The Unification of Gravitation and Electromagnetism

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Abstract

All past efforts to unify gravitation and electromagnetism failed because they considered energy/momentum to be the common denominator in this unification. The problem was that nobody knew how mass curves space and that charge, as we have shown in one of our papers, also curves space. It was naively believed that if mass, namely energy, curves space then so will the electromagnetic energy and in the same way. We took an entirely different approach. Our common denominator, as we show, is the deformation (distortion, curving) of space by both mass and charge. This approach yields the expected result.

Key Words: Gravitation; General Relativity; Riemannian Geometry; Electromagnetism

1 Introduction

We have achieved the unification of gravitation and electromagnetism by reassessing almost all we know about gravitation, electromagnetism and the geometry of space.

1.1 Our Perception of Space and General Relativity (GR)

Space in GR is considered a continuous 3D manifold, **bent** (curved) in a 4D hyperspace - bent by energy/momentum. We, however, consider space to be a 3D **deformed** lattice rather than a bent continuous manifold. We present in another paper a new geometry for this kind of space [1]. This new geometry uses the same terms as Riemannian Geometry, the geometry of bent manifolds. In General Relativity (GR), therefore, instead of considering space distortion by mass to be bending, we consider it a contraction, with no need to change the mathematical formalism, based on Riemannian Geometry.

Note that in this paper we sometimes use the term **curving** although we mean **deforming** or specifically **contraction** or **dilation**. The text will clarify what we mean.

1.2 The Problem with the Right-hand Side Term of the GR Equation

Einstein complained that the left-hand side of the GR equation expresses curving whereas the right-hand side merely expresses energy/momentum and not curving. He expected that future physics will show us how energy/momentum really curves space and will also express it in terms of curvature.

We show that the right-hand side term of the GR equation expresses contraction (curving) as does the left. And we also show that this contraction is due to the angular momentum of the particles that compose a mass.

1.3 Gravitation – Space Contraction - by Angular Momentum of Particles

We show that **the rotation (spin) of an elementary particle is the basic mechanism by which it gravitates - curves space** - in addition to the curving by its charge. This idea came to us following Project "Gravity Probe B" [2] recent validation of the GR predicted "Frame Dragging" [3], [4], phenomenon. This phenomenon is considered to be a result of an additional mechanism by which a rotation of a macroscopic mass contracts (curves) space around it and contributes to gravitation, as the Kerr Metric shows [5], [6]. We asked ourselves: if Frame Dragging is a macroscopic phenomenon could it be that it is also a microscopic phenomenon related to the spin of elementary particles. Our affirmative answer paved the way to unification.

1.4 Electric Charge and its Field as Space Contraction or Dilation

Our paper [7] presents a model of the positive elementary electric charge and its field as a contracted zone of space, and the negative elementary electric charge and its field as a dilated zone of space. Relating to space as a lattice (cellular structure), we define Space Density ρ as the number of space cells per unit volume (denoted ρ_0 for space with no deformations). Based on this definition, we define electric charge density. With this definition alone, we derive electrostatics, without any phenomenology, and together with the Lorentz Transformation - the entire Maxwell theory.

Based on [7] we show how charge curves space [8].

1.5 The Common Denominator Needed for Unification

By attributing space contraction or dilation to charge, and space contraction to mass, we have created a common denominator for both electromagnetism and gravitation. This common denominator enables us to extend Einstein's GR field equation to accommodate electromagnetism.

1.6 Our Work-plan

Prove that the right-hand side term of mass/energy, in the GR equation, expresses contraction (curving). Einstein's dream was to show that this is indeed the case.

Prove that it is the angular momentum of the elementary particles that contract space around them. Thus, revealing how mass creates gravitation.

Present our result that positive charge contracts space whereas negative charge dilates it.

Calculate the ratio of curvature (deformation) - by charge to curvature (deformation) - by mass/energy.

Use the ratio of curvature by angular momentum to the curvature by charge to get the factor needed to incorporate the contribution of charge in the GR equation.

Incorporate the charge/current tensor in the GR equation to extend it.

