Electromagnetism is Space Geometrodynamics Part 3

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Abstract

Part 3: **Electromagnetism as the Geometrodynamics of Space**

Light is a vibration (excitation) of the **Electromagnetic field**, which as we show is **Space** itself. **Light** is thus kind of a vibration of the **Space lattice**:

Adopting the idea that space is a lattice (cellular structure), Space Density ρ is defined as the number of space cells per unit volume (denoted ρ_0 for space with no deformations). Based on this we define Electric Charge Density as: $q=1/4\pi \cdot (\rho-\rho_0)/\rho$. This charge density is positive if $\rho > \rho_0$ (contracted space) and negative if $\rho < \rho_0$ (dilated space). This is all we need to derive electrostatics in this paper Part 1. By adding the Lorentz Transformation to the definition of q, we derive in this paper, Part 3, the entire Maxwell Electromagnetic theory with no phenomenology. Thus, the Electromagnetic field is the Space lattice and Light is its vibration.

Keywords: Electric charge, Space lattice, Electromagnetism, Maxwell equations

1 Introduction

The essence of electric charge has been a mystery. Recently it has been resolved in [1] and [2]. Our definition of charge density, in [1], yields the Maxwell theory of electrostatics with no phenomenology. It also realizes Einstein's vision that in "future physics" there will be no physical separation between a charge and its field. In this paper we derive the entire theory of Electromagnetism by adding the Lorentz Transformation to the definition of.

By relating the field energy density to charge density our theory becomes non-linear. This is the specificity of our theory, which is applicable both for an elementary charge as it is for an ensemble of elementary charges.

Note that reading our papers [1] [2] [3] [4] is helpful for a full understanding of this paper.

2 The Electric Field of a Moving Charge

This section is based on [5]. Let a point charge Q reside at the origin of a frame k, Fig. (1). The electric field **E** has the magnitude HQ/r^2, and for a positive charge it is directed radially outward.

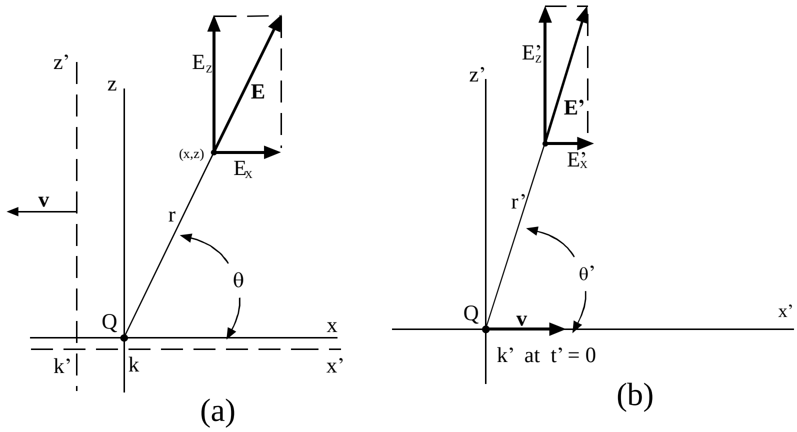
In the xz plane its components at any point (x, z) are:

$$\begin{aligned} E_x &= \frac{HQ}{r^2}\cos\theta = \frac{HQx}{\left(x^2+z^2\right)^{\frac{3}{2}}} \\ E_z &= \frac{HQ}{r^2}\sin\theta = \frac{HQz}{\left(x^2+z^2\right)^{\frac{3}{2}}} \end{aligned} \tag{1}$$

The frame k' moves in the negative x direction, with speed **v**. According to the Lorentz transformation, the relations between the coordinates of an event (a spacetime point) in the two frames are:

$$x = \gamma(x' - \beta ct') \qquad y = y' \qquad z = z' \qquad t = \gamma\left(t' - \frac{\beta x'}{c}\right) \tag{2}$$

$E'_z = \gamma E_z$ and $E'_x = E_x$. We express the field components E'_z and E'_x in terms of the coordinates in k' using equations (1) and (2). At the instant t' = 0, when Q passes the origin in k':



