Virtual Study

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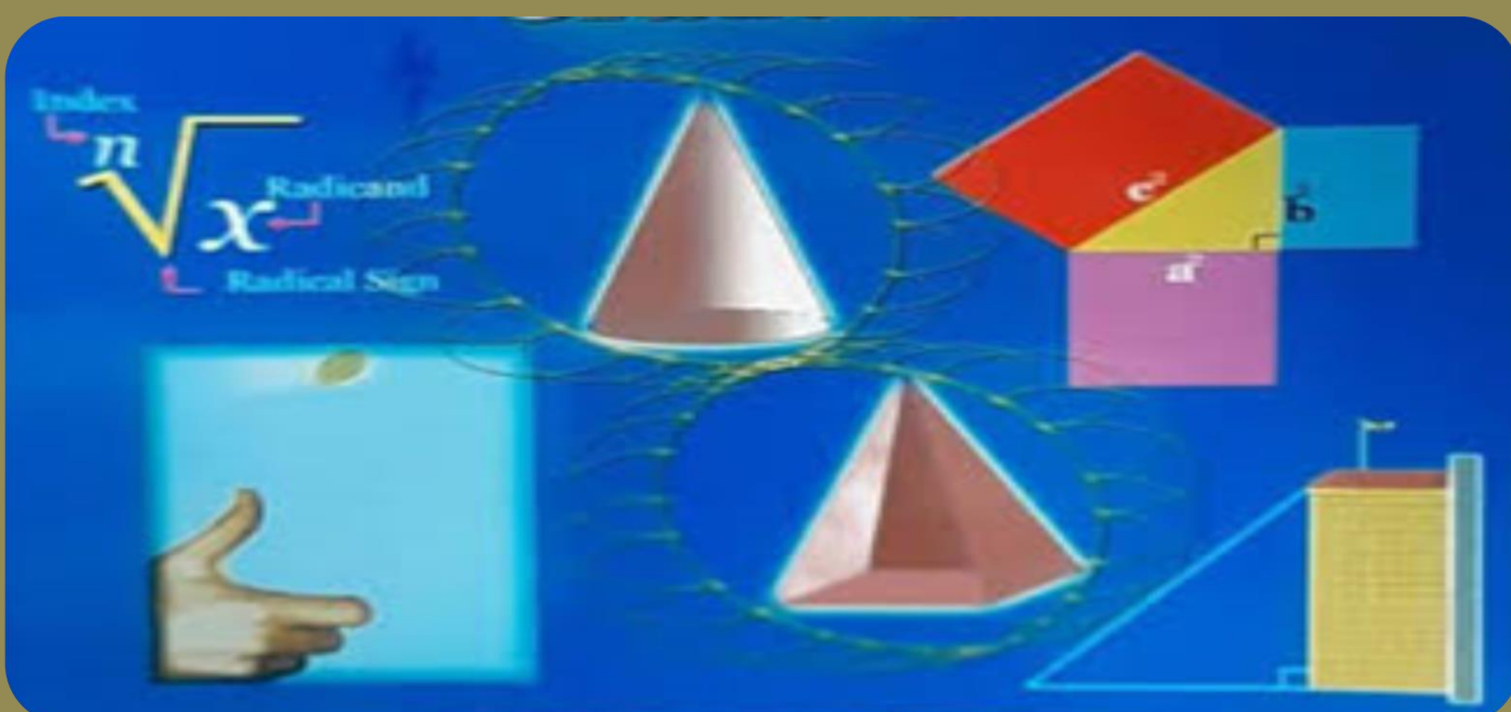
MATHEMATICS

Grade 8



Prepared by: Virtual Study

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UNIT 1

RATIONAL NUMBERS

1.1 Representation of Rational Numbers on a Number line

Revision on Fractions

A fraction represents the portion or part of the whole thing. For example, one- half, three-quarters. A fraction has two parts, namely numerator (the number on the top) and denominator (the number on the bottom).

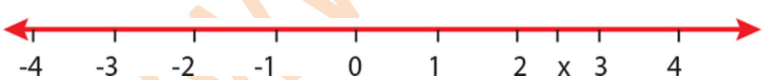
The shaded part is $\frac{1}{9}$ the important ideas about fractions and integers.

i. **Proper fraction:** A fraction in which the numerator is less than the denominator.

ii. **Improper fraction:** A fraction in which the numerator is greater than or equal to the denominator.

If an improper fraction is expressed as a whole number and proper fraction, then it is called mixed fraction.

Integers are represented on number line as shown below .



What number is represented by the marked letter xx on the number line above? You observe that the number xx is greater than 2 but less than 3. So, it belongs to the interval between 2 and 3. Thus xx is not a natural Number, or a whole number, or an integer.

What type of a number is? Using the above discussion, we define a rational number as follow:

Definition

A number that can be written in the form of aa bb where aa and bb are integers and $bb \neq 0$ is called a rational number.

$\frac{1}{5}, \frac{2}{6}, \frac{-2}{7}, \frac{5}{9}$ *are rational numbers.*

Note: The set of rational numbers is denoted by $\mathbb{Q}$. How can we locate rational numbers on number line?

Rational number can be represented on a number line by considering the following facts.

I. Positive rational numbers are always represented on the right side of zero and negative rational numbers are always represented on the left side of zero on a number line.

II. Positive proper fractions always exist between zero and one on number line.

III. Improper fractions are represented on number line by first converting into mixed fraction and then represented on the number line.

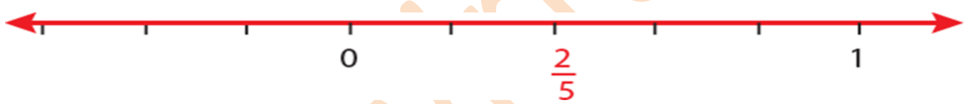
Example

Sketch a number line and mark the location of each rational numbers.

a. $\frac{2}{5}$ b. $\frac{3}{2}$ c. $-\frac{3}{4}$ d. $-\frac{5}{2}$

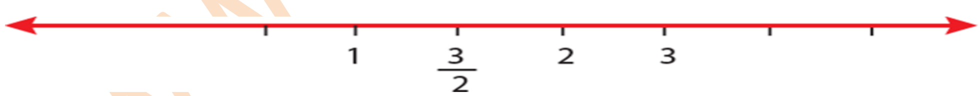
Solution:

a) Since $\frac{2}{5} > 0$, and proper, so it lies on the right side of 0 and on the left side of 1. How can we locate? Divide the number line between 0 and 1 into 5 equal parts. Then the second part of the fifth parts will be a representation of $\frac{2}{5}$ on number line

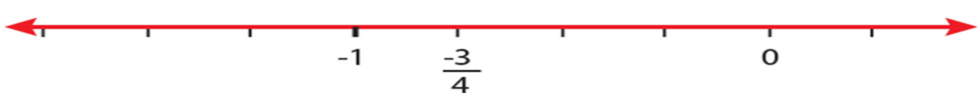


since $\frac{3}{2}$ is an improper fraction, first convert to mixed fraction to find between which whole numbers the fraction exists on the number line.

Thus, $\frac{3}{2} = 1\frac{1}{2}$ the fraction lies between 1 and 2 at $\frac{1}{2}$ point. Now, divide the number line between 1 and 2 in two equal parts and then the 1st part of 2 parts will be the required rational number on the number line.



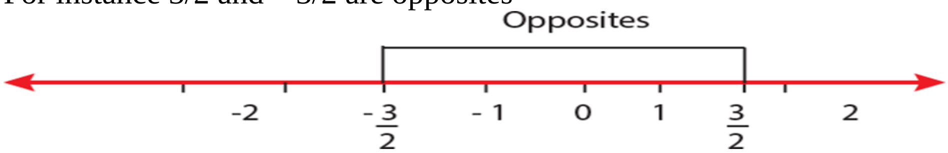
c) Since $-1 < -3/4 < 0$, the fraction will lie between -1 and 0. To represent on the number line, divide the number line between -1 and 0 in to 4 equal parts and the third part of the four parts will be $-3/4$.



d) Since $-5/2 < 0$ and improper, first change in to mixed fraction. That is, $-5/2 = -2\frac{1}{2}$. To represent on the number line divide the number line between −3 and −2 in to two equal parts and the first part of the two parts is – 5/2.

Note:

Two rational numbers are said to be opposite, if they have the same distance from 0 but in different sides of 0. For instance 3/2 and – 3/2 are opposites



Exercise 1.1

1. Consider the following number line.

Select a reasonable value for point B.

a) 0.5 b) 3.6 c) – 0.2 d) 2

2. Between what consecutive integers the following rational numbers exist?

a) 3/7 b) 8/5 c) – 3/5 d) – 9/5

3. Change the following improper fractions to mixed fractions.

a) 32/5 b) – 27/10 c) 7/3

4. Represent the following rational numbers on a number line.

a) 5/6 b) 3/5 c) – 5/6 d) – 8/5 e) $2\frac{2}{5}$

Answer

1. A number line between 1 and 35 divides in 3 equal parts each part represent $\frac{1}{2}$. Then when we add $\frac{1}{2}$., starting from 1 until we get point B, that is 2.
2. a. $\frac{3}{7}$ is between 0 and 1
   b. $\frac{8}{5}$ is between 1 and 2
   c. $\frac{-3}{5}$ is between -1 and 0
   d. $\frac{-9}{5}$ is between -2 and -1

3. divide the numerator by the denominator the quotient uses as the whole number part, the remainder uses as the numerator and the denominator uses as the denominator of the proper fraction

a. $32 \div 5 = 6$ remainder 2 b. $27 \div 10 = 2$ remainder 7

$\therefore \frac{32}{5} = 6\frac{2}{5}$ $\frac{27}{10} = 2\frac{7}{10}$

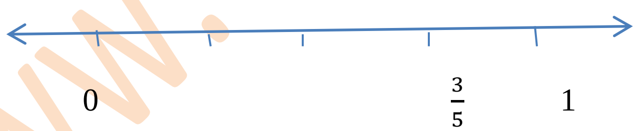
c. $7 \div 3 = 2$ reminder 1

$\mathbf{\frac{7}{3} = 2\frac{1}{3}}$

4. First determine between what consecutive integers the given rational numbers exist.

a. $\frac{5}{6}$ is between 0 and 1 on a number line and divide a number line between 0 and 1 into 6 equal parts , the 5^{th} part represents $\frac{5}{6}$ of the 6 equal parts.

b. $\frac{3}{5}$ is between 0 and 1 and divide a number line between 0 and 1 into 5 equal parts the 3^{rd} part represents $\frac{3}{5}$ of the 5 equal parts.



1.2 Relationship among W, $\mathbb{Z}$ and Q

In this subsection you will discuss the relationship among these set of numbers with the other set of number which is rational numbers.

Recall that:

- ✓ A collection of items is called a set.

- ✓ The items in a set are called elements and are denoted by ∈.
- ✓ A Venn diagram uses intersecting circles to show relationships among sets of numbers.

The Venn diagram below shows how the set of natural numbers, whole numbers, integers, and rational numbers are related to each other.

When a set is contained within a larger set in a Venn diagram, the numbers in the smaller set are members of the larger set. When we classify a number, we can use Venn diagram to help figure out which other sets, if any, it belongs to.

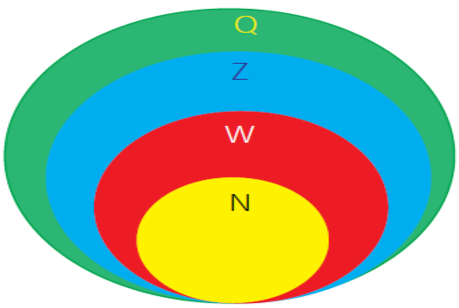


Figure: Venn diagram

Example

Classify the following numbers by naming the set or sets to which it belongs.

a. −13 b. 1/7 c. – 5/76 d. 10

Solution:

a. integer, rational number

b. rational number

c. rational number

d. natural number, whole number, integer, rational number.

Example

Is it possible for a number to be a rational number that is not an integer but is a whole number? Explain.

Solution: No, because a whole number is an integer.

Exercise 1.2

1. Solomon says the number 0 belongs only to the set of rational numbers. Explain his error.

2. Write true if the statement is correct and false if it is not.

a) The set of numbers consisting of whole numbers and its opposites is called integers.

b) Every natural number is a whole number.

c) The number $-3\frac{2}{7}$ belongs to negative integers.

Answer

1). 0 is not only rational number. It is an integer and also whole number.

2. a. True c. False

b. True

1.3. Absolute value of Rational numbers

Definition: The absolute value of a rational number 'x', denoted by $|x|$, is defined as:

$i \ |x| = \{x,$

$i\,i \ x \geq 0 - x,$

$i\,i\,i \ x < 0$

Example

a. $|6| = 6$

b. $|0| = 0$

c. $|-15| = -(-15) = 15$

Example

Simplify each of the following absolute value expressions.

a. $|8-3|$ b. $|-25+13|$ c. $|0-10|$

Solution:

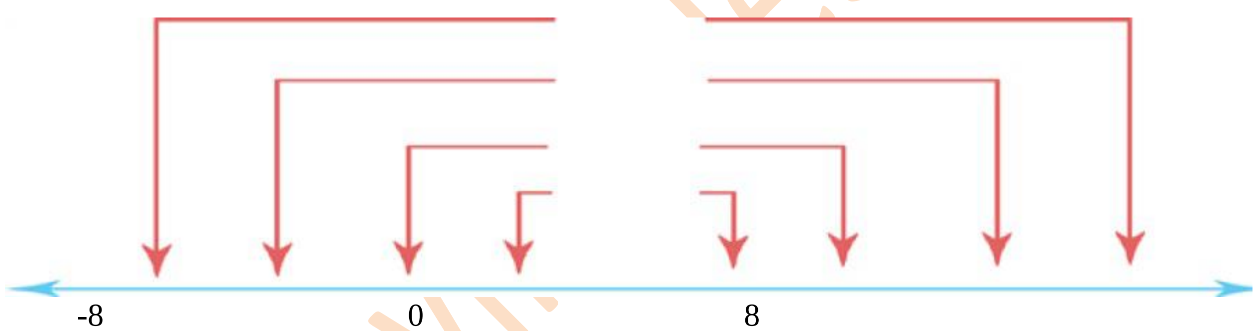
a. Since $8-3=5$ and $5>0$, we have $|8-3| = |5| = 5$

b. Since $-25+13=-12$ and $-12<0$, we have

Equation involving absolute value

Definition: An equation of the form $|x|=a$ for any rational number a a is called an absolute value equation.

Geometrically the equation $|x| = 8$ means that the point with coordinate x is 8 units from 0 on the number line. Obviously the number line contains two points that are 8 units from the origin, one to the right and the other to the left of the origin. Thus $|x| = 8$ has two solutions $x = 8$ and $x = -8$.



Note

The solution of the equation $|x| = a$ for any rational number a, has

i. Two solutions $x = a$ and $x = -a$ if $a > 0$.

ii. One solution, $x = 0$ if $a = 0$ and

iii. No solution, if $a < 0$.

Example:

Solve the following absolute value equations.

a. $|\boldsymbol{x}| = 13$ b. $|\boldsymbol{x}| = 0$ c. $|\boldsymbol{x}| = -6$

Solution:

a. $|\boldsymbol{x}| = 13$

Since, $13 > 0$, $|\boldsymbol{x}| = 13$ has two solutions: ◎

$x = 13$ and $x = -13$

b. $|\boldsymbol{x}| = 0$

If $|\boldsymbol{x}| = 0$, then $x\,x = 0$

c. $|\boldsymbol{x}| = -6$

Since, $-6 < 0$, $|\boldsymbol{x}| = -6$ has no solution.

Exercise 1.3

1. Evaluate each of the following expressions for the given values of x and y.

a. $5x - |x - 3|$, $x = -5$

b. $|x| - x + 9$, $x = 3$

c. $|x + y| - |x|$, $x = -3$ and $y = 6$

d. $|x| + |y|$, $x = 5$ and $y = -10$

e. $-3|x + 6|$, $x = -5$

f. $\frac{|x| - |5y|}{|x + y|}$, $x = 4$ and $y = 8$

2. Solve the following absolute value equations.

a. $|x| = 8$ b. $|x| = 3/5$

Challenge problem

3. Solve the following absolute value equations.

a. $|x + 4| = 10$ d. $|5x - 3| = 5/2$

b. $4\,|x+3| = 1\frac{2}{3}$ e. $|x-5| = 3\frac{3}{2}$

c. $3 - 2\,|x-5| = 9$

Answer

1.a $5x - |x-3|$, x = -5

$= 5 \times (-5) - |-5-3|$

$= -25 - |-8|$

$= -25 - 8$

$= \underline{\underline{-33}}$

b. $|x| - x + 9$, x = 3

$= |3| - 3 + 9$

$= 3 - 3 + 9$

$= 0 + 9$

$= \underline{\underline{9}}$

c. $|x+y| - |x|$, x = -3 and y = 6

$= |-3+6| - |-3|$

$= |3| - 3$

$= 3 - 3$

$= \underline{\underline{0}}$

d. $|x| + |y|$, x = 5 and y = -10

$= |5| + |-10|$

$= 5 + 10$

$= \underline{\underline{15}}$

e. $-3\,|x+6|$, x = -5

$= -3\,|-5+6|$

$= -3\,|1|$

$= \underline{\underline{-3}}$

f. $\frac{|x| - |5y|}{|x+y|}$, x = 4 and y = 8

$= \frac{|4| - |5 \times 8|}{|4+8|}$

$= \frac{|4| - |40|}{|12|}$

$= \frac{4-40}{12} = \frac{-36}{12} =$ **- 3//**

2. a. x = 8 or y = - 8

b. $x = \frac{3}{5}$ or $x = \frac{-3}{5}$

3 . c. $3 - 2\,|x-5| = 9$

$-2\,|x-5| = 9 - 3$

- 2 $|x-5|$ = 6

$|x-5|$ = -3 has no solution because absolute value of any number cannot be negative.

d. $|5x-3| = \frac{5}{2}$

5x -3 = $\frac{5}{2}$ or 5x -3 = $\frac{5}{2}$

5x = $\frac{5}{2}$ + 3 or 5x = $\frac{5}{2}$ + 3

5x = $\frac{11}{2}$ or 5x = $\frac{-5+6}{2}$

$\frac{5x}{5} = \frac{11}{2\times5}$ or $\frac{5x}{5} = \frac{1}{2\times5}$

x = $\frac{11}{10}$ or x = $\frac{1}{10}$

e. $3|x-5| = 3\frac{2}{3}$

$|x-5| = \frac{11}{9}$

x - 5 = $\frac{-11}{9}$ or x- 5 = $\frac{11}{9}$

x = $\frac{-11}{9}$ + 5 or x = $\frac{11}{9}$ + 5

x = $\frac{-11}{9} + \frac{45}{9}$ or x = $\frac{11}{9} + \frac{45}{9}$

x = $\frac{34}{9}$ or x = $\frac{56}{9}$//

1.2. Comparing and Ordering Rational numbers

1.2.1. Comparing Rational numbers Comparing Decimals

A rational number a/b can be expressed as a decimal number by dividing the numerator aa by the denominator b.

Note: Decimal numbers are compared in the same way as comparing other numbers: By comparing the different place values from left to right. That is, compare the integer part first and if they are equal, compare the digits in the tenths place, hundredths place and so on.

Example

Compare the following decimal numbers.

a. 4.25-----12.33 b. 15.52-----15.05 c. 45.667 ----- 45.684

Solution:

a. since 4 <12, then 4.25 < 12.33

b. the integer part of the two numbers are equal, then move to thetenth place and compare: 5> 0, then 15.52 > 15.05

c. here the corresponding integer part and the tenth place numbers are equal, so we move to the hundredth place: 6 < 8. Thus 45.667 < 45.684

Comparing Fractions

Comparing fractions with the same denominator If the denominators of two rational numbers are the same, then the number with the greater numerator is the greater number. That is a/b and c/b are a given rational numbers and $a/b > c/b$ if and only if $a > c$.

Example:

a. $15/7 > 13/7$ because 15 >13

b. $10/3 < 15/3$ because 10 < 15

Fractions that represent the same point on a number line are called Equivalent fractions. For any fraction a/b and m is a rational number different from 0 ($\neq 0$), then $a/b = a/b \times m/m$.

Comparing fractions with different denominators

In order to compare any two rational numbers with different denominators, you can use either of the following two methods:

Method 1:

Change the fractions to equivalent fractions with the same denominators.

Step 1. Determine the LCM of the positive denominators.

Step 2. Write down the given rational numbers with the same denominators.

Step 3. Compare the numerators of the obtained rational numbers.

Example:

Compare the following pairs of rational numbers.

a. $3/5$ and $1/2$ b. $11/16$ and $7/8$

Solution:

a. To compare $3/5$ and $1/2$

i. Find the LCM of 5 and 2 which is 10.

ii. Express the rational numbers with the same denominator 10.

$3/5= 3/5 \times 2/2 = 6/10$ and $1/2 \times 5/5 = 5/10$

iii. Since 6 > 5, $6/10 > 5/10$.

Therefore, $3/5 > 1/2$.

b. To compare $11/16$ and $7/8$

i. Find the LCM of 16 and 8 which is 16.

ii. Express the rational numbers with the same denominator 16.

$11/16 = 11/16 \times 11 = 11/16$ and $7/8 = 7/8 \times 2/2 = 14/16$

Since $11 < 14$, $11/16 < 14/16$

Method 2. (Cross- product method)

Suppose a/b and c/d are two rational numbers with positive denominators. Then

I. $a/b < c/d$, if and only if $a < b$

II. $a/b > c/$, if and only if $a > b$

III. $a/b = c/$, if and only if $a = b$

Example:

a. $\frac{5}{7} < \frac{8}{3}$ because 5 × 3 =15 < 7 × 8 =56

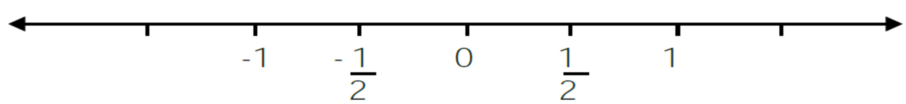
b. $\frac{-12}{9} < \frac{6}{7}$ because −12 × 7 = −84 < 9 × 6 = 54

c $\frac{9}{5} > \frac{11}{7}$ because 9 × 7 =63 > 5 × 11 = 55.

Comparing Rational numbers using number line

Note:

For any two different rational numbers whose corresponding points are marked on the number line, then the one located to the left is smaller.



Thus, $-1 < \frac{-1}{2}$, $\frac{-1}{2} < 0$, $0 < \frac{1}{2}$

From the above fact, it follows that:

- Every positive rational number is greater than zero.
- Every negative rational number is less than zero.
- Every positive rational number is always greater than every negative rational number.
- Among two negative rational numbers, the one with the largest

Absolute value is smaller than the other. For instance, −45 < −23 because |−45| > |−23|.

Exercise 1.4

1. Which of the following statements are true and which are false?

a. $-0.15 < 1.5$

b. $2\frac{3}{5} > 3\frac{2}{5}$

c. $\left|-\frac{3}{5}\right| < \frac{3}{5}$

d. $\frac{12}{8} = \frac{10}{15}$

e. $3\frac{5}{7} > \frac{21}{6}$

f. $\frac{5}{12} > \frac{7}{18}$

g. $6.53 < 6.053$

h. $3\frac{4}{7} = \frac{25}{7}$

2. Insert (>, = o r <) to express the corresponding relationship between the following pairs of numbers.

a. $\frac{15}{9}$----------- $\frac{18}{9}$

b. $-\frac{21}{12}$ --------- $-\frac{28}{16}$

c. $\frac{8}{20}$ ------------ 0.35

d. $3\frac{5}{6}$---------- $3\ \frac{7}{8}$

e. $\left|-\frac{3}{10}\right|$------------ $\frac{3}{10}$

f. $\frac{12}{8}$ ------------- $\frac{13}{9}$

3. k, n, m, x, y, z are natural numbers represented on a number line as follows:

Compare the numbers using > or <.

a. n___________x

b. x_____________y

c. z___________n

d. y___________k

Answer

1. a. True b. False c. True d. False e. True f. True g. False h. True

2. a < b. = c. > d. < e. = f. >

3. a. 3.5 c. 2 $\frac{5}{7}$ e. -1.5 b. $\frac{5}{7}$ d. – 9

1.2.2. Ordering Rational numbers

Ordering rational numbers means writing the given numbers in either ascending or descending order. Ordering a rational numbers of different denominators is a little bit like ordering distance measured in miles and kilometers, where we need all the distances to be in the same unit. For fractions, we need either to rewrite them in such a way that all have the same denominator or to convert them to decimals.

Example

Arrange the following rational numbers in:

i) Increasing (ascending) order

a. −25, 18, −45, 30, 28

b . $\frac{3}{5}, \frac{4}{9}, \frac{7}{3}, \frac{5}{6}, \frac{11}{18}$

ii) Decreasing (descending) order

a. −15, 45, 32, −23,

b. $\frac{7}{3}$, $2\frac{4}{5}$, $\frac{5}{2}, \frac{8}{15},$

c. 4.5, 5.17, 3.75, 4.75c. 2.5, 1.5, 3.21, 1.53, 2.05

Solution:

i) a. −25< −45 < 18 < 28 < 30

b. To order these rational numbers, first change the fractions with the same denominator. Thus, $\frac{54}{90}$, $\frac{40}{90}$, $\frac{210}{90}$, $\frac{75}{90}$, and $\frac{55}{90}$

Compare only the numerators: 40 < 54 < 55 < 75 < 210

Therefore, the numbers in increasing order are

$\frac{70}{30}, \frac{84}{30}, \frac{75}{30}, \frac{16}{30}$

Now compare only the numerators, 84 > 75 > 70 >16

Therefore, $2\frac{4}{5} > \frac{5}{2} > \frac{7}{3} > \frac{8}{15}$

d. 5.17 > 4.75 > 4.5 > 3.75

Exercise 1.5

1. Arrange the following rational numbers in ascending order.

A $\frac{4}{9}$, $\frac{3}{25}$, $\frac{11}{7}$, $-5\frac{2}{3}$, $2\frac{3}{15}$

b. 5.24, 8.13, 6.75, 12.42, −12.51

c. 3.92, $\frac{5}{13}$, $4\frac{6}{7}$, 4.73, $\frac{11}{9}$

2. Arrange the following rational numbers in descending order.

a. 13.72, 23.86, 15.02, 13.05

Answer

1. a .$-5\frac{2}{3}$, $\frac{3}{25}$, $\frac{4}{9}$, $\frac{23}{15}$, $\frac{11}{7}$

b. -12.51 , 5.24 ,6.75 , 8.13 , 12.42

c. $\frac{5}{12}$, $\frac{11}{9}$, 3.92 , 4.73 , $4\frac{6}{7}$

2. a. $23.86 > 15.02 > 13.72 > 13.05$

b. $2\frac{7}{9} > 2\frac{4}{9} > \frac{21}{12} > \frac{9}{7} > \frac{13}{16}$

c.$4.23 > 3\frac{5}{6} > 3.73 > \frac{18}{5} > 3.2$

1.3. Operation and properties of Rational Numbers

1.3.1. Addition of rational numbers

Adding rational numbers with same denominators

To add two or more rational numbers with the same denominators, we add all the numerators and write the common denominator.

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{b}$, $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

Adding rational numbers with different denominators

To find the sum of two or more rational numbers which do not have The same denominator, we follow the following steps:

I. Make all the denominators positive.

II. Find the LCM of the denominators of the given rational numbers.

III. Find the equivalent rational numbers with common denominator.

IV. Add the numerators and take the common denominator.

Example:

Find the sum of the following rational numbers.

a. $\frac{3}{7}+\frac{5}{4}$ b. $\frac{11}{9}+\frac{5}{6}+\frac{3}{4}$

Solution:

a. The LCM of 7 and 4 is 28.

Then write the fraction as a common denominator 28.

a) $\frac{3}{7}\times\frac{4}{4}=\frac{12}{28}$ and $\frac{5}{4}\times\frac{7}{7}=\frac{35}{28}$

$\frac{3}{7}+\frac{5}{4}=\frac{12}{28}+\frac{35}{28}=\frac{12+35}{28}=\frac{47}{28}$

b. The LCM of 9, 6 and 4 is 36.

Then, write the fraction as a common denominator 36.

$\frac{11}{9}\times\frac{4}{4}=\frac{44}{36}$, $\frac{5}{6}\times\frac{6}{6}=\frac{30}{36}$ and $\frac{3}{4}\times\frac{9}{9}=\frac{27}{36}$

$\frac{11}{9}+\frac{5}{6}+\frac{3}{4}=\frac{44}{36}+\frac{30}{36}+\frac{27}{36}=\frac{44+30+27}{36}=\frac{101}{36}$

Note: Always reduce your final answer to its lowest term.

Now you are going to discover some efficient rules for adding any two rational numbers.

Rule 1: To find the sum of two rational numbers where both are negatives:

i) Sign: Negative (−)

ii) Take the sum of the absolute values of the addends.

iii) Put the sign in front of the sum.

Rule 2: To find the sum of two rational numbers, where the signs of the addends are different, are as follows:

i) Take the sign of the addend with the greater absolute value.

ii) Take the absolute values of both numbers and subtract the addend with smaller absolute value from the addend with greater absolute value.

iii) Put the sign in front of the difference.

Example :

Perform the following operation.

a. $\frac{-9}{4}+\frac{5}{4}$

b. $\frac{-5}{6}+\frac{3}{4}$

Solution:

a. $\frac{-9}{4}+\frac{5}{4}=\frac{-9+5}{4}=\frac{-4}{4}=-1$

B . $\frac{-5}{6}+\frac{3}{4}=\frac{-5\times4}{6\times4}+\frac{6\times3}{6\times4}=\frac{-20}{24}+\frac{18}{24}=\frac{-2}{24}=\frac{-1}{12}$

Properties of Addition of Rational Numbers

For any rational numbers aa, bb and cc the following properties of addition holds true:

a. **Commutative:** $a + b = b + a$

b. **Associative:** $a + (b+ c) = (a + b) + c$

c. **Properties of 0:** $a + 0 = a = 0 + a$

d. **Properties of opposites:** $a + (-a) = 0$

Example:

Using properties of addition find the sum: $\frac{1}{3}+\frac{3}{5}+\frac{7}{6}$

Solution:

$\frac{1}{3}+\frac{3}{5}+\frac{7}{6}=\left(\frac{1}{3}+\frac{3}{5}\right)+\frac{7}{6}$ -------- Associative property

$\left(\frac{5+9}{15}\right)+\frac{7}{6}=\ \frac{14}{15}+\frac{7}{6}=\frac{14\times6}{15\times6}+\frac{7}{6}\times\frac{15}{15}=\frac{84}{90}+\frac{105}{90}=\frac{189}{90}=\frac{21}{10}$

Similarly, $\frac{1}{3}+\frac{3}{5}+\frac{7}{6}=\ \frac{1}{3}+\left(\frac{3}{5}+\frac{7}{6}\right)$ -------- Associative property

$=\frac{1}{3}+\frac{18+35}{30}=\frac{1}{3}+\frac{53}{30}=\frac{30}{90}+\frac{159}{90}=\frac{189}{90}=\frac{21}{10}$

Therefore, $\left(\frac{1}{3}+\frac{3}{5}\right)+\frac{7}{6}=\frac{1}{3}+\left(\frac{3}{5}+\frac{7}{6}\right)$

Exercise 1.6

1. Find the sum:

a $\frac{13}{5}+\frac{21}{5}$

b. $\frac{5}{6}+\frac{3}{8}$

c. $\frac{3}{5}+2\frac{3}{5}$

d. $2\frac{1}{3}+\frac{3}{8}+3\frac{5}{6}$

e. $\left|-\frac{2}{5}+\frac{3}{8}\right| + \left|\frac{4}{7}+\frac{2}{7}\right|$

Answer

1. a. $\frac{13}{5}+\frac{21}{5} = \frac{13+21}{5} = \frac{34}{5}$

   b. $\frac{5}{6}+\frac{3}{8}$

L.C.M (6,8) IS 24 Then , $\frac{5}{6}+\frac{3}{8} = \frac{5\times4}{6\times4}+\frac{3\times3}{8\times3}$

$$= \frac{20}{24}+\frac{9}{24}$$

$$= \mathbf{\frac{29}{24}}//$$

c. $\frac{3}{5}+2\,\frac{3}{5} = \frac{3}{5}+\frac{(2\times5)+3}{5} = \frac{3}{5}+\frac{10+3}{5} = \frac{3+10+3}{5} = \mathbf{\frac{16}{5}}//$

d. $2\,\frac{1}{3}+\frac{3}{8}+3\,\frac{5}{6} = \frac{(2\times3)+1}{3}+\frac{3}{8}+\frac{(3\times6)+5}{6}$

$$= \frac{7}{3}+\frac{3}{8}+\frac{23}{6}$$

L.C.M (3,8,6) = 24

$$= \frac{7\times8}{3\times8}+\frac{3\times3}{8\times3}+\frac{23\times4}{6\times4}$$

$$= \frac{56}{24}+\frac{9}{24}+\frac{92}{24} = \frac{56+9+92}{24} = \frac{65+92}{24} = \mathbf{\frac{157}{24}}//$$

e. $\left|\frac{-2}{5}+\frac{3}{8}\right|+\left|\frac{4}{7}+\frac{2}{7}\right| = \left|\frac{-2\times8+5\times3}{40}\right|+\left|\frac{4+2}{7}\right|$

$$= \left|\frac{-16+15}{40}\right|+\left|\frac{6}{7}\right|$$

$$= \left|\frac{-1}{40}\right|+\left|\frac{6}{7}\right|$$

$$= \frac{1}{40}+\frac{6}{7} = \frac{7+240}{40\times7} = \mathbf{\frac{247}{280}}//$$

1.3.2 Subtraction of rational numbers

The process of subtraction of rational numbers is the same as that of addition. Subtraction of any rational numbers can be explained as the inverse of addition: That is, for two rational numbers $\frac{c}{d}$ and $\frac{c}{d}$

Subtracting $\frac{c}{d}$ from $\frac{a}{b}$ means adding the negative of $\frac{c}{d}$ to $\frac{a}{b}$ $=\frac{c}{d}-\frac{a}{b}=\frac{c}{d}+(-\frac{a}{b})$

Example :

Compute the following difference.

a. $\frac{7}{9}-\frac{4}{3}$ b. $\frac{3}{7}$ - $(-\frac{5}{9})$ c. $-\frac{-1}{12}-\frac{9}{8}$

Solution:

a. $\frac{7}{9}-\frac{4}{3}=\frac{7\times9}{9\times3}-\frac{9\times4}{9\times3}=\frac{21}{27}-\frac{36}{27}=\frac{21-36}{27}=\frac{-5}{9}$

b. $\frac{3}{7}$ - $(-\frac{5}{9})=\frac{3\times9+35}{63}=\frac{62}{63}$

c. $\frac{-1}{12}-\frac{9}{8}=\frac{-8-108}{96}=\frac{-206}{96}=\frac{-103}{48}$

Note:

i. The difference of two rational numbers is always a rational number.

ii. Addition and subtraction are inverse operations of each other.