2 The Right-Hand Side of GR Equation Expresses Space Contraction

The right-hand side of Einstein's field equation (1) of GR (below) **should** express curvature exactly as the left-hand side does. This paper shows, for the first time, that this is indeed the case.

The need to express curvature by Riemannian geometry and obtain a covariant formulation of physical laws, by using tensors, led Einstein to the equation of General Relativity (GR):

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = \frac{8\pi G}{c^4} \cdot T_{ij} \tag{1}$$

R_{ij} is the Ricci contracted Riemannian tensor and R is the Ricci scalar, $\frac{1}{2}R$ is the familiar Gaussian Curvature [9] and Λ is the cosmological constant. The $\frac{1}{2}Rg_{ij}$ term is added to give a covariant divergence, which is identically zero, and $T^{00} = j^{(0)0} = \epsilon$ is the energy density of space (see Appendix C). In this paper we ignore the Cosmological Constant term Λg_{ij}.

It was Einstein's vision that "future physics" will show, as we do below, that the right hand-side of (1): $\frac{8\pi G}{c^4} \cdot T_{ij}$ is also curvature (and not simply have the same dimensionality).

In equation (2), see [10] and [11]:

R_c is the **Gaussian radius of curvature**, and:

$K = 1/R_c^2$ is the **Gaussian curvature** at the radial distance r from the center of a mass with Schwarzschild Radius r_S (our notations are different from those in [10] and [11]):

$K=1/R_c^2 = r_s/r^3$ (2)

The right-hand side of the GR equation is $8\pi G/c^4 \cdot T_{ij}$. Relation (2) enables us, for the first time to show that $8\pi G/c^4 \cdot T_{ij}$ expresses curvature.

According to [11] p.290 "*Ignoring curvature effects, the Schwarzschild radius of a body of uniform density m and radius* r *is* $r_S = 2GM/c^2 \sim 8\pi G/3c^2 \cdot mr^3$ " , where m is the mass density. But $m = \epsilon/c^2$ Hence:

$K=1/R_c^2 = r_S/r^3 = (1/3)\ 8\pi Gm/c^2 = (1/3)\ 8\pi G\, \epsilon /c^4$ where $\epsilon = T_{00}$ is the energy density. **We thus conclude** that $8\pi G/c^4 \cdot T_{ij}$ is indeed curvature (ignoring the factor 1/3, which is related to the suggested uniform density).

An elementary particle, besides its direct contraction or dilation of space which is related to its charge, has an additional mechanism by which it contracts space around itself (creates positive curvature). This contraction is. This mechanism of contraction, related to its energy content, is (see Section 3) due to the torsion in space created by the spin of the elementary particle, and hence related to its **inertial mass**.

All this proves that **gravitational mass is inertial mass.**

Note that energy is **purely** electromagnetic [8] and that elementary charges are **kinds of black and white holes** that curve space drastically [7], [8].

3 The Positive Curving of Space by the Angular Momentum of an Elementary Particle

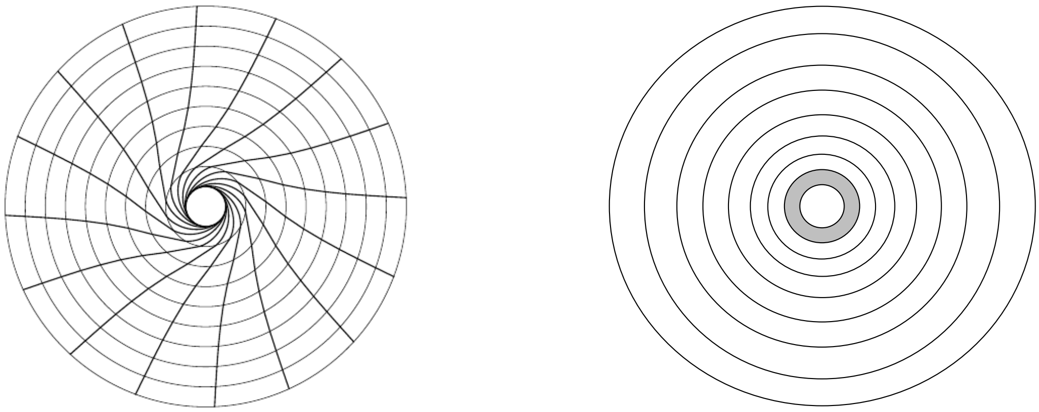
In [8] we consider the electrons/positrons as circulating longitudinal wavepackets of dilation/ contraction, which are their elementary electric charges. Thus, hinted by the validation of "Frame Dragging", it is logical to explore the possibility that gravitation is the result of their angular momentum due to their circulation.