(a) A charge at rest in space

(b) A charge moving with constant velocity v relative to space

Fig. (1) The Electric Field of a Point Charge, Q

$$
\begin{aligned}
E'_x &= E_x = \frac{\gamma HQx'}{\left[(\gamma x')^2 + z'^2\right]^{3/2}} \\
E'_z &= \gamma E_z = \frac{\gamma HQz'}{\left[(\gamma x')^2 + z'^2\right]^{3/2}}
\end{aligned}
\qquad (3)
$$

Note first that $E'_z / E'_x = z'/x'$. This tells us that the vector **E**' makes the same angle with the x' axis as does the radius vector **r**'. Hence **E**' points radially outward along a line drawn from the instantaneous position of Q, as shown in Fig. (1b).

To find the strength of the field, we calculate $E'^2_x + E'^2_z$, which is the square of the magnitude of the field, E'^2.

$$E'^2 = E'^2_x + E'^2_z = \frac{\gamma^2 H^2 Q^2 \left(x'^2 + z'^2\right)}{\left[(\gamma x')^2 + z'^2\right]^3} = \frac{H^2 Q^2 \left(x'^2 + z'^2\right)}{\gamma^4 \left[x'^2 + z'^2 - \beta^2 z'^2\right]^3}$$

$$= \frac{H^2 Q^2 \left(1 - \beta^2\right)^2}{\left(x'^2 + z'^2\right)^2 \left(1 - \frac{\beta^2 z'^2}{x'^2 + z'^2}\right)^3} \tag{4}$$

Let r' denote the distance from the charge Q, which is momentarily at the origin, to the point (x', z') where the field is measured: $r' = \left(x'^2 + z'^2\right)^{1/2}$. Let θ' denote the angle between this radius vector and the velocity of the charge Q, which is moving in the positive x' direction in the frame k'. Then since z' = r' sin θ', the magnitude of the field can be written as:

$$E' = \frac{HQ}{r'^2} \frac{1 - \beta^2}{\left(1 - \beta^2 \sin^2 \theta'\right)^{3/2}} \tag{5}$$

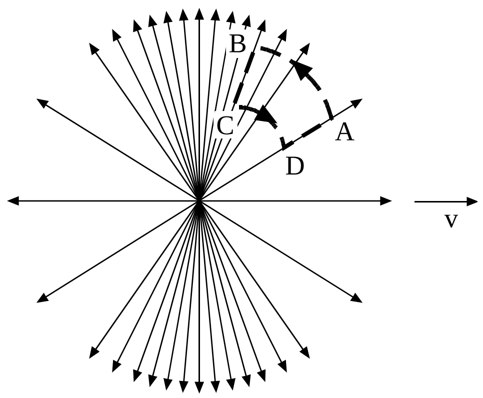


Fig. (2) Representation of the Field of a Uniformly Moving Charge

For low speeds the field reduces simply to $E' \approx HQ/r'^2$. But if β^2 is not negligible, at the same distance from the charge, the field is stronger at right angles to the motion than in the direction of the motion, A simple representation of the field is shown in Fig. (2). A cross-section through the field with some field lines in the x' z' plane is indicated. For the field in the x'y' plane we get an identical representation.

Purcell (1963) [5]: This is a remarkable electric field. It is not spherically symmetrical, which is not surprising because in this frame there is a preferred direction, the direction of motion of the charge. Also, it is a field that **no stationary charge distribution**, whatever its form, could produce. For in this field the line integral of **E**' is not **zero** around every closed path. Consider, for example, the closed path ABCD in Fig. (2). The circular arcs contribute nothing to the line integral, being perpendicular to the field; on the radial sections, the field is **stronger** along BC than along DA, so the **circulation** of **E**' on this path is not zero.

Fig. (2) is a natural result of the charge being a longitudinal wavepacket. As such it becomes contracted in the direction of motion, [3] [6] [7]. Charge density is related to space curvature [2], hence the strong field perpendicular to the direction of motion.

3 Forces between Moving Charges

We consider, [8], two particles of equal charge, Q, moving with equal uniform velocity **v**. Let **v** be along the *x*-axis of frame k, and let the particles have the same *x*-coordinate, and their separation be r, see Fig. (3a). We expect the charges to exert forces upon one another, a repulsive electric force and an attractive magnetic force, as we show.

