Virtual Study

Learn, Connect, Succeed

Mathematics

Grade 9

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Contents

Unit one

Further on sets

Set is a col lection of well- defend object or elements. When we say that a set is well-defined, we mean that , a given object, we are able to determine whether the object is in the set or not

I. set are usually denoted by capital letter. A, B, C x, Z Y

II. The elements of a set are represented by small letter Example :- a, b, c, x, y, z etc

Set Decription

I. Verbal method (Statement form in this method the well-defined description of the elements of the set is written in English statement form (Word

Example

A. The set of all even natural numbers less than 10

B. The set of whole number greater than 1 and less than 20

II. Complete listing method In this method all the elements of the set are completely listed where the elements are separated by commas and enclosed with in braces (}

Example

A. The set of all even integer less than 7 is described in listing method as { 2, 4,6 }

B. The set of all vowels in English alphabet [a , e, I o u }

III. Partial listing method we use this method, If listing of all elements of a set is difficult or Impossible but the elements can be

Indicated clearly by listing A few of them that fully describe the set

Example

A. The set of Natural number less than 100 = 1, 2, 3 ……. 99

B. The set of

B The set of whole number

$W = \{0, 1, 2, 3 \ldots \ldots\}$

Exercise

1. Describe Each of three following sets using a verbal method

a. A= {5, 6, 7, 8, 9

So The set of Natural number gather than 5 and less than 10

B. M= (2, 3, 5, 7, 11, 13 }

The set of prim number less than fiviteen

2. Describe each of the following sets using complete and patria; listing method (If possible

A. The set of primer factor of 36

A= {2, 3, }

B. The set of Natural Number less than 100 and divisible by 5

3

B= {5, 10, 20, 25 … … . 100 }

C. The set of non- negative Integer

C= {0, 1, 2, 3, 4, … … .}

D. The set of even natural n number

D= {2, 4, 6, 8, 10, 12 … . .}

E , f Exercise for you

IV. Set builder method (method of defang prosperity

The set- builder method is described by property that its member must satisfy.

This is the mother of writing the condition to satisfy

Example

I. A= {1, 2, 3 … … . . 10 } can be

A= $\{x: x \in N \ and \ x < 11\}$,

II. let A = {0, 2, 4, … … . }This can be as

A= $\{x: x$ is an element of non negative even integer

Exercise

1. Write the following sets using set Builder method.

   a. D= $\{1, 3, 5 \ldots \ldots\}$

      $x; x \in N$ odd Natural number

b. A= $\{2, 4, \ 6, 8\}$

   A= $xA = \pi r^2; x \in$ even Natural number less than 9,

C. C= $\{1, 4, 9, 16, 25\}$

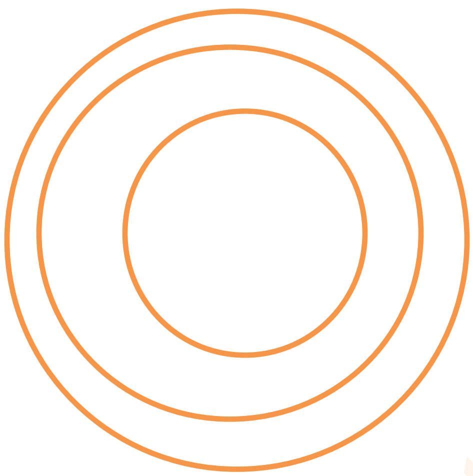
Unit 2

Rational Numbers

Any number that can be written in the form of $\frac{a}{b}$ wher a and ba are Integers and b ≠ 0 is called Rational number .

The set of Rational Donated by Q and is described as

Q= $\{\frac{a}{b}$; a, b, ∈ Z and b ≠ 0}



Suppose x= $\frac{a}{b}$ ∈ Q x is a fraction with number a and denominator b,

I. If a<b then $x$is called proper fraction

II. If a ≥ b then x is called Improper fraction.

III. If Y= C $\frac{a}{b}$ where C∈ Z and $\frac{a}{b}$ proper fraction, then Y is called mixed fraction

IV. x is said to be in simplest (Lowest form) If a and b are relatively prime or GCF (a,b)- 1

➡ For any two natural numbers a and b. G C f (a, b) LCm (a,B)= a,b

Exercise

1. Find G C f and L C m of the following

a, 12, 16 and 24 b, 4, 18 and 30

$12=2^2x3$ $4 = 2^2x\ 1$

$16=2^4x\ 1$ $18 = 2x3^2$

$24=2^3\times 3$ $30 = 2x3x5$

L C m= 2^4x 3 48 L C m = 2^2 x 3^2x5

2. Find GCF (16,24) x LCm (16,24 GCF (16,24) xLCm (16,24)= $16x24$

$2^3\,x$ $2^3\,x$ $2x3 = 16x24$

8 x $8\,x$ 6

$\underline{\underline{384 = 384}}$

Exercise

1. Answer the following True or False

a. Any integer is a rational number. True

b. The simplest form of $\frac{35}{45}$ is $\frac{9}{7}$, False

c. For a, b, c, d $\in$ Z $\frac{a}{b}+\frac{c}{d}$ is rational number . True

d. The simplest form of 5 $\frac{2}{4}$ is $\frac{11}{2}$. True

2. If $\frac{a}{b}$ and $\frac{c}{d}$ are Two rational numbers Show that $(\frac{a}{b})$ x $\frac{c}{d}$ is also a rational numbers.

Answer:- The multiple of any Two rational number is rational number

Example:- $\frac{4}{5}$ x $\frac{3}{2}$ = $\frac{12}{10}$ rational n<u>o</u>

3. Zebiba measures the length of a table and she reads 54 Cm and 4mm express this measurement in terms of Cm in lower form

of $\frac{a}{b}$ Answer 1Cm = 10mm $x=$ 0.4Cm

$x=$ 4mm

$\frac{54Cm}{0.4Cm}$ = $\frac{540}{4}$ = $\frac{270}{2}$

4. A rope of 5 $\frac{1}{3}$ meter is to be cut into 4 pieces of equal length.

What will be length of each piece

Answer:- $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\;x\;\frac{d}{c}$

$5\frac{1}{3} = \frac{16}{3} \div \frac{4}{1} = \frac{16}{3} x \frac{1}{4} = \frac{16}{12} = 1.3$

$\underline{\underline{1.333^{0}}}$

2.1 Representation of rational number by decimals

- Any rational number $\frac{a}{b}$ can be written as decimal form by dividing the numerator a by the denominator b
- When we change a rational number $\frac{a}{b}$ in to decimal form) one of the following cases will occur
  - ✓ The division process ends when

A, remainder of Zero is obtained here the decimal is called Termination Decimal

  - ✓ The division process doors not term igniting but repots as remainder never be com Zero. In this case decimal is called repeating Decimal.

Example

$\frac{4}{3} = 1.33^{0}$ ……

$\frac{1}{3} = 0.3\overline{3}$

$\frac{9}{2} = 4.5$

Exercise

1. Write the following fractions using the notation of repeating decimals

a. $\frac{8}{9}$ = 0.88888889 = 0.8

b. $\frac{5}{6}$ = 0.833

c. $\frac{14}{9}$ = 1.55556 = 1.56

- Converting terminating decimals to fractions

Example

1. Convert each of the following
   - ✓ Decimals to fraction form

a. $7.3 = \frac{73}{10}$

b. $-0.18 = \frac{-18}{100}$

Exercise

1. Write down each of the following as refraction form

   I. a, $0.3 = \frac{3}{10}$ b, $3.7 = \frac{37}{10}$ c. $0.48 = \frac{48}{100}$

2.2 Representing ration numbers on the numbers line

Example:- 1

Locate the rational number -5,3,$\frac{4}{3}$ and $\frac{-5}{2}$ on the number line

Conversion of repeating decimals in to fraction ($\frac{d\times10^{n}}{10^{n}-1}$) -d

a. $0.5 = xA = 0.5$

$10x = 5.5$

$x = 0.5$

$\frac{ax}{a} = \frac{5}{a}$ $x = \frac{5}{a}$

b. $2.\overline{12}$ $x = 2.\overline{12}$

$100x = 212.\overline{12}$ $\frac{99x}{99} = \frac{209}{99} = \frac{209}{99}$

2.3 Represent each of the following decimals as a simplest fraction form

a. $2.6 = x = 2.6$

$10x = 26.6$

$\frac{9x}{9} = \frac{24}{9}$ $x = \frac{24}{9} = \frac{8}{3}$

b. $1.32\ \overline{12}$

$10000\,x = 13212.\ \overline{12}$

$x = 1.32\ \overline{12}$

M= non repeating number

N= repeating number

M= 2

N= 2

$$d = \frac{d \times 10^{m+n} - d \times 10^{m}}{10^{m+n}}$$

$$d = \frac{1.321\overline{2} \times 10^{4} - 1.32\,\overline{12} \times 10^{2}}{10^{4} - 10^{2}}$$

$$d = \frac{13212^{12} - 132.\overline{12}}{10000 - 100}$$

$$d = \frac{13212.12 - 132.12}{10000 - 100} = \frac{13080}{9900}$$

d= $\frac{218}{165}$

2.4 Irrational number

Neither repeating nor terminating numbers.

A. Perfact Squre is a number that can be exprerssed as a product of two equal integers, for instance

1, 4, 9……… $1^2 = 1$

$4 = 2^2$

$9 = 3^2$

A square root of a number × is a number Y such that $Y^2 = x$.

For instance, 5 and -5 are square root of 25 because $5^2 = (-5)^2 = 25$

Exercise

1. Find the numbers with out radicasign
   a. $\sqrt{25}$ =5 c. $\sqrt{0.04}$ = 0.2
   b. -$\sqrt{36}$ = -6 d. -$\sqrt{0.0081}$ = -0.09

When a > 0, b>0, If a <b, then

$\sqrt{a} < \sqrt{b}$

Compare $\sqrt{5}$ and $\sqrt{6}$

5 < 6 then $\sqrt{5} < \sqrt{6}$

Oprations on Irrational numbers

When a > 0,b> 0 then $\sqrt{a} \times 5 = \sqrt{ab}$

Example

Calculate each of the following

a. $\sqrt{2}\ x\ \sqrt{3} = \sqrt{6}$

b. $\frac{2}{\sqrt{3}}\ x\ \frac{1}{\sqrt{3}}$

$= \frac{2}{3}$

c. (2+$\sqrt{2}$) x (1+ $\sqrt{5}$) =

$2x1 + 2x\sqrt{5} + (\bar{2}\ x1)(\sqrt{2}\ x\ \sqrt{5})$

$2 + 2\sqrt{5} + \sqrt{2} + \sqrt{10}$

Exercise

1. Calculate each of the following

a. $\sqrt{3}\ x\ \sqrt{5} = \sqrt{15}$

b. $2\sqrt{5}\ x\ \sqrt{7} = 2\sqrt{35}$

c. $-\sqrt{2}\ x\ \sqrt{6} = -\sqrt{12} = -2\sqrt{3}$

d. $1\frac{1}{\sqrt{5}}\ x\ \frac{10}{\sqrt{5}}$

e. $(2+\sqrt{3})\ x\ (-2+\sqrt{3}) = -4 + 2\sqrt{3} + -2\sqrt{3} + 3$

$= -4 + 0 + 3 = -1$

f. $(\sqrt{3} + \sqrt{2})^2 = (\sqrt{3} + \sqrt{2})(\sqrt{3} + \sqrt{2})$

$3 + \sqrt{6} + \sqrt{6} + 2 \ \underline{\underline{= 5+2\sqrt{3}}}$

2. $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})$

$\sqrt{7}(\sqrt{2} - \sqrt{3}) + \sqrt{3}(\sqrt{7} - \sqrt{3})$

$7 \underline{-\sqrt{21} + \sqrt{21}} -3$

$7 - 3 = \underline{\underline{4}}$

h, $(\sqrt{6} - \sqrt{10})$

$(\sqrt{6} - \sqrt{10})(\sqrt{6} - \sqrt{10}$

$\sqrt{6}(\sqrt{6} - \sqrt{10}) - \sqrt{10}(\sqrt{6} - \sqrt{10})$

$6 - \sqrt{60} - \sqrt{60} + 10$

$16 - 2\sqrt{60} = \underline{\underline{16\text{-}4}}\sqrt{15}$

When a > 0, b>0 then $= \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Calculate

a. $\frac{\sqrt{2}}{3} = \sqrt{\frac{2}{3}}$ b. $-\frac{\sqrt{10}}{\sqrt{2}} = -\sqrt{\frac{18}{2}} = \underline{\underline{\sqrt{9} = -3}}$

Exercise

Calculate

a. $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} = \sqrt{9} = \underline{\underline{3}}$

b. $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} =$

C. $\frac{\sqrt{14}}{\sqrt{7}} = \frac{\sqrt{14}}{\sqrt{7}} \times \sqrt{7} = \frac{7\sqrt{14}}{7} = \underline{\underline{\sqrt{14}}}$

$\times \sqrt{7}$

Example

1. Simplify each of the following
   a. $2\sqrt{3} + 4\sqrt{3} = 2+4 \sqrt{3} = 6\sqrt{3}$
   b. $3\sqrt{5} - 2\sqrt{5} = (3-2)\sqrt{5} = \sqrt{5}$
2. Simplify each of the following
   a. $\sqrt{8} + \sqrt{2} = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$
   b. $\sqrt{12} - 5\sqrt{3} = 2\sqrt{3} - 5\sqrt{3} = -3\sqrt{3}$
   c. $\sqrt{18} - \sqrt{8} = 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

   d. $\sqrt{72} - \sqrt{8} = 6\sqrt{2} - \sqrt{2} = \underline{\underline{4\sqrt{2}}}$

Exercise

a. $\sqrt{3} + 4\sqrt{3} = 5\sqrt{3}$
b. $\sqrt{5} - \sqrt{45} = \sqrt{5} - 3\sqrt{5} = -1\sqrt{5}$
c. $2\sqrt{5} - 4\sqrt{5} = (2-4)\sqrt{5} = -2\sqrt{5}$
d. $\sqrt{18} + 2\sqrt{2} = 3\sqrt{2} + 2\sqrt{2} = \underline{\underline{5\sqrt{2}}}$
e. 0.12345…… - 0.1111….. 0.01234…….
f. $\sqrt{80} - \sqrt{20} = \sqrt{16x5} - \sqrt{4x5}$ =4-2 (5=$\underline{\underline{2\sqrt{5}}}$

2.5 Real Number (Real number

A number is called areal number , If and only If it is either rational number or Irational number . it is doneted by R and it is described as whin of the set of Rational number and Irrational number.

R= { x : x is rational or Irrational }.

Compuring Real numbers.

- ✓ Suppose two real number a and b are given. Then one of the following is true
  A < b or a= b or a > b (troche to my)
- ✓ For any three real number a, b, c
  If a <b and b <c then a <c (called transitive prosperity)
- ✓ For any no negative real numbers a ,b
  If $a^2 < b^2$ then a < b

Exercise

1. Compure each of the following pair (>   <   =)

a. $\frac{22}{7} = \pi$

b. -7 $\sqrt{145} \leq \sqrt{7}^2 = (-7\sqrt{145})^2 \leq \sqrt{7}^2$

c. $2 + \sqrt{3} \leq 4 = (2 + \sqrt{3})^2 \leq (4)^{16}$

2.5.1 Determing Real numbers b/n two numbers

Example:- Find two rational number between

a .   2 and   3   Let $\frac{e}{f}$ and $\frac{2}{h}$ between

$\frac{e}{f} = \frac{1}{2}(2+3) = \frac{5}{2} = 2.5$ Thus 2 < 2.5< 3

$\frac{2}{h} = \frac{1}{2}(2+2.5) = \frac{1}{2}(4.5) = 2.25$

Thus 2 < 2.25 <2.5 < 3

b) $\frac{1}{6}$ and $\frac{1}{5}$

then $\frac{e}{f} = \frac{e}{f}\frac{1}{2}\left(\frac{1}{5} + \frac{1}{6}\right) = \frac{11}{60}$ and $\frac{1}{6} < \frac{11}{60} < \frac{1}{6}$

$\frac{2}{h} = \frac{1}{2}\left(\frac{1}{6} + \frac{11}{60}\right) = \frac{12}{120}$ and $\frac{1}{6} < \frac{12}{120} < \frac{11}{60} < \frac{1}{5}$

Exercise   Solution

1. Find at least two real numbers between

   a. – 0.24 and – 0.246

$\frac{e}{f} = \frac{1}{2}(\ 0.24 – 0.246\ ) = -0.243$

-0.246< < -0.243 < - 0.24

$\frac{2}{h} = \frac{1}{2}$ (-0.246<-0.243) = -0.2445

-0.246 < -0.2445<-0.243<0.24

b. $\frac{3}{5}$ and $\frac{2}{7}$

$\frac{e}{f} = \frac{1}{2}(\frac{3}{5} + \frac{2}{7}) = \frac{62}{35}\ \frac{31}{70} \quad \frac{3}{5} < \frac{62}{35} - \frac{2}{7} < \frac{31}{70} < \frac{3}{5}$

$\frac{2}{h} = \frac{1}{2}\ (\frac{3}{5} + \frac{31}{70}) = \frac{2}{7} < \frac{31}{70} < \frac{365}{700} < \frac{3}{5}$

$\frac{7}{2} < \frac{31}{70} < \frac{365}{700} < \frac{3}{5}$

2.6 Intervales

Case I (Representing real number between two points)

- ✓ Areal Interval is a set whioch contains all real numbers b/n two numbers.

Table

Inequlity	Interval notition	graphic discription
$a < x < b$	(a, b)	 a b
$a \leq x < b$	{ a, b)	 a b
$a < x \leq b$	(a, b]	 a b
$a \leq x \leq b$	a, b	

Case II (Representing real number in numbers with one end point) Symbol ∞ rend as Infinty means end less

- ∞ read as negativ Infinty

Inqulity Interval notation graphic no line Discription

$x > a$ (a, ∞)

a

$x < a$ (- ∞, a)

a

$x \geq a$ (- a, ∞)

a

$x \leq a$ (- ∞, a)

Exercise

a

1. Represent each of the following using Interval notation

a. Real numbers between -3 abd 8 Including both end point

[-3, 8]

-3 -8

b. Real numbers between 4 and 6 with out Including end point. (4 , 6)

C. Real numbers at the right of -1 and Including -1 . (∞ , -1]

2. Represent each of the following intervals on the number line

a. (- ∞ , -5

-5

b. $(\frac{1}{2}, 3) =$

2 3

c. (-2 , 4) [5 , ∞]

-2 4 5

2.7 Absolute Value

The Absolut Value of a real number detonated by, / x / , is defined as

x If x $x \geq 0$

/ x / = x If x < 0

Example:- find the Absolut value of the following

/4/ = 4

1- $\sqrt{2}$) = $\sqrt{2}$

/2 - $\sqrt{8}$ = $\sqrt{8}$ - 2

Exercise

1. Find the absoult value of each of the following

a. / 8 / = 8

b. / 4 + $\sqrt{3}$ = 4 + $\sqrt{3}$

c. / 1 - $\sqrt{2}$ = 1 - $\sqrt{2}$ – 1

d. / 3 - $\sqrt{5}$ / = 3 - $\sqrt{5}$ – 3

F. 7 + $\sqrt{5}$ = 7 + $\sqrt{5}$

2. Find the absolute value of each of the following

2. Find the distance between the given numbers on the number line

a 2 and 10

b -49 and -100 = - 100 + 49 = -149

c -50 and 50 = /50-(-50)= 100 unit

3. Determine the unknown "x" for each

a / x / = 8 x = 8

b / x / = 0 x = 0

c / x / = -4 x = 4

d / x / -3 = -2 ➡ / x / = -2 + 3 ➡ x = 1

2.8 Exponents and radicals

3x3x3 = 27

4x4x4 = 64

2x2x2 2x2= 32

If a is a real number and n is a positive Integer then a×a×a× a×a×a = a^n is an exponential expression, where a is the base and n is the exponent or power

a2^3 2 ×2x2 = 8

b $(-3)^4$ = (-3) x (-3) x (-3) x (-3) = 81

c. = $(-2)^3$ = (-2) x (-2) x (-2) x (-2) = -8

2^n and 2^{-n}

a^n = a^n

$a^{-n} = \frac{1}{an}$

a^0 = 1

If a^2 = b, then a is a square root of b.

If a^3 = b, then a is a cube root of b

Then a is called the n^{th} root of b.

Principle n^{th} root

If b is any real number and n is a positive integer than 1 then the principle n^{th} root of b is denoted by $\sqrt[n]{b}$ is defined as

$\sqrt[n]{b}$ $\sqrt[n]{b}$ - the positive n^{th} root of b, If b > 0

- the negative n^{th} root of b, If b< 0 nte odd

- 0 If b = 0

The $(\frac{1}{n})^{th}$ power

If b ∈ R and n is a positive integer greather than 1, then $b^{\frac{1}{n}} = \sqrt[n]{b}$

a = $\sqrt{a}$ = 3

b $\sqrt[-4]{81}$ = 3

c $\frac{1}{\sqrt[4]{10}}$ = $10^{\frac{-1}{4}}$

Exercise

1. Express each of the following in exponential form

a $(\sqrt[4]{81})^2$ = $(81)^{\frac{2}{4}}$

b $\sqrt{\frac{2}{5}}$ = $(\frac{2}{5})^{\frac{1}{2}}$

c $\sqrt[2]{3^4} = 3^{\frac{4}{2}}$

2. Simplify each of the following

a $(-27)^{\frac{1}{3}} = -3$

b $(32)^{\frac{1}{5}} = 2$

c $\sqrt[3]{125} = \frac{5}{25} = \frac{1}{5}$

Laws of Exponent

For any a, b ∈ R and n, m ∈ N the folloing

- ✓ $a^n x\, a^m = a^{n+m}$
- ✓ $\frac{an}{am} = a^{(n-m)}$, where $a \neq 0$
- ✓ $(a^n)^m = (a^m)^n = a^{nm}$
- ✓ $(a\, x\, b)^n = a^n\ x\ b^n$
- ✓ $\frac{an}{am} = (\frac{a}{b})^n$, for $b \neq 0$

For any $(\frac{1}{n})^{th}$ power and rational power of the form $a^{\frac{1}{n}} = (a^{\frac{1}{n}})\, m$

Using the above rules and simplify

a $7^{\frac{1}{2}} x 7^{\frac{3}{2}} = 7^{\frac{1}{2}+\frac{3}{2}} = 7^2 = 49$

b $(25x\ 9)^{\frac{1}{2}} = 5\, x\, 3 = 15$

c $\frac{3^{\frac{5}{2}}}{3^{\frac{1}{2}}}$ $\quad 3^{\frac{5}{2}-\frac{1}{2}} = 3^{\frac{4}{2}} = 3^2 = 9$

$3^{\frac{1}{2}}$

d $(\frac{8}{27})^{\frac{1}{3}} = \frac{2}{3}$

<u>Exercise</u>

1. Simplify each of the following

a, $2^{\frac{3}{2}}\ x\sqrt{2} = 2^{\frac{3}{2}}\ x\ 2^{\frac{1}{2}} = 2^{\frac{3}{2}+\frac{1}{2}} = 2^2 = 4$

b. $\frac{5\sqrt{24} \div 2\sqrt{50}}{3\sqrt{3}}$

$$\frac{5\sqrt{4\times3\times2}\div2\sqrt{25\times2}}{3\sqrt{3}}$$

$$\frac{10\sqrt{3\times2}\div2\times5\sqrt{2}}{3\sqrt{3}}$$

$$\frac{10\sqrt{3}\times\sqrt{2}\div10\sqrt{2}}{3\sqrt{3}}$$

$\frac{10}{3}$ $\sqrt{2}\div10\sqrt{2}$ $\frac{1}{2}$ - $\frac{1}{2}$

$\frac{10}{3}$ $\sqrt[10]{2}$ = $\frac{100}{3}$ 2 = $\frac{100}{3}x1$

c $(5^{-1})^3\ x\ 5\frac{5}{4}\times\sqrt{25}\ x\ (\sqrt[-3]{125})^3$

$5^{-3}\ x5\frac{5}{4}\times\sqrt{25}\ x\ (\sqrt[3]{125})^3$

$\frac{1}{125}x\ 5^{-3}\ x\ 5\frac{5}{4}\ x\ 5^1\ x5\frac{3}{3}$

$5\frac{-7}{4}x\ 5^1\times5^1$ = $5\frac{-7}{4}+1+1)$

5 $\frac{-7}{4}+2$ -$\frac{-7+8}{54}$ $5\frac{1}{4}$ = $\sqrt[4]{5}$ = $\sqrt[4]{5}$

2.8.1 Addition and Subtraction of radicals

Radical that have the same Index and the same radicand are said to be like radicals.

I $2\sqrt{3}$, $\sqrt{3}$ $\frac{3}{5}$ $\sqrt{3}$ are like radicals

II $\sqrt{2}$, $\sqrt{5}$ aren not like radicals

III $\sqrt{5}$ $\sqrt[3]{5}$ $\sqrt[4]{5}$ are not like tradianal

Example:-

a. $\sqrt{3}$ $+\sqrt{12}$ = $\sqrt{3}$ + $\sqrt{4x3}$ = $\sqrt{3}$ $+2\sqrt{3}$

$3\sqrt{3}$

b. $2\sqrt{18}$ - $\sqrt{2}$ + $\sqrt{8}$ = $2\sqrt{2x9}$ - $\sqrt{2}$ + $\sqrt{4\times2}$

$6\sqrt{2}$ - $\sqrt{2}$ $+2\sqrt{2}$ $=5\sqrt{2}$ = $2\sqrt{2}$ $=7\sqrt{2}$

Exercise

1. Simplify each of the following

a, $\sqrt[3]{54}$ - $\sqrt[3]{2}$ = $\sqrt[3]{2 \times 27}$ - $\sqrt[3]{2}$ = 3 $3\sqrt[3]{2}$ -$\sqrt[3]{2}$

$3\sqrt[3]{2}$ - $\sqrt[3]{2}$ = $2\sqrt[3]{2}$

b, $3\sqrt[3]{27}$ - $\sqrt[3]{125}$ + $\sqrt{169}$

$3 x 3$ - 5 + 13 = 9-5+13 = **17**

Operations on real numbers

Note

- ✓ The sum and difference of rational and Irrational numbers is Irrational.
- ✓ The product, quotient of non-zero rational number and an Irrational number is an Irrational number

Example:-

If $x = 5\sqrt{3} + 2\sqrt{2}$ and $Y = \sqrt{2}$ -4 $\sqrt{3}$ then find

a the sum of X and Y

x+Y = $5\sqrt{3}$ + $2\sqrt{2}$ + $\sqrt{2}$ - $4\sqrt{3}$

$5\sqrt{3}$ -$4\sqrt{3}$ + $3\sqrt{2}$

$\sqrt{3}$ + $3\sqrt{2}$

b, x-Y = $5\sqrt{3}$ + 2 $\sqrt{2}$ - ($\sqrt{2}$ - $4\sqrt{3}$

$5\sqrt{3}$ + $4\sqrt{3}$ + $2\sqrt{2}$ -$\sqrt{2}$

$9\sqrt{3}$ - $\sqrt{2}$

Exercise

1. If $\times = 4\sqrt{2}$ + 7 $\sqrt{5}$ and Y = $\sqrt{2}$ - 3 $\sqrt{5}$ then find

a, x+Y = $4\sqrt{2}$ + $7\sqrt{5}$ + $\sqrt{2}$ - $3\sqrt{5}$

$4\sqrt{2}$ +$\sqrt{2}$ +$7\sqrt{2}$ $\sqrt{5}$ - $3\sqrt{5}$

=5 $\sqrt{2}$ + $4\sqrt{5}$

b. Y- x = $\sqrt{2}$ - $3\sqrt{5}$ - ($4\sqrt{2}$ + $7\sqrt{5}$)

$\sqrt{2}$ $-4\sqrt{2}$ $-3\sqrt{5}$ $-7\sqrt{5}$

$-3\sqrt{2}$ $-10\sqrt{5}$

2. Find the produce of

a, $2\sqrt{5}$ and $4\sqrt{3}$ $= 2\sqrt{5} \times 4\sqrt{3} = 8\sqrt{15}$

b, $3\sqrt{3}$ and $\sqrt{\frac{3}{3}}$ $= 3\sqrt{3} \times \sqrt{\frac{3}{3}} = \underline{\mathbf{3}}$

c. $\sqrt{3} - \sqrt{2}$ and $3\sqrt{3} - 4\sqrt{2}$

$(\sqrt{3} - \sqrt{2})\ (3\sqrt{3} - 4\sqrt{2})$

$\sqrt{3}\ (3\sqrt{3} - 4\sqrt{2}) - \sqrt{2}\ (3\sqrt{3} - 4\sqrt{2})$

$9 - 7\sqrt{6} + 8$

3. <u>Divide</u>

a, $4\sqrt{6}$ by $2\sqrt{2}$ $= \frac{4\sqrt{6}}{2\sqrt{2}}$ $\frac{2\sqrt{6}}{2}$

$\frac{2\sqrt{6}}{\sqrt{2}} = \frac{2\sqrt{6}}{\sqrt{2}} \times \sqrt{2} = \frac{2\sqrt{2}}{2} = \underline{\mathbf{2\sqrt{3}}}$

$\times \sqrt{2}$

b, $10\sqrt{2}$ by $5\sqrt{18}$ $= \frac{10\sqrt{2}}{5\sqrt{18}} = \frac{10\sqrt{2}}{10\sqrt{2}} = \underline{\mathbf{1}}$

c, $\sqrt{5}$ by $(3\sqrt{2}\ x\ 4\sqrt{5})$

$\frac{\sqrt{5}}{3\sqrt{2}} = \frac{\sqrt{50}}{12\times 10} = \frac{5\sqrt{2}}{12\times 10} = \frac{\sqrt{2}}{24}$

4, Compute the following

a, $5\sqrt{2} + 2\sqrt{3} + 3\sqrt{3} - \sqrt{2}$

$5\sqrt{3} + 5\sqrt{2}$ $= \underline{\mathbf{5\sqrt{3} + 4\sqrt{2}}}$

b, $\sqrt{145} - \sqrt{232} + \sqrt{261}$

$\sqrt{5}\ x\ \sqrt{29} - \sqrt{2}\ x\ \sqrt{29} + 3\sqrt{29}$

<u>Closure Property</u>

The set of real number R is closed under addition and multiplication. This means that the sum and product of any two real number is always a real number . In other words, For all a, b ∈ R, a+b ∈ R and ab or a ∈ R

Commutative Property

Addition and multiplication are commutative in R: that is for all a , b $\in$ R

I. a +b = b +a

II. a . b = b .a

✓ **Associative Property**

Addition and multiplication are associative in R: that is for all a, b, c $\in$ R

. I. (a +b) +c = a+ (b +c)

II. (a b) c = a (b c)

Existence of additive and multiplicative identities

There are real numbers 0 and 1 such that:

I) a + 0 = 0 a = a for all $\in$ R

II) a.1 = 1.a = a for all a $\in$ R

Existence of additive and multiplicative Inverses.