Our conjecture is that a longitudinal wavepacket applies pressure on the space lattice and bends its sides perpendicular to the wavepacket propagation direction. This bending is the "Frame Dragging" and it causes the contraction of space around the circulation track. As a result, particles carrying the same or different elementary charges, have an identical minute positive additional curvature that depends on their Angular Momentum. The sum of two additional curvatures created by a pair of particles, carrying different types of charge, is the gravitational curvature of **neutral** matter. Fig. (1a) shows how an anti-clockwise rotating macroscopic mass (the inner circle) drags a radial frame; this is the kind of figure that usually appears in the literature. But this dragging causes space torsion around the mass and hence contraction, which is discussed in the literature on elasticity [12]. This contraction is presented in Fig.(1b); it shows that the concentric frame circles in Fig. (1a) should be corrected to show the contraction due to the torsion that takes place mainly in the gray zone around the mass. These figures represent the dragging and contraction created by a longitudinal wavepacket of radius r_e represented by the thickness of the gray annular ring of the circulating wavepacket with radius R_e .

Note that this type of space contraction, due to lattice bending, is not dependent on the direction of rotation of the wavepacket. Note also that the contraction is spherically symmetric only on average.

This is also relevant to trios or duos of quarks, which are twisted electrons or positrons according to our theory of matter [13].

Our model of quarks [13] leads to the conclusion that in neutral matter the number of electrons equals the number of positrons. Thus, the long-standing issue of **"where is anti-matter?"** is resolved.



a b

Fig. (1) Frame Dragging

The question that we have to ask now becomes "why almost all of the nucleuses in the universe are positive?" It seems that since positive charge curves space positively, atoms with a positive nucleus are much more stable, but this issue is out of the scope of this paper.

Rotation induces Frame Dragging [3], [4], [5], [6], which is a torsional contraction (positive curving). This phenomenon is expressed by the Kerr metric [6] equation (14.22) p.303. We, however, repeat the procedure used in Section 2, in which we rewrote the Schwarzschild radius, $r_S = 2GM/c^2$, by considering mass to be purely electromagnetic [8]. Here we rewrite the Schwarzschild radius by expressing the mass of an elementary particle by its angular momentum $J=1/2\ \hbar$. Since $J= MRv$ we get for the elementary mass:

$M_e=1/2\ \hbar/R_e c$

where R_e is defined in [8]. This gives:

$$r_S = G\hbar/R_e c^3 \tag{4}$$

Using equation (2) we get for the curvature created by spin:

$$K_S=1/R_c^2 = r_S/r^3 = G\hbar/R_e c^3 r^3 \tag{5}$$

Note that M in this discussion is the inertial mass. Thus, it is the inertial mass that creates gravitation, and we are **allowed to eliminate** the concept of **"gravitational mass".**

Note that instead of proving the **equivalence** of inertial mass and gravitational mass we are proving their **identity**.

The Kerr contribution to the **square of the line-element** that appears in equation (14.22 p303) in [6] is:

$4GJ/c^3r^2 \, Sin^2 \, \theta(r \, d\varphi) \, cdt$

By taking J in (6) as $J = 1/2 \cdot \hbar$ and the average of $Sin^2\theta$ as 1/2, we are getting an expression compatible with (4).

Note that, since both charge and gravitation are quantized, the total space curvature is quantized. Thus, "singularity" turns into merely a mathematical issue.