Exercise 1.7

1. Find the difference of each of the following

a. $4\frac{5}{6}-2\frac{3}{4}$

b. $-5.3-3.45$

c. $\frac{6}{13}-\left|-\frac{7}{13}\right|$

d. $\left|-\frac{5}{7}\right|-\left|\frac{3}{4}\right|$

e. $-32.24-\left|-32.24\right|$

f. $-3\frac{2}{5}-2\frac{3}{7}$

2. Evaluate the following expressions:

a. $y-(\frac{2}{7}+5)$, when $y=\frac{9}{4}$

b. $15-(-y-\frac{4}{9}$, when $y = 7$

3. From a rope 23 m long, two pieces of lengths $\frac{12}{7}$ m and $\frac{7}{4}$m are cut off. What is the length of the remaining rope?

4. A basket contains three types of fruits, apples, oranges and bananas, weighing $\frac{58}{3}$ kg in all. If $\frac{18}{7}$ kg be apples,

$\frac{11}{9}$ kg be oranges and the rest are bananas. What is the weight of the bananas in the basket?

Answer

1. a. $4\frac{5}{6} - 2\frac{3}{4} = \frac{4\times6+5}{6} - (\frac{2\times4+3}{4}) = \frac{24+5}{6} - (\frac{8+3}{4})$

$= \frac{29}{6} - \frac{11}{4} = \frac{29\times2}{6\times2} - \frac{11\times3}{4\times3}$

$= \frac{58}{12} - \frac{33}{12}$

$= \mathbf{\frac{25}{12}}$//

b. -5.3 -3.45 = (5.3 + 3.45)

= (8.75)//

c. $\frac{6}{13} - \left|\frac{-7}{13}\right| = \frac{6}{13} - \frac{7}{13} = \frac{6-7}{13} = -\mathbf{\frac{1}{13}}$//

d. $\left|\frac{-5}{7}\right| - \left|\frac{3}{4}\right| = \frac{5}{7} - \frac{3}{4} = \frac{5\times4-7\times3}{7\times4} = \frac{20-21}{28} = -\mathbf{\frac{1}{28}}$//

e. $-32.24 - |-32.24| = -32.24 - 2.24$

$= -(32.24 + 32.24)$ **= -** <u>**64.48**</u>

f. $-3\frac{2}{5} - 2\frac{3}{7} = -\frac{17}{5} - \frac{(2\times7+3)}{7}$

$= -\frac{17}{5} - \frac{17}{7} = -(\frac{17}{5} + \frac{17}{7}$

$= -(\frac{17\times7+5\times17}{5\times7}$

$= \frac{-204}{35} = \mathbf{-5\frac{29}{35}}$//

2.a. $\frac{9}{4} - (\frac{2}{7} + 5) = \frac{9}{4} - (\frac{2}{7} + \frac{5}{1})$

$= \frac{9}{4} - (\frac{2+35}{7})$

$= \frac{9}{4} - (\frac{37}{7}) = \frac{9\times7-4\times37}{4\times7} = \frac{63-148}{28}$

$= -\mathbf{\frac{85}{28}}$//

b. $15 - (-7 - \frac{4}{9}) = 15 - \frac{-7\times9-4}{9} = 15 - (\frac{63-4}{9})$

$= 15- (-\frac{67}{9}) = \frac{15}{1} + \frac{67}{9} = \frac{15 \times 9 \times 67}{9}$

$= \frac{135+67}{9} = \frac{\mathbf{202}}{\mathbf{9}}$//

3. $23- (\frac{12}{7} + \frac{7}{4})$

$= 23 – (\frac{12 \times 4}{7 \times 4} + \frac{7 \times 7}{4 \times 7})$

$= 23 – (\frac{48 + 49}{28})$

$= \frac{23}{1} - \frac{97}{28} = \frac{23 \times 28 - 97}{28} = \frac{547}{28} = \mathbf{19\ \frac{15}{28}}$//

Therefore the length of the remaining rope is $19\ \frac{15}{28}$ meter.

4. $\frac{58}{3} – (\frac{18}{7} + \frac{11}{9}) = \frac{58}{3} – (\frac{18 \times 9 + 7 \times 11}{7 \times 9}) = \frac{58}{3} – (\frac{162+77}{63})$

$= \frac{58}{3} - \frac{239}{63} = \frac{58 \times 21}{3 \times 21} - \frac{239}{63} = \frac{1218-239}{63} = \frac{\mathbf{979}}{\mathbf{63}} = \mathbf{15\ \frac{34}{63}}$//

Therefore, bananas weighing is $15\ \frac{34}{63}$.

1.3.3. Multiplication of rational numbers

To multiply two or more rational numbers, we simply multiply the numerator with the numerator and thedenominator with the denominator. Finally reduce the final answer to its lowest term if it is.

Find the product of $\frac{1}{2}$and $\frac{3}{4}$

Therefore, $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Note: The product of two rational numbers with different signs can be determine in three steps

a. Decide the sign of the product; it is “- “.

b. Take the product of the absolute value of the numbers.

c. Put the sign in front of the product.

Example :

Find the product

a. $\frac{3}{4} \times \frac{7}{9}$

b. $\frac{4}{9} \times (-\frac{5}{2})$

c. $2\frac{5}{7} \times \left(-\frac{3}{2}\right)$

Solution:

a. $\frac{3}{4} \times \frac{7}{9} = \frac{3\times7}{4\times9} = \frac{21}{36} = \frac{7}{12}$

b. $\frac{4}{9} \times \left(-\frac{5}{2}\right) = \frac{4\times(-5)}{9\times2} = \frac{-20}{18} = -\frac{10}{9}$

c. $2\frac{5}{7} \times \left(-\frac{3}{2}\right)$

First change the mixed number to improper fraction.

$2\frac{5}{7} = \frac{2*7+5}{7} = \frac{19}{7}$, then

i. Sign (−)

ii. Multiply the absolute value

$\left|\frac{19}{7}\right| \times \left|-\frac{3}{2}\right| = \frac{19}{7} \times \frac{3}{2} = \frac{57}{14}$

Therefore, $2\frac{5}{7} \times \left(-\frac{3}{2}\right) = -\frac{57}{14}$

Note: The product of two negative rational numbers is a positive rational number.

Example :

Find the product: $-\frac{2}{5} \times \left(-\frac{7}{3}\right)$

Solution: $-\frac{2}{5} \times \left(-\frac{7}{3}\right) = \frac{2}{5} \times \frac{7}{3} = \frac{14}{15}$

The following table 1 summarizes the facts about product of rational numbers.

The two factors	The product	Example
Both positive	Positive	$\frac{2}{3} \times 5 = \frac{10}{3}$
Both negative	Positive	$\left(-\frac{5}{4}\right) \times \left(-\frac{7}{3}\right) = \frac{35}{12}$
Of opposite sign	negative	$-3 \times 5 = -15$
One or both 0	Zero	$-\frac{7}{9} \times 0 = 0$

Properties of multiplication of rational numbers

For any rational numbers a, b and ,the following properties of multiplication holds true:

a. Commutative: $a \times b = b \times a$

b. Associative: $a \times (b \times c) = (\times b) \times c$

c. Distributive: $a \times (b + c) = a \times b + a \times c$

d. Property of 0: $a \times 0 = 0 = 0 \times a$

e. Property of 1: $a \times 1 = a = 1 \times a$

Example 1.28:

Find the cost of $\frac{35}{9}$ m of cloth, if the cost of a cloth per meter is Birr$\frac{162}{4}$.

Solution*:*

Total cost $=\frac{35}{9} \times \frac{162}{4} = \frac{5670}{36} = Birr\frac{315}{2} = Birr\ 157.5$

Note: To get the product with three or more factors, we use the following properties:

1. The product of an even number of negative factors is positive.
2. The product of an odd number of negative factors is negative.
3. The product of a rational number with at least one factor 0 is zero.

1.3.4. Division of Rational Numbers

Note:

- $a \div b$ is read as aa is divided by bb.
- In $a \div b = c$, c is called the quotient, aa is called the dividend and b is called the divisor.
- The quotient $a \div b$ is also denoted by .
- If a, b and cc are integers, $bb \neq 0$ and $a \div b = c$, if and only if $a = c \times b$.

<u>Rules for Division of Rational numbers</u>

When dividing rational numbers:

1. Determine the sign of the quotient:

a) If the sign of the dividend and the divisor are the same, then sign of the quotient is $(+)$.

b) If the sign of the dividend and the divisor are different, the sign of the quotient is $(-)$.

2. Determine the value of the quotient by dividing the absolute value of the dividend by the divisor.

Note: For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ *(Where $c \neq 0$)*

Example 1.31:

Determine the quotient:

a. $\frac{6}{14} \div \frac{3}{7}$ b. $-3 \div \frac{9}{15}$ c. $-\frac{5}{7} \div -4$

Solution:

a. $\frac{6}{14} \div \frac{3}{7} = \frac{6}{14} \times \frac{7}{3} = \frac{42}{42} = 1$

b. $-3 \div \frac{9}{15} = -3 \times \frac{15}{9} = -5$

c. $-\frac{5}{7} \div -4 = -\frac{5}{7} \times -\frac{1}{4} = \frac{5}{28}$

Note: For any rational number ab where $a \neq 0$; $\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ba} = 1$, Then $\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.

Exercise: 1.9

1. Determine the quotient

a. $\frac{5}{8} \div \frac{3}{4}$

b. $-\frac{3}{5} \div \frac{6}{7}$

c. $-\frac{8}{9} \div (-\frac{5}{3})$

d. $2\frac{3}{5} \div (-\frac{4}{3})$

Answer

1. a. $\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{5 \times 4}{8 \times 3} = \frac{20}{24} = \mathbf{\frac{5}{6}}$//

b. $-\frac{3}{5} \div \frac{6}{7} = -\frac{3}{5} \times \frac{7}{6} = -\frac{3 \times 7}{5 \times 6} = \frac{-3 \times 7}{5 \times 6} = \mathbf{\frac{-21}{30}}$//

c. $-\frac{8}{9} \div (\frac{-5}{3}) = \frac{-8}{9} \times (\frac{-3}{5}) = \frac{-8 \times (-3)}{9 \times 5} = \frac{24}{45} = \mathbf{\frac{8}{15}}$//

d. $2\frac{3}{5} \div (-\frac{4}{3}) = \frac{13}{5} \times (-\frac{3}{4}) = \frac{13 \times (-3)}{5 \times 4} = -\mathbf{\frac{39}{20}}$//

2. To calculate the number of bags , we have to divide $\frac{3}{4}$ by $\frac{3}{8}$

$\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3} = \frac{3 \times 8}{4 \times 3} = \frac{8}{4} = \mathbf{2}$//

1.4. Real life applications of rational numbers

1.4.1. Application in sharing something among friends

Rational numbers are used in sharing and distributing something among a group of friends.

Example:

There are four friends and they wanting to divide a cake equally among themselves. Then, the amount of cake each friend will get is one fourth of the total cake.

1.4.2. Application in calculating interest and loans

Simple interest

Interest is a payment for the use of money or interest is the profit return on investment.

- ✓ The money that is borrowed or loaned is called the principal (P).
- ✓ The length of time that money is used or for which interest is paid is called time (T).
- ✓ The interest paid on the original principal during the whole interest periods is called simple interest. Interest can be calculated by: **I = PRT**

Example 1.34:

Abebe borrowed Birr 21100 from CBE five months ago. When he first borrowed the money, they agreed that he would pay to CBE 15% simple interest. If Abebe pays to it back today, how much interest does he owe to it?

Solution:

Given Required

P =Birr 21100 I = ?

R = 15%

I = PRT

I = Birr 21100 × 15%× $\frac{5}{12}$, Where, T = 5 months = $\frac{5}{12}$ years

I = Birr 21100 × 0.15 ×$\frac{5}{12}$

I = Birr 3165 ×$\frac{5}{12}$

I = Birr 1318.75

Therefore, Abebe pay an additional 1318.75 Birr of simple interest as per their agreement.

Exercise 1.10

1. If Birr 1200 is invested at 10% simple interest per annum, then What is simple interest after 5 years?
2. What principal will bring Birr 637 interest at a rate of 7% in 2 years?
3. Find the simple interest rate for a loan where Birr 6000 is borrowed and the amount owned after 5 months is Birr 7500.

Answer

1. P = 1200 $\quad I = p \times R \times T$

R= 10% $\quad = 1200 \times 10\% \times 5 = \frac{1200 \times 10 \times 5}{100} = 120 \times 5$

T= 5 yrs

$\therefore$ *Therefore the simple intereset is birr* 600.

2. $I = P \times R \times T$

$$P = \frac{I}{RT} = \frac{637}{7\% \times 2} = \frac{637}{\frac{14}{100}} = \frac{637\times100}{14} = \frac{637 \times 50}{7} = \frac{3085}{7} = \underline{\underline{\mathbf{4550}}}$$

Therefore, the principal is birr **4550.**

3. P = birr 6,000 T= 5 months $= \frac{5}{12}$ years we need to find the interest rate of a loan

$I = P \times R \times T$, $A = birr\ 7{,}500$

$R = \frac{I}{P \times T}$, but & I = A-P

$= 7{,}500 - 6{,}000 = \underline{\mathbf{1,500}}$ Since $R = \frac{1{,}500}{6{,}000 \times \frac{5}{12}} = \frac{1{,}500}{\frac{5{,}000}{2}} = \frac{1{,}500}{2{,}500}$

$$R = \frac{15}{20} = \frac{3}{5} \times 100\% = \frac{300\%}{5} = \underline{\underline{\mathbf{60\%}}}$$

Therefore, the interest rate is 60%.

REVIEW EXERCISE FOR UNIT 1

1. Which of the following statements are true?

a. $-\frac{5}{3} > -\frac{8}{11}$

b. $3\frac{5}{7} > 2\frac{3}{7} > 1\frac{9}{6}$

c. $|2\frac{3}{5}| = |-2\frac{3}{5}|$

d. $3\frac{5}{6} < 3\frac{4}{5} < \frac{2}{7}$

2. Solve each of the following absolute value equations.

a. $|x| = 7$

b. $|x - 5| = 4$

c. $|2x + 3| = 5$

d. $2|3x - 4| = 6$

e. $5+3|x - 3| = 2$

3. If $x = -8$ and $y = 4$, then fined $\frac{3|x-5|-|4y|}{|x+y|}$

4. Find the sum

a. $\frac{3}{4} + \frac{9}{7} + 2\frac{3}{5}$

b. $-2\frac{1}{3} + 1\frac{4}{7}$

c. $3.35 + 2\frac{3}{7}$

5. Find the difference

a. $\frac{2}{7} - \frac{3}{8}$

b. $-2\frac{6}{7} - \frac{13}{9}$

6. Determine the product

a. $2\frac{3}{7} \times \frac{1}{4}$

b. $2.34 \times \frac{7}{6}$

c. $-3\frac{3}{5} \times 1\frac{2}{3} \times (-\frac{3}{4})$

7. Determine the quotient

a. $\frac{3}{4} \div (-\frac{2}{7})$ c. $2\frac{3}{7} \div 2.3$

b. $-3\frac{5}{8} \div 2\frac{3}{10}$

8. Simplify the following expressions.

a. $\frac{3}{5} + \frac{2}{7}(\frac{4}{5} + \frac{3}{2})$ c$\frac{\frac{1}{6} \div (\frac{1}{3} + \frac{4}{5}}{\frac{5}{2}+\frac{1}{3}(\frac{2}{5}\div\frac{3}{5})}$ b. $\frac{4}{3} \div (\frac{5}{2} - \frac{6}{7})$

9. Eleni baked a batch of 32 cupcakes and iced 24 of them. What fractions of cupcakes were iced?

10. How long will take Birr 500 to get Birr 50 simple interest at rate of 9.5%?

Answer

1. a. False c. True

b. True d. True

2. a. $|x| = 7$ b. $|x - 5| = 4$ has two solutions c. $|2x + 3| = 5$

x=-7 or x= 7

x - 5 = -4 or x - 5 = 4

x = - 4 + 5 or x = 4 + 5

x = 1 or x = 9

2x+3 = -5 or 2x+3 = 5

2x = -5-3 or 2x = 5-3

2x= -8 or 2x =2

x = -4 or x = 1

d. 3x – 4 = 3 or 3x – 4 = -3

3x = 3 + 4 or 3x = -3 + 4

3x = 7 or 3x 1= 1

x = $\frac{\mathbf{7}}{\mathbf{3}}$ or **x** = $\frac{\mathbf{1}}{\mathbf{3}}$//

e. $5 + 3|x - 3| = 2$

$3|x - 3| = 2 - 5$

$3|x - 3| = -3$

$|x - 3| = -1$ has no solution because the absolute value cannot be negative.

3,x = - 8 , y = 4

To solve the expression $\frac{3|x-5|-|4y|}{|x+y|}$

$= \frac{3|-8-5|-|4\times4|}{|-8+4|} = \frac{3|-13|-|16|}{|-4|}$

$= \frac{3|-13|-16}{4} = \frac{3\times13-16}{4}$

$= \frac{39-16}{4} = \mathbf{\frac{23}{4}}$//

4. a. $\frac{3}{4} + \frac{9}{7} + 2\frac{3}{5} = \frac{3}{4} + \frac{9}{7} + \frac{13}{5}$

L.C.M (4,7,5) is 140

$\frac{3\times35}{4\times35} + \frac{9\times20}{7\times20} + \frac{13\times28}{5\times28} = \frac{105}{140} + \frac{180}{140} + \frac{364}{140} = \mathbf{\frac{649}{140}}$//

b. $-2\frac{1}{3} + 1\frac{4}{7} = \frac{-7}{3} + \frac{11}{7} = \frac{-7\times7+3\times11}{3\times7} = \frac{-49+33}{21} = \mathbf{\frac{-16}{21}}$//

c. $3.35 + 2\frac{3}{7} = \frac{335}{100} + \frac{17}{7} = \frac{67}{20} + \frac{17}{7} = \frac{67\times7+20\times17}{20\times7} = \frac{469+340}{140} = \mathbf{\frac{809}{140}}$//

5. a. $\frac{2}{7} - \frac{3}{8} = \frac{2\times8-7\times3}{7\times8} = \frac{16-21}{56} = \mathbf{\frac{-5}{56}}$//

b. $-2\frac{6}{7} - \frac{13}{9} = \frac{-20}{7} - \frac{13}{9} = -\frac{180}{63} - \frac{92}{63} = -\mathbf{\frac{272}{63}}$//

6. a. $2\frac{3}{7} \times \frac{1}{4} = \frac{(14+3)}{7} \times \frac{1}{4} = \frac{17}{7} \times \frac{1}{4} = \mathbf{\frac{17}{28}}$//

b. $2.34 \times \frac{7}{6} = \frac{117}{50} \times \frac{7}{6} = \frac{117\times7}{50\times6} = \frac{819}{300} = \mathbf{\frac{273}{100}}$//

c. $-3\frac{3}{5} \times 1\frac{2}{3} \times \frac{-3}{4} = -\frac{18}{5} \times \frac{5}{3} \times -\frac{3}{4} = \frac{18\times5\times3}{5\times3\times4} = \frac{18}{4} = \mathbf{\frac{9}{2}}$//

7. a. $\frac{3}{4} \div (-\frac{2}{7}) = \frac{3}{4} \times (\frac{-7}{2}) = \frac{-21}{8}$//

b. $-3\frac{5}{8} \div 2\frac{3}{10} = \frac{-29}{8} \div \frac{23}{10} = \frac{-29}{8} \times \frac{10}{23} = \frac{-29\times5}{4\times23} = \mathbf{\frac{-145}{92}}$//

c. $2\frac{3}{7} \div 2.3 = \frac{17}{7} \div \frac{23}{10} = \frac{17}{7} \times \frac{10}{23} = \mathbf{\frac{170}{161}}$//

8. a. $\frac{3}{5} + \frac{2}{7}\left(\frac{4}{5} + \frac{3}{2}\right) = \frac{3}{5} + \frac{2}{7}(\frac{8}{10} + \frac{15}{10})$

$= \frac{3}{5} + \frac{2}{7} \times \frac{8}{10} + \frac{2}{7} \times \frac{15}{10}$

$$=\frac{3}{5}+\frac{16}{70}+\frac{30}{70}=\frac{3}{5}+\frac{46}{70}$$

$$=\frac{14\times3+46}{70}$$

$$=\frac{42+46}{70}=\frac{88}{70}=\frac{\mathbf{44}}{\mathbf{35}}//$$

b. $\frac{4}{3}\div\left(\frac{5}{2}-\frac{6}{7}\right)=\frac{4}{3}\div\left(\frac{5\times7-2\times6}{2\times7}\right)$

$$=\frac{4}{3}\div\left(\frac{35-12}{14}\right)=\frac{4}{3}\div\left(\frac{23}{14}\right)$$

$$=\frac{4}{3}\times\frac{14}{23}=\frac{4\times14}{3\times23}=\frac{\mathbf{56}}{\mathbf{69}}//$$

c. $\frac{1}{6}\div\left(\frac{1}{3}+\frac{4}{5}\right)=\frac{1}{6}\div\left(\frac{1\times5+3\times4}{3\times5}\right)$

$$=\frac{1}{6}\div(\frac{5+12}{15})$$

$$=\frac{5}{2}+\frac{1}{3}(\frac{2}{5}\div\frac{3}{5})$$

$$=\frac{5}{2}+\frac{1}{3}(\frac{2}{5}\times\frac{5}{3})$$

$$=\frac{5}{2}+\frac{1}{3}\times\frac{2}{3}$$

$$=\frac{1}{6}\div\frac{17}{15}=\frac{1}{6}\times\frac{15}{17}=\frac{5}{2\times17}$$

$$=\frac{5}{2}+\frac{2}{9}=\frac{5\times9+2\times2}{2\times9}=\frac{45+4}{18}$$

$$=\frac{\frac{5}{34}}{\frac{49}{18}}=\frac{5}{34}\div\frac{49}{18}=\frac{5}{34}\times\frac{18}{49}=\frac{5\times9}{17\times49}=\frac{\mathbf{45}}{\mathbf{983}}//$$

9. I = 100

P = 500

R = 9.5% We need to find Time t.

I= P × R× T , $T=\frac{I}{P\times R}=\frac{80}{500\times\frac{9.5}{100}}=\frac{80}{9.5\times5}$ $T=\frac{\mathbf{20}}{\mathbf{19}}//$,Therefore, the time is $\frac{20}{19}$ Years .

Unit 2

SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS

2.1 Squares and Square roots

2.1.1Square of a rational number

Definition: The process of multiplying a number by itself is Called squaring of a number.

Example.

a) $1xx1 = 1 = 1^2$

b) $2xx2 = 4 = 2^2$

c) $4xx4 = 16 = 4^2$

d) $5xx5 = 25 = 5^2$

Note:

If the number "a" to be multiplied by itself, then the product is usually written a^2 and read as:

- ✓ A squared or
- ✓ The square of a or
- ✓ A to the power of 2

Example 2.2

Read the following numbers

- 2^2 b. 6^2 c. 10^2

Solution

a. 2^2 read as 2 squared or the square of 2 and 2 to the power of 2

b. 6^2 read as 6 squared or the square of 6 and 6 to the power of2

c. 10^2 read as 10 squared or the square of 10 and 10 to the Power of 2

Example :

Find the square of each of the following numbers

a) 8 c) 20

b) 4/3

d) – 10/16

Solution:

A) $8^2 = 8 \times 8 = 64$

B)$(\frac{4}{3})2 = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$

c) $20^2 = 20 \times 20 = 400$

d) $(-10/16)^2 = -10/16 \times -10/16 = 100/256$

There is a difference between a^2 and $2a$.

i. $a^2 = a \times a$

ii. $2a = a + a$

Consider the following examples to see the difference

a. $10^2 = 10 \times 10 = 100$ while $10 \times 2 = 20$

b. $0.4^2 = 0.4 \times 0.4 = 0.16$ $w$hale $0.4 \times 2 = 0.8$

In general, $a^2 \neq 2a$ for any rational number aa.

Definition : A rational number x is called a perfect square if and only if $x = m^2$ for some $m \in \mathbb{Q}$

Example :

$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$. Thus, 1, 4, 9, 16, 25 and 36 are perfect squares.

Note

i. The square of rational numbers is also rational numbers.

ii. a × a= a^2 Therefore a^2 = a

iii. For any rational numbers a and , $(ba)^2 = a^2b^2$

iv. For any rational number a and, $(\frac{a}{b})^2 = \frac{a}{b} \times \frac{a}{b} = \frac{a^2}{b^2}$

Exercise 2.1

1. Determine whether each of the following statements is true or False.

a) $10^2 = 10 \times 10$

b) $8^2 = 8 \times 2$

c) $x^2 = 2x$, where $x \in \mathbb{Q}$

d) $-60^2 = 3600$

e) $(-30)^2 = 900$

2. Find x^2 in each of the following rational numbers

a) $xx = 6$ e) $x = 0.03$

b) $x = -20$ f) $x = 4.5$

c) $x = 3\frac{1}{4}$

g) $x = 12$

3. Lists out a perfect square numbers from the given list.

4, 9, 12, 16, 18, 24, 36, 1.6, 0.04, $\frac{1}{4}$, $\frac{1}{121}$, 0.01, 225

4. Explain whether the numbers are a square number or not.

a) 144 b) 201 c) 324

5. Which of the following are the squares of even numbers?

a) 196 b) 441 c) 400 d) 324 e) 625

6. Which of the following are the squares of odd numbers?

a) 121 b) 225 c) 196 d) 484 e) 529

7. Evaluate

i. $(38)^2 - (37)^2$

ii. $(75)^2 - (74)^2$

iii. $(92)^2 - (91)^2$

iv. $(105)^2 - (104)^2$

v. $(141)^2 - (140)^2$

vi. $(218)^2 - (217)^2$.

8. Express 64 as the sum of the first eight odd numbers.

Answer

1. a. True c. False e. True

b. False d. False

2 . a. $x = 6$ b. $x = -20$ c. $x = 3\frac{1}{4}$ d. $x = -\frac{5}{4}$

$x^2 = 6^2$ $x^2 = (-20)^2$ $x^2 = (\frac{13}{4})^2$ $x^2 = (-\frac{5}{4})^2$

$x^2 = 36$ $x^2 = 400$ $x^2 = \frac{169}{16}$ $x^2 = \frac{25}{16}$

e. $x = 0.03$ f. $x = 4.5$ g. $x = \frac{12}{4}$

$x^2 = (0.03)^2$ $x^2 = (4.5)^2$ $x^2 = (\frac{12}{4})^2$

$= 0.0009$ $x^2 = 20.25$ $x^2 = 3^2 = 9$

3.4,9, 16, 36, 0.04, $\frac{1}{4}$, $\frac{1}{12}$, 0.01 , 225 are perfect square numbers.

4. a. is a square number

b. bot a square number

c. is a square number

5. 196 , 400 , 324 , we conclude that the square of even number is even.

6. 121 , 225 , 529 , we conclude that the square of odd is odd.

7. a. 38 + 37 = 75 d. 105 + 104 = 209

b. 75 + 74 = 149 e. 141 + 140 = 281

c. 92 + 91 = 183 f. 218 + 217 = 435

As you have seen the pattern, the difference of the square of two consecutive rational number is the sum of the numbers.

Theorem1: Existence theorem

For any rational number x, there is a rational number y ($y \geq 0$) Such that $\boldsymbol{x^2 = y}$.

Example :

Find the square of the following rational numbers by using the Existence theorem.

a) $x = 12$ b) $x = 0.04$ c) $x = \frac{1}{2}$ d) $x = \frac{-1}{4}$

Solution:

a)$x = 12$, then $y = x^2 = 122 = 144$

b) $x = 0.04$, then $y = x^2 = 0.042 = 0.0016$

c) $= \frac{1}{2}$, then $y = x^2 = (\frac{1}{2})^2 = \frac{1}{4}$

d) $= \frac{-1}{4}$, then $y = x^2 = \frac{(-1}{4})^2$

Example :

Find the approximate value of x^2 in each of the following decimals

a) $x = 4.3$

b) $x = 0.026$

c) $x = 2.45$

Solution:

a) x = 4.3 ≈ 4 [since 3 < 5]

x^2 = 42 = 16

Therefore, 4.3 2 ≈ 16

b) x = 0.026 ≈ 0.03

x^2 = 0.032 = 0.0009

Therefore, 0.026 2 ≈ 0.0009

c) x = 2.45 ≈ 2.5

x^2 = 2.52 = 6.25

Therefore, 2.452 ≈ 6.25

Exercise 2.2:

1. Determine whether each of the following statements is true or false

a) $0^2 = 2$

b) 102 > 10.052 > 112

c) $(9.9)^2 = 100$

2. Find the approximate value of x^2 if

a) x = 4.2 b) x = 10.8 c) x = −8.7 d) x = 1.06

Answer

1. a. False b. False c. True
2. a. x = 4.2 ≈ 4 b. x = 10.8 ≈ 11 c. x = - 8.7≈ −9 d. x = 1.06 ≈ 1

$x^2 \approx 4^2 = 16$ $x^2 \approx 11^2 = 21$ $x^2 \approx (-9^2) = 81$ $x^2 \approx 1^2 = 1$

2.1.2 .Use of table values and scientific calculator to find squares

Of rational numbers

- In the table The first column headed by x lists numbers from 1.0 – 9.9, the

Remaining columns are headed respectively by the digits 0 *to* 9.

x	0	1	2	3	4	5	6	7	8	9
1.0										
1.1										
1.2										
.										
.										
.										
9.9										

Example :

By using table of squares evaluate $(3.24)^2$

Solution:

The way how we can find the square of rational numbers from the table as follows:

step1. Find the row which start with 3.2

Step2. Move to right along the row until you get column under 4

Step3. Read the number at the intersection of the row in (1) and Column in (2)

x	0	1	2	3	4	5	6	7	8	9
3.1										
3.2					10.5					
3.3										

Hence $(3.24)^2 = 10.50$

From the table check the value of the following square numbers are true.

a) $(3.10)^2 = 9.610$

b) $(35.2)^2 = (3.52 \times 10)^2 = (3.52)^2 \times 102 = 12.39 \times 100 = 1239$

c) $(0.365)^2 = (3.65)^2 \times (10–1)^2 = 13.32 \times 0.01 = 0.1332$

Exercise 2.3

1. Determine whether each of the following statement is true or

False

a) $(3.22)^2 = 10.30$

b) $(3.56)^2 = 30.91$

c) $(9.9)^2 = (98.01)2$

2. If $(3.67)^2 = 13.47$ then find

a) $(36.7)^2$ b) $(367)^2$ c) $(0.367)^2$

3. If $(8.435)^2 = x$ then determine each of the following in terms Of .

a) $(84.35)^2$ b) $(0.8435)^2$

Answer

1. False b. False c. False
2. If $(3.67)^2 = 13.47$
   a. $(36.7)^2 = (3.67 \times 10)^2 = (3.67)^2 \times 10^2$

   $= 13.47 \times 100$

   $= \mathbf{1347}//$

   b. $(367)^2 = (3.67 \times 10)^2$

   $= (3.67)^2 \times 100^2$

   $= 13.47 \times 10{,}000$

   $= \mathbf{134,700}//$

c. $(0.367)^2 = (3.67 \times \frac{1}{10})^2$

$= (3.67)^2 \times (\frac{1}{10})^2$

$= 13.47 \times \frac{1}{100}$

$= \underline{\underline{0.1347}}$

3. If $(8.435)^2 = x$, then

a. $(8.435)^2 = (8.435 \times 10)^2$

$= (8.435)^2 \times 100$

$= \underline{\underline{100x}}$

b. $(0.8435)^2 = (8.435 \times \frac{1}{10})^2$

$= (8.435)^2 \times (\frac{1}{10})^2$

$= x \times \frac{1}{100} = \frac{x}{100}$

4. a. $8.95 \approx 9$, *then* $(8.95)^2 \approx 9^2 = 81$

c. $(8.95)^2 = (8.95)(8.95) = \mathbf{80.0125}$//

b.80.01

2.1.3. Square Roots of a Rational number

Definition 2.3: For any two rational numbers a and b; if $a^2 = b$, then a is called the square root of b.

