PHYSICS

GRADE 10

Prepared by: Virtual Study

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Unit 1

Vector Quantities

Physical quantity: Any number or sets of numbers used for a quantitative description of a physical phenomenon.

✓ Physical quantities can be divided in two groups: scalars and vectors.

1. Scalars quantities: have only magnitudes
2. Vector quantities: have both magnitude and direction.

1.1. Scalars and Vectors

❖ Scalar quantities are those that can be completely specified by a number together with an appropriate unit of measurement.

Example: say that the length of an object is 1.42 m or that the mass of an object is 12.21 kg. You do not have to add anything to the description of length or mass.

Examples: Time, distance, speed, length, volume, temperature, energy and power are other examples of scalar quantities

❖ Vector quantities require both magnitude and direction for their complete description.

Example: say that the velocity of a train is 100 km/h does not make much sense unless you also tell the direction in which the train is moving

Examples: Displacement, acceleration, momentum, impulse, weight and electric field strength, force and velocity

1.2 Vector representations:

vector quantities are represented either algebraically or geometrically

1. Algebraically representation: they are represented by a bold letter as $\boldsymbol{A}$ or with an

arrow over the letter

2. Graphical representation: are represented geometrically by an arrow, or an arrow-tipped line segment. Such an arrow having a specified length and direction shows the graphical representation of a vector

Example:

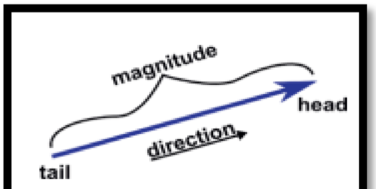


Figure 1.1 Head and tail of a vector.

- ✓ The length of the arrow represents the vector magnitude if it is drawn in scale.
- ✓ The arrow head represents the vector direction.

The procedures used for drawing vectors graphically.

1. Decide upon a scale and write it down.
2. Determine the length of the arrow representing the vector by using the scale.
3. Draw the vector as an arrow. Make sure that you fill in the arrow head.
4. Fill in the magnitude of the vector.

Example

Draw the vector, 16 km East, to scale by indicating the scale that you have used:

Solution first, let us decides upon a scale. Let 1 cm represent 4 km. So if 1 cm = 4 km, then 16 km = 4 cm and the direction is in the East. Using this information, you can draw the vectors as arrows as follows.

Types of vectors.

1. Zero vector or Null vector: a vector with zero magnitude and no direction.
2. Unit Vector: vector that has magnitude equal to one.
3. Equal vectors: vectors that have the same magnitude and same direction.
4. Negative of a vector: a vector that have the same magnitude but opposite direction with the given vector

1.3 Vector addition and subtraction

- ✓ Vector addition is a means of finding the resultant of a number of vectors.
- ✓ Subtraction of a vector is addition of the negative of a vector

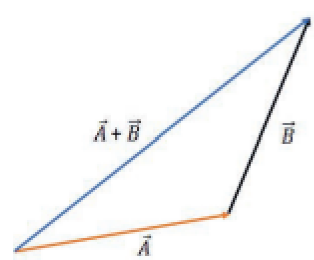


Figure : Addition of vectors A and B.

R = A +B

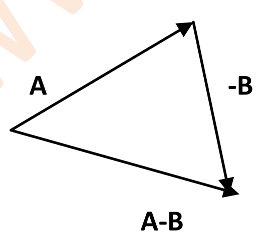


Figure: Subtraction of B from A

Exercise Is it possible to add two vectors in the same way as you did in scalars? Explain

Solution: Like scalars, we cannot add two vectors. This is because when two vectors are added, we need to take account of their direction as well as their magnitude. For example a force and a velocity cannot be added. But only vectors of the same kind are added.

1.4 Graphical method of vector addition

Graphically, vectors can be added:

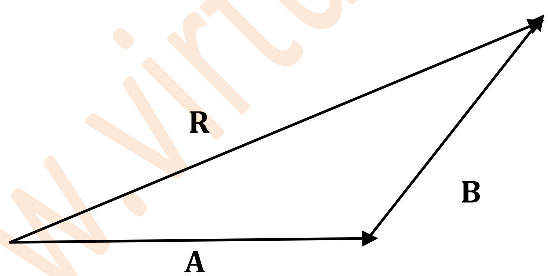
- ✓ triangle
- ✓ parallelogram
- ✓ polygon method of vector addition

Triangle method of vector addition

Procedure:

1. the head of the first vector is joined to the tail of the second vector
2. the tail of the first vector is joined to the head of the second vector to form a triangle

Note: The triangle law of vector addition is also called the head-to-tail method



R= A+B

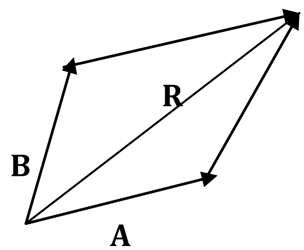
Parallelogram method of vector addition

Procedure:

1. join then tails of the two vectors
2. draw lines at the end of the two vectors parallel to the original vector (we obtain a parallelogram)

3. Draw a diagonal from the origin of the two vectors. The diagonal is the resultant **R** of the two vectors.

R =A +B



R =A+B

Polygon method of vector addition

Procedures

1. Place the tail end of each successive vector to the head end of the previous vector
2. The resultant of all vectors can be obtained by drawing a vector from the tail end of first to the head end of the last vector.

Activity : Use the polygon method of vector addition to find the resultant vector R of the three vectors:

A = 25.0m, 49^o North of East,

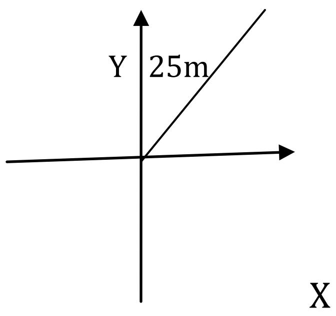
B = 23.0m,15^o North of East and

C = 32.0m, 68^o South of East. Choose a reasonable scale

Solution:

Step1: find the magnitude of each vector by resolving the vectors in to its components

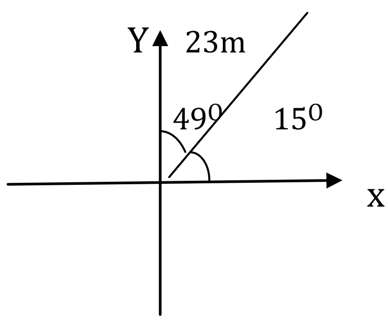
A)



Solution: $\cos(49^o) = x/25m$

$x = 25m*\cos(49^o) = 16.4m$

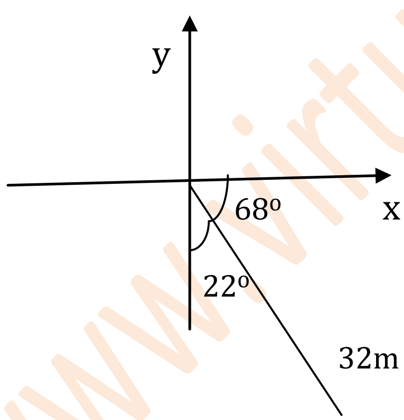
B



solution: $\cos(15^o) = x/23m$

$x = \cos(15^o)* 23m = 22.22m$

c)

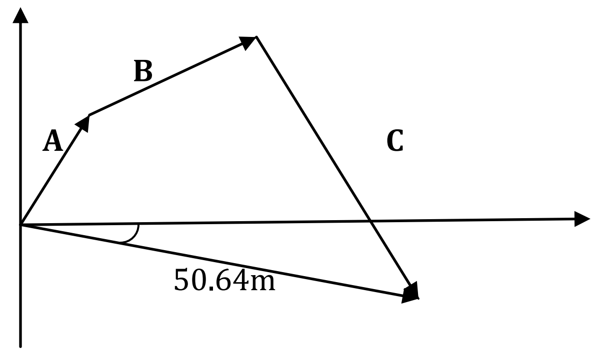


$\sin(22^o) = x/32m$

$x = 32* \sin(22^o) = 11.98m$

R =A+B+C= 16.44m+22.22m+11.98m= 50.64m

Step2: Place the vectors head to tail retaining both their initial magnitude and direction and draw the resultant vector.



Special cases of addition of vectors.

1. When the two vectors are in the same direction (parallel to each other)

- The resultant vector is the sum of the two vectors and takes the common direction. **|R| = |A +B|**

2. When the two vectors are acting in opposite directions

- The resultant vector is the difference between the two vectors and takes the direction of the vector with the greater value.

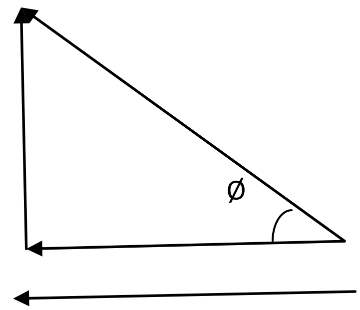
|R| = |A −B|

Note:

- The resultant of two vectors acting on the same point is **maximum** when the vectors are acting in the **same direction**
- The resultant of two vectors acting on the same point is **minimum** when they act in **opposite direction.**

3. When the two vectors are perpendicular If vectors **A** and **B** are perpendicular to each other, then the magnitude of the resultant vector R is obtained using the **Pythagoras theorem**. Hence, the magnitude of the resultant vector is

$\mathbf{R} = \sqrt{A^2 + B^2}$



The direction of the resultant vector is obtained using the trigonometric equation:

$\emptyset = \tan^{-1}\left(\frac{B}{A}\right)$

Exercise

If two vectors A and B are perpendicular, how you can find the sum of the two vectors?

Solution: the sum of the two vectors is determined by using Pythagoras theorem.

Example

Two vectors have magnitudes of 6 units and 3 units. What is the magnitude of the resultant vector when the two vectors are:

(a) In the same direction

(b) In opposite direction

(c) Perpendicular to each other?

Solution: we are given with two vectors of magnitudes 6 units and 3 units.

(a) When the two vectors are in the same direction, |R| = (6+3) units = 9 units.

(b) When the two vectors are in the opposite directions, |R| = (6−3) units = 3 units

(c) When the two vectors are perpendicular to each other,

$|R| = \sqrt{6^2 + 3^2}$ units = 6.7 units.

Review questions

1. Two vectors **A** and **B** have the same magnitude of 5 units and they start from the origin: **B** points to the North East and **A** points to the South West exactly opposite to

vector B. What would be the magnitude of the resultant vector? Why?

Solution: zero: because, the two vectors are exactly opposite in direction and equal in magnitude.

2. If two vectors have equal magnitude, what are the maximum and minimum magnitudes of their sum?

   Solution: the **maximum** magnitude is **twofold** (twice) of the individual magnitude

   The **minimum** magnitude is **zero**

   3. If three vectors have unequal magnitudes, can their sum be zero? Explain

Solution: yes, three vectors of unequal magnitude can add to give the zero vector. For example, a vector 8 N long in the positive x direction added to two vectors of 3 N and 5 N each in the negative x direction will result in the zero vector.

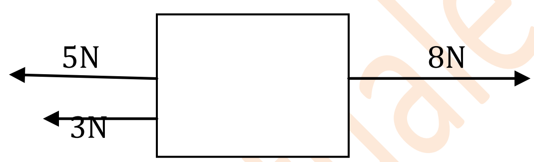
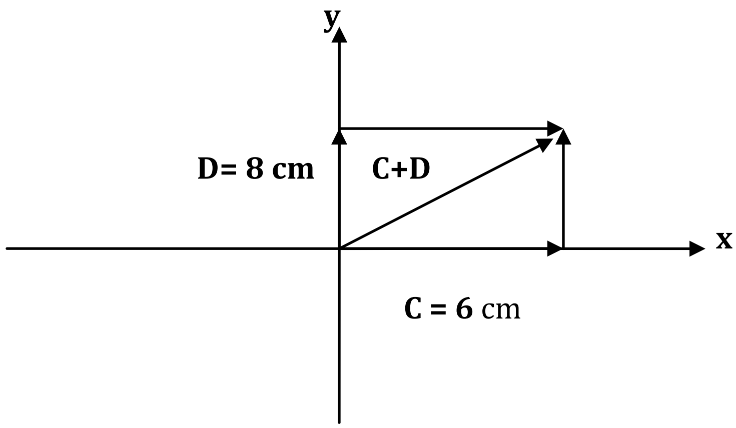


Fig. zero resultant vector

3. Consider six vectors that are added tail-to-head, ending up where they started from. What is the magnitude of the resultant vector?

   Answer: zero. When then tail of the first vector and head of the last vector is joined, the magnitude of the resultant vector is zero.

4. Vector **C** is 6 m in the x-direction. Vector **D** is 8 m in the y-direction. Use the parallelogram method to work out C + D



$$|R| = \sqrt{C^2 + D^2} \text{ units} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ cm}$$

1.5 Vector resolution

Definition: The process of breaking a vector into its components is called resolving into components or resolution

In the rectangular coordinate system shown in Figure 1.13, vector **A** is broken up or resolved into two component vectors. One, $\mathbf{A_x}$, is parallel to the x-axis, and the other, $\mathbf{A_y}$, is parallel to the y-axis

The horizontal and vertical components can be found by two methods:

1. graphical method
2. Simple trigonometry.

1. Graphical method

1. Select a scale and draw the vector to scale in the appropriate direction.

.2. Extend x- and y-axes from the tail of the vector to the entire length of the vector and beyond.

3. From the arrow head of the vector, construct perpendicular projections to the x- and

the y-axes.

4. Draw the x-component from the tail of the vector to the intersection of the perpendicular projection with the x-axis. Label this component as $\mathbf{A_x}$.

5. Draw the y-component from the tail of the vector to the intersection of the perpendicular projection with the y-axis. Label this component as $\mathbf{A_y}$.

6. Measure the length of the two components and use the scale to determine the magnitude of the components.

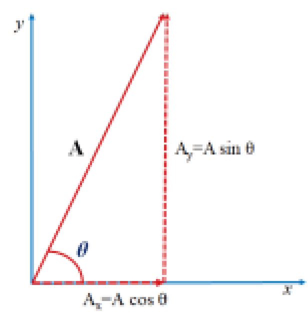


Figure : The horizontal ($\mathbf{A_x}$) and vertical ($\mathbf{A_y}$) components of vector A.

2. ***Trigonometric method of vector resolution:***

Note: The trigonometric method of vector resolution uses the **sine**, **cosine**, and **tangent** functions to resolve the vector into its components.

$$\sin\text{Ø} = \frac{opposite}{hypotenus} = \frac{\boldsymbol{Ay}}{\boldsymbol{A}} \Rightarrow \mathbf{A_y} = \mathbf{A}\sin\text{Ø}$$

$$\cos\text{Ø} = \frac{adjacent}{hypotenus} = \frac{\boldsymbol{Ax}}{\boldsymbol{A}} \Rightarrow \boldsymbol{Ax} = \mathbf{A}\cos\text{Ø}$$

Note: The original vector is the sum of the two component vectors.

$\mathbf{A = A_x + A\,y}$

Because Ax and Ay are at a right angle (90^o), the magnitude of the resultant vector can be calculated using the Pythagorean Theorem. $|\mathbf{A}| = (\sqrt[2]{A_x^2 + A_y^2})$

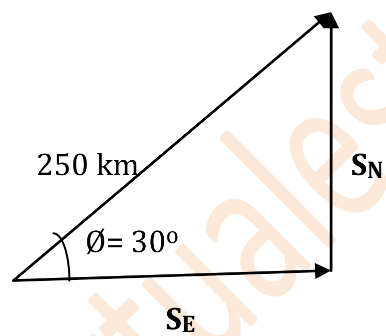
Angle or direction of the resultant

$$\tan\emptyset = \frac{opposite}{adjacent} = \frac{Ay}{Ax} \Rightarrow \emptyset = \tan^{-1}\left(\frac{Ay}{Ax}\right)$$

Example A motorist undergoes a displacement of 250 km in a direction 30º North of East. Resolve this displacement into its components

Given: vector = **S** = 250 km North East $\emptyset = 30^o$

Solution: first, draw the vector along North East direction

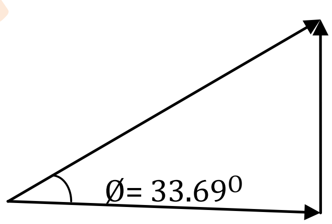


S_N = 250 km*sin30º = 125km

S_E = (250 km)(cos 30º) = 216.5 km

Example : A boy walks 3 km due East and then 2 km due North. What is the magnitude and direction of his displacement vector? Given: S_E = 3 km , S_N =2 km

Required: magnitude and direction of the displacement vector Solution: first, draw a triangle using the paths made during the journey.



S S_N =2 km S_E = 3 km

Using Pythagoras theorem;

$$\mathbf{S} = \sqrt[2]{(\boldsymbol{S}_E{}^2 + \mathbf{S}_N{}^2)} = \sqrt[2]{(3^2+2^2)} = 3.6 \text{ km}$$

The direction or the angle is the tangent that the resultant vector makes with the positive X-axis

$\text{Ø} = \tan^{-1}(\frac{SN}{SE}) = \tan^{-1}(2/3) = 33.69^0$

Exercise: Could a vector ever be shorter than one of its components? Could it be equal in length to one of its components? Explain

Solution: the magnitude of the vector is always greater than the magnitude of its components due to the trigonometric values and its length is greater than any of its components.

Review questions

2. Draw simple vector diagrams and resolve them into their components.

(a) 40 N at an angle of 30º from the horizontal.

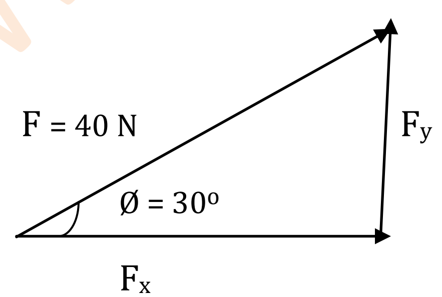
(b) 10 m/s at an angle of 80º from the horizontal.

(c) 1900 km at an angle of 40º from the vertical

Given: F =40 N Ø = 30º

Solution: draw the vector from the horizontal directed at 30º and project its vertical and horizontal components

a)



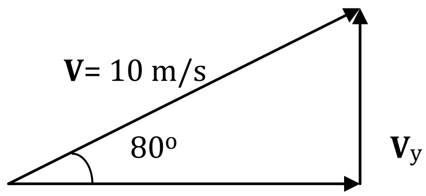
Use trigonometric equation:

$F_y = 40 \text{ N} * \sin 30^o = 20 \text{ N}$

$F_x = 40 \text{ N} * \cos 30^o = 40 \text{ N} * (\sqrt{3}/2)$

$F_x = 20\sqrt{3} \text{ N}$

b)

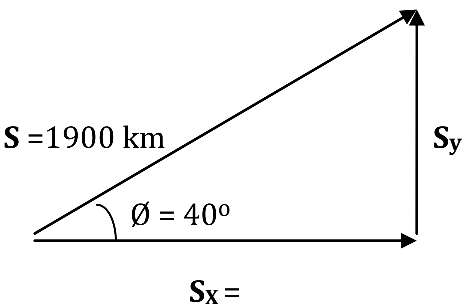


V_x = 10 m/s*cos 80⁰ = 10m/s(0.17) =

V_x =1.7m/s

V_y = **V** sinθ = 10m/s sin80° = 10m/s(0.98) = 9.8m/s

C)



S_y = 1900 km*sin40°

S_y =1900 km* 064 =1,216 km

S_x = 1900 km*cos40°=1444km

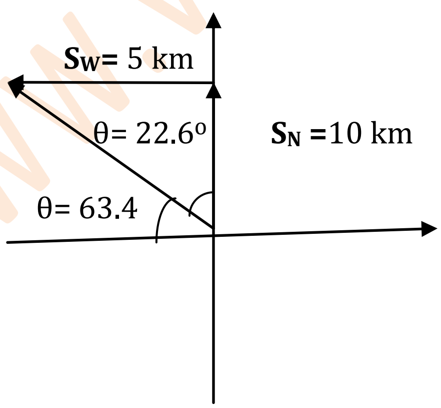
S_x =1900 km*0.76 =1,444 km

3. A car travels 10 km due North and then 5 km due West. Find graphically and analytically the magnitude and direction of the car's resultant vector

Given: S_N =10 km North S_W= 5 km West

Required: S_R =?

θ= ?



use Pythagoras method to find the resultant vector S_R

$S_R = \sqrt{SN^2 + Sw^2}$

$S_R = \sqrt{10^2 + 5^2}$ = 11.18 km

θ= $\tan^{-1}(\frac{SN}{Sw})$ = $\tan^{-1}(10/5)$

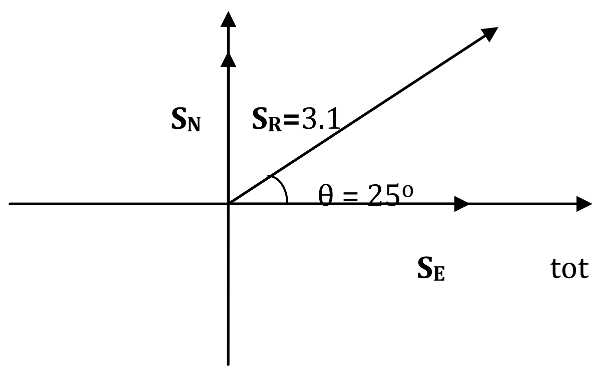
θ =63.4 North of west

or, 26.6° West of North

3. A girl walks 25.0° North of East for 3.10 km. How far would she have to walk due North and due East to arrive at the same location?