4 Space Contraction and Dilation by Charge

In a recent paper [6] titled "On the Essence of Electric Charge" Part 1 we consider positive electric charge to be a contracted zone of space and negative electric charge to be a dilated zone of space. This yields the Maxwell theory of electromagnetism, with no phenomenology. We show that deformed spaces are represented by our geometry [1] in the same way as are bent manifolds by Riemannian geometry. Hence, we can attribute positive curvature to a contracted zone of space, namely to a positive charge, and negative curvature to a negative charge. This attribution enables us, in Part 2 [8], to show that the bivalent charges are kinds of electrical black and white holes, and derive and calculate their radii. We further derive and calculate the masses of the leptons and quarks. All this serves as the basis for our discussion on gravitation in this paper.

According to [7], [8] the energy of an elementary particle is purely electromagnetic. The absolute values of the bivalent charges are the same, and their absolute values of curving space are precisely equal. The curvature, contributed by pairs of these bivalent charges, to a neutral macroscopic body is therefore zero. What is left over is the additional curving due to their angular momentum, which is very small in comparison, as is the gravitation

Note that we consider quarks to be sub-tracks of topologically-twisted electrons or positrons and not elementary particles in their own right. This is how we understand confinement and the assignment of 1/3 Q_e or 2/3 Q_e to quarks. This enables us to derive and calculate the quarks' masses [13].

The equation:

$$1/R_c^2 = r_s\rho/r^3$$

is a corrected (2), that takes into account the space density ρ, see [7] and [8], as explained below.

The absolute charge values of the bivalent elementary charges are equal $|Q_+| = |Q_-|$. Hence for a spherical elementary charge with radius r and charge density q this implies, according to [7], the following: Since $r_+ < r_-$ (dilation versus contraction) and $|Q_+|=|Q_-| = |\int_0^{r_+} q_+ dr| = |\int_0^{r_-} q_- dr|$, necessarily on the average $|q_+| < |q_-|$.

We define, [7], Electric Charge in a given zone of space τ as:

$$Q = \int_\tau q d\tau$$ Electric Charge in the GDM has the dimensions of volume $[Q] = L^3$

Note that by omitting the factor 1/4π in equation (1) in [7] - the definition of charge density - we get for the spherical symmetric case $d\tau = 4\pi r^2 dr$ and hence:

$Q = \int_0^r q4\pi r^2\, dr = 4\pi/3\cdot r^3(1-\rho_0/\rho) = V - V'$. Thus $\rho > \rho_0$ gives $V > V'$ and $Q > 0$ whereas

$\rho < \rho_0$ gives $V < V'$ and $Q < 0$. Thus, Q becomes a pure volumetric change, which is positive for contraction and negative for dilation.

Note also that the equality $|Q_+| = |Q_-|$, of the absolute values of the bivalent elementary charges, means $(1-\rho_0/\rho_+) = -(1-\rho_0/\rho_-)$ and hence $2/\rho_0 = 1/\rho_+ + 1/\rho_-$. It also means that:

$|Q_+| = |Q_-| = |V - V'|$. For example, if $\rho_+ = 2\rho_0$ then $\rho_- = 2/3\cdot\rho_0$ and $|V - V'| = 1/2\cdot V$. This means that $V_+/V_- = 1/3$ and hence $r_+/r_- = (1/3)^{1/3} = 0.69$ whereas according to [8]:

$r_p / r_e = 0.64$

The exact equality $|Q_+| = |Q_-|$ is related to the essence of the photon and Pair Production, and is a theoretical result of our photon model, whereas here it is considered a phenomenological result.

According to [8]: $r_- = 3/2\, r_+$. Hence $|Q_+| = |Q_-|$ implies $q_- = 2/3\, q_+$, as discussed at the beginning of this section. This means that the |curvature| created by Q_+ is the same as that created by Q_-. As a result, the |curvature| for both of the pure bivalent charges, according to (2), and

denoting $r_{S(Charge)}$ as r_Q, is:

$$K_Q = 1/R_c^2 = r_Q / r^3 = \sqrt{2}/2s^2 \cdot \sqrt{G}\, Q / r^3 \qquad (6)$$

where s = 1 and $[s] = LT^{-1}$, see [7] and [8].

The bending of a light beam by charge, due to its induced space curvature, is discussed in Appendix B. In Appendix C we suggest an experiment designed to verify or falsify our theory.