Example :

a) The square root of 4 is 2 because 22 = 4

b) The square root of 9 is 3 because 32 = 9

c) The square root of 16 is 4 because 42 = 16

d) The square root of $\frac{25}{64} = \frac{5}{8}$ because $(\frac{5}{8})^2 = \frac{25}{64}$

Note

i. The operation "extracting square root" is the inverse of the operation squaring.

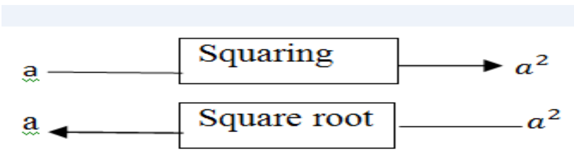
ii. In extracting square roots of rational numbers,

❖ First decompose the number into product consisting of two equal factors and take one of the equal factors as the square root of the given number.

iii. The positive square root of a number is called the principal square root. The symbol "√" called the radical sign, is used to indicate the principal square root.

iv. For $b \geq 0$, the expression $\sqrt{b}$ is called the principal square root of b or radical b, and b is called the radicand.

v. The relation between squaring and square root can be expressed as:



vi. In power form $\sqrt{a}$= a

vii. Negative rational numbers don't have square roots in the set of rational number.

viii. The square root of zero is zero.

Example :

Find the square root of x, if x is

a) 16 b) 121 c) 0.04 d) $\frac{25}{100}$

Solution:

a) $x = 16 = 4 \times 4 = -4 \times -4 \quad x = 4^2 = (-4)^2$

Thus, the square root of 16 is 4 or −4. But √16 = 4 [since "√" indicate the positive square root. To indicate the negative square root.

b) $x = 121 = 11 \times 11 = -11 \times -11$

$x = 112 = (-11)^2$

Thus, the square root of 121 is 11 or −11. But √121 = 11

c) $xx = 0.04 = 0.22 = (-0.2)^2$

Thus, the square root of 0.04 is 0.2 or −0.2. But √0.04 = 0.2

c) $x = 0.04 = 0.22 = (-0.2)^2$

Thus, the square root of 0.04 is 0.2 or −0.2. But √0.04 = 0.2

d) $x = \frac{25}{100} = \frac{5}{10} \times \frac{5}{10} = -\frac{5}{10} \times -\frac{5}{10}$, x =$\left(-\frac{5}{10}\right)^2$= =$\left(\frac{5}{10}\right)^2$

Thus the square root of $\frac{25}{100}$ is $\frac{5}{10}$ or $-\frac{5}{10}$ But, $\sqrt{\frac{25}{100}} = \frac{5}{10}$

Example:

Find the square root of each of the following numbers by using prime factorization.

a) 64 b) 100 c) 400

Solution:

a) 64

Now arrange the factors so that 64 is the Product of two identical factors.

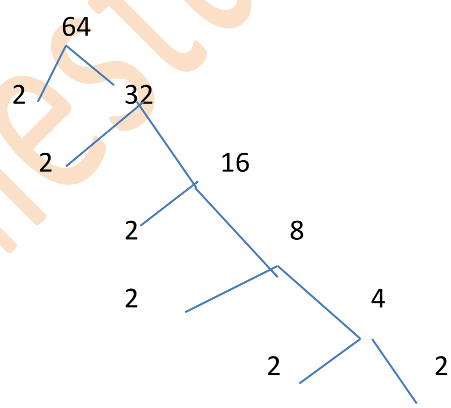
I.e. 64 = 2 × 2 × 2 × 2 × 2 × 2

64 = (2 × 2 × 2) × (2 × 2 × 2)

64 = 8 × 8

64 = 8^2

So √64 = 8



b) 100

Arrange the factors so that 100 is the product of two identical factors

i.e.100 = 2 × 2 × 5 × 5

100 = (2 × 5) × (2 × 5)

100 = 10 × 10

10So √100 = 100 = 10^2

c) 400

Arrange the factors so that 400 is the product of two identical factors i.e. $400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$400 = (2 \times 2 \times 5) \times (2 \times 2 \times 5)$

$400 = 20 \times 20$

400 = 202

So √400 = 20

Exercise 2.4

1. Evaluate the given square root.

a) √0 d) √0.25 b) √169 e) √0.0016 c) √576

2. The area of square is $144cm^2$.What is the length of each side?

3. Using prime factorization technique evaluates the square root of the following numbers.

a) 256 b) 324 c) 1225

Answer

1. a. $\sqrt{0} = 0$

   b. $\sqrt{169} = \sqrt{13 \times 13} = \underline{\mathbf{13}}$

   c. $\sqrt{576} = \sqrt{24 \times 24} = \underline{\mathbf{24}}$

   d. $\sqrt{0.25} = \sqrt{0.5 \times 0.5} = \underline{\mathbf{0.5}}$

   e. $\sqrt{0.0016} = \sqrt{0.04 \times 0.04} = \underline{\mathbf{0.04}}$

2. $A = 144cm^2 = l^2$

   $l = \sqrt{144\ cm^2} = \underline{\mathbf{12\ cm}}$

   $\therefore The\ length\ of\ the\ square\ is\ 12cm$

3. a. 625

5 125 $625 = 5 \times 5 \times 5 \times 5 = (5\times 5) \times (5 \times 5)$

5 25 $= 25 \times 25$

5 5 $\therefore \sqrt{625} = \sqrt{25 \times 25} = 25$

5 1

b. 324 $\sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3} = \sqrt{(2 \times 3 \times 3) \times (2 \times 3 \times 3)}$

2 162 $= \sqrt{18 \times 18}$

2 81 $= \underline{\mathbf{18}}$

2 27 $\therefore \sqrt{324} = 18$

2 9

2 3

2 1

c. 1225 $\sqrt{1225} = \sqrt{5 \times 5 \times 7 \times 7} = \sqrt{(5 \times 7) \times (5 \times 7)}$

5 245 $= \sqrt{35 \times 35}$

5 49

7 7 $= \underline{\mathbf{35}}$

2.1.4. Use of table values and scientific calculator to find square roots of rational numbers

Example:

Find √16.40 from the numeral table

Solution:

step . Find the number 16.40 on the body of the table.

stepii. On the row containing this number move to the left and read 4.0 under x.

step . To get the third digit start from 16.40 move vertically up ward and read 5.

Therefore $\sqrt{16.40} \approx 4.05$

Note:

If the radicand is not found in the body of the table, you can approximate to the nearest square roots of a number

Example:

Find $\sqrt{71.80}$ from numeral table

Solution:

$\sqrt{71.80}$ is not directly in the numerical table. So find two numbers from the table to the left and to the right of 71.80.

That is, 71.74 < 71.80 < 71.91

Find the nearest number to 71.80 from those two numbers. Which is 71.74.

Thus$\sqrt{71.80} \approx \sqrt{71.74} = 8.47$

Therefore$\sqrt{71.80} \approx 8.47$

Exercise 2.5

1. Find the square root of each of the following numbers from numerical table

a) 2.310 b) 4.326 c) 15.68

d) 98.60 e) 95.06

2. If (4.63)2 = 21.44, then find

a) $\sqrt{21.44}$ b) $\sqrt{2144}$ c) $\sqrt{0.2144}$ d)$\sqrt{0.003564}$

Answer

1.a.1.62 b. 2.08 c. 3.96 d. 9.93 e. 9.75

2. . if $(4.63)^2 = 21.44$

a. $\sqrt{21.44} = 4.63$

b. $\sqrt{2144} = \sqrt{2144 \times 100} = \sqrt{21.44} \times \sqrt{100} = 4.63 \times 10 = \underline{\mathbf{46.}}$

c. $\sqrt{0.2144} = \sqrt{21.44 \times \frac{1}{100}} = \sqrt{21.44} \times \sqrt{\frac{1}{100}} = 4.63 \times \frac{1}{10} = \underline{\mathbf{0.463}}$

2.2. Cubes and Cube roots

2.2.1. Cube of a rational number

Definition. A cube number is a number obtained by multiplying the number by itself three times.

Example:

The following are some cube numbers

a) 1 × 1 × 1 = 1

b) 2 × 2 × 2 = 8

c) 3 × 3 × 3 = 27

d) 4 × 4 × 4 = 64

Example:

Find $xx3$ in each of the following rational numbers

a. $x = 2$ b) $x = -4$ c) $x = 0.02$ d) $x = 2$ e) $x = -1$

Solution:

a. $x^3 = x \times x \times x = 2 \times 2 \times 2 = 8$

b. $x^3 = x \times x \times x = -4 \times -4 \times -4 = -64$

c. $x^3 = x \times x \times x = 0.02 \times 0.02 \times 0.02 = 0.000008$

d. $x^3 = x \times xx \times x = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

e. $x^3 = x \times x \times x = -\frac{1}{4} \times -\frac{1}{4} \times -\frac{1}{4} = -\frac{1}{64}$

Definition: A rational number xx is called a perfect cube if and only if $x = m^3$ for some $m \in \mathbb{Q}$. That is, a perfect cube is a number that is a product of three identical factors.

Example:

The numbers 1, 8, 27, 64, 125 and 216 are perfect cubes. Because

$1 = 1^3$, $27 = 3^3$ $125 = 5^3$, $8 = 2^3$, $64 = 4^3$, $216 = 6^3$

Exercise 2.6

1. Determine whether each of the following statement is true or false

a) $2^3 = 2 \times 3$ b) $-20^3 = -400$ c) $8^3 = 64 \times 8$ d) $(\frac{3}{4})^2 = \frac{27}{16}$

e) $(-3)^3 = -27$ f) $12^3 = 144 \times 12$

2. Find x^3 in each of the following

a) $x = 4$ b) $x = 0.5$ c) $x = \frac{1}{2}$ d) $x = \frac{3}{4}$

3. Find the approximate value of x^3 in each of the following

a) $x = -3.45$ b) $x = 4.98$ c) $x = 0.025$ d) $x = 2.75$

4. Identify whether each of the following are perfect cubes?

a) 42 b) 60 c) 64 d) 125 e) 144 f) 216

5. What are the consecutive perfect cubes which added to obtain sum of 100? 441?

Answer

1. a. False b. False c. True d. False e. True f. True
2. a. x = 4 b. x = 0.5 c. x= $-\frac{1}{2}$ d. x= $\frac{3}{4}$

   $x^3 = 4^3 = \mathbf{64}$ $x^3 = (05)^3 = \mathbf{0.125}$ $x^3 = (-\frac{1}{2})^3 = -\frac{\mathbf{1}}{\mathbf{8}}$// $x^3 = (\frac{3}{4})^3 = \frac{\mathbf{27}}{\mathbf{64}}$//
3. a. x = -3.45 b. x = 4.98 c. x = 0.025 d. x = 2.75

   $x^3 \approx (-3)^3 = \mathbf{-27}$ $x^3 \approx (5)^3 = \mathbf{125}$ $x^3 \approx (0)^3 = \mathbf{0}$ $x^3 \approx (3)^3 = \mathbf{27}$
4. $\frac{27}{64}$, -125 and 216 are perfect cubes.

5. $1\ ,8\ ,27\ \ ,64\ ,125\ and\ 216\ are\ consecetive\ perfect\ cube\ numbers\ whose\ sum\ is\ 441.$

$1\ ,8\ \ ,27\ and\ 64\ are\ the\ consecatice\ perfect\ cube\ numbres\ whose\ sum\ is\ 100.$

2.2.2. Cube Root of a rational number

Definition: The cube root of a given number is one of the three identical factors whose product is the given number.

Example:

a) 1 × 1 × 1 = 1, so 1 is the cube root of 1

b) 2 × 2 × 2 = 8 , so 2 is the cube root of 8

c) 4 × 4 × 4 = 64 , so 4 is the cube root of 64

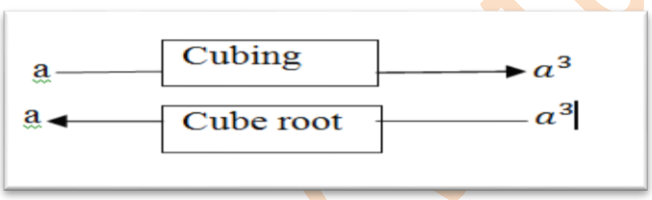
Note: i. 3^3 = 27. Then 27 is the cube of 3 and 3 is the cube root of 27. This is written as 3 = $\sqrt[3]{27}$. The symbol 3 = $\sqrt[3]{27}$ is read as the principal cube root of 27 or simply the cube root of 27.

ii. The symbol $\sqrt[3]{\ }$ is called a radical sign. The expression $\sqrt[3]{a}$ is called a radical, 3 is called the index and a is called the radicand. When no index is written, the radical sign indicates square root.

iii. The relation between cubing and cube root can be expressed as:

iv. $\sqrt[3]{a}$ $=a^{\frac{1}{3}}$ [exponential form]

v. Each rational number has exactly one cube root. Exercise 2.7



Exercise 2.7

1. State another name for $4^{\frac{1}{3}}$

2. Write the following in exponential form

a) $\sqrt[3]{10}$ b) $\sqrt[3]{0.23}$

3. Identify whether each of the following are perfect cube.

3, 6, 8, 9, 12, 64, 216, 729, 625, 400

4. Find the cube root of the following numbers.

a) 216 b) −343 c) 1000 d) −1728 e) 0

Answer

1. Cube root of 4
2. a. $10^{1/3}$ b. $(0.23)^{1/3}$
3. 9 , 64 , 216 , 729 are perfect cube.
4. a. $\sqrt[3]{216} = \sqrt[3]{6^3} = \underline{\mathbf{6}}$ b. $\sqrt[3]{-343} = \sqrt[3]{(-7)^3} = \underline{\mathbf{-7}}$

c. $\sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10} = \underline{\mathbf{10}}$ d. $\sqrt[3]{-1728} = \sqrt[3]{(-12)^3} = \underline{\mathbf{-12}}$ e. $\sqrt[3]{0} = \underline{\mathbf{0}}$

2.2. Applications on squares, square roots, cubes and cube roots

Example: Alemu and Almaz want to make a square patio. They have concrete to make an area of 400 square meters. How long can a side of their patio be?

Solution:

Let x be the length of each sides of a square patio

$A = x^2$

$400 = x^2$ from this xx can be calculated as principal square root:

$x = \sqrt{400} = 20$.

Therefore, the length of each sides of a square patio is 20 m.

Exercise 2.8

1. 1225 students stand in rows in such a way that the number of rows is equal to the number of students in a row, how many students are there in each row?

2. Getaneh's flower garden is a square. If he enlarges it by increasing the width 1 m and the length 3 m, the area will be 19 square meters more than the present area. What is the length of a side now?

3. If the area of square region is 64 square meter, then what is the length sides of a square region?

Answer

1. $x^2 = 1225$

   $x = \sqrt{1225}$

   $x = \underline{\underline{\mathbf{35}}}$

2. $(s+1)\ (s+3) = 5^2 + 19$

   $s^2 + 45 + 3 = s^2 + 19$

   $4s = 19 - 3$

   $\underline{\underline{\mathbf{s = 4}}}$

3. $A = S^2$

   $S = \sqrt{64} = \underline{\underline{\mathbf{8}}}$

REVIEW EXERCISE FOR UNIT 2

1. Determine whether each of the following statements is true or false

a) $8^2 = 8 \times 2$ b) $-2^2 = -4$ c) $(-\frac{2}{3})^2 = \frac{4}{9}$ d) $-3^3 = 27$ e) $0^3 = 3$ f) $(-5)^2 = 25$

2. Find x^2 in each of the following rational numbers

a) $x = 0.03$ b) $x = -2$ c) $x = \frac{1}{4}$ d) $x = -2$ e) $x = -\frac{1}{5}$

3. Identify whether each of the following are perfect square.

a) 216   b) 625   c) 1000   d) 729   e) 900   f) 2025

4. What is the sum of the first 25 odd natural numbers?

5. Find the square root of x if x is

a) 64   b) 81   c) 0.04

6. Using prime factorization technique evaluates the square root of the following numbers.

a) 225   b) 625   c) 2500

7. If $(7.43)^2 = 55.20$ then find

a) $(74.3)^2$ b) $(743)^2$ c) $(0.743)^2$

8. If $(3.42)^2 = 11.70$, then find

a) $\sqrt{11.70}$ b$\sqrt{1170}$ c) $\sqrt{0.117}$

9. Identify whether each of the following are perfect cubes?

a) 27 b) 60 c) 64 d) 25

10. Find x^3 in each of the following rational numbers

a) $x = 2$ b) $x = 0.03$ c) $x = -20$ d) $x = \frac{1}{4}$

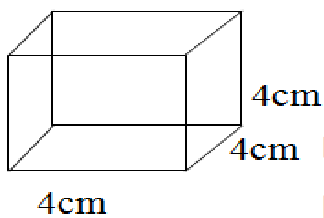
11. Identify the base, exponent, power forms and Standard numeral form for each of the following numbers

a) $2^3 = 8$ b) $3^3 = 27$ c) $10^3 = 1000$

12. Write the following in power form

a) 900 b) 324 c) 216

13. In figure 1.1 as shown find:



a) The surface area of a cube

b) The volume of a cube

c) Compare the surface area and volume of a given cube.

14. Find the area of the square with length sides 10cm?

Answer

1. a. False b. True c. True d. False e. False f. True

2. a. x = 0.09 b. x = 4 c. $(x)^2 = (\frac{1}{4})^2 = \frac{1}{16}$ d. $x = \frac{15}{4}$ $x^2 = (-\frac{15}{4})^2 = \frac{\mathbf{225}}{\mathbf{16}}$//

3. 625 , 729 , 900 and 2025 are perfect square

4. 1 + 3 + 5 + 7 + 9 +11 + 13 +15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 + 31 + 33 + 35 + 37 + 39 + 41 + 43 + 45 + 47 + 49 = **<u>625</u>**

5. a. $8.04 \approx 8$, $8^2 = 64$

b. $(8.04)^2 \approx 64.64$

c. $(8.04) \approx 64.6416$

. a. $\sqrt{64} = 8$ b. $\sqrt{81} = 9$ c. $\sqrt{0.04} = 0.2$

6. $a.\sqrt{225} = \sqrt{3^2 \times 5^2} = 3 \times 5 = \underline{\mathbf{15}}$ b. $\sqrt{625} = \sqrt{5^4} = 5^2 = 25$

c. $\sqrt{2500} = \sqrt{2^2 \times 5^4} = 2 \times 5^2 = 2 \times 25 = \mathbf{50}$//

7. If $(7.43)^2 \approx 55.20$ *then find*

a. $(74.3)^2 = (7.43 \times 10)^2$

$= (7.3)^2 \times 10^2$

$= 55.20 \times 100$

$= \underline{\mathbf{5520}}$

b. $(743)^2 = (7.43 \times 100)^2$

$= (7.43)^2 \times 100^2$

$= 55.20 \times 10{,}000$

$= \underline{\mathbf{552{,}000}}$

c.$(0.743)^2 = (\frac{7.43}{10})^2 = \frac{(7.43)^2}{100} = \frac{55.20}{100} = \underline{\mathbf{0.5520}}$

8. a. $(3.42)^2 = 11.70$

$= \sqrt{11.70} = \underline{\mathbf{3.42}}$

b. $\sqrt{1170} = \sqrt{11.70 \times 100}$

$= \sqrt{11.70} \times \sqrt{100}$

$= 3.42 \times 10$

$= \underline{\mathbf{34.2}}$

c.$\sqrt{0.117} = \sqrt{\frac{11.70}{100}} = \frac{\sqrt{11.70}}{\sqrt{100}} = \frac{3.42}{10} = \underline{\mathbf{0.342}}$

. 27 and 64 are perfect cubes.

9. a. $x^3 = (2)^3$

$= \underline{\mathbf{8}}$

b. $x^3 = (0.03)^3$

$= \underline{\mathbf{0.000027}}$

c. $x^3 = (-20)^3$

$= \underline{\mathbf{-8{,}000}}$

d. $x^3 = (\frac{1}{4})^3 = \frac{1}{4^3} = \mathbf{\frac{1}{64}}$//

10. a. $2^3 = 8$ - The base is 2 b. $3^3 = 27$ - The base is 3 c. $10^3 = 1000$ – The base is 10

- The exponent is 3 - The exponent is 3 - The exponent is 3

- The standard form is 8 - The standard form is 27 – The standard form is 1000

11. a. $900 = 30^2$ b. $324 = 18^2$ c. $216 = 6^3$

12. $31 + 33 + 35 + 37 + 39 + 41 = 216 = 6^3$

Unit 3

LINEAR EQUUANTIIOTNS AND INEQUALITIES

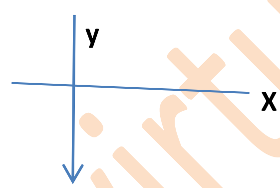
3.1. Revision of Cartesian coordinate system

To determine the position of a point in a Cartesian coordinate plane, you have to draw two intersecting perpendicular number lines. The two intersecting perpendicular lines are called axes, the horizontal line is the x – axis and the vertical line is the y – axis. Usually the arrows indicate the positive direction. These axes intersect at a point called the origin .These two axes together form a plane called the Cartesian coordinate plane. The position of any point in the plane can be represented by an ordered pair of numbers (x, y). These ordered pairs are called the coordinates of the point. The first coordinate is called the x- coordinate or abscissa and the second coordinate is called the y- coordinate or ordinate. The two axes divide the given plane into four quadrants. Starting from the positive direction of the x-axis and moving the anticlockwise (counter clock wise) direction, the quadrants which you come across are called the I, the II, the III, and the IV quadrants respectively.

Example:

The point with coordinates (2, 5) has been plotted on the Cartesian plane as follow. Imagine a vertical line through 2 on the x-axis and a horizontal line through 5 on the y-axis. The intersection of these two lines is the point (2, 5). This point is 2 units to the right of the y-axis and 5 units up from the x-axis.

Solution:

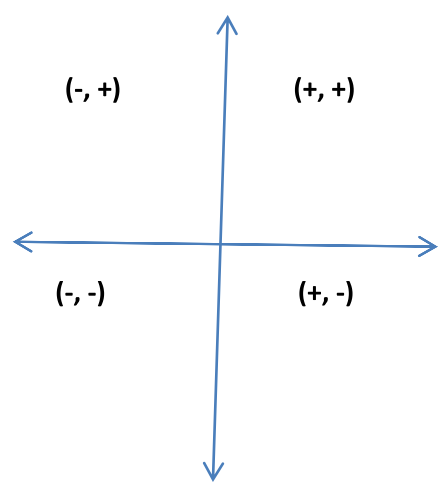


Note: In the I quadrant, all points are (+, +)

In the II quadrant, all points are (-, +)

In the III quadrant, all points are (-, -)

In the IV quadrant, all points are (+, -)



Example 3.2:

In which quadrant the following points lie?

a) (2, −6) b) (−3, 3) c) (5, 0)

Solution:

a) Since x = 2 > 0 and y = −6 < 0 then the point (2, −6) lies in quadrant IV.

b) Since x = −3 < 0 and y = 6 > 0 then the point (−3, 3) lies in quadrant II.

c) The point (5, 0) lies on the positive x- axis. It is neither of any quadrants.

Exercise 3.1:

1. Determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied?

a) x > 0 and y < 0
d) x < 0 and –y > 0
b) x = −4 and y > 0
e) x > 2 and y = 3
c) y < −5
f) x > 0

2. Find the coordinates of the point.

a) The point is located 5 units to the left of the y-axis and 2 units above the x-axis.

b) The point is located on the x-axis and 10 units to the left of the y-axis.

Answer

1. a. quadrant IV c. quadrant III and IV e. quadrant I
   b. quadrant II d. quadrant III f. quadrant and IV.

2. a. (-5,2) b.(-10,0

3.2: Graphs of Linear Equations

Revision on vertical and horizontal lines

In a vertical line all points have the same x- coordinate, but the y coordinate can take any value. The equation of the vertical line through the point (a, b) *is* $x = a$. This line is parallel to the y-axis and perpendicular to the x-axis. Similarly, in a horizontal line all points have the same y – coordinate but the x- coordinate can take any value. The equation of the horizontal line through the point (a, b) s $y = b$. This Line is parallel to the x-axis and perpendicular to the y-axis.

Example:

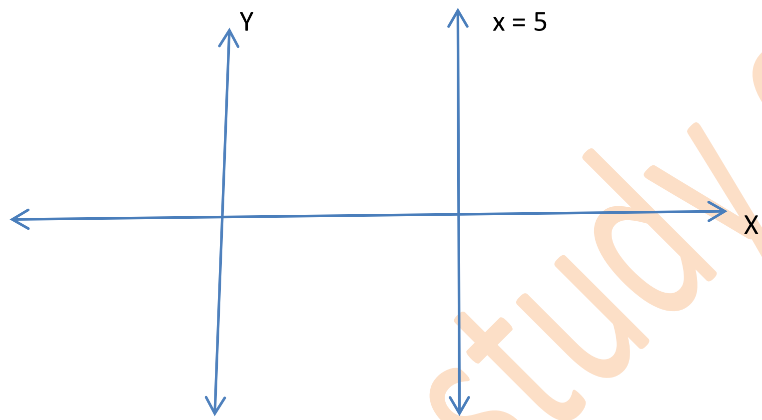
Draw the graphs of

a) x = 5 b) y = −4

Solution: a) First construct tables of values for x and y in which x is constant.

x	5	5	5	5	5	5	5	5	5
y	-4	-3	-2	-1	0	1	2	3	4

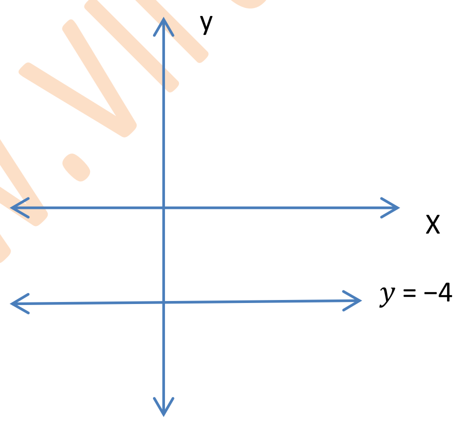
Then, plot these points on the Cartesian coordinate plane and join them. What do you realize? All the points lie on the vertical line.



b) First construct tables of values for x and y in which y is constant.

x	-4	-3	-2	-1	0	1	2	3	4
y	-4	-4	-4	-4	-4	-4	-4	-4	-4

Then plot these points on the Cartesian coordinate plane and join them. What do you realize? All points lie on the horizontal line.



Graph of an equation of the form $\boldsymbol{y} = \boldsymbol{m}$ ($\in \mathbb{Q}$, $\boldsymbol{m} \neq \boldsymbol{0}$)

There are several methods that can be used to graph a linear equation.

The method we used at the start of this section to graph is called plotting points, or the point – plotting method.

Definition: [Graph of a Linear Equation $\boldsymbol{y} = \boldsymbol{m}$ ($\in \mathbb{Q}$, $\boldsymbol{m} \neq \boldsymbol{0}$)]

The graph of a linear equation $\boldsymbol{y} = \boldsymbol{m}$ is a straight line passes through the origin.

Note:

- ✓ Every point on the line is a solution of the equation of a line.
- ✓ Every solution of the equation is a point on the line.

Example:

Sketch the graphs of the following equations on the same Cartesian coordinate plane

a) y = 3x b) y = −3x

Solution:

To sketch the graphs of the equations follow the following steps.

Step 1: Choose some values for x.

Let x = −2, −1, 0, 1, and 2

Step 2: Put these values of x into the equation to get the values of y

a) y =3x

When x = −2, y = 3(−2) = −6

When x = −1, y = 3(−1) = −3

When x = 0, y = 3(0) = 0

When x = 1, y = 3(1) = 3

When x = 2, y = 3(2) = 6

Step 3: Write these pairs of values in a table.

x	-2	-1	0	1	2
y	-6	-3	0	3	6
(x, y)	(−2,−6)	(−1,−3)	(0,0)	(1, 3)	(2,6)

Step 4: Plot the points on the Cartesian plane and join them.

Step 5: Label the line y = mx.

b) y = −3x

When x = −2, y = −3 (−2) = 6 $yy = -3xx$

When x = −1, y = −3 (−1) = 3

When x = 0, y = −3 (0) = 0 Figure 3.5

When x = 1, y = −3 (1) = −3

When x = 2, y = −3 (2) = −6

x	-2	-1	0	1	2
y	6	3	0	-3	-6
(x, y)	(−2, 6)	(−1,3)	(0,0)	(1,−3)	(2,−6)

If the point (k, 5) lies on the line y = −3x, then what is the value of k?

Since, the point is on the line, the point satisfy the equation of the line.

That is, 5 = −3 (k)

k = $-\frac{5}{4}$

Note:

 All ordered pairs that satisfy each linear equation of the form

$\boldsymbol{y} = \boldsymbol{m}$ ($\in \mathbb{Q}$, $\boldsymbol{m} \neq \boldsymbol{0}$) lies on a straight line that passes through the origin.

- ✓ The graph of the line $\boldsymbol{y} = \boldsymbol{m}$ ($\in \mathbb{Q}$, $\boldsymbol{m} \neq \boldsymbol{0}$) passes through the I and III quadrants if m > 0, and the graph passes through the II and IV quadrants if m < 0.
- ✓ In order to draw a straight line, you need to find any two points or coordinates through which the line passes.

Graph of an equation of the form $\boldsymbol{y} = \boldsymbol{m} +$ ($\in \mathbb{Q}$, $\boldsymbol{m} \neq \boldsymbol{0}$)

Example:

1) Given that $yy = 3x - 5$.Decide whether the ordered pairs given below are a solution to the equation?

a) (0, −5) b) (3, 4) c) (−2, −11) d) (−1, −2)

Solution:

Substitute the x- and y- values into the equation to check whether the ordered pair is a solution to the equation.

a) (0, −5)

$y = 3x - 5$.

−5? 3(0) −5

−5 = −5

(0, −5) is a solution.

b) (3, 4)

$y = 3x - 5$.

4? 3(3) −5

4 = 4

(3, 4) is a solution.

c) (−2, −11)

$y = 3x - 5$

−11 ? 3(−2) – 5

−11 = −11

(−2, −11) is a solution.

d) (−1, −2)

$y = 3x - 5$

−2 ? 3(−1) − 5

−2 ≠ −8

(−1, −2) is not a solution.

Example :

a) Sketch the graph of the equation y = 2x + 1 by plotting points.

Solution: Choose any value for x and solve for y.

Let x = -1

y= 2x +1

y= 2(-1) + 1

y= -1

Let x = 0

y= 2x + 1

y = 2(0) + 1

y = 1

Let x = 1

y= 2x + 1

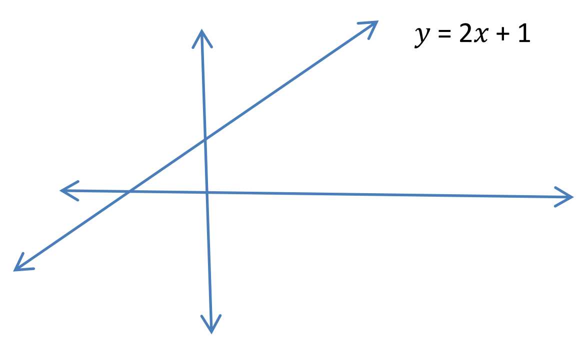
y = 2(1) + 1

y = 3

Then, organize the solutions in a table

x	-1	0	1
y	-1	1	3
(x, y)	(−1, −1)	(0, 1)	(1,3)

Now, we plot the points on the Cartesian coordinate plane and draw the line through these points.



b) Sketch the graph of the equation $y = -2x + 1$ by plotting points.

Solution:

Choose any value for x and solve for y.

Let x = −1 Let x = 0 Let x = 1

y = −2x +1 y = −2x + 1 y = −2x + 1

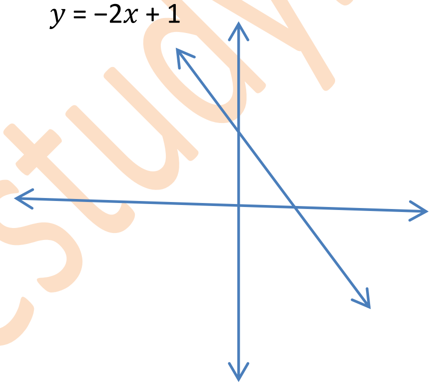
y = −2(−1) + 1 y = −2(0) + 1 y = −2(1) + 1

y = 3 y = 1 y = −1

Then, organize the solutions in a table

x	-1	0	1
y	3	1	-1
(x, y)	(−1, 3)	(0, 1)	(1,−1)

$y = -2x + 1$



Exercise3.2: Sketch the graph of the following equations.

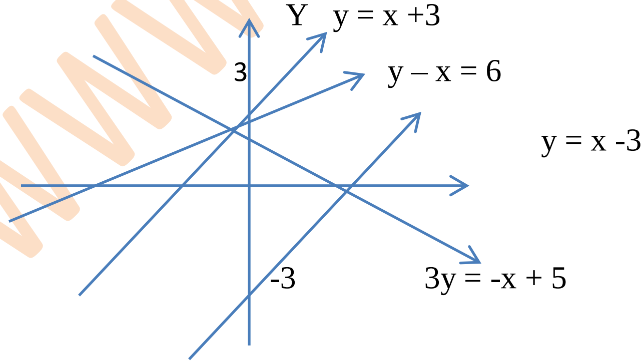
a) $y = x + 3$ c) $y = x - 3$

b) $2y - x = 6$ d) $3y = x + 5$

Answer

a. y = x+3 c. y = x - 3

b. 2y - x = 6 d. 3y = - x + 5



3.3. Solving Linear Inequalities

Definition: A mathematical sentence which contains one of the relation signs: <, >, ≤, ≥ or ≠ are called inequalities.

