Mathematics

Grade 7

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2016 EC

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Unit – One

1. 1 the concept of Set

Definition 1.1 Set is a collection of well-defined objects.

- ✓ The individual objects in a set are called **element (member) of the set.**
- ✓ The braces " { } " are used to on close the members of the set.
- ✓ Set can be represented by capital letters

  Example: The set of all values in English alphabet can be represented by a single letter A and described as

  $A = \{ a, e, I, o, u \}$
- ✓ If a is not a members of set A, then we write $a \notin A$ (read as a is not an elements of set A)

  Example 2: If $M = \{ 2, 4, 6, 8, 10 \}$

  ✓ $2 \in M$ $6 \in M$ $10 \in M$

  ✓ $4 \in M$ $8 \in M$

Then it has exactly 5 elements.

Example 3: How many elements does each of the following sets contain?

a . $B = \{ 1, 2, 4, 1 \}$

b. A = The set of numbers less than 0.

Solution

a. Set B contains 3 elements and the elements are 1, 2 and 4.

Note: Repeating the same element in the set do not increase the number of elements.

b. There is no whose numbers less than 0. So, set A does not have element.

Note:- A set which do not have an element is called empty set and denoted by $\emptyset$ or { }.

Exercise

1. How many elements does each of the following sets contain ?

a. $A = \{ 2, 2, 2, 2 \}$ C. $B = \{ a, b \}$

b. $C = \{ \ \}$ d. D = The set of whole numbers less than 6.

2. Which one of the following are empty set ?

a. A = The set of trees in Addis Ababa.

b. M = The set of students in your class whose age is 30 years.

Explanation:

≠ 1. A . Set A has only one element because repeating the same element does not increase the number of element.

b. C = { }, set C has no element.

C. set B has two elements.

d. D= { 0, 1, 2, 3, 4, 5 } ⇨ This set has 6 elements.

≠ 2. In this grade level it is an empty set.

1.2 Relation among sets

A. <u>Equal Sets</u>

Definition 1.2. Two non – empty sets A and B are said to be equal, if they have exactly the same elements and write, A = B.

Example 1 Set A = { p, q, r, s } and b = { q, r, p, s }.

Are A and B equal ?

Solution:- A and B have exactly the same elements, So A = B.

B. <u>Equivalent Sets</u>

Definition 1. 3 Two sets A and B are said to be equivalent , written A ↔ B, if and only if they have equal number of elements.

Example 1 : Let A = { 2, 3, 4 } and B= { a, b, C }. Are A and B equivalent ?

Solution:- A and B have equal number of elements So A↔ B.

Note: All equal sets are equivalent set, but all equivalent sets are not equal sets.

C. <u>Sub set</u>

Definition 1. 4 A set A is said to be a sub set of set B if every element of A is also an element of B, and write $A \dot{\subseteq} B$.

Example 1: Let A = { 3, 2, 5 } and b = { 2, 3, 5, 7, 9 }.

Is a subset of B ?

Solution:

Every element of A is an element of B.

So $A \dot{\subseteq} B$.

Note: 1. Empty set is a subset of any set. i. e

For any set A, $\emptyset \dot{\subseteq} A$.

2. Any set is a subset of itself. i. e

For any set A, $A \dot{\subseteq} A$.

D. Proper subset

Definition 1. 5 A set is said to be a proper subset of set B, written a $\subset$ B if and only if A is a subset of B and B has one more element.

Example: Let A = { 2, 3, 4 } and B = { 3, 2, 4, 5 }.Type equation here.

Is A is a proper subset of B ?

Solution

All elements in A are also in B, and B has additional element, So A C B.

Note: 1. Empty set is a proper subset of any other set

A. i . e $\emptyset \subset$ A.

2. Any set is not a proper subset of any set A.

i . e $A \not\subset$ A.

Exercise

1. Let A = { O, b } then list all subset of set A.

2. Let B = { m, 2 } then list all proper subset of set B solution :

Note: number of subset can be calculated by 2^n .

1. A = { O, b } ⇨ it has 2 elements.

   no of subset = $2^n = 2^2 = 4$. i . e ∅ { O } , {b } , { O, b}.

   number of proper subset can be calculated by

   $2^n - 1$. where n is no of element.

2. B = { m, 2 } ← it has 2 elements, no proper subset

   $= 2^n - 1 = 2^2 - 1 = 4 - 1 = 3$ i . e

   ∅, {m} and {2}.

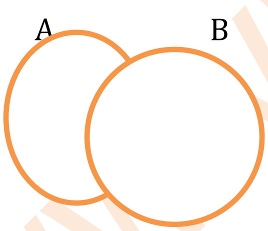
Remark 1: If A = b then $A \dot{\subseteq} B$

2. If A = B then $A \not\subset B$.

1.3. operation On Sets

1.3.1. Union Of Sets

Defination1.6 The union of two sets A and B denoted by A ∪ B. and read "A union B" is the set of all elements with belongs to A or B or both.



The shaded region in the figure below represents

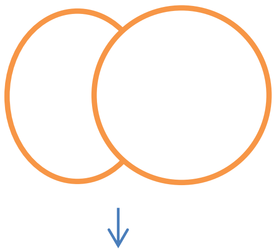
A ∪ B.

Example 1: If A = { 3, 4, 5, 6 } and B = {4, 6. 8,10 } then find A ∪ B ?

Solution: clearly A ∪ B= { 3,4,5,6,8,10}

1.3.2. The intersection of Sets.

Definition 1.7: The intersection of two sets A and B, denoted by A ∩B and read " A intersection B", is the set of all elements which belong to both A and B.



$A \cap B$

Example 1: If A = { 3,4,5,6} and B = { 4,6,8,10}, then find $A \cap B$.

Solution : $A \cap B$ = { 4, 6 }

Exercise

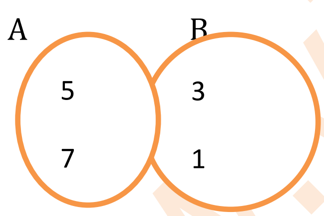
1. Given A = { a, e, i, O } and B = { a, e, i, } , then find $A \cap B$

2. Given A = { 1, 3, 4, 6 } C = { }

B= { 1, 4, 3 } D= { 1, 2, 3, 4, 5, 6 } find

a. $A \cap B$	d. $A \cup B$	g. $(A \cap B) \cup C$
b. $A \cap C$	e. $C \cup B$	h. $(B \cup C) \cap D$
c. $A \cap D$	f. $B \cup D$	i. $(A \cap B) \cup (B \cap D)$

3. The relation between set A and B is shown in Venn diagram below:



a. List all elements of set A

b. List all elements of set B

Solution:

1. $A \cap B$ = A = { a, e, I }

2. a) $A \cap B = B$ b) $A \cap C = C$ C) $A \cap D = A$

$= \{1, 4, 3\}$ $= \{\ \}$ $= \{1, 3, 4, 6\}$

d) $A \cup B = A$ e. $C \cup B = B$ f. $B \cup D = D$

g) $(A \cap B) \cup C = B \cup C$ h. $(B \cup C) \cap D = B \cap D$

$= B$ $= B$

$= \{1, 4, 3\}$ $= \{1, 4, 3\}$

i) $(A \cap B) \cup (B \cap D) = B \cup B = B = \{1, 4, 3\}$.

3. a. $A = \{2, 5, 7\}$

b. $B = \{1, 2, 3\}$

Note: If $A \subset B$ then i. $A \cap B = A$

ii. $A \cup B = B$.

UNIT - 2

Integers

2.1 Revision on whole numbers and natural numbers

Activity:

1. Write : a) The set of natural numbers?

   b) The set of whole numbers?

Solution:

Definition 2.1:

a. The set of natural numbers denoted by "N"

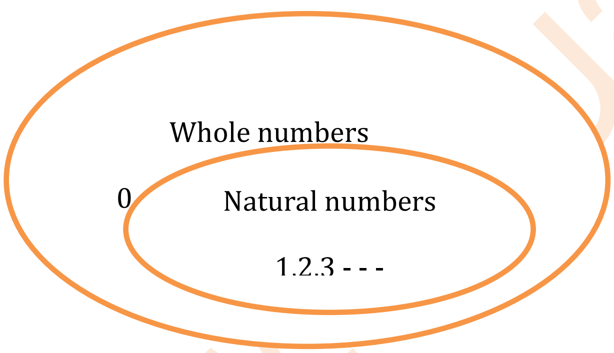
   N = { 1,2,3 - - - - - }

Definition 2.2:

b. The set of whole numbers denoted by "W"

   W = {0, 1,2,3 - - - - - }

Using venn diagram, the relationship between natural and whole numbers is shown below.



Note: 1) All natural numbers are whole numbers.

2) A collection of a natural number and Zero is a whole numbers.

Revision of operations on natural numbers and whole numbers

Example 1: find the following sum:

a. 452 + 323

b. 3234 + 598

Solution

a. 452 b. 3234

+323 +598

775 3,732

Example 2: find the following difference:

a. 452 + 523 C. 456-456

b. 7456+ 352

Solution

a. 453 b. 7456 C. 456

-223 - 352 - 456

230 7,104 = 0

Example 3: find the product of the following nes.

a. 46 x 44 C. 123 x 342

b. 72 x 78 C. 123 x 342 = 42066

Solution: a. 46 x 44 = 2024 b. 72 x 78 = 5616

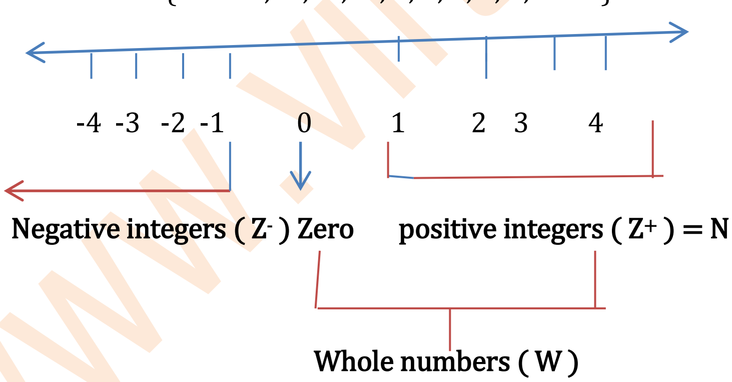
Note:

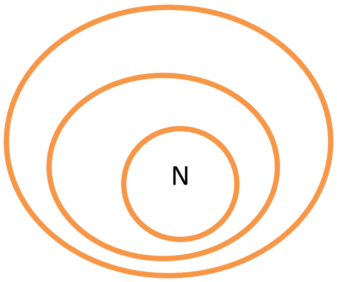
1. The sum of any two natural number is always natural numbers.
2. The product of any two whole numbers is always whole number.
3. The difference of any two natural number is not always natural number.
4. The quotient of any two whole number is not always whole number.

2.2. Introduction number.

Definition 2.3: An integer is a set of numbers consisting of whole numbers and negative numbers. The set of integers is denoted by:

Z = { - - - - 4, -3, -2, -1, 0, 1, 2, 3, 4, - - - - }





Note

1. The set of positive integer is the same as the set of natural number **($Z^+ = N$)**
2. Zero is an integer which is neither negative nor positive integer.
3. Integers, Natural numbers, whole numbers are related as $N \subset W \subset Z$
4. $N \subseteq Z^+$ $\quad$ $W \subseteq Z$ $\quad$ $N \subseteq W$

   $Z^+ \subseteq N$ $\quad$ $W \not\subseteq Z^+$ $\quad$ $W \cap Z = W$

   $N \cup W = W$ $\quad$ $Z^- \cup Z^+ \cup 0 = Z$ $\quad$ $W \cup Z = Z$

Opposite numbers

Definition 2.4 : Opposite numbers are numbers that are at the same distance from zero in opposite direction.

-3 -2 -1 -1 0 1 2 3

Note: for any integer a, $a \neq 0$

i. The opposite of a is – a,

ii. For integer a, - (-a) = a .

iii. The opposite of positive number is negative number.

Exercise

1. find the opposite of each integer given below:

a. 70 $\quad$ b. -23 $\quad$ C, -170

Solution

a. The opposite of 70 is – 70

b. The opposite of -23 is 23

C. The opposite of -170 is 170

2.3. Comparing and ordering integers.

To compare integers, draw number line and indicate the numbers on a number line then,

1. The number which is to the right of the other is bigger number.
2. The number which is to the left of the other is smaller number

We use symbols ">" , "<" , "=" to compare numbers.

- ✓ $a > b$, means a is greatest than b .
- ✓ $a < b$, means a is less than b.
- ✓ $a = b$, means a is equal to b.

⊗ Consider the following number line

1. a is to the left of b. i. e $a < b$

2. b is to the right of b i. e $b > a$

Note:

a. All negative integers are to the left of Zero.

Hence , every negative integer is less than 0.

b. 0 is to the right of every negative integer

hence 055 greater than every negative integers.

C. All negative integers are to the left of positive

integers hence $Z^- < Z^+$

d. All positive integers are to the right of every negative integers . hence $Z^+ > Z^-$.

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Example: compare

a. – 4 ________ 2 C. 4 _____________ -6

b. 5 __________ -6 D. -2 _____________ -4

Solution

a. -4 < 2 (to the left) C. 4>-6 (to the right)

b. 5 > - 6 (to the left) d. -2>-4 (to the right)

Example 2 List all integers that lie between -5 and 2

Solution

-4, -3, -2, -1, 0, 1

Exercise

1. Insert ">" = or "<" to express the corresponding relationship between the following pair of integers.

a. – 150 ______ 0 d. 65 ________ - 20

b. 0 __________ - 300 e. 35 ________ - 45

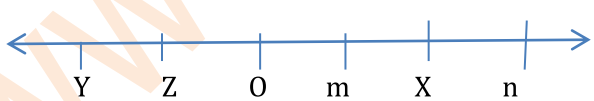
C. – 1200 ____________ -74

Solution

a. – 150 < 0 (to the left)
b. 0 > - 300 (to the right)
c. – 1,200 < - 74 (to the left)
d. 65 > - 20 (to the right)
e. 35 > - 45 (to the right)

2. The five integers X, y, Z, n and m are represented on the number line below:

Using "<" or ">" fill in the blank space.



a. Z ________ X b. m ______ X C. Z_________ n

d. O _______ X

Solution

a. $Z < X$ because Z to the left of X.

b. $m < X$ because m to the left of X

C. $Z < n$ because

d. $0 < X$

Successor and Predecessor of integers

Successor of an integer is an integer that comes after the given integer. $\boxed{n+1}$

Example 1 : Find the successor of the given integer

a. 5 b. – 4 C. – 99 d. 99

Solution

a. The successor of 5 is 5+1 = 6

b. The successor of - 4 is - 4+1 = 3

C. The successor of - 99 is - 99+1 = - 98

d. The successor of 99 is - 99+1 = 100

Predecessor: of an integer is an integer that comes before the given integer.

Let n be an integer the predecessor of n is given by $\boxed{n-1}$

Example 2 : Find the predecessor of the given integers.

a. 5 b. 0 C. – 4 d. - 99

Solution

a. The Predecessor of 5 is 5-1 = 4

b. The Predecessor of 0 is – 0-1 = -1

C. The Predecessor of - 4 is – 4-1 = - 5

d. The Predecessor of - 99 is – 99-1 = -100

Ascending and descending order of integers.

⊗ Ascending order: means arranging the given numbers in increasing order.

✓ **Arranging from the smallest to the largest number.**

⊗ Descending order:- means arranging the given numbers in descending order.

(Arranging from largest to smallest)

Exercise

1. Find the successor of the following integers.

a. 20 b. – 16 C. 999 D. – 1000

2. Find the predecessor of the following integers.

a. 1000 b. – 77 C. - 999

3. Arrange the f.f in ascending order.

a. 0, - 78, 17, - 24, 71

b. 200, - 300, - 757, - 445, 400

4. Arrange the following in descending order

a. -9, - 99, 69, 59, - 89

b. 0, -78, 17, -24, 71

Solution

1. a. n + 1 → successor

20+1 = 21

b. – 16+1 = -15

C. 999+1 = 1000

d. – 1000 = - 100+1

= - 999

≠ 2. Predecessor of

a. 1000 – 1 = 999

b. – 77 – 1 = - 78

C. – 999 – 1 = - 1000

≠3. To arrange ascending → start from smallest

a. – 78, -24, 0, 17, 71

b. – 757, - 445, - 300, 200, 400

≠4. To arrange deceivingly → start from the greatest.

a. 69, 59, - 9, - 89, - 99
b. 71, 17, 0, - 24, - 78

2.4. Addition and subtraction of integers

2.4.1 Addition of integer

⊗ In a+b = C, a and b are called addends and C is called sums.

Rule: Addition of integers On number Line

1. Start the arouse from the first addend.
2. Move the arrows to the **right** with the same magnitude as the second addend, if the sign of the second addend is **Positive**.
3. Move the arrows to the **left** with the same magnitude as the second addend, if the sign of the second addend is **negative.**
4. The sum of the integer is at the point where the arouse ends.

Example 1 : Find the sum of the following integers using number line

a. 3+ (- 6) C. -3+6
b. -3 + (- 6)

a.

C.

b.

Note: The sum of any two opposite integer is zero. Addition of positive and negative integers with out using number line.

Example:

a. 22+(-43)= b. 625+ (-214)

Solution

a. 22 – 43 = - (43 – 22) = **-21**

b. 625+ (-214) = 625 – 214

= **411**

Addition of two negative integers.

⊗ To find the sum of two negative integers.

1. The sign of the sum is always negative.
2. Find the sum of magnitude of the numbers.
3. Put the negative sign in front of the sum.

Example - 15+ (-35) = - (- 15 + 35)

= **- 50**

Exercise

1. You are birr 5 in debt. You Borrow birr 12 more what is the total amount of your debt ?

2. An air plane takes off and then climbs 2500 descends 150 meters. What is the air plane's current height?**Solution**

1. – 5 + - 12 = - (5 + 12)

= - 17 birr

2. 2500 m + (- 150 m) = 2500m

- 150 m

2,350m

2.4.2 Subtraction of integers

Subtracting an integer b from a is the same as adding opposite of b to a

i.e. For any integers a and b,

a – b = a + (-b)

Example Find the difference of the following integer by expressing in the form of sum.

a. 5-2 C. 25- (-30)

b. 4-7 d. -12-(-18)

Solution

a. 5-2 = 5+(-2)=3

b. 4-7 = 4+(-7)=-3

C. 25-(-30) = 25+30

= 55

d. -12 –(-18) = -12+18

= 6

Note: For any two integers a and b

1. a – (-b) = a+b
2. – a – b = - (a+b)

Exercise 2.4.2

1. The melting point of dry ice is -109°F .

The boiling point of dry ice is 109°F then how many degrees is to be boiling point above the melting point?