I) For each a $\in$ R there exists – a $\in$ R such that a + (-a) = 0=(-a) +a) and –a is called the additive Inverse of a.

II) For each non-zero a $\in$ R there exists $\frac{1}{a}$ and $\frac{1}{a}$ is called multiplicative Invers or reciprocal of a.

Distributive Property

✓ Multiplication is distributive over addition: that is, If, b, c $\in$ R

I) a (b +c) = a b + ac

II) (b +c) a = b a + c a

Exercise

1. Find the addativ Invers and multiplication invers

a	$\sqrt{3}$	addativ Invers - $\sqrt{3}$ -$\sqrt{3}$	mutiplation Invers $\frac{1}{\sqrt{3}}$
b	$-\frac{2}{5}$	$\frac{2}{5}$	$-\frac{5}{2}$
c	1.3	-1.3	0.7692

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2.9 Limit of Accuracy

Rounding:- Rounding off is a type of estimation. Estimation is used in every day life and also in subjects like mathematics and physics.

- ✓ If the disit is 0,1,2,3 or 4 do not change the rounding disit. All disits that are on the right hand side of the requsted rounding digit becom zero.
- ✓ If the digit is 5,6,7,8 or 9 the rounding digit round up by one number . All dijits that are on the right hand side of the requested rounding digit becom zero

Exercise

1. Round each of the following to the nearest whole number

a $35.946 \approx 36$

b $45.1999 \approx 45$

c $\frac{7}{8} = 0.875 \approx 1$

d $\sqrt{5} = 2.23606 \approx 2$

2. Express the following decimals to 1 d.p and 2 d.p

a $1.936 \approx 1.90$ 1d.p

≈ 1.94 2d.p

b $4.752 \approx 5$ 1.d.p

≈ 4.75 2d.p

Significant Figure

Numbers can also be approximeted to a given number of significant figures (S.F). In the number 43.25 the 4 is the most significant figure as it has a value of 40.

In contrast, the 5 is the least significant as it only has a value of 5 hand redth.

Example:-

a Write 2.364 to 2 S.F

2.4

b Write 0.0062 to 1 S.F = 0.006

Exercise

Write each of the following to the number of significant figur Indicated in bracket a , 28,645 (1 S.F) = 30000

b, 44,909 (3 S.F) = 41900

c, 4.5568 (3 S.F) = 4.5600

Accuracy

The upper and lower bounds are the maximum and minium values that a number could have been before it was rounded.

A number was given as 10.7 to 1 d.p

step 1. Place value of the degree of accuracy is 0.1
Step2. Divide 0.1 by 2 result 0.05
Step3. The lower bound is 10.7-0.05 and upper bound is 10.7 + 0.05 = 10.75, 10.65

Exercise

The speed of a car is given as 45 m/s to the neaarest integer.

a, Find the lower and upper bounds with in which the car speed can (ie

45 – 0.05 = 44.95 Lower bound

45 + 0.05 = 45.05 upper bound

b, If the car's speed is vmlsi Express this range as Inequality

$v - 0.5 \leq V < V + 0.5$

2. Find the lower and upper bound of

a, 45 rounded to the nearest integer 44

b, 12.6 round to 1 d.p = 11

c, 4.23 round to 2d.p = 4.2

3. Express each of the following in between the upper and lower bound

a x = 34.7 34.7 + 0.05 Upper

34.75 upper 34.7-0.05 = 34.65 lower

b y = 21.36 = 21.30 + 0.05 = **<u>21.41 Upper</u>**

21.36 - 0.05 = **<u>21.31</u>**

c z = 154.134 = 154.134+0.05 = 154.184 Upper

154.134 - 0.05 = **154.084** Lower bound

Exercise

1. Calculat the upper and lower bond for the following calculat If each of the numbers is given to decimal place

a, 5.4 + 6.2 =

4.4 + 5.2 = 9.6 minumum

4+6.2 = 11.6 maxi

Lies between 9.6 and 11.6

b, 4.6 x 2.7

minimum = 3.6 x1.7 = **6.12**

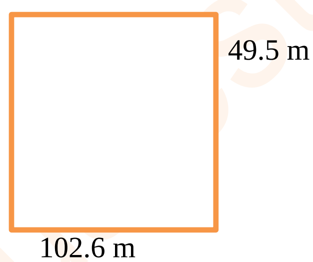
maximum = 4.6 x 2.7 = 12.42

2. Calculat upper and lower bounds for the area of a school , foot ball shown below

A = 4 x w

A = 49.5 x102.6

Minumum



48.5 x 101.6 = 4, 927.6

Maximum = 5, 078.7

Standared natation (Scientific notation)

A number is said to be in scientific notation (for standared notation)
If it is written as a propduct of the form a x 10^n where $1 \leq a < 10$ and n is an integer

Example:-

Express each of the following number in to scientific natation

a 185400000 = 1.854 x 10^8

b 0.000088400 = 8.84 x 10^{-5}

Exercise

1. Express each of the following in standared notation

   a, 158.762 = $1.58762 \ x \ 10^{2}$

   b, 0.000089 = $8.9 \ x \ 10^{-5}$

2. Write each of the following in standared form

   a , $3.3 \ x \ 10^{-3}$ = 0.0033

   b, $1.34 \ x \ 10^{6}$ = 1340000

   c, $\frac{2}{5} \ x \ 10^{-7}$ = 0.0000004

3. Find the simplifed expression in standared notation form

   a , $(4.2 \times 10^{-3}) + 1.6 \times 10^{-3} = 5.8 \times 10^{-3}$

   b, $(21 \ x \ 10^{-3}) \ (1.3 \ x \ 10^{-4} = 2.73 \ x10^{-7}$

   c, $(1.5 \ x \ 10^{-3}) \ (3.1 \ x \ 10^{-3} = 4.65 \ x \ 10^{-3}$

   d. $\frac{5}{2} \ x \ 10^{-3+2} = 2.5 \ x \ 10^{7}$

2.10 Rationalization

The number that can be used as a multiplier to rationalize the denominator is called the rationalized factor whicl is equal

antto 1. $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \times \sqrt{a} = \frac{\sqrt{a}}{a} =$

$\times \sqrt{a}$

Example:-

a, $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \sqrt{3} = \frac{2\sqrt{3}}{3}$

$\times \sqrt{3}$

b, $\frac{\sqrt{2}}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times 5 = \frac{\sqrt{10}}{5}$

$\times 5$

c, $\frac{1}{\sqrt[3]{2}} \times \frac{1}{\sqrt[3]{2}} = \frac{\sqrt[3]{4}}{2}$

$\times \frac{1}{\sqrt[3]{2}}$

Exercise 1. Ra tionalize the denominator for each

a, $\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \sqrt{2} = \frac{6\sqrt{2}}{2}$

$\times\sqrt{2}$

b. $\sqrt[5]{\frac{1}{3}} = \sqrt[5]{3} \quad \sqrt[5]{34} \quad \underline{\mathbf{3}}$

c. $2\frac{\sqrt{2}}{\sqrt{7}}\ x = 2\frac{\sqrt{2}}{\sqrt{7}}\ x\sqrt{7} = 2\frac{\sqrt{2}}{\sqrt{7}}$

$x\sqrt{7}$

More on ratioalization of Denominators

No Given number Rationalize

1. $\frac{1}{\sqrt{a}-\sqrt{b}} \quad \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$
2. $\frac{1}{a+\sqrt{b}} = \frac{a-\sqrt{b}}{a-\sqrt{b}}$
3. $\frac{1}{\sqrt{a}-b} = \frac{\sqrt{\sqrt{a}+b}}{\sqrt{a}+b}$
4. $\frac{1}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

Exercise

1. Rationalize each of the following numbers.

A . $\frac{2}{\sqrt{3}-\sqrt{2}} = \frac{2}{(3-\sqrt{2})}\ \frac{(\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} =$ (6+2

$\frac{(\sqrt{6}+2\sqrt{2}}{\sqrt{3}\ (\sqrt{3}+\sqrt{2})}$

b. $\frac{1+\sqrt{3}}{\sqrt{5}-1} = .\ \frac{1+\sqrt{3}}{\sqrt{5}-1}\ \frac{(\sqrt{5}+1}{\sqrt{5}+1} = \frac{(\sqrt{3}+1+\sqrt{15}+\sqrt{3}}{\sqrt{5}\ (\sqrt{5}+1)-(\sqrt{5}+1}$

$\frac{\sqrt{5}+1+\sqrt{15}+\sqrt{3}}{5-1} = \frac{\sqrt{5}+\sqrt{5}+\sqrt{3}+1}{4}$

C, $\frac{1}{\sqrt{2}+(\sqrt{3}+1}$

Let $\sqrt{2} + \sqrt{3} = x$

$\frac{2}{x+1}$ ratinilze

$\frac{2}{x+1} \quad \frac{(x-1)}{(x-1}$

$\frac{2(x-1)}{x(x-1)}$ + $1(x-1)$ = $\frac{2x-2}{x^2-x+x}$ = $\frac{2x-2}{1x^2-1}$

$\frac{2\times-2}{x^2-1}$ = $\frac{2(\sqrt{2}+\sqrt{3})-2}{(\sqrt{2}(+\sqrt{3})x^2-1}$

$\frac{\sqrt{4}+\sqrt{6}-2}{\sqrt{2}(\sqrt{2}+\sqrt{3})+\sqrt{3}(\sqrt{2}+\sqrt{3})-1}$ $\frac{2-2+\sqrt{6}}{2+\sqrt{6}+\sqrt{6}+3)}$

= $\frac{\sqrt{6}}{2\sqrt{6}+5-1}$ = $\frac{\sqrt{6}}{2\sqrt{6}+4}$ Second

Ratinailized

$\frac{\sqrt{6}}{2\sqrt{6}+4}$ $\frac{(2\sqrt{6}-4}{2\sqrt{6}-4}$ = $\frac{4\sqrt{6}-2\sqrt{6}}{12-8\sqrt{6}+8\sqrt{6}-16}$

$4\sqrt{6}$

$\frac{4\sqrt{36}-4\sqrt{6}}{(2\sqrt{6}+4)2\sqrt{6}+4}$ $4(2\sqrt{6}+4)$ = $\frac{-4\sqrt{6}+12}{24+\sqrt{6}+8\sqrt{6}-16}$

$-4\frac{\sqrt{6}+12}{8}$

Unit Summary Reviw Exercis

1. Write the prim factorization of the following

A, 57 = $57\ x\ 1$

B, 168 = $2^3\ x\ 3\ x\ 7$

1. Find the LCM and GCF of the number given below

A, 15 , 39 , 105

by table method

	15	39	105
3	5	13	35
5	1	13	3
3	1	13	1
13	1	1	1

GCF (Of) 15 , 39, 105, is 3

LCm = 15 , 39, 105, is = $3\times3\times5\times13$

b, 16, 20, 48 =

by prim Factorization

$16 = 2^4 \ x\ 1$

$20 = 2^2 \ x\ 5$

$48 = 2_4 \ x\ 3$

GCF $= 2^2 = \underline{\underline{4}}$

LCm $= 2^4 \ x\ 3\ 5 = \underline{\underline{240}}$

2. The GCF of two number is 9 and the LCm of these number, is 54 If one of the numbers is 27, What is the other number?

LCm (a , b) × GCF (a , b) = a × b

$$\frac{9\ x\ 5^2}{27} 4 = \frac{27\ x\ b}{27} 4$$

$\underline{b = 9x\ \ 2 = 18}$

4. Express each of the following decimals as a fraction in simplest form

a, $2.1\bar{2}$ $100x = 212.2\bar{2}$

$100x = 212.2\bar{2}$

$10x = 21.2\bar{2}$

$$\frac{90x}{90} = \frac{191}{9} \quad x = \frac{191}{90}$$

b, $-0.1\ \overline{123}$

$1000x = 1\,2\,3.\overline{123}$

$x = 0.\ \overline{123}$

$$\frac{90x}{999} = -\frac{123}{999} \qquad x = \frac{123}{999}$$

5. Find the absolut value of each of the following

A $\sqrt[2]{6} - 2 = /\ \sqrt{6} - 2\ / = 2 - \sqrt{6}$

6. Given equivalent expression, conting practional exponent for each of the following

a, $\sqrt{x+y} = (x + y)½$

b, $\sqrt[5]{7}$ = $7^{1/4}$

c, $\sqrt[3]{\frac{5}{6}}$ = $(\frac{5}{3})^{1/3}$

7. Describe the following number as fractions with rational denominator

a, $\frac{4}{3-\sqrt{2}}$ = $\frac{4}{3-\sqrt{2}}$ $\frac{x3+\sqrt{2}}{x3+\sqrt{2}}$

$\frac{12+4\sqrt{2}}{3(3+\sqrt{2})-\sqrt{2}(3+\sqrt{2}}$ = $\frac{12+4\sqrt{2}}{9+3\sqrt{2})-3\sqrt{2}-2}$ = $\frac{12+4\sqrt{2}}{7}$

c, $\frac{2}{\sqrt{3}+\sqrt{2}}$ = , $\frac{\sqrt{2}}{(\sqrt{3}+\sqrt{2}}$ $\times$ $\frac{-\sqrt{3}+\sqrt{2}}{-\sqrt{3}+\sqrt{2}}$ = $\frac{-\sqrt{3}+2}{-3+\sqrt{6}+2}$

$\frac{-\sqrt{6}+2}{-3+2}$ = $\frac{\sqrt{6}+2}{-1}$ = $\underline{\boldsymbol{\sqrt{6}-2}}$

d, $\frac{1}{\sqrt{2})+\sqrt{5}+1}$

$\frac{1}{(\sqrt{2}+\sqrt{5}+1)}$ $\times$ $\frac{(\sqrt{6}-\sqrt{5}+1)}{(\sqrt{2}-\sqrt{5}+1}$ = $\frac{\sqrt{2}-\sqrt{5}+1}{2-\sqrt{10}+1}$

$\frac{\sqrt{2}-\sqrt{5}+1}{-2+\sqrt{2}}$ $\times$ $\frac{(-2-\sqrt{2})}{-2-\sqrt{2}}$

$\frac{-2(\sqrt{2}-\sqrt{5}+1)-\sqrt{2}(\sqrt{2}-\sqrt{5}+1)}{-2(-2-\sqrt{2})+\sqrt{2}(-2-\sqrt{2}}$

$\frac{-2(\sqrt{2}-2\sqrt{5}-2)-2+(\sqrt{10}-2\sqrt{2})}{4+2\sqrt{2}+-2\sqrt{2}-2}$

$\frac{-2\sqrt{2}+2\sqrt{5}-4+\sqrt{10}-\sqrt{2}}{4+2\sqrt{2}-2\sqrt{2}-\sqrt{2}}$ = $\frac{-3\sqrt{2}+2\sqrt{5}-4+\sqrt{10}-4}{2}$

$\frac{-3\sqrt{2}+2\sqrt{5}+\sqrt{10}-4}{2}$

8. Given $\frac{\sqrt{2}+\sqrt{5}}{4+\sqrt{3}}$ = a $\sqrt{2}$ + b $\sqrt{3}$ + c $\sqrt{6}$ + d

Then, find a, b. c and d.

$\frac{\sqrt{2}+\sqrt{3}}{4+\sqrt{3}}$ x $\frac{(4-\sqrt{3}}{4-\sqrt{3}}$ = $\frac{3\sqrt{2}+4\sqrt{3}-\sqrt{6}-3}{13}$

a = 4 b= 4 c = -1 d = -3

9. Simplfy each of the following

a, $7\sqrt{2}$ + $\sqrt{8}$ - $\sqrt{18}$ = $7\sqrt{2}$ + 2 $\sqrt{2}$ - 3 $\sqrt{2}$ = 6 $\sqrt{2}$

b, $\frac{1}{3}$ x ($\sqrt{5}$ - $\sqrt{2}$) x ($\sqrt{5}$ +)=($\sqrt{\frac{5}{3}}-2$ $\sqrt{2}$ - $\sqrt{\frac{2}{3}}$) x ($\sqrt{5}+\sqrt{2}$)

$\sqrt{\frac{5}{3}}$ ($\sqrt{5}+\sqrt{2}$)- $\sqrt{\frac{2}{3}}$ ($\sqrt{5}+\sqrt{2}$) = $\frac{5}{3}+$ $\sqrt{\frac{10}{3}}$ - $\sqrt{\frac{10}{3}}-\frac{2}{3}$

= $\frac{5}{3}$ - $\frac{2}{3}$ = $\frac{3}{3}$ = **1**

C, $3\sqrt{28}$ ÷ 8 $\sqrt{7}$ = 3 $\sqrt{7x4}$ ÷ 8 $\sqrt{7}$ = $\frac{6\sqrt{7}}{8\sqrt{7}}$ = $\frac{6}{8}$

D, $^{-4}\sqrt{b^2}$ x $^2\sqrt{b^2}$ For b ≥ 04

$\sqrt[4]{b^2}$ x $\sqrt{b^2}$ = $\sqrt[4]{b}$ ^2xb = $\sqrt[4]{b^3}$ = $\frac{3}{b4}$

e, $\frac{2\sqrt{72}}{3}$ - $\frac{2\sqrt{128}}{4}$ + 5 $\sqrt{\frac{1}{2}}$ = , $\frac{2\sqrt{2x736}}{3}$, $\frac{-3\sqrt{2x64}}{4}$+$\frac{1}{2}$

= $^{2x6}\sqrt{\frac{2}{3}}$ - $\frac{3x8}{4}$ $\sqrt{2}$ + 5 $\sqrt{\frac{13}{2}}$

$4\sqrt{2}$ - 6 $\sqrt{2}$ + 5 $\sqrt{2}$ = -2 $\sqrt{2}$ + $\frac{5}{\sqrt{2}}$ × $\frac{\sqrt{2}}{\sqrt{2}}$

-2 $\sqrt{2}$ + $\frac{\sqrt{2}}{2}$

10. Each of the following numbers is correct to one decimal place

- ✓ Give the upper and lower bounds of each
- ✓ Using × as the number express the rang in which the number lies as inequality

a, 2.4 = 2.4 - 0.05 = 2.35

2.4 + 0.05 = 2.45

2.35 < x < 2.45

b, 10.6 = 10.6 + 0.05 = 10.65

10.6 - 0.05 = 10.55

10.55 < x < 10.65

11. Express each of the following number in scientific notation

a, 567,200000 = 5.675 x 10^8

b, 0.00000 774 = 7.74 x 10^{-6}

c, $154.6 \;\; x \;\; 10^{5} \; = \; 1.546 \;\; x \;\; 10^{7}$

12. According to the divison algorithim what should be the value of q and r respactivly so that 736 = 12q + r?

A, 60 and -4 b, 54 and 8 C, 60 and 4 D, 61 and 4

13. Suppose X is a rational number and Y is an Irrational number , then which one of the following is necessarily True

A, x,Y is rational number C, $\frac{x}{Y}$ is real number

B, $\frac{x}{Y}$ is Irrational number D, x-Y is Irrational for any x

14. Which one of the following statement is True about the set real number R?

A, Every real number has a multiplication Inverse

B. Subtraction is asssociative over R

C, -a is the additive inverse of a for every a ∈ R

D, Division is commutative over R

15. When the positive Integers a,b and C are divedied by 13, the respective remainders are 9,7 and 10 respactively show

that a+b+c is divisable by 13, 9+7+10 = 26 ÷ 13=2

16. Ther are 14 girls and 12 boys in Q school . They want to give a voluntary traffic safety service .

The school director assign end a task in a group. The team must have an equal number of boy and girls on it. What is

the greatest number of teams the director can make if every student is on a team? How many boys and girls will be in

each team?

	14	21
7	2	3

2 boys and 3 girls for each group

Unir Three /3/

Solving Equationione

Review of linear:- equation in one variable.

- When an equation in one variable has exponent equal to 1, it is said to be a linear equation in one variable. It is of the form ax+b = 0, where × is the variable and a and ba are real coefficient and a ≠ 0. This equstion has only one solution.
- For solving a linear equation in one variable, the following steps
  1, use LCm (Least Common Multipl to clear the fraction If any.
  2, Simplify both sides of the equstion
  3. Isolate the Variable
  4. Verify your answer

Example:-

$3x + 4 = 10 \Rightarrow 3x + 4 - 4 = 10 - 4$

$\frac{3x}{3} = \frac{6}{3}$ $\underline{\underline{x = 2}}$

2. $\frac{5}{4}x + \frac{1}{2} = 2x - \frac{1}{2}$

$\frac{5}{4}x - \frac{2x}{1} = -\frac{1}{2} - -\frac{1}{2}$ Collect like term

$\frac{5x - 8x}{4} = -1 \Rightarrow -\frac{3}{4}x = -1$

$-\frac{4x}{3} - \frac{3}{4}x = -1x - \frac{4}{3}$

$x = \frac{4}{3}$

Exercise

1, Solve following liner equation

A, $5x - 8 = 2x - 2$

$5x - 2x = 8 - 2 = \frac{3x}{3} = \frac{6}{3}$ $x = \underline{\underline{2}}$

B, $10(x + 10) - 7 = 13$

$10x - 100 - 7 = 13$

$\frac{10x}{2} = \frac{120}{2}$ **$\underline{x = 12}$**

C, 7(y – 2) + 21 = (2y) (3)

7 y – 14 + 21 = 6y

7y + 7 = 6y

7y – 6y = -7 **$\underline{y = -7}$**

2, Verfiy when ther Type equation here.x = -3 is a solution of the linear equation $10x+7= 13\text{-}5x$ or not .

$10x+7 = 13\text{-}5x$

$10x+ 5x = 13\text{-}7$ ➡ $\frac{15}{15} = \frac{6}{15} = \frac{3}{5}$

Not **$\underline{x = -3}$**

3, If the sum of two consiecative number is 67, then find the numbers.

The frist no is x

The Second no is $x + 1$

$x+x+1 = 67$

$2x+1 = 67$

$2x = 67 - 1$ ➡ $\frac{2x}{2} = \frac{66}{2}$

x 33 ------------ The second no

- Asystem of linear equstions consist of two or more linear equstion mad up of two or more variables such that all equstions in the system are considered simultanusly. Two find the unique solution to asystem of liner equstions. This to solve linear equstions with two variables solving tables, solving by sub stutation mathod and elemnation method.

<u>Solving System of liner equstions in two Variables using tables</u>

<u>Example:-</u>

$2x + y = 15$ is systems of liner
$3x - y = 15$ equsion in two variable
$Y = -2x + 15$

x	0	1	2	3	4	5	6
y	15	13	11	9	7	5	3

Y = $3x$-5

x	0	1	2	3	4	5	6
y	-5	-2	1	4	7	10	13

In the above Qustion the solution of the order pair is (4 , 7) it is the solution set

Cheek $2 x 4 + 7 = 15$ True

$3x 4 -7 = 5$ True

Exercise

1, Solve the following linear equstion in two varable using table

A, $2x - y = -1$

$x + y = 4$

$2x$ -y= -1➡y = $2x$+1

x	0	1	2	3	4	5	6	7	8
y	1	3	5	7	9	11	13	15	19

x+y = 4 ➡ y = 4-x

x	0	1	2	3	4	5	6	7	8
y	4	3	2	1	0	-1	-2	-3	-4

The solution set of the order

pairs is (1, 3)

Cheek $2 x 1 -3 = -1$ True

$1 + 3 = 4$ True

2, Determine whether the ordered pairs (8 , 5) is a solution to the following linear equstion in two variable or not

A, $5x - 4y = 20$

$2x - 3y = -1$

Take the point (8 , 5)

$5 x 8 - 4x5 = 20$ True

$2 x 8 - 3x 5 = -1$ True

(8 , 5) is not solution set

Defntion

An equstion of the type ax+by = C where a,b and C are aribitraryly constant and a ≠ 0 b ≠ 0 is called a linear equstion in two

variables.

- **Solving Systems of linear equstions in two variables by Elimination method**

The another method of solving systems of linear equstion is the addation method, also called the elemnation method.

Stepes of solued by elemnation method

1. Write both equstion with x and Y variabl on the left and right side
2. Write one equstion above the ather lining up corresponding variables.

If one of thje variables in the top equstion has the opposite cofficent of the sam. Variables in the bottom equstion add

the equstion together elemenating one variable.

If not use multiplicativ by non zero number so that one of the variable in the top equstion has the opposite coefficent of

the same variable in the botem equstion and add the oppostite cofficent.

3. Solve the result equstion for the remining variable
4. Substitution that value in to one of the orginal equstion and solve for the second variable
5. Cheek the solution by substitution the values in to the other equstion

Example:- Solve

x+2Y = 1
x+ Y = 5

\+ x +2 Y = 1
\- x+ Y = 5
0+2 3 Y = 6
$\frac{3Y}{3} = \frac{6}{3}$ **Y= 2**

\- x +2 Y = 5
\- x = 3 = **x= -3**

4x +2 Y = 28
x + 2Y = 13

$4x + 2Y = 28$

$x + 2Y = 13$

$\frac{3x}{3} = \frac{15}{3}$ $\underline{\underline{x = 5}}$

$4x + 2Y = 28$

$2Y = 28 - 20$

$\frac{2y}{2} = \frac{8}{2}$ $\underline{\underline{Y = 4}}$

Exercise

1. Solve the following using elemination method

a, $2x - 3Y = 5$

$x - 2Y = 6$

28

$2x - 3Y = 5$

$-2x + 2Y = -12$ mutipl by (-2)

$-Y = -7$ $\underline{\underline{Y = 7}}$

$2x7 - 3x7 = 5$

$2x = 5+21$

$\frac{2x}{2} = \frac{26}{22}$ $\times = 13$ $\underline{\underline{= 13}}$

$\underline{\underline{x = 13}}$

b, $8x - 26Y = -2Y$

$12x + 3Y = 38$

$8x - 2Y = 26$ Multip conect life term

$12x + 3Y = 38$

$24x + 6Y = 78$ multiply by 3

This equstion can not solve by elemnation method

C, $5x + 15Y = 10$

$2x + 3Y = 2$

$-5x + 3Y = 10$

$-10x + 15Y = -10$

$-15x \quad = \; 0$

$\underline{\boldsymbol{x = \;\; 0}}$

- $5x \quad +15Y \; = \; 10$
- $5x0 \quad +15Y \; = \; 10$

$\frac{15Y}{15} = \frac{10}{5} \quad \Rightarrow \quad \mathbf{Y} = \frac{\mathbf{10}}{\mathbf{5}}$

$(\;0, \frac{10}{15}\;)$ it is true

d, $\quad 2x - 7\,Y \quad = \; -2$

$3x + Y \; = \; -20$

e, $\quad -3x + Y \quad = \; -9$

$-7x + 4Y \; = \; -$

$12x + 4Y \quad = \quad 36$ Multip by 4

$7x + 4Y \; = \; -6$

$-5x + \; 0 \; = \; 30$

$-\frac{5x}{-5} \quad = \quad \frac{-30}{-5}$

$\underline{\boldsymbol{x = \;6}}$

$-3x + \;Y \quad = \; -9$

$-3x6 + Y \; = \; -9$

$-18 + Y \quad = \; -9$

$Y \; = \; -9 + 18$

$Y = \; 9$ $\quad (6, 9)$ True value

Exercise

If twice the age of a son is added to age a father, then sum is 56. If twice the age of the father is added to the age of son then the sum is 82, find the ages of father and son.