Example:

Some examples of inequalities are:

a. $2x > 0$ c) $x - 1 \neq \frac{1}{2}x - 5$

b. $\frac{3}{5}x + 20 \leq 0$ d) $2x^3 - 1 < \frac{1}{2}x + 15$

Definition: A linear inequality in one variable "x" is an inequality that can be written in the form of $ax + b < 0$, $ax + b \leq 0$ or **ax** $+ b > 0$, $ax + b \geq 0$ where a and b are rational numbers and $a \neq 0$.

Example 3.8:

Some examples of linear inequalities are:

a) $x + 8 > 3$ c) $4 - 3x \geq 1 + 3/2\, x$

b) $x - 0.35 \leq 0.25$

Rules of Transformation for Inequalities

Definition: Any two inequalities with the same solution set are called equivalent inequalities.

Example:

$5x + 6 > 2x$ and $x > -2$ are equivalent inequalities.

The following rules are used to transform a given inequality to an equivalent inequality.

Rule 1: If the same number is added to or subtracted from both sides of an inequality, the direction of the inequality is unchanged. That is for any rational numbers a,b and c.

i) If $a < b$, then $a + c < b + c$.

ii) If $a < b$, then $a - c < b - c$.

Rule 2: If both sides of an inequality are multiplied or divide by the same positive number, the direction of the inequality is unchanged. That is for any rational numbers a, b and c.

i. If $a < b$ and $c > 0$, then $ac < bc$.

ii. If a < b and c > 0, then $\frac{a}{c} > \frac{b}{c}$ (c $\neq$ 0)

Rule 3: If both sides of an inequality are multiplied or divided by the same negative number, the direction of the inequality is reversed. That is for any rational numbers a, b and c.

i. If a < b and c < 0, then ac > bc.

ii. If a < b and c < 0, then, $\frac{a}{c} > \frac{b}{c}$ $(provided\ c \neq 0)$

Example:

Which of the following pairs of inequalities are equivalent inequalities in $\mathbb{Q}$?

a) $x + \frac{1}{2} < \frac{5}{2}$ and $x + 1 < 3$

b) $\frac{1}{2}x - 2x > 32$ and x < $-\frac{1}{2}$

Solution: a) $\boldsymbol{x + \frac{1}{2} - \frac{1}{2} < \frac{5}{2} - \frac{1}{2}}$ [$\boldsymbol{Subtracting\ the\ same\ number\ from\ both\ sides}$]

= $\boldsymbol{x}$ **< 2 and x + 1 < 3**

And x + 1 − 1 < 3 – 1 [Subtracting the same number from both sides]

and x < 2

Since they have the same solution, x + $\frac{1}{2} < \frac{5}{2}$ and x + 1 < 3 are equivalent inequalities.

b) $\frac{1}{2}x - 2x > 32$

2($\frac{1}{2}x - 2x > 32$) > 2 x 32

= x - $4x$ > 64

= $-3x$ > 64

= $-\frac{1}{3}(-3x)$ > $-\frac{1}{3}$(64)

$x < -\frac{64}{3}$

Therefore, $\frac{1}{2}x - 2x > 32$ and x < $-\frac{1}{2}$ are not equivalent inequalities.

Exercise 3.3

1. Insert the correct sign (<, >, ≤, ≥ or ≠) in the given blank.

a) If x – 5 ≥ $2x$, then x ------- −5

b) If $\frac{1}{2}x$ + 2 ≤ 3(x – 1), then $\frac{5}{2}x$ ----- 5

c) If −3(x + 1) > 1, then x + 3 ------$\frac{5}{3}$

2. Which of the following pairs of inequalities are equivalent?

a) $3x$ − 2 ≤ x + 1 and 2x + 1 ≤ 4

b) 6(2− x) < 12 and 3(2 – x) < 6

c) $\frac{1}{4}$ − 1 ≥ $\frac{3}{2}$ − x and x + 1 ≥ 3

d) 5x − $\frac{3}{7}$ ≠ $\frac{4}{7}$ − 3x and $x \neq \frac{1}{8}$

Answer

1. a. x-5 ≥ $2x$

= x - 2x ≥ 5

= -x ≥ 5

= **x ≤ 5**

b. $\frac{1}{2}x + 2 \leq 3(x-1)$

= $\frac{1}{2}x + 2 \leq 3x - 3$

= $\frac{1}{2}x - 3x \leq -3 - 2$

= $\frac{-5}{2}x \leq -5$

= $\boldsymbol{\frac{5}{2}x \geq 5}$

c. -3(x+1)≥ 1

= 3x-3≥ 1

= -3x ≥ 1 + 3

= $\frac{-3x}{-3} \geq \frac{4}{3}$

= x ≤ $-\frac{4}{3}$

= x + 3 ≤ $-\frac{4}{3}$+3

= x + 3 ≤ $\frac{-4+9}{3}$

= **x + 3 ≤ $\boldsymbol{\frac{5}{3}}$**

2. a. 3x – 2 ≤ $x + 1$ & $2x + 1 \leq 4$

= 3x ≤ $x + 2 + 2$ & $2x \leq 4 - 1$

= 3x - x ≤ 3 & $2x \leq 3$

b. 6(2 –x) < 12 & $3(2-x) > 6$

= 12 - 6x < 12 &$6 - 3x > 6$

= - 6x < 0 & $-3x > 0$

$= \frac{2x}{2} \le \frac{3}{2} \& \frac{2x}{2} \le \frac{3}{2}$

$= \mathbf{x} \le \frac{\mathbf{3}}{\mathbf{2}} \& \boldsymbol{x} \le \frac{\mathbf{3}}{\mathbf{2}}$

a. $\frac{x}{4} - 1 \ge \frac{3}{2} - x \ \& \ x + 1 \ge 3$

$\frac{x}{4} + x \ge \frac{3}{2} + 1 \ \& \ x \ge 3 - 1$

$= \frac{x+4x}{4} \ge \frac{5}{2} \ \& \ x \ge 2$

$= \frac{5x}{4} \ge \frac{5}{2} \ \& \ x \ge 2$

$= 10x \ge 20 \ \& \ x \ge 2$

$= x \ge 2 \ \& \ x \ge 2$

$\therefore$ *They are equivalent .*

$= x > 0 \ \& \ x < 0$

$\therefore$ ***They are not equvalant***

d. $5x - \frac{3}{7} \ne \frac{4}{7} - 3x \ \& \ x \ne \frac{1}{8}$

$= 5x \ne \frac{4}{7} + \frac{3}{7} \ \& \ x \ne \frac{1}{8}$

$= 5x + 3x \ne \frac{7}{7} \ \& \ x \ne \frac{1}{8}$

$= 8x \ne 1 \ \& \ x \ne \frac{1}{8}$

$= x \ne \frac{1}{8}$

$\therefore$ *They are equivalent .*

Solutions of Linear inequalities by means of equivalent transformations

Example

Solve each of the following inequality in the domain of $\mathbb{Q}$.

a) $x + 2.4 \le 6.4$

Solution:

a) $x + 2.4 \le 6.4$

$x + 2.4 - 2.4 \le 6.4 - 2.4$ [subtracting 2.4 from both sides]

$x \le 4, x \in \mathbb{Q}$

Example :

1. Solve each of the following inequality in the domain of $\mathbb{Q}$.

a) $3x - 7 \ge 11$ d) $-7(x + 1) < 4(x - 3) + 6$

Solution:

a) $3x - 7 \ge \mathbf{11}$

$3x - 7 + 7 \ge 11 + 7$

$3x \ge 18$

$\frac{1}{3} \times 3x \ge \frac{1}{3} \times 18$

$x \ge 6, x \in \mathbb{Q}$

d) $-7(x + 1) < 4(x - 3) + 6$

$-7x - 7 < 4x - 12 + 6$

$-7x - 7 + 7 < 4x - 6 + 7$

$-7x < 4x + 1$

$-7x - 4x < 4x + 1 - 4x$

$-11x < 1$

$\frac{-11}{-11}x > -\frac{1}{11}$

$x > -\frac{1}{11} \quad x \in \mathbb{Q}$

Exercise 3.4

1. Solve each of the following inequality in the domain of $\mathbb{Q}$

   a) $2y - 3 < \frac{1}{2}\ (7 - y$

Answer

a. $2y - 3 < \frac{1}{2}(7 - y)$

$= 2y - 3 < \frac{1}{2}(7 - y) \quad = 2y - 3 < \frac{7}{2} - \frac{y}{2} \ = 2y + \frac{y}{2} < \frac{7}{2} + 3 = \frac{4y+y}{2} < \frac{7+6}{2} = \frac{5y}{2} < \frac{13}{2}$

$= \frac{5y}{5} < \frac{13}{5} \ = \quad , y = \frac{\mathbf{13}}{\mathbf{5}}//$

3.4. Applications of Linear Equations and Inequalities

To solve word problems involving linear equations or inequalities:

i) Carefully read the problem and assign a variable to the unknown.

ii) Interpret the word problem with mathematical statement.

iii) Finally, solve the unknown.

i) Applications of Linear Equations

Example:

Translate the following algebraic expressions in different word phrases.

a) x – 5 b) 8x

Solution:

$x - 5$

The difference of a number and five

- Five subtracted from a number

$8x$

A number multiplied by eight.

The product of a number and eight.

- A number decreased by five
- Five less than a number Eight times a number.

Example: The relationship between the temperature readings in Celsius scale C and Fahrenheit scale F is given by C $\frac{5}{9}(FF - 32)$.

a) Express F in terms of C.

b) Using the above relation of C and F, What interval on the

Celsius scale corresponds to the temperature of 50 < F?

Solution:

a) $C = \frac{5}{9}(F - 32)$.

9C = 9 x $\frac{5}{9}(F - 32)$.

9C = 5$(F - 32)$.

$\frac{9}{5}C = F - 32$

$F = \frac{9}{5}C + 32$

b) $F = \frac{9}{5}C + 32$

Since F > 50, $F = \frac{9}{5}C + 32 > 50$

$\frac{9}{5}C > 50 - 32$

$C > 10$

Example:

Three years ago the sum of the ages of a man and his son was 52 years. Now the man is 18 years older than his son. What is the present age of his son?

Solution: Let M = the man's present age and S = the son's present age.

Then, (M–3) + (S – 3) = 52. But M = S + 18

(M–3) + (S – 3) = 52

M + S – 6 = 52

M + S = 58

S + 18 + S = 58

2S = 40

S = 20

Therefore, the present age of his son is **20 years.**

Example: Samuel can do a certain work in 15 days and Saron can do the same work in 10 days. In how many days do they together to finish the work?

Solution:

Let Samuel and Saron together can finish the work in d days.

By the equation, $\frac{d}{15} + \frac{d}{10} = 1$

$d\left(\frac{1}{15} + \frac{1}{10}\right) = 1$

$d\left(\frac{2+3}{30}\right) = 1$

$5d = 30$

$d = 6$

Therefore, they together can finish the work in 6 days.

Exercise 3.5

1. Write the following sentences in a mathematical symbol.

a) 5 less than 7 times a number is 0.

b) The quotient of a number and nine is 2 less than the number.

C) multiply a number by 2 and add 4, the result you get will be 3 times the number decreased by 7

3. In a class there are 42 students. The number of girls is 1.1 times the number of boys. How many boys and girls are there in the class?

4. The sum of the ages of the mother and her daughter is 68 years. The mother is 22 years older than her daughter. How

old is her mother?

Answer

1. a. $7x - 5 = 0$ b. $\frac{x}{9} = x - 2$ c. $2x + 4 = 3x - 7$

$= 7x = 0 + 5$

$= \frac{7x}{7} = \frac{5}{7}$

$x = \frac{5}{7}$//

$x = 9x - 18$

$= x - 9x = -18$

$= \frac{-8x}{-8} = \frac{-18}{-8}$

$= x = \frac{18}{8} = \frac{9}{4}$//

$= 2x - 3x = -7 - 4$

$= \frac{-1x}{-1} = \frac{-11}{-1}$

x = 11

2. $F = Gm\frac{M}{r^2}$

$\frac{Fr^2}{GM} = \frac{GmM}{Gm}$

$M = \frac{Fr^2}{Gm}$//

3. Let "b" stands for boy "g" stands for girl

$= b + g = 42$ & $g = 1.1b$

$= b + 1.1b = 42$

$= \frac{2.1b}{2.1} = \frac{42}{2.1}$, $b = \frac{420}{21} = \underline{\underline{20}}$ and $g = 42 - b$

$= 42 - 20$

$= \underline{\underline{22}}$

$\therefore$ *There fore there are* 20 *boys and* 22 *girls are there in the class.*

4. Let "a" be the age of the mother and 'b' be the age of her daughter. Then $a + b = 68$

$= a = b + 22$ $\frac{2b}{2} = \frac{46}{2}$, $b = 23$

$= b + 22 + b = 68$ $a = b + 22$

$= 2b + 22 = 68$ $a = 23 + 22$

$= 2b = 68 - 22$ $a = 45$ Therefore, the mother is 45 years old.

ii) Application of Linear Inequalities

Note:

The following mathematical interpretations can be used for the phrase:

- ✓ x is at least aa ------------ $x \geq a$
- ✓ x is not less than aa -------- $x \geq a$
- ✓ x is at most aa ---------------- $x \leq a$
- ✓ x is not more than aa ---------- $x \leq a$

Example 3.19:

Find two consecutive even positive integers whose sum is at most 10.

Solution:

Let x be the first even integer. Then the second will be x + 2.

$$x + (x + 2) \leq 10$$
$$2x + 2 \leq 10$$
$$2x \leq 8$$
$$x \leq 4$$

Since x is a positive even integer, x = 2 or x = 4.

Therefore, the numbers are 2 and 4, or 4 and 6.

Example:

Three years ago a father's age exceeded at least four times his son's age. If the father is 47 years old now, then what is the possible present age of the son?

Solution: Let the son's age be S and father's age be F. Then

$$F - 3 > 4(S - 3)$$
$$F - 3 > 4S - 12$$
$$F > 4S - 9$$
$$47 > 4S - 9$$
$$56 > 4S$$
$$S < 14$$

Exercise 3.6:

1: Translate the following expressions involving inequalities.

a) Three times a number decreased by four is at most twenty.

b) The ratio of a number to seven is less than ten.

c) The height of a roof, h, was no more than 6m.

d) The sum of two consecutive integers is smaller than three times the smaller integer.

2. Five times a certain natural number is decreased by two times the number is less than 12. What are the possible values of this number?

3. Imagine that you are taking a ride while on vacation. If the ride service charges Birr 50 to pick you up from the hotel and Birr 10 per km for the trip. What minimum km's you travel if your cost is not more than Birr 1350.

Answer

1. a. $3x - 4 \leq 20$ c. $h \leq 6m$

   b. $\frac{x}{7} < 10$ d. $x + (x + 1) < 3x$

2. be the number , then

5y - 2y < 12

$\frac{3y}{3} < \frac{12}{3}$, y < 4 , y = 3, 2, 1 S.S = {1,2,3}

3. Let 'x' be the total km covered by the trip , then,

50 + 10x $\leq$ 1350

= 10x $\leq 1350 - 50$

= 10x ≤ 1300

= x $\leq 130km//$

Therefore , x = 130 km

REVIEW EXERCISE FOR UNIT 3

1. Write true for correct statement and false for the incorrect one.

a) The graph of the line $y = -2x$ passes through the II and III quadrants.

b) The graph of the equation $x = a, a \in \mathbb{Q}$, $a \neq 0$, if $a > 0$ is a vertical line that lies to the right of the y-axis.

c) A horizontal line has the equation $x - b = 0$.

d) If $a < b$ and $c = -3$, then $a > b\ c$

e) The inequality $8 \geq 5x, x \in$ W has a finite solution set.

2. Plot the following points in a Cartesian coordinate plane:
   $(0, 6), (2, -3), (-4, 5)$ and $(-4, -5)$. Which point lies in neither of the quadrant?

3. Fill in the blank with an appropriate inequality sign.

   a) If x ≥ 3, then $-4x$ ------- -12

   b) If $x + 4 < 2$, then $\frac{1}{2}x$ ------ 2

   c) If $x < -2$, then $1 - x$ ------ 3

4. Determine whether the given points are on the graph of the equation

   $x - 2y - 1 = 0$

   a) $(0, 0)$ b) $(1, 0)$ c) $(-1, -1)$ d) $(2, 1)$

5. Sketch the graph of the following equations.

   a) $y = -7x$

   b) $y - 11x = 0$

   c) $3\ y = -7x + 6$ d) $y - 7 = 0$

6. Find a and b, if the points P(3, 1) and Q(0,2) lie on the graph of ax$-$ by= 6

7. Consider the equations$yy = x + 1$ and $y = 1 - x$.

a) Determine the values of y for each equation when the values of x are -1, 0 and 1.
b) Plot the ordered pairs on the Cartesian coordinate plane.
c) Which point is a common point, called intersection point?

8. Solve the following inequalities and graph the solution set.

a) $3x + 11 \leq 6x + 8$

c) $4-3x \leq -\frac{1}{2}(2 + 8x)$

b) $6-2x > x + 10$.

9. Solve the following inequalities

a) $\frac{x}{7} \geq \frac{3}{14}x - \frac{1}{7}$

b) $2(12 - x) < 3(1 + 12x) + 5$

c) $\frac{1}{2}x + \frac{1}{3}x - x - 1 \geq 0$

d) $\frac{3}{4}y + \frac{1}{2}(y - 3) < \frac{y+1}{4}$

e) $7(y + 1) - y > 2(3y + 4)$

f) $-4(y - 1) + 3y \geq 1 - y$

10. Solve the following inequalities in the given domain.

a)$\frac{2}{3}x < 4(4 - x), x \in \mathbb{Z}+$

b) $\frac{1}{6} - \frac{1}{4}x \geq 2 + \frac{2}{3}x, x \in$ W

c) $2(3y - 7) - 14 \geq 3(2y - 11), x \in \mathbb{Q}$

11. Translate the following sentences in to mathematical expressions.

a) A year ago a father's age exceeded three times his son's age.

b) Three –fourth of a number is greater than 12.

c) The average mark of Saron is not smaller than 89.

12. A board with 2.5 m in length must be cut so that one piece is 30 cm more than the other piece. Find the length

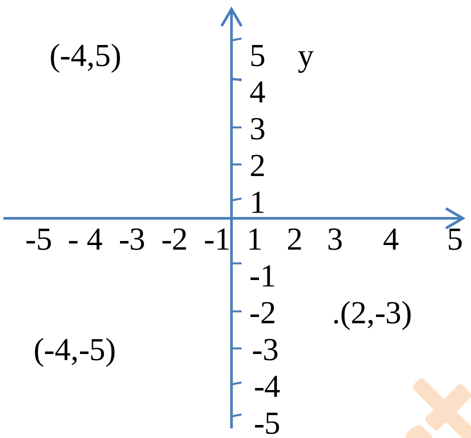
of each piece.

13. Rodas is 25 years old and her brother Mathanya is 10 years old. After how many years will Rodas be exactly twice as old as Mathanya.

Answer

1. a. False b. True c. False d. True e. True

2.



(0,6) does not lie in any quadrant

a. D(0,4) – x-coordinate = 0 , y-coordinate = 4

E(3,-1) – x coordinate = 3 , y-coordinate = -1

F(2,0) - x - coordinate = 2, y-coordinate = 0

b. point A

c. y – coordinate

d. point c III quadrant and point E IV quadrant

3. a. ≤ b. > c. >

4. a. (0,0) – not on the line c. (-1,-1) – on the line

b. (1,0) – on the line d. (2,1) – not on the line

5. a. $\frac{-4x}{-4}$ ________ $\frac{-12}{-4}$ b. x + 4 $< 2x$ c. x< -2

x ≤ 3 $= \frac{-x}{-2} \leq \frac{-4}{-2}$

$= \frac{x}{2} > 2$

6. Given the equation ax- by = 6, subtraction the values of a and y to get two equations in terms of a and b.

The point p (3, 1) in the equation ax – by = 6

a (3) –b(1) = 6

= 3a – b = 6-------------------equ(1)

The point p(0,2) in the equation ax- by = 6

= a(0) - b(2) = 6

= 0 - 2b = 6

= - 2b = 6

b = -3 ----------------- equ (2)

Now substitute the values of b = -3 in the equation

3a- b = 6

= 3a+3 = 6

= 3a= 6-3

a = 1

Therefore, the values of a and b are a = 1, b = -3

7.a. 3x+11 $\leq 6x + 8$

$= 3x - 6x \leq 8 - 11 \ldots\ldots \textit{collect like terms}$

$= \frac{-3x}{-3} \leq \frac{-3}{-3}$

$= x \geq 1$

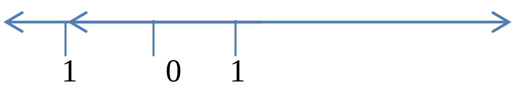
b. $6 - 2x > x + 9$

$= -2x - x > 9 - 6$

$= -3x > 3$

$= \frac{-3x}{-3} > \frac{3}{-3}$

$= x < -1$



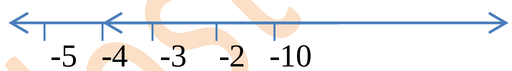
c. $4 - 3x \leq \frac{-1}{2}(2 + 8x)$

$= 4 - 3x \leq -\frac{1}{2}(2) + -\frac{1}{2}(8x)$

$= 4 - 3x \leq -1 - 4x$

$= -3x + 4x \leq -1 - 4$

$= x \leq -5$



8.a. $\frac{x}{7} \geq \frac{3}{14}x - \frac{1}{7}$ – multiply both sides by 14

$14(\frac{x}{7}) \geq 14\left(\frac{3x}{14}\right) - 14(\frac{1}{7})$

$= 2x \geq 3x - 2$

$= 2x - 3x \geq -2$

$= \frac{-x}{-1} \geq \frac{-2}{-1}$

$= x \leq 2$

c. $\frac{1}{2}x + \frac{1}{3}x - x - 1 \geq 0$

$= \frac{3x+2x}{6} - x \geq 1$

$= \frac{5x}{6} - x \geq 1$

$= \frac{5x-6x}{6} \geq 1$

b. $2(\frac{1}{2} - x) < 3\left(1 + \frac{1}{2}x\right) + 5$

$= 2(\frac{1}{2}) - 2(x) < 3(1) + 3\left(\frac{1}{2}x\right) + 5$

$= 1 - 2x < 3 + \frac{3x}{2} + 5$

$= -2x - \frac{3}{2}x < 8 - 1$

$= -2x - \frac{3}{2}x < 7$

$= \frac{-4x-3x}{2} < 7$

$= -7x < 7 \times 2$

$= \frac{-7x}{-7} < \frac{14}{-7}$

$= \underline{\underline{\mathbf{x > -2}}}$

d. $\frac{3}{4}y + \frac{1}{2}(y - 3) < \frac{y+1}{4}$

$= \frac{-1x}{-1} \geq \frac{6}{-1}$

$= \underline{\underline{\mathbf{x \leq -6}}}$

$= \frac{3}{4}y + \frac{y}{2} - \frac{3}{2} < \frac{y+1}{4}$

$= \frac{3}{4}y + \frac{y}{2} - (\frac{y+1}{4}) < \frac{3}{2}$ multiply both sides by 4

$= 4(\frac{3y}{4}) + 4\left(\frac{y}{2}\right) - 4(\frac{y+1}{4}) < 4(\frac{3}{2})$

= 3y + 2y –(y+1)$< 2(3)$

= 3y+2y – y - 1< 6

=5y - y $< 6 + 1$ = $\frac{4y}{y} < \frac{7}{4}$ = $\mathbf{y < \frac{7}{4}}$//

e.7(y+1) - y $> 2(3y + 4)$

= 7y + 7 – y $> 6y + 8$

= 7y – y - 6y $> 8 - 7$

= 6y - 6y > 1

= $0 > 1$ – has no solution

f. – 4(y -1) + 3y $\geq 1 - y$

= - 4y + 4 + 3y $\geq 1 - y$

= - 4y + 3y + 4 $\geq 1 - y$

= - y + y $\geq 1 - 4$

= $0 \geq -3$ – all rational numbers.

9.a. $\frac{2}{3}x < 4(4 - x), x \in z^{+}$

$= \frac{2}{3}x < 16 - 4x$

$= \frac{2}{3}x + 4x < 16$

$= \frac{2x+12x}{3} < 16$

$= \frac{14x}{3} < 16$

= 14x $< 16 \times 3$

$= \frac{14x}{14} < \frac{48}{14}$

= x $< \frac{48}{14}$

= x $< \frac{24}{7}$//

X $<$ 3.42

S.S = {1,2,3}

b. $\frac{1}{6} - \frac{1x}{4} \geq 2 + \frac{2x}{3}$, x $\in w$

$= -\frac{1}{4}x - \frac{2}{3}x \geq 2 - \frac{1}{6}$

$= \frac{-3x-8x}{12} \geq \frac{12-1}{6}$

$= \frac{-11x}{12} \geq \frac{11}{6}$

$= \frac{-66x}{-66} \geq \frac{132}{-66}$

= x $\geq -\frac{66}{33}$ = -2

= x ≤ -2

S.S = {}

c. 2(3y - 7)- 14$\geq 3(2y - 11), x \in Q$

= 6y-14-14 $\geq 6y - 33$

= 6y-28$\geq 6y - 33$

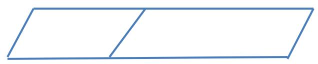
= 6y - 6y $\geq -33 + 28$

= $0 \geq -5$ all rational numbers

that satisfy the inequality

10. a. F- 1 $> 3(5 - 1)$ b. $\frac{3}{4}x > 12$ c. x $\geq$ 89

11.



x y

x + y = 2.5m

y = x + 30cm

Now , x + y = 250cm y = x + 30cm

x + x + 30cm = 250cm y = 40cm+ 30cm

2x = 250cm – 30 cm y = 140cm

$\frac{2x}{2} = \frac{220cm}{2}$, x = 110 cm = 1.1 m y = **1.4 cm**

12. *solution*

Let "m" be the age of mathanya

Let "R" be the age of Radas

M = 10

R = 25

R+ t= 2(m + t)

R+ t = 2m + 2t

25 + t= 20 + 2t 25 - 20 =2t – t 5 = t , after y5 years

UNIT 4

SIMILARIT Y OF FIGURES

4.1. Similar Plane Figures

4.1.1Similar Polygons

Definition: Two polygons are said to be similar if there is a one – to – one correspondence between their vertices such that:

i) All pairs of corresponding angles are congruent

ii) the ratio of the lengths of all pairs of corresponding sides are equal.

In the diagram, ABCD is similar to EFGH.

That is, ABCD ~ EFGH, then

i) $\angle AA \cong \angle EE, \angle BB \cong \angle FF, \angle CC \cong \angle GG, \angle DD \cong \angle HH$

ii) $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EF}$

Example :

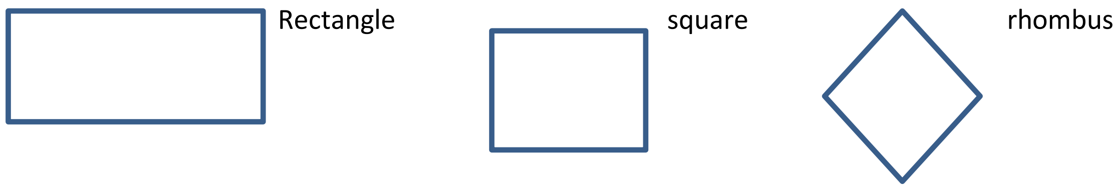
The following pairs of figures are always similar

a) Any two squares.

b) Any two equilateral triangles.

Example:

Consider the following polygons. Which of these are similar?



Solution:

The square and the rectangle are not similar. Because their corresponding angles are congruent but their corresponding sides are not proportional. The square and the rhombus are not similar. Because their corresponding sides are proportional but their corresponding angles are not congruent. The rectangle and the rhombus are not similar. Because their corresponding angles are not congruent and corresponding sides are not proportional.

Definition: The ratio of two corresponding sides of similar polygons is called the scale factor or constant of proportionality (k).

Example:

The sides of a quadrilateral are 2cm; 5cm, 6cm and 8cm. find the sides of a similar quadrilateral whose shortest side is 3cm.

Solution:

Let the corresponding sides of a quadrilateral are 3, x, y, and z. Since the corresponding sides of similar polygons are proportional,

$\frac{2}{3} = \frac{5}{x}$ $\quad$ $\frac{2}{3} = \frac{6}{y}$, $\quad$ $\frac{2}{3} = \frac{8}{z}$

$x = \frac{15}{2}$, $\quad$ $y = 9$, $\quad$ z = 12

Therefore, the sides of the second quadrilateral are 3cm, 7.5cm, 9cm, and 12cm

Exercise: 4.1.

1) Write true if the statement is true or false otherwise.

a) Any geometric figure is similar to itself.

b) Congruent polygons are not necessarily similar.

c) Any two regular polygons that have the same number of sides are similar.

d) Any two parallelograms are similar.

e) Two isosceles triangles are similar.

2) What must be the constant of proportionality of two similar polygons in order for the polygons to be congruent?

3) The sides of a quadrilateral measure 12cm, 9cm, 16cm and 20cm. The longest side of a similar quadrilateral measures 8cm. Find the measure of the remaining sides of this quadrilateral.

Answer

1. a. True b. False c. True d. False e. False
2. when k= 1
3. let x , y and m be the remaining sides of the quadrilateral . then

$\frac{12}{x} = \frac{a}{y} = \frac{16}{m} = \frac{20}{8} = \frac{5}{2}$

$\frac{12}{x} = \frac{5}{2}$ $\frac{9}{y} = \frac{5}{2}$ $\frac{16}{m} = \frac{5}{2}$

$5x = 24$ $5y = 18$ $\frac{5m}{5} = \frac{32}{5}$

$x = \frac{24}{5}$// $y = \frac{18}{5}$// $m = \frac{32}{5}$

4.1.2. Similar Triangles

Definition. Δ ABC is similar to Δ DEF (Δ ABC ~ Δ DEF) if their Corresponding:

I. Sides are proportional.

$$\frac{AB}{DE} = \frac{BC}{FE} = \frac{AC}{DF} = K$$

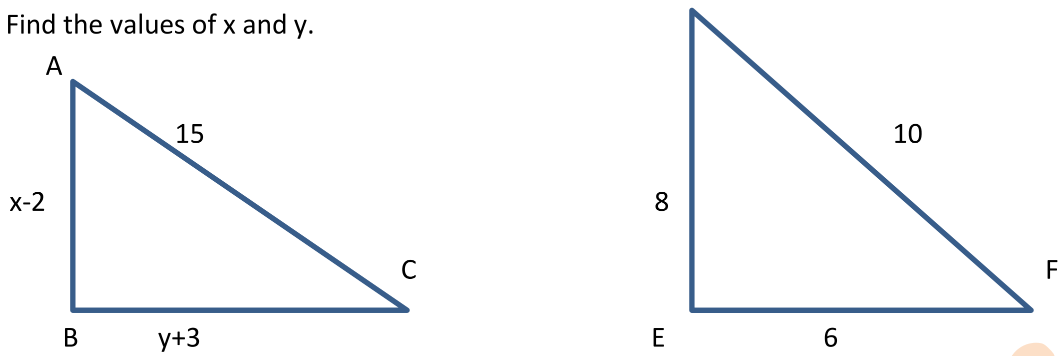
ii) Angles are congruent.

∠A ≅ ∠ D, ∠ B ≅ ∠ E, and ∠ C ≅ ∠ F

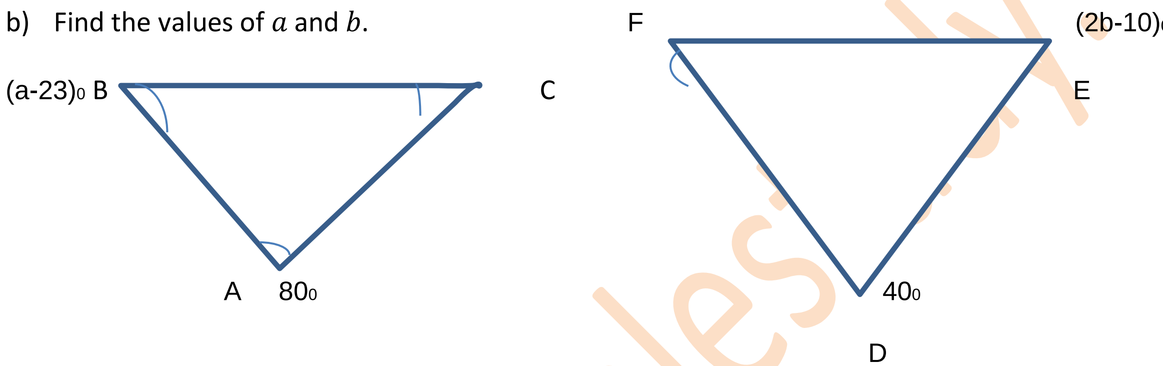
Example:

Suppose Δ ABC ~ Δ DEF. Then D

a) Find the values of x and y.



b) Find the values of a and b.