109°F - (109°F) = 109°F + 109°F

= **218**°F

2.5 Multiplication and division of integers

2.5.1 Multiplication of integers

Note: 1. The produce of two positive integers is positive integer.

E. g : $4x$ 6 = 24 $5x$ 9 = 45

2. The product of negative and positive integers is negative integer.

E. g: - $5x$ 6 = -30 $4x$ (-3) = -12

- $2x$ 6 = -12 $12x$ (-11) = -132

3. The product of two negative integer is positive integer.

E. g: - $6x$ (-8) = $6x$ 8=48

$-9x(-7) = 9x\ 7 = 63$

Properties of Multiplication of integers

1. Commutative properties of multiplication

$a\ x b = b\ x a$

2. Associative property of Multiplication

$a\ x(b\ x C) = (a\ x b)\ x C$

$2\ x\ (5\ x 6) = (2\ x 5)\ x 6$

$2\ x 30 = 10\ x 6$

$60 = 60$

3. Distributive property of multiplication over addition.

Let a, b and C any integers then

$a\ x(b+C) = (a\ x b) + (a\ x\ C)$

E. g: $-6\ x\ (4+3) = (-6\ x 4) + (-6\ x 3)$

$-6\ x 7 = -24 + (-18)$

$-42 = -(24+18)$

$-42 = -42$

4. properties of zero and 1 On multiplication

Note i. The product of any integer and zero is **zero**

Let "a" be any integer, then

$\boxed{a\ x 0 = 0}$ Or $\boxed{0\ x a = 0}$

ii. The product of any integer and 1 is the number it self.

Let "a" be any integer, then

$\boxed{a\ x 1 = a}$ $\boxed{1\ x a = a}$

Exercise

≠1. Simplify each of the following pairs and compare their result.

a. $3\ x\ (-2\ x 3)$ and $(3\ x\ (-2)\ (x 3)$

b. $-6\ x\ (5+2)$ and $(-6\ x 5) + (-6x 2)$

c. -5 x (-3+(-8)) and (-5 x(-3)+ $(-5x$ (-8))

Solution

a. 3 x (-6) and -6 x3

- 18 -18

∴ They are equal

3 x (-2 x3) = (3 x (-2)) x 3)

b. - 6 x (5+2) and (-6 x5) + (- 6 x2)

- 6 x7 and - 30 + - 12

- 42 and – 42

∴ - 6 x (5+2) = (-6 x5) + (-6 x2)

Multiplication of three or more integers

⊗ The following properties are helpful in simplifying products with three or more factors.

- The product of an odd number of negative factors is negative.
- The product of an even number of negative integers factors is positive.

Exercise 2.5.3

1. Find the sign of the product

a. 12 x (-50) x (-61)

b. (-125) x (-3) x (-52)

C. 3 x (-6) x (-9) x (-12)

Solution

a. -600 x (-61) = +<u>36,600</u>

b. -125 x (-3) x (-52 = 125 x3 x (-52)

= 375 x (-52)

= - <u>19, 500</u>

2.5.2 Division Of integers

Division: is an inverse operation of multiplication.

Note : For any number a, b and C where C ≠0

a÷b = C, if and only if a = b xC

⇨ a÷b read as a divided by b. and also denoted by a/b. or $\frac{a}{b}$.

✷ If a÷b = C then

✷ a is called dividend

✷ b is called divisor

✷ C is called quotient

Division of integers table

Dividend	Divisor	quotient
Positive	Negative	Negative
Negative	Negative	Positive
Positive	Positive	Positive
Zero	Positive/ negative	Zero
Positive / negative	Zero	Under find

✷ to divide integers without converting in to multiplication follow these steps:

1. If the sign of the dividend and divisor are the same:-

✷ The sign of the quotient is positive.

2. If the sign of the dividend and the divisor is different.

✷ The sign of the quotient is negative

Exercise

1. identify dividend, divisor and quotient of the following

a. -16÷12 = - 8 b. $\frac{-56420}{-124} = 455$

✷ - 96 is divided ✷ - 56420 is dividend

✷ 12 is divisor ✷ - 124 is divisor

✷ -8 is quotient ✷ 455 is quotient

2. Find the quotient by converting in to products

a. 18÷3 b. 24÷ (-4) C. -40÷ (-5) d. (-44÷ 4)

Solution

a. 18÷3 $= 18\,x\,\frac{1}{3} = \frac{18x1}{3} = \frac{18}{3} = 6$

b. 24÷ (-4) $= 24\,x\,\frac{-1}{4} = \frac{24x-1}{4} = \frac{-24}{4} = \underline{\underline{-6}}$

C. -40÷ (-5) $= -40\,x\,\frac{-1}{5} = \frac{-40x-1}{5} = \frac{40}{5} = \underline{\underline{8}}$

d. (-44÷ 4) $= -44\,x\,\frac{1}{4} = \frac{-44}{4} = \underline{\underline{-11}}$

2.6 Even and Odd integers

Even integer: is an integer that cannot be divisible by 2 without leaving remainder. Numbers and with either 0, 2, 4, 6 or 8. Are integers.

e.g 4256, 3,956, 420, - 13, 988. Are even.

Odd integer: is an integer that cannot be divisible by 2. When we divide any odd integers by 2 its remainder is 1.

Note: For many digit number:

If the unit digit is 0, 2, 4, 6 or 8, then the number is even integer otherwise it is odd.

Exercise

1. What is the greatest odd negative integer?
2. What is the smallest positive even integer?
3. What is the greatest three – digit positive integer?
4. What is the smallest two – digit negative integer?

Solution

≠1. The greatest odd negative integer is **– 1**

≠2. The smallest positive even integer is **10**

≠3. The greatest three – digit positive integer is **999**

≠4. The smallest negative two – digit number is **– 99**

Sum of even and odd integers

1. The sum of two even integer is always **even**

2. The sum of two add integers is always **odd**

3. The sum of even and odd integer is always **odd.**

Different of even and odd integers:

1. The difference of any two even integer is even.

2. The difference of any two odd integer is **even**

3. Odd – even = Odd.

Even – Odd = Odd.

Product of even and odd integers

1. The product of two even integer is even.

2. The product of even and odd integer is **even**

Exercise

1. Fill in the blank space with the correct answer.

a. even + even = ____________

b. even – odd = _____________

C. Odd × even= _____________

d. Odd – even – odd + odd = ______________

e. ecen × odd + even = ______________

Review exercise for unit – 2

1. If – 5 > ×, then list down four possible values of x, where x is an integer

Solution

$x <$ - 5

The possible four values of x are: -6, -7, -8, and – 9

2. decide whether the following statements are Even or odd.

a. The sum of any two even integers.

b. The difference of any two odd integers.

Solution

a. Even b. Even

3. List examples of a pair of integers whose sum is zero.

Solution

(-2,2) -4 and 4 -5 and 5 e t c ..

UNIT - 3

Linear equations:

3.1 Algebraic terms and expressions

3.1.1. Use of variable in formula

Definition 3.1: a variable is any letter or a symbol that represent some unknown number or value.

Such as x, y, z, l, m, n etc

Example 1: in $x + 4$

✓ x is a variable.

Express the following using variables

a. A number x plus 4

b. The difference of 5 and y , where y is greater than 5.

Solution

a. $x+4$ b. $y - 5$, $7 > 0$

Exercise

1. Describe the following using variables.

a. Four times a number

b. One third of a number

C. Ten more than a number

d. The sum two different numbers.

2. Express the following in words.

a. $x - 1$ C. $3+4x$

b. 7 x d. $\frac{x}{3}$ - 1

Solution

1. a. let the number is x, then four times the number is **$4x$**.

   b. let the number is 4, then one third of the number is $\frac{x}{3}$.

   C. let the number is m, then ten more than a number is **m+10**

   d. let × and y be two d/t numbers **×+y**

2. a. × -1 = a number minus one.

   b. 7× = seven times a number.

   C. 3+4× = Three more than four times a number .

   d. One less than one – third of a number.

Definition 3.2: product of (a number) and variable, the numerical factor of the term is called numerical coefficient.

Example 1 : The numerical coefficient of 6 a b is 6.

Definition 3.2: a constant (a number), a variable, product or quotient of a number and variable is called a **term**.

Example 1 : 4 is a term

×y is a term.

× - y is not a term.

Definition 3.4: like terms are terms whose variable and exponent of variable are exactly the same but they may differ in their numerical coefficients. Terms which are not like terms are called **unlike terms**.

Exercise

1. Identify whether each pair of the following are like or unlike terms.

   a. 3× and -5× → They are like terms.

b. 20×y and a b → They are not like terms

Note: Terms which do not have variable are called **constant terms.** All constant terms are called like terms.

Definition 3.5: Algebraic expressions are formed by using numbers, litters and the operation of addition, subtraction, multiple and division.

Note: The term an algebraic expressions are a part of the expression that are connected by plus or minus Signs.

Definition 3.6: An algebraic expression in algebraic which contains one term is called **monomial**.

Definition 3.7: An algebraic expression in algebra which contains two terms is called **binomial.**

Definition 3.8: An algebraic expression in algebra which contains three terms is called **trinomial.**

e. g : ✷ 5× monomial.

12a b

✷ 5×+3y

12-a b binomial.

× - b

- ✓ 2×+4y-2
- ✓ 4+a-b Trinomial.

×-y – z

Simplifying algebraic expressions

✷ To simplify any algebraic expression, follow the following basic steps:

1. Remove brackets.
2. Collecting like terms.
3. Add or subtract like terms.

Note: 1, When we add or subtract like terms, add or subtract their numerical coefficients

2, unlike terms cannot be added or subtracted.

Exercise

1. Simplify the following expressions.

a. $3x$+35y - $8x$

b. 3 ($3x$-6y)+ x+18y-4

C. -3 (- a+6) + x – (Z+4) - $2x$

Solution

a. $3x$+35y - $8x$ = $3x$-$8x$ +35y

= -**<u>5x+35y</u>**

b. 3 ($3x$-6y)+ x+18y-4

= $9x$ – 18y + x+18y-4

= $9x$ – x+18y-18y-4

= **<u>10x – 4</u>**

C. -3 (- a+6) + x – (Z+4) - $2x$

= 3a – 18 + x – 2 – 4 - $2x$

= 3a – 18 – 2+x – 2 x – Z

= **<u>3a – 20 - x – Z</u>**

3.2 Solving linear equations:

3.2.1 Linear equation involving Brackets.

<u>Definition 3.9</u>: Two different algebraic expressions connected by equal (=) sign is called **<u>equation.</u>**

<u>Definition 3.10</u>: A linear equation in one variable × is an equation which can be written in standard form a×+b = 0, where a and b are constant numbers with a ≠ 0 .

Note: an equation of a single variable in which the highest exponent of the variable is one is a linear equation.

e. g : Which of the following equation are linear and which of them are not.

a. $2x = 2$ C. $2x^2+3 = 2x - 1$

b. $x+3 = 2x-1$ d. $x = 2x^3 - 6$

Solution

- a and b are linear equation, because the highest exponent of variable is one.
- C and d are not linear equation – because the highest exponent of the variable is not one.

Exercise

1. Find the value of unknown variable.

a. $-8x = 12$ C. $20 - x = 15$

b. $\frac{2x}{5} = 10$ d. $2x - 4 = 16$

Solution

a. $\frac{-8x}{-8} = \frac{12}{8}$ b. $\frac{2x}{5} = \frac{10}{1}$ C. $20 - x = 15$ d. $2x - 4 = 16$

$x = \frac{-12}{8}$ $2x = 5 \times 10$ $20 - 15 = x$ $2x = 16+4$

$\boldsymbol{x = \frac{-3}{2}}$ $\frac{2x}{2} = \frac{50}{2}$ $\underline{\underline{\boldsymbol{5 = x}}}$ $\frac{2x}{2} = \frac{20}{2}$

$\underline{\underline{\boldsymbol{x = 25}}}$ $\underline{\underline{\boldsymbol{x = 10}}}$

Exercise

1. Solve the following equations and check the result.

a. $3x - 9 = 4x+5$ C. $12x + 7 = 5 - 3x+17$

b. $5x-3 - 4x = 13$ D. $10 = 3x - 5+2x$

Solution

a. $3x - 9 = 4x+5$ b. $5x-3 - 4x = 13$

$3x - 4x = 5+9$ $\underline{\underline{\boldsymbol{x = 16}}}$

$-x = 14$ C. $12x + 7 = 5 - 3x+17$

$\underline{\underline{\boldsymbol{x = -14}}}$ $15x = 15$

$\underline{x = 1}$

Check
3(-14)-9 = 4(-14)+5
-42 – 9 = -56+5
-51 = - 51

Solving linear equations involving Brackets

Note: For any numbers a, b and C

1. A + (b +C) = a +b +C
2. A – (b + C) = a – b – C
3. A – (b – C) = a – b + C
4. A (b + C) = a b + a C
5. A (b – C) = a b – a C
6. A – b = - b + a
7. 8 - 2 x = 2 x + 8

Exercise

1. Solve each of the following equation.

a. 8 (2y – 6) = 5(3y – 7)

b. 7 – (x + 1) = 9 – (2 x – 1)

2. Solve for x in terms of m and n .

a. m (x – 1)= 0

C. n(x-2) = m+ x, n ≠ 1.

b. m (x+n) = mn

Solution

a. 8(2y-6) = 5 (3y-7)

16y – 48 = 15y – 35

16y – 15y = - 35 + 48

$\underline{y = 13}$

b. 7 – (x+1) = 9- (2 x – 1)

7- x -1= 9 -2 x+1

- x+2 x = 9+1+1-7

x = 11 - 7

$\underline{\underline{x = 4}}$

$\neq$2. a. $m\frac{x-1}{x-1} = \frac{0}{x-1}$

$m = 0$

b. $\frac{m}{m}(x + n) = \frac{mn}{m}$

$x + n = n$

$x = n - n$

$\underline{\underline{x = 0}}$

C. $n(x - 2) = m + x$

$n x - 2n = m + x$

$n x - x = m + 2n$

$\frac{x(n-1)}{n-1} = \frac{m+2n}{n-1}$

$x = \frac{m+2n}{n-1}$

Solving linear equations involving fractions:

To solve linear equations follow these steps:

1. Find the L.C.M of the denominators.
2. Multiply both sides of the equation by L.C.M.
3. Solve the linear equation.

Exercise 3.2.4

1. Solve

a. $\frac{2x}{5} - \frac{2}{3} = \frac{x}{2} + 6$

C. $\frac{1}{3}(x+6)\frac{-1}{2}(3x-4)=5$

b. $\frac{n+1}{2} + \frac{n+2}{3} + \frac{n+4}{4} = 3$

d. $\frac{7}{10}x + \frac{3}{2} = \frac{3}{5}x + 2$

Solution

a. L.C.M (5, 3, 2) = 30

$30\left(\frac{2}{5}\times\right) - 30\left(\left(\frac{2}{3}\right) = 30\left(\frac{\times}{2}\right) + 30(6)$

$6(2\times) - 10(2) = 15(\times) + 180$

$12\times - 20 = 15\times + 180$

$12\times - 15\times = 180 + 20$

$\frac{-3\times}{-3} = \frac{200}{-3}$

$\times = \frac{-200}{3}$

b. $\frac{n+1}{2} + \frac{n+2}{3} + \frac{n+3}{4} = 3$ L.C.M (2,3,4) = 12

$12\left(\frac{n+1}{2}\right) + 12\left(\frac{n+2}{3}\right) + 12\left(\frac{n+3}{4}\right) = 12\ (3)$

6(n+1) +4 (n+2) +3 (n+3) = 36

6n+6+4n+8+3n+9= 36

10n+3n+14+9 = 36

13n+23 = 36

13n = 36 – 23

$\frac{13n}{13} = \frac{13}{13}$

n = 1

C. $\frac{1}{3}(\times+6) - \frac{1}{2}(3\times-4) = 5$ L.C.M (3,2) = 6

$6\times\frac{1}{3}(\times+6) - 6\times\frac{1}{2}(3\times-4) = 6(5)$

2(×+6) – 3 (3×-4) = 30

2×+12 - 9×+12 = 30

2× - 9×+12+12 = 30

- 7×+24 = 30

-7× = 30-24

-7× = 6

× = -6/7

Equivalent equations are two or more equations that have the same solution.

e.g : Which of the following pairs of equations are equivalent?

a. × = 0 and 2×-1 = -1

b. 2× -6 = 2 and -× = -5

Solution

a. × = 0 and 2× = -1+1 b. 2×-6=2 and -× = 5

2× = 0 2× = 2+6 & × = 5

× = 0 2× = 8 & × = 5

× = 4

∴ They are equivalent.

ⓑ They are not equivalent.

3.3. Cartesian Coordinate System

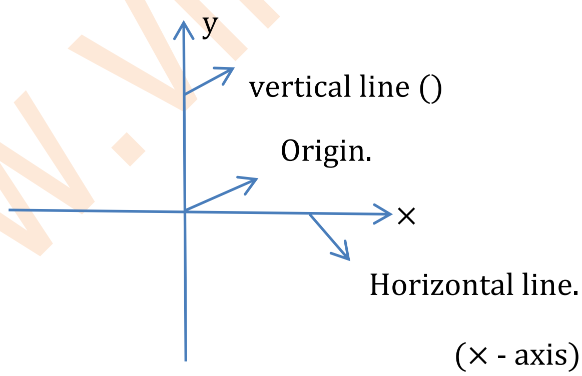
3.3.1. The four quadrants of the Cartesian coordinate plane.

Definition 3.12: The two perpendicular intersecting horizontal and vertical number lines together set up a plane is called Cartesian coordinate plane.

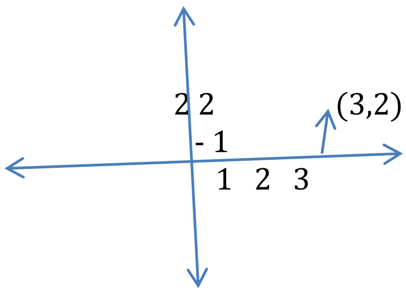
- ✓ The horizontal number line is called × - axis.
- ✓ The vertical number line is called The y – axis.
- ✓ The point where × - axis and y- axis intersect is called Origin.

Locating Points On the Coordinate Plane.

- Points on Cartesian plane is described by two numbers (a, b) that are called **Coordinates.**
- The first number a, is the horizontal position of the point from the origin. It is called × - Coordinate (abscissa).
- The second number b, is the vertical position of the point from the origin. It is called Y – Coordinate (Ordinate).