Given: $\mathbf{S}_R$ = 3.1 km, θ = 25° Required: $\mathbf{S}_N$ and $\mathbf{S}_E$

Solution: sketch the vector first in the direction given.



$\mathbf{S}_E = \mathbf{S}_R$*cos25° = 3.1 km*0.91 =2.81 km

$\mathbf{S}_N = \mathbf{S}_R$*sin25° = 3.1 km*0.423=1.3 km

tot al distance(s) = 2.81+1.3km = 4.11km

End of unit questions and problems

1. A vector drawn 15 mm long represents a velocity of 30 m/s. How long should you draw a vector to represent a velocity of 20 m/s?

Given: Length of **A** = 15 mm, $\mathbf{V}_A$ = 30 m/s required: length@ $\mathbf{V}_A$ = 20 m/s

Solution: by scale ratio or cross multiplication, we can find

Length@ 20 m/s = $\frac{20\ m/s}{30\ m/s}$ * 15 mm =10 mm

2. A vector that is 1 cm long represents a displacement of 5 km. How many kilometers are represented by a 3 cm vector drawn to the same scale?
   Solution: use cross multiplication
   Displacement = $\frac{3\ cm}{1\ cm}$ * 5 km = 15 km
3. Describe how you would add two vectors graphically.
   Solution:
   For adding vectors, place the tail of the second vector at the head of the first vector. The tail of the third vector is placed at the head of the second vector. The resultant vector is drawn from the tail of the first vector to the head of the last vector this method is head to tail method or triangle law of vector addition. The second way is

the parallelogram law of vector addition which is used to add two vectors when the vectors that are to be added form the two adjacent sides of a parallelogram by joining the tails of the two vectors. Then, the sum of the two vectors is given by the diagonal of the parallelogram. The third one is polygon method this law is used for the addition of more than two vectors. According to this law, if you have a large number of vectors, place the tail end of each successive vector at the head end of previous one.

4. Which of the following actions is permissible when you are graphically adding one vector to another?

A) move the vector

B) rotate the vector

C) Change the vector's length.

5. In your own words, write a clear definition of the resultant of two or more vectors.

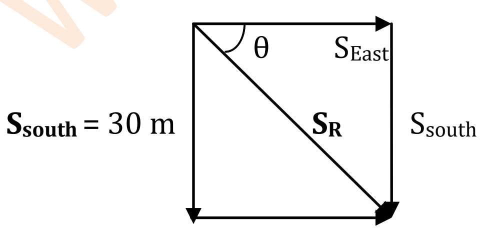
Solution: the resultant is found by adding the tail end of the successive vector to the head end of the previous vector.

6. Explain the method you would use to subtract two vectors graphically.

Solution: first draw the positive vector to the scale and appropriate direction then draw the negative vector to the tip of the first vector then join the tail of the first vector to the head of the negative vector

7. You walk 30 m South and 30 m East. Find the magnitude and direction of the resultant displacement both graphically and algebraically. Given; $\mathbf{S}_{\mathbf{south}}$ = 30 m, $\mathbf{S}_{\mathbf{east}}$ = 30 m , required: $\mathbf{S}_{\mathbf{R}}$ **=**? and θ=?

Solution:



$\mathbf{S}_R = \sqrt{\mathbf{S}east^2 + Ssouth^2}$

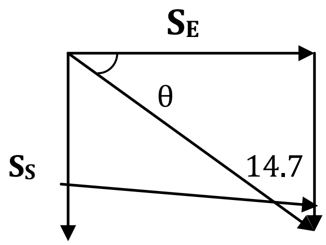
$S_R = \sqrt{30^2 + 30^2} = \sqrt{900} = 30\sqrt{2}$

$S_R = 42.43$ m

S_{east} = 30 m $\theta = \tan^{-1}(\frac{30}{30}) = 45^0$ South of East

8. A hiker walks 14.7 km at an angle 35^o East of South. Find the East and North components of this walk.

Given: resultant displacement(S_R) =14.7 m, θ =35^o



S_E =14.7*cos35^o= 14.7*0.82 =12.04 m

S_S = 14.7*sin35^o= 14.7*0.5735 =8.43 m

9. If two vectors have equal magnitudes, can their sum be zero? Explain.

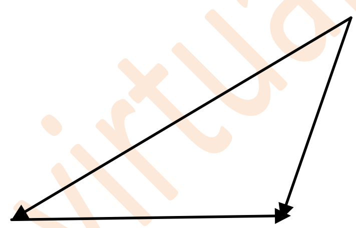
Answer: yes, if two vectors have equal magnitude and are exactly in opposite direction, their sum is zero.

10. Based on the three vectors in Figure 1.16, which of the following is true?

(a) A +B +C = 0

(b) A = C +B

(c) C + A = B

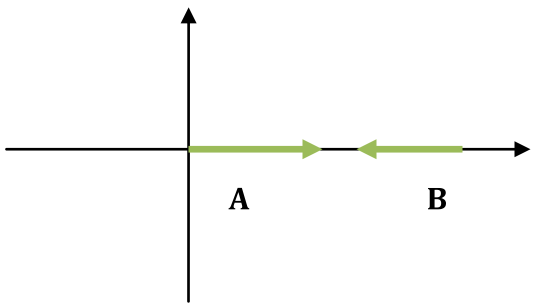


11. For the two vectors A and B with magnitude 6.8 cm and 5.5 cm in Figure 1.17, determine the magnitude and direction of:

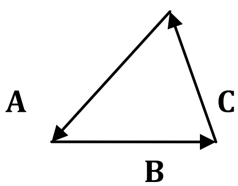
(a) R = A +B

(b) R = A –B

(c) R = B – A



Solution:

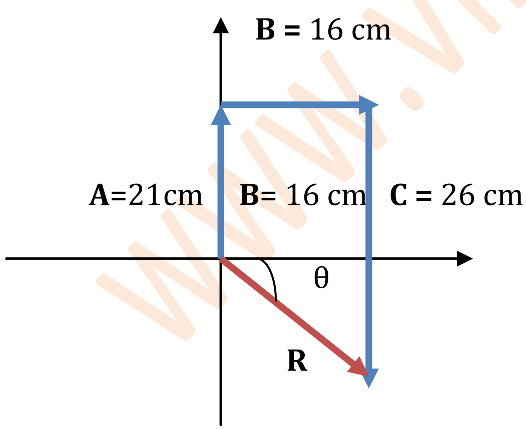


a) **C =A+B** = 6.8 cm+5.8 cm=12.3 cm

b) **C = A +**(**-B**) **=**6.8+(-5.5) = 1.3 cm

c) **C** = **B**-**A** = 5.5-6.8 = -1.3 cm

12. Three vectors A, B and C have a magnitude and direction of 21 unit North, 16 unit East and 26 unit South, respectively. Graphically determine the resultant of these three vectors?



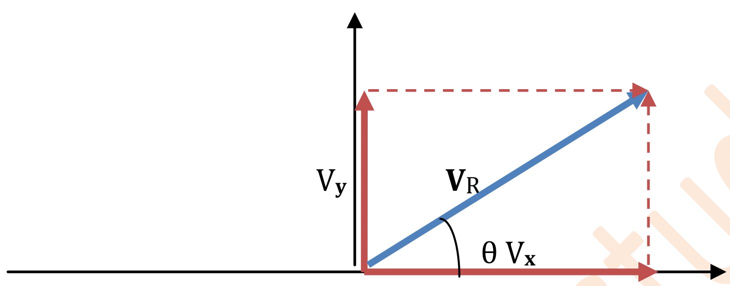
Given: A **=** 21cm C□ = 26cm south , B□ = 16cm east and joined head to tail. A and C□ are anti parallel vectors. Now let us find their difference

(A□−C) = 21+ (−26) = −5cm north (5 cm south) and it is perpendicular to B which is 16 cm east. Using Pythagoras theorem we can find the resultant vector **R**

$\sqrt{b^2 + (c-a)^2}$ = $\sqrt{16^2 + 5^2}$ = $\sqrt[2]{281}$ =16.76 cm

The angle (θ) = tan^{-1}($\frac{\boldsymbol{C-A}}{\boldsymbol{B}}$) = tan^{-1}($\frac{5}{16}$) =17.35^o South of East

13. If $\mathbf{V_x}$ = 9.8 m/s and $\mathbf{V_y}$ = 6.4 m/s, determine the magnitude and direction of **V.**



Given: $\mathbf{V_x}$ = 9.8 m/s and $\mathbf{V_y}$ = 6.4 m/s

Required: $\mathbf{V}_R$ and direction θ Solution: using Pythagoras theorem

$\mathbf{V_R}$ = $\sqrt{9.8^2 + 6.4^2}$ = 11.7 m/s θ = tan^{-1}($\frac{6.4}{9.8}$) = 33.14^o north of East

Unit 2

Uniformly Accelerated Motion

Accelerated motion: is a non-uniformity in the motion of an object caused by a change in the speed or direction of an object.

Example of accelerated motion: A car ride in a city at rush hour during which the car must speed up, slow down, and turn corners

2.1. Position and Displacement

Position: The location of an object in a frame of reference

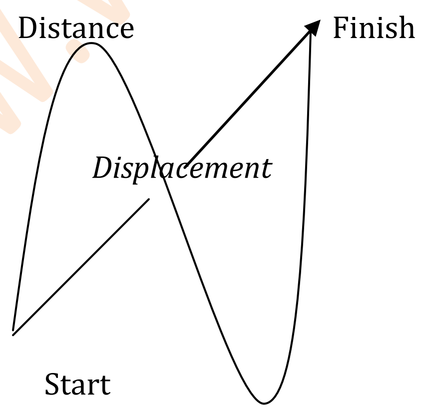
Frame of reference: is an arbitrary set of axes from which the position and motion of an object are described.

Displacement:

Distance: is the total length of the path taken in going from the initial position to the final position. Distance is a scalar.

Displacement: The difference between the initial and final position vectors of a body

Displacement: is the shortest distance between the two positions and has a certain direction.



the curved line= *distance*

straight line= displacement

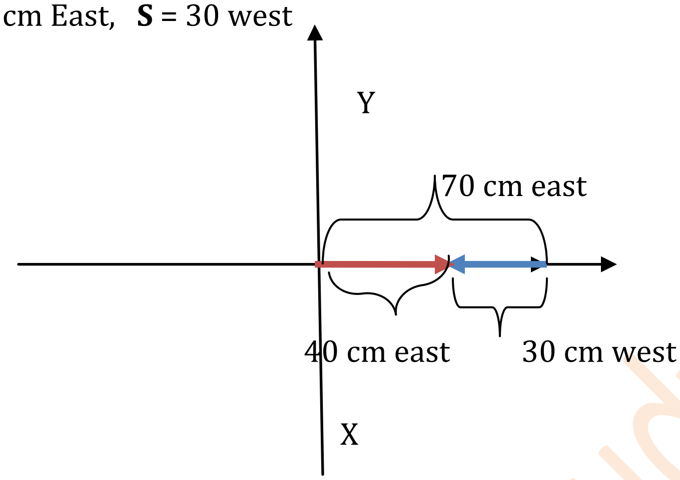
$\Delta s = s - s_0$, where; s_0 = initial position

S = final position

Example: A person walks 70 m East, and then 30 m west. Find the displacement.

Given: $\mathbf{S_0}$=70 cm East, **S** = 30 west

Solution:



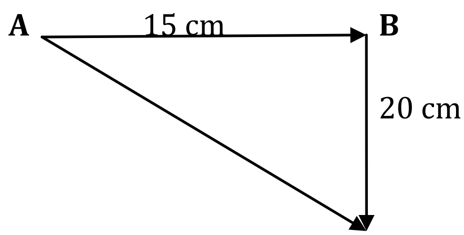
$\Delta s = s - s_0 = 70\text{ m} - 30\text{ m}$ (as both vectors are in an opposite direction with one another. Thus, $\Delta\mathbf{S}$ = 40 m East

Review questions

1. Explain the difference between position and displacement.

Solution: Position is the location of the object in a frame of reference where as displacement is the difference between the initial and final position vectors of a body.

2. Give an example that clearly shows the difference among distance traveled, displacement, and magnitude of displacement. Identify each quantity in your example.
   Solution: a public bus travelling to the office which is 5 km East from its initial point, then the bus is moving 3 km west from the office.
   From this journey
   Distance = 5 km + 3 km = 8 km.
   Displacement = 3 km – 5 km = -2 km west or 2 km East.
3. A body travels a distance of 15 m from A to B and then moves a distance of 20 m at right angles to AB. Calculate the total distance traveled and the displacement.



Distance = 15 cm +20 cm = 35 cm

Displacement = $\sqrt{15^2 + 20^2} = \sqrt{625} =$ 25 cm

$\theta = \tan^{-1}(\frac{20}{15}) =$ 53.1°

Therefore; the magnitude of displacement and direction is: □S = 25 m; θ = 53.1° to the horizontal.

2.2 Average velocity and instantaneous velocity

Average velocity: is defined as the body's displacement (Δs) divided by the time interval (Δt) during which that displacement occurs.

$$V_{avg} = \frac{\Delta s}{\Delta t} = \frac{s - s0}{t - t0}$$

S_0 and **S** be its positions at instants t_0 and t, respectively

- Where t − t_0 is change in time,
- t_0 is the starting time which is commonly zero.
- The SI unit for average velocity is meters per second (m/s or m s^{-1})

The average speed of an object is obtained by dividing the total distance traveled by the total time taken:

$$Vav = \frac{total\ distance}{total\ time\ taken} = S_{tot}/t_{tot}$$

Note: If the motion is in the **same direction** along a **straight line**, the **average speed** is the same as the **magnitude** of the **average velocity**. However, this is always not the case.

Instantaneous velocity: is the velocity at a specific instant in time (or over an infinitesimally small time interval)

- it is the rate of change in displacement as change in time approaches zero.

V = $(S-S_0)/t-t_0$ when $t-t_0$ approaches to zero.

Example

It takes you 10 minutes to walk with an average velocity of 1.2 m/s to the North from the bus stop to the museum entrance. What is your displacement?

Given: Δt = 10 minutes = 600 s and V_{av} = 1.2 m/s , North. Required: Δs

Solution: $\Delta s = V_{av} * \Delta t$ = 1.2 m/s *600 s = 720 m , North

Example

A passenger in a bus took 8 s to move 4 m to a seat on provided place forward. What is his average velocity? Solution:

Given : ΔS = 4 m and Δt = 8 s. required: V_{av}

$V_{av} = \Delta S/\Delta t$ = 4 m/ 8 s = 0.5 m/s

Example

A car travels at a constant speed of 50 km/h for 100 km. It then speeds up to 100 km/h and is driven another 100 km. What is the car's average speed for the 200 km trip?

Given:

First journey: v =50 km/h, s = 100 km

Second journey: v = 100 km/h, s= 100 km

Solution: let us find the time taken for each journey

t_1 = 100 km/50 km/h = 2 h

t_2 =100 km/100 km/h = 1 h

$v_{avg} = s_{tot}/t_{tot}$ = (100 km +100km)/(2+1)h = 200 km/3 h = 66.7 km/h

Exercise

Cheetahs, the world's fastest land animals, can run up to about 125 km/h. A cheetah chasing an impala runs 32 m north, then suddenly turns and runs 46 m west before lunging at the impala. The entire motion takes only 2.7 s.

(a) Determine the cheetah's average speed for this motion.

(b) Determine the cheetah's average velocity.

Given: s_1=32 m north, s_2= 46 m west, total time taken (t) = 2.7 s

Required: a) average speed (v), b) average velocity

Solution:

a) $v_{avg} = s_{tot}/t_{tot}$ = (32+46)/2.7 =78/2.7 =29 m/s

b) $\Delta s = \sqrt{32^2 + 46^2}$ = 56 m, v_{avg} =56/2.7 = 20.7 m/s

Review questions

1. How do you find the average velocity of an object in motion between two points?

Answer: an object moving between two point **a** and **b** ,with initial positions ($\mathbf{S}_o$) and final position ($\mathbf{s}$) in the given interval of time Δt the average velocity can be expressed as

$s = (\mathbf{s} - \mathbf{s}_o)/\Delta t$

4. If an object has the instantaneous velocity of 20 m/s to East, what is its instantaneous speed?

Answer: 20 m/s

5. A car moves with an average velocity of 48.0 km/h to the East. How long will it take him to drive 144 km on a straight highway?

Given: v□$_{av}$ = 48 km/h East, Δ□s = 144 km East, Required: Δt =?

Solution: V□av = Δ□S /Δt , Δt = 144km/h /48km
Δt = 3 h

6. An athlete runs 12 km to the North, then turns and runs 16 km to the East in three hours.
a) What is his/her displacement?

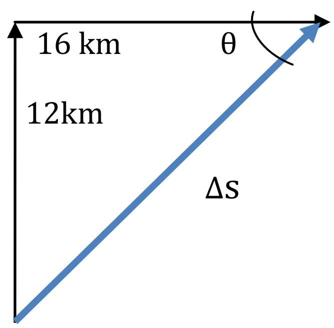
b) Calculate his/her average velocity.

c) Calculate average speed

given: S_1 = 12 km North, S_2= 16 km East, Δt= 3 h

solution:

a)



$\Delta s = \sqrt{12^2 + 16^2} = 20$ km

b) $\Delta v = \Delta s/\Delta t$ = 20 km/3 h =6.7 km/h

The direction can be found as: $\theta = \tan^{-1}(\frac{12}{16}) = 37^o$ N of E

c)= average speed(Δv) = total distance/ total time = (12 km +16 km)/ 3h = 28 km/3 h = 9.3 km/h

2.3 Acceleration

Acceleration: Is the rate of change of velocity in a given time interval. Any change in velocity whether positive, negative, directional, or any combination of these is acceleration.

Average acceleration = $\frac{change\ in\ velocity}{time\ interval}$

$a_{avg} = v - v_o / t - t_o =$

- v_o is the initial velocity of an object and v is the final velocity of an object at instants to and t, respectively.

- $t - t_o$ is the length of time over which the motion changes.
- the SI unit of acceleration is meters per second squared (m/s^2).
- The direction of average acceleration is the direction of change in velocity.

Instantaneous acceleration: is the average acceleration at a specific instant in time (or over an infinitesimally small time interval)

$a = \frac{\Delta v}{\Delta t}$, as Δt approaches to zero.

Example: A car accelerates on a straight road from rest to 75 km/h in 5 s. What is the magnitude of its average acceleration?

Given: $v_0 = 0$, $v = 75$ km/h and $\Delta t = 5$ s. required: average acceleration.

Solution: convert the km/hour in to meter/seconds

75 km/h* 1h/3600 s*1000 m/1 km = 20.83 m/s

$a_{avg} = (v - v_0)/\Delta t = (20.8\ m/s - 0)/5\ s = 4.2\ m/s^2$

Example : An automobile is moving to the right along a straight highway, which you choose to be the positive x-axis. Then the driver steps on the brakes. If the initial velocity (when the driver hits the brakes) is 15 m/s and it takes 5.0 s to slow down to 5 m/s, what was the car's average acceleration?

Given: $v_0 = 15$ m/s, $v = 5$ m/s and $\Delta t = 5.0$ s. The required : average acceleration.

$a = \frac{v - vo}{t - to} = \frac{5 m/s - 15\ m/s}{5 - 0\ s} = -2\ m/s^2$

The negative sign indicates the final velocity is less than the initial velocity. Thus, the direction of the acceleration is to the left (in the negative x-direction) even though the velocity is always pointing to the right. You can say that the acceleration is to the left. That is the automobile is **decelerating**. But be careful: deceleration does not mean that acceleration is necessarily negative. There is a deceleration whenever the magnitude of the velocity is decreasing; thus, the velocity and acceleration point in opposite directions when there is a deceleration.

Review questions

2. A car moves along the x- axis. What is the sign of the car's acceleration if it is moving in the positive x direction with?

(a) Increasing speed

(b) Decreasing speed?

Solution:

a) positive acceleration
b) negative acceleration and deceleration

3. A race horse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

$a = (v - v_0)/ t - t_0 = (15\ m/s - 0)/1.8\ s = 8.33\ m/s^2$ due west

4. A car is traveling at 14 m/s when the traffic light ahead turns red. The car decelerates and comes to a stop in 5.0 s. Calculate the acceleration of the car.