Solution:

a) Since Δ ABC ~ Δ DEF, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = K$

$\frac{X-2}{6} = \frac{15}{10}$, $\frac{15}{10} = \frac{y+3}{8}$

$x - 2 = 6 \times \frac{3}{2}$ $y + 3 = 8 \times \frac{3}{2}$

$x = 11$ $y = 12$

b) Since Δ ABC ~ Δ DEF, then

$m\angle$ A = $m\angle$ D $m\angle$ B = $m\angle$ E

$(a - 23)^0 = 400$ $80^0 = (2b + 10)^0$

$a = 630$ $b = 350$

Exercise 4.2:

1) Given that Δ ABC ~ Δ DEF. Then find x and y.

a) $\frac{X}{DE} = \frac{AC}{DF}$ c) ∠ B ≅ yy

b) A side corresponds to $\overline{CA}$ is x

Answer

1. a. $\frac{x}{DE} = \frac{AC}{DF}$

   x. DF = AC × DE

   In $\Delta ABC \approx \Delta DEF$

   $\frac{\text{AB}}{\text{DE}} = \frac{AC}{DF}$

   $\underline{\therefore X = AB}$

b. The corresponding sides of $\overline{CA}$ is $\overline{FD}$

$\underline{\boldsymbol{\therefore X = \overline{FD}}}$

c. $< B \approx y$

The corresponding angles of B is $< E$

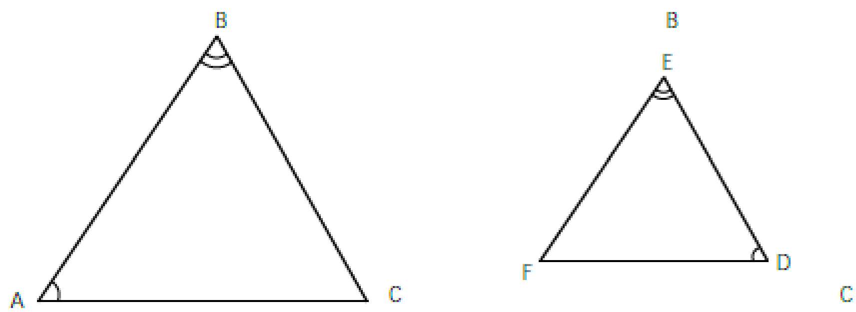
$\underline{\boldsymbol{y = < E}}$

4.1.3. Tests for similarity of triangles [AA, SSS, and SAS]

Theorem. [AA- Similarity Theorem]

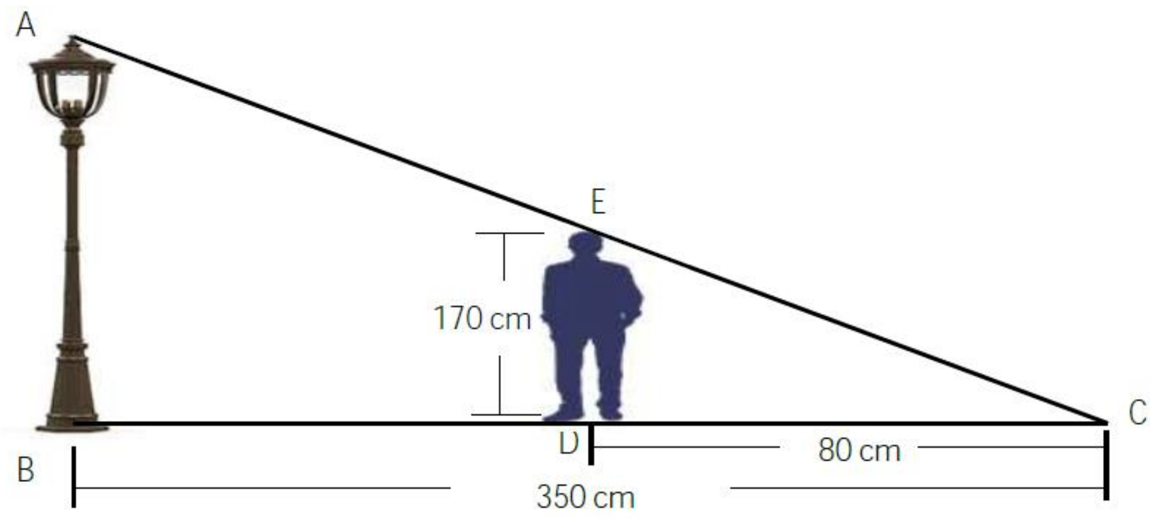
If two angles of one triangle are congruent to the corresponding two angles of another triangle, then the two triangles are similar.

Thus, ∠ A ≅ ∠ D and ∠ B ≅ ∠ E, then Δ ABC ~ Δ DEF



Example:

Mustafa is a 170 cm tall. He is standing 350 cm away from a lamp post and his shadow is 80cm long. How high is the lamp post?



Solution:

Consider Δ ABC and Δ EDC. We can see that ∠ B ≅ ∠ D = 900 and ∠ C ≅ ∠ C (common angle). Hence, by AA similarity,

Δ ABC ~ Δ EDC.

$$\frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$$

$$\frac{h}{DE} = \frac{350}{80}$$

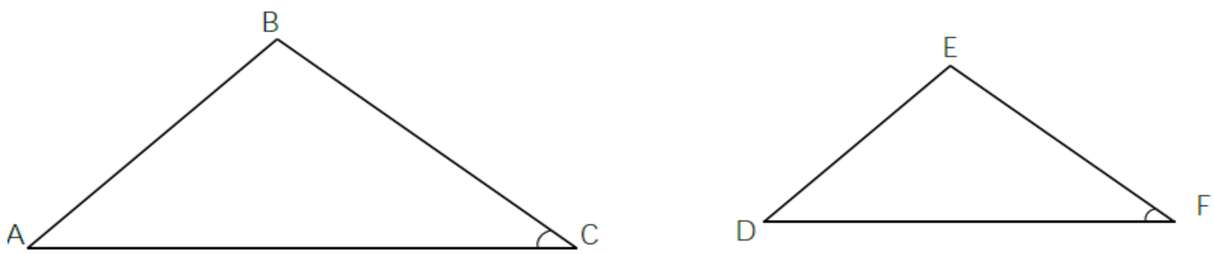
h = 700 cm

Therefore, the height of the lamp post is 700 cm long.

Theorem [SAS similarity theorem]

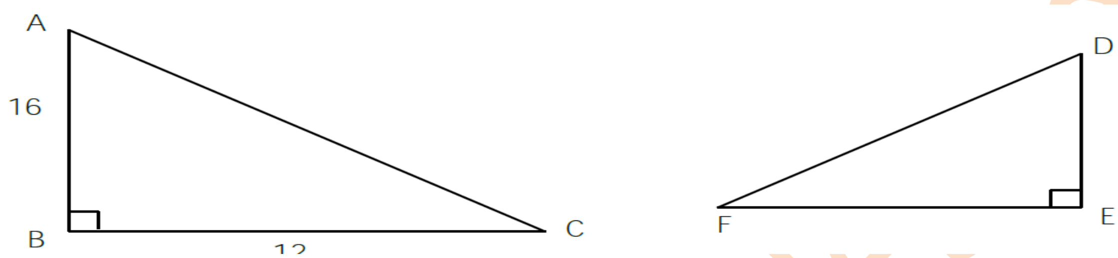
If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are proportional, then the triangles are similar.

Thus, if $\angle C \cong \angle F$ $\frac{AC}{DF}$= $\frac{BC}{EF}$, then Δ ABC ~ Δ DEF.



Example:

Is Δ ABC ~ Δ DEC ? Explain.



Solution:

$\frac{AB}{DE} = \frac{16}{8}$ = 2 and $\frac{BC}{EC} = \frac{12}{6}$ = 2 …… sides are proportional.

$m(\angle B) = m(\angle E) = 90^0$ ….. included angles are congruent.

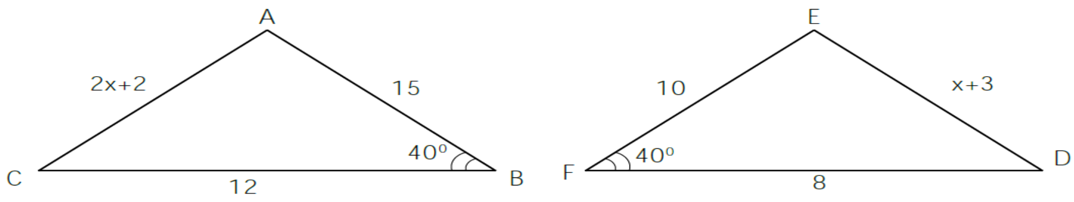
Therefore, by SAS similarity theorem Δ ABC ~ Δ DEC

Example:

In the figure below,

i) Show that Δ ABC ~ Δ EFD.

ii) Find the value of x.



Solution:

i) $\frac{AB}{EF} = \frac{15}{10} = \frac{3}{2}$ and $\frac{BC}{FD} = \frac{12}{8} = \frac{3}{2}$.

$\angle$ B ≅ $\angle$ F …… included angle

Therefore, Δ ABC ~ Δ EFD by SAS similarity theorem.

ii) $\frac{AB}{EF} = \frac{BC}{FD} = \frac{AC}{ED}$

$\frac{2X+2}{X+3} = \frac{3}{2}$

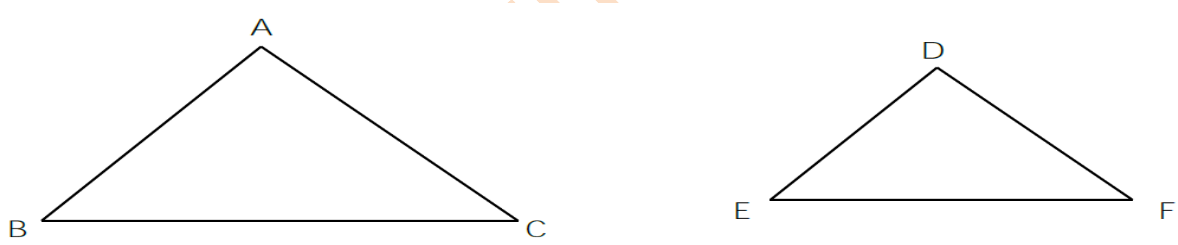
$$2(2X+2) = 3(X+3)$$

$$x = 5$$

Theorem [SSS Similarity Theorem]

If the three sides of one triangle are in proportion to the three sides of another triangle, then the two triangles are similar.

Thus, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, then Δ ABC ~ Δ DEF.



Example :

A man who is 1.80m tall wishes to find the height of a certain four-story building. He measures its shadow and finds it to be 28m long; while his own shadow is 4.2m long. How tall is the building?

Solution:

Let h = the height of the building.

Draw a line from top of a building to the end of its shadow. Similarly draw a parallel line through the head of man and its shadow.

Ratio of height to base in large triangle = $\frac{h}{28}$

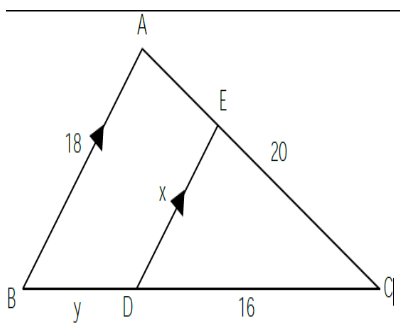
Ratio of height to base in small triangle = $\frac{1.8}{4.2}$

Since the large and small triangles are similar we get the equation

$\frac{h}{28} = \frac{1.8}{4.2}$ Now we solve for h, h = $\frac{1.8 \times 28}{4.2}$

= 12

So the building is 12 m tall.



Exercise 4.3

1. In the figure below, if $AB \| ED$, then

a) Show that Δ ACB ~Δ ECD.

b) Find the values of x and y.

2. Let Δ LMN ~ Δ PQR and the sides of Δ LMN are LM = 12, MN = 15, and LN = 18. If the constant of proportionality is $\frac{2}{3}$,

then find the length of the corresponding sides of Δ PQR.

3. When you shine a flashlight on a book that is 12cm tall and 8cm wide, it makes a shadow on the wall that is 18cm tall and 12cm wide. Then what is the scale factor of the book to its shadow?

5. Δ PQR and Δ LMN are similar such that ∠ P ≅ ∠ L, ∠ Q ≅ ∠ M, LM = 8, MN = 10, PQ = 12, and PR = 18. Find the lengths of LN and QR.

6. The sides of a triangle measures 4cm, 9cm and 11cm. If the shortest side a similar triangle measures 12cm, find the measure of the remaining sides of this triangle.

Answer

1. statements / reasons

statements	reasons
1. $< C \sim < C$	common angle
2. $< B \sim < D$	corresponding angles
3. $\Delta ACB \sim \Delta ECD$	AA similarity theorem.

$b.\ since\ \Delta ACB \sim \Delta ECD$

$\frac{AC}{EC} = \frac{CB}{CD} = \frac{AB}{CD} = \text{K}$

$\frac{AC}{EC} = \frac{CB}{CD}$

$\frac{30}{20} = \frac{y+16}{16}$

y + 16 = 24

y = 24-16

y = 8

$\frac{AC}{EC} = \frac{AB}{ED}$

$\frac{30}{20} = \frac{18}{X}$

$\frac{3X}{3} = \frac{36}{3}$

x = 12

2. $\Delta LMU \sim \Delta PQR$

$\frac{LM}{PQ} = \frac{MN}{QR} = \frac{LN}{PR} = \text{K} = \frac{2}{3}$

$\frac{15}{QR} = \frac{2}{3}$

2QR = 45

QR = $\frac{45}{2}$

QR = 22.5

$\frac{12}{PQ} = \frac{2}{3}$

$\frac{2PQ}{2} = \frac{36}{2}$, **PQ = 18**

$\frac{18}{PR} = \frac{2}{3}$

$\frac{2PR}{2} = \frac{54}{2}$

PR = 27

$\therefore The\ length\ of\ the\ corresponding\ sides\ of\ \Delta PQR\ are\ 18, 22.5\ and\ 27.$

3.K = $\frac{s1}{s2} = \frac{12cm}{18cm} = \frac{2}{3}$

4.b. $\Delta RSQ \sim \Delta RQT$ c. $\frac{SQ}{QT} = \frac{3}{5}$ d. $< RSQ \cong < QST(right\ angle)$

$\frac{RS}{RQ} = \frac{SQ}{QT} = \frac{RQ}{RT}$

$\frac{RQ}{RT} = \frac{15}{25} = \text{k}$, $\text{k} = \frac{15}{25} = \frac{3}{5}$//

$\frac{SQ}{20} = \frac{3}{5}$

$\frac{5SQ}{5} = \frac{60}{5}$

<u>SQ = 12</u>

$\frac{RS}{QS} = \frac{SQ}{ST} = \frac{RQ}{QT}$

$\frac{RS}{QS} = \frac{9}{12} = \frac{3}{4}$

$\frac{SQ}{ST} = \frac{12}{16} = \frac{3}{4}$

Since the two corresponding sides have the same ratio and the included angle is congruent, by SAS.

$< RSQ \sim < QST$

5.$\Delta PQR \sim \Delta LMN$

$\frac{PQ}{LM} = \frac{QR}{MN} = \frac{PR}{LN} = \text{K}$

$\frac{QR}{MN} = \frac{3}{2}$ $\qquad \frac{18}{LN} = \frac{3}{2}$ $\qquad \frac{QR}{10} = \frac{3}{2}$

2QR = 30 $\qquad \frac{3LN}{3} = \frac{36}{3}$

<u>QR = 15</u> **<u>LN = 12</u>**

6. Let the remaining sides of a triangle be x and y. the scale factor of a triangle is $\frac{4}{12} = \frac{1}{3}$. Then

$\frac{9}{x} = \frac{1}{3}$ and $\frac{11}{y} = \frac{1}{3}$

x = 27 y = 33

4. 2. Perimeter and Area of Similar Triangles

Theorem. If the ratios of the corresponding sides of two similar

triangles is k, then the ratio of their perimeters is given by:

$$\frac{P1}{P2} = \frac{S1}{S2} = K$$

Theorem 4.5. If the ratios of the corresponding sides of two similar triangles is k, then the ratio of their areas is given by:

$\frac{A1}{A2} = (\frac{S1}{S2})^2 = K^2$

Example:

The corresponding sides of two similar triangles are 8cm and 6cm. Find the ratio of a) The perimeters of the triangles

c) The areas of the triangles.

Solution:

a)

$\frac{S1}{S2} = \frac{8}{6} = \frac{4}{3}$

b) $\frac{A1}{A2} = (\frac{S1}{S2})^2 = (\frac{4}{3})^2 = \frac{16}{9}$

Example:

The areas of two similar triangles are 64 cm2 and 100cm2. If one side of the smaller triangle is 4cm, then find the corresponding side of the second triangle.

Solution: Let A_1 = 64 cm^2 , A_2 =100 cm^2, and S_1 = 4cm

Then, $\frac{A1}{A2} = (\frac{S1}{S2})^2$

$\frac{64}{100} = (\frac{4}{S2})^2$

$S2^2 = \frac{100\ X16}{64}$

S_2 = 5cm

Example:

The lengths of sides of a triangle are 16cm, 23cm, and 31cm. If the perimeter of a similar triangle is 280cm, find

a) The length of the longest side of the second triangle

b) The ratio of the area of the largest triangle to the smaller.

Solution:

a) Let P_1 be the perimeter of the first triangle. Then

P_1 = 16 + 23 + 31 = 70cm, P_2 = 280cm

$\frac{p1}{p2} = (\frac{s1}{s2})^2$

$\frac{70}{280} = \frac{31}{s2}$

$s2 = \frac{280 \times 31}{70}$

$S2$ =124cm

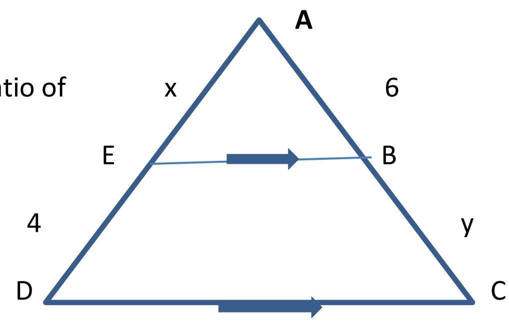
b) $\frac{A2}{A1} = (\frac{P2}{P1})^2 = (\frac{280}{70})^2$ =16, Therefore, $A_2 : A_1$ = 16: 1

Exercise: 4.4

1. In Figure 4.21 given below, find the ratio of

a) Perimeers of Δ ABE to Δ ACD

b) Areas of Δ ABE to Δ ACD



2. The scale factor of Δ ABC to Δ DEF is 4 : 3. The area of Δ ABC is x and the area of Δ DEF is $x-7$. The perimeter of Δ ABC is 8 + y and the perimeter of Δ DEF is $3y-12$.

a) Find the perimeter of Δ DEF.

b) Find the area of Δ ABC.

3. Δ ABC ~ Δ DEF. The length of altitude BP exceeds the length of altitude EQ by 7. If AC = $\frac{5}{4}$ DF, then find the length of each altitude.

Answer

1. a. Since $\Delta ABE \sim \Delta ACD$ *by AAsimilarity* .

$$\frac{AB}{AC} = \frac{BE}{CD} = \frac{AE}{AD}$$

$$\frac{AB}{AC} = \frac{BE}{CD} \qquad\qquad \frac{AE}{AD} = \frac{6}{9}$$

$$\frac{6}{6+y} = \frac{6}{9} \qquad\qquad \frac{X}{X+4} = \frac{6}{9} = \frac{2}{3}$$

36 + 6y = 54 3x = 2x + 8

6y = 54 – 36 3x - 2x = 8

$$\frac{6y}{6} = \frac{18}{6} \qquad\qquad \underline{\mathbf{x = 8}}$$

y = 3

Thus , The perimeter of $\Delta ABE, = AB + BE + AE$

= 6 + 6+ 8 = 20

and perimeter of $\Delta ACD = AC + CD + AD$

= 9 + 9 + 12 = 30

Therefore, $\frac{P(\Delta ABE)}{P(\Delta ACD)} = \frac{20}{30} = \frac{2}{3}$

b. $\frac{a(\Delta ABC)}{a(\Delta ACD)}$ = k^2

$\frac{x}{x-7} = (\frac{4}{3})^2 = \frac{16}{9}$

9x = 16x – 112

9x – 16x = -112

-7x = -

x = 16

$\frac{p(\Delta ABC)}{P(\Delta ACD)}$ = k2

$\frac{8+y}{3y-12} = \frac{4}{3}$

24 + 3y = 12y – 48

3y – 12y = -48- 24

$\frac{-9y}{-9} = \frac{-72}{-9}$

y= 8

a. $p(\Delta DEF) = 3y - 12$

$= 3 \times 8 - 12$

= 24 – 12

= **12 units**

b. $a(\Delta ABC) = X = 16\ square\ unit.$

3.Let ‘x’ be the length of altitude $\overline{EQ}$, Then x + 7 is the length of altitude $\overline{BP}$.

Applying AA similarity theorem gives .

$\frac{x+7}{x} = \frac{AC}{DF}$ 4x + 28 = 5x

$\frac{X+7}{X} = \frac{5}{4}$ 4x – 5x = -28

x = 28

$\therefore\ altitude\ \overline{EQ}$ = 28 Altitude BP = 28 + 7 = 35//

REVIEW EXERCISE FOR UNIT 4

1. Write true if the statement is correct and false if it is not.

a) Two congruent polygons necessarily similar.

b) Two similar polygons are necessarily congruent.

c) Congruence is a similarity when the constant of proportionality is 1.

d) Rectangle ABCD is similar to rectangle EFGH If $\frac{AB}{EF} = \frac{BC}{FG}$.

e) Doubling the side length of a triangle doubles the area.

f) Two triangles with the same area are similar.

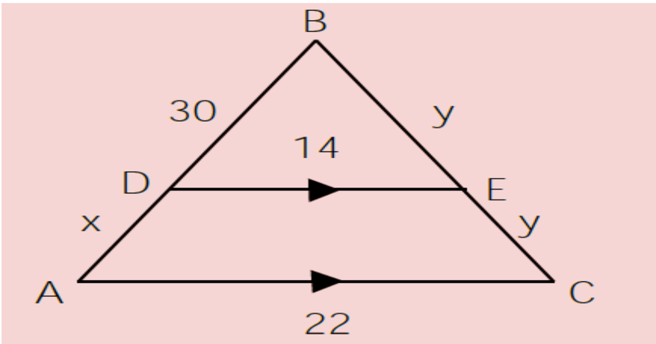
2. Let Δ ABC ~ Δ DEF and the ratio of the respective altitudes is 1:3. If the measures of the sides of Δ ABC are 3cm, 5cm and 7cm, then find the measures of the sides of the larger triangle, Δ DEF.

3. In Δ ABC, the midpoint of $AACC$ is M and the midpoint of $BBBB$ is N.

a) Show that Δ ABC ~ Δ MNC.

b) What is the ratio of the sides of Δ MNC to Δ ABC.

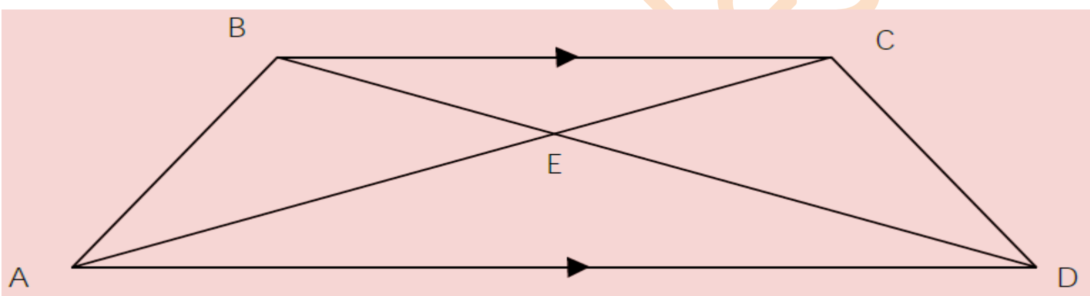
4. In Δ ABC shown below, D E∥ AC. Find the values of x and y



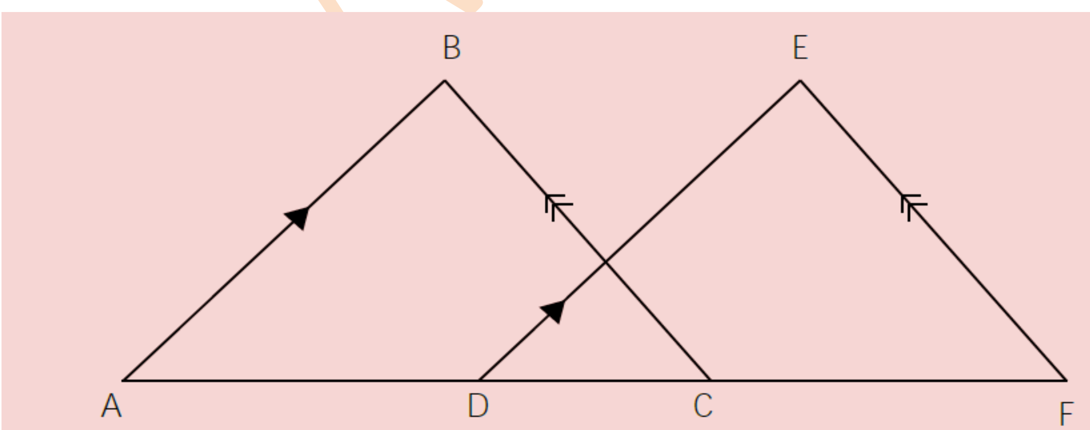
5. The 1.80m tall lady makes a 9m long shadow, and the palm tree makes a 26m long shadow. Find the height of the tree.

6. ABCD is a trapezium with parallel sides BD and AC and its diagonals BD and AC intersect at E. Show that

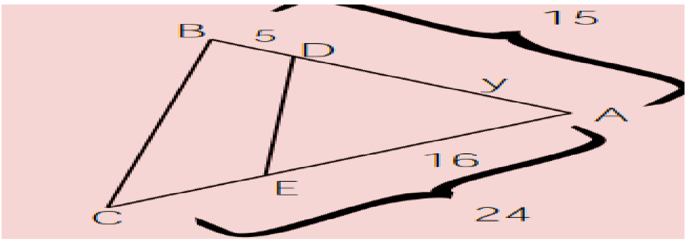
Δ BEC ~ Δ AED.



7. In the figure below, two pairs of angles can be used to prove Δ ABC ~ Δ DEF. Determine the congruent angles.

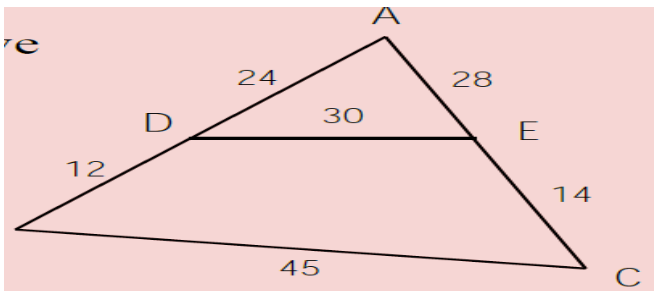


8. What pair of congruent angles and what proportion are needed to prove Δ ADE ~ Δ ABC?

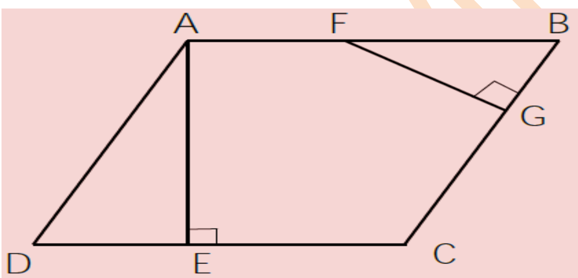


9. Indicate the proportion needed to prove

Δ ADE ~ Δ ABC.



10. What angles can be used to prove Δ AED ~ Δ FGB (ABCD is aparallelogram)



11. The areas of two similar triangles are 64 cm2 and 144cm^2.

a) Find the ratio of their altitudes.

b) If the perimeter of the smaller triangle is 49cm, find the perimeter of the larger triangle.

12. The areas of two similar triangles are 64 cm^2 and 144cm^2.

a) Find the ratio of their altitudes.

b) If the perimeter of the smaller triangle is 49cm, find the perimeter of the larger triangle.

Answer

1. a. True b. False c. True d. True e. False f. False

2. $\Delta ABC \sim \Delta DEF$

$\frac{AB}{DE} = \frac{BC}{CF} = \frac{AC}{DF} = \frac{1}{3}$

$\frac{3}{DE} = \frac{1}{3}$ $\quad$ $\frac{BC}{EF} = \frac{1}{3}$ $\quad$ $\frac{AC}{DF} = \frac{1}{3}$

DE = 9 $\quad$ $\frac{5}{EF} = \frac{1}{3}$ $\quad$ $\frac{7}{DF} = \frac{1}{3}$

EF = 15 $\quad$ **DF = 21**

3. a. $< C \cong C$ ----- Common Angle.

$\frac{BC}{NC} = \frac{AC}{MC}$

$\therefore \Delta ABC \sim \Delta MNC$ *by SAS Similarity theroem.*

4. $\Delta ABC \sim \Delta DBE$ *by AA Similarity theroem.*

$\frac{AB}{DB} = \frac{BC}{BE} = \frac{AC}{DE}$ $\quad$ $\frac{y+10}{10} = \frac{11}{6}$

$\frac{X+12}{12} = \frac{22}{12} = \frac{11}{6}$ $\quad$ 6y + 60 = 110

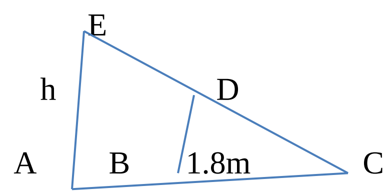
6x + 72 = 132 $\quad$ 6y = 110 – 60

6x = 132 – 72 $\quad$ 6y = 50

6x = 60 $\quad$ y = $\frac{50}{6}$

x = 10 $\quad$ **$y \approx 8.33$**

5. $\Delta ACE \sim \Delta BCD$ *by AA Similarity theroem.*



$\frac{AC}{BC} = \frac{AE}{BD}$

$\frac{26}{9} = \frac{h}{1.8}$ $\quad$ $\frac{9h}{9} = \frac{26 \times 1.8}{9}$ $\quad$, h = $\frac{46.8}{9}$ $\quad$ **h = 5.2**

$\therefore$ *The height of the tree is* 5.2 *meter.*

6. $< BEC \cong < DEA$ ----- Vertical Angles.

$< CBD \cong < ADB --- Alternative\ interior\ angles.$

$\Delta BEC \sim \Delta DEA\ by\ AA\ Similarity\ theroem.$

7. $< A \cong < D$ ⎫ corresponding angles.
$< C \cong < F$ ⎭

8. $< A \cong < A ----Common\ Angle$, $\frac{AD}{AB} = \frac{AE}{AC}$ by SAS Similarity theorem $\Delta ADE \sim \Delta ABC$.

9. $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

10. $< E \cong < C ------ right\ angles.$

$< F \cong < F --- Common\ angle.$

$\frac{FE}{FR} = \frac{FD}{FS} ----Proportional\ Sides.$

$< FED \sim \Delta FRS\ by\ SAS$.

$< B \cong < D ----Opposite\ angles\ of\ a\ parallelogram\ are\ congrunt. by\ AA$ Similarity theorem.

$\Delta AED \sim \Delta\ FGB.$

11. a. $\frac{A1}{A2}\ (\frac{S1}{S2})^2$

$= \frac{64}{144} = (\frac{S1}{S2})^2$

$\frac{S1}{S2} = \sqrt{\frac{64}{144}} = \frac{8}{12} = \frac{2}{3}$ //

$\frac{h1}{h2} = \frac{s1}{s2} = \frac{2}{3}$

b. $\frac{p1}{p2} = \frac{s1}{s2} = \frac{2}{3}$

$\frac{49cm}{p2} = \frac{2}{3}$

$\frac{2p1}{2} = 3 \times 4\ cm$

p2 = **73.5 cm**

12. $\Delta OLP \sim \Delta OMN$

$\frac{OL}{OM} = \frac{LP}{MN} = \frac{OP}{ON}$

$\frac{12}{6} = \frac{5X+3}{4}$

$2 = \frac{5X+3}{4}$

5x + 3 = 8

5x = 8 – 3

x = 1

OP = 5x + 3

= 5(1) + 3

= 5 + 3

= 8

UNIT 5

THEOREMS ON TRIANGLES

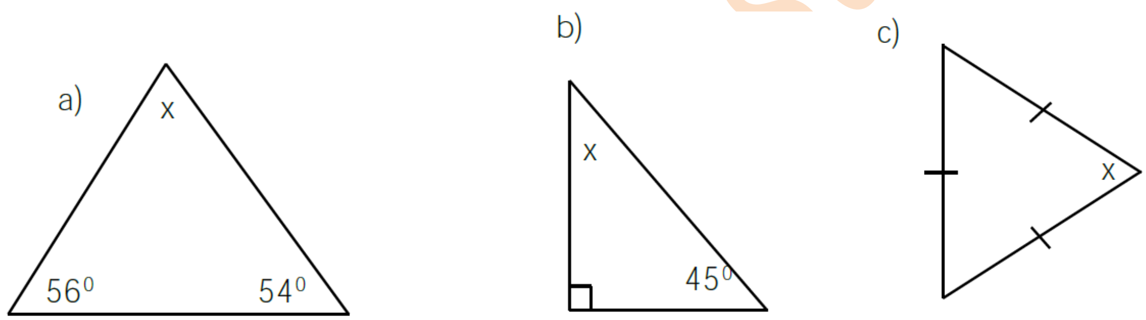
5.1 The three angles of a triangle add up to 180°

Theorem: (Angle- Sum Theorem)

The sum of the degree measures of the interior angles of a triangle is equal to 180°.