5 The Ratio of Curvature-by Charge to Curvature-by Mass/Energy

Dividing (6) by (5) gives the ratio of the curving by charge K_Q to that by spin K_S. This ratio for a given distance r from the center of the elementary particle is the ratio:

$$K_Q / K_S = r_Q / r_S = (\sqrt{2}/2s^2)\sqrt{G}\, Q_e /(G\hbar/R_e c^3) \sim c^2 / s^2 \qquad (7)$$

The ratio of the electric force to the gravitational force between two electrons, at a distance r from each other is:

$F_Q/F_G = Q_e^2 /GM_e^2 \sim 4c^4 / s^4$ (8)

The comparison of equations (7) and (8) reveals the connection between the curving of space and the applied force:

$F_Q/F_G \sim (K_Q/K_S)^2$ which means that **Force is the multiplication of Curvatures.**

For masses M_1 and M_2 the multiplication of their curvatures is:

$K_{S1}K_{S2} = r_{S1}/r_{12}^{\ 3} \cdot r_{S2}/r_{12}^{\ 3} = [4G/(c^4 r_{12}^{\ 4})]\cdot G M_1M_2/r_{12}^{\ 2}$ and thus **gravitational force** is:

$GM_1M_2/r_{12}^{\ 2} = [(c^4 r_{12}^{\ 4})/4G]\cdot K_{S1}K_{S2}$

For charges Q_1 and Q_2 the multiplication of their curvatures is:

$K_{Q1}K_{Q2} = r_{Q1}/r_{12}^{\ 3} \cdot r_{Q2}/r_{12}^{\ 3} = [(½)G/(s^4 r_{12}^{\ 4})]\cdot Q_1Q_2/ r_{12}^{\ 2}$ and thus the **electric force** is:

$Q_1Q_2/ r_{12}^{\ 2} = [2(s^4 r_{12}^{\ 4})/G]\cdot K_{Q1}K_{Q2}$

6 The Curving by Both Charge and Angular Momentum

The total curvature, $K= 1/R_c^2$, created by an elementary particle is the sum of its curvature K_Q, due to charge and its curvature, K_S, due to angular momentum, $K = K_Q + K_S$, or according to (5) and (6) for the same r:

$K = r_Q/r^3 + r_S/r^3 = \sqrt{G}\, Q/s^2r^3 + (2GM/c^2)/r^3$ (9)

Thus, for two very close elementary particles, carrying the bivalent elementary charges, the charge contributions to curvature at a distance r cancel out, whereas the energy/angular momentum contributions add up.

Note that in annihilation of a pair the residual curvature does not cancel out. This is an **open issue** in the current paradigm that asks about the gravitational energy disappearance. We, however, wonder if this very small energy is not carried by a graviton created in the annihilation.

Section 2 has shown that $(2GM/c^2)/\, r^3$ in equation (9) appears in the GR equation (1) as:

$\frac{8\pi G}{c^4} \cdot \epsilon$ where $\epsilon = T_{00}$ and in general $8\pi G/c^4 \cdot T_{ij}$. Similarly, $(\sqrt{2}/2)\sqrt{G}\, Q/\, s^2 r^3$ in (9) becomes:

$(\sqrt{2}/2)\sqrt{G}\, Q/\, s^2 r^3 = (2\sqrt{2}\pi/3)\, \sqrt{G}\, Q/\, s^2(4\pi/3)\, r^3 = (1/3)\; 2\sqrt{2}\pi\sqrt{G}/s^2 \cdot q$ where q is the charge density. Thus:

$$2\sqrt{2}\pi\sqrt{G}\, /s^2 \cdot q \tag{10}$$

becomes analogous to $\frac{8\pi G}{c^4} \cdot \epsilon$ and is used in Section 7 in the extension of the GR equation.

Note that we ignore the factor (1/3), as we did in Section 2.