Let k' move with uniform velocity **v** relative to k along common *x-x'* axis, so that the charges are at rest in k', see Fig.(3b).

(a) Two particles of equal charge Q move with equal uniform velocity v in frame k, their separation being r.

(b) The same situation in k', which moves relative to k with a velocity v.

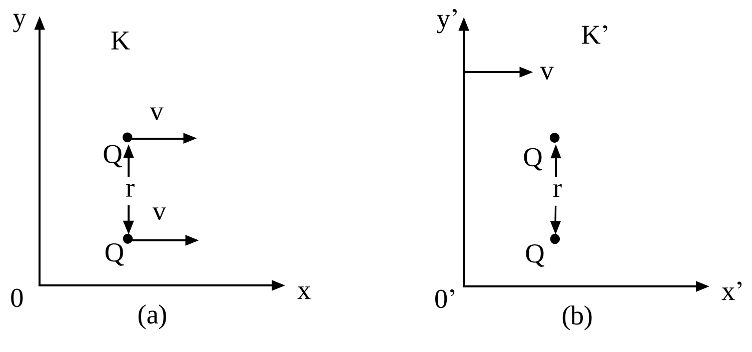


Fig. (3) The Magnetic Field of a Moving Point Charge

Here, there is no magnetic force at all and the electric force is repulsive: the charges would tend to move apart along the *y'*- direction, each exerting a force on the other, of magnitude:

$$F'_y = \frac{HQ^2}{r^2} \tag{6}$$

The force on the upper charge is + F_y' and that on the lower charge is –F'_y. Note that charge invariance is assumed and that the separation r is unchanged by the transformation from k to k'. We now use the general force transformation equations (Equations. 3-33 in [8]) for a force with components F_x', F_y' and F_z', in the S' frame, to obtain the force components F_x, F_y and F_z acting on the particle in the k-frame.

$F_x = F_x' = 0$

$F_z = F_z'/\gamma = 0$ and:

$$F_y = F_y'/\gamma = F_y'\sqrt{1-v^2/c^2} = \frac{HQ^2}{r^2}\sqrt{1-v^2/c^2} = \frac{H}{\gamma}\frac{Q^2}{r^2} \tag{7}$$

Hence, the net force is in the positive *y*-direction, but smaller than in (6). The charged particles repel one another and the coulomb electric force of repulsion must exceed the force of attraction which we call the **magnetic force**.

For $v < c$ the electric force is always greater than the magnetic force. Only when $v = c$ does the net force become zero.

When $v \to 0$, we return to the static result wherein only an electric force exists.

The magnetic force, which exists only when $v \neq 0$, is a second-order effect compared to the electric force, that is, it enters as $(v/c)^2$.

4 The Magnetic Field B, the Vector Potential A and the Lorentz Force

4.1 The Magnetic Field

The electric field of a moving charge at velocity **v**, perpendicular to **v** is:

$$E'_y = \gamma \frac{HQ}{r^2}$$ see equation (5), for the case in which $\theta' = \frac{\pi}{2}$.

This field applies a repulsive force F'_y on another charge Q that runs in parallel, which is:

$$F'_y = \gamma \frac{HQ^2}{r^2}$$ On the other hand, equation (7), shows that the net force is smaller:

$$F_y = \frac{H}{\gamma}\frac{Q^2}{r^2} \tag{7}$$

where: $F_y = F'_y + F''_y$ and we want to find the attractive force F''_y :

$$F''_y = F_y - F'_y = \frac{1}{\gamma}\frac{HQ^2}{r^2} = \left(\frac{1}{\gamma} - \gamma\right)\frac{HQ^2}{r^2}$$

$$\left(\frac{1}{\gamma} - \gamma\right) = \left(\frac{1-\gamma^2}{\gamma}\right) = -\gamma\beta^2$$ hence, for the attractive force (magnetic part) :

$$F''_y = -\gamma\beta^2 \frac{HQ^2}{r^2} = -\beta^2 QE$$ and for the net force:

$$F_y = \gamma \frac{HQ^2}{r^2} - \gamma\beta^2 \frac{HQ^2}{r^2} = QE - Q\beta^2 E = QE\left(1-\beta^2\right)$$

