient'] = np.gradient(df2['level'])

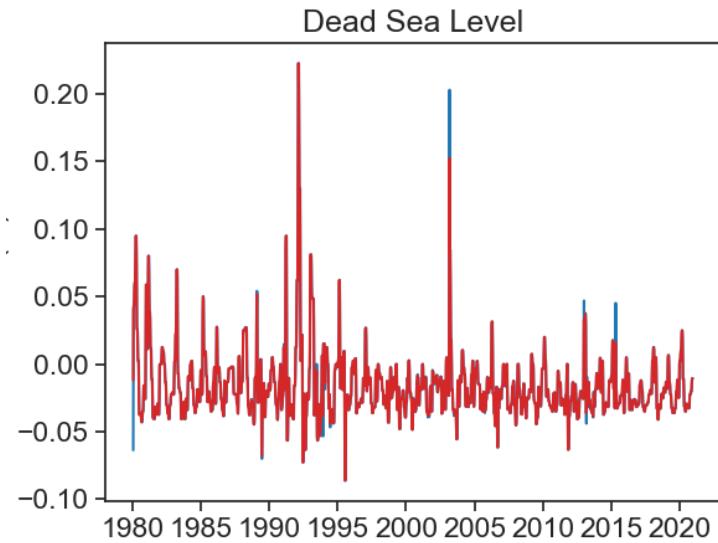
df3['naive'] = df3['level'].diff()
df3['gradient'] = np.gradient(df3['level'])

fig, ax = plt.subplots()
ax.plot(df3.loc["1980":"2020", 'naive'], color="tab:blue")
ax.plot(df3.loc["1980":"2020", 'gradient'], color="tab:red")

```

```
    ax.set(title="Dead Sea Level",
          ylabel="level (m)")

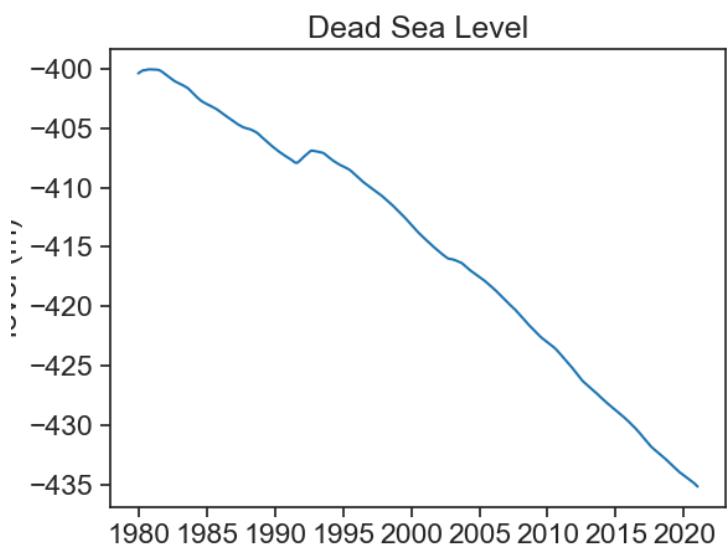
[Text(0.5, 1.0, 'Dead Sea Level'), Text(0, 0.5, 'level (m)')]
```



```
df3 = df2["level"].rolling('365.24D', center=True).mean().to_frame()

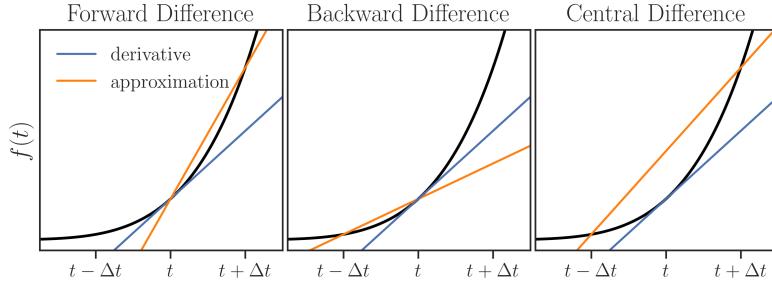
fig, ax = plt.subplots()
ax.plot(df3.loc["1980":"2020", 'level'], color="tab:blue")
ax.set(title="Dead Sea Level",
       ylabel="level (m)")

[Text(0.5, 1.0, 'Dead Sea Level'), Text(0, 0.5, 'level (m)')]
```



38 derivatives

39 finite differences



Definition of a **derivative**:

$$\underbrace{\dot{f} = f'(t) = \frac{df(t)}{dt}}_{\text{same thing}} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

Numerically, we can approximate the derivative $f'(t)$ of a time series $f(t)$ as

$$\frac{df(t)}{dt} = \frac{f(t + \Delta t) - f(t)}{\Delta t} + \mathcal{O}(\Delta t). \quad (39.1)$$

The expression above is called the *two-point forward difference formula*. Likewise, we can define the *two-point backward difference formula*:

$$\frac{df(t)}{dt} = \frac{f(t) - f(t - \Delta t)}{\Delta t} + \mathcal{O}(\Delta t). \quad (39.2)$$

If we sum together Equation 39.1 and Equation 39.2 we get:

$$\begin{aligned} 2 \frac{df(t)}{dt} &= \frac{f(t + \Delta t) - f(t)}{\Delta t} + \frac{f(t) - f(t - \Delta t)}{\Delta t} \\ &= \frac{f(t + \Delta t) - f(t - \Delta t)}{\Delta t}. \end{aligned} \quad (39.3)$$

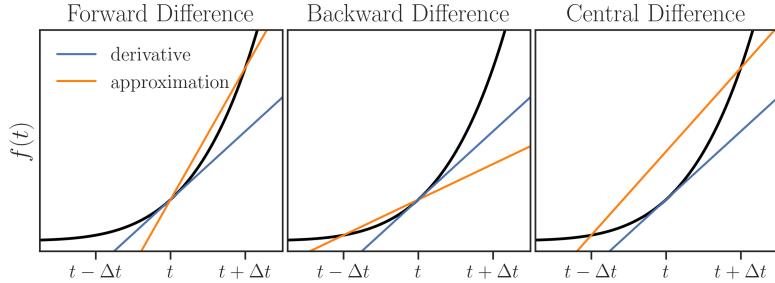
The expression $\mathcal{O}(\Delta t)$ means that the error associated with the approximation is proportional to Δt . This is called “Big O notation”.

Dividing both sides by 2 gives the *two-point central difference formula*:

$$\frac{df(t)}{dt} = \frac{f(t + \Delta t) - f(t - \Delta t)}{2\Delta t} + \mathcal{O}(\Delta t^2). \quad (39.4)$$

Two things are worth mentioning about the approximation above:

1. it is balanced, that is, there is no preference of the future over the past.
2. its error is proportional to Δt^2 , it is a lot more precise than the unbalanced approximations :)



The function `np.gradient` calculates the derivative using the central difference for points in the interior of the array, and uses the forward (backward) difference for the derivative at the beginning (end) of the array.

Check out this [nice example](#).

To understand why the error is proportional to Δt^2 , one can subtract the Taylor expansion of $f(t - \Delta t)$ from the Taylor expansion of $f(t + \Delta t)$. See this, pages 3 and 4.

The “gradient” usually refers to a first derivative with respect to space, and it is denoted as $\nabla f(x) = \frac{df(x)}{dx}$. However, it doesn’t really matter if we call the independent variable x or t , the derivative operator is exactly the same.

40 Fourier-based derivatives

This tutorial is based on Pelliccia (2019).

nice trick: <https://math.stackexchange.com/questions/430858/fourier-transform-of-derivative>

41 LOESS-based derivatives

Part XI

forecasting

42 motivation

43 ARIMA

Part XII

assignments

44 assignment 1

This assignment comes right after the first session, where we discussed resampling. Read the whole instructions.

44.1 task

Go to the [IMS website](#), and choose another weather station we have not worked with yet. Download 10-minute data for a full year, any year.

Make 3 graphs:

1. Daily maximum humidity. Bonus: add another line to the graph, the daily minimum humidity.
2. The number of rainy days for each month
3. For each day of the year, show the number of hours when global solar radiation was above, on average, the threshold 10 W/m^2 . Now add another line, for the threshold 500 W/m^2 .

Make 5 more graphs (total of 8 graphs) of whatever you find interesting. You have the liberty to explore various facets of your dataset that capture your interest. It's essential, however, to maintain a focus on resampling. Each of your plots should effectively showcase and emphasize different aspects or techniques of resampling in your data analysis. To ensure diversity in your visualizations, avoid repetitive themes; for instance, if your first plot illustrates daily wind speed, then your second plot should not simply be a monthly resampling of wind speed. Aim for variety and innovation in each plot to fully explore the potential of resampling in data visualization.

You must download this Jupyter Notebook template. Create a zip file with your Jupyter notebook and with the `.csv` you used. Upload this zip file to the moodle task we created.

44.2 guidelines

1. Always name the axes and add units when relevant.
2. Always give a title to the plot.
3. Make sure that all axis tick labels (the numbers/dates on the axes) are readable.
4. Include a legend if you have multiple lines, colors, or groups.
5. Use appropriate scales for the axes (linear, logarithmic, etc.) depending on the data's nature.
6. Ensure that the plot is adequately sized for all elements to be clear and visible.
7. Choose colors and markers that are distinguishable, especially for plots with multiple elements.
8. If applicable, include error bars to indicate the variability or uncertainty in the data.
9. Use grid lines sparingly; they should not overshadow the data.

44.3 evaluation

All your assignments will be evaluated according to the following criteria:

- Presentation. How the graphs look, labels, general organization, markdown, clean code.
- Discussion. This is where you explain what you did, what you found out, etc.
- Depth of analysis. You can analyze/explore the data with different levels of complexity, this is where we take that into consideration.
- Replicability: Your code runs flawlessly.
- Code commenting. Explain in your code what you are doing, this is good for everyone, especially for yourself!

- Bonus: for originality, creative problem solving, or notable analysis.

45 assignment 2

45.1 Smoothing

In this assignment, you will delve into the application of different smoothing techniques on time series data. Utilizing meteorological data, your task is to create a series of plots that demonstrate the effects of various smoothing methods.

45.1.1 1. Comparative Smoothing Methods Analysis

- **Goal:** Showcase three smoothing techniques – Rolling Average, Savitzky-Golay, and Resampling – on the same time series data.
- **Task:** Overlay these methods over the actual data in a single plot. Ensure each method uses the same window size for consistency. **Describe in a few lines the differences you see.**

```
# code goes here
```

45.1.2 2. Rolling Average Window Size Impact

- **Goal:** Analyze the effect of varying window sizes on the Rolling Average method.
- **Task:** Produce a plot with three lines, each representing the Rolling Average with a different window size. **Describe in a few lines the differences you see.**

```
# code goes here
```

45.1.3 3. Savitzky-Golay Polynomial Order Variation

- **Goal:** Investigate how changing the polynomial order affects the Savitzky-Golay smoothing method.
- **Task:** Create a plot with three lines, where each represents the Savitzky-Golay method with a different polynomial order. **Describe in a few lines the differences you see.**

```
# code goes here
```

45.1.4 4. Kernel Shape Influence in Rolling Mean

- **Goal:** Explore the impact of different kernel shapes on the Rolling Mean.
- **Task:** Generate a plot displaying three lines, each using a different kernel shape in the Rolling Mean. We encourage to use unique kernel shapes that we did not showcase in class. See [this list](#) of kernels. **Describe in a few lines the differences you see.**

```
# code goes here
```

45.1.5 5. Moving Average with Confidence Interval

- **Goal:** Plot a Moving Average along with a 75% confidence interval.
- **Task:** Design a plot illustrating both the Moving Average and its 75% confidence interval.