7 The Extension of Einstein's Field Equation of GR to Incorporate the Curving by Charge / Current

This extension turns the GR equation from an equation that deals with spacetime curving by energy/momentum into a universal equation that also deals with the curving by charge/current. As a result, the equation becomes relevant not only to the **macroscopic** world but also to the **microscopic** world of elementary particles. Note that in this **microscopic** world the quantization of spacetime curvature also becomes relevant, since both angular momentum and charge are quantized.

To construct the extended equation, we adopt the idea expressed by (9) and add a new term to the GR equation, which is the multiplication of (10) by the **tensor**:

charge density/current density.

7.1 The 4 –Vector of Electric Current Density

Electric Current:

$J^{\mu} = \left(cQ, \mathbf{J}\right)$, $\mathbf{J} = Q\mathbf{v}$ hence:

$$J^{\mu} = Q\left(c, \mathbf{v}\right)$$

Electric Current Density:

$j^{\mu} = \left(cq, \mathbf{j}\right)$, $\mathbf{j} = q\mathbf{v}$ hence:

$$j^{\mu} = q\left(c, \mathbf{v}\right) \quad q = \gamma q^{0}$$

$$j^{\mu} = \gamma q^{0}\left(c, \mathbf{v}\right) = q^{0} v^{\mu}$$

Q is not a relativistic quantity, whereas q is a relativistic quantity (note the volume).

7.2 The Tensor $Tq^{\mu\nu}$ of Charge Density / Current Density

The 4-current densities $j^{(0)\mu}$, $j^{(1)\mu}$, $j^{(2)\mu}$, $j^{(3)\mu}$ transform into each other under Lorentz Transformation (LT). Thus, they themselves form a 4-vector, $p^{\mu} = \left(p^{0}, p^{i}\right)$.

$T_q^{\mu\nu}$ is the tensor: (11)

$$T^{\mu\nu} = \begin{pmatrix} j^{(0)\mu} \\ j^{(i)\mu} \end{pmatrix} = \begin{pmatrix} j^{(0)0} & j^{(0)1} & j^{(0)2} & j^{(0)3} \\ j^{(1)0} & j^{(1)1} & j^{(1)2} & j^{(1)3} \\ j^{(2)0} & j^{(2)1} & j^{(2)2} & j^{(2)3} \\ j^{(3)0} & j^{(3)1} & j^{(3)2} & j^{(3)3} \end{pmatrix}$$

$T^{00} = j^{(0)0}$. $\quad T^{0i} = j^{(0)i} =$ the i^{th} component of the current.

$$T^{01} = \gamma \frac{q^{0}k}{c} v_{1} = \gamma q^{0} c v_{1} = q c v_{1}$$

$T^{10} = c\gamma q^0 v_1 = qcv_1$

7.3 The Extended Field Equation of General Relativity for Energy/Momentum and Charge/Current

Using (9), (10) and (11) the **extended Einstein GR field equation** becomes:

$$R^{\mu\nu} - 1/2 R g^{\mu\nu} = 8\pi G/c^4 \cdot T_m^{\mu\nu} + 4\pi\sqrt{G}/s^2 \cdot T_q^{\mu\nu} \qquad (12)$$

8 Summary

We have shown how Charge and Angular Momentum curve space (curve: contract or dilate in the case of charge and contract in the case of angular momentum). Based on this we extended Einstein's field equation to become the equation of both energy/momentum and charge/current. This way the equation becomes an equation of the macroscopic/microscopic reality.

Acknowledgements

We would like to thank Mr. Roger M. Kaye for his linguistic contribution and technical assistance

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Appendix - A The Energy Momentum Tensor

According to [14]:

4-Vectors in Gravitomagnetism

Position: $x^{\mu} = (ct, x, y, z) \qquad x_{\mu} = \eta_{\mu\nu} x^{\nu} = (-ct, x, y, z)$

Interval: $S^2 = x^{\mu} x_{\mu} = -c^2 t^2 + x^2 + y^2 + z^2 = c^2 \tau^2$

Proper time: τ (a Lorentz scalar)

Velocity: $v^{\mu} = \partial_{\tau} x^{\mu}$ but $t = \gamma\tau$ hence $v^{\mu} = \gamma \partial_{\tau} x^{\mu}$

$$v^{\mu} = \gamma(c, v_x, v_y, v_z) = \gamma(c, \mathbf{v})$$

$$\mathbf{v}^2 = v^{\mu} v_{\mu} = \gamma^2(-c^2 + v^2)$$

Quantities which transform a LT, like x^{μ}, are 4-vectors.