This result is for $\mathbf{v} \perp \mathbf{E}$, but we can show that the general expression for the attractive force, F''

, is:

$$\mathbf{F}'' = Q\frac{\mathbf{v} \times (\mathbf{v} \times \mathbf{E})}{c^2} \tag{8}$$

Thus the net force **F is**:

$$\mathbf{F} = Q\left\{\mathbf{E} + \frac{\mathbf{v} \times (\mathbf{v} \times \mathbf{E})}{c^2}\right\} \tag{9}$$

For different velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ of the charged particles, we can show that in general:

$$\mathbf{F} = Q\left[\mathbf{E} + \frac{\mathbf{v}_2 \times (\mathbf{v}_1 \times \mathbf{E})}{c^2}\right] \tag{10}$$

where $\mathbf{v}_1$ is the velocity of the particle that creates the field **E** and $\mathbf{v}_2$ is the velocity of the other particle in this field, and vice versa. The expression:

$$\mathbf{B} = \frac{1}{c}(\mathbf{v}' \times \mathbf{E}) \qquad \textbf{Magnetic Field} \tag{11}$$

is defined as the **magnetic field** created by a particle moving at speed **v**'.

The dimensions of **B** and **E** are the same: [E] = [B] = LT^{-2}, see [1], and both express the elastic displacement.

5 The Lorentz Force

Using (10) and (11) gives the force on a charge Q moving at speed **v**:

$$\mathbf{F} = Q\left[\mathbf{E} + \frac{1}{c}(\mathbf{v} \times \mathbf{B})\right] \qquad \textbf{Lorentz Force} \tag{12}$$

6 The Vector Potential A

The magnetic field is merely the displacement vector **u** created by a moving charge. In this case, **u** is not considered as a gradient of a scalar function φ, namely a polar vector, but an axial vector which is a rotor of a **vector potential A**. In the static case:

$\mathbf{E} = -\nabla\varphi$ whereas, in the general case, we have to add the axial component:

$\mathbf{B} = \nabla \times \mathbf{A}$ $[A] = [\varphi] = L^2 T^{-2}$ Note that:

$$\mathbf{u} = (1/H)(\mathbf{E}+\mathbf{B}) \tag{13}$$

According to (11):

$$\nabla \times \mathbf{A} = \frac{1}{c}(\mathbf{v}' \times \mathbf{E}) \tag{14}$$

and hence:

$$\mathbf{A} = \frac{H}{c}\frac{Q\mathbf{v}'}{r} = \frac{H}{c}\frac{\mathbf{i}}{r}\,. \tag{15}$$

$$\mathbf{i} = Q\mathbf{v}' \quad [i] = L^4T^{-1} \quad \text{and} \quad \mathbf{j} = q\mathbf{v}' \quad [j] = LT^{-1} \tag{16}$$

are defined as the **electric current** and the **current density** respectively. Hence:

$$\mathbf{A} = \frac{H}{c}\int_{\tau'}\frac{\mathbf{j}'\cdot d\tau'}{|\mathbf{r}-\mathbf{r}'|} \tag{17}$$

$$\mathbf{i} = \int_{\sigma}\mathbf{j}\cdot\mathbf{d\sigma} \tag{18}$$

If the electric current flows in a closed circle the displacement **B** creates a torsional space deformation and, hence, space contraction. This phenomenon is related to gravitation [3] and [4].

Examples of **B** and **A** Created by Constant Currents

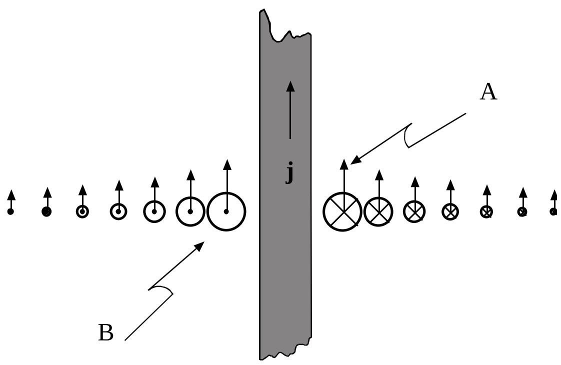


Fig. (4) The Vector Potential A

Fig. (4) shows **B** and **A** created by a constant current in a straight wire.