Solution

$2S \; + F \; = 56$ $\quad S \; = \;$ Son

$2F \; + \; S \; = \; 82$ $\quad F =$ Father

4S + 2F = 112 Multipl by 2

4S + 2F = 112 Substract
S + 2F = 82

$\frac{3S}{3} = \frac{20}{S}\ \frac{30}{3}$ **S= 10 Year**

S= $\frac{20}{3}$ For Son

2S + F = 56 ⇨ $2x$ 10+F = 56⇨ F = 56-20

$\frac{2S}{3}$ + F = 56 ⇨ $2x\frac{20}{3}$ + F = 56

F= 56 -20

$\frac{10}{3}$ + F = 56 **F= 36**

F = 56 - $\frac{52}{3}$

F= $\frac{162-52}{3}$ = $\frac{116}{3}$

$\frac{116}{3}$ = $\frac{116}{3}$ = 38.667

Which mears 38 Year and 6 month

2. In a two digit number, the sum of the digite is 13. Twice the tens digit exceed the units digit by one find the number

$x +$ Y = 13

2 $x +$ 1 = Y

$2x +$ 2Y = 26 Multiply by 2

2 $x +$ 2Y = -1

$2x +$ -Y = -1

$2x +$ 2Y = -26

2 x −Y = -1

0 +3Y = 27

$\frac{3Y}{3}$ = $\frac{27}{3}$ **Y = 9**

$x +$ Y = 13

$x +$ 9 = 13

$x +$ 13 = 9 $\boldsymbol{x}$ = **4**

The number is **49**

3. I am thinking of a two – digit number If I write 3 to the left of my number and double the is three digit number the result is 27

times my orginal number. What is my orginal number?

4, Determine whether each of the following system of equstion has one solution, infinite solutions or no solution

a, $2x - Y = 6$ $\quad$ $2Y = 7$

$-2x + 3Y = 1$ $\quad$ $Y = \frac{7}{2}$

$2x - \frac{7}{2} = 6$

$2x = 6 + \frac{7}{2} = 2x = \frac{19}{2}$

$\boldsymbol{x = \frac{19}{4}}$

b, $x - Y = 7$ $\quad$ $2x$ - Y= 6

$2\ x + 3\ Y = 12$ $\quad$ $2x + \frac{2}{5} = 6$

$2\ x - 2Y = 12$ $\quad$ $2x = 6 + \frac{2}{5}$

$2\ x + 3Y = 12$ $\quad$ $2x = \frac{32}{5}\ x\ \frac{2}{1}$

$\frac{-5y}{-5} = \frac{2}{5}$ $\quad$ <u>$\boldsymbol{x = 64}$</u>

Y= $\frac{-2}{5}$ $\quad$ One Solution

C, $2x$ -+ 5Y= 12

$x - \frac{5}{2}$ Y $= 4$

$2x$ +5 Y= 12 $\quad$ $2x + \frac{20}{10} = 12$

$2x$ - 5Y= 8

$10 + \frac{10Y}{10} = \frac{4}{10}$ $\quad$ $2x = 12-2$

$\frac{2x}{2} = \frac{10}{2}$

Y $= \frac{4}{10}$ $\underline{\underline{x = 5}}$

<u>One Solution</u>

D, $2x$ -5 Y= 6

$2x$ +5 Y= 12 $2x = 12 - 2$

$2x$ -3 = 6

$2x$ - 5 Y= 6 $\frac{2x}{2} = \frac{9}{2}$

$2x$ +5 Y= 12

0 - 10 Y= -6 $x = \frac{9}{2}$

Y = $\frac{6}{10} = \frac{3}{5}$

<u>One Solution</u>

5, What must be the value of K so that the system of linear equstion

x - Y= 3

$2x$ - 2Y= K

i. Infineitely many solution

K = 2

ii, Unique solution

x - Y= 3

$2x$ - 2Y = K

<u>Solving non linear equstion</u>

Equstions involving Absolute value

<u>Example:-</u>

Find the Absolute value of the number

A, - 6 = $|-6| = 6$

B, 7 = $|7| = 7$

C, -0.8 = $|-0.8| = 0.8$

Exercise

Find the Absolut value of the following

A, $|-4| = 4$ B, $| = \frac{1}{2}|$ C, $|0| = 0$

Solve equstions in volving absolute Value

$|x| = 4$

$x = 4$

Properties

1, For any real number P, $|P| = |-P|$
2, For any real number P, /P/ ≥ 0
3, For any non- negitive number P,

$|x| = P$ means $x = P$ or $x = -P$.

Solve the equstion

$|x + 7| = 14$

$x + 7 = 14$ or $x + 7 = -14$

$x = 17 – 7$ or $x = -14\ -7$

$x = 7$ or $x = -21$

Exercise

1. Let A and B be point on a number line with coordinates 6 and -6, respectivivly, How for are the points.

A and B from the origin? How many points are there equal distance from the origin on a number line?

Solution:- The distance of A and B from the origin is 6 unit long on a real line.

a, $|x + 2| = 6$

$x + 2 = 6$ or $x + 2 = -6$

$x = 8$ or $x = -8$

b, $|5 - 3x| = 7$

$5 - 3x = 7$ or $5 - 3x = -7$

$\frac{-3x}{-3} = \frac{2}{-3}$ or $\frac{-3x}{-3} = \frac{-12}{-3}$

$x = \frac{-2}{3}$ or $x = 4$

C, $|x - 2| = 0$

$x - 2 = 0$ $x = 0+2 \Rightarrow$ $\underline{\boldsymbol{x = 2}}$

D, $|2x - 1| - 3 = 6 \Rightarrow |2x - 1| = 6-3$

$|2x - 1| = 3 \Rightarrow$

$2x - 1 = 3$ or $2x - 1 = -3$

$2x = 4$ or $x = -1$

$x = 2$ or $x = -1$

e, $|-8x - 2| - 6 = 10$

$|-8x - 2| = 10+6 = 16$

$-8x - 2 = 16$ or $-8x - 2 = -18$

$\frac{-8x}{-8} = -\frac{-18}{-8}$ or $\frac{-8x}{-8} = \frac{-14}{-8}$

$x = \frac{9}{4}$ or $x = \frac{7}{4}$

F, $-4|x + 3| = -28 \Rightarrow |x + 3| = -7$

$x + 3 = -7$ or x $x + 3 = 7$

$x = -10$ or $x = 4$

h, $\left|\frac{x+9}{2}\right| = 7$

$\left|\frac{x+9}{}\right| = 14$

$x+9 = 14$ or $x + 9 = -14$

$x = 14 - 9$ *or* $x = -14 - 9$

$x = 5$ *or* $x = -23$

K, 4-6 $|4 + 8x|$ = -20

- 6 $|4 + 8x|$ = - 120 ⇨ $|4 + 8x|$ = 20

4 + 8x= 20 Or 4 + 8x = -20

$\frac{8x}{8} = \frac{16x}{8}$ or $\frac{8x}{8} = \frac{-24}{8}$

$x = 2$ Or $x = -3$

L, 3 $|5+3x|$ +1 = 7

$\left|\frac{5+3x}{3}\right| = \frac{6}{3}$ ⇨ $|5+3x|$ =2

5+3x = 2 Or 5+3x = -2

$\frac{-3x}{3} = \frac{-3}{-3}$ Or $\frac{-3x}{-3} = \frac{-7}{-3}$

$x = 1$ Or $x\ \frac{7}{3}$

3.1 Quadratic Equstion

An equstion of the form a x^2 + bx+c =0 where a, b, c, ∈R and a ≠ 0 is a qudratic equstion. Hear are a is leading cofficient , b is the middel term and C is constanterm.

- **Solving Qudratic equstion**

  Ther are three the methods for solving qudratic equstions factorization completing the squre and Quadratic Formula

  - **Steps solve by factorization**

- ✓ Put all terms on one sid of the equal sign living zero on the other side
- ✓ Factorize the equstion
- ✓ Solve each of these equstion
- ✓ Cheek by inserting your Answer in the orginal equstion.

Example:-

1. $x^2 + 3x = 0$

$x\ (x+3) = 0$

$x = 0$ or x-3 = 0 x**= -3**

2, $x^2 - 6x - 16 = 0$

($xA = \pi r^2$-8) (x+2) = 0

x - 8 = 0 and x+2 = 0

$xA = \pi r^2$ = 8 and x = -2

Exercise

1. Solve the following

a, $x^2 - 5x = 0$

$x(x-5) = 0$

x = 0 and x - 5 = 0 x = 5

b, $x^2 - 7x + 10 = 0$

(x+2) (x+5) = 0

x+ 2 = 0 and $xA = \pi r^2$+5 = 0

x = - 2 and x = -5

C, $x^2 + x - 6 = 0$

(x+3) (x- 2) = 0

$xA = \pi r^2$ + 3 = 0 and x - 2 = 0

x**= - 3** and x**= 2**

d, $xA = \pi r^{2 2}$ - 4x + 3 = 0

(x-3) ($xA = \pi r^2$- 2) = 0

x - 3 = 0 and ×-1 = 0

x**= 3** and x**= 1**

Factorization method 2

Example:- $x^2 + bx + 9 = 0$

$(x+3)\ (x+3) = (x+3)^2 = 0$

$x \underline{\underline{= -3}}$

1, $4x^2 + 4x + 1 = 0$

$4x^2 + 2x + 2x + 1 = 0$

$2x\ (2x+1) + 1\ (2x+1) = 0$

$(2x+1)^2 = 0 \Rightarrow \quad \boldsymbol{x} = \frac{-1}{2}$

Exercise

1, Solve the following qudratic equstion using facterization

a, $x^2 + 1 + 2x = 0$

$x^2 + 5x + 5x + 25 = 0$

$x\ (\times + 5) + 5\ ((x+5$

$(x+5)^2 =- 0 \Rightarrow \quad x \underline{\underline{= 5}}$

b, $x^2 - 8x + 16 = 0$

$x^2 - 4x - 4x + 16 = 0$

$x\ (\times - 4) - 4\ (x - 4)$

$(x - 4)\ (x - 4 =$ and $x = -5(x - 4)^2 = 0 \quad x \underline{\underline{= 4}}$

C, $9x^2 - 6x + 1 = 0$

$\frac{9x2}{9} - \frac{6x}{a} + \frac{1}{a} = 0$

$x^2 = \frac{2}{3} \quad x + \frac{1}{9} \Rightarrow x^{2} - \frac{1}{3}\ x - \frac{1}{3}x + \frac{1}{9}$

$x\ (x - \frac{1}{3}) - \frac{1}{3}\ (-\frac{1}{3}) = 0$

$(x - \frac{1}{3})\ (x - \frac{1}{3}) = 0 \quad x = \boldsymbol{\frac{1}{3}}$

Completing the Square method

$x^2 + 6x + 4 = 0$ Using factorization method

a x^2 + b x + c = 0 a $\neq$ 0 and convert in to $(x + \frac{b}{2a})^2 + \frac{4ac - b2}{4a2}$ as follows

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 + \frac{4ac - b2}{4a2} = 0$

Example:-

1, $x^2 + 4x + 4 = 0$

$(x^2 + 4x + 4) + 4 - 4 = 0$

$(x + 2)^2 - 0 = 0$

x**= -2**

2, $3x^2 + 10x + 7 = 0$ Using Completing

$\frac{3x2}{3} + \frac{10x}{3} + \frac{7}{3} = 0$

$x^2 + \frac{10x}{3} + \frac{7}{3} = 0$

$x^2 + \frac{10x}{3} + \frac{100}{3} + \frac{7}{3} - \frac{100}{3} = x^2 + \frac{10x}{36} + \frac{100}{36} + \frac{7}{3} - \frac{100}{36}$

$(x + \frac{10}{6})^2 - \frac{16}{36} = 0 \Rightarrow (x + \frac{10}{6})^2 = \frac{16}{36}$

$x + \frac{10}{6} = \frac{2}{3}$

$x = -\frac{10}{6} \pm \frac{2}{3}$

Ther fore, x= -1 and $x = -\frac{7}{3}$ are solution

Exercise

Using completing squar method to solve the following

a, $x^2\ 2x + 1 = 0$

$x^2 + 2x + 4 + 1 - 4 = 0$

$(x + 2)^2 - 3 = 0$

$(x+2)^2 =- 3$

$x+2 = \pm\sqrt{3}$

$x = 2 \pm \sqrt{3}$

b, $x^2 + 4x + 1 = 0$

$x^2 + 4x + 16 + 1 - 16 = 0$

$(x+4)^2 - 15 = 0$

$(x+4)^2 = 15$

$x + 4 = \sqrt{15}$

$x = -4 \pm \sqrt{15}$

C, $2x^2 + 5x + 3 = 0$

$x^2 + \frac{5}{2}x + \frac{3}{2} = 0$

$x^2 + \frac{5}{2}x + \frac{25}{4} + \frac{3}{2} - \frac{25}{4} = 0$

$x^2 + \frac{5}{2}x + \frac{25}{4} + \frac{3}{2} - \frac{25}{4} = 0$

$(x + \frac{5}{2})2 + \frac{12-50}{8} = 0$

$(x + \frac{5}{2})2 - \frac{38}{8} = 0$

$(x + \frac{5}{2})2 = \frac{38}{8}$

$x + \frac{5}{2} - \frac{5}{2} = -\frac{5}{2} \pm \sqrt{\frac{38}{8}}$

$x = -\frac{5}{2} \pm \sqrt{\frac{38}{8}}$

3.2 The Quadratic Formula Method

A $x^2 + \text{b}\,x + \text{C} = 0$ a$\neq 0$

$(x + \frac{b}{2a})^2 + \frac{4ac - b2}{4a2} = 0$

Solving for x, $(x + \frac{b}{2a})^2 + \frac{b2 - 4ac}{4a2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b2\ -4ac}{4a2}}$

$x - \frac{b}{2a} = \pm\sqrt{\frac{b2\ -4ac}{2a}}$

x - b = $\pm\sqrt{\frac{b2\ -4ac}{2a}}$

For quadratic equation a x^2 + bx+ ⇦

$x = \frac{-b \pm \sqrt{b2-4ac}}{2a}$

Exercise

Cheeck whether the following quadratic equstions have two real roots, one real root or no real roots

a, $x^2 + 2x + 3 = 0$

$b^2 - 4ac < 0$

$4 - 4 x 1 x 3 < 0$

$4 - 12 < 0$

$-8 < 0$ no real root

b, $x^2 + 8x + 7 = 0$

$b^2 - 4ac < 0$

$64 - 4 x 7 < 0$

$6 4 - 28 < 0$

$36 < 0$. Two distinct root

C, $x^2 + 12x + 12 = 0$

$b^2 - 4\ x\ 12$

$144 - 48 < 0$

<u>96 < 0</u> Two distinct root

3.3 Discriminant

When using the quadratic formula, you should be aware of three possibilities.

These three possibilities are distinguished by a part of the formula called the discriminant. The discriminant is the vaw under the radical sign which is b^2 - 4ac.

A quadratic equation with real numbers as coefficients can have the following.

The root of the quadratic equation are $x = \frac{-b \pm \sqrt{b}}{2a}$ where D = b^2 - 4ac.

- ✓ If D > 0 then the roots are real and distinct (Unequal) the quadratic equation has two distinct roots
- ✓ If D = 0 then the root are real and equal (Coincident) . the quadratic equation has exactly one real root
- ✓ If D < 0 , then there are no real roots

Relation Ship between roots and Coefficients of a Quadratic equation

If $r_1 = \frac{-b \pm \sqrt{b2-4ac}}{2a}$ and $r_2 = \frac{-b - \sqrt{b2-4ac}}{2a}$

$r_1 + r_2$ = -b + $\frac{\sqrt{b2-4ac}}{2a}$ + - b - $\frac{\sqrt{b2-4ac}}{2a}$ $= \frac{-b}{a}$

$r_1 . r_2$ = (-b + $\frac{\sqrt{b2-4ac}}{2a}$ ($\frac{-b-\sqrt{b2-4ac}}{2a}$ $= \frac{c}{a}$

Example:-

A $x^2 + 6x + C = 0$ If the sum of the root is $\frac{-6}{5}$ and the product is $\frac{1}{5}$ then find the values of a and C

$r_1 + r_2 = -\frac{6}{5} = -\frac{6}{a}$ a = - $\frac{6x5}{-6}$ = 5

$r_1 x r_2 = \frac{1}{5} = \frac{c}{a} = \frac{1}{5} = \frac{c}{5}$ ⇨ C = $\frac{1x5}{5}$ = 1

Exercise

1. Let $x^2 + bx + C = 0$ If the sum of the roots of the quadratic equation is 10 and the product of the root is 16, Find

b and C

$x^2 + bx + C = 0$

$r_1 + r_2 = 10 = \frac{10}{1} = \frac{-b}{a}$ ⇨ **b= -10**

$r_1 \times r_2 = \frac{10}{1} = \frac{c}{1}$ ⇨ **C= -16**

2. Find the quadratic equation a $x^2 + 3x + C = 0$ when the sum of the root is – 3 , product is 2

$r_1 + r_2 = \frac{-3}{1} = \frac{-3}{a}$ ⇨ a = $\frac{-3x1}{-3}$ = 1

$r_1 xr_2 = \frac{2}{1} = \frac{c}{1}$ ⇨ **C= 2**

3, Solve the following using quadratic formula and cheek using any other method of solving quadratic equstion.

a, x^2 - a x = 0

$x(x - 9) = 0$

$x = 0 \quad x = 9$

b, $3x^2 - 75\ x = 0$

$\frac{3x2}{3} - \frac{75x}{3} = 0$

$x^2 - 25x = 0$

$x(x - 25) = 0 \Rightarrow x = 0$ or $x = 25$

C, $x^2 - 6x + 5$

$x^2 - 6x + 9 + 5 - 9$

$x^2 \quad (x - 3)^2 - 4 \Rightarrow 0 \Rightarrow (x - 3)^2 = 4$

$(x - 3) = \sqrt{4}$

$x = 3 \pm 2 =$ **<u>5 or 1</u>**

D, $x^2 + 6x + 8 = 0$

$(x + 4)(x + 2) = 0$

$x = -4$ Or $x = -2$

E, $10x^2 - 9x + 6 = 0$

$x = \frac{-b \pm\sqrt{b2-4ac}}{2a}$

$\frac{-81\pm\sqrt{81-240}}{20} = \frac{\mathbf{81\pm\sqrt{159}}}{\mathbf{20}}$

<u>Exercise</u>

1. Solve the following Qudratic equstion

a, $x^2 + 8 + 15 = 0$

$(x+3)(x+5) = 0$

$x \quad -3$ Or $x = -5$

b, $x^2 + 6x + 9 = 0$

$(x + 3)(x + 3) = 0$

x - 3 Or $x = 3$

C, $x^2 - 5 + x\ 7 = 0$

$x^2 - 5x + 25 + 7 - 25 = 0$

$(x - 5)^2 - 18 = 0$

$(x - 5)^2 = 18$

$x - 5 = \sqrt{18}$

$x = 5 \pm \sqrt{18} \Rightarrow$ $\mathbf{5 \pm 3\sqrt{2}}$

2. Find the value of K for which the Quadratic expression $(x - K)(x - 10) + 1$ has integral root

$(x - K)(x - 10 + 1) \Rightarrow x(x - 10) - K(x - 10) + 1$

$x^2 - 10x - Kx + 10K + 1$

3, Find the value of K such that the equation $\frac{P}{x+r} + \frac{9}{x-r} = \frac{K}{2x}$ has two equal roots.

$b^2 - 4ac > 0$

$\frac{P}{x+r} + \frac{q}{x-r} = \frac{K}{2x}$

$\frac{P(x-r) + q(x+r)}{(x+r)(x-r)} = \frac{K}{2x}$

$(x+r)(x - r)$

$(P(x-r) + q(x+r)\ 2x = (x+r)(x-r)\ K$

$(Px - Pr + qx + qr)\ 2x = (x^2 - xr + xr - r^2)$

$2Px^2 - 2Prx + 2qx^2 + 2xqr = Kx^2 - Kr^2$

$2Px^2 + 2qx^2 + Kx^2 - 2Prx + 2qrx + Kr^2 = 0$

$(-2Pr + 2qr)^2 - 4(2P + 2q + K)Kr^2 = 0$

Or $\frac{K}{2} > 0$

$\frac{K}{2} = \frac{0}{1}$ $\underline{K = 0}$

4. Find the drastic equation with rational coefficient when one root is $\frac{1}{2+\sqrt{5}}$.

$\frac{1}{2+\sqrt{5}} = \frac{1}{a}$ a= $2+\sqrt{5}$

$r_1 , r_2 = \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{1}{c}$ C = 1

$= \underline{\mathbf{2+\sqrt{5}A\, x^2 - x + 2\sqrt{5}}}$

5, If the coefficient of X in the Quadratic equation $x^2 + b\,x + C = 0$ We taken as 17 in place of 13 its roots were found to be - 2 and 15 find the root of the original quadratic equation

b = 17 C -2

b = 13 C – 15

$\frac{b}{a}$ =

6, For what value of K , b both the Quadratic equations $6x^2 - 17x + 12 = 0$ and $3x^2 - 2x + K = 0$ will be have common root.

$\frac{-17}{12} = \frac{2}{K}$ ⇨ $\frac{24}{-17} = \frac{17\,K}{17}$ ⇨ $\mathbf{K = \frac{24}{17}}$

7, Find the value of K, such that the Quadratic equations

$x^2 - 11\,x + K = 0$ and $x^2 - 14\,x + 2K = 0$

$r_1 + r_2 = \frac{-b}{a}$

$r_1 , r_2 = \frac{C}{a}$

$\frac{-b}{a} = \frac{C}{a}$

$\frac{11}{1} = \frac{K}{1} =$ K = 1

$\frac{-14}{4} = \frac{2K}{2}$ ⇨ $\underline{\mathbf{K = -7}}$

8, Let $x^2 = 17\times + y$

$Y^2 = x + 17y$, $y \neq x$ find $\sqrt{\ } x^2 + Y^2 + 1$

- Equations involving exponents and radical

- Equation's involving exponents for a > 0, $a^x = a^y$ If and only if x=y some rules of exponential equation a , m and n real number and a ≠ 0

  a, $\frac{1}{a^n} = a^{-n}$

  b, $a^m \; a^n = a^{m+n}$

  C, $(a^m)^n = a^{mn}$

  D, $(ab)^n = a^n \; b^n$

  e, $(\frac{a}{b})^n = \frac{a^n}{b^n}$

  F, $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$

Exercise

Solve

a, $3^x = 81 = 3^x = 3^4$ $\underline{x = 4}$

b, $2^x = \frac{1}{8} \Rightarrow 2^x = 2^3 = \underline{x = -3}$

Example:-

A, $2^{x+4} = 32 \Rightarrow 2^{x+4} = 2^5 \Rightarrow x+4 = 5 = x = 1$

b, $9^{x+4} = 27$

$3^2(x^{+4} = 3^3 \Rightarrow 2x + 4 = 3$

$2x = 3-4 \Rightarrow \frac{2x}{2} = \frac{-1}{2} \; x = -\frac{1}{2}$

Exercise

Solve the following

A, $3^{2x+1} = 243$

$3^{2x-1} = 3^5 \Rightarrow 2x - 1 = 5$

$\frac{2x}{2} = \frac{6}{2}$ $\underline{x = 3}$

b, $32^x = 2^{x+4} = 2^{5x} = 2^{x+4}$

$5x = x + 4 \Rightarrow 5x - x = 4 \quad \frac{4x}{4} = \frac{4}{4}$ $\underline{x = 1}$

Equstion involving exponents 2

Example:-

A, $(x^2 + 6x)^{\frac{1}{4}} = 2$

$(x^2 + 6x)^{\frac{1}{4}} x^4 = 2^4$

$x^2 + 6x = 16$

$x^2 + 6x - 16$

$(x + 8)\ (x - 2) = 0$

$x = -8$ Or $\underline{x = 2}$

B, $5Y^{\frac{2}{3}} + 3 = 23$

$5Y^{\frac{2}{3}} = 23 - 3$

$\frac{Y}{5}\frac{2}{3} = \frac{20}{5}$

$Y^{2/3} = 4$

$Y^{2/3} = 4$

$Y = \pm 4^{3/2} = Y = \pm (\sqrt{4})^3 = \underline{\pm 8}$

Exercise

Solve the following

a, $(x^2 + 2x)^{1/2} = 4$

$x^2 + 2x) = 4^2$

$x^2 + 2x = 16$

$x^2 + 2x - 16$

$x^2 + 2x + 1 - 16 - 1 = 0$

$(x1)^2 = 17 = 0$

$(x+1)^2\ 17$

$x+1 = \sqrt{17} \Rightarrow \underline{x = -1 \pm}\ \sqrt{17}$

b, $(6x^{1/2)})^{1/4} = 2$

$(2^4 x^{1/2}) = 2^{x4}$

$2^4 x^{1/2} = 2^4$

$2^{4-4} x^{1/2} = 2^{4-4}$

$x^{1/2} = 1$

$\underline{x = 1}$

Equation involving radicals

- ✓ Radical equstions are equstions that consains variables in the radical (the expression under a radical symbol

Example:-

A, $\sqrt{3x} + 18 = x = x - 3$ and $\sqrt{x+5} - \sqrt{x-3} = 2$ are radical equations.

Steps solving radical equation

- ✓ 1 positive radical number and radiand express under a radical symbol)

2 , I solte the radical expression on one side of the equal sign. Put all remaining terms on the other side

- ✓ 3, If the radical is square root, then square both sides of the equation.
4, Solve the resulting equation.
5, If a radical term still remains repeat steps 1 – 2 .

6. Check solutions by substitution them in to the original equation

Example:- 1, $\sqrt{15-2x} = x$

$(x+3) = (3x-1)^2$

$(x+3) = 9x^2 - 6x$

$x^2 + 2x - 15 = (x-3)(x+5) \quad x = 3$ Or $\underline{x = -5}$

Example:- 2, $\sqrt{x+3} = 3x - 1$

$(x+3) = (3x-1)^2$

$(x+3) = 9x^2 - 6x + 1$

$9x^2 - 6x - +1 - 3 = 0$

$9x^2 - 7x - 2 = 0$

$x^2 - \frac{7x}{9} - \frac{2}{a} = 0$

$(x-1)(9x+2) = 0$

$x = 1 \qquad x = -\frac{2}{a}$

3, $\sqrt{2x+3} + \sqrt{x-2} = 4$

$2x + 3 + x - 2 = 16$ Squaring bother

$3x + 3 - 2 \quad 16$

$3x = 16 - 1$

$\frac{3x}{3} = \frac{15}{3} \Rightarrow \underline{x = 5}$ **Frame new**

Exercise

Solve the following equations involving radicals.

a, $\sqrt{3x-2} \ = x$

$3\,x - 2 \ > 0$ then $x \geq \frac{2}{3}$

$x \geq \ 0$

Squaring Both sides , $3x$-2 $= x^2$,

$x^2 - 3\,x + 2 \ = \ 0 \ \Rightarrow (x-1)\ (x\text{-}2) \ = \ 0$

$x = \ 1\,,\ \ x \ = \ 2$

b, $\sqrt{3x-2} \ \ = \ \sqrt{4x+2}$

$(\,3\,x\,\text{-}2\,) \ = \ 4\,x\,+2$ Squaring Both sides

$3\,x - 4\,x \ = \ 2+2$

$\frac{-x}{-1} \ = \ \frac{4}{-1}$ $\underline{x = \ \mathbf{-4}}$

C, $\sqrt{3x+7} \ \ + \ \sqrt{x+2} \ = 1$

$3\,x+7 \ \ + \ x{+}2 \ = 1$

$3\,x + x + 9 \ = \ 1$ Squaring both said

$4\,x \ = \ 1\text{-}9 \ \Rightarrow \frac{4x}{4} \ = \ \frac{-8}{4}$

$\underline{x \ = \ \mathbf{-2}}$

3.4 Some Applications of Solving Equation

Exercise

1, A ball issnot in to the air from the ground. It intial velocity is 32 meter per second. The equstion h= -16t^2 + 32t + 48 can
be used to model the height of the ball after t second. About how long does it take for the ball to hit the ground?

H = -16t^2 + 32t + 48 = 0

t^2 - 2t – 3 = 0 $\Rightarrow$ (t – 3) (t + 1) = 0 t $= 3$ or $+ = -1$ Answer $\underline{t = 3}$

Exercise

1, The cost of atickes to a foot ball mach is Birr 75,00 for Adult. On acertain day attendance at the foot ball match is 25000and the total gate revenus is Birr 1,375,000. How many children and adults bougt tickets ?