Momentum: $P^{\mu} \equiv M v^{\mu} = \gamma(Mc, P_x, P_y, P_z)$

$$P^0 = \gamma Mc = Mc\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} = Mc\left(1+\frac{1}{2}\frac{v^2}{c^2}+.....\right) = \left(Mc^2+\frac{1}{2}Mv^2+........\right)$$

Rest energy: $cP^0 = Mc^2 = U$

Relativistic energy: $\gamma cP^0 = \gamma Mc^2 = \gamma U$

$$P^\mu = \left(\frac{U}{c}, \mathbf{P}\right)$$

$$\mathbf{P}^2 = P^\mu P_\mu = M^2 v^\mu v_\mu = M^2\left(-c^2\right)$$

Hence: $U^2 = \left(Mc^2\right)^2 + \left(c\mathbf{P}\right)^2$ Covariant force: $F^\mu = d_\tau P^\mu = Md_\tau v^\mu$ or:

$$F^\mu = \gamma d_t P^\mu = \gamma d_t\left(\frac{U}{c}, \mathbf{P}\right) = \gamma\left(\frac{\dot{U}}{c}, \mathbf{F}\right)$$

Rate of energy change – power:

$$W = F^\mu v_\mu = Md_\tau v^\mu \cdot v_\mu = \frac{1}{2}Md_\tau v^\mu v_\mu = 0$$

since $\mathbf{v}^2$ is a constant.

$$F^\mu v_\mu = \gamma^2\left(-\dot{U} + \mathbf{F}\cdot\mathbf{v}\right) = 0$$

hence: $F^\mu = \gamma\left(\frac{\mathbf{F}\cdot\mathbf{v}}{c}, \mathbf{F}\right)$

Mass current: $J^\mu = \left(cM, \mathbf{J}\right)$, $\mathbf{J} = M\mathbf{v}$

hence: $J^\mu = M\left(c, \mathbf{v}\right)$ Mass current density: $j^\mu = \left(cm, \mathbf{j}\right)$, $\mathbf{j} = m\mathbf{v}$

hence: $j^\mu = m\left(c, \mathbf{v}\right)$ $m = \gamma m^0$ $j^\mu = \gamma m^0\left(c, \mathbf{v}\right) = m^0 v^\mu$

Energy current–momentum: $P^0 = \frac{\mathbf{U}}{c}$

$J^{(0)\mu} = \left(U, \frac{\mathbf{U}}{c} \right)$ Energy current - momentum density: $j^{(0)\mu} = \left(\in, \frac{\boldsymbol{\in}}{c} \right)$

The 4-current density for the i th momentum component (p^i):

$j^{(i)\mu} = \left(cp^i, p^i \right)$

Note that in this section M and m stand for inertial mass and mass density. With a proper modification they also stand for gravitational mass and mass density.

The Energy Momentum Tensor

$T^{\mu\nu}$ in (1) is: $T^{\mu\nu} = \begin{pmatrix} j^{(0)\mu} \\ j^{(i)\mu} \end{pmatrix} = \begin{pmatrix} j^{(0)0} & j^{(0)1} & j^{(0)2} & j^{(0)3} \\ j^{(1)0} & j^{(1)1} & j^{(1)2} & j^{(1)3} \\ j^{(2)0} & j^{(2)1} & j^{(2)2} & j^{(2)3} \\ j^{(3)0} & j^{(3)1} & j^{(3)2} & j^{(3)3} \end{pmatrix}$ The 4-current density

$j^{(0)\mu}$, $j^{(1)\mu}$, $j^{(2)\mu}$, $j^{(3)\mu}$ transform into each other under Lorentz Transformation. Thus, they themselves form a 4-vector, $p^\mu = \left(p^0, p^i \right)$.