Example :

Find the values of x for each of the following triangles.



Solution:

a. $x + 56° + 54° = 180°$(Angle Sum Theorem)

$x + 110° = 180°$

$x + 110° - 110° = 180° - 110°$

$x = 70°$

b. Since the triangle is an isosceles right angle triangle

$x + 90° + 45° = 180°$

$x + 135° = 180°$ $\qquad$ $x = 45°$

c. Since the triangle is an equilateral triangle

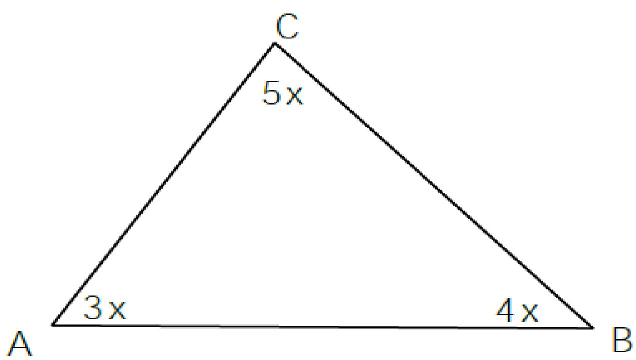
$x + x + x = 180°$

$3x = 180°$

$\frac{3x}{3} = \frac{180}{3}$ $x = 60°$

Example:

If the measure of the angles of a triangle are $3xx$, $4xx$,and $5xx$, then find the measure of each angle.



Solution:

Consider ΔABC with interior angle measures $3x$, $4x$, and $5x$

Then, $mm\angle$ABC + $mm\angle$BCA + $mm\angle CC$AB = 1800 (Angle Sum Theorem)

$4x + 5x + 3x = 1800$

$12x = 1800$

$x = 150$

Hence, $m\angle$ABC = $4x$ = 600

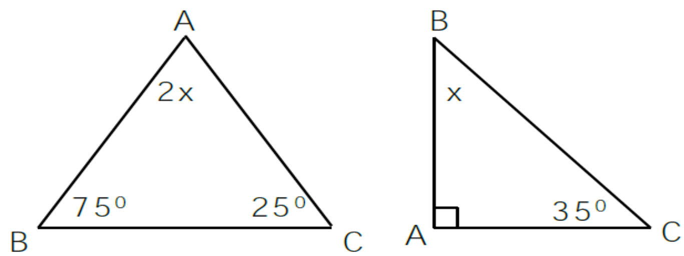
$m\angle$BCA = $5x$ = 750

$m\angle$CAB = $3x$ = 450

Therefore, the angles are 45^0, 60^0, and 75^0

Exercise 5.1

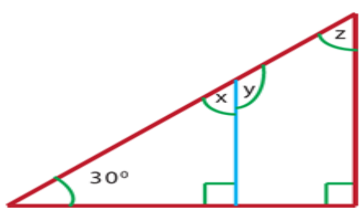
1. Find the values of $x$for each of the following triangles.



2. The measures of the three interior angles of a triangle are in the ratio 2: 3: 4.What is the measure of the largest interior angle of a triangle?

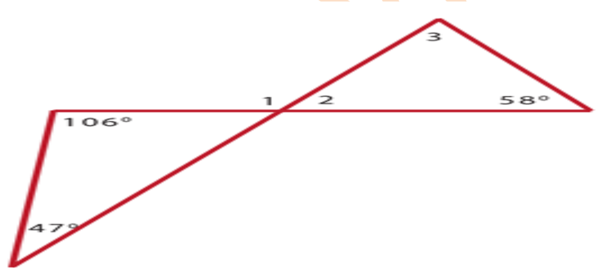
3. In a right angled triangle, the measure of one acute angle of a triangle is two times the measure of the other acute angle. Find the measure of each acute angle.

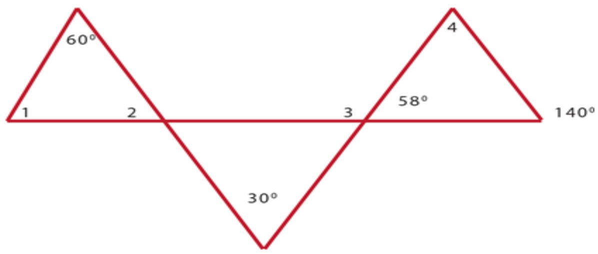
4. Determine the values of x, y and   in the figure below



5. Find the measures of the missing angles.

a)





Answer

1. a. $m(< ABC) + m(< BCA) + m(< CAB) = 180^0$

$2x + 75^0 + 25^0 = 180^0$

$2x + 100^0 = 180^0$

$2x = 80^0$

$\mathbf{x = 40^0}$

b. $m(< ABC) + m(< BCA) + m(< CAB) = 180^0$

$90^0 + x + 35^0 = 180^0$

$x + 125^0$

$\mathbf{x = 55^0}$

2. $2x + 3x + 4x = 180^0$

$9x = 180^0$

$\mathbf{x = 20^0}$, Therefore the largest interior angles of a triangle is $4x = 4 \times 20^0 = \mathbf{80^0}$

3. Let x = the 1st a cute angle

y = the 2nd acute angle so, $90^0 + x + y = 180^0$

$= x + y = 180^0 - 90^0$

$= \mathbf{x + y = 90^0}$

$x = 2y + 6^0$

$y + 2y + 6^0$

$3y = 90^0 - 6^0$

$3y = 84^0$

$y = 28^0$ and $x = 2 \times 28^0 + 6^0$

$\underline{\underline{x = 62^0}}$

$\therefore$ *The two acute angles are* 28^0 *and* 62^0.

4. $x = 60^0$, $y = 120^0$ *and* $z = 60^0$

5. $m(< 1) = 153^0$, $m(< 2) = 27^0$, *and* $m(< 3) = 95^0$

5.2. The exterior angle of a triangle equals to the sum of two remote interior angles.

Theorem : The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example:

Given that for a triangle, the two interior angles 25° and $(x + 15)°$ are remote to an exterior angle $(3x - 10)°$. Find the value of x.

Solution:

Apply the triangle exterior angle theorem

$(3x - 10)° = (25°) + (x + 15)°$,

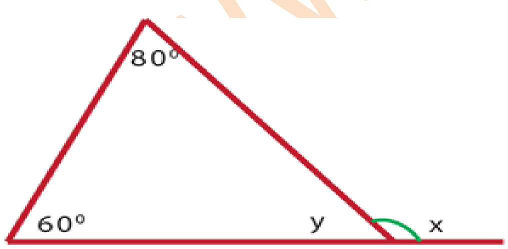
$(3x - 10)° = x + 25° + 15°$

$(3x - 10)° = x + 40°$

$2x = 50°$ $x = 25°$

Example:

Calculate values of x and y in the following triangle.



Solution:

It is clear from the figure that y is an interior angle and x is an exterior angle

So, by triangle exterior angle theorem

x = 80° + 60°

x = 140° and

$y + x$ = 180°

y + 140° = 180°

y = 40°

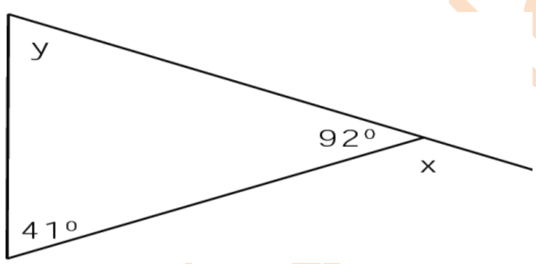
Therefore, the values of x and y are respectively 140° and 40°.

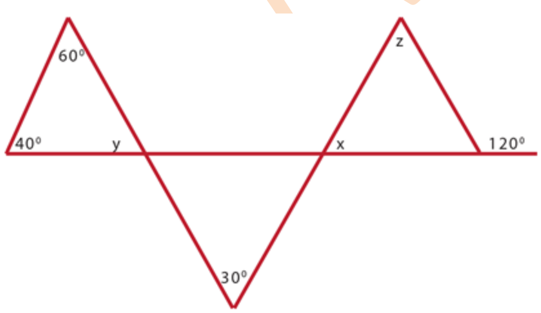
Exercise 5.2

1. The exterior angle of a triangle is120°. Find the value of x if the interior remote angles are $(4x + 40)°$ and 60°.

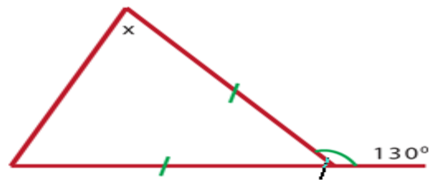
2. Calculate the values of x, and in the following triangle.

a.





c.



Answer

1. $120^0 = 4x + 40 + 60$

   $= 120^0 = 4x + 100$

   $=120 - 100$

   $= 20 = 4x$

   $\underline{\underline{5 = x}}$

2. a. $x + 92^0 = 180^0$

   $x = 180^0$

   $x = 180^0 - 92^0$

   $\underline{\underline{x = 88^0}}$

   $x = y + 41^0$

   $= 88^0 = y + 41^0$

   $= 88^0 - 41^0 = y$

   $\underline{\underline{47^0 = y}}$

b. $x = 70^0$ $y = 65^0$ $z = 50^0$

c. $x = 65^0$

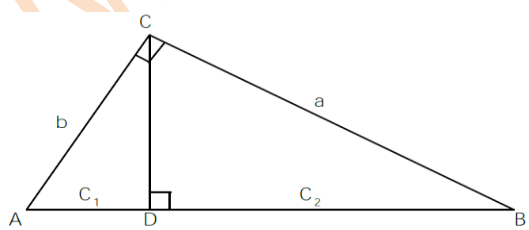
5.3. Theorems on the right angled triangle

5.3.1. Euclid's Theorem and its Converse

Theorem (Euclid's Theorem)

In a right angled triangle with an altitude to the hypotenuse, the square of the length of each leg of a triangle is equal to the product of then hypotenuse and the length of the adjacent segment into which the altitude divides the hypotenuse.

Symbolically



1. $(Bc)^2 = AB \times BD$

or $a^2 = c \times c_2$

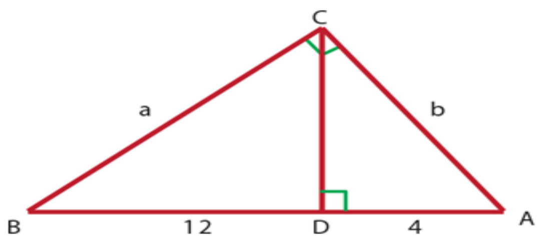
2. $(AC)^2 = AB \times AD$

Or $^{2} = c \times c_1$

Example:

In Figure 5.20 to the right, ΔABC a right angled triangle with $\overline{C}$D the altitude to the hypotenuse

Determine the lengths of AC and BC if AD = 4cm and DB = 12cm.



Solution:

i. $(AC)^2 = AB \times AD$……. Euclid's Theorem

$(AC)^2 = 16cm \times 4cm$….. Since AB = AD + BD

$(AC)^2 = 64cm^2$

AC = $\sqrt{64cm2}$

AC = 8cm

ii. $(BC)^2 = AB \times BD$……. Euclid's Theorem

$(BC)^2 = 16cm \times 12cm$

$(BC)^2 = 192cm^2$

$BC = \sqrt{192cm2}$

$BC = 8\sqrt{3}cm$

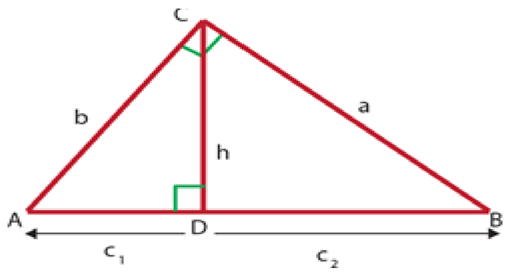
Theorem 5.4 (converse of Euclid's Theorem)

In a triangle, if square of each shorter side of a triangle is equal to the product of the length of the longest side of the triangle and the adjacent segment into which the altitude to the longest sides divides this side, then the triangle is right angled.

Symbolically

1. $a^2 = c \times c_2$ and

2. $b_2 = c \times c_1$ then, ΔABC is right angled triangle.

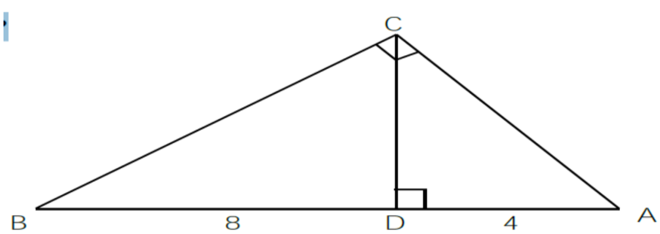


Example :

In figure 5.22 to the right

AD = 4cm, BD = 8cm   AC = 4√3cm and BC = 4√6cm and $\angle A$= 90°

Is ΔABC a right angled?



Solution:

i. $(AC)^2$? AB × AD

$(4\sqrt{3}cm)^2$? 12cm × 4cm Since AB = AD + BD

$48cm^2 = 48cm^2$

Hence $b^2 = c \times c_2$

ii. $(BC)^2$? AB × BD

$(4\sqrt{6}cm)^2$? 12cm × 8cm

$96cm^2 = 96cm^2$

Hence $a^2 = c \times c_1$

Therefore, from (i) and (ii), ΔABC is a right angled triangle.

Exercise 5.3

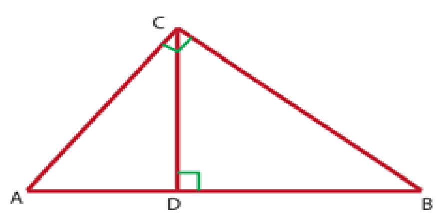
1. In Figure 5.23 to the right if

DB = 8cm and AD = 4cm

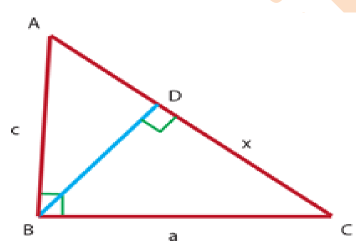
Then find the lengths of

a. AB b. BC

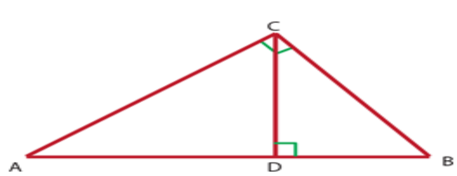
c. AC d. DC



2. In Figure, find the length of x, aa and c If AD = 4cm



3. How long is the altitude of an equilateral triangle if a side of a triangle is a. 4cm? b. 8cm?



4. In figure above , AD = 3.2cm,

DB = 1.8cm, AC= 4cm and

BC = 3cm. Is ΔABC is a right angled?

Answer

1

a. AB = AD + BD = 4cm + 8cm = 12cm

b. $(BC)^2 = \text{BD} \times AB --- Euxlid'theorem.$

$(BC)^2 = 8\text{cm} \times 12cm$

$BC)^2 = 96cm^2$

$BC = \sqrt{96\ cm^2}$

$BC =$ **$4\sqrt{6}$ cm**

c. $(\text{AC})^2 = \text{AD} \times AB --- Euclids's\ theroem.$

$= 4\text{cm} \times 12cm$

$= 48\text{cm}^2$

$\text{AC} = 4\sqrt{3}$ cm

$d.$ $\text{CD}^2 + \text{AD}^2 = \text{AC}^2$

$= CD^2 + 16\text{cm}^2 = 48\text{cm}^2$

$= CD^2 = 48\text{ cm}^2 – 16\text{cm}^2$

$= CD^2 = 32\text{cm}^2$

$= CD = 4\sqrt{2}$ cm

2. $\text{AD}^2 + \text{DB}^2 = \text{AB}^2$

$= (4cm)^2 + (8cm)^2 = \text{AB}^2$

$= 16\text{cm}^2 + 64\text{cm}^2 = \text{AB}^2$

$= 80\text{cm}^2 = \text{AB}^2$

C = AB = $\sqrt{80cm^2} = \sqrt{16 \times 5\ cm^2}$ = **$4\sqrt{5}$ cm**

$(\text{BC})^2 = \text{AC} \times DC$

$= (\text{BC})^2 = \frac{20cm \times 16cm}{\sqrt{64 \times 5cm^2}} = 8\sqrt{5}$ cm = a

$(\text{AB})^2 = \text{AD} \times AC$

$(4\sqrt{5})^2 = 4\text{cm}(\text{x} + \text{k cm})^2 = 4\text{x cm} + 16\text{cm}^2$

$80\text{cm}^2 – 16\text{ cm}^2 = 4\text{xcm} + 16\text{cm}^2$

$\frac{64cm^2}{4cm} = \frac{4xcm}{4cm}$

3. a. $\text{h} = \frac{a\sqrt{3}}{2}$, if a = 4cm

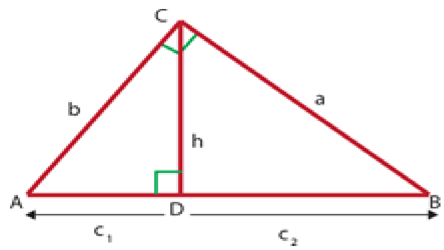
$\text{h} = \frac{4cm\sqrt{3}}{2} = 2\sqrt{3}$ cm x = **16cm** b. if a = 8cm, then h = $\frac{8\sqrt{3}}{2}$ cm = **$4\sqrt{3}$ cm**

5.3.2. The Pythagoras' theorem and its converse

Theorem 5.5 (Pythagoras' Theorem)

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

That is, if legs of lengths aa and bb, hypotenuse of length c, then $\boldsymbol{a^2 + b^2 = c^2}$



Example:

What is the length of the hypotenuse of a right angle triangle with leg lengths are 3cm and 4cm.

Solution:

Let a = 3cm, b = 4cm and c be the length of the hypotenuse. Then

$a^2 + b^2 = c^2$ [Pythagorean Theorem]

$(3cm)^2 + (4cm)^2 = c^2$

$9cm^2 + 16cm^2 = c^2$

$25cm^2 = c^2$

$c^2 = \sqrt{25cm2}$

c = 5cm

Therefore, the hypotenuse is 5cm long.

Example:

Consider a right angle triangle. The measure of its hypotenuse is 13cm. One of the sides of a triangle is 5cm.Find measure of the third sides using the Pythagoras theorem formula?

Solution:

Let the unknown leg be b. Then

$(H\text{hypotenuse})2 = (leg_1)^2 + (leg_2)^2$

$(13cm)^2 = b^2 + (5cm)^2$

$169cm^2 = b^2 + 25cm^2$

$b^2 = 169cm^2 - 25cm^2$

$b^2 = 144cm^2$

$b = \sqrt{144cm}^2$

$b = 12cm$ Therefore, the base right angled triangle is 12cm.

Example 5.10:

Find the length of the leg of the given right angle triangle

Solution:

Let x be the length of the required leg.

Then $(\text{hypotenuse})2 = (leg_1)^2 + (leg_2)^2$

$(10cm)^2 = (8cm)^2 + (x)^2$

$100cm^2 = 64cm^2 + x^2$

$x^2 = 100cm^2 - 64cm^2$

$x^2 = 36cm^2$

$x = \sqrt{36cm}^2$

$x = 6cm$ Therefore, the other leg is $6cm$ long

Note:

When three positive integers can be the length sides of a right triangle, this set of numbers is called Pythagoras triple. The most common Pythagoras triples is 3, 4, and 5. If we multiply each number of a Pythagorean triples by some positive integer xx, then new triples created is also a Pythagorean triples because it satisfy the relation

$a^2 + b^2 = c^2$.

In general patterns of $3x$, $4x$, $5x$ for x is a positive integer forms a Pythagorean triples. There are many other families of Pythagorean triples including $5x$, $12x$, $13x$ or $8x$, $15x$, $17x$ where x is a positive integer.

Theorem (Converse of Pythagoras' Theorem)

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right angled triangle.

Example:

Determine whether the triangle whose sides have lengths 11, 60, and

61 is a right triangle.

Solution:

We want to show that $a^2 + b^2 = c^2$.

$(11)^2 + (60)^2 ? (61)^2$

121 + 3600? 3721

3721 = 3721

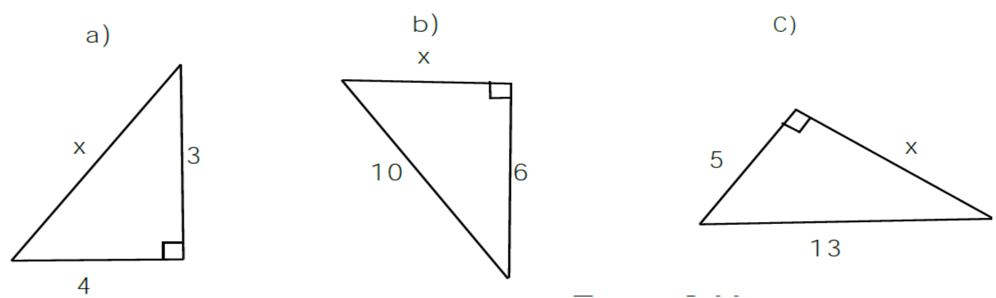
Hence, the triangle is a right triangle.

Exercise 5.4

1. State the Pythagoras theorem in your own words.

2. Find the lengths of the sides of a rhombus whose diagonals are 6cm and 8cm.

3. Find the unknown sides length of each of the following figures



4. What is the length of the hypotenuse of a right angle triangle with leg lengths of 27cm and 36cm?

5. If diagonal of a square is 12cm long, then how long is each sides of a square?

6. A 15m ladder lean against the side of the house and the base of the ladder is 9m away. How high above the ground is the top of the ladder?

7. A man travels 24km due west and then 10km due north. How far is the man now from the starting place?

8. In the figure to the right

i) Find the length of

a. AE

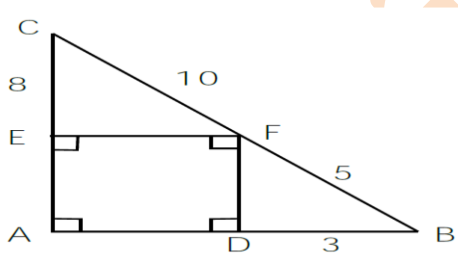
b. DF

c. EF

d. AB

ii) Is ΔCEF is a right- angled?

iii) Is ΔADF is a right- angled?



Answer

1. The length sides of rhombus are 5cm

$S^2 = (\frac{6}{2})^2 + (\frac{8}{2})^2$

$S^2 = 3^2 + 4^2$

$S^2 = 9 + 16$

$S^2 = 25$

$S = \sqrt{25}$

$\underline{S = 5cm}$

2. a. $x^2 = 4^2 + 3^2$

$x^2 = 25$

$\underline{x = 5}$

b. $x^2 + 6^2 = 10^2$

$x^2 + 36 = 100$

$x^2 = 64$

$x = \sqrt{64}$

$\underline{\mathbf{x = 8}}$

c. $x^2 + 5^2 = 13^2$

$x^2 + 25 = 169$

$x^2 = 169 - 25$

$x^2 = 144$

$x = \sqrt{144}$

$\underline{\mathbf{x = 12}}$

3. $C^2 = 27cm^2 + 36^2$

$C^2 = 729cm^2 + 1296cm^2$

$C^2 = 2025cm^2$

$C = \underline{\mathbf{45cm}}$

4. Let "x" be each length side of the square

$x^2 + x^2 = (12cm)^2$

$2x^2 = 144cm^2$

$x^2 = 72cm^2$

$x = \sqrt{72cm^2}$

$x = \sqrt{36 \times 2\ cm^2}$

$x = \underline{\mathbf{6\sqrt{2}\ cm}}$

5. $h^2 + (9m)^2 = (15m)^2$

$h^2 + 81m^2 = 225m^2$ $h^2 = 225m^2 – 81m^2$ $h^2 = 144m^2$ $h = \sqrt{144m^2}$ $h = \underline{\mathbf{12m}}$

$\therefore$ *The height of the top of the ladder is* $\mathbf{12m}$.

6. $(24km)^2 + (10km)^2 = c^2$

$576km^2 + 100km^2 = c^2$

$676km^2 = c^2$

$C = \sqrt{676km^2}$

$C = \underline{\mathbf{26km}}$

$\therefore$ *The distance between the man and starting point is 26 km.*

7. a. $DF^2 + BD^2 = BF^2$

$DF^2 + 9 = 25$

$DF^2 = 25 - 9 = 16$

$DF = \sqrt{16} = \underline{\mathbf{4cm}}$

b. $\overline{AE} = 4cm$

c. $\overline{EF} = 10^2 - 8^2$

$= 100 – 64$

$= 36$

$\overline{EF} = \sqrt{36} = 6cm$

d. $\overline{AB} = \overline{AD} + \overline{DB}$

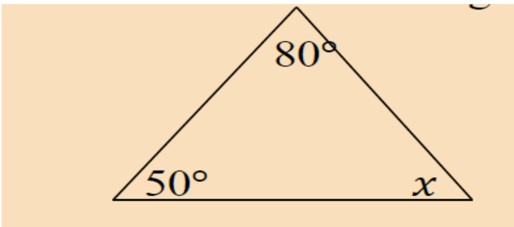
$= 6cm + 3cm = \underline{\mathbf{9cm}}$

REVIEW EXERCISE FOR UNIT 5

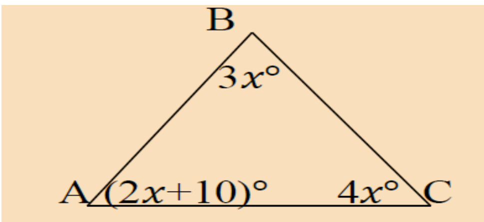
1. If the measure of the angles of a triangle are $6x$, $8x$ and $10x$, then give the measures of each angle.

2. One acute angle of a right triangle measure 37°. Find the measures of the other acute angle?

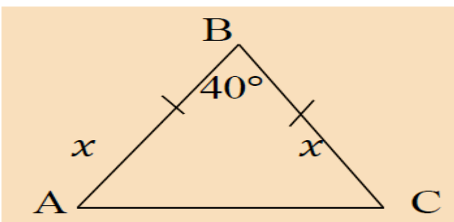
3. What is the value of x in figure below?



4. What is the measure of angle B in the figure below?

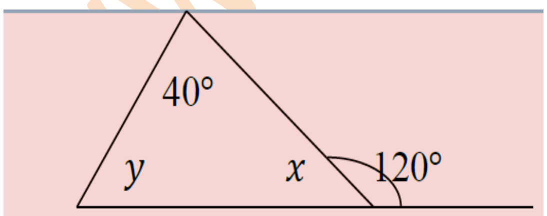


5. Calculate the measure of angle A in the figure below



6. Given that for a triangle the two interior angles $(x + 10)°$ and $(x + 20)°$ are remote to an exterior angle $(6x - 30)°$, then find the value of x?

7. Calculate the values of x and y in the following triangle.

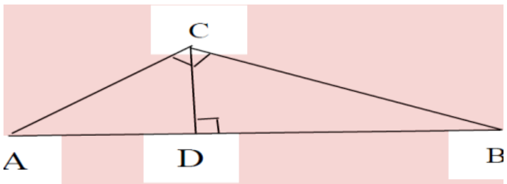


8. In figure 5.42 to the right if DB = 8cm and AD = 4cm

Then find the lengths of

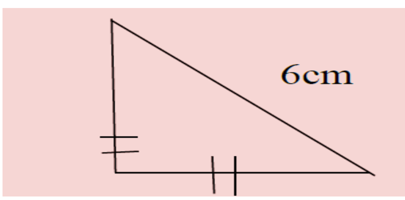
a. AB b. BC

c. AC d. DC



9. A triangle has sides of lengths 16cm, 48cm and 50cm respectively. Is the triangle a right angled triangle?

10. In figure 5.43 below. What is the value of x?



11. A tree 18 meter-high is broken off 3 meter from the ground. How far from the foot of the tree will the top strike the ground?

12. A rectangle has its sides 5cm and 12cm long. What is the length of its diagonal?

13. What is the length of the hypotenuse of a right angle triangle with leg lengths are 9cm and 12cm?

Answer

1. The measure of a triangle is 45^0, 60^0 and 75^0
2. $37^0 + 90^0 + x = 180^0$

   $127^0 + x = 180^0$

   $x = 180^0 - 127^0$

   $\underline{x = \mathbf{53^0}}$

3 . $50^0 + 80^0 + x = 180^0$

130 + x = 180^0

x = $180^0 - 130^0$

4. 2x + 10 + 4x + 3x = 180^0

9x + 10 = 180^0

9x = 170^0

x = **$\underline{\underline{50^0}}$** x = $\underline{\underline{18.89^0}}$

$\therefore$ *The measure of angle* $< B$ *is* 56.667^0

5.ΔABC *is an isoscles triangle , an angle which are opposite to the congrunt sides are congruent.*

$m(<A) + m(<B) + m(<C) = 180^0$

$m(<A) + m(<B) + m(<A) = 180^0$

$2m(<A) + m(<B) = 180^0 - 140^0$

$2m(<A) = 140^0$

$m(<A) = \mathbf{70^0}$

6. x + 10 + x + 20 = 6x – 30

2x +30 = 6x -30

2x - 6x = -30 – 30

- 4x = - 60

x = 15

7. $x + 120^0 = 180^0$

$x = 180^0 - 120^0$

$x = 60^0$

$y = 90^0 = 120^0$

$y = 120^0 - 40^0$ $y = 80^0$

8. a. AB = AD + BD

= 8cm + 4cm

= **$\underline{\underline{12cm}}$**

b. $(BC)^2 = BD \times AB$

$= 8 \times 12$

= **$\underline{\underline{96cm^2}}$**

c. $(AC)^2 = AD \times AB$

$= 4 \times 12$

= $\underline{\underline{48}}$

$AC = \sqrt{48cm^2} = 4\sqrt{3}$ cm

D. $(DC)^2 + (DB)^2 = (BC)^2$

$DC^2 = 96 - 64$

$DC^2 = 32$

$DC = \sqrt{32cm^2}$ = **$\underline{\underline{4\sqrt{2}\text{ cm}}}$**

9. $16^2 + 48^2 = 50^2$

256 + 2304 = 2500

$2560 \neq 2500$

$\therefore$ *It is not right angle triangle.*

10. $x^2 + x^2 = 36$

$2x^2 = 36$

$x^2 = 18$

x = $\sqrt{18}$ = $\sqrt{9 \times 2}$ = **$\underline{3\sqrt{2}\text{ cm}}$**

6. Let x be the length from the foot to the top of tree strike the ground

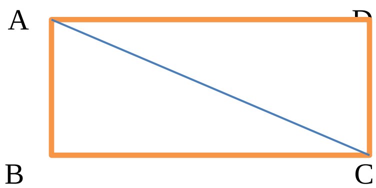
$x^2 = (15)^2 - (3)^2$

$x^2 = 225 - 9$

$x^2 = 216$

x = $\sqrt{216}$ = $\sqrt{36 \times 6}$ x = **$\underline{6\sqrt{6}\text{ m}}$**

7. ABCD be a rectangular with diagonal AC



ΔABC *is a right angled triangle , then*

$AC^2 = AB^2 + BC^2$

$= 5^2 + 12^2$

$= 25 + 144$

$= 169$

AC = $\sqrt{169}$ = **$\underline{13}$**

8. $C^2 = 9^2 + 12^2$

$C^2 = 81 + 144$

$C^2 = 225$

C = $\sqrt{225}$ = **15//**

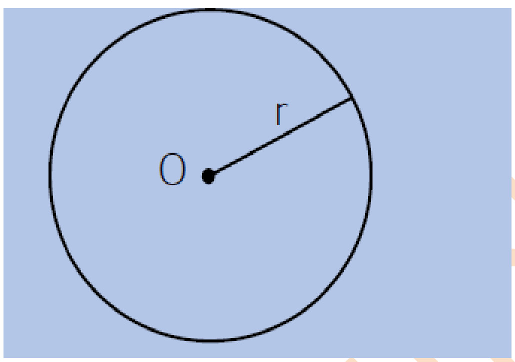
UNIT 6

CIRCLES

6.1. Lines and circles

Definition. A circle is a set of points in a plane having the same distance from a fixed point.

- The fixed point, O is called the center of the circle.
- The distance between the center of the circle and any point of the circle, r is called the radius (plural: radii) of the circle.



Definition

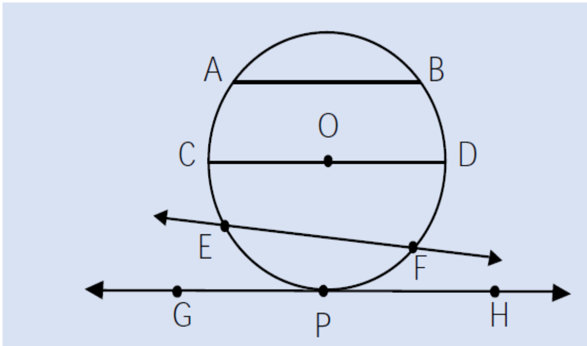
a) A chord of a circle is a line segment joining two points of the circle. Thus, in the figure, AB is a chord.

b) A diameter of a circle is a chord through the center. Thus, CD is a diameter of circle O.

c) Secant of a circle is a line that intersects the circle at two points. Thus, EF is a secant.

d) A tangent of a circle is a line that touches the circle at one and only one point. Thus, GH is a tangent to the circle at P. The point P is called the point of tangency or point of contact.

e) The distance around a circle is called the circumference of the circle.



Note:

- All radii of a circle are congruent.
- A diameter of a circle consists of two radii that lie on the same straight line.