Given: $v_0 = 14$ m/s $v = 0$, $\Delta t = 5$ s

$a_{av} = (v - v_0)/t - t_0 = (0 - 14 m/s)/5 - 0 = -2.8\ m/s^2$

the negative sign indicates the motion is deceleration

2.4 Equations of motion with constant acceleration

If an object travels in a straight line and its **velocity increases** or **decreases** by equal amounts at **equal intervals of time,** the **acceleration** of the object is said to be **uniform.** Such type of motion is said to be a uniformly accelerated motion.

Equations of motion: sets of equations and relations, in a uniformly accelerated motion, that relate the velocity, acceleration and distance covered by the object in the given time interval.

$$a_{av} = \frac{v - vo}{t - to} \text{ ....(1)}$$

Note: Since acceleration is constant in a uniformly accelerated motion, the average and instantaneous accelerations are the same.

Setting to = 0 and rearranging the result gives

$v = v_o + at$ ....(2)

When the acceleration is constant (i.e., when the velocity varies linearly with time), the average velocity is given as:

$$v_{av} = \frac{v + vo}{2} \text{ ....(3)}$$

If the initial position is at the origin, $s_o = 0$, and hence $\Delta s = s$. So from the expression of average velocity,

$$s = v_{av}\, t \text{ ....(4)}$$

Substituting equation 3 into 4 gives:

$S = \left(\frac{v+vo}{2}\right) * t$ ....(5)

Again substituting $v_o + at$ in place of v and making rearrangement gives:

$s = v_0 t + 1/2 * at^2$ ....(6)

Sometimes, there are times when the time of motion is unknown. For such cases, you need to derive an equation that is independent of time as follows:

$s = \left(\frac{v+vo}{2}\right) * t = \left(\frac{v+vo}{2}\right)\left(\frac{v-vo}{a}\right) = (v^2 - v_0^2)/2a$ ....(7)

Rearranging gives:

$v^2 = v_0^2 + 2as$.......................................(8)

Example

A car starts from rest and accelerates uniformly over a time of 5 s for a distance of 100 m.

Determine the acceleration of the car.

Given: $\Delta s = 100$ m, t = 5 s

Required: a

Solution:

$v_{av} = \Delta s/\Delta t = 100 \text{ m}/5 \text{ s} = 20 \text{ m/s}$

since $v = (v+v_o)/2$ …solve for final velocity (v) by making $v_o=0$

$v = v_{av}*2 = 20\text{m/s} *2 = 40 \text{ m/s}$

$a= (v-v_o)/t = (40 \text{ m/s} -0)/5 \text{ s} = 8 \text{ m/s}^2$

Alternatively, we can find the average acceleration (a) from this equation

$s = v_0t + 1/2* at^2$………setting $v_o=0$

$a = 2s/t^2 = 2*100 \text{ m}/25 \text{ s}^2 = 8 \text{ m/s}^2$

Example

An airplane lands with an initial velocity of 70 m/s and then decelerates at 1.5 m/s^2 for 40 s. What is its final velocity?

Given: deceleration (negative acceleration) = -1.5 m/s^2 , v_o= 70 m/s, t = 40 s, t_o= 0

Required: v=?

$a= (v- v_o)/t =$

$v = v_o+ at = 70 \text{ m/s} + (-1.5 \text{ m/s}^2* 40 \text{ s}) = 10 \text{ m/s}$

During deceleration, the final velocity is much less than the initial velocity but is still positive.

Example

An automobile starts at rest and speeds up at 3.5 m/s^2 after the traffic light turns green.

How far did the automobile travel when it was traveling at 25 m/s?

Given: $a = 3.5\ m/s^2$, $v_o = 0$, $v = 25$ m/s, required: s

$s = (v^2 - v_o^2)/2a = ((25\ m/s)^2 - 0^2)/2*3.5\ m/s^2 = 625\ m^2/s^2/7 m/s^2 = 89.3$ m

Free Fall

A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion.

Note: **in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration**

The magnitude of the free-fall acceleration is denoted by the symbol g

Note: At the Earth's surface, the value of g is approximately 9.80 m/s^2

- the value of g is maximum at the earth's surface and decreases with increasing altitude

Neglecting air resistance, the equation for the motion of a free falling object moving vertivcally can be derived from the equation applied for motion in one dimension under constant acceleration

Note: replace a by g and s by h

$v = v_o + gt$……………………….(9)

h= $v_o t + 1/2 gt^2$…………………(10)

$v^2 = v^2 + 2gh$

Where v_0 is the initial velocity, h is the vertical height, g is acceleration due gravity and t is the elapsed time.

Example

A mango fruit has fallen from a tree. Find its velocity and vertical height when it reached the ground if it took 1 s to reach the ground.

Given: v_o= 0, g = 9.8 m/s, time (t) = 1 s, required: v, h

Solution:

$V = v_o + gt = 0 + 9.8\ m/s * 1\ s = 9.8\ m/s$

$h = v_{av}t =$

$v_{av} = (v+v_o)/2 = (9.8\ m/s+0)/2 = 4.9\ m/s$

$h = v_{av}t = 4.9\ m/s * 1\ s = 4.9\ m$

Review questions

1. What type of motion is experienced by a free-falling object?
   Answer: since a free falling object moves under constant acceleration due to gravity and it is a uniformly accelerated motion.

2. A cyclist is traveling at 5.6 m/s when she starts to accelerate at 0.60 m/s^2 for a time interval of 4.0 s.

(a) How far did she travel during this time interval?

(b) What velocity did she attain?

Given: v_o= 5.6 m/s, $a = 0.6\ m/s^2$, $t = 4\ s$

Solution:

a) $s = v_o t + 1/2 at^2 = 5.6\ m/s * 4\ s + ½ * 0.6\ m/s^2 (4s)^2 = 27.2\ m$
b) $v = v_o + at = 5.6\ m/s + 0.6\ m/s^2 * 4\ s = 8\ m/s$

3. A stone that is dropped from the top of a building is in free fall for 8.0 s.

(a) Calculate the stone's velocity when it reaches the ground.

(b) What is the height of the building from which the stone had been dropped?

Given: t = 8 s, $v_o = 0$, $g = 9.8\ m/s^2$

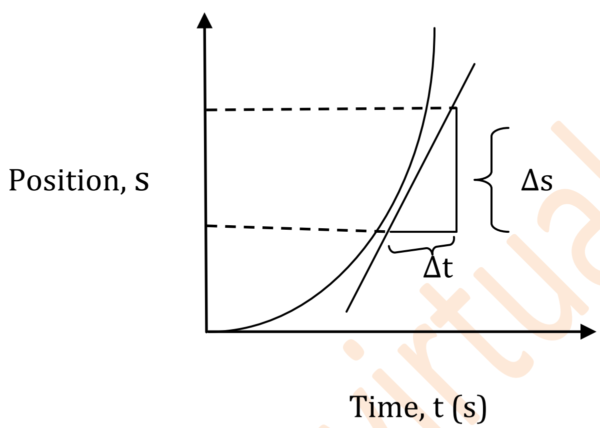
Solution:

a) $v = v_o+gt = 0+9.8\ m/s^2*8\ s = 78.4\ m/s$
b) $h= v_ot+ ½*gt^2= 0*0.8+ ½*9.8*(8s)^2 =313.6\ m$

2.5 Graphical representation of uniformly accelerated motion

Position-time graph

The position-time equation for uniformly accelerated motion along a straight line is $s = v_o t +1/2at^2$. Dependence of s on t^2 shows that it is a quadratic equation or quadratic function of t. So, the position-time graph for uniformly accelerated motion is a parabola,



Slope = rise/run = Δs/Δt= instantaneous velocity

The instantaneous velocity of an object at a specific point in time is thus the slope of the tangent to the curve of the position-time graph of the object's motion at that specific time

Example

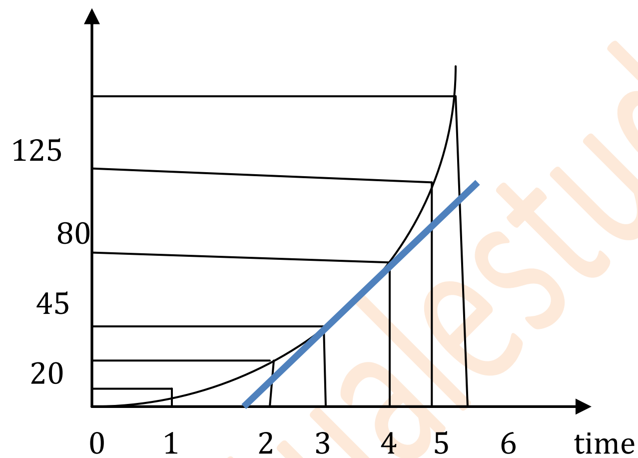
A car starts from rest and accelerates at a 10 m/s^2 for 10 s on the straight, level road. Draw a position-time graph and calculate the instantaneous velocity at 4 s

Given: $v_o = 0$ m/s, $a = 10\ m/s^2$, and t = 10 s. To draw the position-time graph, first solve for **s** using the equation $s = ½*at^2$ for the different values of t. Record and compare the result you obtained with the following table

Position(m)	0	5	20	45	80	125	180
Time(m)	0	1	2	3	4	5	6

S= ½*at^2

Then plot the position on the y-axis and time on the x-axis. The graph is shown as in the Figure



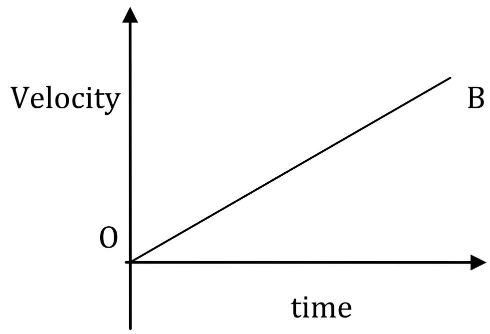
Let us take any two points on the tangent line and determine the slope or the instantaneous velocity

Let us say (3,40), (4, 80)

Slope = instantaneous velocity = (80-40)m/(4-3) s= 40 m/s

Velocity-time graph

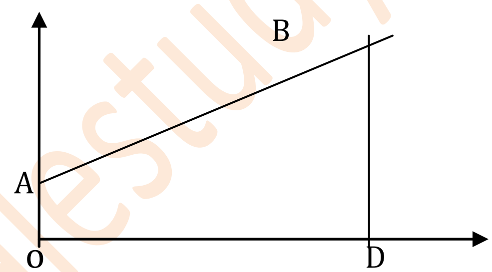
Case 1: If the particle starts from rest and experiences uniform acceleration, the velocity-time graph will be a straight line passing through the origin and having a positive slope.



case 2: If the particle has an initial velocity, the graph will be a straight line, but will not pass through the origin

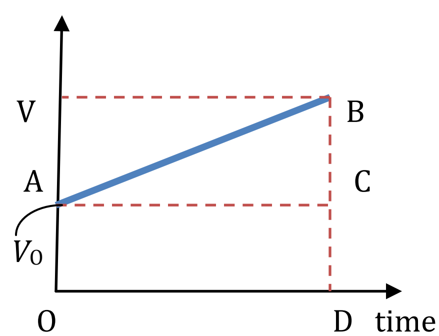
OA= initial velocity

BD = final velocity



The velocity-time graph of a particle moving along a straight line with uniform acceleration may be used to measure the **displacement** of the particle and also the **acceleration**

A) **Area under velocity-time**: Consider a particle moving along a straight line with uniform acceleration a. Let v_0 be the velocity of the particle at the instant t = 0 and v at a later instant t.



✓ The area under the velocity-time graph during the interval 0 to t is ABDO.

Area, ABDO = Area of rectangle ACDO + Area of triangle, ABC.

Area= AO*AC+ ½*AC*BC = displacement

Thus, area under the velocity-time graph gives the total displacement of the particle in that time interval

B) Slope of a velocity-time graph

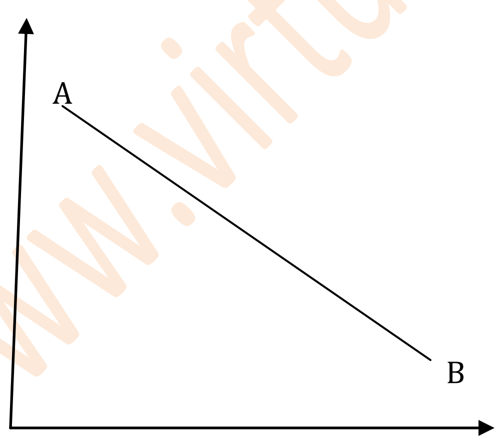
Let v_o be the velocity at $t_o = 0$ and v be the velocity after a time interval t

$a = (v-v_o)/t = BC/AC = slope$

That is, the slope of the velocity-time graph gives the acceleration of the particle

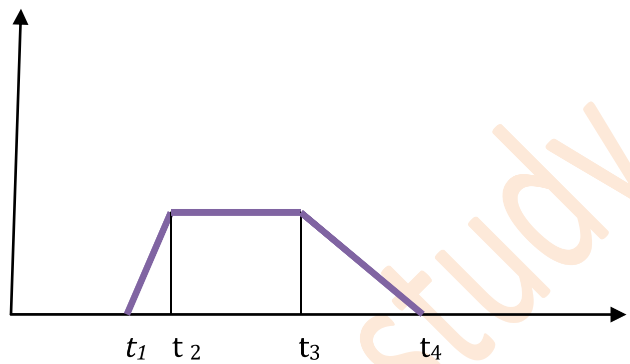
two special cases on velocity time graph

1: For a particle moving with uniform **retardation** or **deceleration**, the **velocitytime** graph will be a straight line with a **negative slope**, as shown in Figure If the body is brought to rest, the graph will touch the time axis.

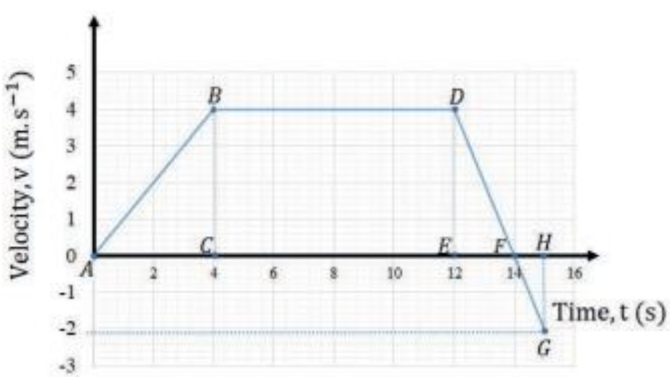


2. Velocity-time graph for non-uniform motion

In the case of a particle moving with variable velocity, the velocity-time curve will be irregular in shape. For example, consider a car starting from point O. Let it be moving along a straight line with uniform acceleration a during the time interval t_1 to t_2 and then start moving with uniform velocity during the interval of time t2 to t3. Thereafter, let the velocity of the car decrease uniformly and the car comes to a stop at the instant t4. The motion of the car can be represented by the velocity-time graph, as shown in Figure . Then the area under the velocity-time graph gives the total displacement of the car.



Example The velocity-time graph of a certain motion is plotted in Figure below. Calculate the total distance and displacement of the truck after 15 s.



Solution: To calculate the total distance and total displacement, you have to find the area under velocity-time graph.

s_1 = area of a triangle ACB

S_1 = ½*base*height= ½*(Δt)* (Δh) = ½*4s*4m/s = 8 m

S_2= base*height= 8 s*4 m/s= 32 m

S_3= ½*base*height = ½*2 s*4 m/s = 4 m

S_4= ½*base*height= ½*1 s* 2 m/s = 1 m

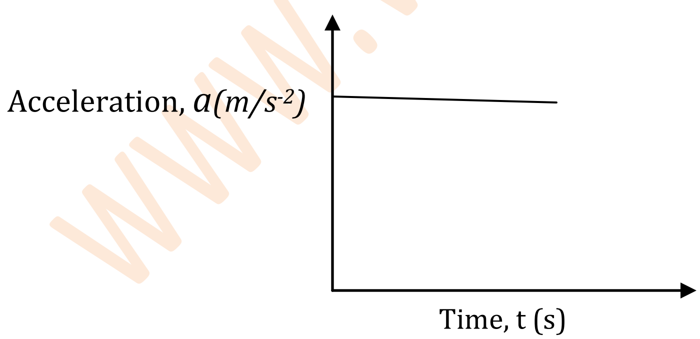
Total distance = s_1+ S_2 + S_3+ S_4 = 8 m+32 m+4 m+ 1 m = 45 m

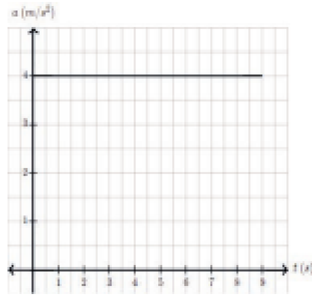
Total displacement = s_1+ S_2 + S_3+ (-S_4) = 8 m+32 m+4 m-1 m (since velocity is negative)= 44 m in the positive direction

Acceleration-time graph: The acceleration-time plots acceleration values on the y-axis and time values on the x-axis.

Note: For a uniformly accelerated motion, acceleration is constant with time.

- The acceleration-time graph will be a straight line parallel to the time axis.
- For a straight line parallel to the time axis, slope equals zero.
- An acceleration-time graph can be used to find the change in velocity during various time intervals. This is accomplished by determining the area under the line on the acceleration-time graph.





Example For the acceleration-time graph shown in Figure 2.16, find the change in velocity of an object. Solution: In this example, the change in velocity can be obtained by finding the area under the acceleration-time graph. Thus,

area = base×height = 9 s ×4 m/s2 = 36 m/s Hence, the change in velocity of the moving object is 36 m/s.

Exercise

2. The velocity-time graph below shows the motion of an air plane. Find the displacement of the airplane at Δt = 1.0 s and at Δt = 2.0 s.

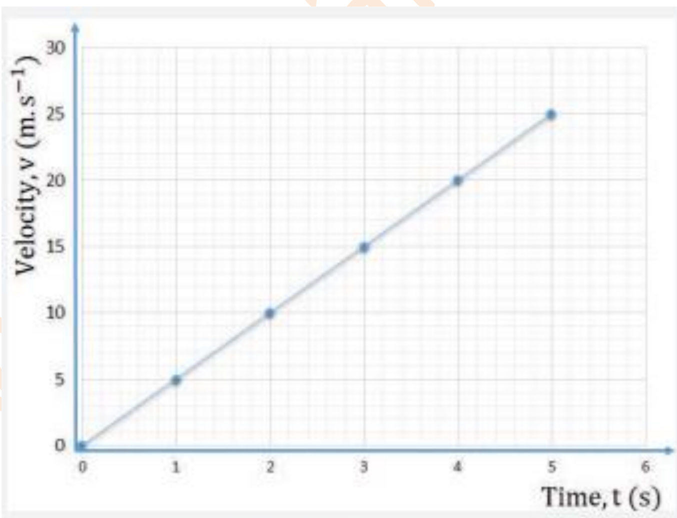


Figure 2.18 Velocity-time graph.

Solution:

The total displacement is the area under the velocity time graph

at $\Delta t = 1.0$ s

Displacement = ½*base*height= ½*1 s * 5 m/s = 2.5 m

Displacement at $\Delta t = 2$ s

½*base*height = ½* 2 s* 10 m/s = 10 m

2.6 Relative velocity in one dimension

Relative velocity is the velocity of an object with respect to another object or observer.

The relative velocity of object A with respect to object B is the rate of change of position of the object A with respect to the object B. If v_A and v_B be the velocities of objects A and B with respect to the ground, then

$v_{AB} = v_A - v_B$

$v_{BA} = v_B - v_A$.

In one-dimensional motion, objects move in a straight line. So there are only two possible cases: objects are moving in the same direction or in opposite directions

Thus, while using the above expressions,

- For bodies that are moving in the same direction, the signs of velocities needs to be the same (say, positive).
- But if they are moving in opposite directions, one of the velocities (say to the right) should be positive while the other one becomes negative.
- Thus, if the two objects are moving in the same direction, the magnitude of the relative velocity of one object with respect to another is equal to the difference in magnitude of two velocities.
- But if the two objects are moving in opposite directions, the above expression becomes

- ✓ $v_{AB} = v_A - (-v_B) = v_A + v_B$
- ✓ $v_{BA} = -v_B - v_A = -(v_A + v_B)$ if object A is moving to the right and object B is moving to the left.

Thus, for objects that are moving in opposite directions, the magnitude of relative velocity of one object with respect to other is equal to the sum of magnitude of their velocities.

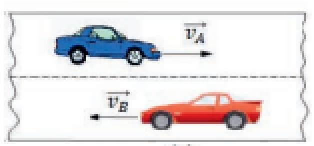
Example

Two cars, A and B are traveling with the same speed of 100 km/h in opposite directions, as in the Figure 2.19

(a). Find the relative velocity of car A with respect to car B and the relative velocity of car B with respect to car A

Solution: In this example, suppose that right is positive and left is negative.