Fig. (5) shows B and A created by a constant current in a circular turn. B is perpendicular to the plane.

Note the relation of the current to the helical motion of a positron (or that of an electron moving in the opposite direction).This helical motion creates **B,** which is a component of the displacement vector **u**.

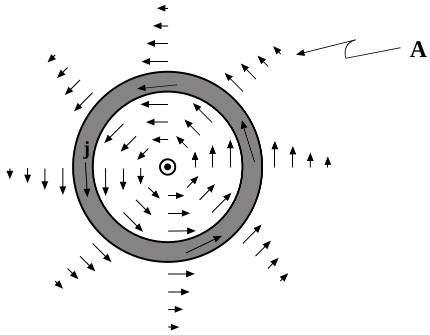


Fig. (5) The Vector Potential A

7 The Current Equation of Continuity

We have seen, (18), that:

$$\mathbf{i} = \int_\sigma \mathbf{j} \cdot \mathbf{d\sigma} \tag{18}$$

For a given zone τ, with a surface σ, in which there is a charge Q we get:

$$\frac{dQ}{dt} = \frac{d}{dt}\int_\tau \frac{\partial q}{\partial t} d\tau$$, but:

$$\frac{dQ}{dt} = -\int_\sigma j_n \cdot d\sigma \tag{19}$$

where **n** is the unit vector perpendicular to the surface σ at a given point.

According to **Gauss Lemma**:

$$\int_\sigma j_n \cdot d\sigma = \int_\tau (\nabla \cdot \mathbf{j}) d\tau = -\int_\tau \frac{\partial q}{\partial t} d\tau$$ This implies the equality of the integrands, namely:

$$\nabla \cdot \mathbf{j} + \frac{\partial q}{\partial t} = 0 \qquad \textbf{Current Equation of Continuity} \tag{20}$$

In the GDM, both **j** and q include the contribution of the field energy.

8 Ampère's Law

$$\mathbf{A} = \frac{H}{c}\int_{\tau'} \frac{\mathbf{j}' \cdot d\tau'}{|\mathbf{r} - \mathbf{r}'|} \tag{17}$$

therefore:

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{H}{c}\nabla \times \int_{\tau'} \frac{\mathbf{j}' \cdot d\tau'}{|\mathbf{r} - \mathbf{r}'|} \qquad (\ \nabla \cdot \mathbf{B} = 0\ ) \tag{21}$$

$$\mathbf{B} = \frac{H}{c}\int \mathbf{j}' \times \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2} d\tau' \tag{22}$$

but:

$$\nabla \times \mathbf{B} = \frac{H}{c}\nabla \times \nabla \times \int \frac{\mathbf{j}' d\,\tau'}{|\mathbf{r} - \mathbf{r}'|} \tag{23}$$

The identity ∇x(∇x**A**) = **-∇(∇·A) -∇^2A** for an arbitrary vector **A**, transforms (23) into:

$$\nabla \times \mathbf{B} = H\nabla \int \mathbf{j}(\mathbf{r}') \cdot \nabla \frac{1}{|\mathbf{r} - \mathbf{r}'|} d\tau' - H \int \mathbf{j}(\mathbf{r}') \cdot \nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) d\tau' \tag{24}$$

If we use, see [9], the relations:

$$\nabla\left(\frac{1}{|\mathbf{r} - \mathbf{r}'|}\right) = -\nabla'\left(\frac{1}{|\mathbf{r} - \mathbf{r}'|}\right) \qquad \text{and:} \qquad \nabla\left(\frac{1}{|\mathbf{r} - \mathbf{r}'|}\right) = -4\pi\delta(\mathbf{r} - \mathbf{r}')$$

the integrals in (24) can be written:

$$\nabla \times \mathbf{B} = H\nabla \int \mathbf{j}(\mathbf{r}') \cdot \nabla'\left(\frac{\mathbf{1}}{|\mathbf{r} - \mathbf{r}'|}\right) d\tau' + 4\pi H \mathbf{j}(\mathbf{r}) \qquad \text{Integration by parts yields:}$$

$$\nabla \times \mathbf{B} = 4\pi H \mathbf{j} + H\nabla \cdot \int \frac{\nabla' \cdot \mathbf{j}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\tau'$$