$T^{00} = j^{(0)0} = \in$ energy density $T^{0i} = j^{(0)i} =$ the i^{th} component of the $\frac{\in}{c}$ current

$T^{01} = \gamma \frac{\in^0}{c} v_1$ or $\gamma \frac{m^0 c^2}{c} v_1 = \gamma m^0 c v_1 = mcv_1$ $T^{10} = c\gamma m^0 v_1 = mcv_1$

We see that the momentum density T^{i0} is equal to the energy density current T^{0i}

T^{ii} = momentum current = $\frac{\text{momentum}}{\Delta S \cdot \Delta t} = \frac{\text{force}}{\Delta S} = \text{pressure}$

The off-diagonal terms are shear forces. $T^{ij} = T^{ji}$ is a symmetric tensor.

$\partial_\mu T^{\mu\nu} = 0$ for an isolated system is the conservation of energy – momentum

Appendix - B The Bending of a Light Beam by an Electric Charge

The bending of a light beam by mass M is given by the known equation [15]:

$\alpha_M = 2GM/bc^2$ (B1)

where α_M is the bending angle in radians and b is the impact parameter (the distance of the passing beam from the center of the mass). Note that 1 milliradian (10^{-3} radian) equals 206" arc second. We now derive a similar equation to (B1) that expresses the bending angle α_Q, caused by the presence of charge Q rather than a mass M.

Equation (7), which is the ratio of the curvatures $KQ / K_S = r_Q / r_S \sim c^2/ s^2$ for the electron/positron, enables us to modify (B1) and convert it to become an equation for α_Q. Since the curving, and hence the bending, by the elementary charge Q_e is c^2/ s^2 times larger than that by the lowest elementary rest mass M_e of the electron/positron we can set a lower limit to α_Q. This limit is achieved by replacing M in (B1) by $Q\cdot c^2/ s^2$, which gives:

$\alpha_Q = 2GQ/bs^2$ (B3)

For a grazing light beam to a sphere of radius b = 10cm the bending by

α_Q=1miliradian $=1\cdot 10^{-3}$ is achieved by charging the sphere by $Q = \alpha_Q\, bs^2/ 2G = 7.5\cdot 10^4$esu. The potential of the surface is then:

$\varphi = Q/b = 7.5\cdot 10^3$esu or $v = 300\cdot 7.5\cdot 10^3 = 2.25\cdot 10^6$ volt.

See also Appendix C.

Appendix C A Practical Experiment to Test Our Theory

We suggest creating an "accumulated flowing charge" at the focal zone of an electron beam. It is a simple task to take a slow beam of several milliamps current and create a focal zone of, say, several microns. Thus, we create at the focal zone a charge, similar to the charge in our example above and observe a similar bending angle. The details are as follows:

1 ampere = (1 coulomb / 1 cm^2)·(1/1sec) = (1 coulomb / 1 cm^3)·(1 cm/1 sec)

$I = Q/(At) = (Q/V) \cdot v$ I - current, A - cross section area, Q - charge, V - volume, v - speed.

$Q = I (V/v)$

The current: I = 0.1 amps = $10^{-1} \cdot 3 \cdot 10^9 = 3 \cdot 10^8$esu $cm^{-2}sec^{-1}$.

The "radius" of the focal zone volume is $r \sim 3 \cdot 10^{-1}$cm.

The velocity of the electrons is $v = 10^4$cm sec^{-1}.

The "accumulated flowing charge" at the focal zone is thus: $Q \sim I\ (r/1cm)\ /v \sim 1 \cdot 10^4$esu

A laser beam, with a radius $r \sim 3 \cdot 10^{-1}$cm, crosses the electron beam perpendicularly, but close to and above the focal zone.

Using (B3) we get the exact (and not the lower limit, since we are dealing with electrons) bending angle α_Q. From the center of the focal zone to the center of the laser beam: b = 2r.

$\alpha_Q = 2GQ/bs^2 = 2 \cdot 6.67 \cdot 10^{-8} \cdot 10^4/(2 \cdot 3 \cdot 10^{-1}) \sim 2 \cdot 10^{-3}$ = 2 milliradians.

This experiment is doable. The result can verify or falsify our theory.