```
# code goes here
```

Part XIII

technical stuff

46 technical stuff

I recommend working with UNIX-based operating systems (MacOS or Linux). Everything is easier.

If you use Windows, consider [installing Linux on Windows with WSL](#).

46.1 software

[Anaconda's Python distribution](#)

[VSCode](#)

46.2 python packages

[Kats — a one-stop shop for time series analysis](#)

Developed by Meta

[statsmodels](#) statsmodels is a Python package that provides a complement to scipy for statistical computations including descriptive statistics and estimation and inference for statistical models.

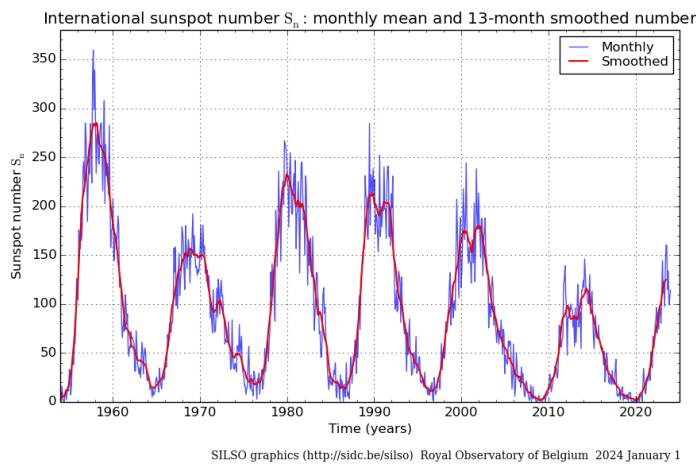
[ydata-profiling](#)

Quick Exploratory Data Analysis on time-series data. [Read also this.](#)

47 datasets

where to find data?

The solar cycle produces varying amounts of sunspots throughout the years.



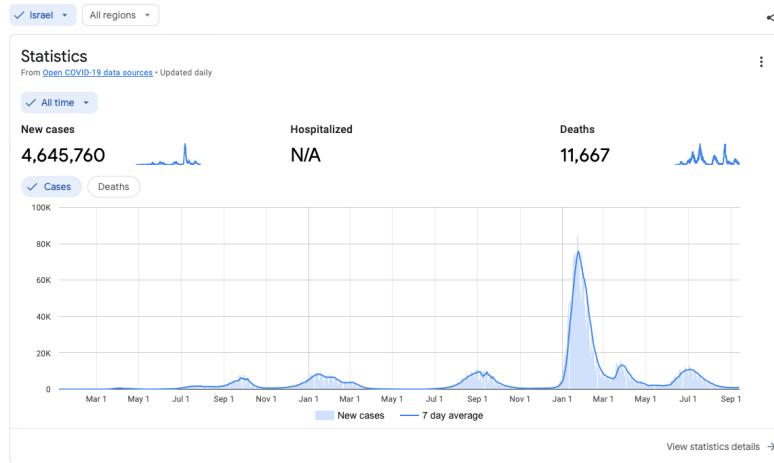
[Download data](#) from the Royal Observatory of Belgium.

Source: <https://www.sidc.be/SILSO/monthlyssnplot>

47.1 Covid-19 Open Data

Download the data into your own tools and systems to analyze the virus's spread or decline, investigate COVID-related deaths, study the effects of different vaccines, and more in 20,000-plus locations worldwide.

Data visualizer



[Click here](#) to go to the download page. Choose desired region under section “Understanding the data”.

Source:

<https://health.google.com/covid-19/open-data/explorer>

Part XIV

behind-the-scenes

sliding window video

Import packages and stuff.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters() # datetime converter for a matplotlib
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import scipy as sp
import json
import requests
import os
import subprocess
from tqdm import tqdm
from scipy import signal

# avoid "SettingWithCopyWarning: A value is trying to be set on a copy of a slice from a Dat
pd.options.mode.chained_assignment = None # default='warn'
```

Download data from the [IMS](#) using an API.

```
# read token from file
with open('../archive/IMS-token.txt', 'r') as file:
    TOKEN = file.readline()
# 28 = SHANI station
STATION_NUM = 28
start = "2022/01/01"
end = "2022/01/07"
```

```

filename = 'shani_2022_january.json'

# check if the JSON file already exists
# if so, then load file
if os.path.exists(filename):
    with open(filename, 'r') as json_file:
        data = json.load(json_file)
else:
    # make the API request if the file doesn't exist
    url = f"https://api.ims.gov.il/v1/envista/stations/{STATION_NUM}/data/?from={start}&to={end}"
    headers = {'Authorization': f'ApiToken {TOKEN}'}
    response = requests.get(url, headers=headers)
    data = json.loads(response.text.encode('utf8'))

    # save the JSON data to a file
    with open(filename, 'w') as json_file:
        json.dump(data, json_file)
# show data to see if it's alright
# data

```

Load and process data.

```

df = pd.json_normalize(data['data'], record_path=['channels'], meta=['datetime'])
df['date'] = (pd.to_datetime(df['datetime'])
              .dt.tz_localize(None) # ignores time zone information
              )
df = df.pivot(index='date', columns='name', values='value')
df

```

name date	Grad	RH	Rain	STDwd	TD	TDmax	TDmin	TG	TW	Time	WD	...
2022-01-01 00:00:00	0.0	77.0	0.0	10.3	11.2	11.2	11.1	10.7	-9999.0	2354.0	75.0	0
2022-01-01 00:10:00	0.0	77.0	0.0	11.2	11.2	11.2	11.1	10.8	-9999.0	1.0	77.0	8
2022-01-01 00:20:00	0.0	75.0	0.0	10.0	11.4	11.5	11.2	10.9	-9999.0	20.0	80.0	8
2022-01-01 00:30:00	0.0	74.0	0.0	9.6	11.5	11.5	11.4	11.0	-9999.0	22.0	76.0	7
2022-01-01 00:40:00	0.0	73.0	0.0	9.1	11.6	11.7	11.5	11.1	-9999.0	34.0	74.0	6
...
2022-01-06 23:10:00	0.0	36.0	0.0	16.1	11.6	12.0	11.1	6.8	-9999.0	2310.0	144.0	1
2022-01-06 23:20:00	0.0	35.0	0.0	10.1	12.1	12.3	11.9	6.3	-9999.0	2320.0	118.0	1
2022-01-06 23:30:00	0.0	36.0	0.0	7.1	12.4	12.6	11.9	7.3	-9999.0	2330.0	113.0	1

name	Grad	RH	Rain	STDwd	TD	TDmax	TDmin	TG	TW	Time	WD	Y
date												
2022-01-06 23:40:00	0.0	37.0	0.0	5.6	12.6	12.7	12.5	7.8	-9999.0	2339.0	119.0	1
2022-01-06 23:50:00	0.0	39.0	0.0	11.5	11.9	12.6	11.5	7.1	-9999.0	2341.0	102.0	1

Define useful functions.

```

def concise(ax):
    """
    Let python choose the best xtick labels for you
    """
    locator = mdates.AutoDateLocator(minticks=3, maxticks=7)
    formatter = mdates.ConciseDateFormatter(locator)
    ax.xaxis.set_major_locator(locator)
    ax.xaxis.set_major_formatter(formatter)

    # dirty trick to have dates in the middle of the 24-hour period
    # make minor ticks in the middle, put the labels there!
    # from https://matplotlib.org/stable/gallery/ticks/centered_ticklabels.html

def center_dates(ax):
    # show day of the month + month abbreviation. see full option list here:
    # https://strftime.org
    date_form = DateFormatter("%d %b")
    # major ticks at midnight, every day
    ax.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax.xaxis.set_major_formatter(date_form)
    # minor ticks at noon, every day
    ax.xaxis.set_minor_locator(mdates.HourLocator(byhour=[12]))
    # erase major tick labels
    ax.xaxis.set_major_formatter(ticker.NullFormatter())
    # set minor tick labels as define above
    ax.xaxis.set_minor_formatter(date_form)
    # completely erase minor ticks, center tick labels
    for tick in ax.xaxis.get_minor_ticks():
        tick.tick1line.set_markersize(0)
        tick.tick2line.set_markersize(0)
        tick.label1.set_horizontalalignment('center')

```

```

def center_dates_two_panels(ax0, ax1):
    # show day of the month + month abbreviation. see full option list here:
    date_form = DateFormatter("%d %b")
    # major ticks at midnight, every day
    ax0.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax1.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax1.xaxis.set_major_formatter(date_form)
    # minor ticks at noon, every day
    ax1.xaxis.set_minor_locator(mdates.HourLocator(byhour=[12]))
    # erase major tick labels
    ax1.xaxis.set_major_formatter(ticker.NullFormatter())
    # set minor tick labels as define above
    ax1.xaxis.set_minor_formatter(date_form)
    # completely erase minor ticks, center tick labels
    for tick in ax0.xaxis.get_minor_ticks():
        tick.tick1line.set_markersize(0)
        tick.tick2line.set_markersize(0)
    for tick in ax1.xaxis.get_minor_ticks():
        tick.tick1line.set_markersize(0)
        tick.tick2line.set_markersize(0)
        tick.label1.set_horizontalalignment('center')

```

We don't need the full month, let's cut the dataframe to fewer days.

```

start = "2022-01-01 00:00:00"
end = "2022-01-06 23:50:00"
df = df.loc[start:end]

```

We now redefine a narrower window, this will be the graph's xlims. We leave the dataframe as is, because we will need some data outside the graph's limits.

```

start = "2022-01-02 00:00:00"
end = "2022-01-05 23:50:00"

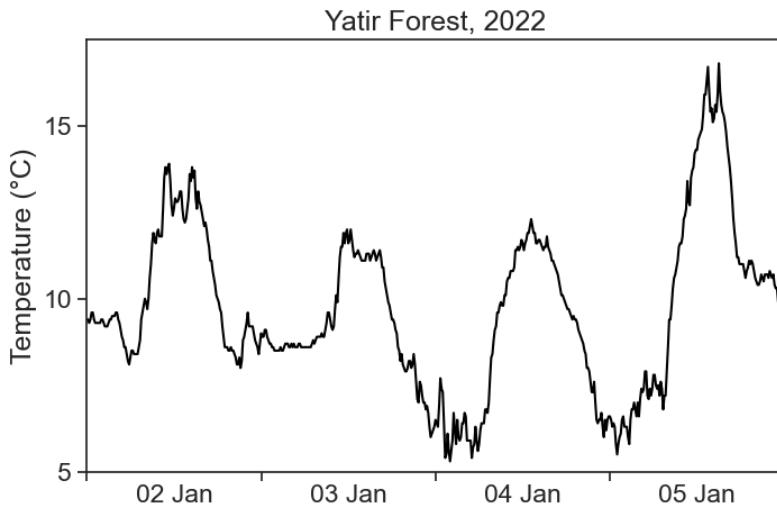
fig, ax = plt.subplots(figsize=(8,5))
ax.plot(df.loc[start:end, 'TD'], color='black')
ax.set(ylim=[5, 17.5],

```

```

        xlim=[start, end],
        ylabel="Temperature (°C)",
        title="Yatir Forest, 2022",
        yticks=[5,10,15])
center_dates(ax)
fig.savefig("sliding_YF_temperature_2022.png")

```



Looks good. Let's move on.