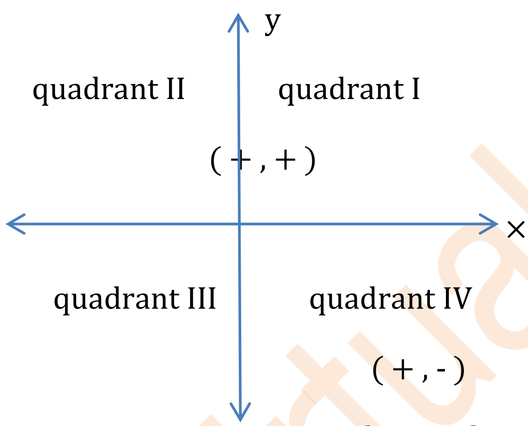
<u>Example 1:</u> locate the point (3,2) on the coordinate plane:



<u>Quadrants</u>

The × - axis and y – axis divides the Cartesian plane in to 4 regions known as **<u>quadrants.</u>**

- ✓ 1st quadrant , 2nd quadrant, 3rd quadrant and 4th quadrant.



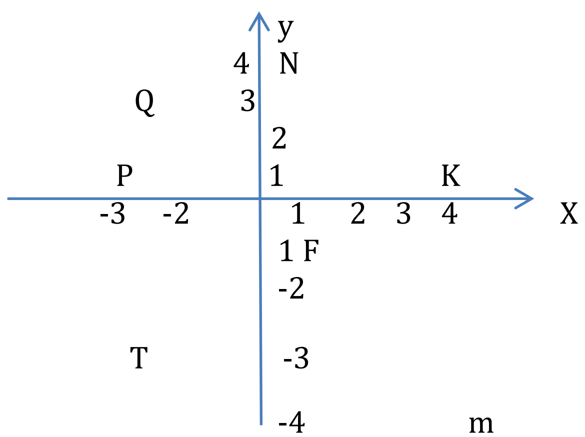
- ✓ Quadrant I contains positive × values and positive y – values
- ✓ Quadrant III contains negative × - values and positive y- values.
- ✓ Quadrant IV. Contains positive × - values and negative y- values.

Exercise

1. Locate the following points on the same Cartesian coordinate plane.

a. m (- 1, 4)	d. P (-3, - 4)
b. n (4 , 6)	e. Q (-5, 0)
C. (4, - 1)	f. R (0, -3)

2. Based on the coordinate plane below answer the following questions.



a) Write the coordinate of the points F, T, P, M, N, Q, and K.

b) Which point has the coordinate (- 2, 3)

Solution

① a. M (-1, 4) → 2nd quadrant

b. N (4, 6) → 1st quadrant

C. C (4, -1) → 4th quadrant

d. P (-3, -4) → 3rd quadrant

e. Q (-5, 0) → X - axis

f. R (0, -3) → y – axis

② a. F (0, - 1) M (2, - 4) K (4, 1)

b. T (-3 , -3) N (0, 4)

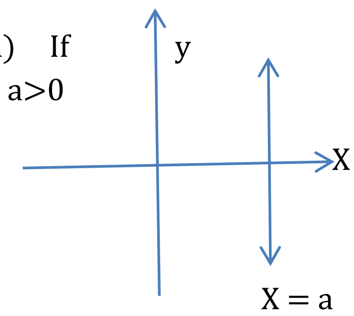
C. P (-3, 0) Q (-2, 3)

3.3.2. Coordinates and graph of linear equations

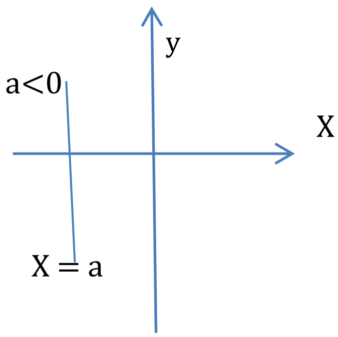
① Graph of an equation of the form X= a, where a is constant.

→ The graph of the equation of the line X = a passes through at X = a which is parallel to the y – axis and perpendicular to the X - axis

a) If a>0



b) If a<0



2. Graph of an equation of the form y = "b" where b is constant.

✷ The graph of the equation of the line y = b (b is constant)

To draw the graph of the line y = b follows the following steps:

<u>Rule 1</u> : Prepare table of values relating X and y in case of y=b, for any different values of X the value of y is constant.

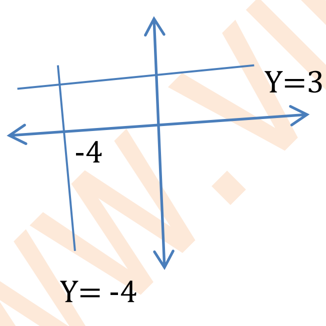
<u>Rule 2 :</u> plot the coordinate of the point in steps.

<u>Rule 3</u> **:** join all points in step 2 using straight line.

e.g : draw the graph of a. y = - 4

b. y = 3

Solution



<u>Note:</u> The graph of the equation of the line y=b is a line parallel to X- axis at a distance of b unit from the origin.

- If b is positive, then the line lies above the X- axis.
- If b is negative , then the line lies below the X- axis.

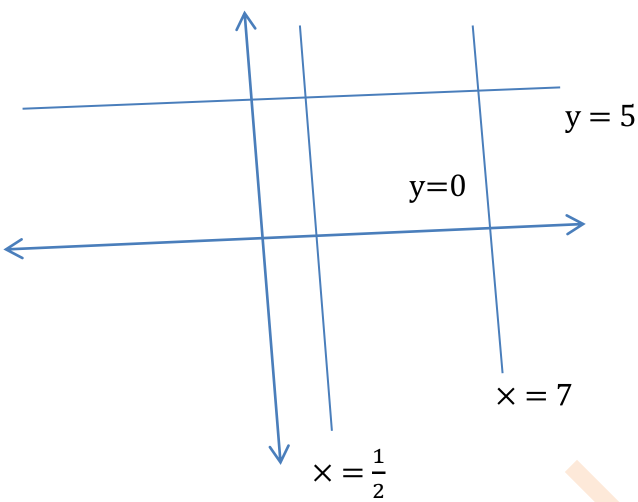
- If b=0, then the line lies on the X-axis.

Exercise

1. Draw the graph of the following equations on the same coordinate plane.

a. $x = 7$ b. $y = 5$ C. $x = \frac{1}{2}$ d. $y = 0$

solution



3. Graphs of equation of the form $y=mx$ where m is constant , and $m \neq 0$.

Note: The graph of $y=mx$ passes through 1st and 3rd quadrant when $m > 0$.

- The graph of $y=mx$ passes though 2nd and 4th quadrant when $m<0$.
- For any values of m the graph of $y=mx$ passes through the origin.
- In the equation of the line $y=mx$, m is called the slope of the line.

Exercise

1. Which of the following equation of lines through 1st and 3rd quadrant.

a. $y = 7x \rightarrow$ 1st and 3rd $m = 7$

b. $y= 10x$ 2nd and 4th $m = -10$

4. If a point (2,8) lies on the line $y=mx$ then find the value of m

$y=mx$ at (2,8)

$8 = m(2)$

$\underline{m = 4}$

3.4. Applications

Solving word problems

To solve problems, follow the following steps

1. Read the problem carefully
2. Select variables for unknown quantities
3. Write a mathematical equations.
4. Solve the equation
5. Interpret the result and write the final answer in words.
6. Check the answer

Exercise

1. A number increased by 7 gives 20, what is the number ?
2. 15 more than twice a number is 37, what is a number ?
3. A number is doubled and the result is increased by 8. If the final result is 36 what is a number?

Solution

$\times + 7 = 20$

$\times = 20 - 7$

$\underline{\underline{\times = 13}}$

$\neq$ 2. Let number be "y "

$2y + 15 = 37$

$2y = 37 - 15$

$2y = 22$

$\underline{\underline{y = 11}}$

$\neq$ 3. Let number be "m "

$2m + 8 = 36$

$2m = 36 - 8$

$2m = 28$ $\quad$ $\underline{\underline{m = 14}}$

Exercise

1. The sum of three consecutive integer is 345. What are the numbers?
2. The sum of two consecutive odd integer is 144. What are the numbers?
3. The sum of two consecutive even integer is 170. What are the integers?
4. The sum of the age of a man and his wife is 83. The man is 3 years older than his wife How old is a man and his Wife?

Solution

≠ 1. Let " × " be the smallest integer

$\times + (\times+1) + (\times+2) = 345$

$3\times+3 = 345$

$3\times = 342$

$\underline{\underline{\times = 114}}$

Then the numbers are 114, 115, and 116.

≠ 3. Let " b " be the smallest even integer.

$b+ (b+2) = 170$

$2b+2 = 170$

$2b = 170\text{-}2$

$2b = 168$

$\underline{\underline{b = 84}}$

Then the two consecutive integers are 84 and 85.

≠ 2. Let " × " be the smallest odd integer.

$\times + (\times+2) = 144$

$2\times+2 = 144$

$2\times = 142$

$\times = 71$

Then the two consecutive odd integers are 71 and 73.

$\neq$ 4. Let " y " the age of man's wife

" " be the age of a man.

$m = y+3$

$y+(y+3) = 83$

$2y+3 = 83$

$2y = 83-3$

$2y = 80$

$\underline{\underline{y = 40}}$ and $y+3 = 43$

$\therefore$ The age of a man is 43 and the age of man's wife is 40.

UNIT - 4

Ratio, Proportion and Percentage

4.1 Ratio and Proportion

4.1.1. Ratio

Definition 4.1: The method of comparing two or more quantities of the same kind and in the same unit is called ratio

ratio of two qualities denoted by a : b

- a is called antecedent
- b is called consequent

✷ Ratio of quantities have no esnit.

e. g: In a class there are 18 boys and 24 girls.

a. What is the ratio of boys to girls.

b. What is the ratio of girls to boys.

C. What is the ratio of boys to total number of students in the class?

Solution

→ Total = no of boys + no of girls

$= 18+24$

$= 42$

a. Ratio of boys to girls $= \frac{number\ of\ boys}{number\ of\ girls}$

$= \frac{18}{24} = \frac{3}{4} = \underline{\underline{\mathbf{3:4}}}$

b. Ratio of girls to boys $= \frac{no\ of\ girls}{no\ of\ boys}$

$$= \frac{24}{18} = \frac{8\times3}{3x6} = 8:6 = \underline{\underline{\mathbf{4:3}}}$$

C. Ratio of boys to total $= \frac{no\ of\ boys}{Total}$

$$= \frac{18}{42} = \frac{9}{21} = \frac{3}{7} = 3:7$$

Exercise

1. Write down the ratio of the first number to the second one in the simplest form:

a. 48 and 80 C. $\frac{2}{21}$ and $\frac{8}{21}$

b. 360 and 72

Solution

a. $48:80 = \frac{48}{80} = \frac{48\div8}{80\div8} = \frac{6}{10} = \frac{3}{5} = \underline{\underline{\mathbf{3:5}}}$

b. $\frac{360}{72} = \frac{180}{36} = \frac{90}{18} = \frac{45}{9} = \underline{\underline{\mathbf{5:1}}}$

C. $\frac{2}{21} \Big/ \frac{8}{21} = \frac{2}{21} \times \frac{21}{8} = \frac{2}{8} = \frac{1}{4} = \underline{\underline{\mathbf{1:4}}}$

Exercise

1. Find two numbers whose ratio is 3 to 5 and whose sum is 192.

2. A wire of length 240 cm is cut in to 3 pieces, in the ratio 1:2:5. Find the length of each pieces.

3. Two numbers have ratio 12:5. Their difference is 98. Find the larger number.

4. Aster, Fatuma, Mohammed and yared contribute the sum of money to renaissance dam in the ratio 1:3:5:7. If the largest amount contributed is birr 1050. Calculate the amount contributed by each person.

Solution

≠ 1. Let 1st number is 3× and the 2nd

number is 5×, then

$3\times + 5\times = 192$

$8\times = 192$

$\times = 24$

∴ The 1st number is $3\times = 3\times 24 = \underline{\underline{72}}$

The 2nd number is $5\times\ 5\times 24 = \underline{\underline{120}}$

≠ 2. | 1× 2× 5× |

______ 240 cm _____________

$1\times+2\times+5\times = 240cm$

$8\times = 240cm$

$\underline{\underline{\times = 30cm}}$

∴ The length of each pieces are 30cm, 60cm, 150cm.

≠ 3. Let the larger number is 12× and the smaller

number is 5×, then

$12\times - 5\times = 98$

$7\times = 98$

$\underline{\underline{\times = 14}}$

∴ The larger number is $12\times = 12\times 14 = \underline{\underline{168}}$

≠ 4. Aster contributed $= 1\times = 1\times\ 150 = \underline{\underline{150}}$

Fatuma contributed $= 3\times = 3\times\ 150 = \underline{\underline{450}}$

Mohammed contributed $= 5\times = 5\times\ 150 = \underline{\underline{750}}$

Yared contributed $= 7\times = \underline{\underline{1050}}$

$7\times = 1050$

$\underline{\underline{\times = 150}}$

Definition 4.2: proportion is the equality of two ratios.

Note: If the four quantities a, b, C and d are in proportion, then a : b = C : d

- The proportion a : b = C : d can be written as $\frac{a}{b} = \frac{c}{d}$, here a and d are called extremes (end terms) and b and C are called means (middle terms)
- In proportion the product of means is equal to the product of extremes. $\frac{a}{b} = \frac{c}{d}$ then $a\times d = b\times C$.

✷ If a : b = C : d then = b×C.

b×C

a×d

Exercise

1. from each pair of ratios below are in proportion.

a. $\frac{2}{3}$ and $\frac{4}{260}$ b. $\frac{1}{3}$ and $\frac{5}{20}$

C. $\frac{4}{5}$ and $\frac{12}{20}$ d. $\frac{8}{4}$ and $\frac{2}{1}$

Solution

a. $\frac{2}{3}$ and $\frac{4}{260}$

$2\times260 = 3\times4$

$520 \neq 12$

b. $\frac{1}{3}$ and $\frac{5}{20}$

$1\times20 \neq 3\times5$

$20 \neq 15$

C. $\frac{4}{5}$ and $\frac{12}{20}$

$4\times20 \neq 60$

$80 \neq 60$

d. $\frac{8}{4}$ and $\frac{2}{1}$

$8\times1 = 4\times2$

$8 = 8$ They are in proportion.

3. Show that the numbers 14, 21, 2 and 3 are in order of proportion.

4. Given the proportion 10 : 18 = 35 : 63, then find

a) The sum of means

b) The product of means

C) The sum of extremes

d) The product of extremes

Solution

≠ 3. $\frac{14}{21} = \frac{2}{3}$

$14\times3 = 21 \times 2$

$42 = 42$ ∴ They are in proportions.

≠ 4. a) sum of means $= 18 + 35 = 53$

b product of means $= 18 \times 35 = 630$

C) sum of extremes $= 10 + 63 = 73$

d) product of extremes $= 10 \times 63 = 630$

Direct and inverse proportionality

A. Direct Proportionality

Definition : y is said to be directly proportional to X (written as y $\alpha \times$), if there is constant K such that $y = k\times$ or $k = y/\times$ The number k is called constant of proportionality.

- If y $\times$, then as $\times$ increase y also increase or as $\times$ decrease y also decrease.

Exercise

1. If y is directly proportional to $\times$:

y= 24 when $\times$=6 then find

a. The constant proportionality

b. The value of y, when $\times$= 3

C. The value of $\times$, when y = 15.

2. y is directly proportional to $\times$, if $\times$= 20 when y= 160 then what is the value of $\times$ when y=3.2

Solution

1. a) $y = k\times$ b) $y = 4\times$ C) $y = 4\times$ $\times = \frac{15}{4}$

$\frac{24}{6} = \frac{6k}{6}$ $\underline{K = 4}$ $y = 4\times 3$ $15 = 4\times$ $\underline{\times = 3.75}$

$\underline{y = 12}$

2. As $\times$ increase y also increase and the ratio $\frac{y}{\times}$ is constant.

$K = \frac{160}{20}$ then $y = 8\times$

$3.2 = 8\times$, $\underline{\times = 0.4}$

B. inverse Proportionality

⇨ **Definition 4.4:** y is said to be inversely proportional to $\times$ (written as y $\alpha \frac{1}{\times}$) if there is constant k such that $y = \frac{1}{\times}$ or $k = y\times$

Exercise

1. If y $\boldsymbol{\alpha} \frac{1}{\times}$ and y = 6 when ×=4, then find the constant proportionality.
2. y is inversely proportional to ×. If ×= 25, then y = 8. What is the value of y when ×= 10?
3. If takes 8 days for 35 laborers to harvest coffees on a plantation. How long will 20 laborers take to harvest coffee on the same plantation.
4. 60 men working in a factory produce 6000 articles in 10 days. How long it takes.

a. 50 men to produce 6,000 articles.

b. 150men to produce 6,000 articles.

5. A contractor appoints 36 workers to build a wall. They could finish the task in 12 days. How many days will 16 workers take to finish the same task?

Solution

1. $y = \frac{k}{\times}$

$K = \times . y$

$K = 4 \times 6$

$\underline{K = 24}$

2. $y = \frac{k}{\times}$

$k = 25 \times 8$

$\underline{k = 200}$

3. No of laborers $\times \frac{1}{Time}$

$L = \frac{k}{t}$

$k = L \times t$

$k = 8 \times 35 = \underline{280}$

⇨ Time $= \frac{280}{20} = 14$ days.

4) a) Time $= \frac{1000}{50} = 20$ days

b) Time $= \frac{1000}{150}$

$= \frac{600}{15} = \frac{20}{3}$

$= 6\frac{2}{3}$ days or $\underline{6}$ days and $\underline{16}$hrs

5. k = 36 (12) = 432

Time $= \frac{432}{16} = \underline{27}$ days

4.2. Revision On percentage:

Definition 4.5: The word percent means " for every hundred "per 100" we use the symbol % to denote percent.

Exercise

1. Convert each of the following percent forms to fractions.

a. 60% b. 2.6% C. d. 0.045%

2. Convert each of the following percent forms in to decimals?

a. 60% b. 2.6% C. $\frac{3}{4}\%$ d. 0.045%

Solution

≠1. a. $60\% = 60 \times \frac{1}{100} = \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$

b. $2.6\% = 2.6 \times \frac{1}{100} = \frac{2.6}{1000} = \frac{26}{10} \div \frac{1}{100} = \frac{26}{10} \times \frac{100}{1}$

$= \frac{13}{500}$ $= \underline{\mathbf{260}}$

C. $\frac{3}{4}\% = \frac{3}{4} \times \frac{1}{100} = \frac{3}{400}$

d. $0.045\% = 0.045 \times \frac{1}{100} = \frac{0.045}{100000} = \frac{45}{100,000} = \frac{9}{20000}$

≠2. a. $80\% = 80 \times \frac{1}{100} = \frac{80}{100} = \frac{8}{10} = \frac{8}{10} = \underline{0.8}$

b. $26\% = 26 \times \frac{1}{100} = \frac{2.6}{100} = 0.26$

C. $12\% = 12 \times \frac{1}{100} = \frac{12}{100} = 0.12$

d. $\% = \frac{2}{5} \times \frac{1}{100} = \frac{2}{500} = \frac{0.4}{100} = \underline{\mathbf{0.004}}$

<u>Note</u> ✷ There are **Three** basic types of Guidelines of percentage; decimal and fraction.