Let a = Adult
C = Children

$a + C = 25000$
So $c + 75a = 1375000$

$a = 25000 - C$

$c = 25000 - a$

So $(25000 - a) + 75a = 1,375,000$

$750000 -$ So $a + 75a = 1375000$

$1250000 + 25a = 1375000$

$25a = 1375000 - 1250000$

$\frac{25a}{25} = \frac{250000}{25}$

$a = 5000$ $\qquad$ $C = 25000 - a$

$C = 25000 - 5000$

$\underline{C = 20,000}$

2, The cost of two tables and three chairs is Birr 705. If the table cost Birr 40 mor than the Chair, Find the cost of the
table and the chair.

Let the cost of table be X birr and that of Chair be Y birr. From the Given,

$2x + 3Y = 705$
$x - Y = 40$

$2x + 3Y = 705$

$2x - 2y = 80$ - multiple by 2

$5Y = 625$

$Y = 125 \Rightarrow 2x + 3Y = 705$

$2x = 705 - 375$

$\frac{2x}{2} = \frac{330}{2}$

$\underline{x = 165}$

Summary and review Exercis

1, If $x = 2$ and $Y = 3$ is a solution of the equstion $8x - ay + 2a = 0$ find the value of a.

$8x - ay + 2a = 0$

$2 x 8 - 3a + 2a = 0$

$16 - a = 0 \Rightarrow \underline{a = 16}$

2, The sum of two numbers is 13 and their product is 42 find the numbers.

$x + Y = 13$

$xY = 42$

$x = 13 - Y$

$(13 - Y)Y = 42$

$-Y^2 + 13Y - 42 = 0$

$Y^2 - 13Y + 42 = 0$

$(Y-6)(Y-7) = 0$

$\underline{Y = 6 \quad Y = 7}$

3, The present age of Gaddis a is 4 Year more than twice the present age of his corssin Chala. Chsla's present age is 2 Years more than $\frac{1}{3}$ of the present age of Gadisa. Find the diffrence of thir ages.

Let G - Gaddisa
C - Chala

$G = 2C + 4$

$C = \frac{1}{3}G + 2$

Using Substitution , $G = 2\left[\frac{1}{3}G + 2\right] + 4$

$G = \frac{2}{3}G + 4 + 4$

$G - \frac{2}{3}G = 8$

$\frac{3G - 2G}{3} = 8$

$\frac{1}{3}G = 8$

$3 x \frac{1}{3} G = 8 x \frac{3}{1} \Rightarrow G = 24 \quad \underline{C = 10}$

4, We want to fance a field whose length is three times the width and whe have 100 meters of Fencing material. If we use
all the dimensions of the field be?
Let the width of the field X and Y

$2x + 2Y = 100$

$Y = 3x$

$2x + 6x = 100$

$\frac{8x}{8} = \frac{100}{8}$ $x = 12.5$ Y= 37.5

5, Three Coffees and Two makiatos cost a total of Birr 17. Two coffees and four makiatos cost atotal of Birr 18.
What is the individual price for a single coffee and a single makiato?
Let C and m represent coffee and makiato

$3C + 2m = 17$

$2C + 4m = 18$

$6C + 4m = 34$ multiply by 2

$-6C + 12m = 54$ multiply by 2

$0 \frac{-8m}{-8} = \frac{-20}{-8}$

$m = 2.5$

$C = \frac{17.5}{3} = 4$

6, Find the Value of k, such that the Integral root $(x - K)(x - 10) + 1 = 0$

$x(x - 10)^{-K}(x - 10) = 0$

$x^2 - 10\times - Kx + 10k = 0$

$x^2 - (k + 10)x + 10k = 0$

$r_1 + r_2 = 10 + K$

$r_1 + r_2 = 10k + 1$

$r_1 + r_2 = k + 10$ -------- i

$r_1 + r_2 = 10k + 1$ --------- ii

$K = r_1 + 12 - 10$

$r_1 + r_2 = 10(r_1 + r_2 - 10) + 1$

$r_1 + r_2 = 10r_1 + 10r_2 - 100 + 1$

$r_1 + r_2 - 10r_1 - 10r_2 + 100 = 1$

$r_1 (r_2 - 10) - 10 (r_2 - 10)$

$(r_1 - 10)(r_2 - 10) = 1$

$r_1 = 11 \quad r_2 = 11 \quad K = 2$ Or

$r_1 = 9 \quad r_2 = 9 \quad K = 8$

7, Find the value of K for the expression the equation $\frac{P}{x+r} + \frac{q}{x-r} = \frac{K}{2x}$ has two equal root.

Remove the denominator

$(2P + 2q - K)x^2 - 2r(P - q)x + kr^2 = 0$

$4r^2 (P - q)^2 K + (P - q)^2 = 0$

$K^2 - 2(P + q)K + (P - q)^2 = 0$

$K = P + q \pm 2Pq$

8, If the coefficient of X in the quadratic equation $x^2 + bx + C = 0$ was taken as 17 in place of 13 it root where found to be

- 2 and – 15 find the root of the original Quadratic equation.

$x^2 + 17x + C = 0$ is – 2 and 15

Theus $C = r_1 + r_2 = 30$

Then, the original equation is $x^2 + 13x + 30 = 0$

$(x + 3)(x + 10) = 0 \; x - 3 \; x = -10$

Solve this equation , we will get $x = -3, -10$.

9, For what value of K, both the Quadratic equations $6x^2 - 17x + 12 = 0$ and $3x^2 - 2x + K = 0$

Will have a common root.

$x = \frac{3}{2}, \frac{4}{3}$

$r_1 + r_2 = \frac{K}{3} = \frac{4}{3} x \frac{3}{2}$

$\frac{K}{3} = \frac{12}{6} \quad \frac{6K}{6} = \frac{3x12}{6}$

$\underline{\underline{K = 6}}$

Unit 4

Solving In Quality

4.1Revision on linear Inequality in one Variable

What is Linear Inequality means an Inequality that contains $>$ $<$ $\geq \leq$ sign and equation is called linear inequality

Example:-

Solve the following Inequality

a, $x - 5 > 2 \quad x > 2+5 \Rightarrow$

$\underline{x > 7}$

b, $2x > 4 \Rightarrow \frac{2x}{2} > \frac{4}{2} \Rightarrow x > 2$

C, $\frac{1}{3}x < 1 \Rightarrow \frac{3}{1} \times \frac{1}{3}\ x < \frac{1}{3} x \frac{3}{1}$

$\underline{x < 3}$

Exercise

Solve the following Inequality

a, $x - 5 > 3 \Rightarrow x > 3+1 \Rightarrow x > 4$

b, $-\frac{1}{5}\ x \leq 2 \Rightarrow - \frac{1}{5}x\ \ \frac{1}{5}\ \times \geq 2\ x\ - \frac{5}{1}$

$\underline{\times \geq -10}$

C, $3x + 1 \geq 7 \Rightarrow 3x > 7+1$

$\frac{3x}{3} > \frac{8}{3} \Rightarrow x > \frac{8}{3}$

Notes

The symbol [a , b] clothed Interval with ended points a and b, it shows the set of all real numbers $x$such that

A $\leq x \leq$ b.

The symbol (a , b) is open interval with end – points a and b, it shows the set of all real numbers x such that

a $< x <$ b the symbol [a , b) and (a , b] is half.

Open Interval They show the set of all real numbers $\times$such that a $\leq x <$ b and a $< x \leq$ b , respectivly.

Exercise

1, Solve the inequality below

a, $x - 2 > 3 \Rightarrow \quad x > 3 \quad (-\infty, 3)$

b, $x + 1 \leq 5 \Rightarrow \quad x \leq 5 - 1$

2, Solve the following in ear inequality

a, $3x - (2x + 2) \leq 7$

$3x - 2x - 2 \leq 7$

$x \leq 7 + 2 \Rightarrow \underline{\underline{x \leq 9}}$

b, $4x - (2x + 8) \geq 0$

$4x - 2x - 8 \geq 0$

$\frac{2x}{2} \geq \frac{8}{2} \Rightarrow \underline{\underline{x \geq 4}}$

C, $5x - 3(x - 1) < 5$

$5x - 3x + 3 < 5$

$2x < 5 - 3 \Rightarrow \frac{2x}{2} < \frac{2}{2} \quad \underline{\underline{x < 1}}$

D, $8x - 5 > 2x + 4$

$8x - 2x > 4 + 5$

$\frac{6x}{6} > \frac{9}{6} \Rightarrow x > \frac{9}{6} \Rightarrow x > \frac{3}{2}$

4.2. Systems of Linear Inequalities in two Variables

Linear inequalities in two variables represent the unequal relation between two algebraic expressions that includes

two distinct variables.

Solve

$2x + 3y \geq 12$

$8x - 4y > 1$

$x < 4$

$2x + 3y = 12$

$\frac{3y}{3} \Rightarrow \frac{12-8}{3} \quad \mathbf{Y = \frac{4}{3}}$

$3Y \geq -2x + 12$

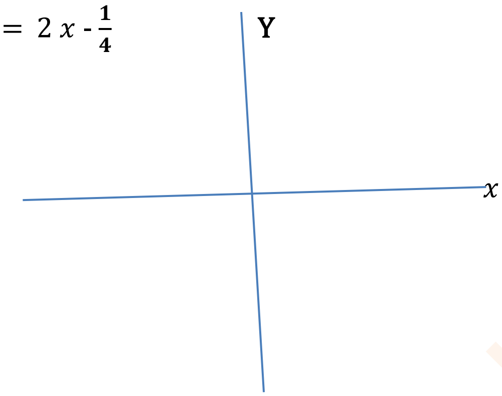
$Y \geq \frac{-2}{3}x + 4$

$\frac{4y}{4} < \frac{8x-1}{4}$

$\mathbf{Y < 2x - \frac{1}{4}}$

$Y = \frac{-2}{3}x + 4$

$Y = 2x - \frac{1}{4}$



4.3 Inequlityes Involving Absolut Value

White graphing absolute value inequality on a number line, you have to keep the following things in mind.

⇨ Use open dots at the end points of the open interval (a , b)

⇨ Use closed dots at the end points of the closed interval [a , b]

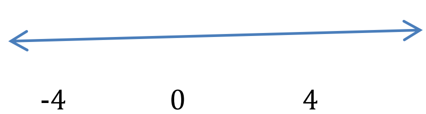
Exercise

Put the following absolute value in equalitys on a number line.

a, $|x| < 2$

b, $|x| \leq 5$

C, $|x| > 4$



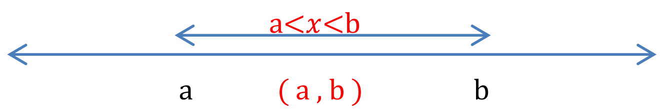
Inequality Involving absolute Value

Graphing Absolute value inequalities

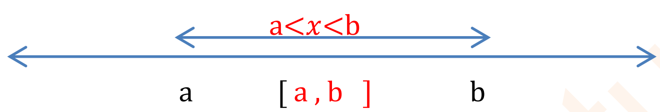
- While graphing absolute value in equities on a number line , you Have to keep the following things in mind.

  - ✓ Use open dots at the end points of the open interval(a , b)

  - ✓ Use closed dots at end points of the closed interval a , b
    Inequalityes with $<$ Or $\leq$



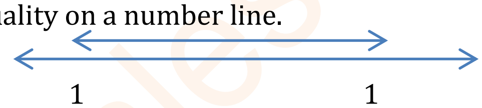
$a \leq x \leq b$



Example:-

Graph the following Absolute value inequality on a number line.

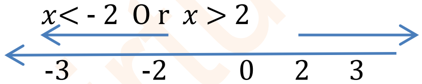
a, $|x| \leq 1 \quad -1 < x < 1$



Exercise

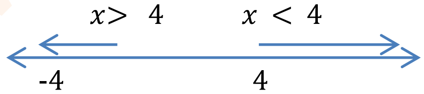
Put the following absolute value in equalitys on a number line.

a, $|x| < 2$ $\qquad x < -2$ Or $x > 2$



b, $|x| \leq 5$ $\qquad -5 < x < 5$

C, $|x| > 4$ $\qquad x > 4 \quad x < 4$



Inequalities Involving Absolut Value

Assum K is an algebraic expression and C is a positive number.

The solution of $|K| < C$ are the numbers that satisfy $-C < K < C$

The solutions of $|K| < C$ are the numbers that satisfy $-C < K < C$

These rules are valid if $<$ is replaced by $\leq$ and $>$ is replaced by $\geq$.

Example:-

Solve $|x+2| < 4$.

$|x+2| < 4$

- $4 < x+2 < 4$
- $4 < x+2$ and $x+2 < 4$
- $6 < x$ and $x < 2$

i, If the problem was $|x+2| \leq 4$ then the solution would have been $[-6, 2]$ Or $-6 \leq x \leq 2$

If $|x+2| > 4$ then the solution would have been $(-\infty, -6) \cup (2, \infty)$ Or $x<-6$ Or $x>2$.

ii, Also If $|x+2| \geq 4$ then the solution would have been $(-\infty, -6] \cup [2, \infty)$ Or $x \leq -6$ 0 r $x \geq 2$,

Example:- 2

Solve $|3x-2| \geq 7$

$3x - 2 \geq 7$ Or $3x - 2 \leq -7$

$3x \geq 7+2$ Or $3x \leq -7+2$

$\frac{3x}{3} \geq \frac{9}{3}$ Or $\frac{3x}{3} \leq \frac{-5}{3}$

Steps For solving Linear Absolute Value Inequalities

1, Identify what the a absolute value inequality is set « equal to -------

a, If the absolute value is less than zero, there is no solution.

b, If the absolute value is less than or equal to zero, there is one solution. Just set the argument equal to zero and solve.

C, If the absolute value is grater than zero, the solution is all real numbers except for the value which makes it equal to zero. This will be written as as a union

D, If the Absolut value is greater than or equal to zero the solution is all real numbers.

2, Graph the Answer on a number line and write the answer in interval notation.

Example:-

$|-x+4| < 0$

$-0 < -x+4 < 0$

$0 < -x+4 < 0$

$0 < -x+4$ and $-x+4 < 0$

$-4 < -x$ and $-x < -4$

$4 < x$ and $x < 4$ There is no, rall number satisfy Inequalities

Example:- 4

$|-5x - 7| \leq 0$

$|5x - 7| = 0$

$\frac{-5x}{x} = 7$

$x = \frac{-7}{5}$

Exercise

1, Solve the following absolut value

a, $|x+3| < 7$

$-7 < x+3 < 7 \Rightarrow -7 < x+3$ Or $x+3 < 7$

$-10 < x$ Or $x < 4$

b, $|x-5| \leq 2$

$-2 \leq x-5 < 2$

$-2 < 5x-5$ and $xx-5 < 2$

$-2+5 < x$ and $x < 2+5$

$3 < x$ and $x < 7$

$3 < x < 7$

C, $|x-7| > 4$

$-4 > x-7 > 4$

$-4 > x-7$ and $x-7 > 4$

$-4+7 > x$ and $x > 4+7$

$3 < x$ and $x < 11$

D, $|4x-5| \geq -2$

$4x-5-2 > 4x-5 > 2$

$-2 > 4-5$ and $4x-5 > 2$

$-2+5 > 4x$ and $4x > 2+5$

$\frac{3}{4} > \frac{4x}{4}$ and $\frac{4x}{4}$ $x > \frac{7}{4}$

e, $|2x-3| \geq 5$

$-5 \geq 2x\ 3 \geq 5$

$-5 \geq 2x-3$ and $2x-3 \geq 5$

$-5+3 > 2x$ and $2x > 5+3$

$-1 > x$ and $x > 4$

4.4 Qudratic Inequalities

The general forms of the qudratic inequlities : are $ax^2 + bx + C < 0$,

$ax^2 + bx + C \leq 0$, $ax^2 + bx + C > 0$ and $ax^2 + bx + C \geq 0$, where $\neq$ a and a , b, c $\in$ R

- **Solving quadratic inequalities Using product Properties**

Product properties

1, m . n > 0 If and only If

i, m > 0 and n > 0 Or ii m< 0 and n< 0

2, m . n < 0, If and only If

ii, m > 0 and n < 0 Or ii m< 0 and n> 0

Example:- 1

Solve $(x-3)(x-1) < 0$

$x-3 > 0$ and $x-1 < 0$

$x > 3$ and $x < 1$

(1 , 3) Interval notation

Example:- 2

$(x-4)(x-2) > 0$

$x-$ $4 > 0$ and $x-2 > 0$

$x > 4$ and $x > 2$ 3 Solution

$(-\infty, 2) \cup (4\ \ \infty)$ interval notation

Exercise

1, Solve the following Qudratic inequalities using product rule

a, $x^2 - 6x + 8 < 0$

$(x-2)\ (x-4) < 0$

$x- < 0$ and $x-4 < 0$

$x < 2$ and $x < 4$

$2 < x < 4$

b, $x^2 - 2x\ - > 0$

$(x-2)\ (x+2) > 0$

i $x-2 > 0$ Or ii $x-2 < 0$

$x+2 > 0$ $x+2 < 0$

From i, $x > 2$ and $x > -2$, thus, $x > 2$

ii, $x < 2$ and $x < -2$, thus, $x < -2$

Ther fore, $x < -2$ Or $x > -2, x > 2$

C, $-2x^2 + 5x + 12 \geq 0$

$-\frac{3}{2} \leq x \leq 4$ There fore $-3 \leq x \leq -\frac{5}{2}$

$(2x+3)\ (x-4) \leq 0$

Using product properties.

$2x + 3 \geq 0$ $\qquad$ $2x + 3 \leq 0$

$x - 4 \leq$ $\qquad$ $x - 4 \geq 0$

From i $x \geq -\frac{3}{2}$ and $x \leq 4$

From ii $x < -\frac{3}{2}$ and $x \geq 4$ thus, there is no solution $-\frac{3}{2} \leq x \leq 4$

4.5 Solving quadratic inequalities Using Sing Chart

- <u>Steps in solving quadratic inequlities by sign Chart</u>

1, Re – write the inequlity to get a 0 on the right – hand side

2, Factor (If possible) the left – hand side

3, « Graph » each factor a sign chart using a display of sign . A root of any facter

<u>Example:-</u> is a key point

1, $|-5-7| \leq 0$

$|-5-7| \leq 0$

$-5 \times -7 \leq 0 \qquad \Rightarrow \frac{-5x}{-5} \leq \frac{7}{-5}$

$x \leq \frac{-7}{5}$

2, $|5+8| > -2$

$6x + 8 > -2$

$8x > -2 - 8 \qquad \Rightarrow \frac{8x}{8} > \frac{-10}{8}$

$x > \frac{-10}{8}$

4, Each key point or interval is displayed using a columen.

<u>Example:-</u> $(x-4)(x+3) < 0$

$x = -3$ Or $x = 4$

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Factor	$x < -3$	$x =-3$	$-3 < x <- 2$	$x = 4$	$x > 4$
$x - 4$	--- ---	-----	--------	0	+++
$x+3$	------	0	+++	++	+++
$x+3)\ (x -4)$	+++	0	-----	0	+++

$(x-3)\ \ (x4)\ < 0$ If $-3 < x<4$

Alternatively $x \in$ **(-3 , 4)**

Solve $(x+3)\ (x+2) \geq 0$

Factor	$x < -3$	$x =-3$	$-3 < x <- 2$	$x = -2$	$x > -2$
$x + 3$	--- ---	0	+++	++	++++
$x+5$	------	--	--	0	+++
$(x+2)\ (x +3)$	+++	0 0	--	0	+++

Interval notation as $(-\infty\ , -3 \quad \cup\ \ [\ -2 , \infty\ )$

<u>Exercise</u>

Using Sign Chart to solve the following quadratic inequlities. Cheek the solution

a, $(x-3)\quad (x+1)\ < \ 0$

$x = \ (3, -1)$ result

b, $x^2 + 2x\ \ -\ \ 15$

> 0 to $(x\text{-}3)\ (x+5) < 0$

Factor	$x < -5$	$x =-5$	$5 < x < 3$	$x = 3$	$x < -1$
$x - 3$	---	--	----	0	+++
$x+5$	--	0	++++	+	++++
$(x\text{-}3)\ (x +5)$	+++	0	---	0	++++

Ther fore $x < -5$ O r $x > 3.$

C, $2x^2 + x\ -1 > 0\ ,\quad (2x - 1)\ (x+1\ ) >0$

Factor	$x < -5$	$x = -1$	$-1 < x < \frac{1}{2}$	$x = \frac{1}{2}$	$x > -\frac{1}{2}$
$2x - 1$	- - -	- -	- - - -	0	+ + +
$x+1$	- -	0	+ + + +	+ + +	+ + + +
$(2x-1)(x+1)$	+ + +	0	- - -	0	+ + + +

Ther for $x < -1$ Or $x > \frac{1}{2}$

d, $10x^2 - 19x + 6 \leq 0$ to $(sx - 2)(2x - 3) < 0$

Factor	$x\ x \leq \frac{2}{5}$	$x = \frac{2}{5}$	$\frac{2}{5}$- $1 < x < \frac{3}{2}$	$x = \frac{3}{2}$	$x > -\frac{3}{2}$
$5x - 2$	- - -	0	+ + + +	+ + +	+ + + +
$2x$-3	- - -	0	- - -	0	+ + + +
$(5x-2)(2x-3)$	+ + +	0	- - -	0	+ + + +

Ther fore $\frac{2}{5} < x < \frac{3}{2}$

Exercise

1, on Adddis Abab interste high way the speed limit is 55 Km/s. The minimum Speed limit is 45 Km/n. write a compond

inequlity represents the allow able speed

Let $\times$ is allowable speed then

45 Km/hr $< x <$ 55 Km/hr

2, The normal number of white blood cell for human blood is between 4800 and 10,800 cell per cubic minmeter, Inclusive

a, Write an ieequlity representing the normal range of white blood cell per cubic millimater.

Let x is normal range

$4800 < x < 10,800$

b, Write a compound ieqlity representing ab normal levels of white blood cell per cubic millimeter

$x < 48000$ Or $x > 10,800$

3, One lege of a right triangle is 3cm longer than the other, How long should the shorter leg be to ensnure the area is at lest

14 Cm^2 Net the snorter leg of the higt angled triangle ($x > 0$) .

Then , the longer leg will be x+3

From the given $\frac{1}{2} \times (x+3) \geq 14$

$x(x+7) > 28$, $x^2 + 3x = 0$

$(x-4)(x+7) \geq 0$

Using product properties

i $\quad x - 4 \geq 0$ $\qquad$ ii $\quad x - 4 \leq 0$

$\quad x + 7 \geq 0$ $\qquad\qquad x + 7 \leq 0$

Form $x \geq 4$ and $x \leq -7$ since $x \geq 0$ then $x \geq 4$

The snourtest leg should be 4 Cm and above

4, Almaz is selling sambusa at school she sells two sit, small (which has 1 scoop of beans inside) and large (which has 2

scoop of beans). She know that she can get amaximum of 70 scoops of beans out of her supply, she charges Birr 3 for

asmall Sambwa and Birr 5 for a larger one. Almaz want to earn at least Birr 120 to given back to school.

Let X be the number of small Sambusa and Y be the number of large one? Write and graph a system of Inequity that

models this sitution

Let X is the number of small Sambusa and Y is the number of larg Sambusa from the given

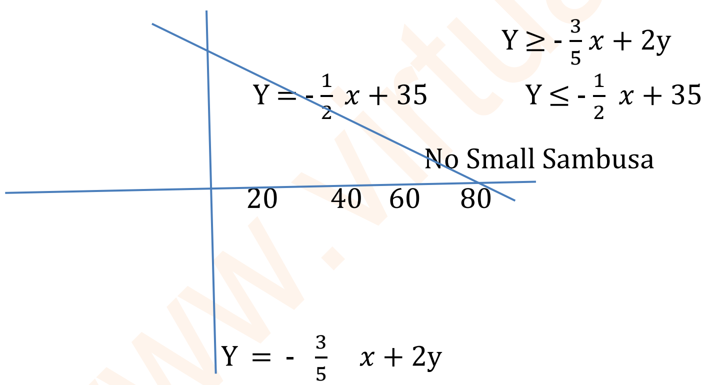
$3x + 5y \geq 120$ This is changed

$x + 2y \leq 70$ to

$Y \geq -\frac{3}{5}x + 24$

$Y \leq -\frac{1}{2}x + 35$

Solve Simultaneously by graphing both on the same x – Y axise



Review Exercise

1, Solve the following simultaneous

a, $3x + 2 > 7$

$x + 1 < 5$

$3x > 7 - 2 \Rightarrow \frac{3x}{3} > \frac{5}{3} \quad x > \frac{5}{3}$

$x + 1 < 5 \Rightarrow x < 5 - 1$

$\underline{x < 4}$

$\frac{5}{3} < x < 4$ The solution set

b, $2x + 1 \geq 9$

$3x + 4 \leq 25$

$2x \geq 9 - 1 \Rightarrow \frac{2x}{3} \geq \frac{8}{2}$ $x \geq 4$

$3x + 4 \leq 25$

$3x \leq 25 - 4 \Rightarrow \frac{3x}{3} \leq \frac{21}{3}$

$\underline{x \leq 7}$

$\underline{4 \leq x < 7}$

C, $\begin{bmatrix} x + 7 > 4 \\ x + 2 < 5 \end{bmatrix}$

$x + 7 > 4 \Rightarrow x > 4 \Rightarrow x > -3$

$x + 2 < 5 \Rightarrow x < 5 - 2 \Rightarrow x < 3$

$\underline{-3 < x < 3}$

2, Solve the following inequalities

a, $|2x - 1| - 7 \leq -5$

$|2x - 1| \leq -5 + 7$

$|2x - 1| \leq 2$

$-2 \leq 2x - 1 \leq 2$

$-2 + 1 < x\ 2x \leq 2 + 1$

$\frac{1}{2} \leq \frac{2x}{2} \leq \frac{3}{2}$

$-\frac{1}{2} \leq x < \frac{3}{2}$

b, $|x - 1| \leq -3$

$-3 \leq x - 1 \leq 3$

$-3+1 < x \leq 3+1$

$-2 \leq x \leq 4$

No Solution

3, A technician measures an electric current which 0.036 A. with a possible error of $\pm$ 0.002 A. write this current .

i, as an inequity with absolute values

$0.036 - 0.002 = 0.034$ A

$0.036 + 0.002 = 0.038$ A

$0.034 \text{ A} \leq I \leq 0.038$

4, A shop owner has determined that the relationship between monthly rent charged for store space r (in hand red birr per square meter) and monthly profite P (r) (in thousands of birr) can be approximate by the function $P(r) = -9r^2 + 8r + 1$. Solve each quadratic equation or inequality. Explain what each answer tells about the relation ship between monthly rent and profit for the shop owner ?

monthly rent charged for store space is r (in hundred Birr per square meter Thus $r > 0$

a, Solving the equation , $r = 1$ and $r = -\frac{1}{a}$, since $r > 0$, then , $r = 1$

This means that when the monthly rent is one hundred Birr per square meter , the profit will be 0

b, Solve Inequality $= 9r^2 + 8r + 1 > 0$

$(ar + 1)(r - 1) < 0$

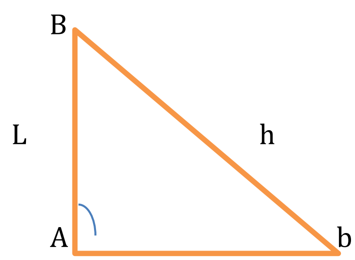
Thus $-\frac{1}{9} < r < 1$ since $r > 0$ $0 < r < 1$

This means that when the monthly rent is between 0 and 1 hand red Birr per square meter, there will be profit.

UNIT 5

Introduction to Trigon no meter

5.1 Revision On Right – angled triangle



$b^2 + L^2 = h^2$

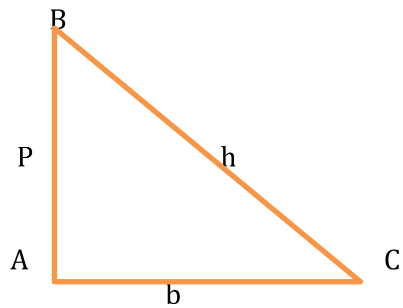
The opposite of right angle is called *Hypotenuse*

Note that

For right – angle triangle , we also have then following Basic properties

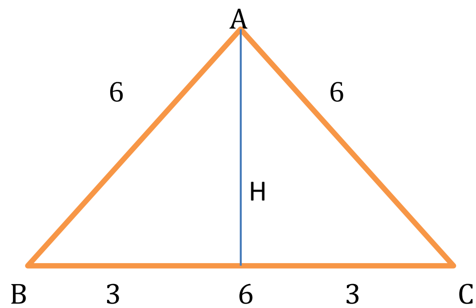
- ✓ One angle is always 90^0 or right angle
- ✓ The opposite side to angle 90^0 is hypotenuse
- ✓ The hypotenuses is always the longest side
- ✓ The sum of the other two interior angles is equal to 90^0 (each angle is acute)
- ✓ The other Two sides adjacent to the right angle are called base and perpendicular side . (b is the base and P is the perpendicular side).
- ✓ If one of the angle is 90^0 and the other two angle are equal to 45^0 each then the triangle is called an Isosceles right – angled triangle, where the adjacent sides to 90^0 are equal in length.

