Equations of Physics and Space Density

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Abstract

We show that the “hidden” variable of the Navier, Maxwell, Einstein and Ricci Flow equations is ρ - the space lattice density. In our GeometroDynamic Model of the physical reality - the GDM (presented in a series of papers during 2017) - we show that sources are merely highly deformed zones of space (highly contracted or dilated) whereas their fields are continuations of these deformations, reduced by orders of magnitudes, with only a small deviation from the normal (standard) space density. Thus the equations of physics express different aspects of the same reality - the geometrodynamics of space.

Key Words: Space, Navier, Maxwell, Einstein, General Relativity, Ricci Flow, GDM

1 Introduction

1.1 Space as a Lattice

By attributing a cellular structure to space we can explain its expansion, its elasticity and can introduce a cut-off in the wavelength of the vacuum state spectrum of vibrations [1].

1.2 Space density ρ

Space density ρ is defined as the number of space cells per unit volume [2]. Space density in a zone of space without deformations (far away from masses and charges) is denoted ρ_0.

Let dn be the number of space cells in a given volume dV. Since dn = ρ_0dV and also dn=ρdV':

$$dV = \frac{dn}{\rho_0} \qquad dV' = \frac{dn}{\rho} \qquad \text{Hence:} \quad \frac{dV' - dV}{dV} = \frac{\rho_0 - \rho}{\rho}$$

1.3 The Elastic Displacement Vector u

In Appendix A of [2] we prove that the relative volumetric change equals the divergence of the Elastic Displacement Vector **u**:

$$\nabla \cdot \mathbf{u} = \frac{dV' - dV}{dV} \tag{1}$$

Thus:

$$\nabla \cdot \mathbf{u} = \frac{\rho_0 - \rho}{\rho} \tag{2}$$

This proof is a cornerstone of the GDM. It is based on the **strain tensor** u_{ik} , which is defined as:

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \frac{\partial u_l \partial u_l}{\partial x_i \partial x_k} \right)$$

1.4 The Small Deformation Strain Tensor as a Fundamental Metric Tensor

The authors of [3], see also [2], conclude:

"……. **the small deformation strain tensor could be used as a fundamental metric tensor, instead of the usual fundamental metric tensor.**"

Note, however, that space deformation is a local feature of curvature, whereas manifold curvature can also be a global feature of curvature (like that of a closed spherical surface).

1.5 Deformed Spaces versus Bent Manifolds

We relate to space not as a passive static arena for fields and particles but as an active elastic entity, which in our model is the one and only entity that exists. Physicists have different, sometimes conflicting, ideas about the physical meaning of the mathematical objects of their models. The mathematical objects of General Relativity, as an example, are n-dimensional manifolds in hyper-spaces with more dimensions than n. These are not necessarily the physical objects that General Relativity accounts for, and n-dimensional manifolds can be equivalent to n-dimensional elastic spaces. Rindler [4] uses this equivalence to visualize bent manifolds, whereas Steane [5] considers this equivalence to be a real option for a presentation of reality. Callahan [6], being very clear about this equivalence, declares: "…in physics we associate curvature with stretching rather than bending". After all, in General Relativity gravitational waves [7] are space waves and the attribution of elasticity to a 3D space is thus a must.

The deformation of space is the change in size, of its cells [8]. The terms positive deformation and negative deformation, around a point in space, are used to indicate that space cells grow or shrink, respectively, from this point outwards. Positive deformation is equivalent to positive curving and negative deformation to negative curving. In [8] we show that the curvature K around a point in space is:

$$K = \frac{4\pi}{45}\left(\frac{\nabla\rho}{\rho}\right)^2 \qquad (3)$$

1.6 The GDM and the Equations of Physics

In our "**G**eometro**D**ynamic **M**odel of reality", the **GDM**, the variable in the equations of physics is directly or indirectly related to the density ρ of the elastic space lattice. These equations yield wave equations that are also related to the density ρ of the elastic space.

In the Navier equation [1], the displacement vector **u** is related to ρ: $\nabla\cdot\mathbf{u} = (\rho - \rho_0)/\rho$.