1. Converting percent to decimal.

To convert percent to a decimal, remove the % symbol and divide by 100 (shift the decimal place two steps to the left).

2. Converting percent to fraction:

To convert a percent to fraction, remove the % symbol and put 10 as denominator and write the fraction in the lowest term.

3. Converting decimal to percent.

To convert a decimal to percent, multiply by 100% which is 1 (and attach the % symbol on the result) or shift the decimal place two steps to the right.

e. g : $0.245 = 0.245 \times 100\% = 24.5\ \%$

Exercise

1. Convert each of the f.f. fractions in percent form:

a. $\frac{3}{5}$ b. $\frac{1}{6}$ C. $2\frac{3}{4}$

2. Express the following decimals in percent form:

a. 0.12 b. 7.5 C. 0.0012

Solution

≠1. a. $\frac{3}{5} = \frac{3}{5} \times 100\% = \frac{300}{5}\ \% = 60\%$

b. $\frac{1}{6} = \frac{1}{6} \times 100\% = \frac{100}{6}\ \% = \frac{50}{3}\ \%$

C. $2\frac{3}{4} = 2\frac{3}{4} \times 100\% = \frac{11}{4} \times 100\% = 11 \times 25\% = \underline{\underline{275}}\%$

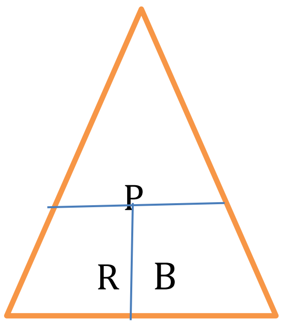
≠2. a. $0.12 = 0.12 \times 100\% = 12\%$

b. $7.5 = 7.5 \times 100\% = 75 \times 10\% = 750\%$

C. $0.0012 = 0.0012 \times 100\% = 0.12\%$

Calculating base , rate and percentage

You can use the following triangle to easily remember the relationship between P,R and B



1. $P = R \times B$

2. $R = \frac{P}{B} \times 100\ \%$

3. $B = \frac{P}{R}$

Exercise

1. Calculate each of the following

   a. What is 10% of 160?

   b. What is 60% of Birr 300?

2. In a class there are 60 students. If 20% of the class are girls, then how many girls and boys are there?

3. 25% of people in addis ababa on TV. How many people watched the foo ball game if the population of the city is 5,000,000

Solution

1. a. $R = R\times B = \frac{10}{100} \times 160 = \frac{160}{10} = \underline{16}$

   b. $p = R\times B = \frac{60}{100} \times 300 = \frac{1800}{10} = \underline{180}$

≠2. Number of girls $= R\times B = \frac{20}{100} \times 60 = \underline{12}$ girl

Students & number of boys $= R\times B$

$$= \frac{80}{100} \times 60 = \underline{48}$$

≠3. Number of people $= R\times B$

$$= \frac{25}{100} \times 5{,}000{,}000 = \underline{250{,}000}$$

Exercise

1. calculate the following:

   a. 6 is 24% of a number, what is the number?

   b. If Birr 700 is 35% of Birr. ×, then what is the value of × ?

   C. If 15% of a number is 18, then what is the number?

2. If 30% of a man's salary is Birr 6300, what is the amount of his salary?

Solution

1. a. $B = \frac{P}{R} = \frac{6}{24\%} = \frac{6 \times 100}{24\%} = \frac{100}{4} = \underline{\underline{25}}$

   b. $B = \frac{P}{R} = \frac{700}{35\%} = \frac{700 \times 100}{35} = \underline{\underline{2{,}000}}$

   C. $B = \frac{P}{R} = \frac{18}{15\%} = \frac{18 \times 100}{15} = \frac{600}{5} = \underline{\underline{120}}$

2. The man's full salary $= \frac{P}{R} = \frac{6{,}300}{30\%} = 6300 \times \frac{100}{30}$

$$= \frac{63{,}000}{3}$$

$$= \underline{\underline{21{,}000}}$$

Exercise

1. A woman saves Br 300 from her monthly salary if her monthly salary is Br 7500, then find her saving in percent?
2. In a class of 48 students 6 of them were absent on Monday. What percent of the class was absent and what percent of class was attended on that day?
3. in a basket of oranges 20% of them are defection and 76 are in good condition find the total number of students present in the class.
4. Tolosa sold 540 eggs. If these are 36% of total eggs, then how many eggs are not sold?
5. A factory has 2400 workers, 900 are males and the rest are females. What percent of the workers are female?

Solution

1. Percent of woman saving $= \frac{P}{B} \times 100\%$

$$= \frac{300}{7500} \times 100\% = \underline{\mathbf{4\%}}$$

2. Percent of Absent student

$$= \frac{P}{B} \times 100\%$$

Attend $= \frac{42}{48} \times 100\%$

$= \underline{\mathbf{87.5\%}}$ $\quad = \frac{6}{48} \times 100\% = 12.5\%$

Percent of attended student $= \frac{P}{B} \times 100\%$

$$= \frac{42}{48} \times 100\%$$

$$= \underline{\mathbf{87.5\%}}$$

3. percent of non – defective oranges = 100% - 20%

$$= 80\%$$

Total number of oranges $= \frac{P}{B} = \frac{76}{80\%} = \frac{76 \times 100}{80} = \underline{\mathbf{95}}$

4. Total number of eggs $= \frac{P}{B} = \frac{540}{36\%} = \frac{540 \times 100}{36} = \underline{\mathbf{1,500}}$

No of egg not sold = Total no of eggs – no of sold

$$= 1,500 - 540 = \underline{\mathbf{960}}$$

5. number of female workers = 1500

Percent of female workers $= \frac{P}{B} \times 100\% = \frac{1500}{2400} \times 100\%$

$$= 62.5\%$$

4.3. Application of Ratio, proportion and percentage

Percent increase and decrease

1. Percent increase $= \frac{increase\ amount}{Original\ quality} \times 100\%$

2. Percent decrease $= \frac{decrease\ amount}{Original\ quality} \times 100\%$

Exercise

1. Find the percent change

   a. from 80 to 100

   b. From 800 to 500

2. The number of students fail in mathematics test decreased from 20 to 12. What is the percent decrease?

3. last year samuel's salary was Birr 8000. If he gets 10% increment this year, what is his current salary?

Solution

1. a. percent decreased $= \frac{100-80}{80} \times 100\% = \frac{20}{80} \times 100\% = \underline{\mathbf{25\%}}$

   b. percent increase $= \frac{800-500}{800} \times 100\% = \frac{300}{800} \times 100\% = \underline{\mathbf{37.5\%}}$

2. Percent decrease $= \frac{20-12}{20} \times 100\% = \frac{8}{20} \times 100\% = \underline{\mathbf{40\%}}$

3. let $\times$ be current salary

$\frac{10}{100} = \frac{\times-8000}{8000}$

$100\times\text{-}\ 800000 = 80000$

$100\times = 80000 + 800,000$

$100\times = 880{,}000$

$\times = \underline{88000}$

$\therefore$ H is current salary is <u>8,8,000</u>

4.3.1. Calculating profit and lass percentage

<u>Definition</u>

<u>Cost price</u> (C . P) ⇨ is the price at which an article is purchased.

<u>Selling price</u> (S . P) ⇨ is the price at which an article is sold.

<u>Profit</u> (Gain): when S . P > C . P

<u>Loss:</u> when S . P < C . P, then there is loss,

Loss = C . P – S . P.

Exercise

1. An article was bought for birr 2,000 and sold by Birr 2,200 . Find the profit or loss percent.
2. A shop keeper bought a jacket for Birr 1500 and gives it clean, then sold it for Birr 1,800 . What is his profit percent?
3. A man bought 100 eggs for birr 800 and sells them for Birr 10 each, then find his profit percent?

Solution

1. Profit = S. P – C.P

$= 2{,}200 - 2{,}000$

$= 200$

$\%\ \text{profit} = \frac{S.P.-C.P}{C.P} \times 100\%$

$= \frac{200}{2{,}000} \times 100\%$

$= \underline{\mathbf{10\ \%}}$

2. $\%\ \text{profit} = \frac{S.P.-C.P}{C.P} \times 100\%$

$= \frac{1800-1500}{1500} \times 100\%$

$= \frac{300}{1500} \times 100\%$

$= \underline{\mathbf{20\ \%}}$

3. S. P = 100×10 = 1,000

C.P = 800

Profit = S. P – C.P

$= \underline{\mathbf{200}}$

$\%\ \text{profit} = \frac{200}{800} \times 100\%$

$\%\ \text{profit} = \frac{200}{8}\ \%\ \frac{100}{4} = \underline{\mathbf{25\%}}$

4.3.2. Simple interest

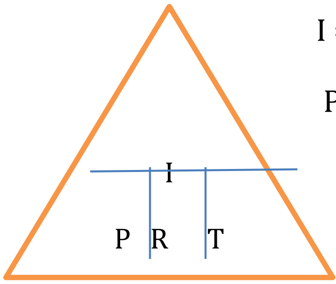
The interest paid on original principal only during the whole interest period is called ***simple interest***

Simple interest is calculated by the formula:

$I = p \times R \times T$

Amount is given by $A = I + P$

You can use the following triangle to easily remember the relation



$$I = p \times R \times T$$

$$P = \frac{I}{R \times T} \qquad T = \frac{I}{P \times R \times T}$$

$$R = \frac{I}{R \times T}$$

Exercise

1. Michael invests Birr 2,000 in the bank that pays simple interest rate of 5% per year for 6 years. Then how much interest will he get in 6 years?
2. Birr 24,000 invested at 11 % simple interest per annum, then what is the amount after 8 years?
3. How long will take for Birr 15000 to double itself, if it is invested at simple interest rate of 10% per year.
4. If Birr 20,000 grows to Birr 28000 after 20 years. Then what is the simple interest rate?
5. An investment earned Birr 2100 interest after 6 years. If the simple interest rate is 7% per year. What was the principal?

Solution:

1. $I = p \times R \times T$

$= 2000 \times \frac{5}{100} \times 6$

$= 100 \times 6$

$= 600$

2. $I = p \times R \times T$

$I = 24000 \times \frac{11}{100} \times 8 = 21{,}120$

$A = I + p$

$= 24000 + 21{,}120$

$A = \underline{\underline{45{,}120}}$

3. $I = A - P$

$I = 30,000 - 15,000 = 15,000$

$T = \frac{I}{P \times R} = \frac{15{,}000}{15000 \times \frac{10}{100}} = \frac{15000}{1500} = \underline{\underline{10}}$ years

4. $I = 28{,}000 - 20{,}000$

$= 8{,}000$

$R = \frac{I}{P \times T}$

$R = \frac{8{,}000}{20{,}000 \times \frac{20}{100}} = \frac{8{,}000}{40{,}000}$

$\underline{R = 20\ \%}$

5. $P \frac{I}{P \times T} = \frac{2100}{\frac{7}{100}} \times 6 = \frac{2100 \times 100}{42} = \frac{210000}{42} =$ **Birr 5,000**

4.3.3 compound interest

Compound interest is the interest on a loan calculated based on the initial principal plus the accumulated interest from perilous periods.

✷ For compound interest amount can be calculated by the formula

$A = P\ (\ I + R\ )^T$

A = Amount R = interest rate, P = principal and T = Time and P = A – I

Exercise

1. Find the compound interest on Birr 8,000 for 2 years at 5% per annum, compounded annually.
2. compare the simple interest and compound interest for Birr 8,000 at 10% per annum for three years if the interest is compounded annually.
3. Find the difference between the simple and the compound interest on Birr 5,000 for 2 years at 6% per annum?

Solution

1. $A = P\ (\ I + R\ )^T$

   $A = 8000\ (\ 1+0.05)^2 = 8820$

   $I = A - P$

   $= 8820 - 8000$

   $= \underline{\mathbf{820}}$

2. $A = P\ (\ I + R\ )^T$

   $= 8000\ (\ 1+0.3)^3 = 10648$

   $I = A - P = 10648 - 8{,}000$

   I = Birr **$\underline{2648}$**

3. Simple interest = P RT

   $= 5000 \times 0.06 \times 2 = 600$

$A = P(I+R)^T$

$= 5000(1+0.06)^2 = 5618$

$I = A - P = 5618 - 5{,}000 =$ Birr 618

The difference between compound interest and simple interest is 618 – 600 = **18**

4.3.4 Ethiopian income tax, Turn over Tax, VAT

Taxes are imposed by the governments on their citizens to generate income for under taking projects to boost the economy of the country.

VAT ⇨ value added Tax

VAT is a tax imposed by government on sales of some goods and services.

Note:

1. In Ethiopia VAT rate is 15%
2. Amount of VAT = 15% of original cost
3. Cost including VAT = original cost + amount of VAT

Example: The price of a machine is Birr 3,000 before VAT.

a. Calculate the amount of VAT

b. Calculate the total cost of machine

Solution:

a. Vat = 15% of original cost

$= \frac{15}{100} \times 3{,}000$

= Birr 450

b. Total cost of machine = 3,000 + 450

= Birr **3,450**

Turn over Tax (TOT)

- Turn over tax is imposed or merchants who are not required to register for VAT. But supply goods and services in the country.
- In Ethiopia turn over tax rate is 2% on goods sold and Services rendered locally.

Note 1. A merchant his annual income below Birr 500,000 will be registered to collect turn over tax.

2. A merchant whose annual income above Birr 500,000 will be registered to collect turn over tax.

E.g 1: Calculate turn over tax on sales of Birr 10,000

Solution

Turn over tax = 2% of 10,000

$= \frac{2}{100} \times 10{,}000$

$= \underline{200}$ birr

Employment income tax

Employer deduct income tax from the employed before paying monthly salary based on the following tax rate (according to Ethiopian income tax rate).

Employment income (per month) in Birr	Employment income Tax rate
Above 0 up to 600	0%
Above 600 up to 1650	10%
Above 1,650 up to 3,200	15%
Above 3,200 up to 5,250	20%
Above 5,250 up to 7,800	25%
Above 7,800 up to 10,900	30%
Over 10,900	35%

Exercise

1. Find the income tax and net income of the following employees of commercial bank of Ethiopia.

a. A to Ahmed with monthly salary of Birr 7,500.

b. W/ro Mekds with monthly salary Birr 11,600

Solution:

a . ⇨ Birr 7,500 Falls on 25%

for interval 0 to 600, the tax rate is 0%

hence $Tax_1 = 0$

⇨ For interval 600 to 1,650, the tax rate is 10%. Hence tax on this in

$\text{Tax}_2 = \frac{10}{100} \times 1050 = 105$ birr

⇨ For interval 1650 to 3,200, the tax rate is 15%

Hence tax on this in

$\text{Tax}_3 = \frac{15}{100} \times 1{,}550 = \text{Birr } 232.5$

⇨ For interval 3,200 to 5,250, the tax vate is 20%, Hence tax on this.

$\text{Tax}_4 = \frac{20}{100} \times 2050 = \text{Birr } 410$

⇨ For interval 5,250 to 7,500 , the tax rate is 25% , Hence tax on this in

$\text{Tax}_5 = \frac{25}{100} \times 2250 = \text{Birr } \underline{\mathbf{562.5}}$

$\text{Income tax} = \text{Tax}_1 + \text{Tax}_2 + \text{Tax}_3 + \text{Tax}_4 + \text{Tax}_5$

$= 0 + 105 + 232.5 + 410 + 562.5$

$= \text{Birr } \underline{\mathbf{1320}}$

Net income = 7,500 – 1,320 = Birr **<u>6180</u>**

b. For interval 0 to 600, the tax rate is 0% , Hence Tax, = 0.

⇨ For interval 600 to 1,650, the tax rate is

10%, Hence tax on this

$\mathbf{Tax_2} = \frac{10}{100} \times 1050 = \text{Birr } 105$

⇨ For interval 1,650 to 3,200, the tax rate is 15% Hence tax on this

$\mathbf{Tax_3} = \frac{15}{100} \times 1550 = \text{Birr } \underline{\mathbf{232.5}}$

⇨ For interval 3,200 to 5,250, the tax rate is 20%, Hence tax on this

$\mathbf{Tax_4} = \frac{20}{100} \times 2050 = \text{Birr } 450$

⇨For interval 5,250 to 7,800 the tax rate is 25% , Hence tax on this

$\mathbf{Tax_5} = \frac{25}{100} \times 2550 = \text{Birr } 637.5$

⇨For interval 7,800 to 10,900 the tax rate is 30%, Hence tax on this

Tax$_6 = \frac{30}{100} \times 3100 =$ **<u>Birr 930</u>**

- For interval 10,900 to 11,600 the tax rate is 35% , Hence tax on this

  Tax$_7 = \frac{35}{100} \times 700 =$ Birr 245

- Income tax $= \text{Tax}_1 + \text{Tax}_2 + \text{Tax}_3 + \text{Tax}_4$

  $+ \text{Tax}_5\ \text{Tax}_6 + \text{Tax}_7$

  $= 10 + 232.5 + 410 + 637.5 +$

  $930 + 245 \quad =$ Birr 2,560

  Net income $= 11{,}600 - 2{,}560 =$ **<u>Birr 9,040</u>**

  Net income $= 11{,}600 - 2{,}560 =$ **<u>Birr 9,040</u>**

Review exercise For Unit – 4

I . True/ False

1. False	3. True	5. False	7. False
2. True	4. True	6. True	8. False

II. Choice

9. A	11. C	13. A	15. B	17. B	19. B
10. B	12. D	14. B	16. D	18. D	20. C

III. Work Out

1. Birr 6000 is invested at rate of 5% compound interest compounded annually, Find

a. The amount at the end of 2 years.

b. the interval at the end of 2 years.

Solution

a. $A = p\,(1+0.05)^2$

$= 6000\,(1.05)^2$

$= 6000\,(1.1025)$

$=$ <u>**6615**</u>

b. Interest $= 6615 - 6{,}000$

$=$ **<u>Birr 615</u>**

2. A person wants to buy a car from Toyota Company . If the price car including VAT is Birr 5,750,000 then

a. What is the price of car before VAT?

b. What is the value of VAT

Solution

a. Price of car before VAT $= \frac{Perice\ including\ VAT}{1.15}$

$$= \frac{5{,}7500}{1.15} = \underline{\text{Birr } 500{,}000}$$

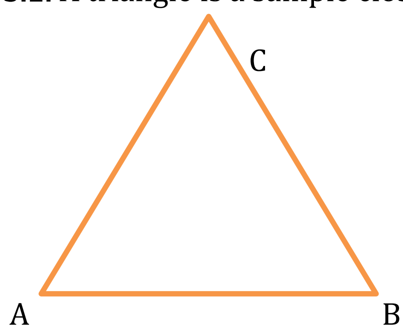
b. VAT $= 500{,}000\ (\ 0.15\ ) = \underline{\text{Birr } 75{,}000}$

UNIT - 5

Perimeter and area of plane figures

5.1 Revision of triangles

Definition 5.1: A triangle is a sample closed plane figure made of three line segment



✷ The interior angles of the above triangle are:

i. The angle at vertex A, <BAC or <A.
ii. The angle at vertex B, <ABC or <B.
iii. The angle at vertex C, <BAC or <C.