Rectangular kernel

```

%matplotlib widget
fig, ax = plt.subplots(2, 1, figsize=(8,5), sharex=True,
                      gridspec_kw={'height_ratios':[1,0.4], 'hspace':0.1})

class Lines:
    """
    empty class, later will be populated with graph objects.
    this is useful to draw and erase lines on demand.
    """
    pass
lines = Lines()

```

```

# rename axes for convenience
ax0 = ax[0]
ax1 = ax[1]
# sm = df['TD'].rolling(10, center=True).mean()
# ga = df['TD'].rolling(10, center=True, win_type="gaussian").mean(std=100.0)

# set graph y limits
ylim = [3, 22]
# choose here window width in minutes
window_width_min = 200.0
window_width_min_integer = int(window_width_min) # same but integer
window_width_int = int(window_width_min // 10 + 1) # window width in points
N = len(df) # df length
# time range over which the kernel will slide
# starts at "start", minus the width of the window,
# minus half an hour, so that the window doesn't start sliding right away at the beginning of
# ends an hour after the window has finished sliding
t_swipe = pd.date_range(start=pd.to_datetime(start) - pd.Timedelta(minutes=window_width_min),
                        end=pd.to_datetime(end) + pd.Timedelta(minutes=60),
                        freq="10min")
# starting time
t0 = t_swipe[0]
# show sliding window on the top panel as a light blue shade
lines.fill_bet = ax0.fill_between([t0, t0 + pd.Timedelta(minutes=window_width_min)],
                                  y1=ylim[0], y2=ylim[1], alpha=0.1, zorder=-1)
# this is our "boxcart" kernel (a rectangle)
kernel_rect = np.ones(window_width_int)
# calculate the moving average with "kernel_rect" as weights
# this is the same as a convolution, which is just faster to compute
df.loc[:, 'con'] = np.convolve(df['TD'].values, kernel_rect, mode='same') / len(kernel_rect)
# create a new column for the kernel, fill it with zeros
df['kernel_plus'] = 0.0
# populate the kernel column with the window at the very beginning
df.loc[t0: t0 + pd.Timedelta(minutes=window_width_min), 'kernel_plus'] = kernel_rect
# plot kernel on the bottom panel
lines.kernel_line, = ax1.plot(df['kernel_plus'])
# plot temperature on the top panel
ax0.plot(df.loc[start:end, 'TD'], color="black")
# make temperature look gray when inside the sliding window

```

```

lines.gray_line, = ax0.plot(df.loc[df['kernel_plus']==1.0, 'TD'],
                           color=[0.6]*3, lw=3)
# calculate the middle of the sliding window
window_middle = t0 + pd.Timedelta(minutes=window_width_min/2)
# plot a pink line showing the result of the moving average
# from the beginning to the middle of the sliding window
lines.pink_line, = ax0.plot(df.loc[start:window_middle, 'con'], color="xkcd:hot pink", lw=3)
# emphasize the location of the middle on the window with a circle
lines.pink_circle, = ax0.plot([window_middle], [df.loc[window_middle, 'con']],
                             marker='o', markerfacecolor="None", markeredgecolor="xkcd:dark pink", markeredgewid
                             markersize=8)
# some explanation
ax0.text(0.99, 0.97, f"kernel: boxcar (rectangle)\nwidth = {window_width_min:.0f} minutes",
         horizontalalignment='right', verticalalignment='top',
         fontsize=14)
# axis tweaking
ax0.set(ylim=ylim,
        xlim=[start, end],
        ylabel="Temperature (°C)",
        yticks=[5,10,15,20],
        title="Yatir Forest, 2022")
ax1.set(ylim=[-0.2, 1.2],
        xlim=[start, end],
        ylabel="kernel"
        )
# adjust dates on both panels as defined before
center_dates_two_panels(ax0, ax1)

def update_swipe(k, lines):
    """
    updates both panels, given the index k along which the window is sliding
    """
    # left side of the sliding window
    t0 = t_swipe[k]
    # middle position
    window_middle = t0 + pd.Timedelta(minutes=window_width_min/2)
    # erase previous blue shade on the top graph
    lines.fill_bet.remove()
    # fill again the blue shade in the updated window position

```

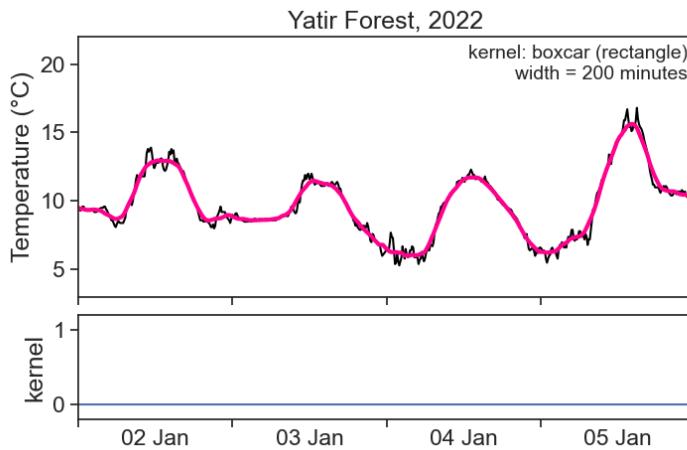
```

lines.fill_bet = ax0.fill_between([t0, t0 + pd.Timedelta(minutes=window_width_min)],
                                 y1=ylim[0], y2=ylim[1], alpha=0.1, zorder=-1,
                                 # update pink curve
                                 lines.pink_line.set_data(df[start:window_middle].index,
                                                          df.loc[start:window_middle, 'con'].values)
                                 # update pink circle
                                 lines.pink_circle.set_data([window_middle], [df.loc[window_middle, 'con']])
                                 # update the kernel in its current position
                                 lines.kernel_rect = np.ones(window_width_int)
                                 df.loc[:, 'kernel_plus'] = 0.0
                                 df.loc[t0: t0 + pd.Timedelta(minutes=window_width_min), 'kernel_plus'] = kernel_rect
                                 # update gray line
                                 lines.gray_line.set_data(df.loc[df['kernel_plus']==1.0, 'TD'].index,
                                                          df.loc[df['kernel_plus']==1.0, 'TD'].values)
                                 # update kernel line
                                 lines.kernel_line.set_data(df['kernel_plus'].index, df['kernel_plus'].values)

# create a tqdm progress bar
progress_bar = tqdm(total=len(t_swipe), unit="iteration")
# loop over all sliding indices, update graph and then save it
for fignum, i in enumerate(np.arange(0, len(t_swipe)-1, 1)):
    update_swipe(i, lines)
    fig.savefig(f"pngs/boxcar{window_width_min_integer}/boxcar_{window_width_min_integer}min"
    # update the progress bar
    progress_bar.update(1)
# close the progress bar
progress_bar.close()

```

100% | 604/605 [05:27<00:00, 1.85iteration/s]



Combine all saved images into one mp4 video.

```
# Define the path to your PNG images
pngs_path = f"pngs/boxcar{window_width_min_integer}"
pngs_name = f"boxcar_{window_width_min_integer}min_%03d.png"

# Define the output video file path
video_output = f"output{window_width_min_integer}.mp4"

# Use ffmpeg to create a video from PNG images
# desired framerate. choose 24 if you don't know what to do
fr = 12
# run command
ffmpeg_cmd = f"ffmpeg -framerate {fr} -i {pngs_path}/{pngs_name} -c:v libx264 -vf fps={fr} {subprocess.run(ffmpeg_cmd, shell=True)}
```

```
ffmpeg version 6.0 Copyright (c) 2000-2023 the FFmpeg developers
built with Apple clang version 14.0.3 (clang-1403.0.22.14.1)
configuration: --prefix=/usr/local/Cellar/ffmpeg/6.0 --enable-shared --enable-pthreads --enable
libavutil      58. 2.100 / 58. 2.100
libavcodec     60. 3.100 / 60. 3.100
libavformat    60. 3.100 / 60. 3.100
libavdevice    60. 1.100 / 60. 1.100
libavfilter     9. 3.100 /  9. 3.100
libswscale      7. 1.100 /   7. 1.100
```

```

libswresample 4. 10.100 / 4. 10.100
libpostproc 57. 1.100 / 57. 1.100
Input #0, image2, from 'pngs/boxcar200/boxcar_200min_%03d.png':
  Duration: 00:00:50.33, start: 0.000000, bitrate: N/A
  Stream #0:0: Video: png, rgba(pc), 4800x3000 [SAR 23622:23622 DAR 8:5], 12 fps, 12 tbr, 12 t
Stream mapping:
  Stream #0:0 -> #0:0 (png (native) -> h264 (libx264))
Press [q] to stop, [?] for help
[libx264 @ 0x7fa027f2e300] using SAR=1/1
[libx264 @ 0x7fa027f2e300] using cpu capabilities: MMX2 SSE2Fast SSSE3 SSE4.2 AVX FMA3 BMI2 AVX2
[libx264 @ 0x7fa027f2e300] profile High 4:4:4 Predictive, level 6.0, 4:4:4, 8-bit
[libx264 @ 0x7fa027f2e300] 264 - core 164 r3095 baee400 - H.264/MPEG-4 AVC codec - Copyleft 200
Output #0, mp4, to 'output200.mp4':
  Metadata:
    encoder : Lavf60.3.100
  Stream #0:0: Video: h264 (avc1 / 0x31637661), yuv444p(tv, progressive), 4800x3000 [SAR 1:1 D
    Metadata:
      encoder : Lavc60.3.100 libx264
    Side data:
      cpb: bitrate max/min/avg: 0/0/0 buffer size: 0 vbv_delay: N/A
frame= 604 fps= 23 q=-1.0 Lsize= 1412kB time=00:00:50.08 bitrate= 231.0kbit/s speed=1.91x
video:1404kB audio:0kB subtitle:0kB other streams:0kB global headers:0kB muxing overhead: 0.56%
[libx264 @ 0x7fa027f2e300] frame I:3 Avg QP: 9.98 size:135751
[libx264 @ 0x7fa027f2e300] frame P:154 Avg QP:14.75 size: 2507
[libx264 @ 0x7fa027f2e300] frame B:447 Avg QP:22.66 size: 1440
[libx264 @ 0x7fa027f2e300] consecutive B-frames: 1.0% 0.7% 1.0% 97.4%
[libx264 @ 0x7fa027f2e300] mb I I16..4: 55.5% 38.8% 5.7%
[libx264 @ 0x7fa027f2e300] mb P I16..4: 0.4% 0.3% 0.0% P16..4: 0.2% 0.1% 0.0% 0.0% 0.0%
[libx264 @ 0x7fa027f2e300] mb B I16..4: 0.1% 0.0% 0.0% B16..8: 1.0% 0.2% 0.0% direct: 0.0%
[libx264 @ 0x7fa027f2e300] 8x8 transform intra:37.1% inter:49.0%
[libx264 @ 0x7fa027f2e300] coded y,u,v intra: 3.6% 0.4% 0.6% inter: 0.1% 0.0% 0.0%
[libx264 @ 0x7fa027f2e300] i16 v,h,dc,p: 90% 10% 0% 0%
[libx264 @ 0x7fa027f2e300] i8 v,h,dc,ddl,ddr,vr,hd,vl,hu: 41% 3% 56% 0% 0% 0% 0% 0% 0%
[libx264 @ 0x7fa027f2e300] i4 v,h,dc,ddl,ddr,vr,hd,vl,hu: 51% 14% 20% 3% 2% 3% 2% 3% 2%
[libx264 @ 0x7fa027f2e300] Weighted P-Frames: Y:0.0% UV:0.0%
[libx264 @ 0x7fa027f2e300] ref P L0: 56.6% 3.8% 28.4% 11.3%
[libx264 @ 0x7fa027f2e300] ref B L0: 85.6% 13.4% 1.0%
[libx264 @ 0x7fa027f2e300] ref B L1: 95.9% 4.1%
[libx264 @ 0x7fa027f2e300] kb/s:228.44