But for steady-state magnetic phenomena $\nabla \cdot \mathbf{j} = 0$, hence:

$$\nabla \times \mathbf{B} = 4\pi H\mathbf{j} \qquad \textbf{Maxwell equation} \tag{25}$$

From (25), by applying Stokes' theorem, we obtain the integral equation:

$$\int_\sigma (\nabla \times \mathbf{B}) \cdot \mathbf{d\sigma} = 4\pi H \int_\sigma \mathbf{j} \cdot \mathbf{dr} \tag{26}$$

and transforming (26) into: $\oint_c \mathbf{B} \cdot \mathbf{dl} = 4\pi H \int_\sigma \mathbf{j} \cdot \mathbf{d\sigma}$ gives:

$$\oint_c \mathbf{B} \cdot \mathbf{dl} = 4\pi H\mathbf{i} \qquad \textbf{Ampère's law} \tag{27}$$

Maxwell "fixed up" (25) by adding to it the **displacement current** $\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$:

$$\nabla \times \mathbf{B} = \frac{4\pi H}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \qquad \textbf{Maxwell equation} \tag{28}$$

9 Comments on the Electric Field E, Magnetic Field B, Potential Vector A and Lorentz Force

Equation (13), notated here (29), is:

$$\mathbf{u} = (1/H)\,(\mathbf{E}+\mathbf{B}) \tag{29}$$

E expresses the polar displacement and **B** the axial displacement. This is compatible with **E** and **B** being the components of **the four-field anti-symmetric EM tensor:**

$$F_{oi} = -E_i \qquad F_{ij} = \varepsilon_{ijk} B_k \tag{30}$$

where ε is the anti-symmetric Levi-Civita symbol, see [9].

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix} \tag{31}$$

Space has an interesting feature: the vector potential **A** expresses a kind of **potential momentum**, that creates an axial displacement $\mathbf{B} = \nabla \times \mathbf{A}$. This situation is similar to the well-known behavior of a spring: load it and it twists (the radius increases), see Fig. (6).

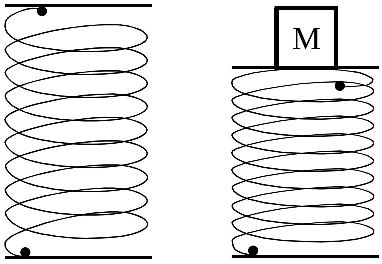


Fig. (6) The Vector Potential as a Potential Momentum

Note that equation (12), which here is notated (32), can be written as (33):

$$\mathbf{F} = Q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \qquad \textbf{Lorentz force} \tag{32}$$

$$\mathbf{F} = Q\left[\nabla\varphi + \frac{\mathbf{v}}{c} \times (\nabla \times \mathbf{A})\right] \tag{33}$$

10 Faraday's Law of Induction

When currents change, there is also a change in **A** and **B**. But a change in **B** means a change in the displacement **u**. In this case, a charge is not displaced because of a direct force, but because of a displacement of the zone of space in which it is immersed. In other words, the background in which the charge is immersed moves.

In this case, we should look upon q**A** as the **momentum potential density** in space of the charge density, q, immersed in it, and on the time derivative of q**A** as the force required to move it. A good guess of the force density is, therefore, the expression:

$$\mathfrak{f} = -q\nabla\varphi - \frac{q}{c}\frac{\partial A}{\partial t} \tag{34}$$

and for the force:

$$\mathbf{F} = -Q\nabla\varphi - \frac{Q}{c}\frac{\partial \mathbf{A}}{\partial t} \tag{35}$$