✷ The sum of interior angles of at triangle is 180^0

$$<A + <B + <C = 180^0$$

<u>**Note:**</u> A triangle has three sides, three angles and three vertices

2. The sum of all interior angles of a triangle is always equal to 180^0 .

<u>Types of triangles</u>

- Triangles can be classified in 2 different ways.
- Classification of triangle according to interior angles.

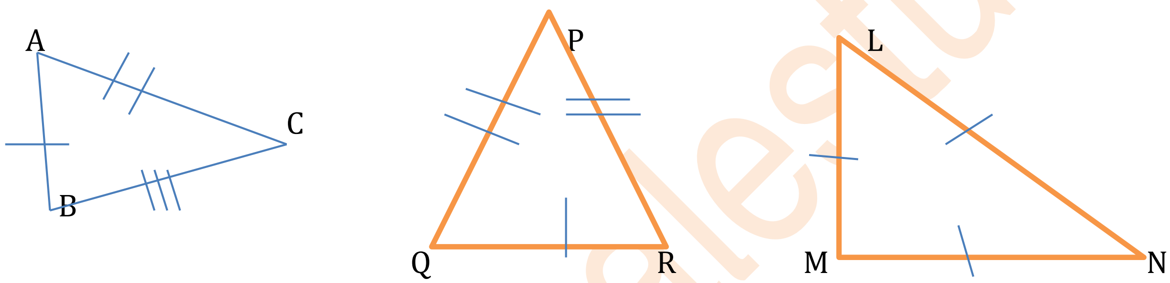
- Classification of triangle based on side length.

Classification of triangle based on sides length

✷ Based on side length, triangles are classified in to three:

1. Scalene triangle
2. Isosceles triangle
3. Equilateral triangle

- A triangle in which all three sides are unequal in length is called *a scalene triangle*
- A triangle in which two of its sides are equal is called isosceles triangle.
- A triangle in which all its three sides are equal in length is called an *equilateral triangle*



Scalene triangle Isosceles Triangle Equilateral Triangle

Classification of a triangle according to interior angles

Exercise

1. Fill in the blank space with the correct answer.

a. The triangle in which all asides are equal is called _________

b. The triangle in which all its sides are different length.

C. Each angle of equilateral triangle is ________

d. ________ is a triangle with two equal sides.

2. Classify the following triangle.

a. Sides of the triangle are 4cm, 4cm, and 7cm.

b. Angles of the triangle are 90^0, 60 and 30^0

C. Angles of the triangle is 110^0, 40^0 and 30^0

<u>Solution (Answer)</u>

1. a. equilateral triangle

   b. Scalene triangle

   C. 60^0

   d. isosceles triangle

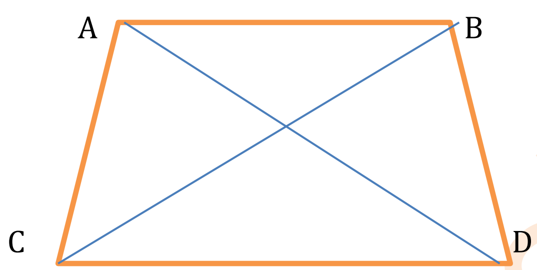
2. a. isosceles triangle

   b. right angle triangle

   C. obtuse angle triangle

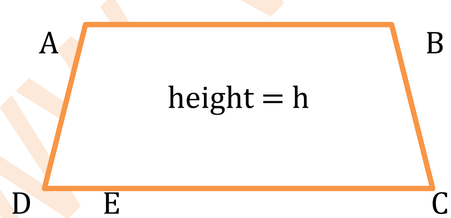
5.2 Four – Sided figures

Definition 5.2: A quadrilateral is a four – sided geometric figure bounded by line segments



- A quadrilateral is named by using its four vertices in clock wise or anti – clock wise direction. It can be named quadrilateral ABCD or CDBA
- The sides of the quadrilateral ABCD above are $\overline{AB}$, $\overline{AC}$, $\overline{CD}$ and $\overline{BD}$.
- A line segment that connects two opposite vertices of the quadrilateral is called **diagonal.** The diagonals of quadrilateral ABCD above are $\overline{AD}$ and $\overline{CB}$.

Definition 5.3: A trapezium is a special type of quadrilateral in which exactly one pair of opposite sides are parallel.



- The parallel sides of trapezium are called the bases of trapezium. In the above figure, the parallel sides $\overline{AB}$, and $\overline{DC}$ are bases.
- The distance between the bases is called the height (altitude) of the trapezium. $\overline{AE}$, is the height.

- The non-parallel sides of the trapezium are called legs of the trapezium. the non-parallel sides $\overline{AD}$ and $\overline{BC}$ are legs.

- It the legs of trapezium are congruent, then trapezium is called **isosceles trapezium.**

isosceles trapezium

Exercise

1. Fill in the blank space with the correct answer.

   a. Four-sided geometric figure is called _______________

   b. A line segment that joins opposite vertex of a quadrilateral is _____________

   C. The point where the sides of a quadrilateral meat is called________________

Answer

1. a. quadrilateral C. vertex

   b. Diagonal

Properties of Parallelogram

1. Opposite sides of parallelogram are long rent.

   AD = BC and AB = DC

2. Opposite angles of a parallelogram are congruent

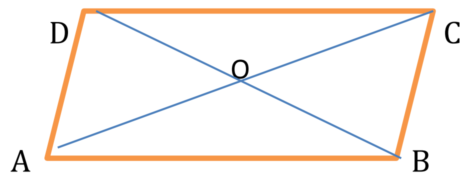
   m (<A) = m (<C)

   m (<B) = m (<D)

3. Consecutive (adjacent) angles of a parallelogram are supplementary.

$m(<A) + m(<B) = 180^0$,

$m(<B) + m(<C) = 180^0$,



4. The diagonals of parallelogram bisect each other.

AO = OC, DO = OB.

Note: Bisect means " divides exactly in to two equal parts.

Properties of rectangle

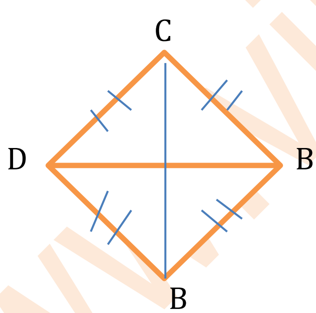
1. Rectangle satisfies all properties of parallelogram.
2. The diagonals of rectangle are equal in length and bisect each
3. All angles of rectangle are right angle.

Note:

1. Right angle is an angle that measures 90^0.
2. All rectangles are parallelogram, but all parallelograms are not rectangles.
3. A quadrilateral with congruent diagonals is not necessarily rectangle.
4. A parallelogram with congruent diagonals is rectangle.

B. Rhombus

Definition 5.6: A rhombus is a parallelogram in which all its sides are congruent.



Properties of rhombus

i. All properties of parallelogram are properties of rhombus.

ii. All sides of rhombus are congruent.

(AB = BC = CD = AD)

iii. The diagonals of rhombus are perpendicular to each other. (AC $\perp$ DB)

i V. The diagonals of rhombus bisects the angles at the vertices.

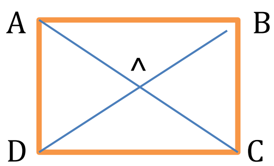
m (< CDB) = m (< ADM)

Note: 1. All rhombus are parallelogram, but all parallelogram are not rhombus.

2. The diagonals of rhombus are perpendicular and bisect each other.

C. Square

Definition 5.7: Square is a parallelogram with four congruent sides and four right angles.



Properties of Square

1. Square satisfies all properties of parallelogram, rectangle and rhombus.

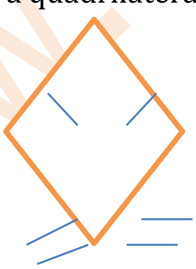
2. The diagonals of a square are:

- Perpendicular to each other.
- Bisect each other.
- Congruent.

3. The diagonals bisect the angles at the vertices. Hence, the diagonals 45^0 with its side.

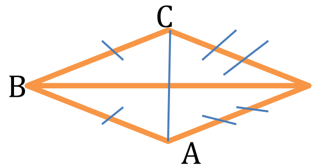
D. kite

Definition 5.8: kite is a quadrilateral that has two pairs of concessive congruent sides, but opposite sides is not congruent.



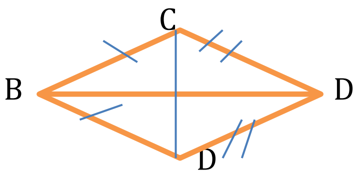
Properties of Kite

1. The diagonals of kite are perpendicular to each other, but they do not bisect each other.



$\overline{AC} \perp \overline{BD}$

2. One pair of opposite angles of kite are congruent



$<A \equiv <C$

$<B \neq <D$

Exercise

I. Write "True" if the statement is correct and write "False" if not.

a. All rectangles are parallelogram.

b. The diagonals of rhombus are congruent

C. The opposite sides of kite are congruent.

d. All squares are rhombus.

e. A quadrilateral with congruent diagonals is necessarily rectangle.

Answer

1. a. True C. False e. False

b. False d. True

Definition 5.9: Area of a closed figure is the of square units inside that closed figure. Perimeter is the length of the boundary of a closed figure.

1. Area and perimeter of rectangle.

- Area of rectangle is the product of its base and height.

A = base × hight

$\boxed{A = b\times h}$

h

b

- The perimeter of rectangle is calculated as:

$P = b + h + b + h$

$P = 2b + 2h$

$$\boxed{P = 2(b + h)}$$

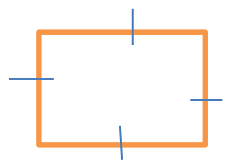
2. Area and perimeter of Square

- Square is a rectangle whose base and height are equal.

The area of a square whose side length "S" is

$$\boxed{A = S^2}$$

and its perimeter is $\boxed{P = 4S}$



Example 1: Calculate the area and the perimeter of rectangle given below.

8cm

6cm

Solution:-

$A = b \times h$

$A = 6cm \times 8cm$

$\underline{A = 48cm^2}$

$P = 2(b + h)$

$= 2(6cm + 8cm)$

$= 2(14cm)$

$= \underline{28cm}$

2. Calculate the area and perimeter of square whose side length is 10m.

Solution

$A = S^2$

$A = (10m)^2$

$\underline{A = 100m^2}$

$p = 4S$

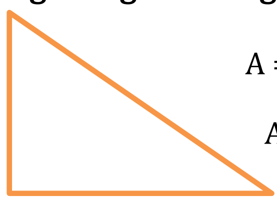
$= 4 \times 10m$

$= \underline{40m}$

3. Area and perimeter of triangle.

a. Area of right angled triangle.

height



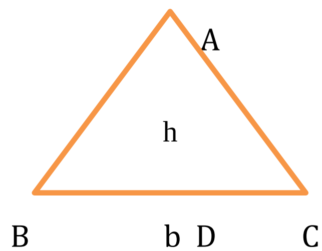
$A = \frac{1}{2}$ (base × height)

$A = \frac{1}{2}$ (b × h)

Base

b. Area of cute angle triangle.

To calculate the area of such triangle, draw perpendicular line from one of the vertices to the opposite Side.



$A = \frac{1}{2}\ b \times h$

$A = \frac{1}{2}$ (AD × BC)

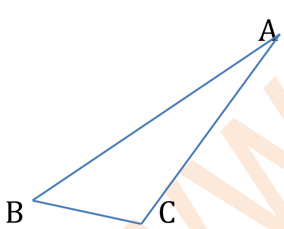
Area of Δ ABC = area of Δ ABD + area of Δ ADC

$A = \frac{1}{2}\ BD \times AD + \frac{1}{2}\ AD \times DC$

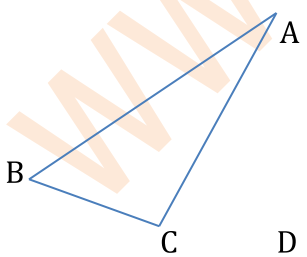
$A = \frac{1}{2}\ AD \times\ (BD + DC)$

$A = \frac{1}{2}\ AD \times BC$ - - - BC = DC + BD

C. area of obtuse angle triangle.



- To calculate the area Of obtuse angled triangle Draw height from vertex A to side extension of base BC.



- Area of Δ ABC = area of Δ ABD - area of Δ ADC

$$= \frac{1}{2}\ BD \times AD - \frac{1}{2}\ AD \times DC$$

$$= \frac{1}{2}\ AD\ \ (BD - DC)$$

$$= \frac{1}{2}\ AD \times BC \text{ - - - } (BD - DC = BC)$$

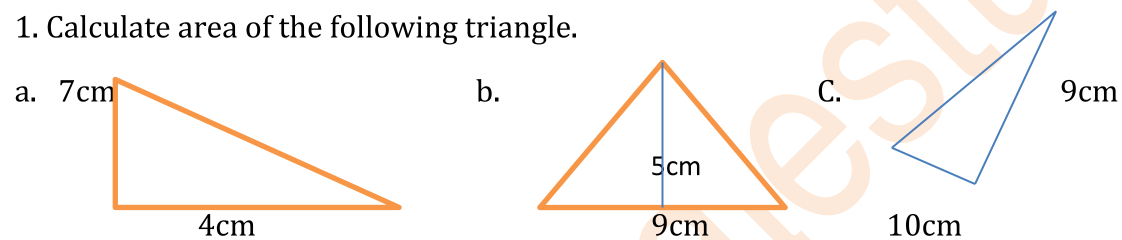
$A = \frac{1}{2} b\times h$ – - - where Ad is height (h) and BC is base (b).

Perimeter of triangle is given by

$$\boxed{P = a + b + c}$$

Examples

1. Calculate area of the following triangle.



Solution

a. $A = \frac{1}{2} b\times h$

$A = \frac{1}{2} \times 7cm \times 4cm$

$A = \underline{\underline{14cm^2}}$

b. $A = \frac{1}{2} b\times h$

$A = \frac{1}{2} \times 9cm \times 5cm$

$A = \underline{\underline{22.5\ cm^2}}$

C. $A = \frac{1}{2} b\times h$

$A = \frac{1}{2} \times 10cm \times 9cm$

$A = \underline{\underline{45\ cm^2}}$

2. The area of triangle is 64 cm^2. If the base is 16 cm long, then calculate the height of the triangle

Solution

$A = 64\ cm^2$, $b = 16cm$, $h = ?$

$A = \frac{1}{2} b\times h$

$64cm^2 = \frac{1}{2}\ (16cm)\ h$

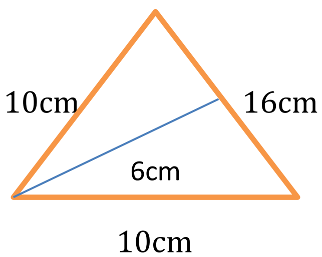
$\frac{64cm^2}{8cm} = \frac{8cm}{8cm} \times h$

$\underline{\underline{h = 8cm}}$

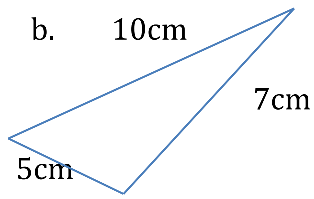
Exercise

1. Calculate the area and perimeter of the following triangles.

a.

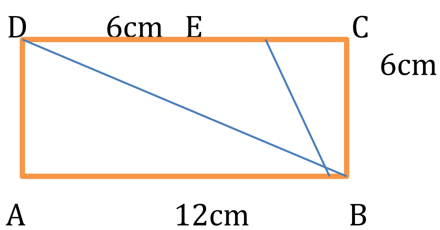


b.



2. The area of triangle is $63cm^2$. if the height is 9cm long, then calculate the length of its base.

3. Based on the figure below answer the following questions.



a. Calculate the area of shaded region.
b. Calculate the area of un shaded region.

Solution

1. a. $A = \frac{1}{2}\ b \times h$

$A = \frac{1}{2}\ 16cm \times 6cm$

$A = 8cm \times 6cm$

$\underline{A = 48cm^2}$

b. $A = \frac{1}{2}\ b \times h$

$A = \frac{1}{2}\ 5cm \times 7cm$

$\underline{A = 17.5cm}$

② $A = 63cm^2$

$h = 9cm$

$b = \frac{63cm^2}{9cm}$

$\underline{b = 7cm}$

③ a. $A\ \text{shaded} = \frac{1}{2}\ (12cm \times 6cm) + \frac{1}{2}\ 16cm \times 6cm = 36cm^2 + 18cm^2$

$= \underline{54cm^2}$

b. $A\ \text{un shaded} = A\ \text{rectangle} - A\ \text{shaded}$

$= 72cm^2 - 54cm^2$

$= \underline{18cm^2}$

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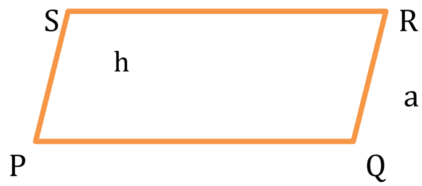
5.4 Perimeter and area of four – sided figures

A. Perimeter and area of parallelogram

1. The area of parallelogram with base "b" and altitude "h" is given by the formula:

$$\boxed{A = b\,h}$$



2. The Perimeter of the parallelogram is calculated by adding all its sides.

$$\boxed{p = PQ + QR + RS + PS}$$

$$P = b + a + b + a$$

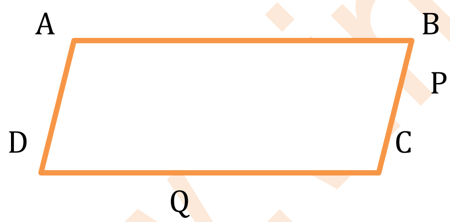
$$P = 2a + 2b$$

$$P = 2(a + b)$$

<u>Example: 1</u> In the figure AP, AQ are altitude of the parallelogram ABCD. If AQ = 4cm, CD= 5cm and AP = 8cm, then calculate

a) Area of the parallelogram
b) Length of BC
c) Perimeter of the parallelogram

Solution



a. $A = b\,h$

$A = DC \times AQ$ Using base DC And height AQ

$A = 5cm \times 4cm$

<u>$A = 20cm^2$</u>

b. $A = b \times h$

$A = BC \times AP$

$\frac{20cm^2}{8cm} = BC$

<u>$BC = 2.5cm$</u>

C. $P = A\,b + BC + CD + AD$

$P = 5cm + 2.5cm + 5cm + 2cm$

<u>$p = 15cm$</u>

Exercise

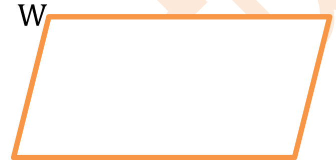
1. Calculate the area of the following parallelogram

2. ABCD is a parallelogram of area $18cm^2$. Find the length of the corresponding altitudes if AB = 5cm.

3. In the figure WQ, QP, are altitudes of the parallelogram WXYZ, If WP = 6cm, XY = 5cm and WQ = 8cm, then

Calculate

a. Area of the parallelogram
b. Length of ZY
c. Perimeter of the parallelogram



Solution

1. $A = b \times h$

$A = (15cm)\ (8cm)$

$A = \underline{120cm^2}$

2. $A = b \times h$

$18cm^2 = 5cm\ (h)$

$h = \underline{3.6cm}$

3. a. $A = b \times h$

$= 6cm\ (5cm)$

$= \underline{30cm^2}$

b. $A = b \times h$

$30cm^2 = 2y\ (8cm)$

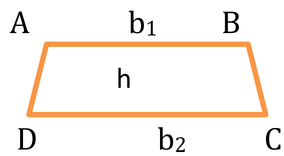
$ZY = \underline{3.75cm}$

C. $P = 3.75cm + 3.75cm + 5cm + 5cm = \underline{17.5cm}$

B. Perimeter and Area of trapezium

1. Area of trapezium with base b_1 and b_2 and altitude h is given by the formula

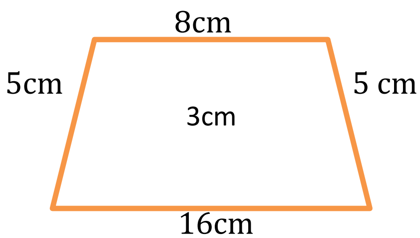
$$A = \frac{h}{2}(b_1+b_2)$$



2. The perimeter of trapezium is the sum of the two base and its legs

$$P = AB + BC + DC + AD$$

<u>Example 1:</u> Calculate the area and perimeter of the trapezium given below.