In the Maxwell equations, [9] and [2], the charge density q is defined as $q = 1/4\pi\cdot(\rho - \rho_0)/\rho$, and the fields **E** and **B** are related to **u** in the following way: $\mathbf{E} = H\mathbf{u}$ and $\mathbf{B} = 1/c\ \mathbf{v}\times\mathbf{E}$, where **v** is the velocity of the charge that creates **E** and hence **B**.

In the General Relativity (GR) equation, [10], the variable is the metric g_{ij} and as shown in Section 1.4 it is related to the **strain tensor** $u_{ik,}$ which is related to ρ . The Ricci tensor R_{ij} in the GR equation expresses curvature. But the curvature K is also related to space density:

$K = 4\pi/45\cdot(\nabla\rho/\rho)^2$.

In the Ricci Flow equation:

$\partial_t g_{ij} = -2R_{ij}$ (Section 6)

with the Ricci tensor and the metric, the situation is the same.

Necessarily, all waves are space waves of different kinds.

Note that transversal waves **do not** require a time-variable space density ρ, whereas longitudinal waves require a time-variable space density ρ.

The Navier equation yields both transversal and longitudinal waves, whereas the Maxwell and Einstein equations seem to yield "only" transversal waves. We relate to this situation in Sections 3 and 5.

1.7 The GDM Postulate

The elastic and vibrating three-dimensional Space Lattice is all there is. Elementary particles are transversal or circulating longitudinal wavepackets of this vibrating space [1].

2 Navier Equation - The Vibrating Space

2.1 Elastic Waves – a Reminder

The equation of equilibrium in an elastic media, with displacement vector, **u**, (53.6) in [11], is the **Navier equation**:

$$(\lambda+2\mu)\Delta\mathbf{u}-\mu\nabla\times(\nabla\times\mathbf{u})-\mathrm{m}\frac{\partial^2\mathbf{u}}{\partial t^2}=0 \tag{4}$$

μ and λ are the **elastic Lamé coefficients**, and m is the media mass density.

Mass in the GDM is only a **practical** attribute [3]. The relevant fundamental attribute is energy U or energy density $\in$. Hence, for space as the elastic media, m represents the energy density of vibrations:

$$\mathrm{m}=\frac{\in}{c^2} \tag{5}$$

To solve equation (4), we adopt the Kelvin [11] method and decompose **u** as follows:

$\mathbf{u}=\mathbf{u}_L+\mathbf{u}_T$ Where:

$\nabla\times\mathbf{u}_L=0$ $\nabla\cdot\mathbf{u}_T=0$ Therefore:

$\mathbf{u}_L=-\nabla\varphi$ $\mathbf{u}_T=\nabla\times f$

φ stands for the scalar potential and **f** for the vectorial potential. This decomposition is true under the boundary condition $\mathbf{u}\to 0$ at ∞. The known equations for $\mathbf{u}_L$ and $\mathbf{u}_T$ are obtained by substituting $\mathbf{u}=\mathbf{u}_L+\mathbf{u}_T$ in (4).

For $\mathbf{u}_L$:

$$(\lambda+2\mu)\nabla^2\mathbf{u}_L - m\frac{\partial^2\mathbf{u}_L}{\partial t^2}=0 \qquad \text{or:} \qquad \nabla^2\mathbf{u}_L - \frac{1}{c_L^2}\frac{\partial^2\mathbf{u}_L}{\partial t^2}=0$$

which is the vector wave equation for waves which move at a speed c_L, where:

$$c_L = \left(\frac{\lambda+2\mu}{m}\right)^{\frac{1}{2}} \tag{6}$$

Since $\nabla\times\mathbf{u}_L = 0$ this is the contractional /dilational, **longitudinal wave**.