✓ If b , P and h are sides of a right – angled triangle as shown below figure we can write a relation using the *Pythagoras theorem* that is $b^2 + P^2 = h^2$,



Example:-

Find the height of
Equatorial triangle ABC
With length



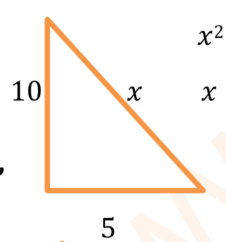
$BH^2 + AH^2 = BA^2$
$3^2 + AH^2 = 6^2$
$AH^2 = 36 - 9$

$AH^2 = 27 \Rightarrow AH = \sqrt{27} = \underline{5}$
$AH^2 = 27$ Or $\sqrt{27} = \underline{3\sqrt{3}}$

Exercise

1, Find the value of X. express your answer in simplest radical form

a,

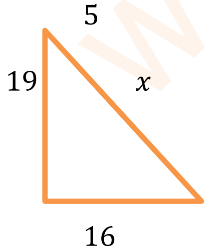


$x^2 + 5^2 = 10^2 \Rightarrow x^2 + 25 = 100$

x $\quad x^2 = 100 - 25$

$x^2 = 75 \Rightarrow x = \sqrt{75} = \underline{5\sqrt{3}}$

b,



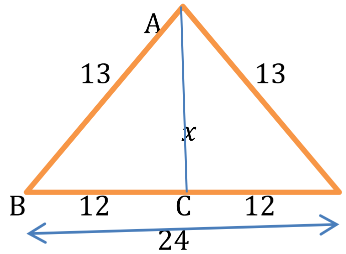
$16^2 + x^2 = 19^2$

$x^2 = 19^2 - 16^2$

$x^2 = 361 - 256$

$x^2 = 105 = \underline{x = \sqrt{105}}$

2, Find the value of X, If your answer is not an integer give it in the simplest radical form



a $\quad x^2 + 12^2 = 13^2$

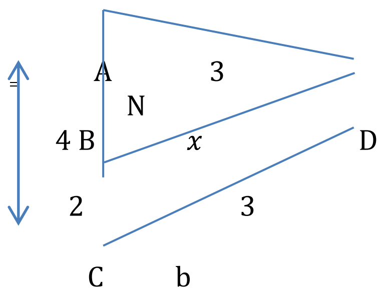
$x^2 + 12^2 = 13^2$

$x^2 + 144 = 169$

$x^2 = 169 - 144$

$x^2 = 25$

$\underline{\underline{x = 5}}$

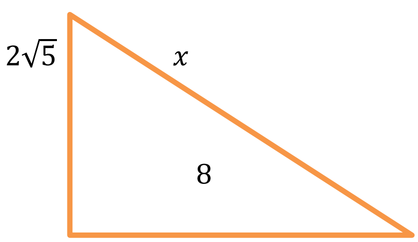


$2^2 + x^2 = 3^2$

$4^2 + x^2 = 9$

$x^2 = 9 - 4$

$x^2 = 5 \quad \Rightarrow \quad \underline{\underline{x = \sqrt{5}}}$



$2^2 = (2\sqrt{5})^2$

$Y^2 = 4 x 5 - 4$

$Y^2 = 16$

$\underline{\underline{Y = 8}}$

$Y^2 + 8^2 = x^2$

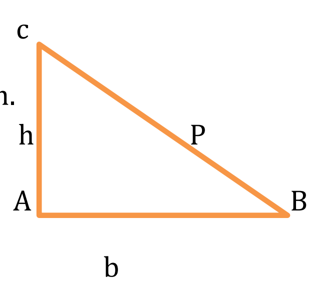
$8^2 + 8^2 = x^2$

$64 + 64 = x^2$

$x^2 = 128$

$x = \sqrt{128} = \underline{\underline{8\sqrt{2}}}$

- Conversion of the Pythagoras theorem.
  If $b^2 + P^2 = h^2$, then $< B = 90^0$



Example:-

Which one of the following can be sides of a right – angled triangle

a, 5 , 6 ^{2}

$5^2 + 2^2 = 6^2$

$25 + 4 = 36$ not triangle

$29 \neq 36$

b, 3 , 4 , 5

$3^2 + 4^2 = 5^2$

$4 + 16 = 25$

$25 = 25$ There right angle

Exercise

1, Identify whether the given sides can form a right – angled triangle or not

a, 4 , 6 , 8 $4^2 + 6^2 = 8^2$

$16 + 36 = 64$

$52 \neq 64$ not right angle

b, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

$(\sqrt{2})^2 + (\sqrt{3})^2 = (\sqrt{5})^2$

$2 + 3 = 5$

$5 = 5$ it is right angle triangle

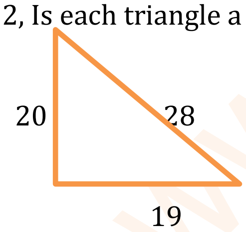
C, $\sqrt{6}$, 3 , $\sqrt{3}$

$(\sqrt{6})^2 + (\sqrt{3})^2 = 3^2$

$6 + 3 = 9$

$9 = 9$ right angled

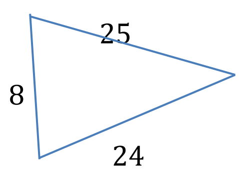
2, Is each triangle a right – angled triangle ?



$20^2 + 19^2 = 28^2$

$400 + 301 = 784$

$761 \neq 784$ not



$8^2 + 24^2 = 25^2$

$64 + 576 = 625$

$640 \neq 625$ not right angle

5.2Trigonometric Ratios

The trigonometric ratios of a given angle are defined by the ratios of two sides of a right – angled triangle. This trigonometric ratios remains unchanged as long as the angle remains the sam that is they are independent of the size of the triangle provided the angle remains the sam.

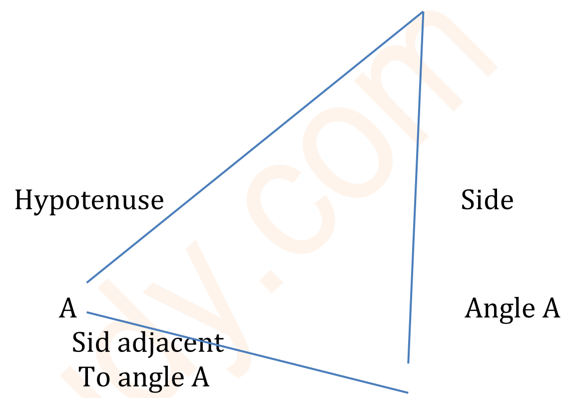
There are six trigonometric ratios. But here we will define only the first three trigonometric ratio as follow

$\text{Sin of} < A = \frac{\text{Side opp to } A}{\text{hypotenes}} = \frac{BC}{Ac}$

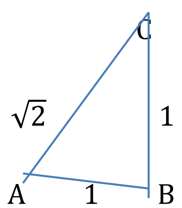
$\text{Cosine of} < A = \frac{\text{Side adj } A}{\text{hypotenes}} = \frac{Ab}{Ac}$

$\text{Tangent of} < A \frac{\text{opp } A}{\text{hyadj } A} = \frac{BC}{AB}$

opposite to



Example:- Find the trigonometric ratio (Sin , Cosine , tangent



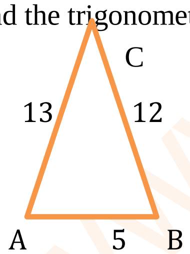
$\text{Sin } A = \frac{Opp}{Hpp} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\text{Cos } A = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}$

$\tan A = \frac{Opp}{adj} = \frac{1}{1} = \underline{1}$

Exercise

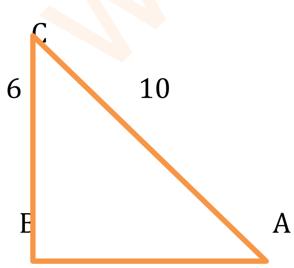
Find the trigonometric ratio (Since , Cosin and tangent) of the Angle A



$\text{Sin } A = \frac{Opp}{hyp} = \frac{12}{13} = \frac{12}{13}$

$\text{Cos } A = \frac{adjc}{hyp} = \frac{5}{13} = \frac{5}{13}$

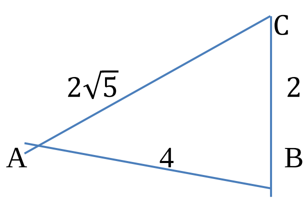
$\tan A = \frac{opp}{adj} = \frac{12}{5} = \frac{12}{5}$



$\text{Sin } A = \frac{Opp}{hyp} = \frac{6}{10} = \frac{3}{5}$

$\text{Cos } A = \frac{adjc}{hyp} = \frac{8}{10} = \frac{4}{5}$

$\tan A = \frac{opp}{adj} = \frac{6}{8} = \frac{3}{4}$



$$\text{Sin A} = \frac{Opp}{hyp} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\text{Cos A} = \frac{adj}{hyp} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan \text{A} = \frac{opp}{adj} = \frac{2}{4} = \mathbf{\frac{1}{2}}$$

<u>Notice:-</u> If one of the three trigonometric ratioes is given , it is possible to determine the other two trigonometric ratios.

<u>Example:-</u> Determine the remaining two trigonometric ratios of an acute angle A of a right Angled triangle If

a, $\text{Sin A} = \frac{3}{5} =$

$\text{Sin A} = \frac{BC}{AC}$ $\quad BC = 3 \quad AC = 5$

$AB^2 + BC^2 = AC^2$

$AB^2 + 3^2 = 5^2$

$AB^2 = 25 - 9 \Rightarrow AB^2 = 16 \quad \underline{\underline{AB = 4}}$

$\tan \text{A} = \frac{BC}{AB} = \frac{3}{4}$ and $\text{Cos A} = \frac{AB}{AC} = \frac{4}{5}$

<u>Exercise</u>

1, Determin the remaing two trigono mtric ration of an acute angle A of a right – angled triangled If

a, $\text{Sin A} = \frac{1}{2}$

$\text{Sin A} = \frac{BC}{AC} = \frac{1}{2}$ $BC = 1 \quad AC = 2$

$AB^2 + BC^2 = AC^2$ AC $\Rightarrow AB^2 = 4 - 1 = \underline{\underline{AB = \sqrt{3}}}$

$\text{Sin A} = \frac{1}{2}$

$\tan a = \frac{Opp}{adj} = \frac{AC}{AB} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\text{Cos} = \frac{adj}{hyp} = \frac{\sqrt{3}}{2} = \mathbf{\frac{\sqrt{3}}{2}}$

b, $\text{Cos A} = \frac{2}{3}$

$\text{Cos A} = \frac{AB}{AC} = \frac{2}{3}$ $\quad AB = 2 \quad AC = 3$

$AB^2 + BC^2 = AC^2$

$2^2 + BC^2 = 3^2 \Rightarrow 4 + BC^2 = 9$

$BC^2 = 9 - 4$

$\underline{\underline{BC^2 = 5}} \qquad \underline{\underline{BC = \sqrt{5}}}$

$\text{Sin A} = \frac{BC}{AC} = \frac{\sqrt{5}}{3}$

$\tan A = \frac{\sqrt{5}}{2} = \frac{\sqrt{5}}{2}$

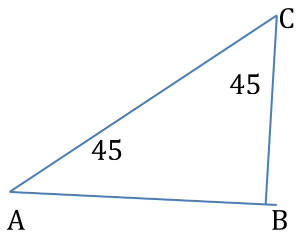
5.3 Trigon metric Values of Basic angles

Basic angles

From lower grade mathematics lesson you are already familiar with the construction of angles of

0^0 , 30^0 , 45^0 , 60^0 , and 90^0 , Now you will learn how to find the values of the trigonometric ratios for this angles

Example:- Find Sin 45^0 , Cos 45^0 and tan 45^0



BC = AB

BC = AB = a

$AC^2 \sqrt{AB2 + BC2}$

$A = \sqrt{a^2 + a^2} = AC = \sqrt{2a^2} = \sqrt{2a}$

$\text{Sin } 45^0 = \frac{\text{Side opposite to angle } 45}{hyp}$

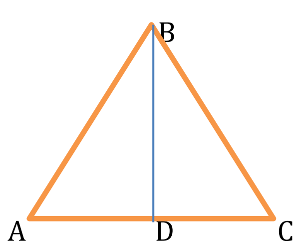
$\frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\text{Cos } 45 = \frac{adj}{hyp} = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}} x \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

$\tan 45^0 = \frac{Opp}{adj} = \frac{BC}{AB} = \frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = \underline{\underline{1}}$

Example:- 2 i sin 30, Cos 30 , and tan 30

Ii Sin 60^0 , Cos 60^0 , and tan 60^0



$AD^2 + DB^2 = AB^2$

$1^2 + BD^2 = 2^2$

$BD^2 = 4 - 1$

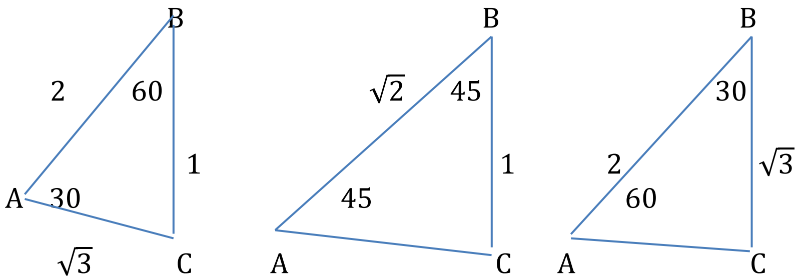
$\text{Sin } 30^0 = \frac{Opp}{hyp} = \frac{1}{2}$ $\text{Sin } 60 = \frac{Opp}{hyp} = \frac{\sqrt{3}}{2}$

$\text{Cos } 30^0 \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$ $\text{Cos } 60 = \frac{adj}{hyp} = \frac{1}{2}$

$\tan\ 30^0 = \frac{Opp}{hyp} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\tan 60 = \frac{adj}{hyp} = \frac{\sqrt{3}}{1} =$

Exercise

1, Complete the following table column wise using the give triangles



$\text{Sin } 30 = \frac{Opp}{hyp} = \frac{1}{2}$

$\text{Cos } 30 = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$

$\tan\ 30 = \frac{Opp}{adj} = \frac{1}{\sqrt{3}} = \frac{\sqrt{1}}{3} x \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\text{Sin } 45^0 = \frac{Opp}{hyp} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\text{Cos } 45^0 = \frac{adj}{hyp} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45^0 = \frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} x \frac{2}{\sqrt{2}} = 1$

$\text{Sin } 60^0 = \frac{Opp}{hyp} = \frac{\sqrt{3}}{2}$

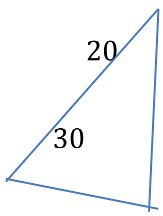
$\text{Cos } 60^0 = \frac{adj}{hyp} = \frac{1}{2}$

$\tan 60^0 = \frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{\sqrt{3}}{2} xA = \pi r^2\ \frac{2}{1} = \underline{\underline{\sqrt{3}}}$

	$<A = 30^0$	$<A = 45$	$<A = 60$
Sin A	$\frac{1}{2} = 0.5$	$\frac{\sqrt{2}}{2}$	$\frac{3}{2}$
Cos A	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan A	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

2, Find the un known value of the following figure. If the value is not all integer, express it in simple radical form.

a,



Sin 30 $= \frac{x}{20}$

$\frac{\sqrt{3}}{2} = \frac{x}{20} = x$

$\frac{20\sqrt{3}}{2} = x$

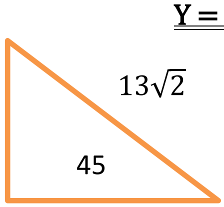
Cos 30 $= \frac{Y}{20}$ $\qquad x = \underline{10\sqrt{3}}$

$\frac{1}{2} = \frac{Y}{20}$

$\frac{2Y}{2} = \frac{20}{2}$

$\underline{\mathbf{Y = 10}}$

b,



Sin 45 $= \frac{Y}{13\sqrt{2}}$

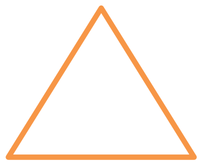
$\frac{\sqrt{2}}{2} = \frac{Y}{13\sqrt{2}}$ $\qquad 2Y = 13\, x\sqrt{2}\ \ x\ \sqrt{2}$

$\frac{2Y}{2} = \frac{13x\,2}{2}$ $\qquad \underline{\mathbf{Y = 13}}$

Cos 45 $= \frac{x}{13\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2} = \frac{X}{13\sqrt{2}}\ \ \frac{2x}{2} = \sqrt{2}\ x\ \sqrt{2}\ \ x13$

$\frac{2X}{2} = \frac{2X13}{2}$

$\underline{x = 13}$

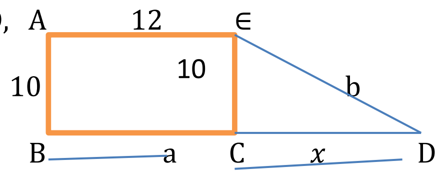
C, 

$Y^2 = \sqrt{7}^2 + \sqrt{7}^2$

$Y^2 = 7 + 7$

$Y^2 = 14$

$\underline{\mathbf{Y = \sqrt{14}}}$

D, 

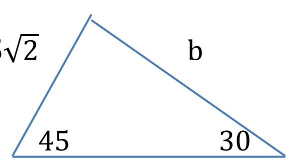
$\text{Sin } 45 = \frac{Opp}{hyp} = \frac{10}{b} = \frac{\sqrt{2}}{2} = \frac{10}{b} = \frac{\sqrt{2b}}{\sqrt{2}} = \frac{20}{\sqrt{2}}$

$b = \frac{20}{\sqrt{2}} = \frac{20}{\sqrt{2}} x \frac{\sqrt{2}}{\sqrt{2}} \frac{20\sqrt{2}}{2} = \underline{\mathbf{10 \sqrt{2}}}$

$\text{Cos } 45 = \frac{adj}{hyp} = \frac{x}{10\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{x}{10\sqrt{2}}$

$2x = 10x\sqrt{2} \times \sqrt{2} \Rightarrow \frac{2x}{2} = \frac{20x}{2} \quad x = 10$

$a = x + BC = 10 + 12 = \underline{\mathbf{22}}$

e, $5\sqrt{2}$ 

$\text{Sin } 45 = \frac{a}{5\sqrt{2}}$

$\frac{\sqrt{2}}{2} = \frac{a}{5\sqrt{2}}$

$\tan 30 = \frac{a}{b}$

$\frac{\sqrt{3}}{3} = \frac{5}{b} \qquad \mathbf{b = \frac{15}{13}} \qquad \frac{\mathbf{5x2}}{\mathbf{2}} = \frac{2a}{2} \qquad \underline{\mathbf{a = 5}}$

$\text{Sin } 30 = \frac{Opp}{hyp} =$

$\text{Sin } 30 = \frac{a}{b} = \frac{5}{b}$

$\frac{1}{2} = \frac{5}{b} \qquad \underline{\mathbf{b = 10}}$

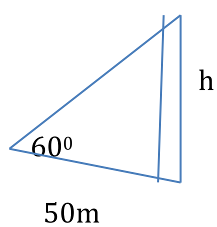
Trigonometric Ratios of 0^0 to 90^0

$< A$	0^0	30^0	45^0	60^0	90^0
Sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

- Remark:- From the above table you can observe that as $< A$ Increase from 0^0 to 90^0 , Sin A Increase from 0 to 1 while Cos A decreases from 1 to 0 , On the other hand , the value of tan A increase from 0 , become 1 at 45^0 and continues to increase until 90^0

Review exercise

1, The angle formed by the top of the building at a distance of 50m from its foot on horizontal plane is found to be 60 degree. Find the height of the building .



Tan 60 $= \frac{Opp}{adj} = \frac{h}{50} = \sqrt{3} = \frac{h}{50}$

$\underline{h = 50\sqrt{3}}$

2, A building with height of 60m above sea level is located at Bahirdar city hear to lake Tana. The angle between the line

segment from the sailing boat to the toap of the building and surface of the water is 25^0 as shows in figure 5.13.

a, How for is the boat from the base of the building to the nearest meterc

$\tan 25^0 = \frac{PB}{SB} = \frac{60m}{xm}$ so $x = \frac{60}{\tan 25}$ 128.62

128 . 67 $=$ $\underline{\mathbf{129}}$

b, What is the distance between the man at the top of the building and the boat ?

If SB is determind , we can get the distance between P and S using Pythagoras thermo

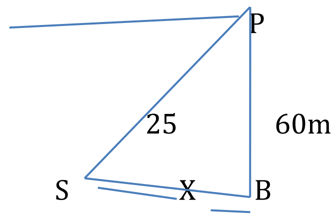
That is $PS^2 = SB^2 + PB^2$

$PS = \sqrt{(PS)2\,(PB)^2}$

$= \sqrt{129^2 + 60^2} \approx \underline{\mathbf{142}}$

Or Sin 25 $= \frac{PB}{SB} = \frac{60}{Y}$, So Y $= \frac{60}{\sin 25}$

Y $= 141.97 \approx \underline{\mathbf{142\ m}}$



4, Two women A and B lie on the leveled ground at opposite sides of 150m tell tower . If a observes the top of the tower at

an angle of 60^0 and B observes the same point at all angle of 30^0 , then how fare the two women can be a way from each

other? A , $100\sqrt{3}$ m B, $600\sqrt{3}$ m C, $200\sqrt{3}$ m D, $450\sqrt{3}$ m

AB 1 Cd and let AC = x and CB = Y

What we need to determine x+Y

Using trigonometric ratio

$\tan 60^0 = \frac{150}{x}$ and $\tan 30^0 = \frac{150}{Y}$

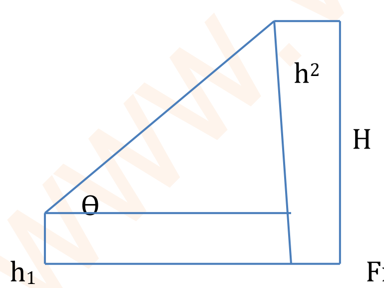
$x = \frac{150}{\tan 60}$ and $Y = \frac{150}{\tan 30}$

$x = 50\sqrt{3}$ $Y = 150\sqrt{3}$

$x + Y = 50\sqrt{3} + 150\sqrt{3} = \underline{\mathbf{200\sqrt{3}}}$

5, In the finger below If the height of the man is 1.74m , $\theta = 63^0$ and he is located at 100m away from the building find the

height of the building.



$H = h_1 + h_2$

$h_1 = 1.74$m

$\theta = 63^0$

From the right angle

triangle ABC , $\tan 63^0 = \frac{h2}{100}$

$h^2 = 100\text{m } x \tan 63 = 196.26$m

hence $H = h_1 + h_2 = 1.74 + 196.26 = \underline{\mathbf{198m}}$

UNIT 6

Regular Polygons

6.1 Sum Of Interior Angles Of Convex Polygon

The sum of the measure of interior angle of n – sided polygon is equal to (n – 2) 180

- ✓ You will start from a triangle and reach to every n- son polygon
- ✓ A polygon is simple closed plane figure formed by three or marlin segment joined end to end no two of which in succession are collinear.
- ✓ The line segments forming the polygon called sides.
- ✓ The common end point of any two sides is called vertex of the polygon.
- ✓ The angle formed inside the polygon are called interior angel

Common naming of polygon

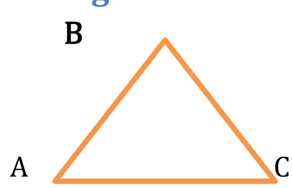
Name of Polygon	Number of sides	No of vertices
Triangle	3	3
Quadrilateral	4	4
Pentagon	5	5
hexagon	6	6
Heptagon	7	7
Octagon	8	8
Nonagon	9	9
Decagon	10	10

- ✓ A polygon is said to be convex polygon when the measure of each interior angle is less than 180^0 . The vertics of a convex polygon are always out wared.
- ✓ A polygon is said to be concave polygon when there is at least one interior angle whose measure is more than 180 degree. The vertices of a concave polygon present in wareds and also out wards.
- If none of the side extension intersect the polygon then is convex, otherwise it is concave.

6.2 Sum of interior angles

a, Convex Polygon

For Δ ABC not that
$A+b+C = 180^0$



The sum of interior angle of any polygon is n x180 n – means any number of side

Deriving sum of the Interior angles of a polygon

No sides of polygon	Name of polygon	No triangle	Sum of measured of interior angle
3	triangle	1	$1x180^0$
4	Quadrilateral	2	$2x180^0$
5	Pentagon	3	$3x180^0$
6	hexagon	4	$4x180^0$
n	n-gon	(n-2)	(n-2 x 180)

The sum of the measure of interior angles of n- sided polygon is equal to (n-2) $x180^0$

Example:- Find the measure of interior angle of

A, hexagon $= (6 . 2)\ 180$

$4\ x\ 180 = 720$

B, Octagon $= (8 - 2)\ x\ 180 = 6\ x180 = 1080$

C, Nonagon $= (9 - 2)\ x\ 180 = 7\ x180 = 1{,}260$

Exercise

1, Find the sum of the measure of interior angle of a, 12 – sided polygon $(12 - 2)\ x\ 180 = 1800$

2, The sum of interior angles of apolygon is 1440^0 . how many sided does the polygon have

$(n - 2)\ 180 = 1440$

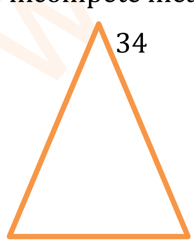
$180\ n - 360 = 14440$

$180n = 1440 + 360$

$\frac{180n}{180} = \frac{1800}{180}$

$\underline{n = 10}$

3, Find the incompete measure of interior angle of the given polygons.

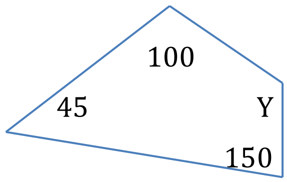


$x + 34 + 50 = 180$

$x + 250 = 180$

$x = 180 - 84$

$x = 96$



$Y + 100 + 45 + 150 = 360$

$Y + 250 = 360$

$Y = 360 - 250$

$Y = 110$

4, all architect design to constrict recreation area of a school with a heptagonal shape with each interior angle are put in

increasing order, each differs from the next by 25^0 . find the measure of the smallest interior angle of the given heptagon to the nearest tenth of degree.

Let , the solution smallest interior angle of a heptagon (7 – sided polygon) be X degree,

The sum of the measure of the interior angle of any n – sided polygon is (n – 2) 180^0 . Hence The sum equtie the left side

(sum of interior angles that is

$$x + (x+25) + (x+50) + (x+75) + x + 100) + (x + 125) + (x + 150) = (n - 2)\,180$$

$$7x + 525 = 5\,x 180$$

$$7x + 525 = 900$$

$$7x = 900 - 525 \Rightarrow \frac{7x}{7} = \frac{375}{7} = 53.57$$

$$x = 53.6^0$$

6.3 Sum of Exterior Angle of a convex Polygon

An exterior angle of a convex polygon is an angle out sid the polygon formed by one of its sides and the extension of an adjacent side.

Exercise

1, Write **True** Or **False**

a, For atriangle , ther are – 6 exteror angle **False** it has 3 extiror angle

b, each exterior angle of angle of rectangl is 90^0 **True**

- **Sum of the exterior angle of polygon**

  interior angle of Δ ABC The sum of a triangle is 180

  $a+b+C = 180$

  $(a+b+C) + (a+B+\gamma) = 3\,x 180$

  $A + B + \gamma = 360$

  Heinc the sum of the measure of exterior angles of a triangle is 360^0 .

Example:-

Find the sum of the measure of exterior angles of quadrilateral.

The interior angle of Quadrilateral is $(n - 2)\,180 = 2x\,180 = 360^0$

The exterior angle is 360^0

Exercise

1, Find the sum of the measure of exterior angles of a hexagon

$(n - 2) 180 = 4x\ 180 = 720^0$

The exterior angle is $= 720^0$

2, The exterior angle of a pentagon are

$(n + 5)^0$, $(2n + 5)^0$, $(3n + 2)^0$, $(4n + 1)^0$

and $(5n + 4)^0$ respectively. Find the measur of each angle

$(n - 2) 180 =$

$(5 - 2) 180 = 3\ x\ 180 = 540$

$(n + 5) + (2n+3) + (3n+2) + (4n+1) + (5n+4)$

$= 360^0$

$15n + 15 = 360$

$\frac{15n}{15} = \frac{345}{15} \Rightarrow \frac{15n}{15} = \frac{345}{15}$ n $= 23$

$n = 23$

- ✓ $n+5 = 23 + 5 = 28^0$
- ✓ $2n + 3 = 2x23+3 = 460 + 3 = 49^0$
- ✓ $3n+2 = 69 + 2 = 71^0$
- ✓ $4n+1 = 4x23 + 1 = 93^0$
- ✓ $5n+4 = 5x23 + 4 = 119^0$

6.4 The Sum of the exterior angle of n – sided Polygon

If we take an n – sided polygon , the sum of the interioe angle of a polygon is $(n - 2) 180^0$.