  Example 6.1:

In circle O, radius OA = $3x - 10$ and radius OB = $x + 2$. Find the length of a diameter of circle O.

Solution:

Since all radii of a circle are congruent, OA = OB

$3x - 10 = x + 2$

$3x = x + 12$

$x = 6$

OA = OB = $x + 2 = 6 + 2 = 8$

Therefore, the length of a diameter of circle O is 2 x 8 = 16.

Arcs and its Type

Definition: An arc of a circle is a part of the circumference of a circle. The symbol for an arc is $\frown$.

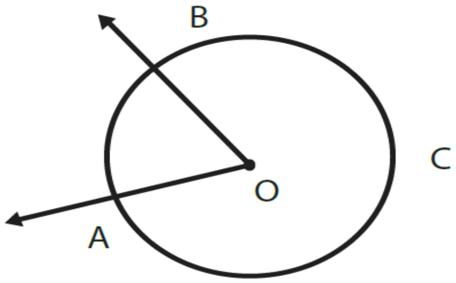
.Thus, AB stands for arc AB.

In the figure below, A, B, C, and D are points on circle O and ∠ AOB intersects the circle at two distinct points, A and B, separating the circle in to two arcs.

1) If $m\angle$ (AOB) < 180^0, points A and B and the points of the circle in the interior of ∠ AOB make up minor arc AB, written as AB.

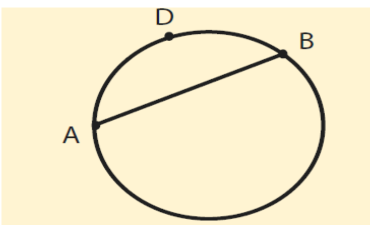
2) Points A and B and the points of the circle not in the interior of ∠ AOB make up major arc AB. Usually a major arc is named by three points: The two end points and any other point on the major arc. Thus, the major arc with end points A and B is written as ACB.

3) A semi-circle is an arc of a circle whose end points are the end points of a diameter of the circle; its measure is 180^0.



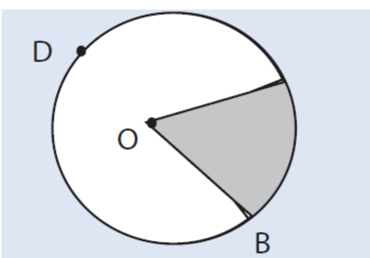
Note:

A chord of a circle subtends the arcs which it cut off on the circle. The arc of a circle cut off by a chord is subtended by the chord. In figure below, arc ADB is subtended by chord AB.



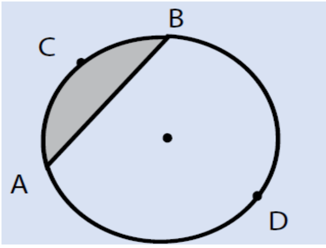
Definition : A sector of a circle is the part of a circle bounded by two radii and their intercepted arc.

In the figure, region AOB is called a minor sector and region ADB is called a major sector.



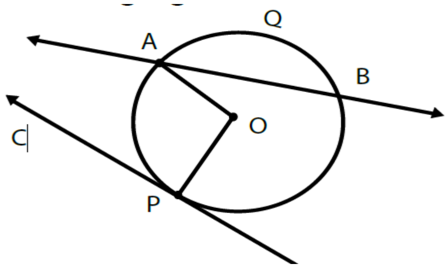
Definition: A segment of a circle is the part of a circle bounded by a chord and its subtended arc.

Regions ABC and ADB are called segments. Region ABC is the minor segment and region ADB is called the major segment.



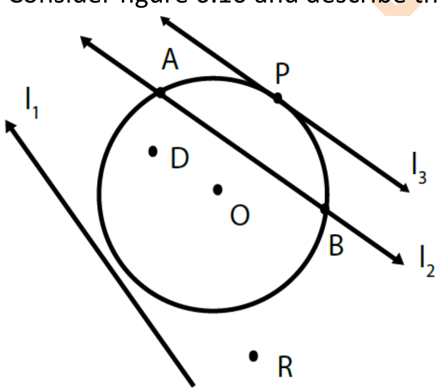
Exercise 6.1

1) Identify a chord, a secant, a tangent, point of tangency, a sector and a segment for the following figure.



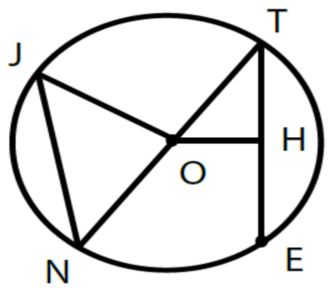
2) Write true if the statement is correct and false if it not.

a) A secant of a circle contains chord of the circle.
b) Tangent is a segment whose end point is on a circle.
c) If point A is an interior point of circle O, then it is possible to draw a tangent line that contains point A.
d) Sector is a region bounded by two radii and the intercepted arc.
3) Consider figure 6.10 and describe the relationship between lines and circle O, points and circle O.



3) Consider figure 6.11 and answer the following questions.

a) Name four minor arcs of circle O.
b) Name four different major arcs of circle O.
c) Name major arc TEJ in three different ways.



Answer

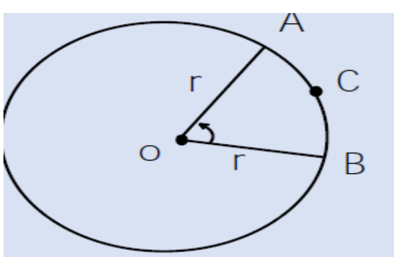
1. Regions AOP and ABP are sectors. region APB and AQB are segments of the circle.
2. Line L_2 and circle o intersects at A and B the line L_2 is a secant line.
   - line L_1 and circle o do not intersect at all.
   - line L_3 and circle o intersect at exactly one point p , this line is a tangent.
   - Point D is an interior point of circle o.
   - Point R is an exterior point of circle o.

3. a. in circle O the minor arcs are NJ ,$\widehat{JT}$, $\widehat{TE}$, and $\widehat{EN}$

b. in circle O the Major arcs Are $\widehat{NETJ}$, $\widehat{JTN}$, $\widehat{TEJ}$ and $\widehat{EJT}$

6.1.2. Central angle and inscribed angle

Definition. A central angle is an angle whose vertex is the center of the circle and whose sides are radii of the circle. In figure below, ∠AOB is a central angle. The reflex angle AOB is called the reflex central angle.



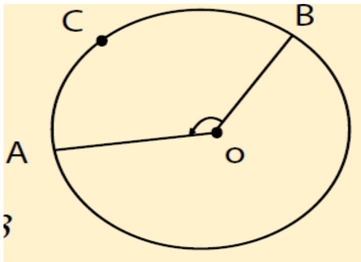
Note: From figure above,

Arc ACB is said to be intercepted by ∠AOB and ∠AOB is said to be subtended by arc ACB.

A central angle is measured by its intercepted arc.

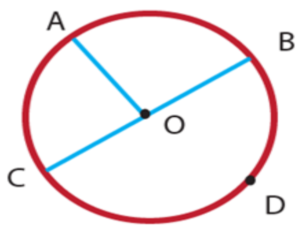
The measure of the central angle ∠AOB is equal to the measure of the intercepted arc ACB. That is,

$m\angle(AOB) = m$ arc(ACB).



Example: For the following figure, if $(\angle AOB) = 67^0$ and CB is a diameter of circle O, then find

a) $m(\widehat{AB})$ c) $m(\widehat{ADB})$
b) $m(\widehat{AC})$ d) $m(\widehat{BDC})$



Solution:

b. (arc)= $m(\angle AOB) = 67^0$

c. (arc)= $m(\angle AOC) = 180^0 - 67^0 = 113^0$

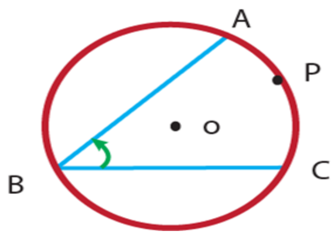
d. (arc ADB) = 360^0 – (arc AB) = $360^0 - 67^0 = 293^0$

e. (arc BDC) = 180^0

Let us now consider the situation in which the vertex of the angle is a point on the circle and the sides of the angle are chords of the circle.

Definition: An inscribed angle is an angle with its vertex on the circle and whose sides contain chords of the circle.

In the figure, $\angle ABC$ is inscribed angle for circle O.

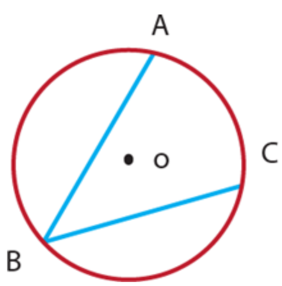


Note:

The arc formed by the intersection of the two sides of the angle and the circle is called an intercepted arc. From figure 6.15, ∠ABC is an inscribed angle and arc AC is an intercepted arc. We say that ∠ABC is inscribed in the arc ABC and ∠ABC is subtended by arc APC.

Theorem . The measure of an inscribed angle is one-half of the measure of its intercepted arc.

In the figure, (∠ABC)= $\frac{1}{2}$m (arc AC).



Example:

Let AC be the diameter of circle O and (∠AOB) = 1120 as shown in the figure. Then find (∠OAB).

Solution:

($\angle A$OB) = (arc AB). Hence, (arc AB) = 112^0

(arc BC) = (arc ABC) − (arc AB)

= $180^0 - 112^0$

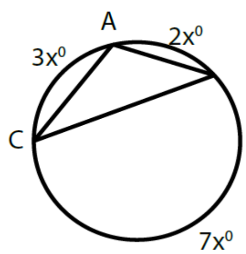
= 68^0

($\angle O$AB) = $\frac{1}{2}$m (arc BC).

$=\frac{1}{2} \times 68^0$ = 34^0

Example:

ΔABC whose vertices lie on a circle so that its sides divide the circle into arcs whose measures have the ratio of 2: 3: 7. Find the measure of the smallest angle of the triangle.



Solution:

First, determine the measures of the arcs of the circle.

(arc AB) + (arc BC) + (arcAC) = 360^0

$2x + 7x + 3x = 360^0$

$12x = 360^0$

$x = 30^0$

The measures of the arcs of the circle are:

(arcAB) = $2x = 60^0$

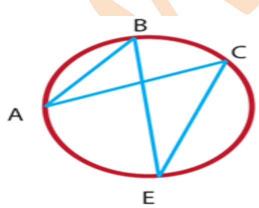
(arc BC) = $7x = 210^0$

(arcAC) = $3x = 90^0$

Each angle of the triangle is an inscribed angle. The smallest angle lies opposite the arc having the smallest measure.

Hence, $(\angle C) = \frac{1}{2}$m (arc AB) = $\frac{1}{2} \times 60^0 = 30^0$

Theorem: In a circle, inscribed angles subtended by the same arc are congruent. That is m $(\angle ABE)$ = $(\angle ACE)$



Exercise 6.2

1. Write true if the statement is correct and false if it is not.

a) If the measure of the central angle is halved, then the length of the intercepted arc is doubled.

b) An angle inscribed in a semi-circle is a right angle.

c) Angles inscribed in the same arcs are congruent.

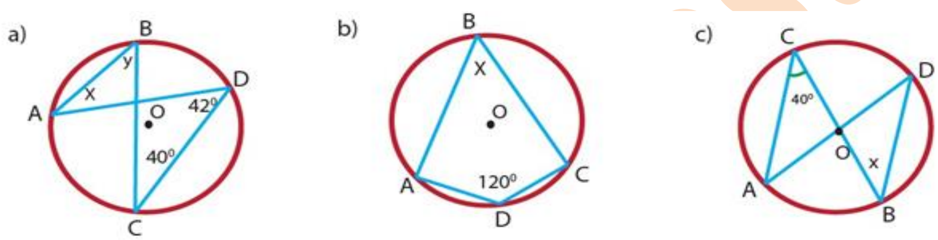
d) A central angle is measured by its intercepted arc.

e) A central angle of a circle is an angle whose vertex is on a circle and whose sides contain chords of the circle.

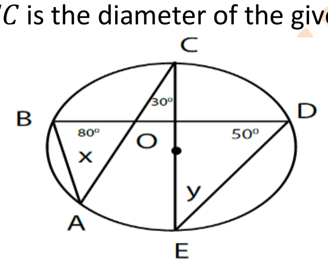
f) The measure of the central angle is twice of the measure of the inscribed angle.

2. A circle is divided into three arcs in the ratio 3:2:5. The points of division are joined to form a triangle. Find the largest angle of the triangle.

3. Find the degree measure of the angle which is indicated by variables.



4.If EC is the diameter of the given circle in figure below, find the degree measure of angle x and y.



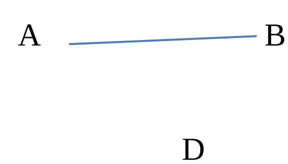
Answer

1. a. False b. True c. True d. True e. False f. True

2. A circle measures 360^0 , since $3x + 2x + 5x = 360^0$

$10x = 360^0$

$\underline{\underline{x = 36^0}}$



$\therefore$ The mesure of the arcs with the ratio $3:2:5$ are 108^0 , 72^0 *and* 180^0 *resepectively*

$m(< CAB) = \frac{1}{2}m(\widehat{CPB})$

$= \frac{1}{2}(180^0)$

$= \underline{\underline{\mathbf{90^0}}}$

The largest measure of a triangle 108^0 is $\underline{\underline{\mathbf{90^0}}}$

3. A). since $< BAD \cong < DCB$ *becuase they intercept the same arc*.

$x = m(< DCB) = 40^0 = m(< BAD)$

$< ADC \cong < CBA$ *beacuse they* Intersect the same arc

$Y = (< ADC) = 40^0 = m(< CDA)$

B). $(< ADC) = \frac{1}{2}m(\widehat{ABC})$

$m(\widehat{ABC}) = 2(120^0) = 240^0$

$m(\widehat{ABC}) = 360^0 - 240^0$

$m(\widehat{ABC}) = 120^0$

$m(< ADC) = \frac{1}{2}m(\widehat{ABC})$

C). $m(\widehat{AB}) = 2m(< ABC)$

$= 2 \times 40°$

$= 80^0$

similarly, $m(\widehat{AC}) + m(\widehat{CD}) = 180^0$

$= m(\widehat{CD}) = 180^0 - 100^0$

$= 80^0$

$m(\widehat{AB}) + m(\widehat{AC}) = 180^0$

$= m(\widehat{AC}) = 180^0 - 80^0$

$= m(\widehat{AC}) = 100^0$

And Hence, $m< CBD = \frac{1}{2}m(\widehat{CD})$

$X = \frac{1}{2}(80^0) = 40^0$

Or m($\widehat{AB}$)= m($<\widehat{AOB}$)=80^0 and m($<\widehat{AOB}$) $\cong$ $< COD$

Vertical opposite angle then m(COD)=2 m($< CBD$)

$$m(< CBD) = \frac{1}{2}m< COD)$$

$$= \frac{1}{2}(80^0) = 40^0$$

4. m($\widehat{AB}$) = m($\widehat{BAE}$) – m($\widehat{AE}$)

$= 100^0 - 60^0$

$= 40^0$

NOW Let us find m($\widehat{BC}$)

m($\widehat{BC}$) = $180^0 - 60^0 - 40^0$

$= \underline{\underline{\mathbf{80^0}}}$

m(<(BAC)) = $\frac{1}{2}m(\widehat{BC})$ $= \frac{1}{2}(80^0) = 40^0$//

m(< $\widehat{DCA}$) + m($\widehat{AB}$) + m($\widehat{BC}$) + m($\widehat{CD}$) =360^0

m($\widehat{CD}$) = $360^0 - 280^0$

$= 80^0$

M($< CED$) $= \frac{1}{2}m(\widehat{CD})$

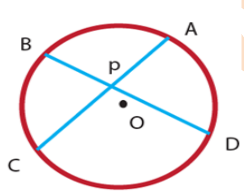
$= \frac{1}{2}(80^0)$

$= \mathbf{40^0}$//

6.1.4. Angles formed by two intersecting chords

Theorem

The measure of an angle formed by two chords intersecting inside a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

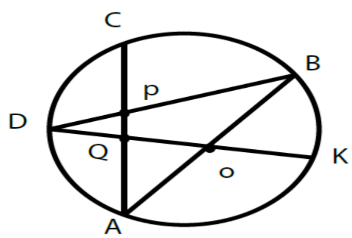


(∠APD) =$\frac{1}{2}$m (arc BC)+ $\frac{1}{2}$m (arc AD).

Example:

An angle formed by two chords intersecting within a circle is 54^0, and one of the intercepted arcs measures 40^0. Find the measures of the other intercepted arc.

Solution:



Consider in the figure above.

$(\angle BPC) = \frac{1}{2}m$ (arc BC)$+ \frac{1}{2}m$ (arc AD).

$54^0 == \frac{1}{2} \times 40^0 + \frac{1}{2}m$ (arc AD).

$108^0 = 40^0 + m$ (arc AD)

m (arc AD) $= 68^0$

Exercise 6.3

1. Write true if the statement is correct and false if it is not.

a) A diameter perpendicular to a chord bisects the chord.

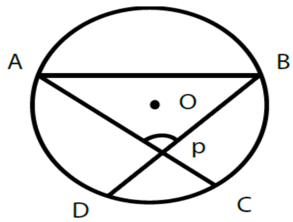
b) A perpendicular bisector of a chord passes through the center of the circle.

c) The measure of the angle formed by two chords of a circle is equal to the sum of the measure of the arc subtending the angle and its vertically opposite angle.

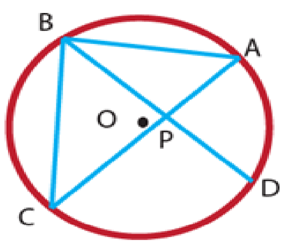
2. In the figure 6.27, $(\angle APB) = 110^0$ and $(\angle BAC) = 40^0$. Then,

a) Find the sum of (arc AD) and (arc BC).

b) $(\angle ACD)$.



3. In the figure below, (∠CBD) = 45^0, (∠ABD)= 40^0 and m(arc AB) = 90^0. Then find (∠APD).



Answer

1. a. True b. True c. False
2. a. m$(< APB) = 180^0 - 110^0 = 70^0$

   m$(< BPC) = \frac{1}{2}(\widehat{BC})$ + m$(\widehat{AD})$)

   m.$(\widehat{BC})$ + m$(\widehat{AD})$ = R. m$(< BPC)$

   m$(\widehat{BC})$ + m$(\widehat{AD})$ = 2×70^0

   = 140^0

   b. first find the m$(< ABD)$

   m$(< BAC) + m(< APB) + m(ABD) = 180^0$

   m$(< ABD) = 180^0 - 150^0 = 30^0$ and

   $< ABD \cong < ACD - - - They\ intercept\ the\ same\ arc$

   $\therefore m(< ACD) = 30^0$

3.m$(\widehat{AD})$ + m$(\widehat{DC})$ + m$(\widehat{CB})$ + m$(\widehat{BA})$ = 360^0

m$(\widehat{AD})$ = 2m$(ABD) = 2(40^0) = 80^0$

m$(\widehat{DC})$ = 2m$(< CBD) = 2(45^0) = 90^0$

m$(\widehat{AB})$ = 90^0

m$(\widehat{CB})$ = $360^0 - 260^0$

= $\mathbf{100^0}$//

$\therefore m(APD) = \frac{1}{2}(m(\widehat{AD}) + \mathrm{m}(\widehat{BC}))$

$= \frac{1}{2}(80^0 + 100^0)$

$= \frac{1}{2}(180^0) = 90^0$

6.2 Applications of Circle

Example:

Dawit observes that the time is 5.00 hrs. What is the angle between the hands of the clock?

Solution:

The small hand on a clock shows the hours on the face of a clock. So, if the hour hand lies on 6.00 hrs. Then the angle between the hands is 180^0.

6hr = 180^0. $x = \frac{1hr \times 1800}{6hr} = 30^0$

1hr = x

Thus, 1hr measures 30^0 Therefore, a 5 hr Hand measures $5 \times 30^0 = 150^0$.

REVIEW EXERCISE FOR UNIT 6

1. Write true if the statement is correct and false if it not.

a) Minor arc is the part of a circle greater than a semi-circle.

b) Inscribed angles intercepted by the same arc are congruent.

c) A central angle is not measured by its intercepted arc.

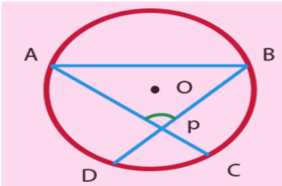
d) A circle O has a radius of 6cm. Then the length of the longest chord of circle O is 12cm.

d) An angle is inscribed in a circle and intercepts an arc with measures 108^0, then the measure of the inscribed angle is 54^0.

f) 180^0 cannot be the measure of an inscribed angle.

2. In the figure below, if $m\angle$ABD = 45^0, (arc BC) = 100^0. Then

i) Find a) ($\angle$ACD)



b) ($\angle$BAC)

c) ($\angle$BPC)

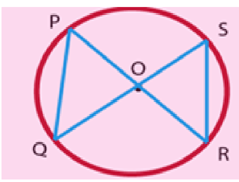
ii) Is the point P lies at the center of the circle? Explain.

Iii)What type of triangle is formed by CD, PD and PC?

3. In the figure below, O is the center of the circle. If $mm\angle$POS = 95^0, then find

a) ($\angle$QSR)

b) ($\angle$PQS)



c) ($\angle$QPR)

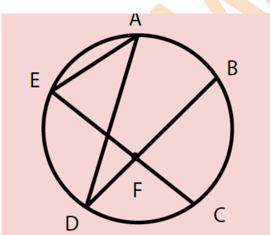
4. In figure 6.34, ($\angle$AEC) = 70^0, ($\angle$ADB) = 25^0, and

($\angle$BFC) = 80^0. Find

a) ($\angle$DAE)

b) (arc DE)

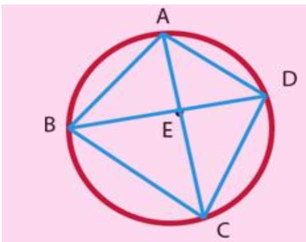
c) (arc BC)



5. In figure 6.35, (∠BCD) = 90^0, (∠BAC) = 49^0 and (∠ADB) = 61^0. Find

a) (∠ACB) c) (∠CAD)

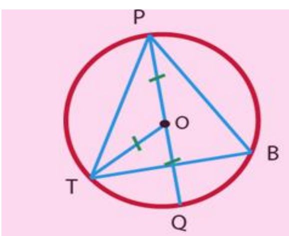
b) (∠ABC) d) (∠BEC)



6. In figure below PT is a chord and O is the center of the circle and (∠PBT) = 70^0, then calculate

a) (∠TPO)

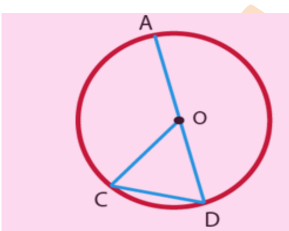
b) (TQ)



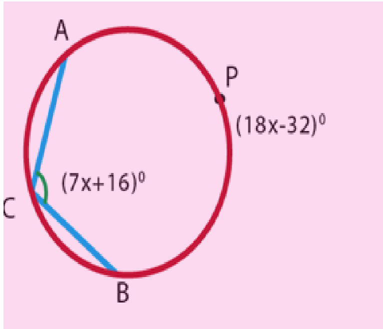
7. In figure below, AD is a diameter and (arc AC) = 132^0. Find

a) (∠OCD)

b) (arc CD)



8. In the figure below, what is the measure of the inscribed angle?



Answer

1. a. False b. True c. False d. True e. True f. True
2. i.

a. $< ACD \cong < ABD$ — — — *They intercept the same arc.*

m$(< ACD) = m(ABD) =$ $\mathbf{45^0}$//

b. M$(< BAC) = \frac{1}{2} m(\widehat{BC})$.

$= \frac{1}{2}(100^0)$

$= \underline{\underline{\mathbf{80^0}}}$

c. First find m$(\widehat{AD})$

m$(\widehat{AD}) = 2 \times m(< ABD) = 2 \times 45^0 = \mathbf{90^0}$//

m$(< BPC) = \frac{1}{2} m(\widehat{BC}) + m(\widehat{AD})$

$= \frac{1}{2}(100^0 + 90^0$

$= \frac{190^0}{2}$

$= \underline{\underline{\mathbf{95\ ^0}}}$

ii. No , because m$(< bpc) \neq m(\widehat{AD})$ that is m$(< BPC)$ *is not a centeral angle.*

iii. m$(< ACD) = m(< ABD) = 45^0$

m$(< BDC) = m(< BAC) = 50^0$

m$(< DPC) = 180^0 - 95^0 = 85^0$

$\therefore$ *the triangle is a cute angle triangle.*

3. m$(< POS) = 95^0$ *, then* $m(< QOR) = 95^0$

a. m$(< QSR) = \frac{1}{2} m(< QOR)$

$= \frac{1}{2}(95^0)$

$= \underline{\underline{\mathbf{47.5^0}}}$

b. m $(< PQS) = \frac{1}{2}m(< POS)$

$= \frac{1}{2}(95^0)$

$= \underline{\underline{\mathbf{47.5^0}}}$

c. m$(< QPR)$ = m$(< QSR) = 47.5^0$

4. first find m$(\widehat{DE})$,

m$(< BFC) = \frac{1}{2}m\left(\widehat{BC}\right) + m\left(\widehat{DE}\right))\ but$

m$(\widehat{BC})$ = $140^0 - 50^0 = 90^0$

m$(\widehat{DE})$ = 2m$(< BFC) - m(\widehat{BC})$

$= 2(180^0) - 90^0$

$= 160^0 - 90^0$

$= \underline{\underline{\mathbf{70^0}}}$

m$(< DAE) = \frac{1}{2}m(\widehat{DE})$

$= \frac{1}{2}(70^0)$

$= \underline{\underline{\mathbf{35^0}}}$

5. m$(< BCD) = 90^0$

m(BAD) = 2m$(< BCD) = 2(90^0) = \mathbf{180^0}$//

Since $\overline{BD}$ is a diameter.

a. M$(< ACB) = m(< ACD) = 61^0$

b. First , find m$(\widehat{CD})$

m$(\widehat{BC})$ = $2 \times m(< BAC) = 2(49^0) = 98^0$

m $(\widehat{CD})$ = $180^0 - m(\widehat{BC})$

$= 180^0 - 98^0$

$= \underline{\underline{\mathbf{82^0}}}$ and m$(\widehat{ADC})$ = m$(\widehat{CD})$ + m$(\widehat{AD})$

$= 82^0 + 58^0 = \underline{\underline{140^0}}$

$m(<ABC) = \frac{1}{2}m(\widehat{ADC})$

$= \frac{1}{2}(140^0)$

$= \underline{\underline{\mathbf{70^0}}}$

c. $m(<CAD) = \frac{1}{2}(\widehat{CD})$

$= \frac{1}{2}(82^0)$

$= \underline{\underline{\mathbf{41^0}}}$

d. $m(<BEC) = \frac{1}{2}m(\widehat{BC}) + m(\widehat{AD}))$

$= \frac{1}{2}(98^0 + 58^0)$

$= \frac{156^0}{2}$

$= \underline{\underline{\mathbf{87^0}}}$

6. $m(\widehat{TP}) = 2\times m(<PBT)$

$= 2 \times 70^0$

$= \underline{\underline{\mathbf{140^0}}}$

Since , $\overline{PQ}$ is a diameter.

$m(\widehat{TQ}) = 180^0 - m(\widehat{TP})$

$= 180^0 - 140^0$

$= \underline{\underline{\mathbf{40^0}}}$

$M(<TPQ) = \frac{1}{2}m(\widehat{TQ})$

$= \frac{1}{2}m(40^0)$

$= \underline{\underline{\mathbf{20^0}}}$

7. $m(<AOC) = m\left(\widehat{AC}\right) = 132^0$

$= m(COD) = 180^0 - 132^0 = \underline{\underline{\mathbf{48^0}}}$

$= m(<CDA) = \frac{1}{2}m(\widehat{AC})$

$= \frac{1}{2}(132^0)$

$= \underline{\underline{\mathbf{66^0}}}$

a. $m(<CDO) + m(<COD) + m(<OCD) = 180^0$

$= m(< OCD) = 180^0 - 114^0$

$= \underline{\underline{\mathbf{66^0}}}$

b. $m(\widehat{CD}) = m(< \widehat{COD}) = \mathbf{48^0}$//

8. $m(< ACB) = \frac{1}{2}m(APB)$

$7x \times 16 = \frac{18x}{2} - \frac{32}{2}$

7x + 16 = 9x -16

7x – 9x = -16 – 16

= -2x = -32

$\boldsymbol{X = 16}$ //

∴ The measure of the inscribed angle is 7x + 16 = $\mathbf{128^{\,0}}$//

UNIT 7

SOLID FIGURES AND MEASUREMENTS

Solid Figures: Solid figures are three-dimensional objects. What this means is that solid figures have a width, a length, and a height. For instance, computer, laptop, phone etc. has a width, a length, and a height.

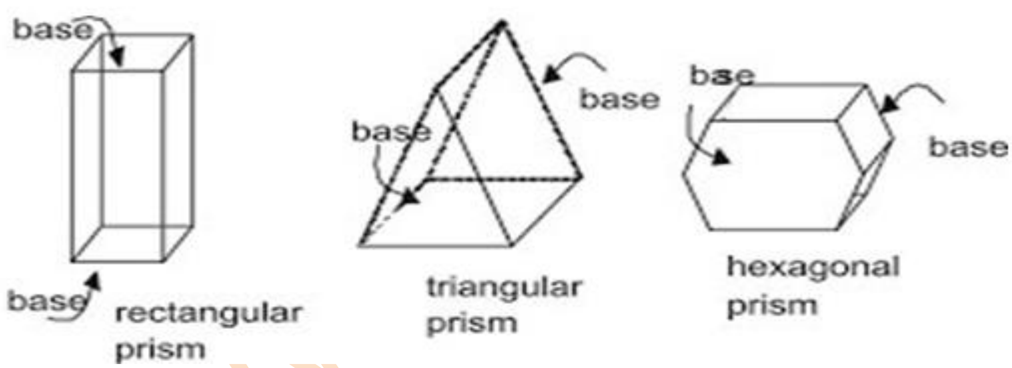
In mathematics, there are many solid figures. Most of them are prisms, Cylinders, pyramids and Cones.

7.1 Prisms and Cylinders

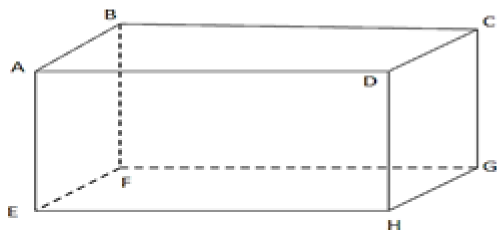
1. Prisms

Definition: A prism is a three dimensional figure in which two of the faces, called the bases of the prism, are congruent polygons in parallel plane.

Depending on the shape of its base a prism can be triangular prism, rectangular prism, pentagonal prism, hexagonal prism and so on. For instance,



Consider the rectangular prism as shown below



- The vertices of the rectangular prism are A, B, C, D, E, F, G, and H
- A prism has two bases: upper base and lower base. The rectangular region ABCD is the upper base and rectangular region EFGH is the lower base.
- The line segments; AB, BC ,D ,DA ,EF ,FG,GH, and HE are edges of the bases.
- The line segments; AE , BF, CD and DH (the segments that connect the vertices) are the lateral edges of the prism.
- The rectangles ABFE, BCGF, CDHG, and ADHE (the surfaces between corresponding sides of the bases) are called the lateral faces of the prism
- An altitude of a prism is a line segment perpendicular to each of the bases with an end point on each base. The height of a prism is the length of an altitude.

Definition. A right prism is a prism in which the lateral sides are all perpendicular to the bases.

Example:

The bases of a prism are equilateral triangles. The length of one edge of a base is 4cm and its height is 5cm.

a) How many lateral edges does this prism have?

b) What is the shape of the lateral face?

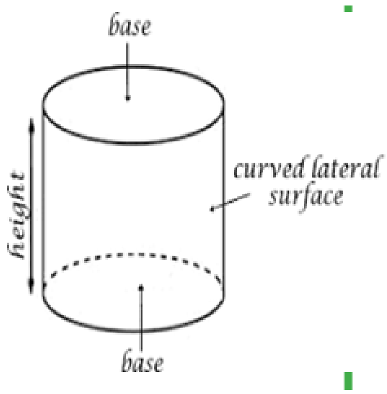
Solution:

a) Because this is a prism with a triangular base, the prism has three lateral edges.

b) Because it is a right prism, the lateral sides are rectangles.

2. Cylinders

Definition: a cylinder is a solid figure with congruent non polygonal bases that lie in parallel planes. If the bases are circular, the cylinder is circular cylinder.



- The line passing through the centers of the two circular bases is called the axis of the cylinder.
- The altitude of a cylinder is the perpendicular distance between its bases.
- The radius of the base is also called the radius of the cylinder.
- A cylinder is called a right cylinder if the segment joining the centers of the bases is perpendicular to the bases.

Exercise 7.1

1. Write true if the statement is correct and false otherwise.

a) The intersection of the faces of a prism are the edges of the prism.

b) A line through the centers of the bases of a circular cylinder is perpendicular to the diameter of the bases.

c) The intersection of the edges of a prism are the vertices of the prism.

d) The upper and the lower bases of circular cylinder are circles ofequal radii.

e) A triangular prism has a triangular lateral face.

f) the bases of a prism lie on parallel planes.

g) a cylinder is a circular prism.