Given: v_A = 100 km/h and v_B = - 100 km/h



(a)

- The relative velocity of A with respect to B is $v_{AB} = v_A - v_B$ = [100− (−100)] km/h = 100+100 km/h = 200 km/h.
- The relative velocity of B with respect to A is $v_{BA} = v_B - v_A$ = [(−100)−100] km/h = −200 km/h.

In the same question, if both bodies are moving in the same direction (say to the right) as in the Figure 2.19 (b) with the same speed, then

- The relative velocity of A with respect to B is $v_{AB} = v_A - v_B$ = [100−100] km/h = 0km/h

• The relative velocity of B with respect to A is $v_{BA} = v_B - v_A$ = [100−100] km/h = 0 km/h.

That means A is at rest with respect to B, and B is at rest with respect to A; but both are moving at 100 km/h with respect to the ground. This shows that, when objects are moving in the same direction, the magnitude of the relative velocity between them is equal to the difference between the magnitudes of their velocities.

Exercise

1. A motorcycle traveling on the highway at a velocity of 120 km/h passes a car traveling at a velocity of 90 km/h. From the point of view of a passenger on the car, what is the velocity of the motorcycle?

   Given: velocity of motorcycle= V_A=120 km/h, velocity of car=v_B= 90 km/h, required: v_{AB}

   Solution:

   $v_{AB}= V_A-V_B$= 120 km/h-90 km/h = 30 km/h

2. An automobile is moving at 80 km/h, and a truck is moving at 60 km/h, approaching an automobile. What is the relative velocity of an automobile with respect to a truck when the observer on the automobile measures it?

Given: V_A= 80 km/h, v_T= -60 km/h (since the truck is moving in opposite direction to the motion of the automobile), required: V_{AT}

Solution: $V_{AT}= V_A-V_T$ = 80 km/h –(- 60 km/h) = 80 km/h + 60 km/h =140 km/h

3. A thief is running away on a straight road on a jeep moving with a speed of 9 m/s. A police man chases him on a motor cycle moving at a speed of 10 m/s. If the instantaneous separation of jeep from the motor cycle is 100 m, how long does it take for the policemen to catch the thief?

Given: velocity of thief = V_T= 9 m/s, velocity of police = 10 m/s,

Separation Distance= 100 m , required: time (t)

Solution: first let us find the relative velocity of the policeman with respect to the thief

$V_{PT}=V_P-V_T$ = 10 m/s-9 m/s =1 m/s = instantaneous velocity

Time required(t) = distance/instantaneous velocity = 100 m/1 m/s = 100 s

End of unit questions and problems

1. A runner travels around rectangular track with length 50 m and width 20 m. After traveling around the rectangular track twice, the runner back to its starting point. Determine the distance and displacement of the runner.

   Given: length = 50 m, width = 20 m, required: distance and displacement

   Solution:

2. How do intervals of constant acceleration appear on a velocitytime graph?

3. A truck on a straight road starts from rest, accelerating at 2 m/s^2 until it reaches a speed of 20 m/s. Then the truck travels for 20 s at constant speed until the brakes applied, are stopping the truck in a uniform manner in an additional 5 s.

(a) how long is the truck in motion?

(b) What is the average speed of the truck for the motion described?

Given: $V_0 = 0$, $a = 2\ m/s^2 = v = 20\ m/s$, $t_2 = 20$ s, $v_2 = 20$m/s and constant, $t_3 = 5$ s

Solution:

for constant acceleration motion;

$t_1 = (v-v_o)/a = (20\ m/s-0)/2m/s^2 = 10\ s$

$s_1 = v_o t + ½*at^2 = ½*2\ m/s^2*(10\ s)^2 = 100\ m$

for constant speed period:

$s_2 = v_{av}*t = 20\ m/s*20\ s = 400\ m$

for Deceleration

$(a_3) = v_2/t_3 = 20\ m/s/\ 5\ s = -4\ m/s^2$

$S_3 = vot + a_3 t^2{}_3/2 = 20\ m/s*\ 5\ s - 4\ m/s^2*(5\ s)^2/2 = 50\ m$

a) Total time = $t_1+t_2+t_3$= 10 s+ 20 s+5 s = 35 s

Total distance = $s_1+s_2+s_3$= 100 m+400 m + 50 m = 550 m

b) Average velocity = total distance/ total time = 550 m/ 35 s = 15.7 m/s

4. If a student rides her bicycle in a straight line for 15 minutes with an average velocity of 12.5 km/h south, how far has she ridden?

Given: v_{av}=12.5 km/h, t = 15 min= 0.25 h, required: displacement ***s***

Solution:

S = v_{av}*t = 12.5 km/ h * 0.25 h = 3.125 km south

5. A race car travels on a racetrack at 44 m/s and slows at a constant rate to a velocity of 22 m/s over 11 s. How far does it move during this time?

Given: v_o= 44 m/s, v_f = 22 m/s, t= 11 s, required= displacement ***s***

Solution: $s = (\frac{vo+vf}{2})*t = (\frac{44\frac{m}{s}+22\ m/s}{2})*11\ s = 363\ m$

6. A truck is traveling at 22 m/s when the driver notices a speed limit sign for the town ahead. He slows down to a speed of 14 m/s. He travels a distance of 125 m while he is slowing down. (a) Calculate the acceleration of the truck. (b) How long did it take the truck driver to change his speed?

Given: v_o= 22 m/s, v_f= 14 m/s, s = 125 m

Solution:

$S= (\frac{vo+v}{2})*t$

$T= 2s/(v_o+v)= (2*125\ m)/(22\ m/s + 14\ m/s)= 250\ m/36\ m/s =6.94\ s$

$a=\frac{vf-vo}{\Delta t}$ = (14 m/s- 22 m/s)/6.94 s = -8 m/s/6.94 s = -1.15 m/s^2

8. A brick is dropped from rest from a height of 4.9 m. How long does it take the brick to

reach the ground?

Given: for free fall questions, h =4.9 m, v_o= 0, gravitational acceleration (g) = 9.8 m/s^2
required: time

Solution: $h = v_o t + 1/2 * gt^2$

t = $\sqrt[2]{2h/g}$ = $\sqrt[2]{2 * 4.9m/9.8m/s2}$ = 1 s

9. The velocity of a car changes over an 8 s time period as shown in the following table.

(a) Plot the velocity-time graph of the motion.

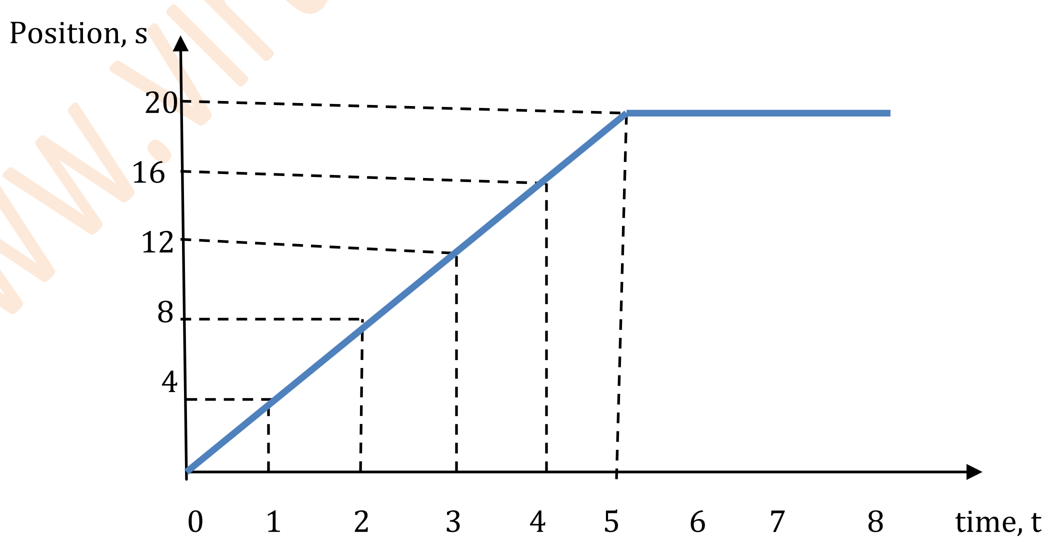
(b) What is the displacement of the car during the entire 8 s?

(c) Find the slope of the line between t = 0 s and t = 4 s? What does this slope represent?

(d) Find the slope of the line between t = 5 s and t = 7 s. What does this slope represent?

Position (m)	0	4	8	12	16	20	20	20	20
Time (s)	0	1	2	3	4	5	6	7	8

A)



B) to find the total displacement, we have to find the velocity during each time interval and then plotting the velocity time graph to calculate the are under velocity time graph to get the displacement.

Velocity between 0 and 1 s

V= Δs/Δt = (4-0) m/ (1-0)s = 4 m/s

Between 1& 2 s

V =Δs/Δt =(8-4) m/ (2-1) s = 4 m/s

Between 2 &3 s

V=Δs/vt = (12-8) m/ (3-2) s = 4 m/s

Between 3 & 4 s

V= Δs/vt = (16-120 m/(4-3) s = 4 m/s

Between 4 & 5 s

V=vs/Δt = (20-16) m/(5-4) s = 4 m/s

Between 5 & 6 s

V=Δs/Δt = (20- 20)/(6-50 s = 0

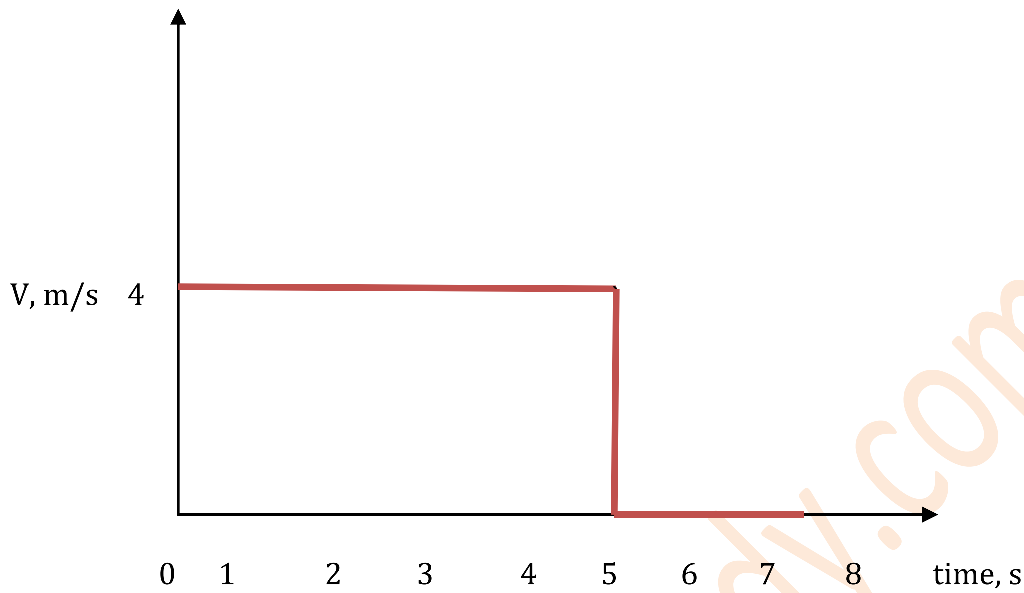
Between 6 & 7 s

V = (20-20) m/(7-6) s = 0

Between 7 & 8 s

V= (20-20) m/(8-7) s = 0

A)



B) S = length *width = 4 m/s * 5 s = 20 m

C) slope= $a = \Delta v/\Delta t$ = (4-4) m/(4-0) s = 0 m/s²

Indicates that the car is moving with constant velocity and the motion is uniform on straight line surface.

D) slope =a= $\Delta v/\Delta t$ = (0-0)/(7-5) = 0 m/s²

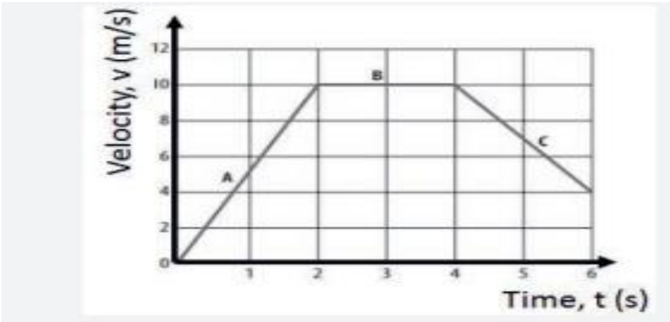
Indicates that the car is stationary or not moving

10. For the motion described by the velocity-time graph below,

A) What is the acceleration between 0 and 2 s?

B) During what time period does the object have a constant speed?

C) What is the displacement of the motion?



Solution:

A) $a = \frac{\Delta v}{\Delta t}$ = *(10-0) m/s/ 2-0) s = 5 m/s*^{2}

B) between 2 and 4 seconds

c) the displacement is the area under the velocity time graph

between 0 and 2 s, area = Δs= Δv*Δt =1/2*b*h= ½*(2-0) s*(10-0) m/s =10 m

between 2 and 4 s, area =Δs =b*h = (4-2) s*(10-10) m/s = 0 m

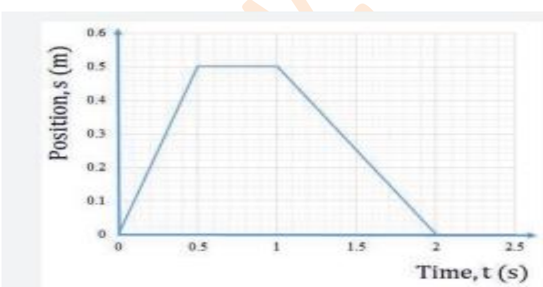
between 4 and 6 s (there are triangle and rectangle areas)

for the triangle, area=Δs = ½*b*h = ½(6-4) s*(10-4) m/s = 6 m

for the rectangle, area= Δs =b*h = (6-4) s* (4-4) m/s = 0 m

Therefore ; total displacement = 10 m +0 m+ 6 m +0 m = 16 m

11. Given the position-time graph below, find the velocity-time graph



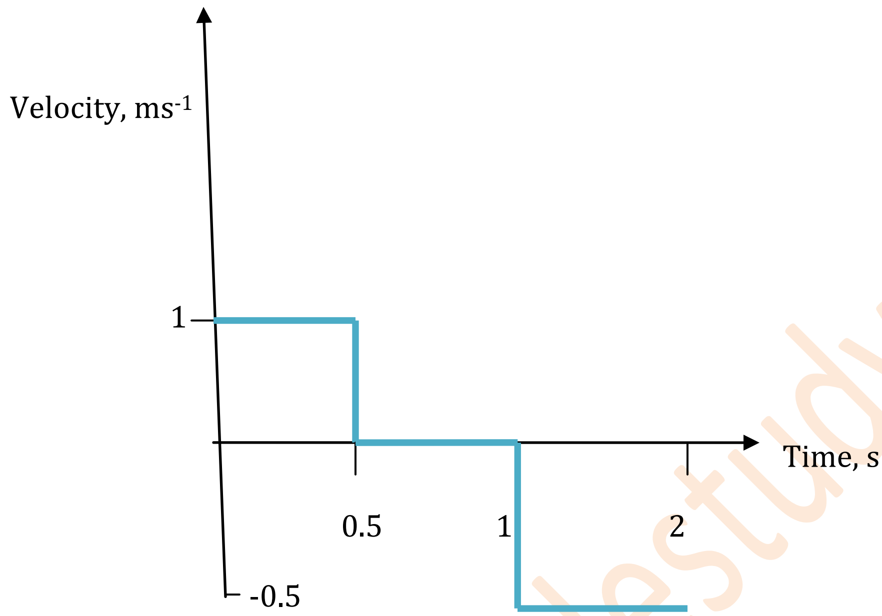
By taking the slope of the line using the grid

Between the intervals 0 & 0.5 s, Δv= Δs/Δt = (0.5-0) m/(0.5-0) s = 1 m/s

Between the intervals 0.5 & 1, $\Delta v = \Delta s/\Delta t = (0.5-0.5)\ m/(1-0.5)\ s = 0\ m/s$

Between the intervals 1& 2, $\Delta v = \Delta s/\Delta t = (0-0.5)\ m/(2-1)\ s = -0.5\ m/s$

Therefore, the graph of the given position- time graph is shown in figure below



12. A jet cruising at a speed of 1000 km/h ejects hot air in the opposite direction. If the speed of the hot air with respect to the jet is 800 km/h, then find its speed with respect to the ground.

Given: speed of the jet= $v_j = 1000\ km/h$

Let speed of hot air be $= v_h$

Relative speed of hot air with respect to the jet = $v_{hj} = -800$ km/h

Required: v_h

Solution:

Let the direction of motion of the jet be in the positive x direction so that the hot air direction is in the negative x- direction.

$-v_{hj} = -v_h - v_j = -800\ km/h = -v_h - 1000 \qquad v_h = -200\ km/h$

Note: the minus sign indicates the direction of motion of the hot air (exhaust gas)

Unit 3

Elasticity and Static Equilibrium of Rigid Body

3.1 Elasticity and plasticity

Rigid body: is a hard solid object having a definite shape and size.

Note: all bodies can be **stretched, compressed** and **bent** when a force is applied

Deforming force: is a force required to change (or deform) the shape or size of a body.

Elasticity: is a property when a body regains its original shape and size after the removal of a deforming force. The process is called elastic deformation.

Example: if you stretch a helical spring shown in Figure 3.1 by gently pulling its ends, the length of the spring increases slightly. When you leave the ends of the spring, it regains its original size and shape

Examples: rubber, metals and steel ropes.

Fig helical spring

Plasticity: is a property when a body does not regain its original shape and size after removal of the deforming force. The process is called plastic deformation

Plastic deformation: is defined as the persistent deformation or change in the shape of a solid body caused by a **sustained force**. This happens when a **great amount of tension** is applied to a material.

Note: Plastic deformation is **permanent** and **irreversible**. Plasticity is the ability to be permanently formed or molded

Elastic limit: The maximum deforming force up to which a body retains its property of elasticity

Note: Beyond the elastic limit, the solid does not regain its original shape and size.

Key concepts

- The elastic limit of ductile material is the beginning point of plastic deformation.
- The elastic limit of a solid is the utmost amount to which it may be stretched without permanently changing size or form
- If the tension is placed beyond the elastic limit, the substance will deform plastically

Elastic behavior of materials plays an important role in our day to day life. The following are some of the practical applications of elasticity.

1. The metallic parts of machinery are never subjected to a stress beyond elastic limit; otherwise they will get permanently deformed.

2. The thickness of the metallic rope used in the crane in order to lift a given load is decided from the knowledge of elastic limit of the material of the rope and the factor of safety.

3. The bridges are declared unsafe after long use because during its long use, a bridge undergoes quick alternating strains continuously. It results in the loss of elastic strength.

4. Maximum height of a mountain on earth can be estimated from the elastic behavior of earth.

3.2 Density and specific gravity

Density: is defined as the mass of an object per unit of its volume.Density: is the ratio between mass and volume

- The symbol of density is "ρ".

$$\rho = \frac{m}{v}$$

Where;

m = the mass of the substance

V = the volume of the substance

- The SI unit of density is kg /m3. Sometimes densities are given in g /cm3.
- $1kg/m^3 = 10^{-3} g/cm^3$

Factors affecting density of the substance

1. Temperature 2. Pressure 3. Size

4. Mass 5. Arrangement of atoms

Examples of density;

1. Rock sinks in the water, while wood, being less dense than the water, floats on the surface.

2. Oil is less dense than water. It rises to the surface in cases of an oil spill in the ocean creating an oil slick on the surface of the water.

Specific gravity: is defined as the ratio of the density of the given substance to that of water at 4 ^oc

Note: water is used as a reference substance at 4 ^oC because at 4 ^oC, water has the **highest density** and not at 0 ^oC or 100 ^oC.

The specific gravity (SG) of a substance can thus be calculated using the expression:

$SG = \rho_{substance} / \rho_{water}$

Note: specific gravity (SG) is a unitless quantity

The following are some of the applications of specific gravity.

1. Geologists and mineralogists use the concept of specific gravity in determining the

mineral content of the rock.

2. You apply the concept of specific gravity in comparing the purity of the newly found gem with the standard one.

3. Specific gravity helps us in urinalysis and extracting the contents information of the urine.

Table 3.1 Density and specific gravity of substance at 0 ^oC and 1 *atm*.

Material type	Material name	Density (Kg/m^3)	Relative density
Gas	Helium	0.179	1.79×10^{-4}
	Air	1.29	1.29×10^{-3}
	Carbon dioxide	1.98	1.98×10^{-3}
Liquid	Alcohol	7.9×10^2	0.79
	Gasoline	8.6×10^2	0.86
	Water (4^oC)	1×10^3	1
	Mercury	13.6×10^3	13.6
Solid	Glass (common)	$(2.4-2.8) \times 10^3$	2.5
	Aluminum	2.7×10^3	2.7
	Iron	7.86×10^3	7.86
	Copper	8.92×10^3	8.92
	Silver	10.5×10^3	10.5
	Uranium	19.07×10^3	19.07
	Gold	19.3×10^3	19.3

Note: relative density (RD) is the same as specific gravity (SG)

Note: An object made of a particular pure substance such as pure gold, can have any size or mass, but the density will be the same for each.