Solution

$$A = \frac{h}{2}(b_1+b_2)$$

$$A = \frac{3cm}{2}(16cm + 8cm)$$

$$A = \frac{24cm+3cm}{2}$$

$$A = \underline{\mathbf{36cm^2}}$$

$$P = b_1+b_2 + L_1+L_2$$

$$P = 8+ 16+ 5 + 5$$

$$P = \underline{\mathbf{34cm}}$$

<u>Exercise</u>

1. The area of trapezium is $170cm^2$. If its height and one of the bases are 17cm and 12cm respectively. then calculate the other base of the trapezium.
2. One of the bases of the trapezium exceeds the other by 2cm. If the altitude and area of trapezium are 6cm and $42cm^2$ respectively, then calculate the larger base of the trapezium.

<u>Solution</u>

1. $A = \frac{h}{2}(b_1+b_2)$

$$170cm^2 = \frac{17cm}{2}(12cm + b_2)$$

$$b_2 = \underline{\mathbf{8cm}}$$

2. Let $b_1 = b_2 + 2$

$$A = \frac{1}{2}h\,(b_1+b_2)$$

$42cm^2 = \frac{6cm}{2}(b_2 + 2 + b_2)$

$\frac{42cm^2}{3} = \frac{3cm}{3}(2b_2 + 2)$

$2b_2 + 2 = 14$, $2b_2 = 12$

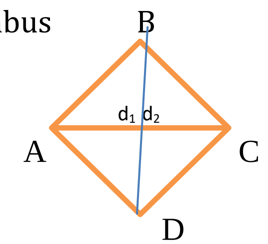
$b_2 = \underline{6cm}$

C. Perimeter and area of rhombus and kite

a. The area of rhombus and kite is given by the formula : $\boxed{A = \frac{1}{2} d_1 d_2}$ where d_1 and d_2 are diagonals.

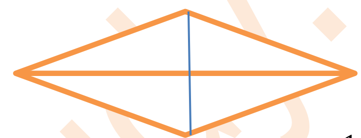
The perimeter of rhombus and kite is calculated by adding the length of all the four sides.

Rhombus



$A = \frac{1}{2} d_1 \times d_2$

kite

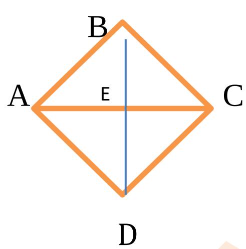


$A = \frac{1}{2} d_1 \times d_2$

Exercise

1. Calculate the area and perimeter of the following rhombus if

AB = 5cm, AC = 8cm and BD = 6cm



2. Calculate the area of kite, whose diagonals are 12cm and 16cm.

3. Calculate the perimeter of kite , whose adjacent sides are 18mm and 10mm.

4. The area of rhombus is $144cm^2$. If one of its diagonals is 8cm long, then calculate the length of the other diagonal.

Solution

1. $A = \frac{1}{2}(8cm)(6cm) = \frac{48cm^2}{2} = 24cm^2$

$P = 4(5cm) = \underline{20cm}$

2. $A = (12cm)(16cm) = 96cm^2$

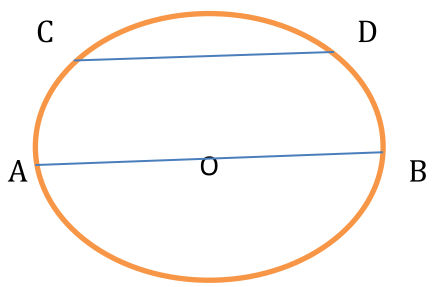
3. $P = 2(10mm + 18mm) = 56mm$

4. $144cm^2 = \frac{1}{2}(18cm)\, d_2$

$d_2 = \underline{16cm}$

5.5 Circumference and area of a circle

Definition 5.10: A circle is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.



Note:

1. **A circle is:** usually named by its center. The above circle is named as circle O.

2. **A chord is:** a line segment whose and points are on the circle. In the above circle $\overline{CD}$ and $\overline{AB}$ are chord of the circle.

3. **a diameter is:** any chord that passes through the center. The chord $\overline{AB}$ is the diameter of the circle. Diameter if the longest chord of the circle.

4. **a radius is:** a line segment from the center to any point on the circle.

5. **Diameter is:** twice of the radius. (d = 25)

6. **Circumference is:** the complete path around the circle. It is the perimeter of the circle

$C = \pi \times d$ or $\frac{C}{\pi} = d$ or $\boxed{C = 2\pi r}$

Example 1: Find the circumference of the circle with diameter 6cm (use $\pi = 3.14$

Solution

$C = \pi \times d$

$C = \pi \times 6cm$

$C = 3.14 \times 6cm$

$= 18.84cm$

Exercise

1. Find the circumference of the circle with each of the given diameter below (use π = 3.14)

a. 4m b. 10cm

2. a piece of land has a shape of semicircular region a shown below find the perimeter of the land

(use π = 3.14)

______ 20m________

Solution

1. a. $C = d \times \pi$

$C = 4m \times 3.14$

$C = 12.56$ cm

b. $C = \pi \times d$

$= 3.14 \times 10cm$

$C = 31.4cm$

2. P = diameter + circumference of half circle

$P = 20m + \frac{1}{2} \times \pi \times 20m$

$P = 20m + 10\,\pi$ m

$P = 20m + 10 \times 3.14m$

$P = 20m + 31.4m$

P = **41.4m**

Formula for area of a circle

$A = \pi r^2$ Or $A = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

Since, $r = \frac{d}{2}$

Example 1: Find the area of a circle with radius 10cm (use $\pi = 3.14$)

Solution

$A = \pi r^2 = 3.14 \times (10\text{m})^2$

$= 3.14 \times 100\text{m}^2$

$= \underline{\underline{314\text{m}^2}}$

Example 2: Find the area of a circle with diameter is 8m.

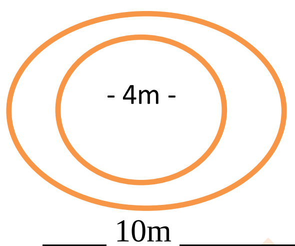
$$A = \frac{\pi d^2}{4} = \frac{\pi\,(8m)^2}{4} = \frac{\pi 64 m^2}{4} = \underline{\underline{16\pi\text{m}^2}}$$

Exercise 5.5.2

1. Find the area of a circles with each of the given diameter below (leave your answer in terms of π)

a. 10cm b. 16cm

2. Find the area of the shaded region below (use $\pi = 3.14$)



Solution

1. $A = \frac{\pi d^2}{4}$

$= \frac{\pi (10m)^2}{4}$

$= \frac{100\pi\ cm^2}{4}$

$A = \underline{\underline{25\ \pi\text{cm}^2}}$

2. Area of shaded = area of big circle minus area of small circle.

$= \frac{\pi (10m)^2}{4} - \frac{\pi (4m)^2}{4}$

$= \frac{100\ \pi m^2}{4} - \frac{16\pi m^2}{4}$

$= 25\ \pi\text{m}^2 - 4\ \pi\text{m}^2$

$= \underline{\underline{21\ \pi\text{m}^2}} = 21\ (3.14)\ \text{m}^2$

$= \underline{\underline{65.94\text{m}^2}}$

5.6. Applications

Example 1: A class room has length of 9m and width of 6m. The flooring is to be replaced by terazo tiles of size 30cm by 30cm. How many terazo tiles are needed to cover the class room?

Solution

Area of class room = 9m ×6m = 54m² = **540,000cm²**

Area of terazo tile, A = 30cm × 30cm = **900cm²**

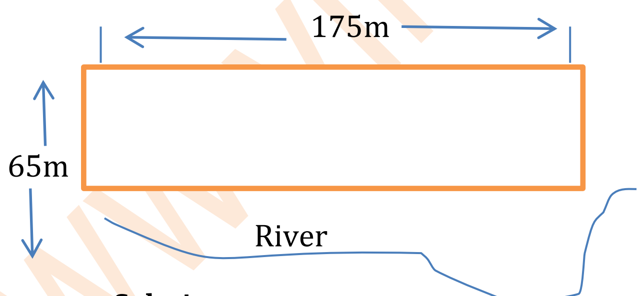
No of terazo tiles $= \frac{area\ of\ class\ room}{area\ of\ terazo\ tile}$

$= \frac{540{,}000cm^2}{900cm^2} = \underline{600}$

∴ 600 terazo tiles are required for flooring the class room.

Exercise

1. The length and width of rectangular football field are 100m and 80m respectively. 1m² artificial grass costs birr 500, then how much it costs to cover the field by artificial grass.
2. a farmer wants to fence the following plot of land. If no fencing material is not required a long river side, then calculate.
   a) The length of fencing material required
   b) If the cost of 1m fencing material is Birr 150, then calculate the cost to fence the land



Solution

1. A = 80m (100m) = 800m²

   Cost = 800 (500) = Birr 400,000

2. a) length of fencing material

$= 65m + 175m + 65m = \underline{\underline{305m}}$

b) $305m$ (Birr 150) $=$ **Birr 45, 750**

Answer for Review exercise for unit 5

I. True / False

1. True 3. False 5. True 7. False

2. False 4. False 6. True

II. Fill in the blank

1. Equilateral triangle
2. Right angle triangle
3. Kite
4. Rhombus
5. Squares

III. Choose the correct answer

1. B 3. D 5. B
2. A 4. C

IV. Work Out

1. a. $A = (8cm)\ (10cm) + \frac{1}{2}(6cm \times 8cm\ ) = \underline{\underline{104cm^2}}$

$P = 8cm + 10cm + 16cm + 10cm = \underline{\underline{44cm}}$

b. $A = 6cm\ (\ 10cm\ ) = 60cm^2$

$BC = \frac{60cm^2}{12cm} = 5cm,$

$P = 5cm + 5cm + 6cm + 6cm = 22cm$

C. $A = \frac{1}{2}(\ 8cm\ )\ (\ 11cm\ ) = 44cm^2$

$P = 5cm + 5cm + 9cm + 9cm = \underline{\underline{28cm}}$

d. $A = \frac{1}{2}(24cm)\ (\ 10cm) = \underline{\underline{120cm^2}}$

$P = 4\ (\ 13cm\ ) = 52cm$

e . $A = \frac{1}{2} 8cm\ (12cm + 24cm) = 144cm^2$

$P = 12cm + 24cm + 10cm + 10cm = 56cm$

f. $A = \frac{1}{2}(5cm)(12cm) = 30cm^2$

$P = \underline{1}\ 3cm + 12cm + 5cm = 20cm$

2. a. $A = \frac{1}{2}(12cm \times 9cm) - 10cm \times 3cm$

$= 54cm^2 - 30cm^2$

$= \underline{\underline{24cm^2}}$

b. area of shaded $A = 13cm \times 8cm - \frac{1}{2}(4cm \times 4cm)$

$= 104cm^2 - 8cm^2$

$= 96cm^2$

C. area of shaded $A = \frac{1}{2}(12cm + 10cm)(7cm) - \frac{1}{2}(7cm \times 10cm)$

$= 77cm^2 - 35cm^2 = \underline{\underline{42cm^2}}$

d. area of shaded $A = 12cm \times 6cm$

$= \underline{\underline{72cm^2}}$

UNIT - 6

Congruency of plane figures

6.1 Congruent of plane figures

6.1.1. Definition and illustration of congruent figure

Definition 6.1: Congruent figures are figures that have the same size and shape.

Note:

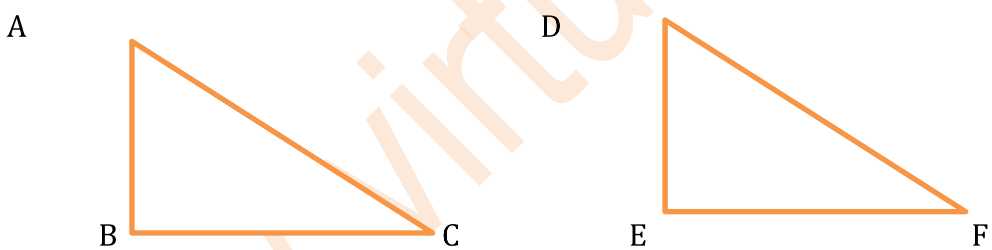
1. Two line segments are congruent if they have the same length.
2. Two circle are congruent if they have the same radius.
3. Two angles are congruent if they have the same measure.

6.1.2 Congruency of triangles

Definition 6.1: Two triangles are congruent if they are copies of each other and when you place one triangle on another , they cover each other completely.

Note:

1. If ΔABC is congruent to ΔDEF, then symbolically. Written as ΔABC ≅ ΔDEF.
2. If ΔABC ≅ ΔDEF, then when you place ΔDEF on ΔABC it should satisfy the following conditions.



i. D falls on A , $< D \cong <A$.

ii. E falls on B , and $<E \cong <B$

iii. F falls on C and $<F \cong <C$

iv. $\overline{DE}$ falls along $\overline{AB}$ and $\overline{DE} \cong \overline{AB}$

v. $\overline{EF}$ falls along $\overline{BC}$ and $\overline{EF} \cong \overline{BC}$

vi. $\overline{DF}$ falls along $\overline{AC}$ and $\overline{DF} \cong \overline{AC}$

Example 1:

1. If $\Delta ABC \cong \Delta DEF$, then find the six congruent corresponding parts of the triangles.

i. $<A \cong <D$ iv. $\overline{AB} \cong \overline{DF}$

ii. $<B \cong <E$ v. $\overline{BC} \cong \overline{EF}$

iii. $<C \cong <F$ vi. $\overline{AC} \cong \overline{DF}$

2. Let $\Delta ABC \cong \Delta DEF$, and m $(<A) = 70^0$, DE = 6cm, then find m (<D) and the length of AB.

Solution

Since the triangles are congruent, their corresponding parts are congruent

So, m (A) = m $(<D) = 70^0$ and $\overline{DE}$ = AB = 6cm

Exercise 6.1.2

1. Write *True* if the statement is correct and **False** if not.

a. If $\Delta ABC \cong \Delta DEF$, then m $(<A)$ = m $(<D)$.

b. If equilateral triangles are congruent.

C. If $\Delta ABC \cong \Delta DEF$, then AB = ED

D. If $\Delta ABC \cong \Delta DEF$, then $<B \cong <D$

Solution

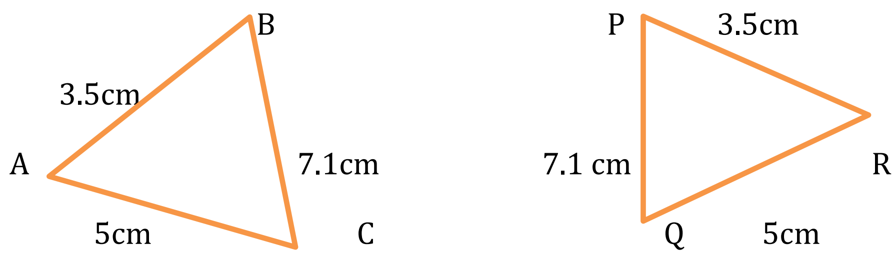
a. False C. True
b. False D. True

6.1.3. Tests for congruency of triangles (ASA, SAS, SSS)

A. Side – Side – Side (SSS) congruence tests

If the three sides of one triangle is congruent to the three corresponding sides of another triangle, then the triangles are congruent.

Example 1: determine whether the two triangles are congruent or not.



Solution

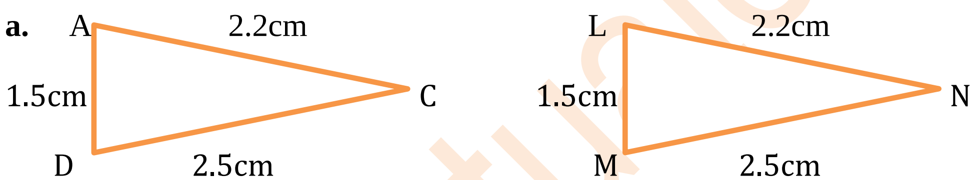
- ✓ AB = PR = 3.5cm
- ✓ AC = RQ = 5cm
- ✓ BC = PQ = 7.1cm

- The three sides of one triangle are congruent (equal)to the three sides of other triangle. So by SSS congruence test the two triangles are congruent. Hence $\Delta ABC \cong \Delta RPQ$ by SSS congruence test.

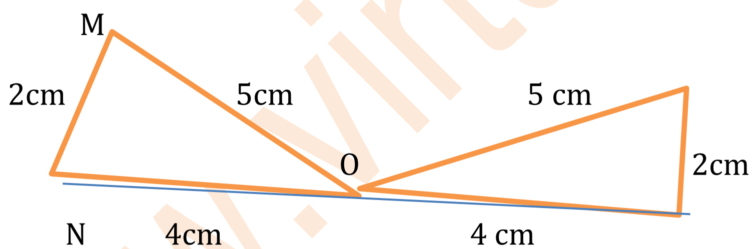
Exercise

1. In the given figure below, lengths of the sides of the triangles are indicated by applying SSS congruence test. State which pairs of triangles are congruent.