```

```
CompletedProcess(args='ffmpeg -framerate 12 -i pngs/boxcar200/boxcar_200min_%03d.png -c:v libx264 -b:a 128k -t 50 -pix_fmt yuv444p output200.mp4')
```

The following code does exactly as you see above, but it is not well commented. You are an intelligent person, you'll figure this out.

Triangular kernel

```
%matplotlib widget
fig, ax = plt.subplots(2, 1, figsize=(8,5), sharex=True,
                      gridspec_kw={'height_ratios':[1,0.4], 'hspace':0.1})

class Lines:
    pass
lines = Lines()

ax0 = ax[0]
ax1 = ax[1]
ylim = [3, 22]
window_width_min = 500.0
window_width_int = int(window_width_min / 10) + 1
N = len(df)
t_swipe = pd.date_range(start=pd.to_datetime(start) - pd.Timedelta(minutes=window_width_min),
                        end=pd.to_datetime(end) + pd.Timedelta(minutes=60),
                        freq="10min")
t0 = t_swipe[200]
window_middle = t0 + pd.Timedelta(minutes=window_width_min/2)
# fill between blue shade, plot kernel
lines.fill_bet = ax0.fill_between([t0, t0 + pd.Timedelta(minutes=window_width_min)],
                                  y1=ylim[0], y2=ylim[1], alpha=0.1, zorder=-1)
half_triang = np.arange(1, window_width_int/2+1, 1)
kernel_triang = np.hstack([half_triang, half_triang[-2::-1]])
kernel_triang = kernel_triang / kernel_triang.max()
df.loc[:, 'con'] = np.convolve(df['TD'].values, kernel_triang, mode='same') / len(kernel_triang)
df['kernel_plus'] = 0.0
df.loc[t0: t0 + pd.Timedelta(minutes=window_width_min), 'kernel_plus'] = kernel_triang
lines.kernel_line, = ax1.plot(df['kernel_plus'], color="tab:blue")
ax0.plot(df.loc[start:end, 'TD'], color="black")
lines.gray_line, = ax0.plot(df.loc[df['kernel_plus']!=0.0, 'TD'],
                           color=[0.6]*3, lw=3)
```

```

lines.pink_line, = ax0.plot(df.loc[start>window_middle, 'con'], color="xkcd:hot pink", lw=3)
lines.pink_circle, = ax0.plot([window_middle], [df.loc[window_middle, 'con']], 
                             marker='o', markerfacecolor="None", markeredgecolor="xkcd:dark pink", markeredgewid
                             markersize=8)
ax0.text(0.99, 0.97, f"kernel: triangle\nwidth = {window_width_min:.0f} minutes", transform=
           horizontalalignment='right', verticalalignment='top',
           fontsize=14)
ax0.set(ylim=ylim,
        xlim=[start, end],
        ylabel="Temperature (°C)",
        yticks=[5,10,15,20],
        title="Yatir Forest, 2022")
ax1.set(ylim=[-0.2, 1.2],
        xlim=[start, end],
        ylabel="kernel"
       )
center_dates_two_panels(ax0, ax1)

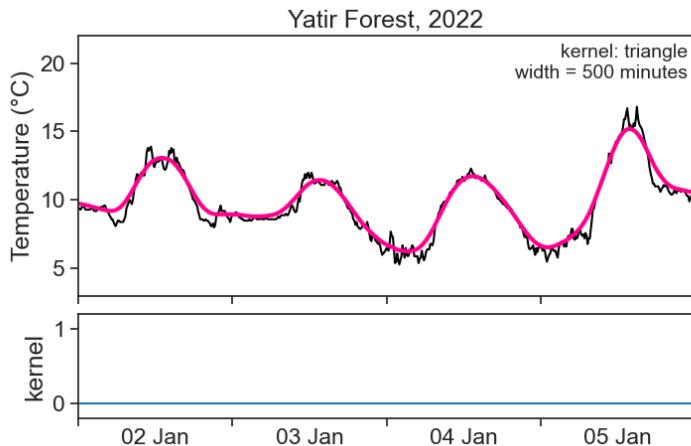
def update_swipe(k, lines):
    t0 = t_swipe[k]
    window_middle = t0 + pd.Timedelta(minutes=window_width_min/2)
    lines.fill_bet.remove()
    lines.fill_bet = ax0.fill_between([t0, t0 + pd.Timedelta(minutes=window_width_min)],
                                      y1=ylim[0], y2=ylim[1], alpha=0.1, zorder=-1,
    lines.pink_line.set_data(df[start>window_middle].index,
                            df.loc[start>window_middle, 'con'].values)
    lines.pink_circle.set_data([window_middle], [df.loc[window_middle, 'con']])
    lines.kernel_rect = np.ones(window_width_int)
    df['kernel_plus'] = 0.0
    df.loc[t0: t0 + pd.Timedelta(minutes=window_width_min), 'kernel_plus'] = kernel_triang
    lines.gray_line.set_data(df.loc[df['kernel_plus']!=0.0,'TD'].index,
                            df.loc[df['kernel_plus']!=0.0,'TD'].values)
    lines.kernel_line.set_data(df['kernel_plus'].index, df['kernel_plus'].values)

    progress_bar = tqdm(total=len(t_swipe), unit="iteration")
    for fignum, i in enumerate(np.arange(0, len(t_swipe)-1, 1)):
        update_swipe(i, lines)
        fig.savefig(f"pngs/triangle/triangle_{fignum:03}.png", dpi=600)
        # update the progress bar

```

```
    progress_bar.update(1)
# close the progress bar
progress_bar.close()
```

```
100%| 634/635 [05:35<00:00, 1.89iteration/s]
```



```
# Define the path to your PNG images
pngs_path = "pngs/triangle"
pngs_name = "triangle_%03d.png"

# Define the output video file path
video_output = "output_triangle.mp4"

fr = 12
# run command
ffmpeg_cmd = f"ffmpeg -framerate {fr} -i {pngs_path}/{pngs_name} -c:v libx264 -vf fps={fr} {video_output}"
subprocess.run(ffmpeg_cmd, shell=True)
```

```
ffmpeg version 6.0 Copyright (c) 2000-2023 the FFmpeg developers
built with Apple clang version 14.0.3 (clang-1403.0.22.14.1)
configuration: --prefix=/usr/local/Cellar/ffmpeg/6.0 --enable-shared --enable-pthreads --enable
libavutil      58. 2.100 / 58. 2.100
libavcodec     60. 3.100 / 60. 3.100
```



```
[libx264 @ 0x7fa9b0807f80] ref B L1: 96.6% 3.4%
[libx264 @ 0x7fa9b0807f80] kb/s:203.87
```

```
CompletedProcess(args='ffmpeg -framerate 12 -i pngs/triangle/triangle_%03d.png -c:v libx264 -v
```