Example

A mining worker gets an unknown mineral with a volume of 20 cm3 and a mass of 54 g. Determine the density and the specific gravity of the mineral.

Given: volume (v) = 20 cm^3, mass (m) = 54 g

Required: density (ρ) and specific gravity (SG)

Solution:

$\rho = m/v = 54\ g/20\ cm^3 = 2.7\ g/cm^3 = 2.7\ g/cm^3 * 1\ kg/1000\ g * 10^6\ cm^3/1\ m^3 = 2700\ kg/m^3$

$SG = \rho/\rho_{water} = 2700\ kg/m^3/(1000\ kg/m^3) = 2.7$

From table 3.1 above, this unknown rock is aluminium (Al)

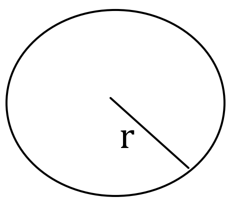
Example

What is the mass of a solid iron ball of radius 18 cm?

Given: radius of solid iron ball(r) = 18 cm = 0.18 m

From table 3.1 above, the density (ρ) of solid iron ball = $7.86*10^3$ kg/m^3

Solution: assuming the solid iron ball is perfectly spherical in shape



$$V = \frac{4}{3} * \pi r^3 = \frac{4}{3} * 3.14 * (0.18\ m)^3 = 0.024\ m^3$$

$\rho = m/v$

$m = \rho * v = 7.86*10^3\ kg/m^3 * 0.024\ m^3 = 188.64\ kg$

Review questions

1. What is the approximate mass of air in a living room of 5.6 m× 3.6 m ×2.4 m?

Given: from table 3.1, $\rho_{air} = 1.29\ kg/m^3$, v = (5.6 m* 3.6*2.4 m)

Required: mass of air

Solution;

$m = \rho * v$

v = (5.6 m* 3.6*2.4 m) = 48.4 m^3

$m = \rho * v = 1.29\ kg/m^3 * 48.4\ m^3 = 62.4\ kg$

2. You have a sample of granite with density 2.8 g /cm^3. The density of water is 1.0 g/cm^3.

What is the specific gravity of your granite?

Given: $\rho_{substance} = 2.8\ g/cm^3$, $\rho_{water} = 1\ g/cm^3$, required: specific gravity (SG)

Solution:
$SG = \rho/\ \rho_{water} = 2.8\ g/cm^3/(\ 1\ g/cm^3) = 2.8$

3. Calculate the average density and specific gravity of the Earth given that the mass and radius of the Earth are $m_E = 5.98\times10^{24}$ kg and $r_E = 6.37\times106$ m, respectively.

Given: radius of earth=$r_E = 6.37 * 10^6$ m

Mass of earth = $m_E = 5.98*10^{24}$ kg

Required: ρ_E and SG

Solution: assuming earth is perfectly spherical in shape

$v_E = \frac{4}{3} * \pi r^3 = \frac{4}{3} * 3.14 * (6.37 * 10^6\ m)^3 = 1{,}082.15 * 10^{18}\ m^3$

$\rho_E = m/v = 5.98*10^{24}\ kg/(1{,}082.15 * 10^{18}\ m^3) = 5{,}526\ kg/m^3 = 5.53 * 10^3\ kg/m^3$

$SG = \rho_E/\ \rho_{water} = 5.53 * 10^3\ kg/m^3/(1*10^3\ kg/m^3) = 5.53$

3.3 Stress and Strain

Stress

Stress: is a quantity that describes the magnitude of forces that cause deformation. If the magnitude of deforming force is F and it acts on area A, stress is generally defined as force per unit area.

- Stress is the external force acting on an object per unit cross-sectional area.

Stress = $\frac{F}{A}$

- The SI unit of stress is N/m^2

Tensile stress: is a pulling force that causes an elongation or stretching of an object.

Compressive stress: is a pushing force that causes an object to shrink or compress.

Bulk stress (volume stress): is a force that causes a squeeze on an object from all sides. Example: a submarine in the depths of an ocean.

Strain: Strain is a dimensionless quantity that gives the amount of deformation of an object or medium under stress. Strain is given as a fractional change of length or volume

- Strain under a tensile stress is called tensile strain
- Strain under bulk stress is called bulk strain (or volume strain)
- Strain under shear stress is called shear strain.

Their equations are given as follows

1. **Tensile/Linear strain**: If on application of a longitudinal deforming force, the length L_o of a body changes by ΔL as in Figure 3.5, then

Tensile (or compressive) strain = $\Delta L / L_0$

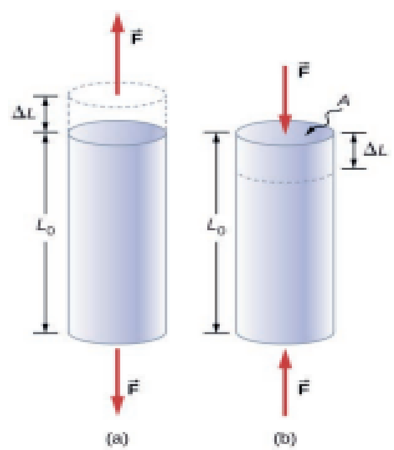


Figure an object under (a) tensile stress (b) compressive stress.

3. **Volumetric strain**: If on application of the deforming force, the volume Vo of the body changes by ΔV without change of shape of the body as in Figure 3.6,

Volumetric strain = $\Delta V / V_0$

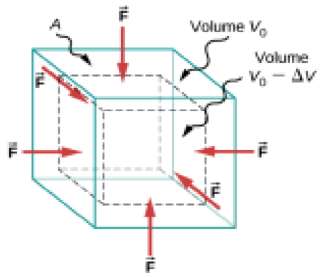


Figure an object under volume strain.

4. **Shearing strain**: When the deforming forces are tangential, the shearing strain is given by the angle through which a line perpendicular to the fixed plane is turned due to deformation.

Shearing strain = $\Delta x / L_0$

Hooke's law : within elastic limit, stress is directly proportional to corresponding strain.

Stress $\propto$ Strain

$\Rightarrow$ Stress = k ×Strain

K (modulus of elasticity) = constant of proportionality

- It is a measure of elasticity of the substance and is called modulus of elasticity.
- has the same dimension (or units) as stress.
- Its value is independent of the stress and strain but depends on the nature of the material

Note:

- When elastic modulus is large, the effect of stress is small or the stress produces small strain (deformation).
- When elastic modulus is small, the stress produces large strain (deformation).

For example, for the same stress and dimensions:

A stress on a rubber band produces larger strain (deformation) than on steel

- The elastic modulus for tensile stress is called the Young modulus;

- The elastic modulus for bulk stress is called the bulk modulus; and
- Elastic modulus for shear stress is called the shear modulus.

Example

Find the tensile stress when a force of 9.8 N acts over a cross-sectional area of 2×10−3 m^2.

Given: force= 9.8 N, Area = $2*10^{-3}$ m^2, required: tensile stress

Solution:

Stress = F/A = 9.8 N/ (2 $*10^{-3}$ m^2) = $4.9*10^3$ N/m^2

Example

When a weight of 98 N is suspended from wire of length 3 m and diameter 0.4 mm, its length increases by 2.4 cm. Calculate tensile stress and tensile strain.

Given: F_W = 98 N , L_0 = 3 m= 300 cm , diameter = 0.4 mm

ΔL = 2.4 cm, required: tensile stress and strain.

Solution: D= 0.4 mm= 2r

r= 0.2 mm

A = πr^2 = (3.14)* $(2*10^{-4}\ m)^2$ =1.256* 10^{-7} m^2

Tensile stress = F_W/A = mg/A =98 N/(1.256* 10^{-7} m^2) =$7.8*10^8$ N/m^2

tensile strain = $\Delta L/L_0$ =2.4 cm/300 cm = 0.008

The amount of elongation of the wire due to the suspended load is 0.008.

Review questions

2. Can compressive stress be applied to a rubber band?

Yes, because compressive stress squeezes the rubber at its ends towards each other to shorten its length

3. A nylon string that has a diameter of 2 mm is pulled by a force of 100 N. Calculate the tensile stress.

Given: diameter=D= 2 mm, force=F= 100 N, required: tensile stress

D=2r =2 mm, r = 1 mm =

Solution: A= πr^2 = 3.14* $(1*10^{-3}$ m$)^2$ = 3.14 * 10^{-6} m^2

stress= F/A =100 N/(3.14 * 10^{-6} m^2) = 3.185 *10^7 N/m^2

4. A load of 2.0 kg is applied to the ends of a wire 4.0 m long, and produces an extension of 0.24 mm. If the diameter of the wire is 2.0 mm, find the stress on the wire and the strain it produces

Given: m=2 kg , L_0 =4 m = 4000 mm, ΔL =0.24 mm, diameter = D= 2 mm

Required: tensile stress and strain

Solution:

Diameter= 2r = 2 mm

R= 1 mm

F_W = mg = 2 kg*9.8 m/s^2 = 19.6 N

A= πr^2 = 3.14*$(1*10^{-3}$ m$)^2$ =3.14 *10^{-6} m^2

Stress = F_W/A = 19.6 N/ (3.14*10^{-6}) = 6.24*10^6 N/m^2

Strain = ΔL/ L_0 = 0.24 mm/ 4000 mm = 6*10^{-5}

3.4 The Young Modulus

The Young modulus is the elastic modulus when deformation is caused by either tensile or compressive stress

Young modulus: is the ratio of tensile (or compressive) stress to the longitudinal.

- it is denoted by the symbol Y.

$$\text{Young modulus(Y)} = \frac{tensile\ stress}{tensile\ strain}$$

Note: Since strain is a dimensionless quantity, the unit of the Young modulus is the same as that of stress i.e., N/m^2 or Pascal (Pa).

Note: The relation between the tensile stress and the tensile strain is linear when the rod is in its elastic range.

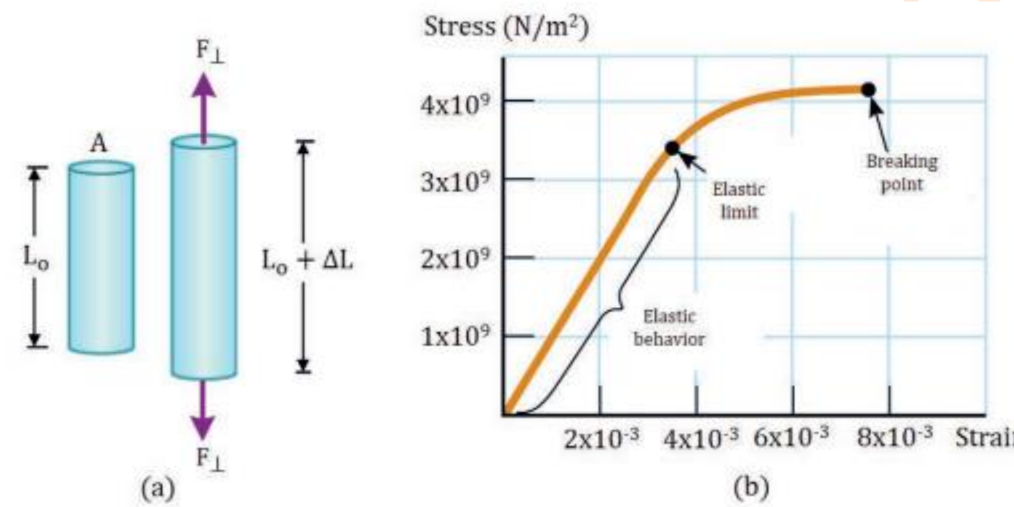


Figure 3.8 (a). A rod of length L_0 can be stretched by an amount ΔL after application of a tensile stress $F_\perp$. (b) The stress versus strain diagram for a ductile material.

Young modulus(Y) = $F/A/(\Delta L/L_0) = (F^* L_0)/(A^*\Delta L)$

If the wire of radius r is suspended vertically with a rigid support and a mass m hangs at its lower end, then $A = \pi \times r^2$ and $F = mg$.

$$Y = \frac{mg * L0}{\pi r2 * \Delta L}$$

Table 3.2 The Young modulus of different substance in N/m^2

Substance	Young modulus (N/m^2)
Tungsten	35×10^{10}
Steel	20×10^{10}
Copper	11×10^{10}
Brass	9.1×10^{10}
Aluminum	7.0×10^{10}
Glass	$6.5 - 7.8 \times 10^{10}$
Quartz	5.6×10^{10}
Water	-
Mercury	-

Note:

- Young moduli are large for metals. Therefore, these materials require a large force to produce a small change in their length.
- The value of Young modulus is maximum for steel and thus steel is more elastic in comparison to other materials mentioned in the table.

Exercise

A copper wire is 1.0 m long and its diameter is 1.0 mm. If the wire hangs vertically, how much weight must be added to its free end in order to stretch it 3.0 mm?

Given: L_0= 1 m , D = 1 mm , $Y_{Copper} = 11*10^{10}\ N/m^2$, ΔL =3 mm, required: weight

Solution: $Y = (F*L_0)/(A*\Delta L)$

D=1mm= 2r

$r = 0.5\ mm = 5*10^{-4}\ m$

$A = \pi r^2 = 3.14*(5*10^{-4}\ m)^2 = 7.85*10^{-7}\ m^2$

$Y = (F*L_0)/(A*\Delta L)$

$F = (A*\Delta L*Y)/(L_0) = (7.85*10^{-7}\ m^2*3*10^{-3}\ m*11*10^{10}\ N/m^2)/(1\ m)$

F = weight = 259 N

Example: For the same cross sectional area of 0.1 cm^2, to produce the same strain (deformation) by 0.1 percent

Steel requires a force (stress) = 2000 N

Copper requires a force (stress) = 1100 N

Brass requires a force (stress) = 900 N

Aluminium requires a force (stress) = 690 N

The above condition shows that steel is more elastic than copper, brass and aluminium. It is for this reason that steel is preferred in heavy-duty machines and in structural designs.

Example

A 1.60 m long steel piano wire has a diameter of 0.20 cm. How great is the tension in the wire if it stretches 0.25 cm when tightened?

Given: original length (L) = 1.6 m , change in length(ΔL) = 0.25 cm

Diameter (D) = 0.2 cm , from table 3.2, $Y_{Steel} = 20*10^{10}\ N/m^2$

Required: tension (force) =?

Solution:

D = 0.2 cm=2r

r= 0.1 cm

$A = \pi r^2 = (3.14)*(1*10^{-3}\ m)^2 = 3.14*10^{-6}\ m^2$

$Y = (F * L)/(\pi r^2 * \Delta L)$

$F = (Y*\Delta L* \pi r^2)/(L) = (2*10^{11}\ N/m^2 *0.0025\ m*3.14*10^{-6}\ m^2)/(1.6\ m) = 981.25\ N$

Example

A pendulum consists of a big sphere of mass m = 30 kg hung from the end of a steel wire that has a length of 15 m, a cross-sectional area of $9\times10{-}6\ m^2$, and the Young modulus of $200\times10^9\ N/m^2$. Find the tensile stress on the wire and the increase in its length.

Given: m = 30 kg , L_0= *15 cm , A =* $9*10^{-6}\ m^2$

$Y = 200\times10^9\ N/m^2$, gravitational acceleration (g) = $9.8\ m/s^2$

Required: 1. tensile stress, 2. ΔL

Solution :

Tensile stress = F/A

F = mg = 30 kg * 9.8 m/s^2 = 294 kgm/s^2 = 294 N

Tensile stress = 294 kgm/s^2/(9*10^{-6} m^2) = 3.2710^7 N/m^2

Y = tensile stress/ strain= F/A/ (ΔL/L) = F*L/($A * $ΔL)

ΔL = (tensile stress*L_o)/(*Y) = (3.2710^7 N/m^2*$15\ m$)/(2×10^{11} N/m^2) = 2.45*10^{-3} m

ΔL =2.45 mm

Review questions

1. In the Young experiment, if the length of the wire and the radius are both doubled, what will happen to the value of the Young modulus?

Given: r_1= initial radius, L_1 =initial length, and r_2 = second radius, L_2 = second length

Solution: let us calculate two young modulus values using the above conditions

Y_1 = stress/ strain = (F/A)/ (ΔL/L) = (F*L_1)/(π*r_1^2 ΔL)……………..1

Y_2 =(F*L_2)/(πr_2^2 *ΔL)

When L_2 = 2L_1 and r_2 = 2r_1

Y_2=(F*(2L_1))/(π(2r_1^2)*ΔL) = 2/4[(F*L_1)/(πr_1^2 *ΔL)]

Y_2 = ½(Y1)

Therefore; the value of young modulus will be half

2. A wire increases by 10^{-3} of its length when a stress of 1×10^8 N/m^2 is applied to it. Calculate the Young modulus of material of the wire.

Given: ΔL =1*10^{-3}L_0 ,   tensile stress = 1 × 108 N/m^{-2} , required: Y

Y= tensile stress/ strain = $(1 \times 10^8\ N/m^2 *L_0)/(1*10^{-3}L_0) = 1*10^{11}\ N/m^2$

3. A wire is stretched by 0.01 m by a certain force F. Another wire of same material whose diameter and length are double to the original wire is stretched by the same force. What will be its elongation?

   Given: ΔL_1= 0.01 m, relation: $L_{02}= 2L_{01}$, $r_2 =2r_1$, *required:* ΔL_2

   Solution:

   $Y_1 = (F*L_{01})/(A_1 * \Delta L_1) = (F*L_{01})/(\pi r_1^2 * \Delta L_1)$

   $Y_2 = (F*L_{02})/(\pi r_2^2 * \Delta L_2)$, $F_1=F_2$, $Y_1=Y_2$,

   Substitute $L_{02}= 2L_{01}$ and $r_2 =2r_1$

$Y_2 = (F*2L_{01})/(\pi(2r_1^2) * \Delta L_2) = (F*2L_{01})/4\pi(r_1^2*\Delta L_2) =(F*L_{01})/2\pi(r_1^2*\Delta L_2)$

$Y_2=Y_1 =(F*L_{01})/2\pi(r_1^2*\Delta L_2) = (F*L_{01})/(\pi r_1^2 * \Delta L_1)$

$Y_2/Y_1 = (F*L_{01})/2\pi(r_1^2*\Delta L_2)/(F*L_{01})/(\pi r_1^2 * \Delta L_1)$

$1= (\Delta L_1)/(2\Delta L_2)$

$\Delta L_2 =1/2*\Delta L_1$

$(F*L_{01})/2\pi(r_1^2*\Delta L_2) = (F*L_{01})/(A_1 * \Delta L_1) = (F*L_{01})/(\pi r_1^2 * \Delta L_1)$

4. A 200 kg load is hung on a wire having a length of 4.0 m, cross-sectional area 0.20×10^{-5} m^2, and the Young modulus 8.00×1010 N/m^2. What is its increase in length?

Given: mass= 200 kg, length = 4 m, $A = 0.2*10^{-5}\ m^2$

$Y = 8 * 10^{10}\ N/m^2$,$= 8*10^{10}\ kg/ms^2$, required: ΔL

Y = stress/ strain

$Y= F*L/A*\Delta L$

$\Delta L = (F*L)/(A*Y)$

$F = mg = 200\ kg * 9.8\ m/s^2 = 1960\ kgm/s^2$

$\Delta L = (1960\ kgm/s^2 * 4\ m)/(0.2*10^{-5}\ m^2 * 8*10^{10}\ kg/ms^2)$

$\Delta L = (7840\ m)/(1.6*10^5) = 4.9*10^{-2}\ m$

3.5 Static equilibrium

Static equilibrium occurs when an object or a system remains at rest and does not tilt nor rotate.

Static means that the body is not in motion,

Equilibrium indicates that all opposing forces are balanced.

Thus, a system is in static equilibrium if it is at rest and all forces and other factors influencing the object are balanced.

Two conditions need to be satisfied for a system to be in static equilibrium: the first condition of equilibrium and the second condition of equilibrium. Firstly, the net force acting upon the object must be zero. Secondly, the net torque acting upon the object must also be zero. In other words, both static translational and static rotational equilibrium conditions must be satisfied.

3.5.1 First condition of equilibrium

The first condition of equilibrium states that for an object to remain in equilibrium, the net force acting upon it in all directions must be zero.

- Simply put, the above statement means that the body must not be experiencing acceleration.

Since force is a vector, both the magnitude and the direction of its components should also be considered. For example, a negative sign should be used if an object moves in the opposite direction. Therefore, for static equilibrium to be reached, the condition of static equilibrium should be fulfilled such that the component of the forces in all dimensions should be equal to zero. Thus,

• For one dimensional forces applied on an object (for instance, if the forces are along the x-axis), the first condition of equilibrium is given by:

$\sum F = 0$

✓ For two dimensional forces applied on an object, the first condition of equilibrium is given by:

$F_{net} = 0$,

1. Summation of forces in the y- direction should be zero

$\sum F = 0$

2. Summation of forces in the x- direction should be zero

$\sum F = 0$

✓ The above condition is true for both static equilibrium, where the object's velocity is zero, and dynamic equilibrium, where the object moves at a constant velocity

3.5.2 Second condition of equilibrium

The second condition of equilibrium states that for an object to remain in equilibrium the net external torque on an object must be zero.