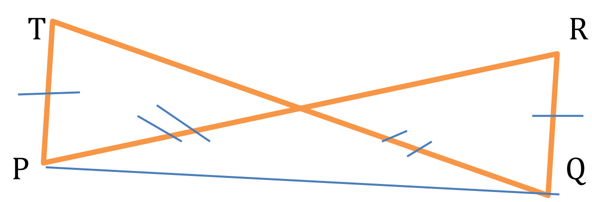
a.



b.



2. In the figure PR = QT and PT = QR . Which one of the following statements is correct?



a. $\Delta PQR \cong PQT$

b. $\Delta PQR \cong \Delta QPT$

c. $\Delta TPQ \cong \Delta RPQ$

d. $\Delta PRQ \cong \Delta QTP$

Solution

1. a. $\overline{AB} \cong \overline{LM}$, $\overline{BC} \cong \overline{MN}$, $\overline{AC} \cong \overline{LM}$

Hence, they are congruent by SSS congruence rule.

Symbolically, ΔABC ≅ ΔLMN

b. $\overline{MN} \cong \overline{PQ}$, $\overline{NO} \cong \overline{QO}$, $\overline{MO} \cong \overline{PQ}$

Hence, they are congruent by SSS congruence rule.

ΔMNO ≅ ΔPQO

2. $\overline{PR} \cong \overline{QT}$ - - - Given

$\overline{PT} \cong \overline{QR}$ - - - Given

$\overline{PQ}$ is common side of ΔPQR and ΔQPT.

Hence, ΔPQR ≅ ΔTPQ by SSS congruence test.

a. Not correct C. Not correct
b. Correct d. correct

B. Side – Angle– Side congruence Tests(SAS)

If the two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle then the triangles are congruent.

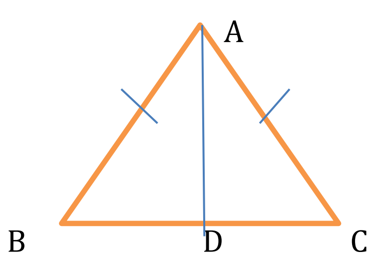
Example 1: In figure AB = AC, and $\overline{AB}$ is the bisector of <BAC

i. State the three pairs of equal parts in ΔADB and ΔADC

ii. Is ΔADB ≅ ΔADC ?

iii. Is < B ≅ < C ?

v. Is BD = DC ?



Solution

i. The three pairs of parts are:-

AB = AC - - - - Given

AD = AD - - - - Common side for both.

m (< BAD) = m (< CAD) = ($\overline{AD}$ bisects < BAC)

ii. yes, ΔADB ≅ ΔADC (by SAS)

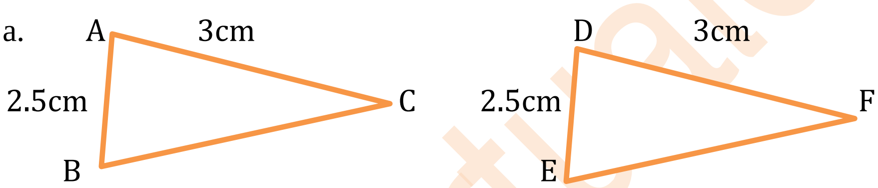
iii. yes, < B ≅ < C, because they are corresponding parts of congruent triangles.

iv. yes, BD = DC, because they are corresponding parts of congruent triangle.

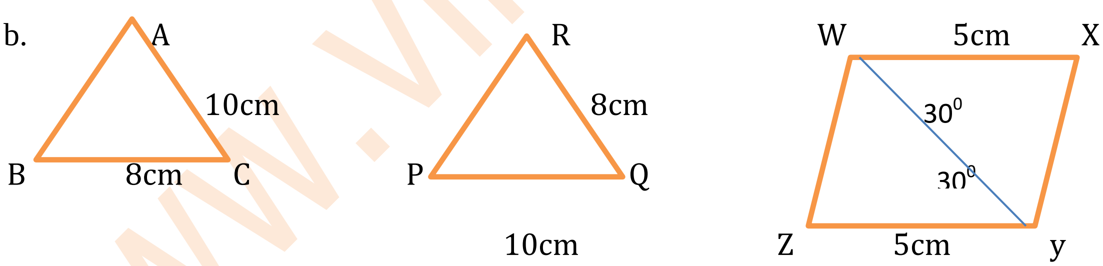
Exercise

1. In the given figure bellows, by applying SAS congruence test, state the pairs of congruent triangles, incase of congruent triangles, write them in symbolic form.

a.



b.



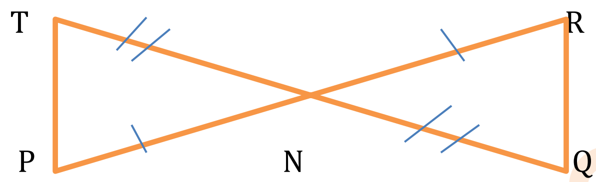
2. In the given figure, PN = RN and TN = QN which one of the following statement is correct.

a. $\Delta PNT \cong \Delta PNQ$

b. $\Delta PNT \cong \Delta QNR$

c. $\Delta TPN \cong \Delta RQN$

d. $\Delta NTP \cong \Delta NQR$



Answer

1. a. $\overline{AB}$ $\overline{DE}$ - - - - Given

$< A \cong < D$ - - - - Given

$\overline{AC}$ $\overline{DF}$ - - - - Given

Hence , ΔABC and ΔDEF are congruent by SAS congruence rule.

$\Delta ABC \cong \Delta DEF$

b. $\overline{BC} \cong \overline{RQ}$ - - - - Given

$< C \cong < Q$ - - - - Given

$\overline{AC} \cong \overline{PQ}$ - - - - Given

Hence , ΔABC and ΔPQR are congruent by SAS congruence rule.

$\Delta ABC \cong \Delta PQR$

C. $\overline{WX} \cong \overline{YZ}$ - - - - Given

$< XWY \cong < ZYW$ - - - - Given

WY is common side

$\overline{XWY} \cong \overline{ZYW}$

2. $\overline{PN} \cong \overline{RN}$ - - - - Given

$\overline{TN} \cong \overline{QN}$ - - - - Given

$< TNP \cong \overline{QN}$ - - - - by VOA

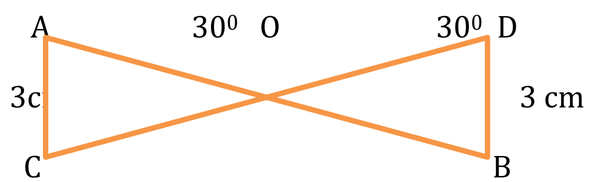
Hence , ΔTNP ≅ Δ QNR by SAS congruence rule.

a. correct C. Not correct
b. Not Correct d. correct

C. Angle – Side– Angle congruence Tests(ASA) congruence Tests

If two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.

Example 1: Is ΔABC ≅ Δ QRP?



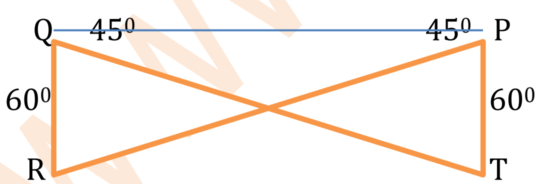
Using the figure shows that ΔAOC ≅ Δ BOD?

Solution

✓ In ΔAOC, m (<A) + m (<C) +m (<O) = 180^0
m (< A) + 70^0 + 30^0 = 180^0
m (<A) = 180^0
m (<AOC) = m (< DOC) = 30^0 – VOA
✓ In ΔDOB, m (<D) + m (<O) +m (<B) = 180^0
70^0 + 30^0 + m (<B) = 180^0, m (<B) = 80^0
<A ≡ <B and < c ≅ <D
By ASA ΔAOC ≅ Δ BOD

Exercise 6.1.5

1. In the figure below by applying ASA congruence test, state which pairs of triangles are congruent and write the result in symbolic form:



< RQP ≅ < RPQ - - - - Given

QP is common side

$< RQP \equiv < TPQ$ - - - - Given

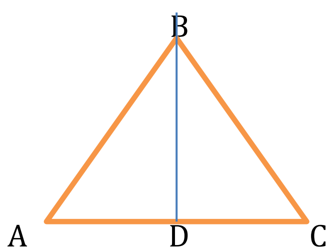
Hence , $\Delta TPQ \cong \Delta RQP$ by ASA congruence rule

6.2. Applications

Exercise

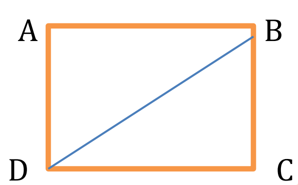
1. Show that the diagonal of rectangle divides the rectangle in to two congruent triangles.

2. In the figure given below, ΔABC is Isosceles triangle with $AB = BC$ and BD bisects $<ABC$, then show that D is the mid – point of AC.



Answer

1.



✓ ABCD is rectangle and BD is diagonal of rectangle

BD is diagonal of rectangle

$BA = DC$ - - - Opposite side of rectangle are congruent

$AD = BC$ - - - opposite sides of rectangle are congruent.

BD is common angle.

$\therefore$ The diagonals of rectangle divides rectangle in to two congruent triangles.

2. $AB = BC$ - - - Given

$< ABD = CBD$ - - - BD is bisector of $<ABC$.

BD is common side.

Hence, $\Delta ABD \cong \Delta CBD$ by SAS congruence rule.

AD = DC - - - - corresponding sides of congruent triangles.

∴ D is the mid – point of AC.

Answer for review exercise on unit – 6

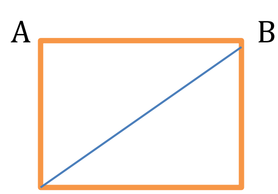
I. True / False

1. True
2. True
3. True
4. True
5. False
6. True
7. True
8. False
9. True
10. False

I. Choice

11. A 12. C 13. B 14. B 15. A

16. . ABCD is a square and BD is diagonal of square



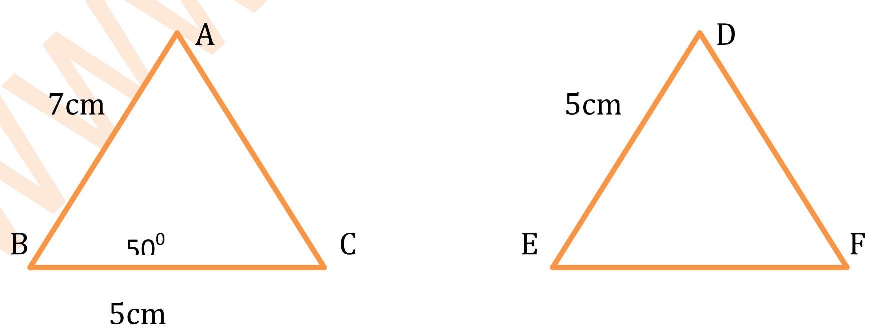
BA = DC - - - All sides of square are congruent.

AD = BC - - - All sides of square are congruent

BD is common angle.

$\Delta BAD \cong \Delta DCB$ - - BY SSS congruence rule.

∴ The diagonals of square divides square in to two congruent triangle



$\Delta BAD \not\equiv \Delta DCB$ but

$\Delta BAD \cong \Delta DCB$ by SAS congruence rule.

21. a. FG = FH - - - Given

GM = HM - - - Given

FM is common side.

Hence, $\Delta FGM \cong \Delta FHM$ by SSS congruence rule.

UNIT - 7

Data handling

7.1 Organization of data using frequency table

Definition 7.1: Data is a collection of facts, such as numbers, words, measurements, observations or even just descriptions of things.

You can collect data:

- ✓ by using a questionable
- ✓ by making observations and recording the results.
- ✓ By carrying out an experiment
- ✓ From records or data base
- ✓ From the internet.

Note: one method of presenting data is ✓ rally chart or

✓ frequency table.

- A tally chart is a simple way of recording or counting frequencies
- A tally chart or frequency table is a quick and easy way of recording data.
- Frequency is the number of times a data value occurs.

Example 1: Draw tally chart or frequency table using the following data. In a school, 40 students were asked

what size of shoes does they wear. Hence are the results:

33	32	34	37	37	35	38	36	37	36
36	38	35	33	36	37	38	38	31	37
36	35	37	36	39	37	36	35	38	33
34	33	37	37	38	35	34	37	39	36

Solution

No of friends	Tally	No of Frequency
31	|	1
32	|	1
33	|||	3
34	~~||||~~	5

35	卌 III	8
36	卌 II	7
37	卌 卌	10
38	卌 I	6
39	II	2

Exercise

1. Display the following information more clearly by drawing a tally chart or frequency table.

The following are weights in Kg of 40 students in class.

40	**45**	**45**	**46**	**44**	**43**	**45**	**47**	**46**	**49**
44	51	47	45	44	46	46	43	44	50
48	43	45	46	44	44	47	43	44	45
45	43	45	46	44	47	45	46	44	47

2. The table below shows the favorite color of grade 7th students:

White	Red	White	Yellow	Green	Black	Green	Blue
Green	White	Black	Red	Yellow	Blue	Blue	Red
Yellow	Blue	White	Blue	Green	White	White	White
Yellow	Blue	Green	Green	White	Blue	Black	Red
Red	Blue	Yellow	Red	Green	White	White	Green

Solution

1.

Weight of student	Tally marks	Frequency
40	I	1
43	卌	5
44	卌 IIII	9
45	卌 IIII	9
46	卌 II	7
47	卌	5
48	I	1
49	I	1
50	I	1
51	I	1
	I	1

2.

Favorite Colors	Tally marks	Frequency
White	𝍸 𝍸	10
Green	𝍸 |||	8
Yellow	𝍸	5
Red	𝍸		6
Blue	𝍸 |||	8
Black	|||	3

7.2 Construction and interpretation of the graphs and pie charts:

7.2.1 Line graphs

Definition 7.2: line graph that uses lines to connect individual data points on a Cartesian coordinate plane .

⊗ A line graph is most commonly used to represent two related facts.

⊗ The following points are important to making a line graph.

1. Construct a Cartesian coordinate plane and label an appropriate scale.
2. Make a table of data arranged in order pairs and mark the points on a Cartesian coordinate plane.
3. Connect the points by a straight line or smooth curve.

Exercise

1. Draw a line graph to represent each of the following data.

a) The number of letters delivered to an office in one week.

Days	Mon	Tue	Wed	Thu	Fri	Sat	Sun
no of letters	10	0	5	8	12	15	10

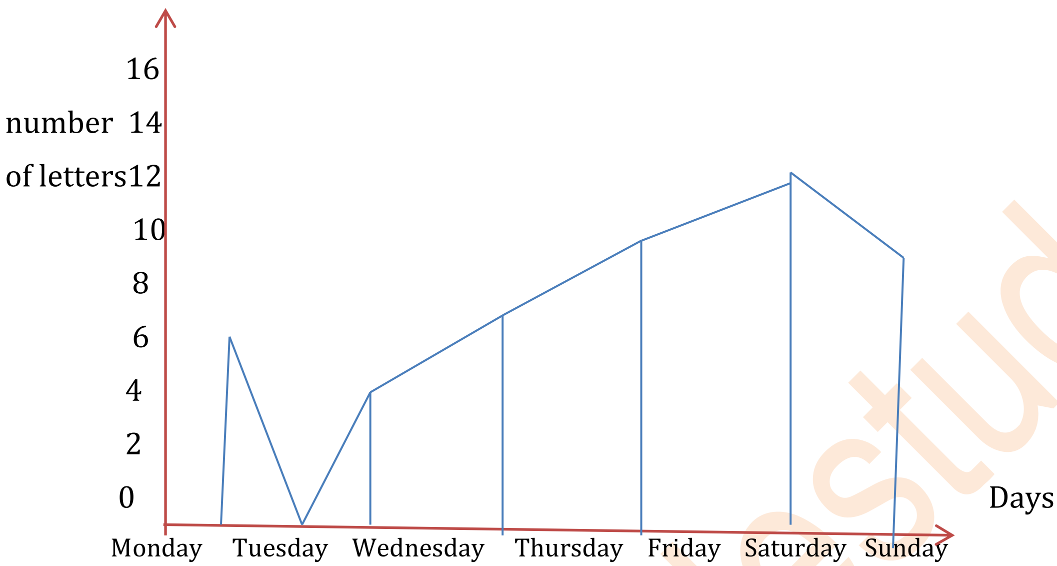
b) The temperature in addis Abeba at midday during the last week in April.

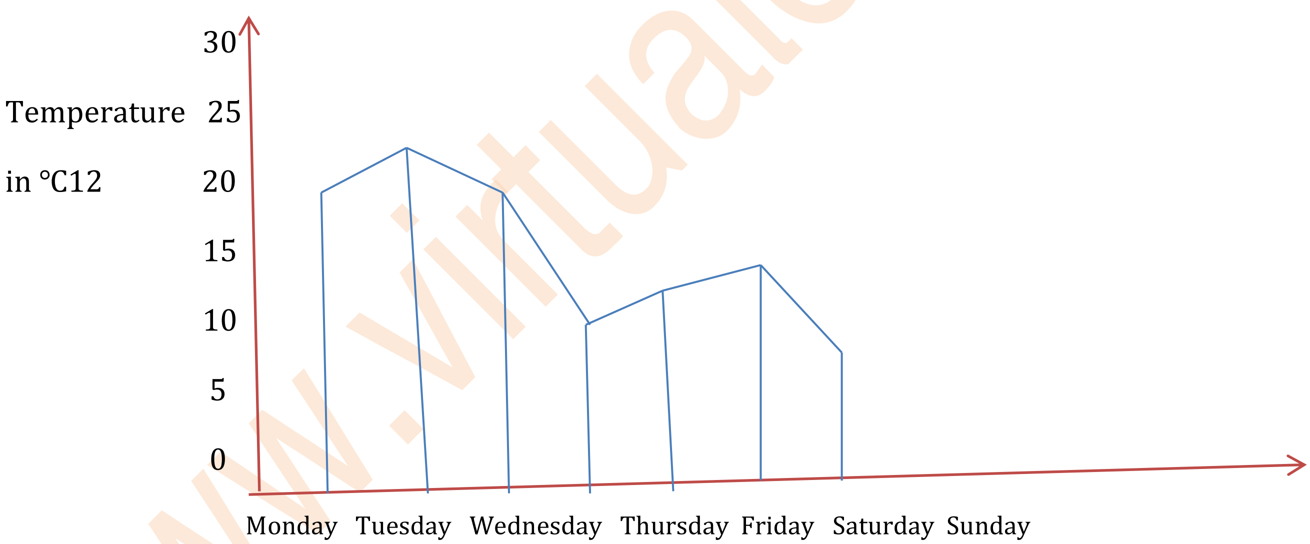
Days	Mon	Tue	Wed	Thu	Fri	Sat	Sun
no of letters	20	22	21	20	23	24	25

Solution

1. a. The no of letters delivered to office in one week.