For $\mathbf{u}_T$: $\quad \mu\nabla^2\mathbf{u}_T - m\frac{\partial^2\mathbf{u}_T}{\partial t^2}=0 \quad$ or:

$$\nabla^2\mathbf{u}_T - \frac{1}{c_T^2}\frac{\partial^2\mathbf{u}_T}{\partial t^2}=0$$

This is again a vector wave equation for waves with speed c_T, where:

$$c_T = \left(\frac{\mu}{m}\right)^{\frac{1}{2}} \tag{7}$$

Since $\nabla\cdot\mathbf{u}_T = 0$, $\mathbf{u}_T$ and c_T, correspond to the shear, **transverse wave**.

2.2 On the Transversal and Longitudinal Wave Velocities

Historically, to account for the absence of electromagnetic longitudinal waves, Cauchy (19th century) suggested that $\lambda+2\mu=0$, see [12] P.108. Hence $\lambda=-2\mu$, but the **bulk modulus** is $k=\lambda+\frac{2}{3}\mu$ and since $\mu>0$ a negative **compressibility** (1/k) of the aether was required.

In the GDM, material (non-zero rest energy) elementary particles are circulating longitudinal wavepackets [13], [14]. This circulation is complex [13] and its basic motion is at the longitudinal velocity $c_L > c_T = c$. Hence we require:

$\lambda+2\mu > \mu$ or $\lambda+\mu > 0$ see (6) and (7).

In [13] we show that

$$c_L = 1.6068\ c \tag{8}$$

hence $(\lambda+2\mu)/\mu = 1.6068^2 = 2.5818$ and:

$$\lambda = 0.5818\ \mu \tag{9}$$

3 Maxwell Equations

3.1 Classical Electromagnetism

Electromagnetic theory can be fully expressed by the four **Maxwell's equations** [2] and [9]:

I $\nabla \cdot \mathbf{E} = 4\pi H q$ **Coulomb's Law**

II $\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$ **Faraday's Law**

III $\nabla \cdot \mathbf{B} = 0$ **No magnetic monopoles**

IV $\nabla \times \mathbf{B} = \frac{4\pi H}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$ **Ampère's Law**

In these equations **B** is the vector [9]:

$$\mathbf{B} = \frac{1}{c}(\mathbf{v} \times \mathbf{E}) \qquad \textbf{Magnetic Field} \tag{10}$$

The **magnetic field** is created by a charge, with the field **E**, moving at speed **v**.

The dimensions of **B** and **E** are the same, and in the GDM they are: $[E] = [B] = LT^{-2}$, see [2], [9], and both express the elastic displacement.

In the GDM the charge density q is defined as $q = 1/4\pi\cdot(\rho - \rho_0)/\rho$, and the fields **E** and **B** are related to **u** in the following way: $\mathbf{E} = H\mathbf{u}$ and $\mathbf{B} = 1/c\ \mathbf{v}\times\mathbf{E}$, where **v** is the velocity of the charge that creates **E**. The displacement vector **u**, though, is related to ρ, as follows:

$\mathbf{\nabla\cdot u} = (\rho - \rho_0)/\rho$.

The Maxwell electromagnetic equations are, thus, related to space density ρ only.

On electromagnetic longitudinal waves see [15].

4 The Einstein General Relativity (GR) Equation

4.1 The Equation

The right hand-side of Einstein's field equation (1) of GR (below) **should** express curvature exactly as the left-hand side does. In [10] we show, for the first time, that this is indeed the case.

The need to express curvature by Riemannian geometry and obtain a covariant formulation of physical laws, by using tensors, led Einstein to the equation of General Relativity (GR):

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = \frac{8\pi G}{c^4} \cdot T_{ij} \qquad (11)$$

R_{ij} is the Ricci contracted Riemannian tensor and R is the Ricci scalar, $\frac{1}{2}R$ is the familiar Gaussian Curvature and Λ is the cosmological constant. The $\frac{1}{2}Rg_{ij}$ term is added to give a covariant divergence, which is identically zero, and $T^{00} = j^{(0)0} = \in$ is the energy density of space. In this paper we ignore the Cosmological Constant term Λg_{ij}.

It was Einstein's vision that "future physics" will show that the right-hand side of (1):

$\frac{8\pi G}{c^4} \cdot T_{ij}$ is also curvature.