F = Q**E**, and therefore:

$$E = -\nabla\varphi - \frac{1}{c}\frac{\partial A}{\partial t} \tag{36}$$

By applying the operator $\nabla\times$ to both sides, we get the:

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \textbf{Maxwell equation} \tag{37}$$

From the Maxwell differential equation, we get the integral relation:

$$\int_\sigma (\nabla \times \mathbf{E}) \cdot \mathbf{d\sigma} = \oint_c \mathbf{E} \cdot \mathbf{dl} \tag{38}$$

and on the other hand:

$$\int_\sigma (\nabla \times \mathbf{E}) \cdot \mathbf{d\sigma} = -\frac{1}{c}\frac{\partial}{\partial t}\int_\sigma \mathbf{B} \cdot \mathbf{d\sigma} \tag{39}$$

and therefore:

$$\oint_c \mathbf{E}\cdot\mathbf{dl} = -\frac{1}{c}\frac{\partial}{\partial t}\int_\sigma \mathbf{B}\cdot\mathbf{d\sigma} \qquad \textbf{Faraday's law of induction} \tag{40}$$

$$\int_\sigma \mathbf{B} \cdot \mathbf{d\sigma} = \phi \tag{41}$$

where ϕ is the **magnetic flux** through the surface σ.

11 The Vector Potential as the Momentum Potential in Space

We show that $\frac{Q}{c}\mathbf{A}$ can be considered as the potential momentum of the electromagnetic field, i.e., the momentum stored in space, momentum available for conversion to kinetic energy, see [10].

By substituting: $\mathbf{B} = \nabla \times \mathbf{A}$ and: $\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$ in the Lorentz law of force:

$\mathbf{F} = Q\mathbf{E} + Q\frac{\mathbf{v}}{c} \times \mathbf{B}$ we obtain:

$$\mathbf{F} = -Q\nabla\varphi - \frac{Q}{c}\frac{\partial \mathbf{A}}{\partial t} + \frac{Q}{c}\mathbf{v} \times \nabla \times \mathbf{A}. \tag{42}$$

Note identity (8) in the book [11]:

$$\mathbf{v} \times (\nabla \times \mathbf{A}) = \nabla(\mathbf{v} \cdot \mathbf{A}) - (\mathbf{A} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \mathbf{A})\mathbf{A} - \mathbf{A} \times (\nabla \times \mathbf{v}) \tag{43}$$

Assuming that Q moves at a constant velocity**, v,** in a field with a uniform **A**, the second, third and fourth terms in (43) equal zero. We therefore get:

$$\mathbf{v} \times (\nabla \times \mathbf{A}) = \nabla(\mathbf{v} \cdot \mathbf{A}) \tag{44}$$

Substituting (44) in (42) enables the reorganization of (42) such that:

$$\frac{d}{dt}\left(M\mathbf{v} + \frac{Q}{c}\mathbf{A}\right) = -Q\nabla\left(\varphi - \frac{1}{c}\mathbf{v} \cdot \mathbf{A}\right) \tag{45}$$

We can relate to (45) as follows:

The force is the time derivative of the generalized momentum:

$\mathbf{P} = \left(M\mathbf{v} + \frac{1}{c}Q\mathbf{A}\right)$ that is equal to the gradient of the potential energy:

$U = Q\left(\varphi - \frac{1}{c}\mathbf{v} \cdot \mathbf{A}\right)$ hence: $\frac{\partial \mathbf{P}}{\partial t} = -\nabla U$ Let us adopt the known expressions:

$M\mathbf{v} + \frac{Q}{c}\mathbf{A}$ **Conjugate Momentum** (46)

$Q\left(\varphi - \frac{1}{c}\mathbf{v} \cdot \mathbf{A}\right)$ **Interaction Energy** (47)

To the **Potential Energy**:

$$Q\varphi \tag{48}$$

we can add **Potential Momentum**:

$$\frac{Q}{c}\mathbf{A} \tag{49}$$

Which, in quantum theory, is related to the operator $\frac{\hbar}{i}\nabla$.