The sum of interior and exterior angle of n – sided polygon is $n\ x\ 180^0$

i, The sum of interior angle + sum of exterior angl

$= n\ x\ 180^0$

ii, $(n - 2)\ x180$ + sum of exterior angle $= n\ x\ 180$

iii, $n \times 180 - 360$ + sum of exterior angl $= n\ x\ 180^0$

iv, The sum of the measure of exterior angles os a polygon $= 360^0$

hince we can conculide that for any n – sided polygon, the sum of the measure of exterior angle of the polygon is

360^0.

At each vertex of a polygon, the sum of interior and exterior angle is 180^0

Example:-

Consider a triangle whose two interior angles are 60^0 and 40^0 , then find

a, The reaming interior angle of the triangle

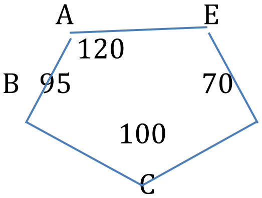
$x + 60 + 40 = 180$

$x + 100 = 180$

$x = 180 - 100$

$x = 80^0$

Exercise 1, Given a pentagon with four of its interior angles are 120^0 , 95^0 , 100^0 and 70^0 find the measure of exterior angle at each vertex.



180

$(n-2)\,180 =$

$(5-2)\,180 =$

$3\,x\,180 = \underline{\mathbf{540}}$

$3\,x\,180 = 540$

$120 + 95 + 100 + 70 + E = 540$

$385 + E = 540$

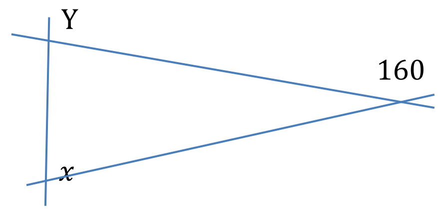
$\in\ = 540 - 385$

$\in\ = 155$

$(n-2)\,180 + 360 =$

Now the measure of each exterior angle corresponding to the interior angle 120^0 , 95^0 , 100^0 , 70^0 , and 155^0 is 60^0 85^0 80^0 110^0 an 25^0

2, find the measure of angle X and Y from the figure below , The dasnes on each line are to mean the line segmants are equal in length.



$x + Y = 180$

The supplement angle

160 is 20 $\Rightarrow x + Y = 160$

$x + x = 160 \Rightarrow \frac{2x}{2} = \frac{160}{2} \quad x = \underline{\mathbf{80}}$

6.5 Measures of each interior angle and Exterior angle of a Regular Polygon

A polygon is said to be regular If it is equiangular (having equal angles) and equilatrial (having equal sides).

For n – sidedn polygon , recall that

i, The sum of the measure of interior angles is $(n-2)\,x180^0$

ii, The sum of the measure of exterior angles is 360^0

Using this and the defintion of regular polygon , the measur of each interior angle of n – sided regular polygon is determined by $\frac{n-2\,)\,x\,180}{n}$ (Since a regular polygon is equian gular?

and the measure of each exterior angle is $\frac{360}{n}$ (Since there are n exterior angles with same measure)

Example:- Find the measure of each interior and each exterior angle of a regular pentagon.

$$\frac{(n-2)}{n}\,x80 = \frac{5-2}{5}\,x\,180 = 108^0$$

Measure of each exterior angle is $\frac{360}{5} = \underline{72}$

Exercise

1, Find the measure of each interior and Exterior angle of

i, regular hexgon

n = 6 $\quad \frac{(n-2)}{n} \times 180 = \frac{6-2}{6}\,x\,180 = 120^0$

measure of each exterior is $\frac{360}{6} = 60^0$

ii, regular octagon

n = 8 $\frac{(n-2)}{n}\,x180 = \frac{(8-2)}{8}\,x180 = \frac{6x180}{8} = \underline{\underline{135}}$

measur of each exterior is $\frac{360}{6} = 45^0$

2, What is the sum of each interior and its corresponding exterior angle of the above two regular polygon ?

At each vertex, we can observe the sum of the measur of interior and exterior angle is 180^0 ($120 + 60 = 180$ for I and

$135 + 45 = 180$ for ii)

3, A mathematics teacher gave a group work students to constract a convex regular polygon with one of the interior angle

measures 135^0 . What is the name of this polygon ?

The measur of each interior angle of n – sided regular polygon is obtaned by

$\frac{(n-2)x180}{n}$ Now we need to determin n of result is 135^0

$\frac{(n-2)}{n}\,x180 = 135$

(n – 2) x 180 = 135 n

180 n – 360 = 135 n

180 n – 135 n = 360

$\frac{45\,n}{45} = \frac{360}{45}$ $\quad \underline{\underline{n = 8}}$

Hence the regular polygon is octagon

4, for n – sided regular polygon , the measur of each interior angle is 5 tims the measur of each exterior angle.

What is the number of sides of this polygon?

Let the measur of each exterior angle of n – sided regular polygon be X degree then each entiror angle measur 5 x.

$x + 5x = 6x = 180$

$\frac{6X}{6} = \frac{180}{6}$ $x = 30$

The sum of extiror angle of each polygon

$\frac{360}{n} = 30$ n $= \frac{360}{30}$ $\underline{n = 12}$

6.6 Properties of Regular polygons pentagon , hexagon , Octagon , and Decagon

- ✓ Line of symmetry for regular polygon
- ✓ If two part of a figure are Identical after folding or flipping , then it is said to be **Symmetric** , to be symmetrical one half must be the mirror Image of the other , If the figure is not symmtrical then it is said to be **a symmetrical**
- ✓ A figur can have more than one line of symmetry. An a symmetrical figure has no line of symmetry.
- ✓ All regular polygons are symmetrical shapes so that we hav lines of symmetry for each.

DiD you Know ?

- ✓ An equilatrial triangle has 3 line of symmtry
- ✓ Squre / rectangl / has 4 line of symmetry
- ✓ Pentagon has 5 line of symmetry
- ✓ Regular hexagon has 6 line of symmetry

The line of symmetry meet at a point in side the polygon called center of the polygon

- ✓ If « n » is the number of a regular polygon
- ✓ If n is odd the line of symmetry connect vertix to side
- ✓ If n is even the line of symmetry connect vertex to vertex and side to of symmetry

Exercise

1, How many line of symmetry does a regular octagon have ?

8 line of symmtry

2, Say **True** or **False**

A, If a polygon is not regular then it is a symmetric. **True**

B, A right angled triangle has at most one line of symmetry . **True**

C, A rombes has 4 line of symmetry **True**

Inscribed and circumscribed polygon

- ✓ The n – sided polygon is inscribed in the bigger circle whole radius is $\overline{OB}$ sduch a circle is called **a circum circle** . and it connect all vertics (corner points) of the polygon.
- ✓ The line segment from the center perpendicular to the sides of a regular polygon is called **a pothem** of polygon
- ✓ In side circle (the smaller circle) called an **in circle** and it just tuches each side of the polygon at its mid point the radius of the incircle is called the a pothem of the polygon.

- ✓ A circle is inscribed in and circum scribed about any regular polygon

<u>Measure of Centeral angle</u>

The central angle for a regular polygon is an angle formed at the center of a circle by two consecative radiifrom then vertex of a polygon

- ✓ The measur of each centeral angle of n – sided regular polygon is $\frac{360}{n}$

<u>Example:-</u> Find the measure of central angle of a regular polygon in scribed in a circle which has number side

a, 5 $= \frac{360}{5} = 72$

b, 6 $= \frac{360}{6} = \frac{360}{6} = 60$

C, 9 $= \frac{360}{9} = 40$

<u>Exercise</u>

1, Determine the measure of each centeral angle of regular polygon

A, 10 sided $= \frac{360}{10} = 36$

B, 15 sided $= \frac{360}{15} = 24$

2, find the number of sided of a regular polygon whes measure of centeral angle is 12^0

$\frac{360}{n} = 12 \qquad \frac{12\,n}{12} = \frac{360}{12}$

n = 30 sided

3, Which of the following measure of centeral angle yield a regular polygon

a, $6^0 \quad \frac{360}{9} = \frac{6}{1} \quad \frac{6n}{6} = \frac{360}{6} \quad n = 60$

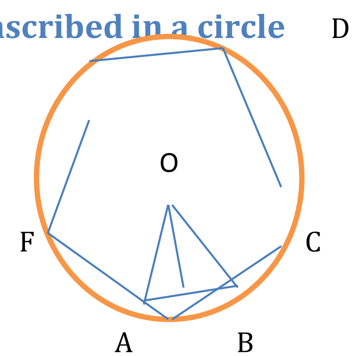
b, $14^0 \quad \frac{360}{n} = \frac{14}{1} \quad \frac{14n}{14} = \frac{360}{14} \quad n = 25.7$

C, $80^0 \quad \frac{360}{n} = 80 \quad \frac{80n}{80} = \frac{360}{80} \quad n = 4.5$

D, $40^0 \quad \frac{360}{n} = \frac{40}{1} \quad \frac{360}{40} = \frac{40}{40}$

<u>**n = 9**</u>

6.7 Permeter, area and a pothem of regular Polygons Inscribed in a circle



<u>Example:-</u> The regular hexagon is inscribed in a circle with center 0 and radius repress

The side length5

Premaster P , a pot hem a,

And area A of the regular hexagon using ABCDE F is regular hexagon . it has 6 sides. Triangle A O B is equal trial

Triangle $< AOB = \frac{360}{6} = 60^0$

$< OAB = < OBA = (180 - 60) = 2 = 6$

There are 6 congruent equilateral triangle in side this regular hexagon. So that

$5 = r \;, P = 65 = 6r$

2, Then figure below show that an equilateral triangle is Inscribed in a circle with the radius of r show the length of the

side , the a pot hem the perimeter and the area of the triangle in terms of the radius r .

Consider triangle OGC

$m < OGC = 30$

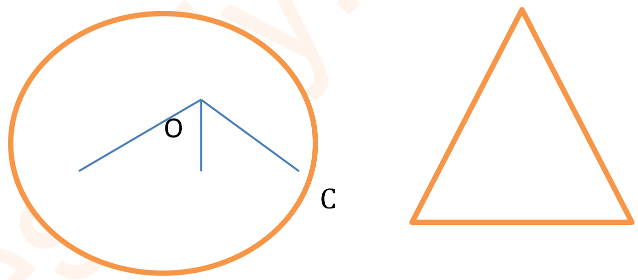
$\sin 30 = \frac{a}{r} \Rightarrow a = \frac{r}{2}$

$\text{Cos } 30 = \frac{GC}{OC} = \frac{GC}{r} \; GC = \frac{\sqrt{3}}{2} r$

Their for $5 = BC = 2 \, x \, \frac{\sqrt{3}}{2} r = \sqrt{3} \, r$

The perimeter of the triangle is $= 35$

$= 3x\sqrt{3} \, r = \underline{\mathbf{3\sqrt{3}\ r}}$



The Area of the triangle is

$A = 3x\frac{1}{2} \, x \, a \, x \, 5 = 3 \, x \, \frac{1}{2} \, x \, \frac{1}{2} r \, x \sqrt{3} \, r$

$= \frac{3\sqrt{3}}{4} r^2 \quad = \frac{3\sqrt{3}}{4} r^2$

For n – sided regular polygon Inscribed in a circle of radius r. length of sided S, apothem a, perimeter P , and area A

are determined by $S = 2r \sin\left(\frac{180}{n}\right)$

$P = 2n \, r \sin\left(\frac{180}{n}\right)$

$a = r \cos\left(\frac{180}{n}\right)$

$A = \frac{1}{2} \, a \, , p$

Example:-

a, Find the length of side of a squre inscribed in a circle of radius 5 cm

$5 = 2r \sin\left(\frac{180}{n}\right)$

$5 = 2\,x5\,x\sin\left(\frac{180}{4}\right) \quad = 10\,\frac{\sqrt{2}}{2} = \underline{5\ \ \sqrt{2}}$

b, Find the apothem of a regular patagon Inscribed in a circle of radius 6 cm

$a = r\cos\left(\frac{180}{n}\right) = 6\cos\left(\frac{180}{5}\right)$

$6\cos 36 = 4.854$

$\text{Cos } 36 = 0.8090$

Exercise

1, Find the perimeter, area and a pothem of a decagon, which is inscribed in a circl of radius r = 8 cm

$S = 2r\sin\left(\frac{180}{n}\right) = 2x8\sin 18$

$S_1 = 16\sin 18$

$a= r\cos\left(\frac{180}{n}\right) = 8\cos\frac{180}{10} = 7.608 \text{ cm}$

Per mater $= 2nr\sin\frac{180}{n} = 2x10x8\sin 18$

≈ 49.443 cm

Area $= \frac{1}{2}$ a,p $= 7.608 \text{ cm } x49.443 \text{ cm} = 18808 \text{ cm}^2$

2, Suppose central angle of a regular polygon is 60^0 . Then find the radius of a circle circums cribbing the given polygon

If it side is of length 9 cm

$\frac{360}{n} = 60 \qquad \frac{60n}{60} = \frac{360}{60} \qquad \underline{n = 6}$

$S = 2\,r\sin\frac{180}{6} = 9 \text{ cm } \ 2\,r\sin 30$

$9 \text{ cm} = \frac{2r}{2} \qquad \underline{r = 9 \text{ cm}}$

3, One student from your class states that the radius of a regular polygon is never less than it's apothem, Do you agree? If

so provided justification

Yet it is correct. In the consnstraction of apothem the radius is the hypotenuse of a right – angled triangle and apothem is one of the perpendicular leg of this right angled triangles.

Review Exercise

1, Find the number of sides of regular polygon If each exteririr is equal to

i, its adjacent interior

ii, twic its adjacent interior angle

Answer:- For the above Question

2, One of the interior angles of a polygon is 100^0 and each of the other angles is 110^0 . find the number of sid of the polygon.

Answer:- For the above Question

i, $E + I = 180$ Or $2E = 180$

$E = 90 \quad I = 90$

$n = \frac{360}{9} = 4$

hence the regular polygon is = square

ii, $E = 2I \quad E + I \ 180$

$2I + I = 180$

$\frac{3I}{3} = \frac{180}{3} \quad I = 60 \quad E = 120$

$n = \frac{360}{120} = 3$

since the regular polygon is equilateral triangle.

Answer for Question no 2

The sum of exterior angles of any polygon is 360^0 $\quad 80^0 + 70^0 n = 360 \quad n = 4$

3, The interior angles of a polygon are in the ratio 2: 3: 5: a: 11 find the measure of each angle. Is the polygon convex or

concave?

Let the interior angle of pentagon be $2x$, $3x$, $5x$, $9x$ and $11x$ so that the corresponding ratio of interior angles be

2 : 3 : 5 : 9 : 11 since it is pentagon the sum of the measure of interior of angle of polygon is 540^0 , ther for

$2xA = \pi r^2 + 3x + 5x + 9x + 11xA = \pi r^2 = 540^0$

$\frac{30x}{30} = \frac{540}{30}$

$\underline{x = 18}$

Interior angle of a polygon

36^0, 54^0, 90^0, 162^0, 198^0, The given polygon is concave one of the interior angle is greater than 180^0

4, How many sides does a regular polygon have If the measure of an interior angle is 165^0?

$165 + \theta = 180 \quad \theta = 180 - 165$

$\theta = 15$

$\frac{360}{15} = \frac{360}{n} = 15$

$n = \frac{360}{15} = \underline{24}$

5, Ther are two regular polygons with number of sides equal to (n – 1) and (n – 2) . Their exterior angles differ by 6^0 ,

find the number of side of the two polygons.

The measure of the first interior angle is $\frac{360}{n-1}$ and the measure of each interior angle of the second

regular polygon is $\frac{360}{n+2}$, Also the difference between the two exterior angle is 6^0

$\frac{360}{n-1} - \frac{360}{n+2} = 6 = 360\left(\frac{1}{n-1} - \frac{1}{n+2}\right) = 6$

$n^2 + n - 182 = 0 = (n - 13)(n + 14) = 0$

$n = 13$ and $n = -14$

So the correct Answer is n = 13 the result the number of sides of the regular polygon are

$n - 1 = 13 - 1 =$ **12** and $n + 2 = 13 + 2 =$ **15**

6, A regular polygon has a perimeter 143 unit and the sides are 11 units long how many sides does the polygon have?

P = ns where n is number of sided of a polygon and S is the length of side given that

P = 143 unit and S = 11 unit. So that $n = \frac{143}{11} = 13$ Hence the polygon is 13 – son .

7, Find the possible measure of minimum interior angle and maximum exterior angles in a regular polygon . Given areson to support your answer. For each interior Angle, we use a formula $\frac{(n-2)+180}{n}$ and each exterior angle we use $\frac{360}{n}$ where n is the number of sides of a polygon .

The maximum exterior angle is possible when n is the last possible number.

The last possible number of sides of a polygon is n = 3 using this in the interior angle formula. We find each interior angle of an equilateral triangle is 60^0 , so that the maximum possible exterior angle is 120^0 . No other regular polygon has such property.

8, The school pedagogical center is making a design of a regular octagon stop sign. Which will be placed at the main gate of the school to prevent car accidents. Each side of the sign is 40 cm long . what is the area of the sign?

The central Angle is $\frac{360}{8} = 45^0$ So that $\Theta = 22.5^0$. The length of the sides of the stop sign I AB = 40^0cm

So that AD = 20cm now the apothem (a) will be determine as

$\tan 22.5^0 = \frac{AD}{a}$ $a \approx 48.2843$ cm

$A = \frac{1}{2}a, p$ $p = ns = 8x40 = 320$

$A = \frac{1}{2}(48.2843 \text{ cm})(320\text{cm}) \approx 7725.488 \text{ cm}^2$

So the Area of stop sign is 7725 cm^2

UNIT 7

CONGRUNCY AND SIMILARITY

7.1 Revision On Congruency Of triangle

When two triangles have exactly the same three sides and the same three angles they are called **Congruent triangle**

- ✓ Same three sides means the corresponding sides have equal length and same three angles means the Corresponding angles have equal measure.

  Not that Δ ABC $\equiv$ Δ DEF means $\overline{AB} \equiv \overline{DE}$

  $\overline{BC} \equiv \overline{EF}$, $\overline{AB} \equiv \overline{DE}$ and $\overline{AC} \equiv \overline{DF}$

  $<A \equiv <D$, $<B \equiv <E$, $<C \equiv <F$

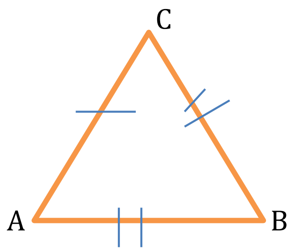
1, SAS Congruency

Side Angle – side Congruency when two triangles have two pair of Congruent side and one pair of congruent angles

between the sides, then the triangles are congruent .

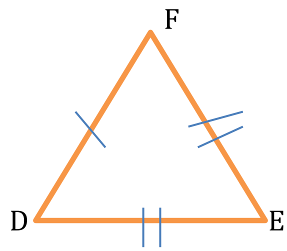
2, S S S congruency

Side – side – Side congruency – when two triangles have equal corresponding sides , then the triangles are congrent.



S S S

Δ ABC ≡ Δ DEF

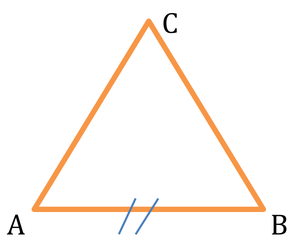


3, A S A congruency

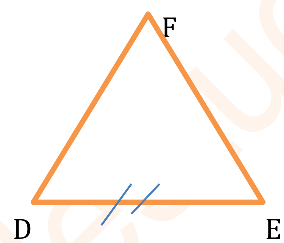
Angle – side – Angle Congruency

When two triangles have two pair of congruent angles and one pair of congruent sides between the angles then the

triangles are congruent.

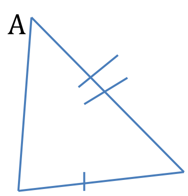


Δ ABC ≡ Δ DEF

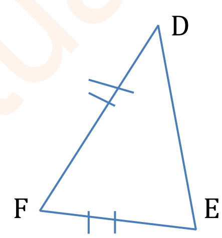


4, R H S Congruency : when two right triangle have equal hypotenuse and equal one side (leg), then the triangles are

Congruent



Δ ABC ≡ Δ DEF



Definition Of Similar Figures

Two plane figures are similar If their corresponding angles are congruent and the ratios of their corresponding sides are proportional. This common ration is called the scale factor Two plane figure are similar means that

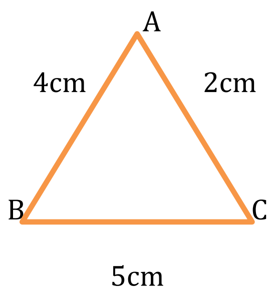
- ✓ All corresponding angle are congruent
- ✓ All corresponding sides are proportional when we magnify or diminish similar figure one exactly places on the other and vice versa.

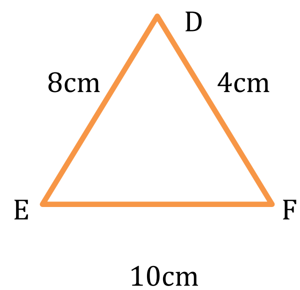
1, If the Two triangles , the two rectangles and the two pentagon below satisfy the above properties then they are similar each other

2, any two circles (of any radii) have the same shape and hence they are always similar.

Example:-

Δ ABC $\sim$ Δ DEF , find the common ratio of their corresponding sides.



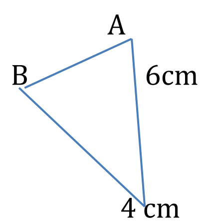


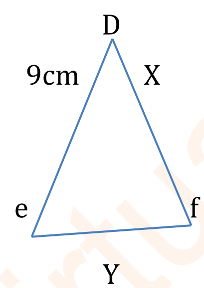
$\frac{DE}{AB} = \frac{8}{4} = 2$, $\frac{EF}{BC} = \frac{10}{5} = 2$

$\frac{FD}{AC} = \frac{4}{2} = \underline{\mathbf{2}}$

Exercise 7.6

a, Given Δ ABC $\sim$ Δ DEF , find X and Y .





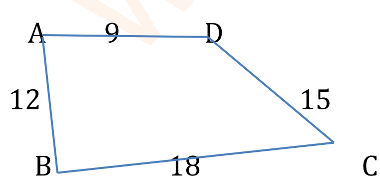
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{3}$

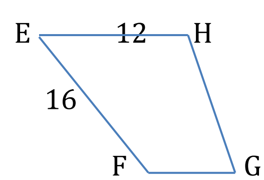
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{3}$

$\frac{1}{3} = \frac{4}{Y} = \frac{6}{x}$ $\frac{1}{3} = \frac{6}{X}$ $x = 18$

Y = 12 $x = 18$

b, figure ABCD is similar to figure EFGH find the perimeter of EFGH





ABCD $\sim$ EFGH

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH} = \frac{P1}{P2}$$

$$\frac{S1}{S2} = \frac{P1}{P2} \text{ and } \frac{A1}{A2} = (\frac{S1}{S2})^2 = (\frac{P1}{P2})^2$$

$$\frac{12}{16} = \frac{18}{FG} = \frac{15}{GH} = \frac{9}{12}$$

$$\text{FG} = \frac{16x18}{12} = \frac{6x\,6}{3} = 24$$

$$\frac{15}{GH} = \frac{9}{12} = \frac{12x15}{9} \quad \text{GH} = \frac{4x5}{3} = 12$$

$$\text{GH} = 20$$

$$\frac{S1}{S2} = \frac{P1}{P2} \quad \text{P}_1 = 12x18+15+9 = \underline{54}$$

$$\frac{12}{16} = \frac{54}{P2} = \frac{16x54}{12} = \underline{\underline{72}}$$

Exercise

For the following Questions determine whether each of the following **True** Or **False**

a, all equilateral angles are similar **True**

b, All isosceles triangles are similar **True**

c, All Isosceles right triangles are similar . **True**

d, All rectangles are similar. **False**

e, All Rhome use are similar. **True**

f, All squares are similar. **True**

g, All congruent polygons are similar . **True**

h, All similar polygons are congruent. **False**

i, All regular pentagons are similar. **True**

7.2 Theorems On Similar Plane Figures

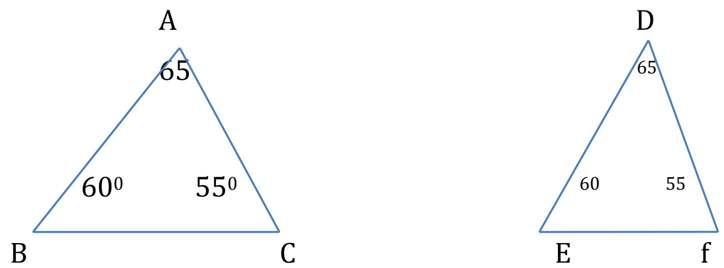
A A Angle - Angle

S S S - Side - Side - side

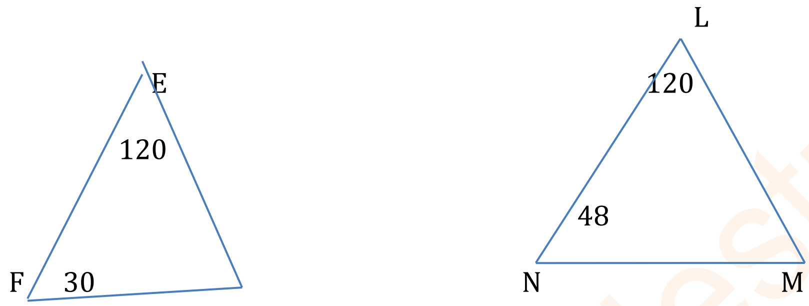
S A S- are Important for determing similarity in angle

A , A Similarity theorem

The A , A similarity thermo for triangle states that If the two angles of one triangle are respectively congruent to the two angles of the other. Then the triangles of the other, then the triangles are similar. In short equianqular triangles are similar



Δ ABC ~ ADEF by A A Similarity Determine whether or not the following two triangles are similar



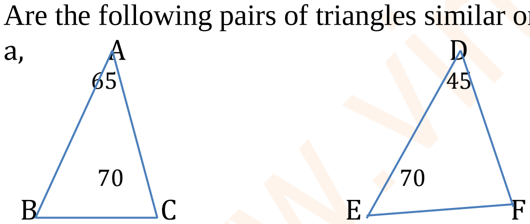
Comur the angles to see If we can use the A A Similarity theorem using triangle sum theorm m(<G) $= 48^0$

& m (<m) = 30^0 , Hence , Δ EFG ~ Δ LMN by A A Similarity theorem .

Exercise

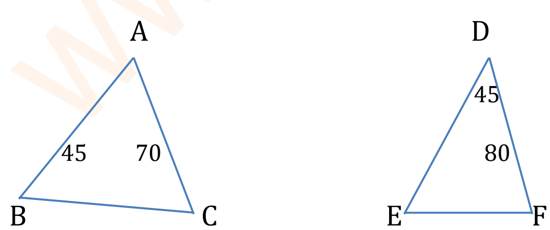
Are the following pairs of triangles similar or not by A A similarity therom

a,



ABC ~ FDE by A . A similarity

B,

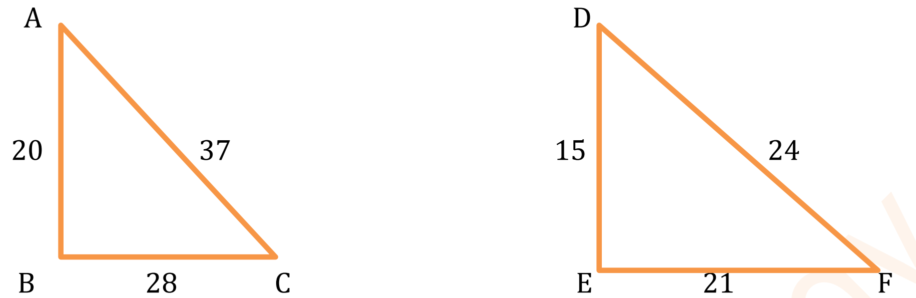


Not Similar

S S S Similarity Theorem

If the corresponding three sides of two triangles are proportional to each other , then the triangles are similar. This essentially means that means any such pair of triangle are equiangular (all corresponding angle pair are congruent)

Example:-



Similar by S S S similarity theorem

S A S Similarity Theorem

If the corresponding two sides of two triangles are proportional and the included angles are congruent then the two triangles are congruent then.

The two triangles are similar

Example:-

Given Δ ABC ~ Δ DEF , and < D , <E , 4 and the lengths of DE.