Torque: is the twisting force or the amount of force that causes an object to rotate when it is applied to a certain distance from the axis of rotation

Factors affecting torque are:

✓ The force
✓ the location of the force relative to the pivot point (radius)

The magnitude of torque is thus given by the equation

$\tau = Fr \sin \theta$

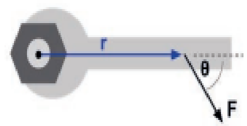


Figure 3.9 A force applied on one side of a nut at an angle makes it to rotate.

where

τ = the symbol for torque in Nm,

F= the magnitude of the force in N

r =the distance from the pivot point to the point where the force is applied in m

θ = the angle between the force and the vector directed from the point of application to the pivot point.

Note: If r is perpendicular to F

$\theta = 90^o$, $\sin\theta = \sin 90^o = 1$

the magnitude of the torque can be obtained by:

$\tau = F\, r$

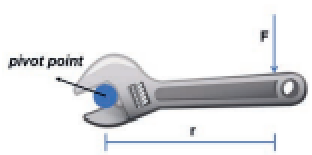


Figure 3.10 A force applied perpendicularly on one side of a nut makes it to rotate.

Rotational equilibrium, the second condition for static equilibrium, ensures that either there is no torque acting on the body or that the torques present are balanced and sum up to zero.

$\sum \tau = \tau_{net} = 0$

It means that the clockwise torque acting on the object is also equal to the counter clockwise torque. This is rotational equilibrium, which is the second condition for static equilibrium.

Static Equilibrium Examples

1. A book placed on top of a table

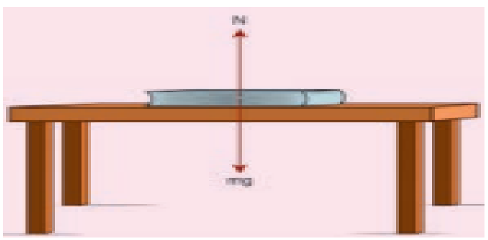


Figure 3.11 A book at rest on the top of a table.

2. A seesaw balanced by two children

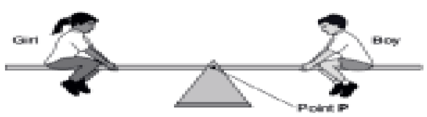


Figure 3.12 A balanced seesaw in static equilibrium.

The following general procedure is recommended for solving problems that involve objects in equilibrium

1. Draw free body diagram by showing all the forces acting on that object, including gravity and the points at which these forces act. If you aren't sure of the direction of a force, choose a direction; if the actual direction of the force (or component of a force) is opposite, your eventual calculation will give a result with a minus sign
2. Choose a convenient coordinate system, and resolve the forces into their components using:

   $F_x = F\cos\theta$

   $F_y = F\sin\theta$

where θ is given in an anticlockwise direction from the positive x-axis.

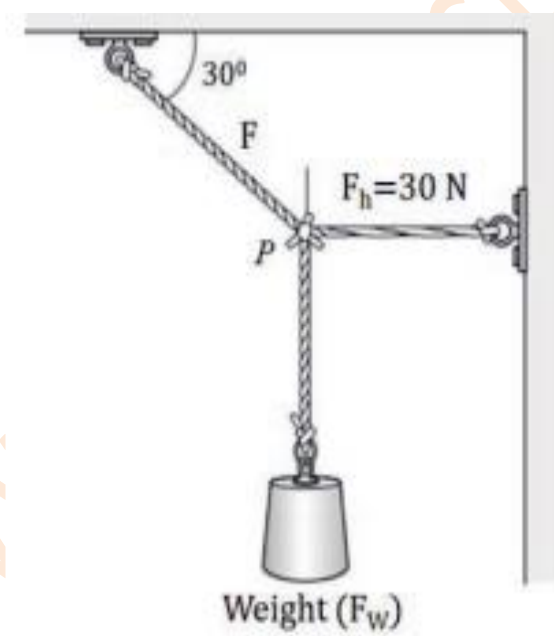
3. Using letters to represent unknowns, write down the equilibrium equations for the forces: and assuming the entire forces act in a plane.
   $\sum fx = 0$
   $\sum fy = 0$

4. For the torque equation, $\sum \tau = 0$

5. Solve these equations for the unknowns. Three equations allow a maximum of three unknowns to be solved for. They can be forces, distances, or even angles

Example

An object shown in Figure below is in static equilibrium. The horizontal cord has a force of 30 N. Find the force F of the cord and weight F_W of the object.

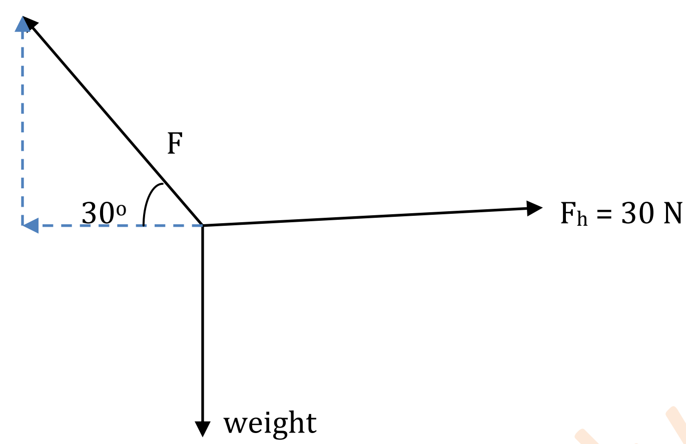


Given: F_h = 30 N and diagram

Required: F and F_W

Solution: draw the free body diagram showing all forces acting on the object representing their direction

A. Free body diagram



B. Resolving vectors in to their components: the only force that has components is F. Hence,

$F_x = F\cos\theta$

$F_X = F\cos 30^o$

$F_x = 0.866F$

$F_y = F\sin\theta$

$F_y = F\sin 30^o$

$F_y = 0.5F$

$F = 30\ N/.866 = 34.64\ N$

$F_y = F\sin\theta$

C. By applying the first condition of equilibrium,

$\sum fx = 0 = 30\ N - F\cos 30^o = 0$

$30N - 0.866F = 00.866F = 30N$

$F = 34.64N$

$\sum fy = 0 = F_w - F\sin 30^o = 0$

$F_w - 0.5F = 0$

$F_W = 0.5F$

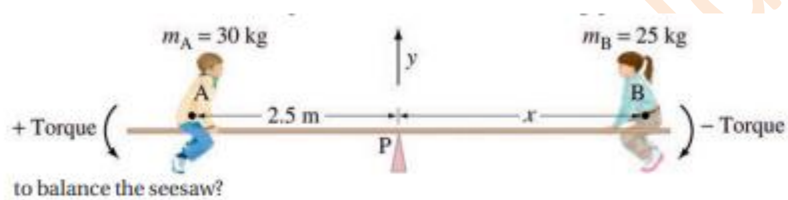
Substitute 34.64 N in F

$F_W = 0.5*34.64$ N

FW = 17.32 N

Example

A uniform board of mass 'M' serves as a seesaw for two children as shown in Figure below. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P. At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw.

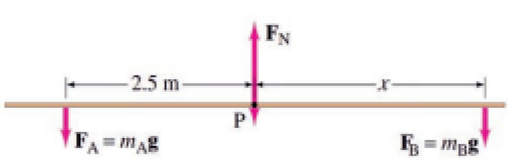


Solution: given $m_A = 30$ kg , $m_B = 25$ kg , and $x_A = 2.5$ m.

Required: $x_B = x$

You can easily solve this problem using the above stated steps.

(a) Draw free body diagram. The forces acting on the board are the forces exerted downward on it by each child, and the upward force exerted by the pivot and the force of gravity on the board which acts at the center of the uniform board. This is indicated in the Figure shown below.



(b) Coordinate system. You choose y to be vertical, with positive upward, and x horizontal to the right, with origin at the pivot.

(c) Force equation. All the forces are in the y-(vertical) direction. So,

$\sum fx = 0$

$\sum fy = 0 = FN\text{-}mAg - m_Bg\text{-}mg$

30 kg*9.8 m/s^2 $-F_N$ -25 kg *9.8 m/s^2 =0

F_N= 49 N

(D) Torque equation: Let us calculate the torque about an axis through the board at the pivot point, P. Then, the lever arms for the weight of the board are zero, and they will contribute zero torque about point P. Thus, the torque equation will involve only the forces FA and FB which are equal to the weights of the children. The torque exerted by each child will be mg times the appropriate lever arm, which is the distance of each child from the pivot point. FA tends to rotate the board counterclockwise (+) and FB clockwise (−) so the torque equation is

$\sum \tau = \tau_{net} = 0$

$F_A*X_A - F_N*(0) - F_B*X_B$

$F_A = m_A*g$, $F_B = m_B*g$

$= m_A*g\ (2.5\ m)\text{-}\ m_B*g(x_B)=0$

$m_A*g\ (2.5\ m)= m_B*g(x_B)$

$x_B = \frac{mA}{mB} * 2.5\ m = \frac{30\ kg}{25\ kg} * 2.5\ m = 3\ m$

Thus, to balance the seesaw, child B must sit 3.0 m from the pivot point.

End of unit questions and problems

3. You have a rock with a volume of 15 cm^3 and a mass of 45 g. What is its density?

Given: v= 15 cm^3 , m= 45 g required: density = ?

Density = m/v = 45 g/15 cm^3 = 3 g/m^3

4. A bar measures 12 mm x 20 mm x 1 m. It has a specific gravity of 2.78. Determine its mass

   Given: volume = $12*10^{-3}$ m*$2*10^{-2}$ m *1 m= $2.4*10^{-4}$ m^3

   Specific gravity(SG) =2.78, required: mass(m)

   Solution:

   Specific gravity = density of substance (bar)/density of water

   2.78 = density of substance (bar)/1000 kg/m^3

   Density of substance (bar) = 2780 kg/m^3

   Density of substance= m/v

   2780 kg/m^3 = m/($2.4*10^{-4}$ m^3)

   M=0.67 kg

6. A wire is stretched 3 mm by a force of 150 N. Assuming the elastic limit is not exceeded; calculate the force that will stretch the wire 5 mm

Given: ΔL_1 = 3 mm, F_1 = 150 N, ΔL_2 = 5 mm, required= F_2

SOLUTION:

Since the material is the same, the young modulus, length and cross sectional area are constant.

$Y_1 = Y_2$

$F_1L/(A \Delta L_1)= F_2L/(A \Delta L_2)$ ....MULTIPLY BOTH SIDES BY (A ΔL_2/(L)

$F_2 = (F_1 \Delta L_2)/ (\Delta L_1)$ = (150 N * 5 mm)/(3 mm) = 250 N

7. A circular rod of cross-sectional area 100mm^2 has a tensile force of 100 kN applied to it. Calculate the value for the stress in the rod.

Given: A= 100 mm^2 =$1*10^{-4}$ m^2, tensile force = 100 kN , required: stress

Stress = F/A = 100 kN/10^{-4} m^2 =$1*10^9$ N/m^2 =1Gp*a*

8. A steel wire of length 6 m and diameter 0.6 mm is extended by a force of 60 N. The wire extends by 3 mm. Calculate:

a) The applied stress.

Diameter =2r= 0.6 mm

r= 0.3 mm $=3*10^{-4}$ m

A= $\pi r^2 = 3.14*(3*10^{-4})^2 = 28.3*10^{-8}\ m^2$

Stress = F/A = 60 N/($28.3*10^{-8}\ m^2$) = $2.12 *10^8\ N/m^2$

b) the strain on the wire.

Strain= $\Delta L/L_0$ = 3 mm/ 6000 mm = $5*10^{-4}$

c) the Young Modulus of the steel.

Y = stress/strain = $2.12 *10^8\ N/m^2/(5*10^{-4}) = 4.24*10^{11}\ N/m^2$

9. A wire increases by 10^{-3} of its length when a stress of $1x10^8\ Nm^{-2}$ is applied to it. Calculate the Young modulus of the wire.

Given: $\Delta L = 10^{-3}L_0$, tensile stress = $1x10^8\ Nm^{-2}$

Y= tensile stress/$\Delta L/L$ = (tensile stress*L)/(ΔL) = $1x10^8\ Nm^{-2}$*L/(0.001L)= $1x10^{11}\ Nm^{-2}$

10. Two wires are made of the same metal. The length of the first wire is half that of the second and its diameter is double that of the second wire. If equal loads are applied on both wires, find the ratio of the increase in their lengths?

Given: $L_1 = 0.5L_2$, $D_1 = 2D_2$, $F_1 = F_2$, required: $\Delta L_2/\Delta L_1$

Solution: since the material is the same their youngs modulus values is the same also.

$Y_1 = Y_2$

$Y_1 = F_1L_1/(\Delta L_1A_1)$

$Y_2 = F_2L_2/(A_2\ \Delta L_2)$

Take the ratio of the youngs modulus (Y_1/Y_2)

$Y_1/Y_2 = (F_1L_1A_2\Delta L_2/(\Delta L_1A_1F_2L_2)=1$

$(F_1/F_2)*(L_1/L_2)*(\Delta L_2/\Delta L_1)*(A_2/A_1)=1$

But, $L_1 = 0.5L_2$, $D_1 = 2D_2$, $F_1 = F_2$

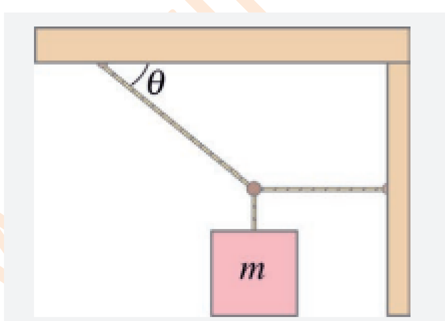
$1*(L_1/2L_1)*(\Delta L_2/\Delta L_1)*(\pi(D_2/2)^2)/(\pi(2D_2^2/2)^2)=1$

$1/8*(\Delta L_2/\Delta L_1)=1$

$(\Delta L_2/\Delta L_1) = 8$

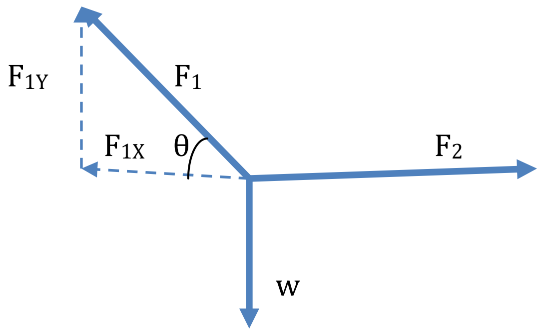
$\Delta L_2 = 8\Delta L_1$

11. Find the tension in the two cords shown in the Figure below. Neglect the mass of the cords, and assume that the angle is 33^o and the mass m is 190 kg



Solution:

1. Draw free the body diagram



2. Force resolving

$F_{1X} = F_1\cos\theta = F_1\cos33^o = 0.84F_1$

$F_{1Y} = F_1\sin\theta = F_1\sin33^o = 0.545F_1$

Apply First condition of equilibrium

$\sum fx = 0$
$F_2 - 0.84F_1 = 0$

$\sum fy = 0$

$0.54F_1 - mg = 0$

$0.54F_1 - 190\ kg*9.8\ m/s^2 = 0$

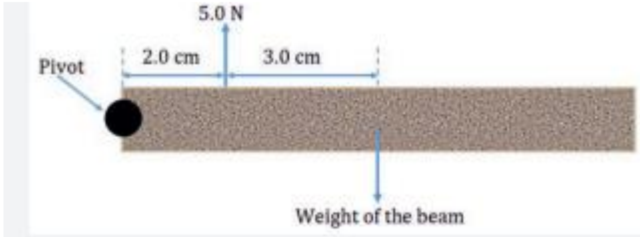
$0.54F_1 = 1862\ N$

$F_1 = 3{,}448\ N$

$F_2 - 0.84F_1 = 0$

$F_2 = 0.84*3{,}448\ N = 2896\ N$

12. A beam pivoted at one end has a force of 5.0 N acting vertically upwards on it as shown in the Figure below. What is the weight of the beam?



Solution: apply second condition of equilibrium

$\tau net = 0$

$\sum \tau = \tau_{net} = 0$

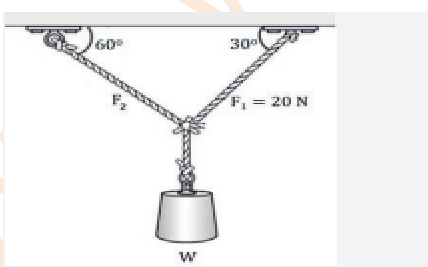
$\tau 1 = \tau 2$

$(F)(r_1) = (W)(r_2)$

$(5N)(2cm) = (W)(5cm)$

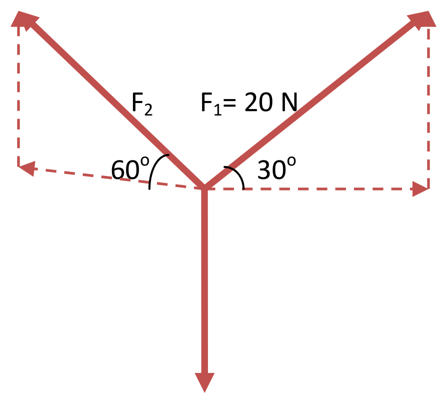
$W = 2N$

13. A force of 20 N at angle of 30 ^{o} to the horizontal and a force F_2 at an angle of 60^o to the horizontal are applied on an object as shown in the Figure below so as to make the object in equilibrium. Calculate the magnitude of the force F_2 and weight of the object.



Solution:

A) Draw the free body diagram



B) Force resolving

$F_{1X} = F_1 cos\theta = F_1 cos30^o = 20\ N*0.86 = 17.32\ N$

$F_{1Y} = F_1 sin\theta = 20\ N*sin30^o = 20\ N*\ 0.5 = 10\ N$

$F_2x = F_2 cos\theta = F_2 cos60^o = F_2*0.5 = 0.5F_2$

$F_{2Y} = F_2 sin\theta = F_2 sin60^o = 0.866F_2$

$\sum fx = 0$

$F_{1X} - F_2x = 0$

$17.32\ N - 0.5F_2 = 0$

$0.5F_2 = 17.32\ N$

$F_2 = 34.64\ N$

$F_{2Y} = 0.866F_2 = 0.866*34.64\ N = 30\ N$

$\sum fy = 0$

$F_{1Y} + F_{2Y} - W = 0$

$W = F_{1Y} + F_{2Y} = 10\ N + 30\ N = 40$

Unit 4

Static and Current Electricity

Physical phenomenon associated with the presence and flow of electric charge is known as electricity

4.1 Charges in Nature

Objects surrounding us (including people) contain large amounts of electric charge. There are two types of electric charge: positive charge and negative charge. Protons have a positive charge, and electrons have a negative charge.

Unit of Charge

- ✓ The SI unit of electric charge is Coulomb (C).
- ✓ One coulomb (1 C) of charge is carried by 6.25×10^{18} electrons.
- ✓ An electron possesses a negative charge of 1.6×10^{-19} C. In electrostatics, you often work with charge in micro Coulombs ($1\mu C = 1\times10^{-6}$ C) and nano coulombs ($1nC = 1\times10^{-9}$ C).

Conservation of Charge

The law of conservation of charge: During electrification, electric charges are neither created nor destroyed, but are transferred from one material to another.

Example: charge transfer between comb and hair

Quantization of charge:

- ✓ The smallest charge that is possible to obtain is that of an electron or proton. The magnitude of this charge is denoted by *e*.
- ✓ Charge is said to be quantized when it occurs as the integral multiples of *e*. This is true for both negative and positive charges

Charge is expressed as;

q = ne

Where n is positive or negative integer

Exercise

A conductor possesses a positive charge of 3.2×10^{-19} C. How many electrons does it have in excess or deficit (use: $e = 1.60 \times 10^{-19}$ C)?

Solution

q = ne

3.2×10^{-19} C =n*1.60×10^{-19} C

n = 3.2×10^{-19} C/1.60×10^{-19} C

n= 2e

4.2 Methods of Charging a Body

There are three (3) methods of charging a body:

i. **Charging by rubbing**: occurs when two different neutral materials are rubbed together and electric charges are transferred from one object to the other

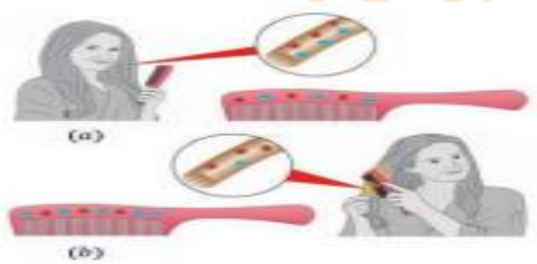


Figure 4.1 (a) The comb and the hair are both neutral. (b) After being rubbed together, the comb is negatively charged and the hair is positively charged.

ii. **Charging by conduction**: Charging by conduction occurs when a charged object makes contact with a neutral object.