12 The Electromagnetic Equations

12.1 Classical Electromagnetism

Electromagnetic theory can be fully expressed by the four **Maxwell's equations**:

I	$\nabla \cdot \mathbf{E} = 4\pi Hq$	**Coulomb's Law**	see [1]
II	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	**Faraday's Law**	(37)
III	$\nabla \cdot \mathbf{B} = 0$	**No magnetic monopoles**	
IV	$\nabla \times \mathbf{B} = \frac{4\pi H}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$	**Ampère's Law**	(28)

These equations can also be expressed in terms of differential equations for the scalar and vector potentials:

$$\Box\, \varphi = -4\pi Hq \tag{48}$$

$$\Box\, \mathbf{A} = -\frac{4\pi H}{c}\mathbf{j}$$

(49)

Using the potential 4-vector $A^\mu = (\varphi, \mathbf{A})$ and the current density 4-vector $j^\mu = (j^0, \mathbf{j}) = (cq, \mathbf{j})$ we get:

$\Box A^\mu = -\frac{4\pi H}{c} j^\mu$ as the potential 4-vector equation. Since:

$F_{\lambda\rho} = \partial_\lambda A_\rho - \partial_\rho A_\lambda$

the **covariant Maxwell's equation** can also be written as:

$$\partial_\mu F^{\mu\nu} = -\frac{4\pi}{c} \mathbf{j}^\nu \tag{50}$$

12.2 Gauge Transformation

The correspondence between $F_{\mu\nu}$ and A_μ is not unique. Maxwell's equations are invariant under the gauge transformation:

$$A_\mu \rightarrow A_\mu' = A_\mu - \partial_\mu \psi \tag{51}$$

where ψ is an arbitrary spacetime dependent scalar function (the gauge function). A change of A_μ does not change the EM representation by **E** and **B** fields. The relevant electric field **E**, in this case, is $\mathbf{E} = -\frac{1}{c}\frac{\partial A}{\partial t}$ and the magnetic field is simply $\mathbf{B} = \nabla \times \mathbf{A}$.

13 The Electromagnetic Equations

13.1 The Equivalence of Energy Density and Charge Density

Our charge density is $q = 1/4\pi \cdot (\rho-\rho_0)/\rho$. This charge density is positive if $\rho > \rho_0$ and negative if $\rho < \rho_0$. But space density ρ depends on the space energy density ϵ, [12].

13.2 Our EM equations

To obtain our EM non-linear equations we use Maxwell's equations and modify the expression $j^{\mu} = (cq, \mathbf{j}) = (cq, q\mathbf{v})$ by adding the contribution of the field energy (this, however, is out of the scope of this paper).

$$q \to q_{source} + \frac{1}{8\pi H s^2} (\nabla\varphi)^2$$

, where H = 1 [H] = T^{-2} see [1], [$\nabla\varphi$] = LT^{-2}, s = 1, [s] = LT^{-1}, see [2] and [q] = 1.

Thus both gauge invariance and charge conjugation are lost.

13.3 On Charge Conjugation and Gauge Invariance

The modified classical potential field equation includes the contributions of the energy density and the current of energy density of the field, which are equivalent to the charge density and current density, respectively. Maxwell's equations are the approximated equations of our Electromagnetism where **the energy density of the field is neglected**. The electric charge, Q, of an elementary particle is not a constant. As the distance from the particle increases, the absolute value of the charge decreases. **E** is, therefore, proportional not only to$1/r^2$ but also to the changing Q.

In the field there is a change in space density that affects c. Thus, c depends on the potential as in the case of gravitation, and we can therefore expect light bending. However, this bending is different for positive and negative charges. Thus, there is no **charge conjugation**, nor **gauge invariance** close to a charged particle.

Charge conjugation: the results of experiments are independent of alternating the sign of all charges.

Gauge invariance: the results of experiments are independent of the choice of the gauge for the potentials.

14 Summary

In the classical Maxwell EM, space does not play any role, whereas in our theory EM is merely the Geometrodynamics of space. This new understanding enables the turning of the form of the equations from linear into non-linear, which is also the case for QED. This is the specificity of the theory. The new understanding adds tangibility and enables the derivation of the theory from merely the definition of charge density and the Lorentz Transformation.

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Data available with the paper

The authors declare that the data supporting the findings of this study are available within the paper.

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