$\Delta ABC \sim \Delta DEF \quad \angle D = \angle A = 45^0$

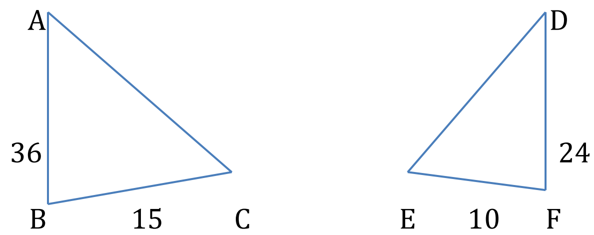
$\angle E = \angle B = 90^0 \qquad \frac{DE}{AB} = \frac{DF}{AC}$

$\angle F = \angle C = 45^0$

$\frac{DE}{3} = \frac{6\sqrt{2}}{3\sqrt{2}} = DE = \frac{6\sqrt{2}\,x3}{3\sqrt{F}} = \underline{\underline{6}}$

Exercise

a, Are these two triangles similar two triangles are similar by
S S S



$36^2 + 15^2 = AC^2$ $\qquad$ $10^2 + 24^2 = DE^2$

$AC^2 = 36^2 + 15^2$ $\qquad$ $DE^2 = 100 + 576$

$AC^2 = 1296 + 225$ $\qquad$ $DE^2 = 676$

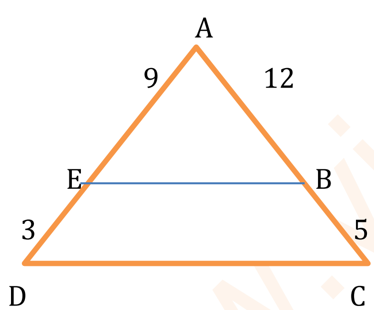
$AC = \underline{\underline{39}}$ $\qquad$ $DE = \underline{\underline{26}}$

ABC $\sim$ DEF by SAS

$$\frac{AB}{DF} = \frac{BC}{FE} = \frac{AC}{DE}$$

$$\frac{36}{24} = \frac{15}{10} = \frac{39}{26}$$

b, Are there similar triangles in the figure below ?



There are no two similar no two similar triangles, Because $\frac{AE}{AD} \neq \frac{AB}{AC}$

7.3 Ratio of Perimeters of Similar plane figures

If two figure are similar then the ratio of their perimeter is equal to ratio of their corresponding side lengths.

$$\frac{P1}{P2} = \frac{S1}{S2}$$

Example:- Δ ABC $\sim$ Δ DEF and the ratio of the lengths of their side is 2 . Then find the ratio of the corresponding peri meters . The perimeter of Δ ABC is $6 + 8 + 10$ and the perimeter of Δ DEF is $3 + 4 + 5$.

Perimeter of Δ ABC is $6 + 8 + 10 = 24$

Perimeter of Δ DEF is $3 + 4 + 5 = 12$

$$\frac{Perimater\ of\ \Delta\, ABC}{Perimeter\ of\ \Delta\, DEF} = \frac{24}{12} = 2$$

$$\left(\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}\right)$$

Not : $\frac{AB}{DE}$ is the same as AB : DE

<u>Ratio of areas of Similar plan Figures</u>

If two plane figures are similar then the ratio of the ratio of their corresponding side lengths.

$$\frac{A1}{A2} = \left(\frac{P1}{P2}\right)^2 = \left(\frac{S1}{S2}\right)^2$$

<u>Example:- 1</u>

Consider a pair of similar rectangle 1 cm by 2 cm and 3 cm by 6cm below. The ratio of the corresponding sid is 3, Find the ratio of their areas .

$$\frac{A1}{A2} = \left(\frac{P1}{P2}\right)^2 = \left(\frac{S1}{S2}\right)^2 = \frac{A1}{A2} = \left(\frac{6}{18}\right)^2 =$$

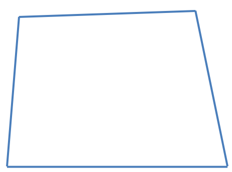
$$\frac{A1}{A2} = \frac{36}{324} = \frac{1}{9} = 1:9$$

<u>Exercise</u> 7.12

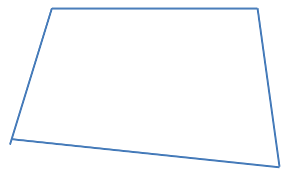
a, Find the ratio of the areas of rhombi below . The rhombi are similar.

The ratio of the area of the rhombi is $(\frac{6}{10})^2 = \frac{9}{25}$ Or $(\frac{10}{6})^2 = \frac{25}{9}$

b, Find the ratio of areas of similar qudirelatrials below.



3cm



6cm

$(\frac{6cm}{3cm})^2 = \frac{36}{9} = 4$ Or $\frac{3}{6} = \frac{9}{36} = \frac{1}{4}$

C, The ratio of the area of the two square is 64 : 49 . The scale factor is <u>8 : 7</u> and the ratio of there perimeter is <u>8 : 7</u>

d, Two circles has radii 8 cm and 12 cm respectively . The ratio of their circus ferences is <u>2 : 3 Or 3:2</u> and the ratio of their area is <u>4 : 9 or 9 : 4</u>

e, A triangle with area of $10m^2$ has a base of 4m . A similar triangle has an area of $90m^2$. what is height of the larger triangle ?

$$\frac{Area\ of\ the\ smaller\ triangl}{Area\ of\ the\ larger\ triangle} = (\frac{base\ b1}{base\ b2})^2$$

$$(\frac{4}{b2})^2 = \frac{10}{90}$$

Hence , the larger triangle has base $b_2 = 12$ meter . hence area of triangle which is $90 = \frac{1}{2}$ b (height

hence $90 = \frac{1}{2}(12)$ height.

Therefore the height of the larger triangle is equal to **<u>15</u>** meter

7.4 Application Of Similarity

Similar triangles are used to work out the height of tall objects such as tree buildings and towers without climbing which are difficult to measure for us.

<u>Example:-</u>

Almaz is 1.6 meter tall and is standing outside next to her younger brother. She notices that she can see both of their shadows and decides to measure each shadow. Her shadow is 2 meter long and her brother's shadow is 1.5 meter long . how tall is Almaz's brother:

Solution

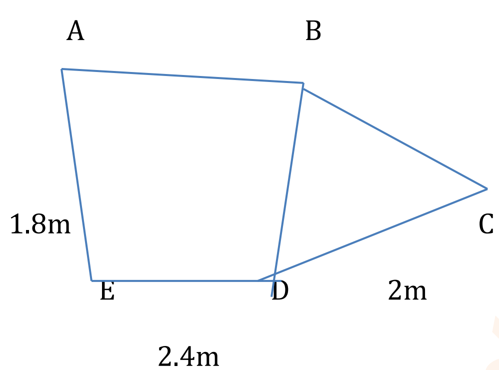
Let x be the height of the younger brother . using proprtinally of corresponding heighs and length of Shadow. We have

$\frac{1.5}{2} = \frac{x}{1.6}$ hence $x = 1.2$ meter

Ther for Almaz's brother is 1.2 m long

Exercise

a, Abdi is 1 . 8 meters tall and is standing out side Next to his younger brother. He notice that he can see both of their Shadow and decides to measure each Shadow his Shadow is 2 . 4 meters long and his brother's Shadow is 2 meter long . how tall abdi brother?



Solution Find two similar triangles Δ AEC $\sim$ Δ BDC . Then $\frac{1.8}{BD} = \frac{2.4}{2}$ Which Implies that , the height of Abdi's brother is BD = 1.5m

b, Abdisa boat has come united and floated a way on lake hawasa he is standing on the top of building that is 75m above the water from the lake . If he stands 15m from the edge of the building, he can visually align the top of the building

with the water at the back of his boat. His eye level is 1.75 m about the top of the building. Approximately, how far is abebe's boat from the building?

Solution

Let $\times$ be the distance from the boat to the building. Using proportionally of corresponding sides of the two triangles , we have

$\frac{75}{1.75} = \frac{x}{15}$ hence $x \sim 642.86$m

Therefore the boat is approximatry 642.86m from the building

Review Exercise

1, Two poles of heights 12m and 17m stand on a plane ground and the distance between their feet is 12m . Find the

distance between their top

Solution

$\frac{A1}{A2} = \frac{17}{12} = \frac{x+12}{x} \Rightarrow 17x = 12x+144$

$17x - 12x = 144$

$\frac{5x}{5} = \frac{144}{5}$

$x = \frac{144}{5} \quad = \underline{x = 28.8}$

2, Aladder is placed against a wall and it top reaches a point at hight of 8m from the ground . If the distance between the

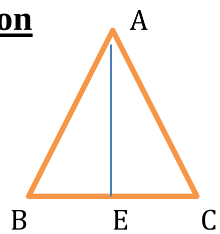
wall and foot of the ladder is 6m, Find the length of the ladder.

Solution

Let L be the length of the ladder Then $L^2 = 6^2 + 8^2 = 100$ Implies $L = 10$

3, In all equilateral atrial triangle show that three times the square of a side equals four times the square of a median.

Solution



Using Pythagoras theorem for Δ ABE $AB^2 = AE^2 + BE^2$, Let x be side of the equilateral triangle , $AB = x$,

$BE = \frac{x}{2}$, then $AE^2 = AB^2 - BE^2$,

$AE^2 = x^2 - (\frac{x}{2})^2$ Implies $AE^2 = 3\frac{x2}{4}$

Hence $4\,AE^2 = 3x^2$. The for , three times the square of a side equals four times the square of median.

4, A triangle with an area of 60 square meters has a base of 4 meters. A similar triangle has an area of 50 square meters.

Find the height of the smaller triangle.

Solution

If two triangles are similar then the ratio of their areas equals the square of the ratio of their corresponding base .

Let b be the base of the smaller triangle there for $\frac{60}{50} = \frac{4}{b}$

$\frac{60}{50} = \frac{4}{b}$ b $= \frac{50x40}{60} = \frac{20}{6} = \frac{10}{3}$

5, If Δ ABC $\sim$ Δ DEF , then which of the following is not true?

A, $\frac{AB}{DE} = \frac{BC}{EF}$ B, $\frac{AB}{DE} = \frac{EF}{BC}$ C, $\frac{AC}{DF} = \frac{BC}{FE}$ D, $\frac{DE}{AB} = \frac{EF}{BC}$

6, If the ratio of side of triangles ABC and DEF is K, The n the ratio of their primaries.

A, K B, K^2 C, $\frac{1}{K}$ D, K+1

UNIT 8

Vectors in Two Dimensions

8.1 Vector and Scalar Quantities

- Scalars are Quantities that are fully described by a magnitude (or numerical value) alone.

Example:-

- ✓ The mass of a stone is 10 kg.
- ✓ The density of pur water is 1 g/cm^3
- ✓ Temperature of a room is 27^{0}c

- Vector are quantities that are fully described by both magnitude and a direction.

There are many engineering application where vector are Important. Force acceleration, velocity, electric and magnetic field are vector Quantity.

Exercise

1, Which statement describes avector

A, it has direction but not magntiud

B, it has constant magntiud but no

C, it has magnitude diraction but no direction

D, it has both magnitude diraction

2, Scalar quantities are completely described by their

A, area B, Unit C, magnitiud D, direction

3, Consider the following Quantity and Identify whether each is scalar quntity Or vector Quntity Or

i, Amount of rainfall in mm = scalar

ii, Tempratur in a room = scalar

iii, Acceleration of a car = scalar

iv, Area of a rectangle = scalar

v, The speed of a care = scalar

vi, Velocity of the boat = vector

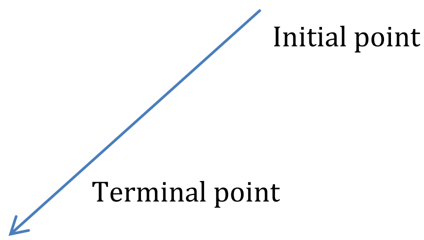
vii, The height of abuilding = scalar

viii, The weight of the body = vector

Representation Of Vector

A vector can be represented by either algebraically or geometrically .

A vector is represented geometrically by a directed line segment (a line segment with direction).



A vector also represented by using letters with an arrow bar over it such as $\vec{a}$, $\vec{u}$, $\vec{v}$

or bold letters lik **a** , **u** , **v**

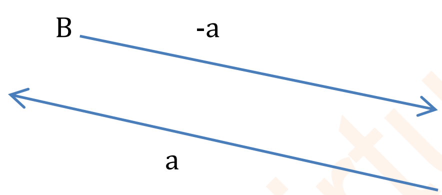
Equality Of Vectors

Two vectors are said to be equal If and only If they have the sam magnitude and direction

Opposite Of Vector

The vector which has the sam magnitude as that of a vector $\overrightarrow{AB}$ but opposite in direction is called opposite vector of $\overrightarrow{AB}$

and is denoted by - $\overrightarrow{AB}$

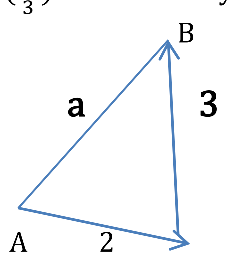


Parallel Vectors

Vector that have the same or opposite direction are called **Parallel Vector**

Column Vector in two dimension

Any vector **a** can be represented by a column vector **a** = $\left(\frac{x}{Y}\right)$, where × is the horizontal and Y is the vertical component of **a** vector $\left(\frac{2}{3}\right)$ is donated by **a** $\overline{or\ \vec{a}}$



We write **a** = $\left(\frac{2}{3}\right)^2$ Or $\vec{a} = \left(\frac{2}{3}\right)$

Magnitude Of Vectors

The magnitude of a given vector $\overrightarrow{AB}$ Or $\vec{a}$ as show in the length of the line segment from its initial point A to terminal point $\vec{B}$. it is denoted by as $|\overrightarrow{AB}|$ Or $|\vec{a}|$.

Example:- $|\vec{U}| = \left(\frac{4}{3}\right)$

$$|\vec{U}|\ u^2 = 4^2 + 3^2 \quad \text{(Pythagoras theorem}$$

$$= 16 + 9$$

$$= 25$$

Ther for $|\vec{U}| = |\vec{U}| = \sqrt{25}$

- ✓ if a vector is represented on a plane sheet of paper , we can determine its magnitude (length) by me assuring its length using a ruler and express it with appropriate unite.
- ✓ any vector a = $\left(\frac{x}{Y}\right)$ has a magnitude (length) $|a| = \sqrt{x2 + Y2}$.

Example:-

Find the magnitude of each of the following vectors on the coordinate system.

$$|\overrightarrow{PQ}| = \sqrt{(x2 - x1)2 + (Y2 - Y1)}^2$$

(4 , 6) (-3 , 4)

$$|\overrightarrow{AB}| = \sqrt{(4 - (-3)2 + (6 - 4)}^2$$

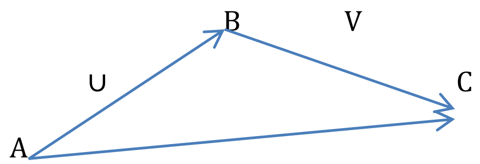
$$\sqrt{49 + 4} = \sqrt{53} = \underline{\sqrt{\mathbf{53}}}$$

Vector Operation

Addition Of Vector

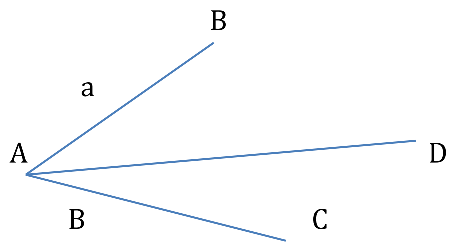
i) Triangle law of addition of vectors

Consider two vectors AB = u and $\overrightarrow{BC} = \vec{V}$ in a coordinate system. The sum of $\overrightarrow{AB} + \overrightarrow{BC} = \vec{U} + \vec{V}$ is a directed line segment connecting A to C say $\vec{W} = \overrightarrow{AC}$ such that $\overrightarrow{AC} + \overrightarrow{BC} = \overrightarrow{AC}$ Or $\vec{U} + \vec{V}$. such a vector AC is called resultant vector.



$W = U + V$

ii) Parallelogram Law Of Vector Addition



$\overrightarrow{AB}$ & $\overrightarrow{CD}$ $\overrightarrow{AC}$ & $\overrightarrow{BD}$ are equal vector by triangle law of vector addition , the resultant vector will be

$\overrightarrow{AB} + \overrightarrow{BD} = AC + \overrightarrow{CD} = \overrightarrow{AD}$. that is $a + b = \overrightarrow{AD}$. This method of deterring the sum of vector is called

parallelogram law of vector addition.

- ✓ If two vectors have different inti all point you need to bring the two initial points placed at a point by constriction before apply <u>parallelogram method</u>.

Addition and Sub traction of Column Vectors

For two vector in component form

$\binom{P}{q}$ and $\binom{r}{s}$ we law

$$\binom{P}{q} + \binom{r}{s} = \binom{P+r}{q+s} \text{ and } \binom{P}{q} - \binom{r}{s} = \binom{P-r}{q-s}$$

Example:-

Then find

a) $a + b = \binom{1}{-3}$

b) $a - b = \binom{-9}{-3}$

Exercise

1, For the vector $a = \binom{1}{3}$, $b = \binom{2}{-1}$ $C, = \binom{-4}{5}$ find

a, $a + b = \binom{1}{3} + \binom{2}{-1} \equiv \binom{3}{2}$

b, a – b $= \left(\frac{1}{3}\right) - \left(\frac{2}{-1}\right) = \left(\frac{-1}{4}\right)$

C, a + C $= \left(\frac{1}{3}\right) + \left(\frac{-4}{5}\right) = \left(\frac{-3}{8}\right)$

d, b – C $= \left(\frac{1}{3}\right) - \left(\frac{-4}{3}\right) = \left(\frac{5}{0}\right)$

e, a + b+C $\left(\frac{1}{3}\right) + \left(\frac{2}{-1}\right) + \left(\frac{-4}{5}\right) = \frac{-1}{7})$

Multiplication Of a vector by Scalar

Let $\overrightarrow{AB}$ be any given vector and K be any real number. The scalar multiple vector K $\overrightarrow{AB}$ is the vector whose magnitude (Length) is K times the magnitude of AB and

i) The direction of K $\overrightarrow{AB}$ is the same as the direction of $\overrightarrow{AB}$, If $K > 0$

ii) The direction of K AB is the opposite of $\overrightarrow{AB}$, If $K < 0$

Example:-

Given a vector $\overrightarrow{AB}$ as shown in figure. 2 AB , $\frac{1}{2}$ AB , -2 AB and - $\frac{3}{2}$ AB

- Two vector are said to be parallel If one can be written as a scalar multiple of the other . If b is parallel to a then it can be expressed as b = Ka

Example:-

Consider Column Vector a = $\left(\frac{2}{4}\right)$ then find - a , 2a and $\frac{1}{2}$ a

- a = -1 $\left(\frac{2}{4}\right) = \left(\frac{-2}{-4}\right)$

2a = $2\left(\frac{2}{4}\right) = \left(\frac{4}{8}\right)$

$\frac{1}{2}$a $= \frac{1}{2}\left(\frac{2}{4}\right) = \left(\frac{1}{2}\right)$

Exercise

1, Given vector a $= \left(\frac{-1}{4}\right)$ and b $= \left(\frac{3}{6}\right)$

a, 4 a $= 4\left(\frac{-1}{4}\right) = \left(\frac{-4}{-16}\right)$

b, -3 a $= -3\left(\frac{-1}{4}\right) = \left(\frac{3}{-12}\right)$

C, $\frac{1}{3}$ b $= \frac{1}{3}\left(\frac{-1}{4}\right) = \left[\frac{-1}{3}\right]$

4/3

Position Vector

Vectors that start at the origin (0) are called **Position Vector** the position vectors are also said to be vectors which are written in standard form. These are used to determine the position of a point with reference to the origin.

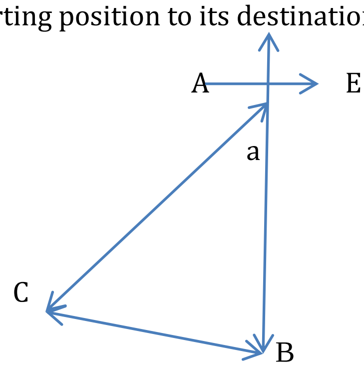
8.2 Applications Of Vector in two Dimensions

Suppose a boat start moving on the sea 8km to the south and then 8km to the west . What is the displacement of the boat from its starting position to its destination.

$AC = \sqrt{(AB)2 + (BC)^2}$

$= \sqrt{64+64} = 8\sqrt{2}$

$\tan a = \frac{OPP}{adj} = \frac{8}{8} = 1$



$\tan a = 45^0$, Hence the displacement of AC is $8\sqrt{2}$ Km in the S 45^0 W .

<u>Exercise</u> 8.12

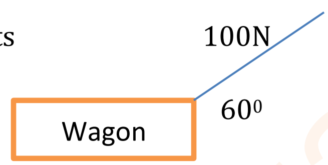
2, A wagon is being poiied by a rope that makes a 60^0 with the ground. A person is pulling with a force of 100N along the

rope. Determine the horizontal and vertical components of the vector

<u>Solution</u>

Find the components

Fx and $\frac{Fy}{100}$



0.87

Fy $= 100 \sin 60 = 87$N

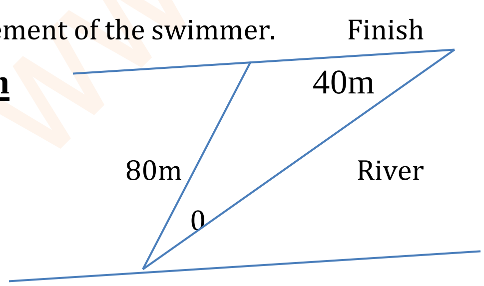
added vertically

Fx $= 100 \cos 60 = 50$N

3, A swimmer head directly to the north across a river swimming at 1.6 m/s relative to still water which move to the east.

She arrives at a point 40m sown stream from the point directly across the river is 80m wide. Determine the displacement of the swimmer.

<u>Solution</u>



Start

AC $= \sqrt{(80)2 + (40)^2} = 40\sqrt{5}m$

tan $\Theta = \frac{40}{80} = \frac{1}{2}$ hence $\Theta = \approx 27^0$

So the swimmer is located $40\sqrt{5}$ m to N $27^0 \in$.

Important not

- ✓ Vector that starts from the origin 0 is called a position vector written as $\vec{U} = x\text{i} + \text{yj} = \binom{x}{y}$

  Where $\vec{I} = \binom{1}{0}$ ad $\vec{J} = \binom{0}{1}$

Review Exercise

1, The vector $\vec{V}$ has initial point P = (1 , 0) and terminal point Q that on the Y – axis and above the initial point . find

coordinates of terminal point Q such that the magnify of the vector $\vec{V}$ of the vector $\vec{V}$ is $\sqrt{10}$

V = $\overline{PQ}$ where P = (1 , 0) and the point Q lies on the Y – axis. So that it coordinate has a form

Q = (0 , Y) where Y to be determent

$\vec{V} = \sqrt{10}$ $\quad \sqrt{10} = \sqrt{1 + Y^2}$

$(\sqrt{10}) = (\sqrt{1 + Y^2})$ $\quad y^2 = 10 - 1$

$Y^2 = 9$ $\quad \underline{\underline{y = 3}}$

Hence the point Q = (0 , 3)

2, Let $\vec{\boldsymbol{a}}$ be a standard – position vector with terminal point (- 2 , - 4). Let $\vec{\boldsymbol{b}}$ be a vector with intial point (1 , 2) and

terminal point (-1 , 4) find the magnitude of

$-3\ \vec{a} + \vec{b}$ - 4 it j

$\vec{a}$ = -2i - 4j also $\vec{b}$ = (-1-1 , 4-2) = (-2, 2)

(-2 , 2) = 2i + 2j so that we can

find- 3 $\vec{a} + \vec{b}$ - 4i + j = -3 (2i – 4j) +

2j – 4i + j = 0i + 15j

Hence the magnitude of -3 $\vec{b} + \vec{b}$ - 4i + j is **15**

3, A merchant start to move in a city . he drives to the book store which is 17km to the north.

Then he moves to the music shop 6 Km to east and finally he drives 5Km to south . Find the displacement of the merchant.

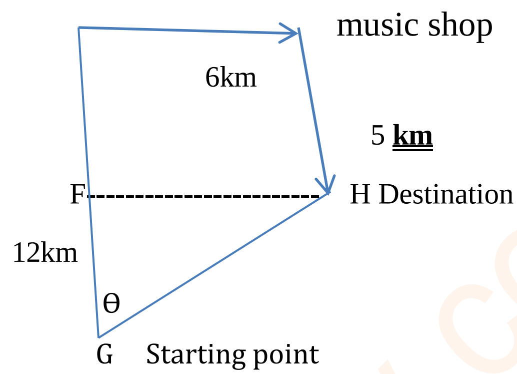
Solution

Book store

$|HG| = \sqrt{6 + (12)^2} = \underline{\mathbf{6\sqrt{5}\ Km}}$

$\tan \Theta = \frac{FH}{GH} = \frac{6}{12} = 0.5$

$\Theta = \approx 2.6\ .57$



hence the merchant is located $6\sqrt{5}$ km in the direction N 26.56^0 E

4, A plane flies 100km , n 30^0 w and after a brief stopover flies 150 km, N 60^0 E . Determine the plane 's displacent

x - Component

$\text{Sin } 30^0 = \frac{x1}{100km}$

$x_1 = 100 \sin 30$

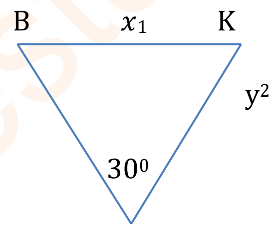
$x = 50$ km

y – component

$\text{Cos } 30 = \frac{y1}{100km}$

$y_1 = 100\ x\ \cos 30$

$y_1 = 87$



y - component

$\text{Sin } 60 = \frac{x2}{150km}$

$x_2 = 150 \times \sin 60$

$Y_2 \approx 130$ km

y – component

$\text{Cos } 60 = \frac{y2}{150km}$

$y_2 = 150\ x \cos 60$

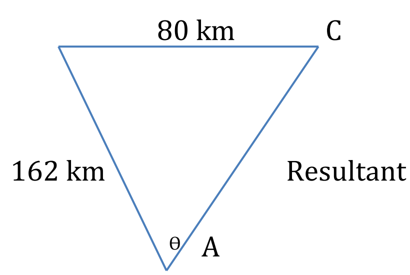
$y_2 = 75$ km

$AC^2 = (AD)^2 + DC^2$

$AC = \sqrt{(162) + (80)^2}$

$AC = (81 \text{ km})$

$\tan \theta = \frac{80km}{162km} \Rightarrow \theta \approx 26.28^0$



Hence , the plane displacement is 181 km in the direction of N 26.28^0 E .

UNIT 9

Statistics And Probability

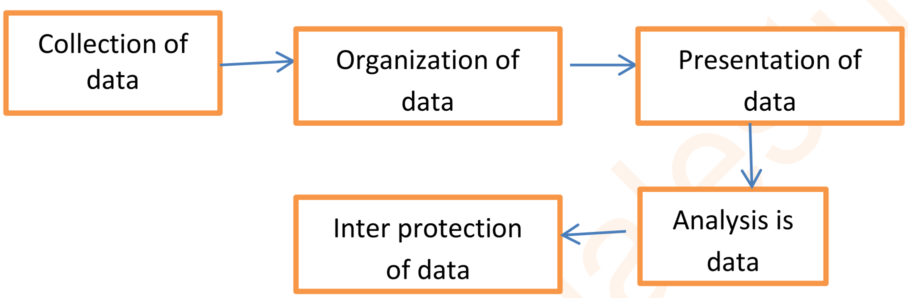
9.1 Statistical Data

- ✓ Collection of numerical data. it is the mathematical science that deals with the gathering, evolution , Interpretation of numerical facts using the concept of probability of a population from examination of a random pattern.

Collection and tabulation of Statically data

Statics is a science of collection organizing, presenting analyzing and interpreting data so that one can make a

generalization.



Data collection

In statics , data collection is a process of gathering Information from all the relevant source to find a solution to the research problem. it helps to evaluate the outcome of the problem.

9.2. Types of Data

Qualitative data and Quantitative data

Data can be classified as either qualitative or quantitative

Qualitative Data is a means for exploring and understanding the meaning individual or groups a scribe based on some character whose value are not numbers, such as their color , Sex, religion.

Quantitative :- data is a means for resting objective theories by examining the relationships among numerical variable such as score in exam, height , weight age, wealth.

Continues data and discrete data

Continuous:- data is information that could be meaning fully divided in to finer level. It can be measured on a scale or

continuum and can have almost any numeric value.

Discrete Data:- is a count that usually involve integer. Example

The number of children in a school is discrete data.

Sources Of collecting data

i, **Primary Data:-** is a data that has been generated by the researcher himself / herself , Survey , Interview , experiment

ii, **Secondary data:-** is data used after it is generated by large government institutions, health care facility, organization record keeping.

iii, **Interview:-** The main purpose of an interview as a tool of data collection is to gather data extensively and intensively. Exchanging ideas and experiences.

iv, **Direct observations:-** A technique that involves system apically selecting watt chins, listing , reading, touching and recording behavior and characteristics of living things objects, or phenol mean.