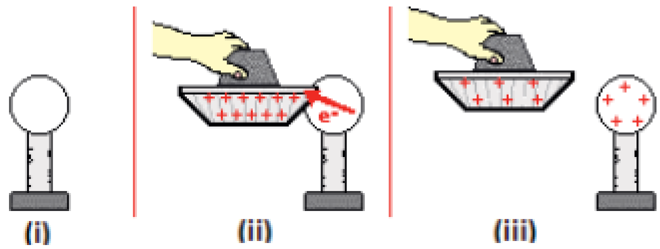


Figure 4.2 Charging by conduction.

Note: Charging by conduction leaves the charged body and the uncharged body with the **same sign of charge** but a weaker strength than the original object

iii .**Charging by induction**: Charging by induction is a process where the charged object is held near to an uncharged conductive material that is grounded on a neutrally charged material.

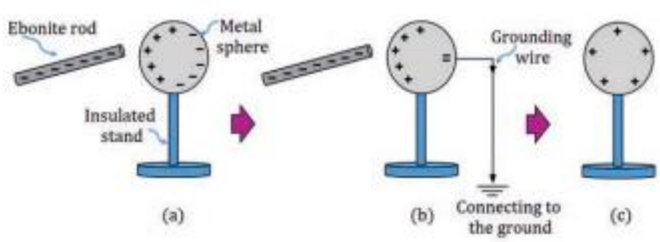


Figure 4.3 Charging by induction.

Note: charging by induction leaves the charged body and the uncharged body with the opposite sign of charge

4.3 The electroscope: is a very sensitive instrument which can be used to:

- ✓ detect the type of electric charge,
- ✓ to identify whether an object is charged or not,
- ✓ to measure the quantity of charge
- ✓ to know whether an object is conductor or insulator.

4.4 Electrical Discharge

Lightning: is a very large electrical discharge caused by induction.

Process steps of lightning

1. .In a thunderstorm, a **charged area**, usually negative, builds up at the base of a cloud.

2. When enough charge has built up, **a path of charged particles is formed**.

3. The cloud then discharges its excess electrons along the path to the ground, creating a huge spark

4. This discharge also causes a rapid expansion of the air around it, causing thunder.

Note: Air is normally an insulator. If it were not, lightning would occur every time that cloud formed. For lightning to happen, charges in the clouds must build up to the point where the air cannot keep them separated from the ground. Then, the air stops being an insulator and becomes a fair conductor, resulting in a lightning strike.

- The process of providing a pathway to drain excess charge into the earth is called **grounding.**

4.5 Coulomb's law of electrostatics

Charles Coulomb: a French scientist who studied about **electrostatic force**

Coulomb's law: states that the electrostatic force between the two charges is proportional to the product of the charges and is inversely proportional to the square of their distance apartCoulomb's law can be stated in mathematical terms as

$F \propto |q1*q2|$, $F \propto 1/ r^2$

$F \propto |q_1*q_2| / r 2$

where F is the magnitude of the electric force between the two charges q_1 and q_2, and r is the distance between the two charges

$F = K * (q_1*q_2)/r^2$

$k = \frac{1}{4\pi\varepsilon} \approx 9\times10^9\ Nm^2/C^2$ is the electrostatic constant;

$\varepsilon_o = 8.85\times10^{-12}\ C^2/Nm^2$ is called permittivity of free space.

- The SI unit of force is the Newton.
- The electrostatic force is directed along the line joining the charges,
- it is attractive if the charges have unlike signs
- repulsive if the charges have like signs

Example Charges $q_1 = 5.0\mu C$ and $q_2 = -12.0\mu C$ are separated by 30 cm on the x-axis. What is the magnitude of the force exerted by the two charges?

Solution: given $q_1 = 5.0\mu C$, $q_2 = -12.0\mu C$ and r = 30 cm=0.3 m. required: magnitude of the force F

Using Coulomb's law,

$F = K * (q_1*q_2)/r^2 = 9\times10^9\ Nm^2/C^2 * (5*10^{-6}\ C) * (-12*10^{-6}\ C)/(0.3\ m)^2$

F= 6 N

Since the two charges are of opposite sign, the force between the charges is an attractive force.

Review questions

1. Two charges 1 C and - 4 C exist in the air. What is the direction of the force between them?

Answer: the two charges attract each other.

2. Two charges $q_1 = 2\times10^{-6}$ C and $q_2 = -4\times10^{-3}$ C are placed 30 cm apart. Determine the magnitude and direction of the force that one charge exerts on the other.

Given: $q_1 = 2\times10^{-6}$ C , $q_2 = -4\times10^{-3}$ C , r= 30 cm = 0.3 m , k = = $9\times10^9\ Nm^2/C^2$

Required: F

$F = K * (q_1*q_2)/r^2 = 9\times10^9\ Nm^2/C^2 * (2\times10^{-6}\ C) * (-4\times10^{-3}\ C)/(0.3\ m)^2$

F= -800 N. The minus sign shows that the force is an attraction force.

4.6 The electric field

- An electric field is a region where an electric charge experiences a force.

Test charge (point positive charge): it tests for the existence of electric fields

- It has to be small so that its presence does not disturb the electric field it is trying to detect.

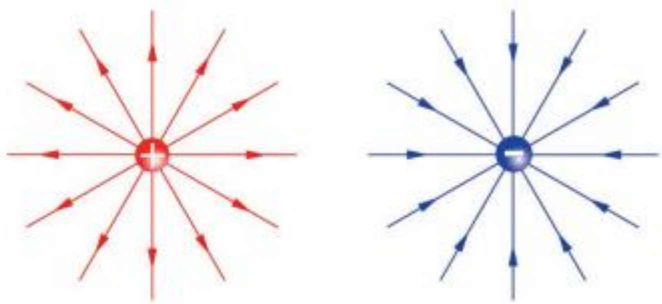


Figure 4.7 Electric field lines from positive and negative charges.

• Field lines originate at a positive charge and terminate at a negative charge

Properties of electric field lines

• The field lines never intersect or cross each other.

• The field lines are perpendicular to the surface of the charge.

• The magnitude of the charge and the number of field lines are proportional to each other.

• Field lines originate at a positive charge and terminate at a negative charge.

• The lines of force bend together when particles with unlike charges attract each other. The lines bend apart when particles with like charges repel each other.

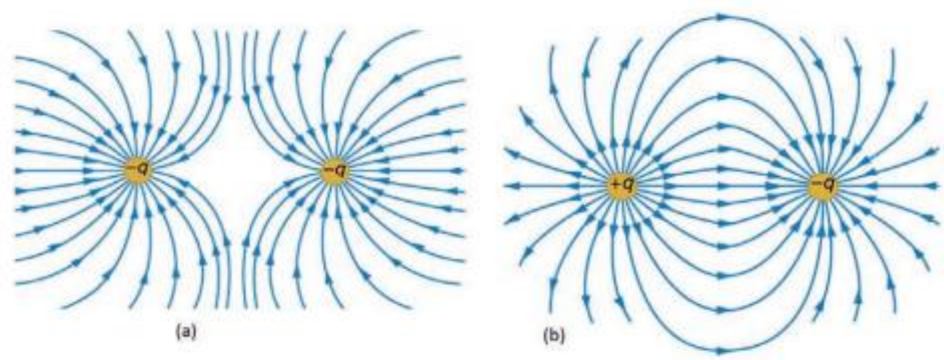


Figure 4.8 Electric field lines between (a) similar charges (b) opposite charges.

Electric field strength: The strength of the electric field, E, at any point in space is equal to the force per unit charge exerted on a positive test charge.

Mathematically

$E = F/q$ or $F = E*q$

- E is a vector
- If q is positive, the electric field E has the same direction as the force acting on the charge.
- If q is negative, the direction of E is opposite to that of the force F .
- the SI unit of electric field is Newton per Coulomb (N /C).

<u>Electric field strength due to a point charge</u>

Figure 4.9 Electric field at a distance *r* from a charge.

$E= F/q= K*(q_0*q)/r^2/q_0 =k*q/r^2$

Where, $F= K*q_0*q/r^2$

Example

Calculate the strength and direction of the electric field E due to a point charge of 2.0 nC at a distance of 5.0 mm from the charge.

Solution:

$q = 2.00\times10^{-9}$ C

$r = 5.00\times10^{-3}$ m , required: E

9×10^{9} Nm^2/C^2

$E = k*q/r^2 = 9\times10^{9}\ Nm^2/C^2 *2.00\times10^{-9}\ C/(5.00\times10^{-3}\ m)^2 = 7.2*10^{5}$ N/C

Review questions

2. What is the magnitude and direction of the force exerted on a 3.50μ C charge by a 250 N/C electric field that points due East?

   Given: $q = 3.5*10^{-6}$ c, E= 250 N/C

   Required; F

   E= F/q

   $F = E*q = 250\ N/C *3.5*10^{-6}\ c = 8.75*10^{-4}$ N East

3. Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC

$q = 5*10^{-3}$ C , r= 2 m ,

$k = 9\times10^{9}\ Nm^2/C^2$ required: E

Solution: $E = k*q/r^2 = 9\times10^{9}\ Nm^2/C^2 *5*10^{-3}\ C/(2m)^2 = 11.25*10^{6}$ N/C

4.7 Electric circuits

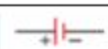
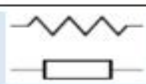
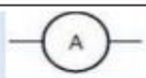
Electric circuit is a path through which charges can flow

Figure 4.10 Simple electric circuit.

Load: Any element or group of elements in a circuit that dissipates energy.

Components of electrical circuits

Table 4.1 Names of electrical components and their circuit symbols

Components	Symbol	Usage
Bulb or lump		bulb glows when charge moves through it
Battery		provides energy for charge to move
Switch	 Open Closed	allows a circuit to be open or closed
Resistor		resists the flow of charge
Voltmeter		measures potential difference
Ammeter		measures current in a circuit
connecting lead	——	connects circuit elements together

4.8 Current, Voltage, and Ohm's Law

The flow of charge particles or the rate of flow of electric charge through a point in a conducting medium is called electric current.

$I = \Delta Q/\Delta t$

The SI unit for electric current is the ampere (A), 1 A = 1 C/s. Small quantities of current are expressed in milliampere (1mA =10^{-3}A)

Example

A current of 0.5 A is drawn by a filament of an electric bulb for 10 minutes.

Find the amount of electric charge that flows through the circuit.

Solution:

You are given I = 0.5 A, and Δt = 10 minutes = 600 s.

Required: ΔQ?

From the equation for current, you have

$\Delta Q = I \times \Delta t = 0.5\ A * 600\ s = 300\ C$

Potential Difference(V) : is defined as the work done to move a unit

charge from one point to the other.

V =Work done (W)/Charge (Q)

=W/Q

The SI unit of electric potential difference is the volt (V).

Example

How much work is done in moving a charge of 2 C across two points having

a potential difference 12 V?

Given : Q = 2 C, V= 12 V.

required: W?

$W = V*Q = 12\ V \times 2\ C = 24\ J$

Ohm's Law

Resistance opposes the flow of charge through it.

States the relationship between current, voltage and resistance.

$I= V/R$

$V= IR$

$R= V/I$

Note: resistance is proportional to the conductor's length, and is inverselyproportional to its cross sectional area A.

$R \propto L/A$

$R = \rho L/A$

ρ = resistivity

L= length

A= cross sectional area.

Example

How much current will an electric bulb draw from a 220 V source, if the

resistance of the bulb filament is 1200 Ω?

Solution:

Given: V = 220 V and R = 1200 Ω.

required: I ?

I= V/R = 220 V/ 1200 Ω = 0.18 A

Example

The potential difference between the terminals of an electric heater is 60 V when it draws a current of 4 A from the source. What current will the heater draw if the potential difference is increased to 120 V?

Solution:

Given V = 60 V and I = 4 A.

Required: R ?

According to Ohm's law,

R =V/I

=60 V/4 A

= 15 Ω

When the potential difference is increased to 120 V, the current is given by

I =V/R

=120 V/15 Ω= 8 A

Example

The resistance of a metal wire of length 1 m is 26Ω at 20^{o}C. If the diameter of the wire is 0.3 mm, what will be the resistivity of the metal at that temperature?

Solution:

Given: resistance(R)of the wire = 26Ω, the diameter (d) = 0.3 mm =

$3×10^{-4}$m, length(L)of the wire = 1 m.

Required: resistivity(ρ) ?

ρ = R*A/L

$=R\pi d^2/4l$ = 260Ω* 3.14 * ($3×10^{-4}$m$)^2$ /(4*1m)

ρ = $1.84×10^{-6}$Ωm.

4.9. Combination of resistors in a circuit

Resistors in Series

A series circuit is a circuit that has only one path for the electric current to

follow. If this path is broken, then the current will no longer flow and all the devices in the circuit will stop working.

In series circuit total the current is the same thoughout.

$I_1= I_2= I_3= I_{tot}$

V = V1+ V2+ V3.......... (1)

$V = I_{tot}Req$

By making use of equation (1)

IReq= V1+ V2+ V3

Since V1= IR1,V2= IR2,V3= IR3,

I_{tot}*Req= I_{tot}(R1+ IR2+ IR3) 2

∴Req= R1+ R2+ R3......... (3)

Resistors in Parallel

Because the potential difference across each bulb in a parallel arrangement

equals the terminal voltage (V) = V1= V2= V3, you can divide each side of

the equation by V to get the following equation

$V_1 = V_2 = V_3$

$I = I_1 + I_2 + I_3 ...$

Therefore, the total current I is equal to the sum of the separate currents

through each branch of the combination.

Let Req be the equivalent resistance of the parallel combination of resistors.

By applying Ohm's law to the parallel combination of resistors, you have

$I = V/R_{eq}$

Since I = I1+ I2+ I3, applying Ohm's law to each resistor gives you

$V/R_{eq} = (V_1/R_1 + V_2/R_2 + V_3/R_3)$

where $I_1 = V/R_1$

, $I_2 = V/R_2$

, and $I_3 = V/R_3$

$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$

Example

For series circuit with R1= 12 Ω, R2= 3.0 Ω, R3= 4.0 Ω, R4= 5.0 Ω and V = 12 V .

Find:

a) the equivalent resistance,

b) the current through each resistor.

Solution:

Given R1= 12 Ω, R2= 3.0 Ω, R3= 4.0 Ω, R4= 5.0 Ω and V = 12 V .

required : Req and I.

for four resistors connected in

series.

a) Since all the four resistors are in series combination,

Req= R1+R2+R3+R4= 12 Ω+3.0 Ω+4.0 Ω+4.0 Ω = 24 Ω.

b) The current through all resistors in a series circuit is the same. Thus,

using Ohm's law,

I =V/Req=V/R

=12V/24 Ω= 0.50 A

Example

In the circuit shown in Figure above and find:

a) the equivalent resistance,

b) the current through the battery and each resistor.

Solution:

The given quantities are $R1=12\Omega$, $R2=12\Omega$, $R3=6.0\Omega$ and $V=12V$.

a) The three resistors are in a parallel combination. So

$1/Req=1/R1+1/R2+1/R3$

$=1/12\ \Omega+1/12\ \Omega+1/6.0\ \Omega$

$=1/3.0\ \Omega$

$Req=3.0\ \Omega$

Therefore, the equivalent resistance should be less than the smallest resis-

tance as expected.

b) From Ohm's law,

$I=V/Req=12\ V/3.0\ \Omega=4.0\ A$

Since the voltage is constant in a parallel connection,

$I1=V/R1=12\ V/12\ \Omega=1.0\ A$

$I2=V/R2=12\ V/12\ \Omega=1.0\ A$

$I3=V/R3=12\ V/6.0\ \Omega=2.0\ A$

Series-parallel combination circuit

•Start from the resistor combination farthest from the voltage source,

find the equivalent series and parallel resistances.

•Reduce the current until there is a single loop with one total equiva-

lent resistance.

•Find the total current delivered to the reduced circuit using Ohm'slaw.

Example

Find the equivalent resistance and the current across the 4.0Ωresistor

shown in Figure

Solution:

Given R1= 4.0 Ω, R2= 5.0 Ω, R3= 9.0 Ω, and V = 6 V.

Required:Req and I.

Since the 5.0 Ω and 9.0 Ω resistors are connected in parallel,

$R_{parallel}$=1/R2+1/R3=1/5 Ω+1/9 Ω=14/45 Ω

$R_{paralle}$l= 3.21 Ω

Figure 4.20Circuit diagram for

series-parallel combination of resistors.

Now the 4.0Ω and $R_{parallel}$ resistors are connected in series. Therefore,

Req= 4.0 Ω+$R_{parallel}$= 4.0 Ω+3.21 Ω = 7.21 Ω.

The current through the circuit can be calculated by

I =V/R=V/Req=6 V/7.21 Ω= 0.830 A.

End of unit questions

8. Two charges q1=2µC and q2= −4µC are placed 20 cm apart. Determine the magnitude and direction of the force that one charge exerts over the other.

Solution:

$F = kq_1q_2/r^2 = (9\times10^9 Nm^2/C^2(2\times10{-}6C)(-4\times10^{-9}C)/(0.2m)^2$

= −6.0×10−4N

9.Two spheres; 4.0 cm apart, attract each other with a force of1.2×109N. Determine the magnitude of the charge on eachto see if one has twice the charge (of the opposite sign) as the other.

Solution:$F = kq_1q_2/r^2$

Required: q

$1.2\times10^{-9}N = 9\times10^9\ (Nm^2)/C^2(q)(2q)/(0.2m)^2$

$q = 1.03\times10^{-11}C$ and $2q = 2.06\times10^{-11}C$

10.Two equal charges of magnitude 1.1×107C experience an electrostatic force of 4.2×10^4N. How far apart are the centers the two charges?

Solution:F = kq1q2/r2

Required: r

$4.2\times10^4N = 9\times109\ (Nm^2)/C^2\ (1.1\times10^7C)^2\ /r^2$

r= 0.51 m

16.A copper wire has diameter 0.5 mm and resistivity of $1.6\times10^{-8}\Omega m$. What will be the length of this wire to make its resistance 10Ω? How much does the resistance change if the diameter is doubled?

Given, diameter, d=0.5 mm resistivity, $\rho=1.6\times10-8\Omega m$

Resistance, R=10 Ω

7Let the length of wire be *l*.

$A = \pi d^2/4$

$R = \rho l/A = 4\rho l/\pi d^2$

$l = R\pi d^2/4\rho = 10\Omega\times3.14\times(0.5\times10^{-3}\ m)^2\ /(4*1.6\times10^{-8}\Omega m)$

= 122.7m

20.A battery of 9 V is connected in series with resistors of 0.2Ω,0.3Ω, 0.4Ω, 0.5Ω, and 12Ω. How much current would flow through the 12 Ω resistor? Solution : For resistors in series,

Required: current (I)

Req= R1+R2+R3+R4+R5= 0.2+0.3+0.4+0.5+12 = 13.4Ω

By ohm's Law:

I= V/Req   I= 9 V/13.4Ω= 0.67A

Unit 5

Magnetism

Magnetism is an interaction that allows certain kinds of objects, which are called magnets, that exert forces on each other without physically touching.

5.1Magnet

A magnet is a material or object that produces a magnetic field that is re- sponsible for a force that pulls or attracts other materials. Magnets attract objects made of iron or steel, such as nails and paper clips. Magnets can also attract or repel other magnets.

Types of Magnets

There are three categories of magnets. These are:

1. **Permanent Magnets**:are made up of magnetic material (such as steel) that is magnetized and has its own magnetic field. They are known as permanent magnets because they do not lose their mag- netic property once they are magnetized.

•Striking one magnet with the other in an inappropriate manner will reduce the magnetic strength.

2. **Temporary Magnet**:can be magnetized in the presence of a mag- netic field. When the magnetic field is removed, these materials lose their magnetic properties. Iron nails and paper-clips are examples of temporary magnets.

3**. Electromagnets**:consist of a coil of wire wrapped around a metal core made of iron.

Properties of magnet

The following are the basic properties of magnet:

•When a magnet is dipped in iron filings, you can observe that the iron filings cling to the end of the magnet as the attraction is greatest at the ends of the magnet. These ends are known as the poles of the magnets.

•Whenever a magnet is suspended freely in mid-air, it always points in a North-South direction. The pole pointing towards geographic North is known as the North Pole, and the pole pointing towards geographic South is known as the South Pole.

• Like poles repel while unlike poles attract.

•The magnetic force between two magnets is greater when the dis- tance between them is smaller. Note: A magnet has a North pole and a South pole.

•Like magnetic poles repel each other; unlike poles attract each other.

5.2. Magnetic Field

it is a region around a magnetic material or a moving electric charge within which a a force of magnetism acts.