Gaussian kernel

```
%matplotlib widget
fig, ax = plt.subplots(2, 1, figsize=(8,5), sharex=True,
                      gridspec_kw={'height_ratios':[1,0.4], 'hspace':0.1})

class Lines:
    pass
lines = Lines()

ax0 = ax[0]
ax1 = ax[1]
ylim = [3, 22]
window_width_min = 500.0
window_width_int = int(window_width_min / 10) + 1
N = len(df)
t_swipe = pd.date_range(start=pd.to_datetime(start) - pd.Timedelta(minutes=window_width_min),
                        end=pd.to_datetime(end) + pd.Timedelta(minutes=60),
                        freq="10min")
t0 = t_swipe[0]
window_middle = t0 + pd.Timedelta(minutes=window_width_min/2)
# fill between blue shade, plot kernel
half_triang = np.arange(1, window_width_int/2+1, 1)
kernel_triang = np.hstack([half_triang, half_triang[-2::-1]])
kernel_triang = kernel_triang / kernel_triang.max()
df['con'] = np.convolve(df['TD'].values, kernel_triang, mode='same') / len(kernel_triang) *
df['kernel_plus'] = 0.0
df.loc[t0: t0 + pd.Timedelta(minutes=window_width_min), 'kernel_plus'] = kernel_triang

# array of minutes. multiply by 10 because data is every 10 minutes

std_in_minutes = 60
g = sp.signal.gaussian(window_width_int, std_in_minutes/10)#, sym=True)
```

```

df.loc[t0: t0 + pd.Timedelta(minutes=window_width_min), 'kernel_plus'] = g
gaussian_threshold = np.exp(-2**2) # two sigmas
lines.kernel_line, = ax1.plot(df['kernel_plus'], color="tab:blue")
window_above_threshold = df.loc[df['kernel_plus'] > gaussian_threshold, 'kernel_plus'].index
lines.fill_bet = ax0.fill_between([window_above_threshold[0], window_above_threshold[-1]],
                                 y1=ylim[0], y2=ylim[1], alpha=0.1, zorder=-1, col

# gaussian convolution from here: https://stackoverflow.com/questions/27205402/pandas-rolling
df.loc[:, 'con'] = np.convolve(df['TD'].values, g/g.sum(), mode='same')
ax0.plot(df.loc[start:end, 'TD'], color="black")
lines.gray_line, = ax0.plot(df.loc>window_above_threshold[0]:window_above_threshold[-1], 'TD',
                             color=[0.6]*3, lw=3)
lines.pink_line, = ax0.plot(df.loc[start>window_middle, 'con'], color="xkcd:hot pink", lw=3)
lines.pink_circle, = ax0.plot([window_middle], [df.loc>window_middle, 'con']],
                             marker='o', markerfacecolor="None", markeredgecolor="xkcd:dark pink", markeredgewid
                             markersize=8)
ax0.text(0.99, 0.97, f"kernel: gaussian\nwidth = {window_width_min:.0f} minutes\nstd = {std_
                             horizontalalignment='right', verticalalignment='top',
                             fontsize=14)
ax0.set(ylim=ylim,
        xlim=[start, end],
        ylabel="Temperature (°C)",
        yticks=[5,10,15,20],
        title="Yatir Forest, 2022")
ax1.set(ylim=[-0.2, 1.2],
        xlim=[start, end],
        ylabel="kernel"
        )
gauss = df['TD'].rolling(window=window_width_int, center=True, win_type="gaussian").mean(std_
center_dates_two_panels(ax0, ax1)

def updateSwipe(k, lines):
    t0 = t_swipe[k]
    window_middle = t0 + pd.Timedelta(minutes=window_width_min/2)
    lines.fill_bet.remove()
    lines.pink_line.set_data(df.loc[start>window_middle].index,
                            df.loc[start>window_middle, 'con'].values)
    lines.pink_circle.set_data([window_middle], [df.loc>window_middle, 'con']])
    lines.kernel_rect = np.ones(window_width_int)

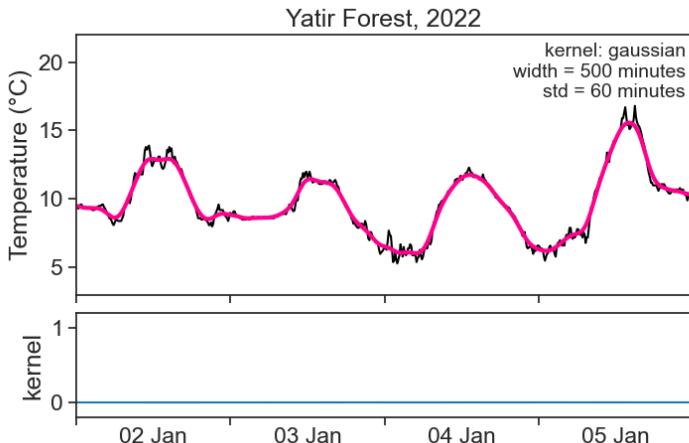
```

```

df['kernel_plus'] = 0.0
df.loc[t0: t0 + pd.Timedelta(minutes=window_width_min), 'kernel_plus'] = g
window_above_threshold = df.loc[df['kernel_plus'] > gaussian_threshold, 'kernel_plus'].index
lines.gray_line.set_data(df.loc[window_above_threshold[0]:window_above_threshold[-1], 'T'])
df.loc[window_above_threshold[0]:window_above_threshold[-1], 'T']
lines.kernel_line.set_data(df['kernel_plus'].index, df['kernel_plus'].values)
window_above_threshold = df.loc[df['kernel_plus'] > gaussian_threshold, 'kernel_plus'].index
lines.fill_bet = ax0.fill_between([window_above_threshold[0], window_above_threshold[-1],
y1=ylim[0], y2=ylim[1], alpha=0.1, zorder=-1, color="black")
progress_bar = tqdm(total=len(t_swipe), unit="iteration")
for fignum, i in enumerate(np.arange(0, len(t_swipe)-1, 1)):
    update_swipe(i, lines)
    fig.savefig(f"pngs/gaussian/gaussian_{fignum:03}.png", dpi=600)
    progress_bar.update(1)
# close the progress bar
progress_bar.close()

```

100% | 634/635 [05:47<00:00, 1.83iteration/s]



```

# Define the path to your PNG images
pngs_path = "pngs/gaussian"
pngs_name = "gaussian_%03d.png"

```

```

# Define the output video file path
video_output = "output_gaussian.mp4"

fr = 12
# run command
ffmpeg_cmd = f"ffmpeg -framerate {fr} -i {pngs_path}/{pngs_name} -c:v libx264 -vf fps={fr} {subprocess.run(ffmpeg_cmd, shell=True)}

ffmpeg version 6.0 Copyright (c) 2000-2023 the FFmpeg developers
built with Apple clang version 14.0.3 (clang-1403.0.22.14.1)
configuration: --prefix=/usr/local/Cellar/ffmpeg/6.0 --enable-shared --enable-pthreads --enable
libavutil      58. 2.100 / 58. 2.100
libavcodec     60. 3.100 / 60. 3.100
libavformat    60. 3.100 / 60. 3.100
libavdevice    60. 1.100 / 60. 1.100
libavfilter     9. 3.100 /  9. 3.100
libswscale      7. 1.100 /  7. 1.100
libswresample   4. 10.100 /  4. 10.100
libpostproc    57. 1.100 / 57. 1.100
Input #0, image2, from 'pngs/gaussian/gaussian_%03d.png':
Duration: 00:00:52.83, start: 0.000000, bitrate: N/A
Stream #0:0: Video: png, rgba(pc), 4800x3000 [SAR 23622:23622 DAR 8:5], 12 fps, 12 tbr, 12 t
Stream mapping:
Stream #0:0 -> #0:0 (png (native) -> h264 (libx264))
Press [q] to stop, [?] for help
[libx264 @ 0x7ff6d8907580] using SAR=1/1
[libx264 @ 0x7ff6d8907580] using cpu capabilities: MMX2 SSE2Fast SSSE3 SSE4.2 AVX FMA3 BMI2 AVX512
[libx264 @ 0x7ff6d8907580] profile High 4:4:4 Predictive, level 6.0, 4:4:4, 8-bit
[libx264 @ 0x7ff6d8907580] 264 - core 164 r3095 baee400 - H.264/MPEG-4 AVC codec - Copyleft 200
Output #0, mp4, to 'output_gaussian.mp4':
Metadata:
encoder       : Lavf60.3.100
Stream #0:0: Video: h264 (avc1 / 0x31637661), yuv444p(tv, progressive), 4800x3000 [SAR 1:1 D
Metadata:
encoder       : Lavc60.3.100 libx264
Side data:
cpb: bitrate max/min/avg: 0/0/0 buffer size: 0 vbv_delay: N/A
frame= 634 fps= 21 q=-1.0 Lsize= 1386kB time=00:00:52.58 bitrate= 215.9kbits/s speed=1.77x
video:1378kB audio:0kB subtitle:0kB other streams:0kB global headers:0kB muxing overhead: 0.60%
[libx264 @ 0x7ff6d8907580] frame I:3 Avg QP:10.13 size:140267

```

```
CompletedProcess(args='ffmpeg -framerate 12 -i pngs/gaussian/gaussian_%03d.png -c:v libx264 -v
```

Comparison

Let's plot in one graph the smoothed temperature for each kernel shape we calculated above (rectangular, triangular, gaussian), all of which with a 500-minute-wide window.

```
window_width_min = 500.0
window_width_int = int(window_width_min // 10 + 1)

# rectangular, 500 min
kernel_rect = np.ones(window_width_int)
rect = np.convolve(df['TD'].values, kernel_rect, mode='same') / len(kernel_rect)

# triangular
half_triang = np.arange(1, window_width_int/2+1, 1)
kernel_triang = np.hstack([half_triang, half_triang[-2::-1]])
kernel_triang = kernel_triang / kernel_triang.max()
triang = np.convolve(df['TD'].values, kernel_triang, mode='same') / len(kernel_triang) * 2

# gaussian
```

```

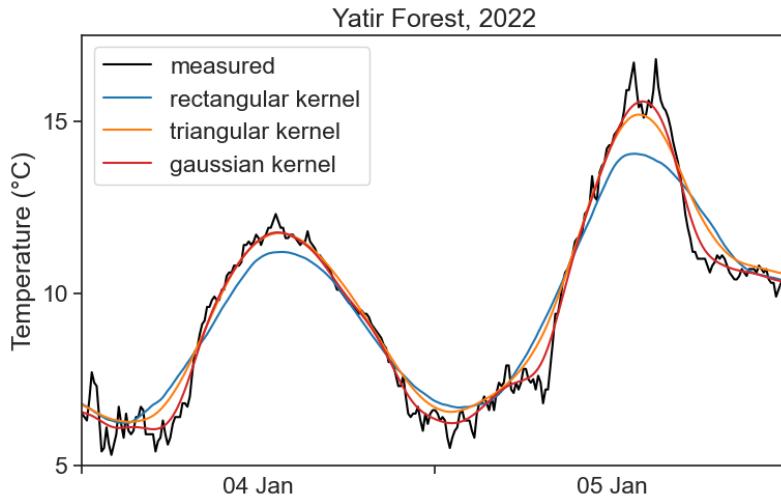
gauss = df['TD'].rolling(window=window_width_int, center=True, win_type="gaussian").mean(std)

fig, ax = plt.subplots(figsize=(8,5))
ax.figure.subplots_adjust(top=0.93, bottom=0.10, left=0.1, right=0.95)

ax.plot(df.loc[start:end, 'TD'], color='black', label="measured")
ax.plot(df.index, rect, color="tab:blue", label="rectangular kernel")
ax.plot(df.index, triang, color="tab:orange", label="triangular kernel")
ax.plot(df.index, gauss, color="tab:red", label="gaussian kernel")
ax.legend()

ax.set(ylim=[5, 17.5],
       xlim=['2022-01-04 00:00:00', '2022-01-05 23:50:00'],
       ylabel="Temperature (°C)",
       title="Yatir Forest, 2022",
       yticks=[5,10,15])
center_dates(ax)
fig.savefig("kernel_comparison.png")