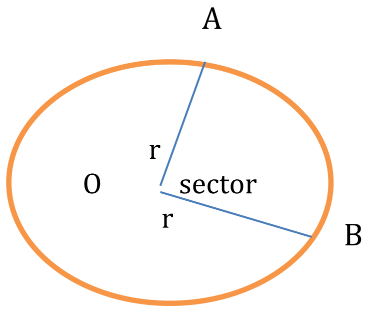
Days	Mon	Tue	Wed	Thu	Fri	Sat	Sun
no of letters	10	0	5	8	12	15	10





7.2.2 Pie Charts

Note: The portion of a circular region enclosed between two radii and part of circumference is called a sector of the circle.



r = radius

O is the center of the circle

$\overset{\frown}{AB}$ is an arc of the sector AaB.

- The size of the sector is determined by the size of the angle formed by the two radii.

Note:

360^0 Covers 100% of a circle.

$100\% = 360^0$ $\quad\quad 1^0 = \frac{10}{36}\%$

$1\% = 3.6^0$

$$\text{Rectangle} = \frac{\textit{Measure of centeral angle} \times \textit{total value}}{360^0}$$

$$\text{Measure of central angle} = \frac{\textit{percentage} \times 360^0}{\textit{total value}}$$

Definition 7.3: Pie chart is a type of graph that represents the data in the circular graph. It is very common and accurate way of representing data especially useful for showing the relation of one item with another and one item with the whole items.

Example 1: The expenditure on different budget title of a family in a month is given below table draw a pie chart for the given data.

Budget title	food	Education	clothing	house rents	other	saving	total
Expenditure in birr	2,400	1080	800	1800	720	400	7200

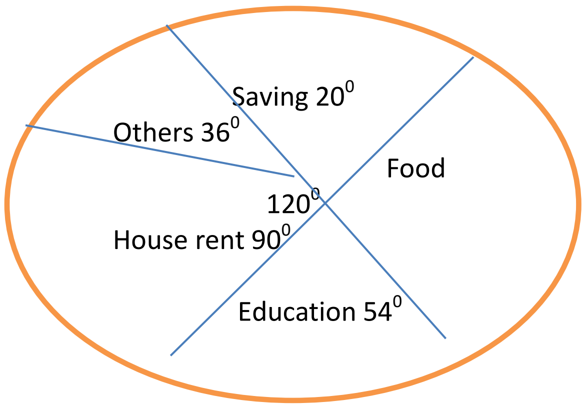
Solution:

$$\text{Measure of an angle} = \frac{\textit{Expenditure on the given Budget} \times 360^0}{\textit{Total expenditure}}$$

⊗ Food: $\frac{2400 \times 360^0}{7200} = 120^0$ $\quad\quad$ House rent $= \frac{180 \times 360^0}{7200} = 90^0$

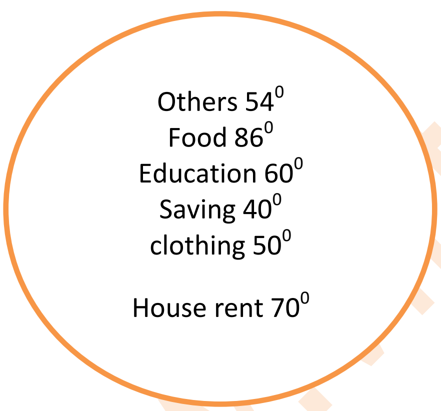
⊗ Education: $\frac{1080 \times 360^0}{7200} = 54^0$ $\quad$ Other $= \frac{720 \times 360^0}{7200} = 36^0$

Clothing: $\frac{800 \times 360^0}{7200} = 40^0$ Saving $= \frac{400 \times 360^0}{7200} = 20^0$



<u>Example 2:</u> The pie chart given below shows w/ro Tseday's expenses and saving for last month. If the monthly in come was Birr 10800, then find

a) her food expense
b) her house rent
c) her clothing expense
d) her saving
e) her education expenses
f) for other expenses



Solution: Each expense $= \frac{measure\ of\ angle\ of\ sector \times Total\ of\ Buaget}{360^0}$

a) Food expense $= \frac{86^0 \times 10800}{360^0} =$ Birr 2580

b) House rent $= \frac{70^0 \times 10800}{360^0} =$ Birr 2100

C) Clothing expense $= \frac{50^0 \times 10800}{360^0} =$ Birr 1500

d) Saving $= \frac{40^0 \times 10800}{360^0} =$ Birr 1200

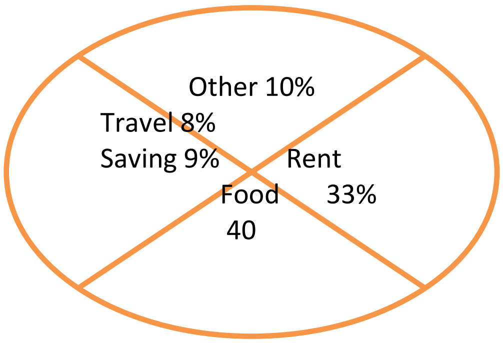
e) Education $= \frac{60^0 \times 10800}{360^0} =$ Birr 1800

e) for other $= \frac{54^0 \times 10800}{360^0} =$ Birr 1620

Exercise 7.2.2

1. The following pie chart shows a family budget based on a net income of Birr 8400 per month.

Family Budget



a) Determine the a moving spent on rent ?

b) Determine the amount spent on food?

C) Determine the amount saving money ?

d) How much more money is spent?

Solution

1. a. Spent on rent $= \frac{33 \times 8400}{100}$ = Birr **2772**

b. Spent on food $\frac{40 \times 400}{100}$ = Birr **3360**

C. Saving $= \frac{9 \times 8400}{100} =$ Birr **756**

d. The family more spent on food .

7.3. The mean, mode, median and Range of data

A. mean

Definition 7.4: The mean of a given data is the sum of an values divided by the number of values :

$$\text{Mean} = \frac{\text{sum of all values}}{\text{no of values}}$$

Mean $(\overline{x}) = \frac{x_1 + x_2 + x_3 + --- + x_n}{n}$, where $\overline{x}$ mean of data.

Example 1: Find the mean of the following data:

a. 22, 20, 14, 12, 27 b. 100, 200, 120, 320, 150, 160

Solution

a. Mean $(\overline{x}) = \frac{x_1+x_2+x_3+x_4+x_5}{5}$

$= \frac{22+20+14+12+27}{5}$

$= \frac{95}{5}$

$= \underline{\mathbf{19}}$

b. Mean $(\overline{x}) = \frac{100+200+120+320+180+160}{6}$

$= \frac{1050}{6}$

$= \underline{\mathbf{175}}$

Example 2: The mean of five numbers is 30. Four of the numbers are 32, 28, 40 and 27, then find the values of the other numbers.

Solution:

Let x be the missing number.

Mean $(\overline{x}) = \frac{sum\ of\ all\ values}{no\ of\ values}$

$30 = \frac{32+28+40+27+x}{5}$

$x + 127 = 150$

$x = 150 - 127$

$\underline{\boldsymbol{x = 23}}$

Exercise

1. Find the following given data.

a. 10 14 16 19 25 12

b. 28 35 70 140 160

2. If the age of 8 students are 11, 12, 14, 17, 15, 13, 14 and 16, then find the mean of age of students.

Solution

1. a) $\frac{96}{6} = 16$ b) $(\overline{x}) = \frac{28+35+70+140+160}{5} = \underline{\mathbf{86.6}}$

2. $(\overline{x})(mean) = \frac{11+12+14+17+15+13+14+16}{8}$

$$= \frac{112}{8} = \underline{\underline{14}}$$

B. Mode

Definition 7.5: The mode of list of data is the values which occurs most frequency.

Example 1: Find the mode of given data below.

a. 20 30 40 20 20 30 50

b. 12 14 12 14 13 14 12

Solution

a) 20 occurs more frequently than any other value of data. Then the mode is 20.

b. 12 and 14 occurs three times , hence three are two modes 12 and 14.

Note:

- ✓ A data that has one mode is called unimodal.
- ✓ A data that has two mode is called bimodal.
- ✓ A data that has three mode is called trimodal.
- ✓ If each value occurs only one, so there is no mode.

Exercise

1. Find the mode of the following given data below and identify it is unimodal, bimodal, trimodal and no mode.

a. 8 11 9 14 9 15 18 6 9 10

b. 24 15 18 20 18 22 24 26 18 26 24

Solution

a. mode = 9, it is unimodal.

b. mode = 18 and 24, it is bimodal.

C. Median

Definition 7.6: The median is the middle value when data is arranged in order of size.

To find the median of list of data.

i. Arranged data in increasing or decreasing order.
ii. Median = the middle value of arranged data.
iii. If the middle value is two data, then median is the mean of two middle data.

Example 1: Find the median of the following data.

a. 4 5 6 10 14

b. 20 30 65 70 15 90 45

Solution

a. The increasing order of given data is $\boxed{4 \quad 5}$ $\boxed{10 \quad 14}$

The middle value is 6. The median is 6.

b. The increasing order of given data is

$\boxed{15 \quad 20 \quad 30}$ 45 60 $\boxed{65 \quad 70 \quad 90}$

The middle values are 45 and 60.

Exercise

1. Find the median of the following list of data.

a. 14 12 24 36 23

b. 104 112 100 150 102 160

Solution

1. a. 12 14 <u>23</u> 24 36

Median = 23

b. median = 108 =

D. Range

Definition 7.7: The range of the listed data is the difference between the highest value and the lowest value.

Range = highest value – lowest value.

Example 1: Find the range of the following data.

a. 70 55 74 63 80 40

b. -800 -200 -600 0 -300

Solution

a. The lowest value is 40 and the highest value is 80.

Range = H.V – L.V

= 80 – 40 = **40**

b. The lowest value is – 800 and the highest value is 0.

Range = H.V – L.V

= 0 – (- 800)

= **800**

Exercise

1. Find the range of the following given data.

a. 61 23 13 90 72 30 50

b. -900 -300 -600 -460 0 -500 - 250

2. In a class of 20 students the highest score in mathematics exam was 94 and the lowest values 41. What was the range?

Solution

1. a. Lowest value is 13 and highest value is 90

Range = 90 – 13

= **77**

b. Lowest value is -900 and highest value is 0.

Range = H.V – L.V

= 0 – (- 900)

= **900**

2. Range – highest score – lowest score

= 94 – 41

= **53**

7.3. Application

1 The score of 9 grade seventh students in mathematics exam listed below. Find the mean mode , median and range of given data.

82, 92, 78, 5, 91, 92, 89, 95, 100, 86

Mean $(\overline{x}) = \frac{x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9}{9}$

$$= \frac{82+92+78+5+91+92+89+95+100+86}{9}$$

$$= \frac{805.5}{9} = \underline{\underline{89.5}}$$

✳ mode – 92 occurs more frequently than any other values of data , then the mode is 92.

✳ The increasing order of data is

$\boxed{78.5 \quad 82 \quad 86 \quad 89}$ $\underline{\underline{\mathbf{91}}}$ $\boxed{92 \quad 92 \quad 95 \quad 100}$

The middle value is 91. The median is 91.

✳ The lowest value is 78.5 and the highest value is 100.

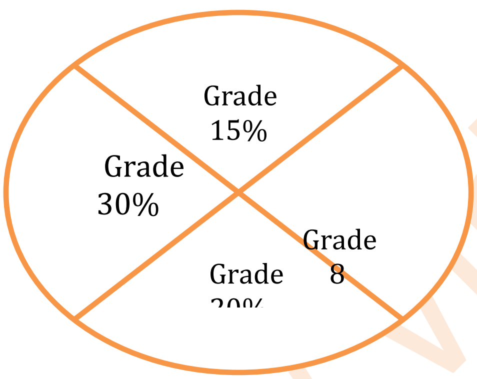
Range = HV – L.V

= 100 – 78.5

= $\underline{\underline{21.5}}$

② The pie chart shown below is the number of students in a certain school.

There are 1200 students in the school, then what is the number of students in grade 6,7 and 8?



Grade 6 = $1200 \times \frac{30}{100} = 360$

Grade 7 = $1200 \times \frac{20}{100} = 240$

Grade 8 = 100% - 30% - 15% - 20%

= 35%

Grade 8 = $1200 \times \frac{35}{100} = 420$

Exercise

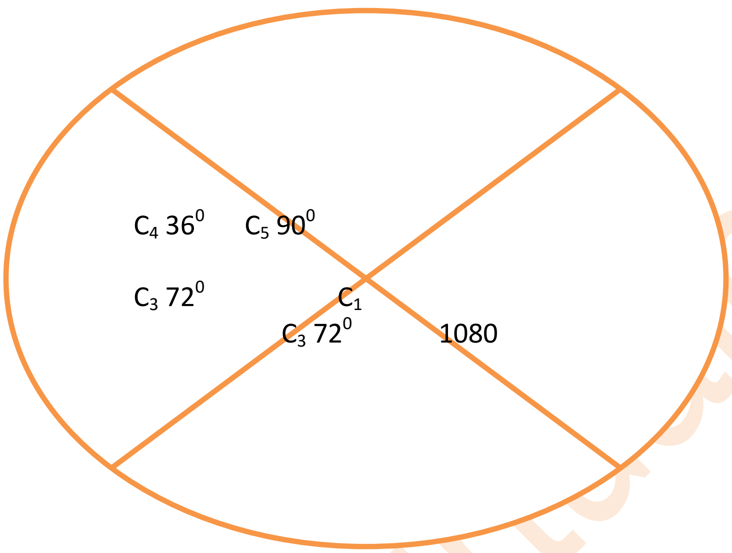
1. The height (in cm) of the members of a school football team has been listed below. 142, 140, 130, 150, 160, 135, 158, 132. Find the mean , mode median and range of the above given data.

2. The table shows the daily earnings of a store for five days in Birr.

Day	Mon	Tues	Wed	Thurs	Fri
learning	300	450	200	400	650

a. Construct a line graph for the frequency table.
b. On which days were the earnings above Birr 400.

3. 3000 students appeared for an examination from five different centers. C_1, C_2, C_3, C_4, and C_5, of a city. From the given pie chart, find the number of students appearing for the examination from each center.



Solution

1. mean $= \frac{142+140+130+150+160+135+158+132}{8}$

$= \frac{1147}{8} = \underline{\mathbf{143.375}}$

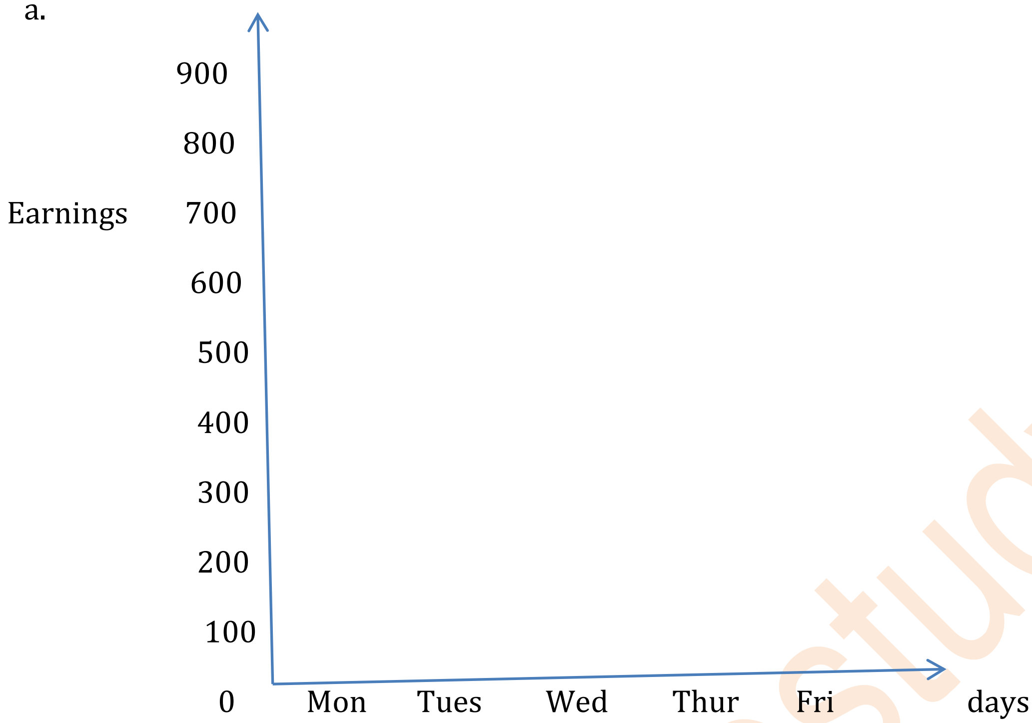
Mode = has no mode.

Median = 141.

Range = h value – lowest value

= 160 – 130 = **<u>30</u>**

2. a.



b. Tuesday and Friday.

3. $C_1 = \frac{108\times3000}{360} = 900$ $C_3 = \frac{72\times3000}{360} = 600$

$C_2 = \frac{54\times3000}{360} = 450$ $C_4 = \frac{36\times3000}{360} = 750$

$C_5 = \frac{90\times3000}{360} = 750$

ANSWER FOR REVIEWEXERCISE UNIT – 7

I. True or False

1. False
2. False
3. True
4. True
5. True

II. Work Out

6. a. $\frac{56\times1440}{360} = \underline{\underline{224}}$

b. $\frac{204\times1440}{360} = \underline{\underline{816}}$

7. a. For Education $= \frac{54 \times 720,000}{100}$

$= 388, 800, 000$

b. For public health $= \frac{24 \times 720,000}{100} = 172, 800,00$

C. For Social Service $= \frac{14 \times 720,000}{100} = 100, 800,00$

8. The sum of 7 numbers = 20×7 = 140

The sum of 5 numbers = 44×5 = 220

The sum of 12 numbers = 140+220 = 360

The mean of 12 numbers $= \frac{360}{12} = 30$

9. a. mean = 5, mode = 3, median = 4.5 , Range = 8

b. mean = 9.9, mode = 7, median = 9.5 , Range = 11

C. mean = 16.25, mode = 17, median = 16.5 , Range = 10

15 a. $\frac{5+7+4+1+n+5}{6} = 6$

n+22 = 36

n = 36-22

n = 14

b. n = 4

C . n = 2

d. $2.6 \frac{+3.5+n+6.2}{4} = 4$

n+12.3 = 16

n = 16-12.3 = 3.7

16 a. 28-18 = 10

b. mean $= \frac{168}{7}$

$= \underline{24}$

C. Friday