Properties of magnetic field lines

- Magnetic field lines never intersect with each other
- Magnetic field lines form a closed-loop
- Outside a magnet, magnetic field lines appear to emerge or start from the North pole and merge or terminate at the South pole. Insidea magnet, the direction of the magnetic field lines is from the South pole to the North pole
- The closeness or density of the field lines is directly proportional to the strength of the field. In areas where the magnetic field is strong, the field lines are closer together. In a place where the field is weaker, the field lines are drawn further apart
- The magnetic field is stronger at the poles because the field lines are denser near the poles.

Similarities between magnetic and electric fields

•Electric fields are produced by two kinds of charges, positive and negative. Magnetic fields are associated with two magnetic poles: North and South, although they are also produced by charges (mov- ing charges).

• Like poles repel, but unlike poles attract each other.

•The electric field points in the direction of the force experienced by a positive charge. The magnetic field points in the direction of the force experienced by the North pole.

Differences between magnetic and electric fields

•Positive and negative charges can exist separately. The north and south poles always come together. Single magnetic poles, known as magnetic monopoles, have been proposed theoretically, but a magnetic monopole has never been observed.

•Electric field lines have definite starting and ending points. Magnetic field lines are continuous loops. Outside of a magnet, the field is directed from the North Pole to the South Pole. Inside a magnet, the field runs from south to north.

5.3The Earth's magnetic field and the compass

Earth's magnetic field (Geomagnetic field) is a magnetic field that extends from the Earth's interior out in to the space and causes the compass needle rotate.

A compass

A compass is an instrument that is used to find the direction of a magnetic field. It can do this because a compass consists of a small metal needle that is magnetized itself and that is free to turn in any direction. Therefore, in the presence of a magnetic field, the needle is able to line up in the same direction as the field.

5.4 Magnetic field of a current-carrying conductor

Current is generally defined as the rate of flow of charge. a current-carrying wire produces a magnetic effect around it. This magnetic field is generally attributed to the sub-atomic particles in the conductor, for example, the moving electrons in the atomic orbitals.

The strength of a magnetic field, **B**, some distance d away from a straight wire carrying a current, I, can be found using the equation

$B = \mu o I/2\pi d$

where, $\mu o = 4\pi \times 10^{-7}$ Tm/A

μo= refers to the permeability of free space

I =the magnitude of the current

d= the distance from the source current

to the magnetic field

B=magnetic field strength. It's SI unit is Tesla (T)

Another smaller unit, called the gauss (G)

$1G = 10^{-4} T$.

Magnetic field strength produced due to a current carrying conductor has the following characteristics:

- It encircles the conductor.
- It lies in a plane perpendicular to the conductor.
- A change in the direction of the current flow reverses the direction of the wire.

Example

An infinitely long wire has a current of 3 A passing through it. Calculate the magnetic field at a distance of 2 cm from the wire ($\mu 0 = 4\pi \times 10-7$ Tm/A).

Solution:

Given: I= 3 A, d = 2 cm= 0.02 m

Required: magnitude and direction of a magnetic field.

Solution:

B =μ0I/2πd

Substituting the given values into the equation gives

$B = (4\pi\times10-7\ Tm/A)\times(3\ A)/2\pi\times(0.02\ m) = 3\times10^{-5}T$

The direction of the magnetic field can be determined using the right-handrule.

5.5. Magnetic force on a moving charge placed in a uniform magnetic field.

Consider a positively charged particle that is moving in a uniform mag- netic field. Then, the magnitude of the force (magnetic force) is directly proportional to the magnitude of the charge, the component of the ve- locity which is acting perpendicular to the direction of this field and the magnitude of the generated magnetic field..

$F = qvB\sin\theta$ Magnetic force (F) = 0, when the speed or velocity of the particle and the magnetic field are parallel to each other.

F= **maximum**, when the velocity of the particle and the magnetic field are perpendicular to each other

a magnetic force on a moving charge.

Example

Determine the magnitude of the magnetic force of a 50 C charged particles moving with the velocity of 3 m/s in the same direction to a magnetic field of magnitude 1 T.

Solution:

Given q= 50 C, v= 3 m/s, B = 1 T and $\theta = 0^{o}$. Required: magnitude of the magnetic force. The magnitude of the force is obtained by $F = qvB\sin\theta = 50\ C \times 3m/s \times 1\ T \times \sin 0^{o} = 0t$

5.6. Magnetic force on a current-carrying wire

The force on a wire carrying a current I with length in a uniform magnetic field B is given by:

$F = ILB\sin\theta$

In general, the force on a current-carrying conductor is:

i)always perpendicular to the plane containing the conductor and the direction of the field in which it is placed and

ii) greatest when the conductor is at right angles to the field.

Example

A wire 25 cm long is at right angles to a 0.30 T uniform magnetic field. The current through the wire is 6.0 A. What is the magnitude of the force on the wire?

Solution:

Given L = 25 cm = 0.25 m, B = 0.3 T, $\theta = 90^o$ and I =6 A.

Required : magnitude of the force (F) The magnitude of the magnetic force can be calculated by

$F = ILB\sin\theta = 6\ A \times 0.25\ m \times 0.3\ T \times \sin 90o = 0.45\ N$.

5.7. Magnetic force between two parallel current- carrying wires.

1.When current in both wires are in the same direction, you will need to draw a diagram to get a clear idea of the particular situation. Here, you have two parallel current-carrying wires, separated by a particular distance d, such that one of the wires is carrying current, such that one of the wires is carrying current $I_1$and the other is carrying I_2 which are in the same direction. Therefore, you can say that they are attractive.

2. When the current in the two wires is going in opposite directions, carrying current I_1 and the other is carrying I_2, the force acting on themwill be repulsive.

Review Questions

1.A wire 0.50 m long carrying a current of 8.0 A is at right angles to a uniform magnetic field.

The force on the wire is 0.40 N.

What is the strength of the magnetic field?

L = 0.5m,I = 8.0A, F = 0.4N, B =?

F= ILB

B=F/IL=0.40N/8.0A×0.5m =0.1T

2.The current through a wire 0.80 m long is 5.0 A. The wire is per- pendicular to a 0.60 T magnetic field. What is the magnitude of the force on the wire?

L = 0.5m,I = 8.0A, B = 0.6T, F =?

F = ILB = 8A ×0.5m ×0.6T =2.4 N

3.An electric wire in the wall of a building carries a current of 25 A vertically upward. What is the magnetic field at a distance of 10 cm from the wire?

4. I = 25A, d= 10cm

B =μ0I/2πd

$=(4\pi\times10^{-7}$ Tm/A)×(25A)/2π×(0.01m)

$= 1.5\times10^{-11}$T

The direction of the magnetic field can be obtained using the right hand rule.

16. q =1.6×10−19C,v =8.75×105m/s,B =0.75Tandθ =90o. The Magnitude of the force is obtained by

F = qvBsinθ = $(1.6\times10^{-19}$C)(8.75×105m/s)(0.75T)sin90^{o}= 1.05×10^{-13}N

The direction is perpendicular to the plane containing v and B.

Unit 6

Electromagnetic Waves and Geometrical Optics

6.1Electromagnetic (EM) waves

A wave transfers energy from one place to another without transferring matter. Waves, such as water waves and sound waves, transfer energy by making particles of matter move. The energy is passed along from particle to particle as they collide with their neighbors. Depending on their medium of propagation, waves are categorized into mechanical waves and EM waves.

•Mechanical waves are the types of waves that use matter to transfer energy. They cannot travel in almost empty space between the Earth and the Sun.

•An EM wave is a wave that can travel through empty space or through matter. They are produced by charged particles, such as electrons, that move back and forth or vibrate. ate outward from the source at the speed of light.

•An EM wave is a wave that can travel through empty space or through matter. They are produced by charged particles, such as electrons, that move back and forth or vibrate. EM waves are transverse waves. In transverse waves, the direction of oscillation is perpendicular to the direction of propagation of waves while in a longitudinal wave, the direction of oscillation is parallel to the direction of propagation of the wave. a wave on a rope is a transverse wave that causes the rope to move at right angles to the direction the wave is traveling.

Radiant Energy from the Sun

The Sun emits EM waves that travel through space and reach Earth. The energy carried by EM waves is called radiant energy.

★ Infrared and visible light waves carry the radiant energy that reaches the Earth from the Sun.

6.2 EM Spectrum

EM spectrum is the complete range of EM wave frequencies and wavelengths.

- ★ When the wave length is short, the frequency is higher and the energy is also higher.
- ★ When the wavelength is long, the frequency is low and the energy is small.

  Radio waves, microwave waves, infrared waves, visible light, ultraviolet waves, X-ray and gamma rays.
- ★ **Radio waves** have the lowest frequency and smallest energy and the longest wavelength.
- ★ **Gamma rays** have the highest frequency and the highest enegery but the shortest wavelength.

6.3 Light as a wave

As light is an EM wave, it cantravel in a vacuum as well as through materials such as air, water, andglass.

- ★ EM waves move at a constant velocity (in a vacuum), which is known as the speed of light.

Speed of light

- ★ The speed of light has the symbol **c** and is equal to

2.99792458×10^8m/s $\approx 3.00 \times 10^8$m/s.

Because all EM waves in a vacuum have the same speed c (i.e., speed of light), it follows that:

$c = \lambda \times f$

Example

Find the frequency of red light with a wavelength of 700 nm.

Given: λ =700nm, c =3.00×10^8m/s.

Required: the frequency (f)

Solution:

$c = \lambda \times f$

Solve for f,

$f = c/\lambda$

$= 3\times10^{8}\text{m/s}/700\times10^{-9}\text{m} = 4.29\times10^{14}\text{Hz}$

Example

An FM radio station broadcasts EM radiation at a frequency of 103.4 MHz.

Calculate the wavelength of this radiation.

Givenf =103.4MHz , $c = 3.00\times10^{8}$m/s.

Required: wavelength, λ.

$c = \lambda \times f$,

Solve for ,λ.

$\lambda = c/f$

$= 3\times10^{8}\text{m/s}103.4\times10^{6}\text{Hz} = 2.9\text{ m}$

Propagation of light

•Light travels in all directions from its source, in straight lineswith arrows to show the path of light.

6.4 Laws of reflection & refraction

Reflection of light

reflection: is the change in direction of light rays at a surface that causes them to move away from the surface.

ii.The incident ray, the normal to the mirror at the point of incidence and the reflected ray all lie in the same plane.

These laws are applicable to all types of reflections, i.e., specular and dif- fuse reflection. The following is the discussion of the distinction between these two types of reflection.

The incoming light ray is called the incident ray. The light ray moving away from the surface is the reflected ray. The most important characteristic of these rays is their angles in relation to the reflecting sur- face. These angles are measured with respect to the normal of the surface.

$\theta i = \theta r$

ii.The incident ray, the normal to the mirror at the point of incidence and the reflected ray all lie in the same plane.

Specular Reflection and Diffuse Reflection

specular reflection: is the reflection of light from a smooth shiny surface in the same direction.

Diffuse reflection: is the reflection of light from a rough surface in different directions.

Refraction of light: Refraction is the bending of light as it moves from one optical medium to another.

Note:

- ★ If the speed of light in the first medium (V1) is higher than the speed of the second medium (V2), then the ray bends away from the normal.
- ★ If the speed of light in the first medium (V1) is lower than the speed of second medium (V2), then the ray bends towards the normal.

The laws of refraction of light:

•The incident ray, the refracted ray, and the normal to the interface of two transparent media at the point of incidence all lie in the same plane.

•**Snell's law**:The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant.

Note: Ifθ_1 is the angle of incidence and θ_2 is the angle of refraction then, sin θ1/sin θ2 = constant. This constant value is called the refractive index of the second medium with respect to the first medium.

Refractive Index : measure of how difficult it is for light to get through a material Note:The speed of light and the degree of bending of the light depend on the refractive index of the material through which the light passes.

Let $v_1$be the speed of light in medium 1, and v_2 be the speed of light in medium 2. The refractive index of medium 2 with respect to medium 1 is given by the ratio of the speed of light in medium 1 and the speed of light in medium 2. This is usually represented by the symbol **n21** n21=speed of light in medium 1/speed of light in medium 2 n21=v1/v2 By the same argument, the refractive index of medium 1 with respect to medium 2 is represented as **n12** and it is given by n12=speed of light in medium 2/speed of light in medium 1 **n12**=v2/v1 n =speed of light in vacuum/speed of light in medium

n =c/v

Table . Absolute refractive index of some material mediums

Material medium	Refractive index	Material medium	Refractive index
Air.	**1.0003.**	**Canada Balsam**	**1.53**
Ice	**1.31**		
Water.	**1.33.**	**Rock salt**	**1.54**
Alcohol	**1.36**		
Kerosene	**1.44**	**Carbon disulphide**	**1.63**
Fused quartz	**1.46**	**Dense flint glass**	**1.65**
Turpentine oil	**1.47.**	**Ruby**	**1.71**
Benzene.	**1.5.**	**Sapphire**	**1.77**
Crown glass.	**1.52.**	**Diamond.**	**2.42**

If a light ray is incident on the surface between these materials with an angle of incidenceθ1, the refracted ray passes through the second medium with an angle of refractionθ2. Snell's law which was discussed above can thus be written as follows:

$n_1 \sin\theta 1 = n_2 \sin\theta 2$ where, n1= refractive index of material 1 ,n2= refractive index of material 2,

θ1= angle of incidence

θ2= angle of refraction. Remember that angles of incidence and refraction are measured from the normal, which is an imaginary line perpendicular to the surface.

Example

If a light ray with an angle of incidence of 35^o passes from water to air, find the angle of refraction using Snell's Law. Given angle of incidence: 35^o

As depicted in Table 6.1, the refractive index is 1.33 for water and about 1 for air.

Solution: use Snell's law to find the value for the angle of refraction.

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.33\times \sin 35^o = 1\times \sin\theta 2$

$\sin\theta 2 = 1.33\times 0.57 = 0.763$

$\therefore \theta 2 = \sin-1(0.763) = 49.7^o$

Total internal reflection

- ★ When light passes from a dense medium (larger index of refraction) to a less dense medium (smaller index of refraction)the refracted ray bends away from the normal. for example, from water to air.

As the angle of incidence increases, the angle of refraction also increases.

90^o angle of refraction: occurs When the angle of incidence reaches a certain value, called the critical angle θc

- ★ the refracted ray points alongthe surface.

Total reflection: When the angle of incidence exceeds the critical angle, there is no refracted light. All the incident light is reflected back into the medium from which it came.

For total internal reflection to take place, the following two conditions must be satisfied.

•Light must travel from an optically denser medium (i.e., a medium having a high refractive index) to an optically rarer medium (i.e., a

medium having a lower refractive index). It does not occur when light propagates from a less dense to a denser medium, for example, from air to water.

•The angle of incidence in the denser medium must be greater than the critical angle.

$n1\sin\theta1 = n2\sin\theta2$ For total internal reflection, you know that the angle of incidence is the

critical angle (i.e.,$\theta1 = \theta c$). You also know that the angle of refraction at the critical angle is 90^o(i.e., $\theta c = 90o$). You can then write Snell's law as:

$n1\sin\theta c = n2\sin 90^o$

Solving for θcgives:

$\sin\theta c = n2/n1$

$\therefore \theta c = \sin-(n2/n1)$

The Dispersion of Light: Prisms and Rainbows

dispersion: is the splitting of light into its component colors.

6.5. Mirrors and lenses

mirror: is a reflective surface that does not allow the passage of light and instead bounces it off, thus producing an image.

Examples: Plane and spherical mirrors are the two types of mirrors.

Plane Mirrors

plane mirror:A mirror that has a flat reflective surface. Number of images formed by two plane mirrors inclined to each other

If two plane mirrors are placed inclined to each other at an angleθ, the number of images formed by mirrors is :

Number of images$\approx(360^o/\theta)-1$, if $360^o/\theta$ is an even integer.

Number of images$\approx360^o/\theta$, if $360^o/\theta$ is an odd integer.

Spherical Mirrors

Some mirrors are not flat. A spherical mirror is formed by the inside (concave) or outside (convex) surfaces of a sphere.

1. concave mirror has a surface that is curved inward.

Example: bowl of a spoon.

- ★ concave mirrors cause light rays to come together, or converge.

2. **convex mirror**: has a surface that curves outward.

Example :the back of a spoon.

- ★ Convex mirrors cause light waves to spread out, or diverge.

•The center of the sphere, of which the mirror is apart is called the center of curvature (C) of the mirror and the radius of this sphere defines its radius of curvature (R).

•**pole**:The middle point P of the reflecting surface of the mirror

•**principal axis**: The straight line passing through the center of curvature and the pole.

•The circular outline (or periphery) of the mirror is called its aperture.

- ★ Aperture is a measure of the size of the mirror.

•A beam of light incident on a spherical mirror parallel to the prin- cipal axis converges to or appears to diverge from a common point after reflection. This point is known as principal focus (F) of the mirror.

• **Focal length:**The distance between the pole and the principal focus.

the radius of curvature(R) is equal to twice the focal length.

R =2f.

Mirror Formula and Magnification

object distance (u): the distance of the object from its pole is called the

image distance (v) :The distance of the image from the pole of the mirror is called the.

$1/v+1/u=1/f$

Magnification

m =height ofthe image (h')/height of the object (h)

$m=h'/h$

Magnification (m) $=h'/h= -v/u$

Example

A convex mirror used for rear-view on an automobile has a radius of cur- vature of 3.00 m. If a bus is located at 5.00 m from this mirror, find the position, nature and size of the image.

Solution:

Given radius of curvature, R = - 3.00 m

object distance, u = +5.00 m;

The focal length (f) =R/2

$f= -3/2= -1.50$m(as the principal focus of a convex mirror is behind the mirror).

Required: image distance(v) height of the image(h')

Using the mirror equation,

$1/v+1/u=1/f$

$1/v=1/f-1/u$

$= -1/1.50-1/5.00$

$=-5.00-1.50$

7.50

$\therefore v = -1.15m$

Therefore, the image is 1.15 m at the back of the mirror.

Magnification (m) $=h/h= -v/u$

$= -(-1.15m)/5.00m$

$= +0.23$

The image is thus virtual, erect and smaller in size by a factor of 0.23.

Example

An object, 6.0 cm in size, is placed at 25.0 cm in front of a concave mirror of 15.0 cm focal length. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Determine the nature and the size of the image.

Solution:

The given quantities are object size, h = + 6.0 cm, object distance, u = + 25.0 cm and focal length, and f = + 15.0 cm.You are asked to find the image distance, v and image size, h'.

Since

$1/v+1/u=1/f$

$1/v=1/f-1/u$

$1/v=1/15.0-1/25.0$

$\therefore v = 37.5$ m

The screen should be placed at 37.5 cm in front of the mirror. The image is real. Also,

Magnification (m) $=h'/h= -v/u$

Solving for h' gives

$h'= (-h*v)/u$

$= -(37.5\text{ cm})(6.0\text{ cm})/(25.0\text{cm})$

$= -6.0$ cm Thus, the height of the image is - 6.0 cm. The negative sign implies that the image is inverted and enlarged.

lens:is bound by at least onespherical surface.

6.7 Primary colors of light and human vision

Primary colors: red, green, and blue light

6.8 Color addition of light

Red + Green = Yellow

Red+ Blue = Magenta

Blue + Green = Cyan

6.9 Color subtraction of light using filters

Cyan - Blue = (Green + Blue) - Blue = Green

Yellow - Green = (Red + Green) - Green = Red

Magneta - Red = (Red + Blue) - Red = Blue

End of unit questions

1.An object 5.0 cm in length is placed at a distance of 20 cm in front of a convex mirror of radius of curvature 30 cm. Find the position of the image, its nature, and its size.

Solution: Radius of curvature (R) = 30 cm

f = R/2 = 30/2 = 15 cm

u = -20 cm, h= 5 cm.

Since

$1/v+1/u=1/f$

$,1/v=1/f-1/u$

$=1/15.0-1/(-20.0)$

v= 8.57 cm

Image is virtual and erect and formed behind the mirror.

m = -v/u= h'/h

h'/5= 8.57/20= 0.428

h' = 0.428 x 5 = 2.14cm

Position of Image: Behind the mirror.

Nature of Image: Virtual and Erect.

Size of the Image: Diminished.

2.An object of size 7.0 cm is placed at 27 cm in front of a concave mirror of 18 cm focal length. At what distance from the mirror should a screen be placed, so that a sharp focused image can be obtained? Find the size and the nature of the image.

Solution: $h=+7cm$, $u=-27cm$, $f=-18cm$

Since

$1/v+1/u=1/f$ $1/v=1/18-1/27$

$v=-54cm$

3.A convex lens produces a virtual image which is four times larger than the object. The image is 15 cm from the lens. What is the focal length of the lens?

Solution :

$h'= 4h$,$v = -15cm$; $m =-v/u$

$=h'/h$

$u =-1/4\times v = -(-15/4)=15/4cm$ $1/f=1/v+1/u =4/15-3/15$ $f = 5cm$