```



```

fig, ax = plt.subplots(figsize=(8,5))
ax.figure.subplots_adjust(top=0.93, bottom=0.15, left=0.1, right=0.95)

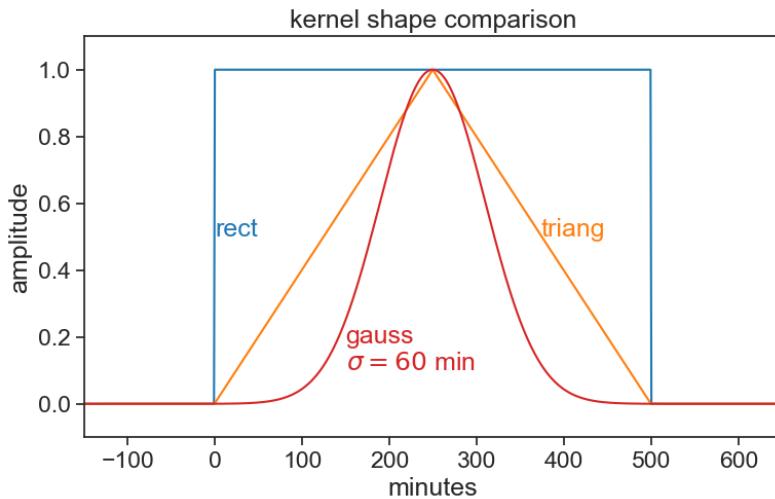
N=500

```

```

rec_window = np.zeros(800)
rec_window[150:150+N] = signal.windows.boxcar(N)
tri_window = np.zeros(800)
tri_window[150:150+N] = signal.windows.triang(N)
gau_window = np.zeros(800)
gau_window[150:150+N] = signal.windows.gaussian(N, std=60)
t = np.arange(-150, 650)
ax.plot(t, rec_window, color="tab:blue")
ax.plot(t, tri_window, color="tab:orange")
ax.plot(t, gau_window, color="tab:red")
ax.text(0, 0.5, "rect", color="tab:blue")
ax.text(373, 0.5, "triang", color="tab:orange")
ax.text(150, 0.1, "gauss\n"+r"$\sigma=60$ min", color="tab:red")
ax.set(ylim=[-0.1, 1.1],
      xlim=[-150, 650],
      ylabel="amplitude",
      xlabel="minutes",
      title="kernel shape comparison",
fig.savefig("kernel_shapes.png")

```



savgol video

Import packages and stuff.

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters() # datetime converter for a matplotlib
import seaborn as sns
sns.set(style="ticks", font_scale=1.5)
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import scipy as sp
import json
import requests
import os
import subprocess
from tqdm import tqdm
from scipy import signal
from scipy.signal import savgol_filter

# avoid "SettingWithCopyWarning: A value is trying to be set on a copy of a slice from a Dat
pd.options.mode.chained_assignment = None # default='warn'
```

Download data from the [IMS](#) using an API.

```
# read token from file
with open('../archive/IMS-token.txt', 'r') as file:
    TOKEN = file.readline()
# 28 = SHANI station
STATION_NUM = 28
```

```

start = "2022/01/01"
end = "2022/01/07"
filename = 'shani_2022_january.json'

# check if the JSON file already exists
# if so, then load file
if os.path.exists(filename):
    with open(filename, 'r') as json_file:
        data = json.load(json_file)
else:
    # make the API request if the file doesn't exist
    url = f"https://api.ims.gov.il/v1/envista/stations/{STATION_NUM}/data/?from={start}&to={end}"
    headers = {'Authorization': f'ApiToken {TOKEN}'}
    response = requests.get(url, headers=headers)
    data = json.loads(response.text.encode('utf8'))

    # save the JSON data to a file
    with open(filename, 'w') as json_file:
        json.dump(data, json_file)
# show data to see if it's alright
# data

```

Load and process data.

```

df = pd.json_normalize(data['data'], record_path=['channels'], meta=['datetime'])
df['date'] = (pd.to_datetime(df['datetime'])
              .dt.tz_localize(None) # ignores time zone information
              )
df = df.pivot(index='date', columns='name', values='value')
df

```

name	Grad	RH	Rain	STDwd	TD	TDmax	TDmin	TG	TW	Time	WD	...
date												
2022-01-01 00:00:00	0.0	77.0	0.0	10.3	11.2	11.2	11.1	10.7	-9999.0	2354.0	75.0	0
2022-01-01 00:10:00	0.0	77.0	0.0	11.2	11.2	11.2	11.1	10.8	-9999.0	1.0	77.0	8
2022-01-01 00:20:00	0.0	75.0	0.0	10.0	11.4	11.5	11.2	10.9	-9999.0	20.0	80.0	8
2022-01-01 00:30:00	0.0	74.0	0.0	9.6	11.5	11.5	11.4	11.0	-9999.0	22.0	76.0	7
2022-01-01 00:40:00	0.0	73.0	0.0	9.1	11.6	11.7	11.5	11.1	-9999.0	34.0	74.0	6
...
2022-01-06 23:10:00	0.0	36.0	0.0	16.1	11.6	12.0	11.1	6.8	-9999.0	2310.0	144.0	1

name	Grad	RH	Rain	STDwd	TD	TDmax	TDmin	TG	TW	Time	WD	Y
date												
2022-01-06 23:20:00	0.0	35.0	0.0	10.1	12.1	12.3	11.9	6.3	-9999.0	2320.0	118.0	1
2022-01-06 23:30:00	0.0	36.0	0.0	7.1	12.4	12.6	11.9	7.3	-9999.0	2330.0	113.0	1
2022-01-06 23:40:00	0.0	37.0	0.0	5.6	12.6	12.7	12.5	7.8	-9999.0	2339.0	119.0	1
2022-01-06 23:50:00	0.0	39.0	0.0	11.5	11.9	12.6	11.5	7.1	-9999.0	2341.0	102.0	1

Define useful functions.

```

def concise(ax):
    """
    Let python choose the best xtick labels for you
    """
    locator = mdates.AutoDateLocator(minticks=3, maxticks=7)
    formatter = mdates.ConciseDateFormatter(locator)
    ax.xaxis.set_major_locator(locator)
    ax.xaxis.set_major_formatter(formatter)

# dirty trick to have dates in the middle of the 24-hour period
# make minor ticks in the middle, put the labels there!
# from https://matplotlib.org/stable/gallery/ticks/centered_ticklabels.html

def center_dates(ax):
    # show day of the month + month abbreviation. see full option list here:
    # https://strftime.org
    date_form = DateFormatter("%d %b")
    # major ticks at midnight, every day
    ax.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax.xaxis.set_major_formatter(date_form)
    # minor ticks at noon, every day
    ax.xaxis.set_minor_locator(mdates.HourLocator(byhour=[12]))
    # erase major tick labels
    ax.xaxis.set_major_formatter(ticker.NullFormatter())
    # set minor tick labels as define above
    ax.xaxis.set_minor_formatter(date_form)
    # completely erase minor ticks, center tick labels
    for tick in ax.xaxis.get_minor_ticks():
        tick.tick1line.set_markersize(0)
        tick.tick2line.set_markersize(0)

```

```

    tick.label1.set_horizontalalignment('center')

def center_dates_two_panels(ax0, ax1):
    # show day of the month + month abbreviation. see full option list here:
    date_form = DateFormatter("%d %b")
    # major ticks at midnight, every day
    ax0.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax1.xaxis.set_major_locator(mdates.DayLocator(interval=1))
    ax1.xaxis.set_major_formatter(date_form)
    # minor ticks at noon, every day
    ax1.xaxis.set_minor_locator(mdates.HourLocator(byhour=[12]))
    # erase major tick labels
    ax1.xaxis.set_major_formatter(ticker.NullFormatter())
    # set minor tick labels as define above
    ax1.xaxis.set_minor_formatter(date_form)
    # completely erase minor ticks, center tick labels
    for tick in ax0.xaxis.get_minor_ticks():
        tick.tick1line.set_markersize(0)
        tick.tick2line.set_markersize(0)
    for tick in ax1.xaxis.get_minor_ticks():
        tick.tick1line.set_markersize(0)
        tick.tick2line.set_markersize(0)
        tick.label1.set_horizontalalignment('center')

```

We don't need the full month, let's cut the dataframe to fewer days.

```

start = "2022-01-01 00:00:00"
end = "2022-01-06 23:50:00"
df = df.loc[start:end]

```

We now redefine a narrower window, this will be the graph's xlims. We leave the dataframe as is, because we will need some data outside the graph's limits.

```

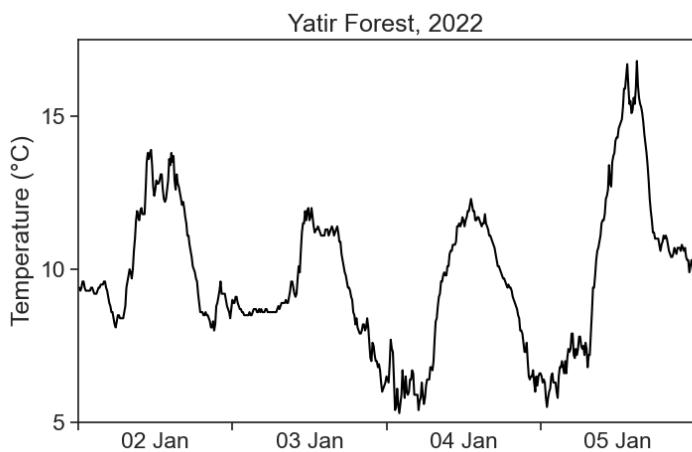
start = "2022-01-02 00:00:00"
end = "2022-01-05 23:50:00"

```

```

fig, ax = plt.subplots(figsize=(8,5))
ax.plot(df.loc[start:end, 'TD'], color='black')
ax.set(ylim=[5, 17.5],
       xlim=[start, end],
       ylabel="Temperature (°C)",
       title="Yatir Forest, 2022",
       yticks=[5,10,15])
center_dates(ax)
# fig.savefig("sliding_YF_temperature_2022.png")

```



Looks good. Let's move on.

Savgol filter

```

# Function to fit and get polynomial values
def fit_polynomial(x, y, degree):
    coeffs = np.polyfit(x, y, degree)
    poly = np.poly1d(coeffs)
    return poly(x), coeffs

# Function to fit and get polynomial values
def poly_coeffs(x, y, degree):

```

```

coeffs = np.polyfit(x, y, degree)
return coeffs

%matplotlib widget
fig, ax = plt.subplots(figsize=(8,5))
ax.plot(df.loc[start:end, 'TD'], color='black')

sg = savgol_filter(df['TD'], 13, 2)

i = 500
ax.plot(df.index[:i], sg[:i], color='xkcd:hot pink')

window_pts = 31
p_order = 3

window_x = np.arange(i - window_pts // 2, i + window_pts // 2)
window_y = df['TD'][i - window_pts // 2:i + window_pts // 2]