In equation (2) of [5], R_c is the Gaussian radius of curvature, and $1/R_c^2$ is the Gaussian curvature at the radial distance r from the center of a black hole with Schwarzschild Radius r_S (our notations are different from those in [5]):

$$1/R_c^2 = r_S/r^3 \tag{12}$$

Equation (12) enables us, for the first time to show, [10], that $8\pi G/c^4 \cdot T_{ij}$ expresses curvature.

4.2 Curvature

The Ricci tensor R_{ij} in the GR equation expresses curvature. But the curvature K is also related to space density [8]:

$$K = 4\pi/45 \cdot (\boldsymbol{\nabla}\rho/\rho)^2$$

4.3 The Metric

In the GR equation the variable is the metric g_{ij} which, as shown in Section 1.4, is related to the **strain tensor** $u_{ik,}$ which is itself related to ρ.

Thus the GR equation relates to space density only.

5 The Extended GR Equation of the GDM

5.1 The Extended Field Equation of General Relativity for Energy/Momentum and Charge/Current

Using (9), (10) and (11) in [10] the extended Einstein GR field equation becomes:

$$R^{\mu\nu}\text{-}1/2Rg^{\mu\nu} = 8\pi G/c^4 \cdot T_m^{\mu\nu} + 4\pi\sqrt{G}\,/s^2 \cdot T_q^{\mu\nu} \tag{13}$$

And in the GDM, by inserting the constant, H, it takes the form:

$$R^{\mu\nu}\text{-}1/2Rg^{\mu\nu} = 8\pi G/c^4 \cdot T_m^{\mu\nu} + 4\pi\sqrt{HG}\,/s^2 \cdot T_q^{\mu\nu} \tag{14}$$

Electromagnetism relates to space density only, Section 3, and necessarily this is also the case for the extended GR equation.

Note that the gravitational weak field approximation yields the gravitational Poisson equation.

Similarly the electric weak field approximation should yield the electrical Poisson equation.

Note that the weak force is electromagnetic (Salam and Weinberg) and according to the GDM so is the strong force.

On gravitational longitudinal waves see the introduction of [16].

6 The Ricci Flow

The Ricci Flow in topology, [17], is governed by the equation:

$$\partial_t g_{ij} = -2R_{ij} \tag{15}$$

where g_{ij} is the metric of space and R_{ij} is the Ricci tensor. This equation expresses the idea that the rate of change in the metric in a given zone is dependent on the local curvature. This is kind of an "ironing" process of space or a curved manifold, analogues to heat diffusion. We consider this idea in topology to be also relevant to physics. Curved zones of space (contracted or dilated) or bent manifolds have a built-in tension, hence higher energy, due to the elasticity of space. We assume that in order to ease the tension and lower the energy the rate of change, with time, of the metric $\partial_t g_{ij}$ is dependent on the amount of curvature $2R_{ij}$. This is the idea of space being "ironed". Thus two zones of space, both curved positively or negatively will repel each other, whereas if one of them is curved positively and the other negatively they will attract each other. This is the case for the bivalent electrical charges [2]. We contend that this should also be the case for masses since they curve space positively. Thus, in contrast to the current understanding, masses should repel each other. Free Fall appears as an attraction force but, as we have seen, it is not. Locally Free Fall is stronger than the repulsion, but on the large scale of the universe it is the repulsion that takes over.

We know that the Hubble Flow (space expansion) takes place in the inter-galactic space and not within the galaxies themselves. Thus one can argue that galaxies, with their inner black holes, grip space locally and that their mutual repulsion is the reason for space expansion.

We have already explained that both the metric and the curvature are the expression of space density, and necessarily so is also the Ricci Flow.

7 Summary

Equations of physics express different aspects of the same reality. This reality is the geometrodynamics of space, and the same "hidden" variable of these equations is the density of space.

Laws of physics express the geometrodynamics of the elastic space and there is no need to ask where they come from.

Our papers can be found by inserting *shlomo barak* in the HAL Search:

HAL (https://hal.archives-ouvertes.fr)

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