Organization Of Data

The process of condensing data and presenting it in compact form , by putting data in to statistical table is called tabulation of statically data.

Presentation Of Data

The main purpose of data presentation is to facilitate statically analysis. This can be done by illustrating the data using graph and diagrams like bar graph, histograms, pie chart, pictograms , frequency polygon etc..

Analysis Of Data

The primary objective of analysis data is to grasp the tendency and characteristics of data from the represented values (mean , median and mode) and from how it is disturbed (dispersion), through the previous step organizing and presorting data.

Interpretation Of Data

Based on analyzed data conclusion have to be drawn. This step usually involve decision making about a large collection of objects (the population) based on information gathered from a small collection of similar object (the sample).

Example:- Recording annual dropout rated of students in schools that help ministry of Education of Ethiopia to reform an appropriate policy.

9.3 Population and Sampling

Population:- in statics refers to all over collection of Individuals , objects or measurement that have common characteristic.

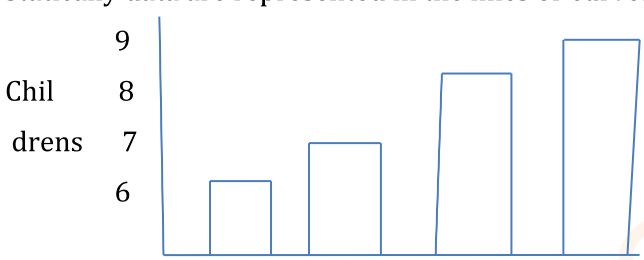
Census:- an enumeration of people, house firms or other important items in a country or region at a particular time. The term usually refers to a population census. However many countries take census of housing manufacturing and agriculture.

Censuses:- being expensive are taken only at infrequent interval every 10 years in money countries, every 5 years or at Irregular intervals in other countries.

Sampling frame:- is the actual set of units from which a sample has been drawn, in the case of a sample random sample , all units from the sampling frame have an equal chance to be drawn and to occur in the sample.

Graphical Presentation of Satirical data

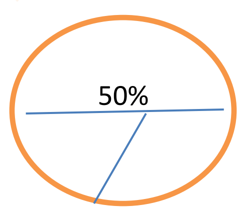
Graphical representation is another way of analyzing numerical data. A graph is a sort of chart through which statically data are represented in the lines or curves drawn a cross the coordinated points plotted on its surface.



Pie - Charts

Data can be displayed on a pie – charts a circle divided in to sectors. The size of the sector is in direct proportion to the frequency of the data.

The sector size does not show the actual frequency.



Pi – Chart

Example:- In a survey of 120 children, they were asked to choose their favorite color. The total is 120 is represented by

100% . it follows that If 120 represent 100% , then.

a, 50% represent $120\ x\ \frac{50}{100} = 60$ children white b, 30% represent 120 $x\ \frac{30}{100}$ = 36 children read

C, 20 % represent 120 $x\ \frac{20}{100}$ = 24 children blue

9.4 Methods of to represent a frequency distribution

Distribution and histograms

Data should be organized once it is collected so that it can be manageable. Data that is not organized is called **raw data**.

A variable:- refers to a characteristics or attribute of an individual or an organization that can be measured or observed and that varies among the people or organization being studied. A variable typically will vary in two or more categories or on a continuum of scores, and it can be measured.

Example:- Suppose there are 12 people in a village whose weights in kilograms were measured as follows: 55, 62 , 49 , 67 , 55 , 62 , 49 , 67 , 62 , 55 , 49 , This data are raw. Organizing as in table.

Weight in kg (v)	49	55	62	67	
Number of people (f)	3	3	4	2	

This table is called frequency distribution table.

Exercise 9.3

Complete the frequency table with the data below.

a, 1, 1 , 1 , 2 , 3 , 3 , 4 , 4 , 4 , 4 , 5 , 5 , 6 , 6 , 6

Value (v)	1	2	3	4	5	6
frequency (f)	3	1	2	4	2	3

b, 2 , 3 , 4 , 6 , 8 , 9 , 10 , 11 , 13 , 13 , 15 , 17

Class interval (x)	Frequency
$0 \le x < 5$	3
$5 \le x < 10$	3
$10 \le x < 15$	4
$15 \le x < 20$	2

A histogram is a graphical representation of a frequency distribution in which the variable (V) is plotted on the x - axis and the frequency (f) is plotted on the y – axis.

9.5 Measures Of central tendency

Measure of central tendency is a single value that attempts to describe a set of data by Identifying the central position with in that set of data.

- ✓ The mean, median and mode are all valid measures of central tendency,

Mean

The Arithmetic mean (or average) is the most popular and well – known measure of central tendency. It can be used with both discrete and continuous data Although it use is most often with continuous data (see our types of variable guid for data types). The mean is equal to the sum of all the values in the data set divided by the number of values in the data set, so we have value x_1 , x_2 , --- x_n the mean usually denoted by $\overline{x}$

$$\overline{x} = \frac{x1+x2+x3 + -- xn}{n}$$

Example:-

Seven children have the following number of text book 5 , 4 , 6 , 4 1 3, 6 and 7

$$\overline{x} = \frac{5+4+6+4+3+6+7}{7} = \frac{35}{7} = \underline{\underline{5}}$$

- The mean can also be calculated from its frequency distribution .

  So the value x_1 , x_2 , --- x_n occurs f_1 , f_2 , f_3 --- f_n

$$\overline{x} = \frac{f1\, x1 + x2\, f2 + f3\, x + --fn\, xn}{f1+f2+f3+ - - - fn}$$

Example:-

Calculate the mean test score from the frequency distribution table below shows the test scor of 10 student

Test scores (v)	10	20	30	40	50
frequency (f)	0	4	1	3	2

Solution:-

$$\overline{x} = \frac{10(0)+20(4)+30(1)+40(3)+50(2)}{10}$$

$$\frac{\mathbf{330}}{\mathbf{10}} = \mathbf{33}$$

Exercise

a, calculat the mean of the weights in kg of 6 peoples below

45 , 50 , 55 , 60 , 70 , 80

$\frac{45+50+55+60+70+8}{6} = \frac{360}{6} = \underline{60}$

b, Calculate the mean height of the people shown in frequency distribution below.

Height (cm) (v)	160	170	175
frequency (f)	3	2	1

$\bar{x} = \frac{160(3)+170(2)+175(1)}{6}$

$\bar{x} = \frac{480+240+175}{6} \quad \frac{935}{6} = \underline{155.8}$

Properties Of Mean

- ✓ The sum of the deviations from the mean is zero. If $\bar{x}$ is the arithmetic mean of n observe x_1 , x_2 , x_3 - - - x_n then

  $(x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) \; --- (x_n - \bar{x}) = 0$
- ✓ The mean of "n" observations x_1 , x_2 , x_3 - - - x_n is x . If each observation is decreased by P , the mean of the new observation is $(x + \bar{P})$.
- ✓ The mean of n observations x_1 , x_2 , x_3 - - - x_n is $\bar{x}$. If each observation is decreased by P , the mean of the new observation is $\bar{x}$- P.
- ✓ The mean of n observation x_1 , x , x_3 - - - x_n is $\bar{x}$. If each observation is multiplied by a nonzero number P , then the mean of the new observation is P $\bar{x}$

  $\bar{x} = \frac{x1+x2-- \; xn}{n}$

  x_1 , x - - - + x_n = n $\bar{x}$
- ✓ The mean of n observations x_1 , x_2 --- x_n is $\bar{x}$. if each observation is divided by a no zero number P , then the mean of the new observation is $\frac{\bar{x}}{P}$

Exercise

1, If $\bar{x} = 4$ find the new mean in case

a, each value Increased by 1 .

$x + 1 = 4 + 1 = 5$

b, each value decreases by 2.

$x - 2 = 4 - 2 = 2$

C, each value multiple by 3.

$x \; x\, 1 \quad = 4\, x\, 3 \; = \underline{12}$

d, each value is divided by 3.

$\frac{x}{2} \quad = \frac{4}{2} \; = \; 2$

2, The weekly mean number of customer who use a bus stop is 100 per day . If 10 more customers use it every day. What
is the weekly mean number of the customers?

Solution:-

$\bar{x} = \; 100 \qquad \Rightarrow \; \bar{x} \; + 10 \; = \; 100 + 10 \; = \; \underline{110}$

3, a car factory manufactures 1000 cars every day on average. One day a new machine is installed, and they can now
produce twice as many cars every day. What is the mean number of car that they produce every day?

$\bar{x} \quad = 1000 \qquad\qquad 2\bar{x} \; = \; 2x\, 1000 \; = \; \underline{2000}$

Median

The median is the middle score for a set of data that has been arranged in order of magnitude, The median is less affected by extreme values and skewed data.

Example:-

The following are the scores of 11 peoples . find the median.

65 , 55 , 89 , 56 , 35 , 14 , 56 , 55 , 87 , 45 , 92.

$\text{Median} \; = \qquad \frac{11+1}{2} \; = \; \underline{6}$

14 , 35 , 45 , 55 , 55 , 56 , 56 , 65 , 87 , 89 , 92 **median**

Example:- Find the median 65 , 55 , 89 , 56 , 35 , 14 , 56 , 55 , 87 , 45 ,

We again rearrange that data

14 , 35 , 45 , 55 , 55 , 56 , 56 , 65 , 87 , 89 , 92

The total is 10

$\text{Median} \; = \frac{10}{2} \; = \; 5 \qquad \frac{5^{th} + 6^{th}}{2} \qquad\qquad \frac{55+56}{2} \quad 55.5$

$\frac{55+56}{2} \quad = \; \underline{55.5}$

Properties Of the median

1, If is not affected by extreme values

2, it is unique for a given data set .

Example:- Find the median of the numbers

5 , 3 , 99 , x , 4 where x is greather than 10 less than 99

Solution:-

arrange 3, 4 , 5 , x , 99 hence 5 is the median and is not affected by extremism value , 99 and is unique.

Exercise

a, find the median of the data below 5, 7, 3, 10, 1, 5, 9

Arrenge 1, 3, 5, 5, 7, 9, 10

Median is $\frac{n+1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4^{th}$

The median is 5

b, Calculat the mean and median compure the value of the two data below.

15, 10, 2, 6, 7, 20, 3, 18, 100

Solution:-

Arrange 2, 3, 6, 7, 10, 15, 18, 20, 100

mean is $\frac{x_1+x_2+x_3+x_9}{9}$

$\frac{2+3+6+7+10+15+18+20+100}{9} = \frac{181}{9} = \underline{20.11}$

Median $= \frac{9+1}{2}$ $\frac{10}{2}$ 5^{th} $=$ **10 median**

Mode

The mode is the most frequent value in a set of data.

Example:- Find the mode of the following data.

5, 10, 20, 20, 30, 35, 35, 35, 40, 50, 70, 70

hence frequent value is 35 the mode is **35**

Properties Of the mode

1, The mode is not always unique

2, The mode can also be used for qualitative data.

Exercise

1, a boy recorded the number of book pages that he read over the last 5 days. 24, 10, 16, 7, 33

Arrenge 7, 10, 16, 24, 33

a, Find the mean number of pages read $\overline{x} = \frac{7+10+16+24+33}{5} = \frac{90}{5} = 18$

b, Find the madian $\frac{5+1}{2} = 5^{th} = 16$

C, Find the mode = No mode

2, a women was recording her weight in kg in the last six months as follow

88, 86, 89, 90, 91, 85

Arreng the data 85, 86, 88, 89, 90, 91

a, What is the mean weight of the women in the last six month?

$$\overline{x} = \frac{85+86+88+89+90+91}{6} = \frac{529}{6} = 88.1$$

b, Find the median $= \frac{6}{2} = \frac{3^{rd}+4^{th}}{2}$

$\frac{88+89}{2} = \underline{88.5}$

3, is it possible to find the arithmetic mean of qualitative data? **No**

4, Answer the following by referring to the frequency distribution given.

a, Find the mean , median and mode

$$\overline{x} = \frac{x_1 f_1 + x_2 f_2 + x_3 F_3}{F^1 + F^2 + F^3 + F_n} \frac{(-4x2)+(-1x3)+(0x4)}{2+3+4}$$

$$+\frac{(1x5)+(2x3)+(3x3)}{+5+3+3} = \frac{-8+(-3)+0+5+6+9}{20}$$

$$-\frac{8\pm3+5+6+9}{20} = \frac{9}{20} = \underline{0.45}$$

b, How many of the values are greater than or equal to = 2 = **16**

v	-4	-1	0	1	2	3
f	2	3	4	5	3	3

5, If the mean is 5 for the data 2, 3, ×, 5, 6, 12, Find the value ×

$$\frac{2+3+\times+5+6+12}{6} = 5$$

$\frac{28}{6} + \times \quad = 5 \qquad\qquad 28 + \times = 30$

$\times = 30 - 28$

$\underline{\times = 2}$

6, If the mean of a , b, C, d, is k then what is the mean of, a+ b, 2b C+ b , d+ b ? The mean is k+b

7, If the mean of y+2, y+4, y+6 and y+10 is 13. Find the value of y.

Solution:- $\frac{y+2+y+4+y+6+y+10}{4} = 13$

$\frac{4y+22}{4} = 13 \Rightarrow 4y + 22 = 52$

$4y = 52 - 22$

$\frac{4y}{4} = \frac{30}{4} \Rightarrow y = \frac{30}{4} = \underline{7.5}$

8, The mean monthly salary of 12 employees of a firm is Birr 1450. If one more person joins the firm who gets birr 1645

per month, what will be the mean monthly salary of 13 employees?

9.6 Measures Of dispersion

When comparing sets of data it is use full to have a way of measuring the scatter or sqread of the data. There are several measures of dispersion that can be calculated for a set of data .

1, Range

The simplest and crudest measure of the dispersion of quantitative data is the range.

- ✓ The difference between the largest and smallest values of a set of numerical data is called the **range** R.
  Range = largest value - smallest value

Example:- 6, 7, 8, 5, 3, 8, 9, 5, 4

Arranged = 3, 4, 5, 6, 7, 8, 8, 9

Rang = 9 – 3 = $\underline{6}$

2, Variance and standard deviation

- ✓ Variance is the mean of the squared deviations of each value from the arithmetic mean and is denoted by 8^2
- ✓ Standard deviation is the square root of variance

Example:-

Calculate the variance 8^2 and standard deviation , 8 If the number of television sold in each day of a week is

13, 8, 4, 9, 7, 12, 10.

Solution:- $x = \frac{13+8+4+9+7+12+10}{7} = \frac{63}{7} = 9$

x i	$\overline{xi}$ - ×	(xi - $\overline{xi}$)^2
13	13-9 = 4	16
8	8 – 9 = 1	1
4	4 – 9 = -5	25
9	9 – 9 = 0	0
7	7 – 9 = -2	4
12	12 – 9 = 3	9
10	10 – 9 = 1	1

$8^2 = 16$

$8 = \sqrt{8^2} = \sqrt{8}$

$\underline{8 = 2\sqrt{2}}$

$8^2 = \frac{16+1+25+0+4+9+1}{7} = \frac{56}{7} = \underline{8}$

Example:- 48 students were asked to write the total number of hour. Per week they spent watching television. With

this information, find the standard division of hours spent watching television.

Hour (h) per week	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

$\overline{x} = \frac{6x3+8x9+7x610x8+11x5+12x4}{3+6+9+13+8+5+4}$

$\overline{x} = \frac{18+42+72+80+117+55+48}{48}$

$\overline{x} = \frac{432}{48} = 9$

x i	F_i	x i F_1	$\overline{xi}$ - ×	(x i - x)^2	F (x i - x)^2
6	3	18	-3	9	27
7	6	42	-2	4	24
8	9	72	-1	1	9
9	13	117	0	0	0

10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36

$\delta^2 = \frac{27+24+9+0+8+20+36}{48} = \frac{124}{48} = \frac{31}{12}$

$\delta = \sqrt{\delta^2} = \sqrt{\frac{31}{12}} = \sqrt{2.58} = \underline{1.6}$

Properties of Standard deviation

1, If a constant K is added to each value of a data, then the new variance is the same as the old variance. The new standred

deviation is also the same as the old.

2, If each value of data is multiplied by constant C, then

a, The new variance is C^2 times the old variance

b, The new standard deviation is $|C|$ times the old standard deviation.

Exercise

1, Find the mean median and mod of the following data 3, 4, 5, 5, 7, 8, 9, 9, 10

$\bar{x} = \frac{3+4+5+5+7+8+9+9+10}{9} = \frac{60}{9} = \underline{6.6}$

$= 6.6667 = \underline{6.6}$

median $= \frac{9+1}{2}\ \frac{10}{2} = 5^{th} = \underline{7}$ is median

mod is has two mode **5 and 9**

2, find the rang, variance and standared deviation of the distribution in the table below.

V	-3	-2	-1	0	1	2	3
f	3	2	1	8	1	2	3

$\bar{x} = \frac{0}{20} = 0$

x i	F_1	x i F_1	$\overline{xi} - x$	$(\overline{xi} - x)^2$	F $(x\text{ i} - x)^2$
-3	3	-9	-9	81	243
-2	2	-4	-4	16	32
-1	1	-1	-1	1	1
0	8	0	0	0	0
1	1	1	1	1	1
2	2	4	4	16	32

3	3	9	9	81	243
n = 20					

$$\delta^2 = \frac{243+32+1+0+1+32+243}{20}$$

$$\delta^2 = \frac{552}{20} = \underline{27.6} \qquad \delta = \sqrt{\delta^2} = 5.25 = \underline{5.2}$$

2, If the standard deviation of the data 5, y, 5, 5, 5, is 5 what is the value of y?

V	5	Y
f	4	1

$$\overline{x} = \frac{5+y+5+5+5}{5} = \frac{y+20}{5}$$

x i	F_1	x i F_1	$\overline{x\,i} - x$	$(\overline{x\,i - x})^2$	Fi $(\times i - \overline{\times})^2$
5	4	20	$5-\frac{y+20}{5}$	$(5-y-\frac{y}{5})^2$	
y	1	y	$y-\frac{y+20}{5}$	$(y-\frac{y+20}{5})^2$	

$$(x\,i - \overline{x})^2 = (5 - \frac{y+20}{5})^2 = (5 - 4 - \frac{y}{5}) =$$

$$(1 - \frac{y}{5})^2 = 1(1 - \frac{y}{5})\ \frac{-y}{5}\ (1 - \frac{y}{5})$$

$$1 - \frac{y}{5} - \frac{y}{5} + \frac{y^2}{25} \qquad = \frac{y^2}{25} \qquad \frac{2y}{5} + 1$$

$$(y - \frac{y+20}{5})^2 = (y - \frac{y}{5} + 4)^2 = (\frac{4y}{5} + 4)^2$$

$$\frac{4y}{5}(\frac{4y}{5} + 4) + 4(\frac{4y}{5} + 4)$$

$$\frac{16y^2}{25} + \frac{16y}{5} + \frac{16y}{5} + 16$$

$$\Rightarrow \frac{16y^2}{25} + \frac{32y}{5} + 16$$

$$\text{Fi}\ (x\,i - \overline{x})^2 = 4\,(y^2 - \frac{2y^2}{25} + 1) =$$

5

$$\frac{4y^2}{25} \quad 4y^2 \qquad - \frac{8y}{5} + 4$$

25

$F\,i\,(x\,i - \overline{x})^2 = 1\,(\frac{16y^2}{25} + \frac{32y}{5} + 16\,)$

$= \frac{16y^2}{25} + \frac{32y}{5} + 16$

$\delta^2 = \frac{4y^2}{25} - \frac{8y}{5} + 4 + \frac{16y^2}{25} + \frac{32y}{5} + 16$

25

$\delta^2 = \frac{4y^2}{25} + \frac{16y^2}{25} \frac{-8y}{5} + \frac{32y}{5} + 16 + 4$

2

$\frac{20y^2}{25} + \frac{24y}{5} + 20 = 5$

$\frac{20\,y^2}{25} + \frac{24\,y}{5} + 20 = 5$

$120y^2 + 120\text{ y} + 500 = 125$ multiply by **25**

$20y^2 + 120y + 375 = 0$

$4y^2 + 24y + 75$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\sqrt{576 - 4\,(4)75}$

$\frac{-b \pm \sqrt{576-1200}}{2a} = \sqrt{-624} = 24.9$

$\frac{-\,b \pm (-24.9)}{2a} = \frac{24 \pm (-24.9)}{2a}$

$\frac{-\,24+24.9}{8}$ Or $\frac{-\,24-24.9}{8}$

$\frac{-\,24+24.9}{8} = \underline{0.1125}$ **Or**

$\frac{-\,24-\,24.9}{8} = \frac{48.9}{8} = \underline{6.11}$

3, Twenty students were asked to write the total number of hours per week they spent watching Ethiopian television. With

this information find the standard deviation of hours spent watching Ethiopia television.

Hour (h) per week	4	3	2	1
f	2	6	5	7

x i	F_1	x i F_1	x i - $\bar{x}$	(x i - $\bar{x}$)^2	Fi (x - x)^2
4	2	8	2.15	4.6225	9.245
3	6	18	1.15	1.3225	7.935
2	5	10	0.15	0.0225	0.1125
1	7	7	-1.15	1.3225	9.2575

$$\bar{x} = \frac{8+18+10+7}{20} = \frac{43}{20} = \underline{\underline{2.15}}$$

$$\delta = \frac{9.245+7.935+0.1125+9.25\ 75}{20}$$

$$\delta = \frac{26.55}{20} = \underline{1,3275} \quad \delta = \sqrt{1.3245} \quad \underline{\underline{1.15}}$$

9.7 Probability

- ✓ Probability is numerical value that describes the likely hood of the occurrence of an event in an experiment
- ✓ An experiment is a trial by which an observation is obtained by but whose out come can not be predicted in advance.
- ✓ Experimental probability – is a probability determined by using data collected from a repeated data.
- ✓ If an experiment has n equally likely out comes and If m is a particular outcome, then the probability of this out come occurring is $\frac{m}{n}$

Example:- All the possible outcomes that can occur when a coin is tossed twice are HH, HT, TH, TT what is

the probability of having?

a, Exactly one tail ?

b, at least one head

Solution:-

a, Two (HT & TH) out of thr four possibility $P = \frac{2}{4} = \frac{1}{2}$

b, Three (HH, HT & TH) out of four possibility $P = \frac{3}{4} = \frac{\mathbf{3}}{\mathbf{4}}$

Sample Space and event

Sample space for an experiment is the set of all possible outcomes of an experiment.

Example:-

a, What is the sample space in throwing two coins?

Set of possibility out comes is

S = { HH, HT, TH, TT

- ✓ An event is a subset of a sample space set or a sample space.

Probability of an events

Example:-

Tosss a coin 10,000 times and you obtained 5010 heads.

i, if the event was tail, how many times did this event occur?

ii, According to the experiment , what was the probability of tails ?

Solution:-

i, The times you get a tail is

$10000 - 5010 = 4990$

4990 times

ii, $\frac{4990}{10000} = 0.499$

probability of tails is **0. 499**

Exercise

1, in a class there are 20 boys out of 40 classing a class mate at random, what is the probability of choosing a boy

$P = \frac{20}{40} = \frac{1}{2}$

2, There are 12 males and 8 females in room . If one person is randomly selected. what is the probability that the selected

one is a female?

$P(e) = \frac{fmal}{Total} = \frac{8}{20} = \frac{2}{5}$

3, Suppose you are planning a reserch for public and privete schools and want to sample / school / at random.

If the ratio of the number of public school to that of private schools is 9 : 1 what is the probabilityof sampling privet

school ? $P(e) = \frac{1}{10} = \frac{1}{10}$

Exercise

1, Calculate the probability of getting the following events when three fair coins are tossed by drawing a three diagram.

a, Exactly one head $= \frac{3}{8}$

b, three heads $= \frac{1}{8}$

C, at least two tails $= \frac{4}{8} = \frac{1}{2}$

The Orifice Probability

The theoretical probability of an event E , written a P (E) is defined as follow.

$$P(E) = \frac{number\ of\ out\ com\ favourable\ to\ event}{Total\ number\ of\ possible\ out\ com}$$

$$P(E) = \frac{n(E)}{n(S)}$$

Exercise

1, A bag contains ared, blue and black ball. You run an experment where you pick a ball from the bag at random and put it

back to the bag after confirmind which color is picked. You run this experment twice.

a, Find the probability of getting a red ball once $= \frac{1}{3}$

b, Find the probability of getting two balls of the sam color $= \frac{1}{3}$

C, Find the probablity of getting two balls of different color $= \frac{2}{6} = \frac{1}{3}$

$1 - \frac{1}{3} = \frac{2}{3}$

Review Exercise

1, The Amount of money in birr that 20 peoples have in there pocket are given below :

4, 9, 2, 3, 9, 1, 2, 4, 2, 4, 6, 6, 7, 2, 5, 9, 3, 5, 8, 9

a, Constrict a frequency distribution table

1	2	3	4	5	6	7	8	9
1	4	2	3	2	2	1	1	4

b, What percent of the people have less than Birr 4 ?

$\frac{7}{20}\ x\ 100 = 3.5$ Or 35%

2, a fair die is rolled . what is the probability that a, 1, 4, 5 or 6 will be on the upper face?

S $= \{1, 2, 3, 4, 5, 6\}$. So the probability that a 1, 4, 5, or 6 will be on the upper face is $\frac{4}{6} = \frac{2}{3}$

3, Which one of the following is true ?

A, The mean , mode and median of a data cannot be equal ? **False**

B, The rang of a data cannot be a non- positive number. **False**

C, The sum of the deviation of each value of a population from the mean will always be zero. **False**

4, A card is drawn from a well shuffled pack of 5^2 cards, what is the probability of getting given of club or king of hearts

card ? The probability of getting a queen of clubs $= \frac{1}{5^2}$

The probability of getting a king of hearts $= \frac{1}{5^2}$

Since only one card is drawn – getting both is an Impossible outcome. Hence , the probability of getting either a queen

of clubs or a king of heart is $\frac{1}{5^2} + \frac{1}{5^2} = \frac{2}{5^2} = \frac{\mathbf{1}}{\mathbf{26}}$

5, Find the mean, medium, mode, rang variance and standard deviation of the scores of an exam out of 20 % whose distribution is given in the table below

V	11	12	13	14	15	16
F	6	7	5	7	3	2

Solution:-

$\overline{x} = \frac{11\,x6+12\,x7+13\,x5+14\,x7+15\,x3+16\,x2}{30} = \frac{390}{30} = \underline{\mathbf{39}}$

$\overline{x} = 39$

x i	F_1	x i F_1	$(x\,i - \overline{\times})^2$	Fi $(x - \overline{x})^2$
11	6	66	4	24
12	7	84	1	7
13	5	65	0	0
14	7	98	1	7
15	3	45	4	12
16	2	32	9	18

ƐFi = 30 Ɛ ×I Fi = 390 $\Sigma F\,(xi - \overline{x})^2 = 68$

The variance $= \frac{\Sigma F\,(xi - x)^2}{\Sigma Fi} = \frac{68}{30} = 2.26$

$= \underline{\mathbf{2.267}}$

Standard deviation , $\delta = \sqrt{\frac{\Sigma F\,(xi - x)^2}{\Sigma Fi}}$

$= \sqrt{2.267} = 1.50554 = \underline{\mathbf{1.5}}$

6, The point received from the first 7 games of a football club were respectively 1, 3, 1, 3, 3, 1 and 3 . What must be the point in the 8th game to have a mean of 2 point ?

A, 0 B 1 C, 3 D, Non

7, Refer to the frequency distribution table below to answer question that follows $\Sigma F\,(xi - x)^2\,\Sigma Fi$

V	9	6	5	4	3	2
F	6	7	5	7	3	2

Which one of the following true

A, the mean is 5.4 C, The medians 5.5

B, the mode is 9 D, No Answer

Solution:-

$$\overline{\times} = \frac{9\,x6+7\,x6+5\,x5+4\,x7+3\,x3+2\,x2}{30}$$

Mean =

$$\overline{\times} = \frac{54+42+25+28+9+4}{30} = \frac{162}{30} = \underline{5.4}$$

$\times$ i	F_i	$Fi \times i - x$	x i - $\bar{x}$	(x i - $\times$)^2	Fi (x i - x)^2
9	6	54	3.6	12.96	77.76
6	7	42	0.6	0.36	2.52
5	5	25	-0.4	0.16	0.8
4	7	28	-1.4	1.96	13.72
3	3	9	-2.4	5.76	17.28
2	2	4	-3.4	11.56	23.12

mode is = 4

median is $= \frac{30}{2} = \frac{15^{th}+16^{th}}{2} = \frac{5+5}{2} = \underline{\underline{5}}$

The Answer is $\underline{\underline{\textbf{A}}}$

Factor	$x < 3$	$x = 3$	$-1 < x < 2$	$x = -1$	$x < -1$
$x - 3$	- - -	0	- - -	- - -	+ + +
$x+1$	- -	+ + +	+ + + + +	0	+ + +
$(x+2)\ (x+3)$	+ +	0	- -	0	+ + +