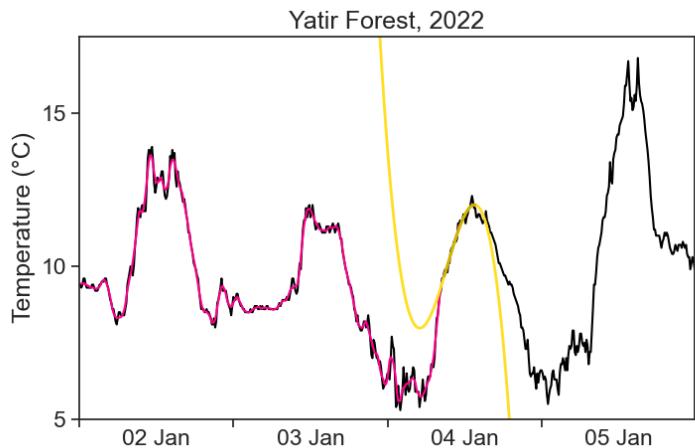
# Fit and plot polynomial inside the window
fitted_y, coeffs = fit_polynomial(window_x, window_y, p_order)

whole_x = np.arange(len(df))
whole_y = df['TD'].values
poly = np.poly1d(coeffs)
whole_poly = poly(whole_x)

ax.plot(df.index, whole_poly, color='xkcd:sun yellow', lw=2)
# ax.plot(df.index[window_x], fitted_y, color='0.8', lw=3)
ax.plot(df.index[window_x], fitted_y, color='xkcd:mustard', lw=2)

ax.set(ylim=[5, 17.5],
       xlim=[start, end],
       ylabel="Temperature (°C)",
       title="Yatir Forest, 2022",
       yticks=[5,10,15])
center_dates(ax)
# fig.savefig("sliding_YF_temperature_2022.png")

```



```

p_order = 3

%matplotlib widget
fig, ax = plt.subplots(figsize=(8,5))

class Lines:
    """
        empty class, later will be populated with graph objects.
        this is useful to draw and erase lines on demand.
    """
    pass
lines = Lines()

# set graph y limits
ylim = [3, 22]
# choose here window width in minutes
window_width_min = 500.0
window_width_min_integer = int(window_width_min) # same but integer
window_width_int = int(window_width_min // 10 + 1) # window width in points

```

```

N = len(df) # df length
t_swipe = pd.date_range(start=pd.to_datetime(start) - pd.Timedelta(minutes=window_width_min),
                        end=pd.to_datetime(end) + pd.Timedelta(minutes=60),
                        freq="10min")
# starting time
t0 = t_swipe[0]
ind0 = df.index.get_loc(t0) + window_width_int//2 + 1
# show sliding window on the top panel as a light blue shade
lines.fill_bet = ax.fill_between([t0, t0 + pd.Timedelta(minutes=window_width_min)],
                                 y1=ylim[0], y2=ylim[1], alpha=0.1, zorder=-1)

sg = savgol_filter(df['TD'], window_width_int, p_order)
df.loc[:, 'sg'] = sg
# plot temperature
ax.plot(df.loc[start:end, 'TD'], color="black")

# define x,y data inside window to execute polyfit on
window_x = np.arange(ind0 - window_width_int // 2, ind0 + window_width_int // 2)
window_y = df['TD'][ind0 - window_width_int // 2:ind0 + window_width_int // 2].values
# fit and plot polynomial inside the window
fitted_y, coeffs = fit_polynomial(window_x, window_y, p_order)
# get x,y data for the whole array
whole_x = np.arange(len(df))
whole_y = df['TD'].values
poly = np.poly1d(coeffs)
whole_poly = poly(whole_x)

# calculate the middle of the sliding window
window_middle = t0 + pd.Timedelta(minutes=window_width_min/2)
# plot a pink line showing the result of the moving average
# from the beginning to the middle of the sliding window
lines.pink_line, = ax.plot(df.loc[start:window_middle, 'sg'], color="xkcd:hot pink", lw=3)

lines.poly_all, = ax.plot(df.index, whole_poly, color='xkcd:sun yellow', lw=2)
lines.poly_window, = ax.plot(df.index[window_x], fitted_y, color='xkcd:mustard', lw=2)

# emphasize the location of the middle on the window with a circle
lines.pink_circle, = ax.plot([window_middle], [df.loc[window_middle, 'sg']],
                           marker='o', markerfacecolor="None", markeredgecolor="xkcd:dark pink", markeredgewidth=2,
                           markersize=8)

```

```

# some explanation
ax.text(0.99, 0.97, f"savitzky-golay\nwidth = {window_width_int:.0f} pts\npoly order = {p_order}\n    horizontalalignment='right', verticalalignment='top',\n    fontsize=14)
# axis tweaking
ax.set(ylim=ylim,
       xlim=[start, end],
       ylabel="Temperature (°C)",
       yticks=[5,10,15,20],
       title="Yatir Forest, 2022")
# adjust dates on both panels as defined before
center_dates(ax)

def update_swipe(k, lines):
    """
    updates both panels, given the index k along which the window is sliding
    """
    # left side of the sliding window
    t0 = t_swipe[k]
    # middle position
    window_middle = t0 + pd.Timedelta(minutes=window_width_min/2)
    ind0 = df.index.get_loc(window_middle)
    # erase previous blue shade on the top graph
    lines.fill_bet.remove()
    # fill again the blue shade in the updated window position
    lines.fill_bet = ax.fill_between([t0, t0 + pd.Timedelta(minutes=window_width_min)],
                                      y1=ylim[0], y2=ylim[1], alpha=0.1, zorder=-1,
                                      edgecolor='blue')
    # update pink curve
    lines.pink_line.set_data(df[start:window_middle].index,
                             df.loc[start:window_middle, 'sg'].values)
    # update pink circle
    lines.pink_circle.set_data([window_middle], [df.loc[window_middle, 'sg']])
    # define x,y data inside window to execute polyfit on

    window_x = np.arange(ind0 - window_width_int // 2, ind0 + window_width_int // 2)
    window_y = df['TD'][ind0 - window_width_int // 2:ind0 + window_width_int // 2]
    # fit and plot polynomial inside the window
    fitted_y, coeffs = fit_polynomial(window_x, window_y, p_order)
    poly = np.poly1d(coeffs)

```

```

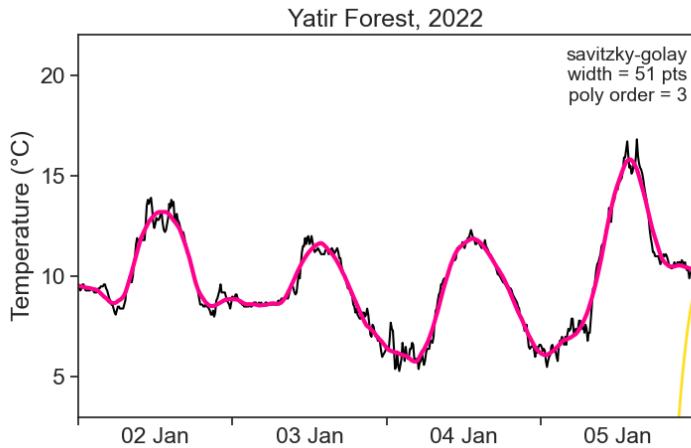
whole_poly = poly(whole_x)
lines.poly_all.set_data(df.index, whole_poly)
lines.poly_window.set_data(df.index[window_x], fitted_y)

fig.savefig(f"pngs/savgol{window_width_int}/savgol_zero.png", dpi=600)

# create a tqdm progress bar
progress_bar = tqdm(total=len(t_swipe), unit="iteration")
# loop over all sliding indices, update graph and then save it
for fignum, i in enumerate(np.arange(0, len(t_swipe)-1, 1)):
    updateSwipe(i, lines)
    fig.savefig(f"pngs/savgol{window_width_int}/savgol_{window_width_int}_{fignum:03}.png",
    # update the progress bar
    progress_bar.update(1)
# close the progress bar
progress_bar.close()

```

100% | 634/635 [13:07<00:01, 1.24s/iteration]



Combine all saved images into one mp4 video.

```

# Define the path to your PNG images
pngs_path = f"pngs/savgol51"
pngs_name = f"savgol_51_%03d.png"

```

```

# Define the output video file path
video_output = f"output_savgol51.mp4"

# Use ffmpeg to create a video from PNG images
# desired framerate. choose 24 if you don't know what to do
fr = 12
# run command
ffmpeg_cmd = f"ffmpeg -framerate {fr} -i {pngs_path}/{pngs_name} -c:v libx264 -vf fps={fr} {subprocess.run(ffmpeg_cmd, shell=True)}
```

```

ffmpeg version 6.1.1 Copyright (c) 2000-2023 the FFmpeg developers
built with Apple clang version 15.0.0 (clang-1500.1.0.2.5)
configuration: --prefix=/usr/local/Cellar/ffmpeg/6.1.1_2 --enable-shared --enable-pthreads --
libavutil      58. 29.100 / 58. 29.100
libavcodec     60. 31.102 / 60. 31.102
libavformat    60. 16.100 / 60. 16.100
libavdevice    60.  3.100 / 60.  3.100
libavfilter     9. 12.100 /  9. 12.100
libswscale      7.  5.100 /  7.  5.100
libswresample   4. 12.100 /  4. 12.100
libpostproc    57.  3.100 / 57.  3.100
Input #0, image2, from 'pngs/savgol51/savgol_51_%03d.png':
  Duration: 00:00:52.83, start: 0.000000, bitrate: N/A
  Stream #0:0: Video: png, rgba(pc, gbr/unknown/unknown), 4800x3000 [SAR 23622:23622 DAR 8:5],
Stream mapping:
  Stream #0:0 -> #0:0 (png (native) -> h264 (libx264))
Press [q] to stop, [?] for help
[libx264 @ 0x7f9cb7906dc0] using SAR=1/1
[libx264 @ 0x7f9cb7906dc0] using cpu capabilities: MMX2 SSE2Fast SSSE3 SSE4.2 AVX FMA3 BMI2 AVX2
[libx264 @ 0x7f9cb7906dc0] profile High 4:4:4 Predictive, level 6.0, 4:4:4, 8-bit
[libx264 @ 0x7f9cb7906dc0] 264 - core 164 r3108 31e19f9 - H.264/MPEG-4 AVC codec - Copyleft 2005
Output #0, mp4, to 'output_savgol51.mp4':
  Metadata:
    encoder         : Lavf60.16.100
  Stream #0:0: Video: h264 (avc1 / 0x31637661), yuv444p(tv, progressive), 4800x3000 [SAR 1:1 DAR 8:5]
    Metadata:
      encoder         : Lavc60.31.102 libx264
    Side data:
      cpb: bitrate max/min/avg: 0/0/0 buffer size: 0 vbv_delay: N/A
```

```
[out#0/mp4 @ 0x7f9cb7806000] video:3513kB audio:0kB subtitle:0kB other streams:0kB global headers:0kB muxing overhead: 0.000000%
frame= 634 fps=3.2 q=-1.0 Lsize= 3521kB time=00:00:52.58 bitrate= 548.5kbit/s speed=0.27x
[libx264 @ 0x7f9cb7906dc0] frame I:3 Avg QP:13.70 size:146882
[libx264 @ 0x7f9cb7906dc0] frame P:232 Avg QP:19.02 size: 7856
[libx264 @ 0x7f9cb7906dc0] frame B:399 Avg QP:24.04 size: 3341
[libx264 @ 0x7f9cb7906dc0] consecutive B-frames: 11.4% 10.7% 10.4% 67.5%
[libx264 @ 0x7f9cb7906dc0] mb I I16..4: 32.8% 61.0% 6.2%
[libx264 @ 0x7f9cb7906dc0] mb P I16..4: 0.5% 0.7% 0.3% P16..4: 0.4% 0.2% 0.1% 0.0% 0.0%
[libx264 @ 0x7f9cb7906dc0] mb B I16..4: 0.1% 0.0% 0.0% B16..8: 1.9% 0.3% 0.0% direct: 0.0%
[libx264 @ 0x7f9cb7906dc0] 8x8 transform intra:51.3% inter:35.8%
[libx264 @ 0x7f9cb7906dc0] coded y,u,v intra: 7.2% 6.5% 4.4% inter: 0.1% 0.1% 0.0%
[libx264 @ 0x7f9cb7906dc0] i16 v,h,dc,p: 89% 10% 1% 0%
[libx264 @ 0x7f9cb7906dc0] i8 v,h,dc,ddl,ddr,vr,hd,vl,hu: 31% 3% 65% 0% 0% 0% 0% 0% 0%
[libx264 @ 0x7f9cb7906dc0] i4 v,h,dc,ddl,ddr,vr,hd,vl,hu: 54% 7% 23% 3% 1% 4% 1% 5% 1%
[libx264 @ 0x7f9cb7906dc0] Weighted P-Frames: Y:0.0% UV:0.0%
[libx264 @ 0x7f9cb7906dc0] ref P L0: 58.7% 5.7% 25.2% 10.4%
[libx264 @ 0x7f9cb7906dc0] ref B L0: 85.9% 11.8% 2.4%
[libx264 @ 0x7f9cb7906dc0] ref B L1: 96.9% 3.1%
[libx264 @ 0x7f9cb7906dc0] kb/s:544.58
```

```
CompletedProcess(args='ffmpeg -framerate 12 -i pngs/savgol51/savgol_51_%03d.png -c:v libx264 -vcodec libx264 -crf 23 -pix_fmt yuv420p -f mp4 out#0/mp4 @ 0x7f9cb7806000')
```

API to download data from IMS

```
# TOKEN = "f058958a-d8bd-47cc-95d7-7ecf98610e47"
# STATION_NUM = 28 # 28 = "SHANI"
# DATA = 10 # 10 = TDmax (max temperature)
# start = "2022/01/01"
# end = "2022/02/01"
# url = f"https://api.ims.gov.il/v1/envista/stations/{STATION_NUM}"
# url = f"https://api.ims.gov.il/v1/envista/stations/{STATION_NUM}/data/?from={start}&to={end}"
# url = f"https://api.ims.gov.il/v1/envista/stations/{STATION_NUM}/data/{DATA}/data/11/?from={start}&to={end}"
# headers = {'Authorization': 'ApiToken f058958a-d8bd-47cc-95d7-7ecf98610e47'}
# response = requests.request("GET", url, headers=headers)
# data= json.loads(response.text.encode('utf8'))

# # Save the JSON data to a file
# with open('shani_2022_january.json', 'w') as json_file:
#     json.dump(data, json_file)

# data

# # https://ims.gov.il/he/ObservationDataAPI
# # https://ims.gov.il/sites/default/files/2021-09/API%20explanation.pdf
# # https://ims.gov.il/sites/default/files/2022-04/Python%20API%20example.pdf
# TOKEN = "f058958a-d8bd-47cc-95d7-7ecf98610e47"
# STATION_NUM = 23 # 23 = "JERUSALEM CENTRE"
# DATA = 9 # 9 = TDmax (max temperature)
# start = "2022/01/01"
# end = "2022/02/01"
# url = f"https://api.ims.gov.il/v1/envista/stations/{STATION_NUM}/data/{DATA}/?from={start}&to={end}"
# headers = {'Authorization': 'ApiToken f058958a-d8bd-47cc-95d7-7ecf98610e47'}
# response = requests.request("GET", url, headers=headers)
# data= json.loads(response.text.encode('utf8'))
```

```
# print(url)

# url = "https://api.ims.gov.il/v1/envista/stations/28/data/10/data/11/?from=2022/01/01&to=2022/01/31"
# response = requests.request("GET", url, headers=headers)
# data = json.loads(response.text.encode('utf8'))

# # RH = 8
# # TDmax = 10, max temperature
# # TDmin = 11, min temperature
# url = "https://api.ims.gov.il/v1/envista/stations/28/data/10/?from=2022/01/01&to=2022/01/31"
# response = requests.request("GET", url, headers=headers)
# data = json.loads(response.text.encode('utf8'))

# df = pd.json_normalize(data['data'], record_path=['channels'], meta=['datetime'])
# df['date'] = pd.to_datetime(df['datetime']).dt.tz_localize(None) # ignore time zone information
# df = df.set_index('date')

# df
# data['data']
```

remove consecutive values

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.dates import DateFormatter
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import warnings
# Suppress FutureWarnings
warnings.simplefilter(action='ignore', category=FutureWarning)
warnings.simplefilter(action='ignore', category=UserWarning)
import seaborn as sns
sns.set(style="ticks", font_scale=1.5) # white graphs, with large and legible letters

# %matplotlib widget
```

create data, put some defective windows here and there...

```
steps = np.random.randint(low=-2, high=2, size=500)
data = steps.cumsum()
date_range = pd.date_range(start='2023-01-01', periods=len(data), freq='1D')
df = pd.DataFrame({'series': data}, index=date_range)

# make sequence of consecutive values
df.loc['2023-06-05':'2023-07-20', 'series'] = 2
df.loc['2023-10-05':'2023-10-25', 'series'] = -150
```

plot

```
def concise(ax):
    locator = mdates.AutoDateLocator(minticks=3, maxticks=7)
    formatter = mdates.ConciseDateFormatter(locator)
    ax.xaxis.set_major_locator(locator)
```

```

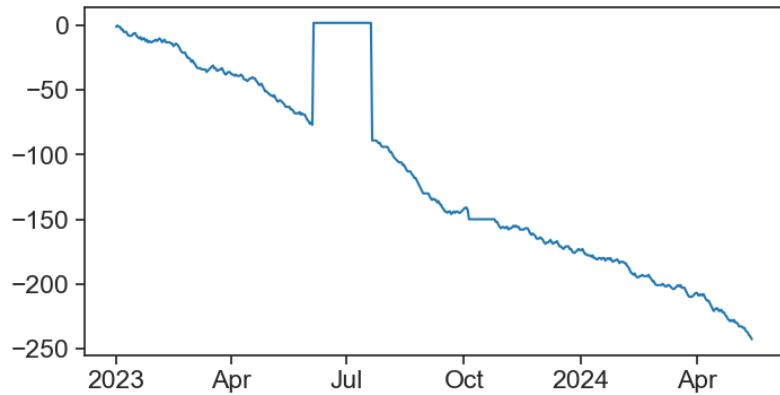
    ax.xaxis.set_major_formatter(formatter)

fig, ax = plt.subplots(figsize=(8,4))
ax.plot(df['series'], color="tab:blue")
concise(ax)
ax.legend(frameon=False)

```

No artists with labels found to put in legend. Note that artists whose label start with an un

<matplotlib.legend.Legend at 0x7fe480a66230>



nice function, keep that for future reference

```

# function to copy paste:
def conseq_series(series, N):
    """
    part A:
    1. assume a string of 5 equal values. that's what we want to identify
    2. diff produces a string of only 4 consecutive zeros
    3. no problem, because when applying cumsum, the 4 zeros turn into a plateau of 5, that's
       so far, so good
    part B:
    1. groupby value_grp splits data into groups according to cumsum.
    2. because cumsum is monotonically increasing, necessarily all groups will be composed of
    3. what are those groups made of? of rows of column 'series'. this specific column is now
    4. count 'counts' the number of elements inside each group.

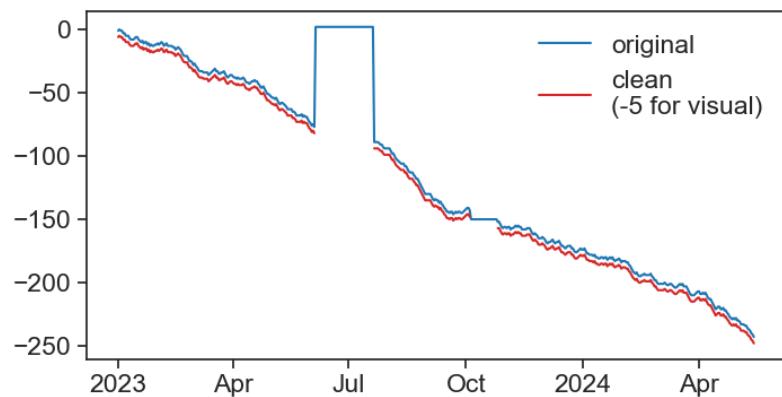
```

5. the real magic here is that 'transform' assigns each row of the original group with t
6. finally, we can ask the question: which rows belong to a string of identical values g
zehu, you now have a mask (True-False) with the same shape as the original series.

```
"""
# part A:
sumsum_series = (
    (series.diff() != 0)           # diff zero becomes false, otherwise true
        .astype('int')           # true -> 1 , false -> 0
        .cumsum()                # cumulative sum, monotonically increasing
)
# part B:
mask_outliers = (
    series.groupby(sumsum_series)      # take original series and group
        .transform('count')          # now count how many are in each
        .ge(N)                      # if row count >= than user-defined N
)
# apply mask:
result = pd.Series(np.where(mask_outliers,
                            np.nan, # use this if mask_outliers is True
                            series), # otherwise
                   index=series.index)
return result
```

plot results. it works :)

```
fig, ax = plt.subplots(figsize=(8,4))
ax.plot(df['series'], color="tab:blue", label='original')
ax.plot(conseq_series(df['series'], 10)-5, c='tab:red', label='clean\n(-5 for visual)')
concise(ax)
ax.legend(frameon=False);
```



McDonald, Andy. 2022. “Creating Boxplots with the Seaborn Python Library.” *Medium*. Towards Data Science. <https://towardsdatascience.com/creating-boxplots-with-the-seaborn-python-library-f0c20f09bd57>.

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