2. Describe the differences between a prism and a cylinder. Describe their similarities.

3. Give a mathematical name of the solids given below.

a) Soup can   b) Shoe box

Answer

1. a. True b. True c. True d. True e. False f. True g. True

2. a. circular cylinder

b. rectangular prism

3. there are a total of 6 faces

12 faces

8 Vertices in a rectangular prism.

There are a total of 3 faces, 2 curved edges and no vertices in circular cylinder.

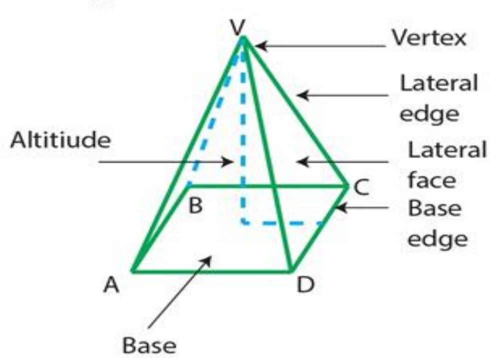
7.2 Pyramids and Cones

1. Pyramids

Definition: A pyramid is a polygon in which the base is a polygon and the lateral faces are triangles with a common vertex.

- The polygonal region ABCD is called the base of the pyramid.
- The point V outside of the plane of the base is called the vertex of the pyramid.
- The triangles VAB, VBC, VCD, and VDA are called lateral faces of the pyramid.
- The intersection of two lateral faces is called lateral edges. Thus, VA , VB, VC , and VD are lateral edges of the pyramids.
- The intersection of the base and a lateral face is a base edge. Thus, AB , BC, CD , and DA are the edges of the base of the pyramid.
- The altitude of the pyramid is the perpendicular distance between the base and the vertex.

A C



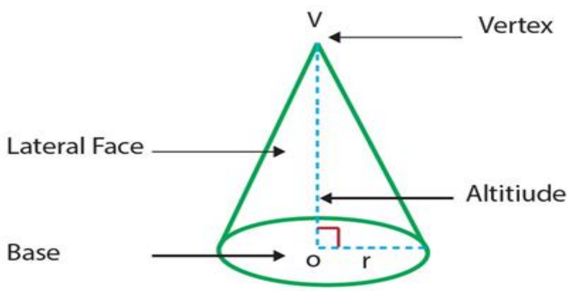
3. Cones

Definition: A circular cone is a solid figure formed by joining all points of a circle to a point not on the plane of the circle.

- The point outside the plane and at which the segments from the circular region joined is called the vertex of the cone.
- The flat surface, the circle, is called the base of the cone and the curved closed surface is called the

lateral face of the cone.

- The perpendicular distance from the base to the vertex is called the altitude of the cone.
- A circular cone with the foot of its altitude is at the center of the base is a right circular cone.



Exercise 7.2

1) Write true if the statement is correct and false if it not.

a) Cone is a pyramid.

b) A cone has a flat surface and a curved surface.

c) A triangular pyramid has three vertices and three faces.

d) A cone has one vertex and one curved edge.

e) A pyramid cannot have any parallel faces.

Answer

1. a. False b. True c. False d. True e. True

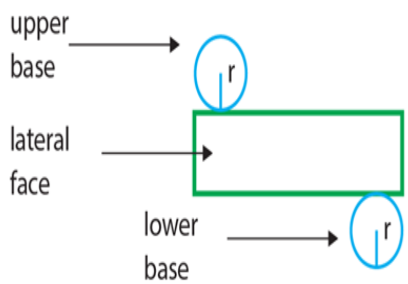
7.3. Surface Area and Volume of Solid Figures

7.3.1. Surface area of Prisms and Cylinders

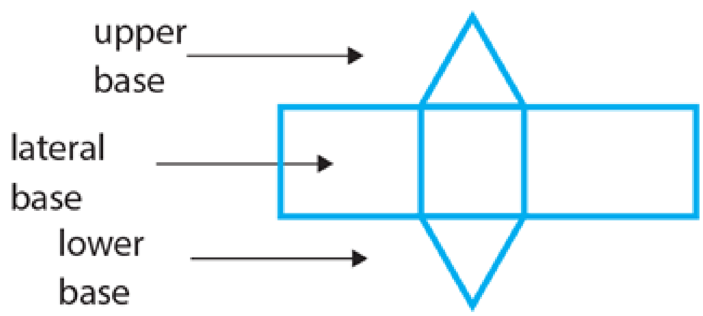
Definition: A net is a pattern of shapes on a piece of paper that are arranged so that the net can be folded to make a hollow surface.

Example:

a) The net of a cylinder looks like a rectangle with two circles attached at opposite ends.



b) The net of a triangular prism consists of two triangles and three rectangles. The triangles are the bases of the prism and the rectangles are the lateral faces.



A. Surface area of Prisms.

There are two types of surface areas: Lateral surface area and Total surface area.

- The lateral surface area, denoted by AL of the prism is the sum of the areas of all lateral faces of the prism.
- The total surface area, denoted by AT is the sum of the areas of all the faces.

  A_L= area of front face + area of back face +area of left face+ area of right face

  $= lh + lh + wh + wh$

  $= 2\ lh + 2\ wh$

  $= 2h\ (l + w)$

  $= ph$ where $p = 2(l + w)$ is the perimeter of the base.

Total surface area (A_T) = A_L + area of top face + area of bottom face

$= ph + lw + lw$

$= ph + 2lw$

$= A_L + 2A_B$ where $A_B = 2lw$ is the area of the base.

In general,

The lateral surface area of a prism is AL = ph where p is the perimeter of the base. The total surface area of a prism is AT = AL + 2AB where AB is the area of the base.

Example:

The base of a rectangular prism is ABCD, where AB = 6cm, BC = 3cm and height h = 4cm. then

Find a) Lateral surface area of the prism

b) Total surface area of the prism

Solution:

a) $A_L = ph$

$= 2(6 + 3) \times 4 = 72\ cm^2$.

b) $A_T = AL + 2\ A_B$

$= 72 + 2\ (6 \times 3)$

$= 108cm^2$

Example :

The base of a triangular prism is ΔABC, where AB = 6cm, BC = 8cm and $m\angle B$ = 900. If the height h of the prism is 7cm. Find

a) The lateral surface area

b) The total surface area

Solution:

a) First determine the length of AC.

Using Pythagoras theorem,

$AC^2 = BC^2 + AB^2$

$= 8^2 + 6^2$

AC = 10. Then,

$A_L = ph$

$= (6 + 8 + 10) \times 7$

$= 168cm^2$

b) $A_T = A_L + 2\ AB$

$= 168 + 2\frac{1}{2}(AB.\ BC)$

$= 168 + 6 \times 8$

$= 216\ cm^2$

B) Surface area of cylinders

To derive a formula to the surface area of a circular cylinder, consider the net of the cylinder. It has two congruent and parallel circular bases and a rectangular face of length $2\pi r$ and width h as shown in figure below

Therefore,

The lateral surface area (A_L) = area of the rectangle

= $2\pi\pi\pi\pi$ x h

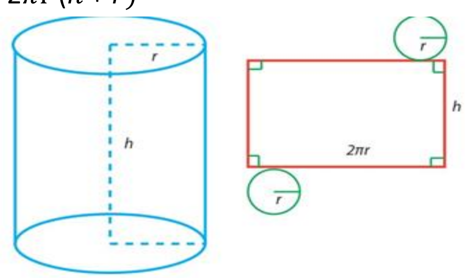
AL = $2\pi\pi\pi\pi\ h$

Similarly, the total surface area (A_T) = A_L + 2. Area of the bases

$A_T = A_L + 2\pi r^2$

$A_T = 2\pi r h + 2\pi r^2$

$A_T = 2\pi r\ (h + r)$



In general,

The lateral surface area of a circular cylinder is $A_L = 2\pi r\ h$ and

The total surface area of a circular cylinder is $A_T = 2\pi r\ (h + r)$

Example:

Find the total surface area of a right circular cylinder whose radius is 8cm and height is 12cm.

Solution:

AT = $2\pi r\ (h + r)$

= $2\pi \times 8 \times (12 + 8)$

$A_T = 320\pi$ cm^2

Example:

Find the height of the right circular cylinder if its lateral surface area is $108\pi cm^2$ and radius of the cylinder is 6cm.

Solution:

$A_L = 2\pi rh$

$108\pi cm^2 = 2\pi . 6. h = 12\pi h$

$h = \frac{108\pi}{12\pi} = 9cm$

Example:

The total surface area of a right circular cylinder of radius 7cm is $112\pi cm^2$. Then find the height of the cylinder.

Solution:

$AT = 2\pi\pi\pi\pi\ (h + rr)$

$112\pi = 2\pi \times 7\ (h + 7)$

$\frac{112\pi}{14\pi} = h + 7$

$8 = h + 7$

$h = 1cm.$

Therefore, the height of the cylinder is 1cm.

Example:

The sum of the height and the radius of a right circular cylinder is 9cm.

If the surface area of the cylinder is $54\ \pi cm^2$, then find the height of the cylinder.

Solution:

Let h be the height and rr be the radius of the cylinder.

$h + r = 9$

$A_T = 2\pi\ (h + r)$

$54\ \pi = 2\pi\ (9 - h)\ 9$

$\frac{54\pi}{18\pi} = 81 - 9h$

$27 = 81 - 9h$

$h = 6cm$

Therefore, the height of the cylinder is 6cm.

Exercise 7.3:

1) Write true if the statement is correct and false if it is not

a) The total surface area of a triangular prism can be calculated by adding the areas of 5 faces.

b) A 3D object with two parallel and congruent circular bases is a cylinder.

c) Surface area is measured in cubic units.

d) The dimensions of the rectangular prism are increased 2 times, and then the surface area will increase 8 times.

e) In a cylinder, if the radius is doubled and height is halved, then its lateral surface area will be the same.

f) The two-dimensional representation of all of the faces is a net.

2) Find the lateral surface area of a triangular prism whose height is 10cm and the dimensions of each of its bases are 3cm, 6cm, and 7cm.

3) Find the lateral surface area of the prism if the perimeter of the base is 100cm and its height is 5cm. Also, find the total surface area of the same prism if its base area is 50cm^2.

4) The lateral surface area of a right circular cylinder of height 8cm is 88πcm^2. Then find the diameter of the base of the cylinder.

5) The radius of two similar right circular cylinders is 3cm and 12cm. find the ratio of their altitudes.

Answer

1. a. True b. True c. False d. False e. True f. True
2. AL = Ph
   = (3 cm + 6 cm + 7cm) (10 cm)
   = 16 cm $\times$ 10 cm
   = **160 cm^2**
3. AL = Ph
   = 100 cm $\times$ 5 cm
   = **500 cm^2**

AT = At + 2 AB

= 500 cm^2 + 2(50 cm)^2

= 500 cm^2 + 100 cm^2

= **600 cm^2**

4. AL = $2\pi rh$

$88\,\pi\, cm^2 = 2\pi r \times 8\, cm$

r = $\frac{88cm^2}{2\times 8\, cm} = \frac{11}{2}$ cm = **5.5 cm** but d = 2r = $2 \times 5.5\, cm$ = **11cm**

$\therefore$ *The diameter of circular cylinder is* $11cm$

5. Let r_1 = 3 cm and r_2 = 12 cm

$\therefore$ *The ratios of the two radius is* $1:4$ *or* $4:1$

7.3.2. Volume of Prisms and Cylinders

Definition: The volume of a solid is the number of cubic units contained in its interior. That is, the amount of space it occupies.

Volume is measured in cubic units such as m^3, cm^3 etc.

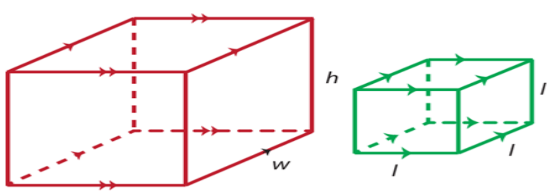
A) Volume of Prism

The volume V of a rectangular prism is equal to the product of its length (l), width (w) and height (h). That is,

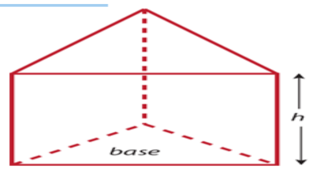
V = $l \times w \times h$.

By using this relationship, special formula can be derived for determining the volume of a cube. That is,

V = $l \times l \times l = l^3$.



The volume V of a right triangular prism equals the product of its base area AB and its height h. That is, V = AB × h.



The volume V of a prism is V = ABh, where AB is the area of the base and h is the height.

Example:

Determine the height of a rectangular prism that has a base dimension of 8cm, 6cm and a volume of 312cm^3.

Solution:

Let l = 8cm and w =6cm.

V = $l \times w \times h$

312cm^3 = 8 × 6 × h

$h = \frac{312}{48}$ $\quad h$ = 6.5 cm

Therefore, the height of the prism is 6.5cm

Example:

The base of a prism is an equilateral triangle each of whose sides measures 4cm. If the altitude of the prism measures 5cm, then find the volume of the prism.

Solution:

Since the base is an equilateral triangle, first determine the height, say y of the base.

$y^2 + 2^2 = 4^2$

$y^2 = 16 - 4$

$y = \sqrt{12}$. So,

AB $= \frac{1}{2}$. AB. y

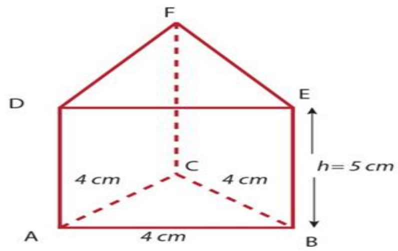
$= \frac{1}{2}$. 4. $\sqrt{12}$

$= 2\sqrt{12}$

Then, V = AB. h

$V = 2\sqrt{12}. 5$

$= 10\sqrt{12}\text{cm}^3$



B) Volume of cylinder

The volume V of a cylinder is $V = \pi r^2 h$

Example:

A right circular cylinder of height 12cm has a volume of 972 $\pi\pi$ cm3. Find the radius of the base of the cylinder.

Solution:

$V = \pi r^2 h$

$972\,\pi = \pi r^2. 12$

$\frac{972\pi}{12\pi} = r^2$

$r^2 = 81$

r =9cm

Therefore, the radius of the base is 9cm.

Exercise 7.4

1. Write true if the statement is correct and false if it is not.

a) The volume of a cylinder with base diameter dd and height h is given by $V = \frac{\pi d2h}{4}$.

b) If the volume of a rectangular prism whose dimensions , $\frac{4}{l}$, and h is 1cm^3, then the value of h is $\frac{1}{4}$.

c) Measuring the space region enclosed by the solid figure is the volume of the solid figure.

d) If two solid figures are congruent, then they have the same volume.

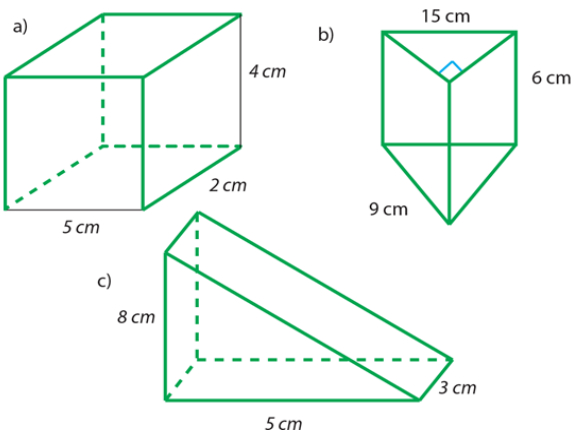
e) Measuring the surface constituting the solid is the area of the solid figure.

2. The volume of a triangular prism is 204cm3. If its height is 17cm, then find the area of its base.

3. Given the length, width, and height of the rectangular prism as 8cm, 5cm, and 16cm respectively. Find the volume of the rectangular prism.

4. Find the volume of a circular cylinder if its diameter is 12cm and its height is 20cm.

5. Find the volume of the following figures.



Answer

1. a. True b. True c. True d. True e. False

2. $V = A_B \times h$

$204^3 = A_B \times 17\ cm$

$A_B = \frac{204\ cm^3}{17\ cm} = \underline{\mathbf{12\ cm^2}}$

3. $V = L \times w \times h$

$V = 8\ \text{cm} \times 5\ cm \times 16\ cm$

$V = 40\ \text{cm}^2 \times 16\ cm$

$V = \underline{\mathbf{640\ cm^3}}$

4. V = $\pi r^2 h$ but d = 25 r = $\frac{12\ cm}{2}$ = **6 cm**

V = $\pi(6\ cm)^2 \times 20\ cm$

V= 36 π $cm^2 \times 20\ cm$

V= **$720\ \pi\ cm^3$**

5. a. V = 40 cm^3

V = $L \times w \times h$

= 5 cm $\times$ 2 cm $\times$ 4 cm

= 10 $cm^2 \times 4\ cm$

= **40 cm^3**

b. v = $L \times w \times h = A_B \times h$

= $(9cm)^2 + x^2 = (15\ cm)^2$

= 81 $cm^2 + x^2$ = 225 cm^2

x^2 = 225 cm^2 - 81 cm^2

x^2 = 144 cm^2

x = $\sqrt{144\ cm^2}$ = **12 cm**

Now A_B = $\frac{1}{2}(9\ cm \times 12\ cm)$ = 9 cm $\times$ 6 cm = **54 cm^2**//

V = $A_B \times h$ = 54 $cm^2 \times 6\ cm$

= **324 cm^3**

c. V = $L \times w \times h$

= 5 cm $\times$ 3 cm $\times$ 8 cm

= 16 $cm^2 \times 8\ cm$

= **120 cm^3**

7.3. Applications on Solid Figures and Measurements

Example :

A truck that delivers gasoline has a circular cylindrical storage space.

The diameter of the bases of the cylinder is 34cm and the height of the

cylinder is 46cm. How many gallons of gasoline does the truck hold?

(Use 1cm3 = 0.000264 gal)

Solution:

V = $\pi r^2 h$

= π. 17^2. 46

= 13,294 cm^3

$1cm^3 = 0.000264$ gal

V = 13,294 x 0.000264 gal

V = 3. 51 gal

Therefore, the truck holds 3.51 gallons of gasoline.

Example :

The walls, floor, and ceilings of a room form a rectangular solid. The total surface area of the room is 992 m2. The dimensions of the floor are 12m by 20m.

a) What is the lateral surface area of the room?

b) What is the height of the room?

Solution:

a) Since the solid is a rectangular prism, then

$A_T = A_L + 2A_B$

$A_T = A_L + 2 . l. w$

992 = AL + 2. 20. 12

$A_L = 992m^2 – 480m^2$

$A_L = 512m^2$

Therefore, the area of the walls of the room is $512m^2$

b) Since the solid is a rectangular prism, then

AL = ph = 2(l + w)h

$512m^2 = 2(20 + 12)$ m. h

$h = \frac{512}{64}$

h = 8m Therefore, the height of the room is 8m.

Exercise 7.5

1. Find the volume and surface area of a can of soda if the radius is 6cm and the height is 11cm.

2. A cans goods company manufactures a cylindrical can of height 10cm and radius 4cm.

a) Find the surface area of the can

b) Find the volume of a can whose radius and height are twice that of the given can.

3. A food company stores food item A in a cylindrical can that is 14 cm tall and has a diameter of 10cm. A company stores food item B in a rectangular can of dimensions 12cm by 13 cm by 5cm. Which can accommodate more food?

Answer

1. V = $\pi r^2 h$ but r = 6 cm and h = 11 cm

V= $\pi(6\ cm)^2 \times 11\ cm$

V= 36 $\pi cm^2 \times 11\ cm$

V = $\underline{\underline{\mathbf{396\ \pi\ cm^3}}}$

$A_T = A_L + 2A_b$ but $A_L = 2\pi rh$

$= 2\pi \times 6\ cm \times 11\ cm$

$= \underline{\underline{\mathbf{132\ \pi\ cm^2}}}$

$A_B = \pi r^2$

$= \pi(6\ cm)^2$

$= \underline{\mathbf{36\ \pi}}\ cm^2$

$A_T = 132\ \pi\ cm^2 + 2 \times 36\ \pi\ cm^2$

$= 168\ \pi\ cm^2 + 36\ \pi\ cm^2$

$= \underline{\underline{\mathbf{204\ \pi\ cm^2}}}$

2. $A_T = A_L + 2AB$ but $A_L = 2\pi rh = 2\pi \times 4\ cm \times 10\ cm$

$= \underline{\underline{\mathbf{80\ \pi\ cm^2}}}$

$A_B = \pi r^2 = \pi(4cm)^2 = 16\pi\ cm^2$

$A_T = A_L + 2AB$

$= 80\pi\ cm^2 + 2(16\ \pi\ cm^2) = 80\pi\ cm^2 + 32\ \pi\ cm^2$

$= 112\ \pi\ cm^2$//

V = $\pi r^2 h$ but r = 8 cm and h = 20 cm

V = $\pi(8\ cm)^2 \times 20\ cm$

V = $64\pi\ cm^2 \times 20\ cm$

V = $\underline{\underline{\mathbf{1280\ \pi\ cm^3}}}$

3. $V_A = \pi r^2 h$ but r = 5 cm and h = 14 cm

$V_A = \pi(5cm)^2 \times 14\ cm$

$V_A = 25\,\pi\ cm^2 \times 14\ cm$

$= 350\pi\ cm^3 = 1099\ cm^3$ and

$V_B = 12\ \text{cm} \times 13\ cm \times 5\ cm$

$V_B =$ **$\underline{780\ cm^3}$**

REVIEW EXERCISE FOR UNIT 7

1. Write true if the statement is correct and false if it is not.

a) A cube has 6 faces, 12 edges and 8 vertices.

b) A cone has one circular face, one vertex and has no edges.

c) A prism always has 2 parallel faces.

d) The net of a rectangular prism is made from 4rectangles.

e) A triangle is a possible base of a prism.

f) A pyramid can have a circular base.

2. How many edges has

a) a squared pyramid?

b) a cylinder?

c) a triangular prism?

d) a cube?

3. Given that the surface area of a rectangular solid is the sum of the area of its six faces. Then

a) What is the expected type of solid?

b) Which type of area is referred?

4. A right prism has bases that are squares. The area of one base is$81cm^2$. The lateral surface area of the prism is $144cm^2$. What is the length of the altitude of the prism?

5. The bases of a prism are right triangles whose edges measure 9cm, 40cm and 41cm. The lateral sides of the prism are perpendicular to the bases. The height of the prism is 15cm.

a) What is the shape of the lateral sides of the prism?

b) What are the dimensions of each lateral sides of the prism?

c) What is the total surface area of the prism?

6. If the total surface area of a right circular cylinder is $884\pi\pi cm^2$ and its radius is 2cm, then what is the length of the altitude?

7. The sum of the height and radius of a right circular cylinder is 9cm. if the total surface area is $81\pi\pi$cm2, then what is the radius of the cylinder?

8. The length, width and height of a rectangular solid are in the ratio of 3: 2: 1. If the volume of the box is 138cm3, what is the total surface area of the box?

9. The dimensions of a box are 16cm by 11cm by 9cm. a small cubical box measures 2cm long each side. How many of these small boxes fit into the bigger box?

10. The volume of a right circular cylinder is $252cm^3$ and the radius of the base is 4cm. Find is the height of the cylinder.

11. The areas of the bases of a cylinder are each 124cm2 and the volume of the cylinder is $116\pi\pi$cm3. Find the height of the cylinder.

12. Arega built a wooden, cubic toy box for his son. Each side of the box measures 13cm.

a) How many square centimeter of wood did he used to build the box?

b) How many cubic centimeters of toys will the box hold?

Answer

1. a. True b. True c. True d. False e. True f. False
2. a. 8
   b. has no edge
   c. 9
   d. 12.
3. a. cubes or rectangular prism
   .b. total surface area.
4. $S^2 = (9\text{ cm})^2 = 81^{\;2}$

   S = $\sqrt{81cm^2}$ = **9 cm**

   $A_L = Ph$

   $144\text{ cm}^2 = (9\text{ cm} + 9\text{ cm} + 9\text{ cm} + 9\text{cm}) \times h$

$=\frac{144\ cm^2}{36\ cm} = \frac{36\ cm \times h}{36\ cm}$, **h = 4 cm**

5. a. rectangular

b. 9 cm , 40 cm , 41 cm and 15 cm

c. $A_L = Ph$

$A_L = (9\ cm + 40\ cm + 41\ cm) \times 15\ cm$

$A_L = 90\ cm \times 15\ cm$

$= \underline{1350\ cm^2}$

$A_T = A_L + 2\ AB$ but $A_B = \frac{1}{2} \times 9\ cm \times 40\ cm$

$= 1350\ cm^2 + 2(180\ cm^2)$ $= \underline{180\ cm^2}$

$A_T = 1350\ cm^2 + 360\ cm^2$

$= \underline{1{,}710\ cm^2}$

6. $A_T = 2\pi r(h + r)$

$884\ \pi cm^2 = 4\pi cm(h + 2cm)$

$884\ cm^2 = 4cm\ (h + 2\ cm)$

$h + 2cm = \frac{884 cm^2}{4\ cm}$

$h + 2\ cm = 221\ cm - 2\ cm$

$h = \underline{219\ cm}$

7. r be radius and h be the height of cylinder.

$h + r = 9cm$

$A_T = 81\ cm^2$

$81\pi\ cm^2 = 2\pi r\ (h + r)$

$= 2\pi r(9\ cm)$

$\frac{81\pi\ cm^2}{9\ cm} = 2\pi r$

$9\pi\ cm = 2\pi r$

$r = \underline{4.5\ cm}$

8. $3x \times 2x \times x = 384\ cm^3$

$\frac{6x^3}{6} = \frac{384\ cm^3}{6}$

$x^3 = 64\ cm^3$

$L = 3x$ $w = 2x$ $h = x = \underline{4\ cm}$

$= 3 \times 4\ cm$ $= 2 \times 4\ cm$

$= \underline{12\ cm}$ $= \underline{8\ cm}$

$A_L = Ph = 2(l + w)h$

$= 2(12\ cm + 8\ cm)\ 4cm$

$= 2(20\ cm)\ 4\ cm$

$= 40\ cm \times 4\ cm$

$= \underline{160\ cm^2}$

$x = \sqrt[3]{64cm^3}$

$x =$ **4 cm**

$A_T = A_L + 2AB$

$= 160cm^2 + 2(96\ cm^2)$

$= 160\ cm^2 + 192\ cm^2$

$=$ **352 cm²**

9. $V_L = l \times w \times h$

$V_L = 16\ cm \times 11\ cm \times 9\ cm$

$= 1584\ cm^3$ and

$V_S = l \times w \times h$

$= 2\ cm \times 2\ cm \times 2\ cm$

$=$ **8 cm³**

The number of small boxes fit in to the bigger box is $\frac{1584\ cm3}{8\ cm^3} = 198$

10. $V = \pi r^2 h$

$V = \pi(4cm)^2 \times h$

$= 252\ \pi\ cm^3 = 16\pi\ cm^2 \times h$

$h = \frac{252cm^3}{16\ cm^2} \approx \mathbf{5\ cm}//$

11. $V = A_B \times h$

$116\pi\ cm^2 = 124\ cm \times h$

$h = \frac{116\ \pi\ cm^2}{124\ cm}$

$h \approx 3\ cm$

12. $A_T = 6S^2$

$A_T = 6(13\ cm)^2$

$=$ **1014 cm²**

$V = L^3$

$V = 13\ cm \times 13\ cm \times 13\ cm$

$=$ **2197 cm³**

UNIT 8

INTRODUCTION TO PROBABILITY

8.1. The concept of probability

Definition: An activity involving chance in which results are observed is called an experiment.

Definition: Each observation of an experiment is called a trial and each result of the experiment is an outcome.

Example:

Toss a coin once.

Experiment: Tossing a coin

Trial: one time

Outcome: Head (H), Tail (T)

Definition: The set of all possible outcomes of an experiment is called a sample space or possibility set of the experiment and is denoted by S.

Definition: Any set of outcomes of the experiment is called an event. That is, event is a subset of the sample space. Events will be denoted by the capital letters A, B, C, D, E, and so on.

Example:

Consider the experiment "throwing a die once".

(A die is a cube with each of its six faces marked with a different number ofdots from

one to six)

What are the possible outcomes?

a) Write the sample space.
b) b) Write the event of a number 5 appearing on the upper face.
c) c) Write the event of getting "the number shown on the upper face is even".
d) d) Write the event of getting "a number different from three".

e) e) Write the event of getting “a prime number and an even number”.

Solution:

a) The possible outcomes: 1, 2, 3, 4, 5, 6

b) S = {1, 2, 3, 4, 5, 6}

c) E = {5}

d) E = {2, 4, 6}

e) E = {1, 2, 4, 5, 6}

f) E = {2}

Exercise 8.1:

1) Discuss the following terms

a) Experiment

b) Possibility set

c) Event

2) Write a number 1 through 10 on a ten identical cards. Then select one card randomly from the cards.

a) What is the experiment?

b) Write the sample space.

c) Write the event of getting an odd number.

Answer

1. a. an activity involving chance in which results are observed is called an experiment.
   b. The set of all possible outcomes of an experiment are called possibility set.
   c. Any subset of outcomes of the experiment is called an event.
2. a. select one card randomly from the cards
   b. The sample space { 1,2,3,4,5,6,7,8,9,10}.
   c. The card number with {1, 3, 5,7,9}

Definition: Events (two or more) of an experiment are said to be equally likely, if any one of them cannot be expected to occur in preference to the others.

Example:

a) Getting a 1, 2, 3 on the toss of a die and getting a 4, 5, 6 on the toss of a die are equally likely because the chance of occurrence of each event are equal.

b) A bag has 10 balls of different colors and sizes. If you pick up a ball randomly from the bag, the chance of occurrence of all the balls are not necessarily the same. Such outcomes are not equally likely outcomes.

Definition: An event which is sure to occur at every performance of an experiment is called a certain event to the experiment.

Definition: An event which cannot occur at any performance of the experiment is called an impossible event to the experiment.

Example:

a) A teacher chooses a girl from a class of 30 girls is a certain event. Because all the students in the class are girls, the teacher is certain to choose a girl.

b) A spinner has 4 equal sectors colored yellow, red, green and blue. It is impossible to land on purple after spinning the spinner.

Exercise 8.2:

1) A glass jar contains 3 red, 6 blue and 4 green chocolates. If a chocolate is chosen at random from the jar, then which of the following is an impossible event?

a) Choosing a red chocolate.

b) Choosing a yellow chocolate.

c) Choosing a green chocolate.

2) A spinner has 6 equal sectors numbered 1 to 6. If you spin the spinner, then which of the following is a certain event?

a) Landing on a number greater than 1.

b) Landing on a number less than 7.

c) Landing on a number between 6 and 9.

3) Identify certain and impossible events

a) You will live to be 300 years.

b) Two lines intersect at one point.

c) The earth revolves around the sun.

Answer

1. Choosing a yellow chocolate is impossible because there is no yellow chocolate in a glass jar.
2. B.
3. a. impossible

   b. certain

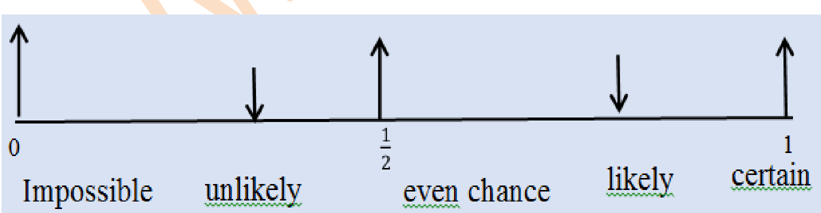
   c. certain

Definition: The probability of an event is the measure of the chance that the event will occur as a result of the experiment.

The probability of an event E , denoted by P(E), is a number between 0 and 1, inclusive, that measures the likelihood of an event in the following way:

- If P(E_1) > P(E_2) , then event $E1$ is more likely to occur than event $E2$
- If P(E_1) = P(E_2), then the events $E1$ and $E2$ are equally likely to occur.
- If event $E1$ is impossible, then P($E1$) = 0.
- If event $E1$ is certain, then P($E1$) = 1.

This can be shown on a probability scale, starting at 0 (impossible) and ending at 1(certain)



Example :

Decide whether or not each of the statements below is reasonable.

a) The probability that you will go to bed before midnight tonight is 0.99.

b) The probability that your pocket money is doubled tomorrow is 0.01.

Solution:

a) This is a reasonable statement as it is very likely that you will go to bed before midnight, but not certain that you will.

b) This is a reasonable statement, as it is very unlikely that your pocket money will be doubled tomorrow, but not totally impossible.

Note: probabilities are given on a scale of 0 to 1, as decimals or fractions; sometimes probabilities are expressed as percentages using a scale of 0% to 100% particularly on weather forecasts.

8.2. Probability of Simple events

Definition

Let S be the possibility set of an experiment and each element of S be

equally likely to occur. Then the probability of the event E occurring

is given by p(E) = $\frac{number\ of\ favourable\ out\ comes}{total\ posible\ out\ comes} = \frac{n(E)}{n(S)}$

Example:

A box contains 5green balls and 3black balls. If one ball is drawn at random, what is the probability of getting a

a) Black ball b) Green ball

Solution:

Let event B = a black ball appears and event G = a green ball appears.

There are 8 possible outcomes. Then

a) P(B) = $\frac{n(E)}{n(S)} = \frac{3}{8}$ b) P(G) = $\frac{n(EE)}{n(s)} = \frac{5}{8}$

Example:

A die is thrown once. What is the probability that the number appearing on the upper face will be

a) 6? b) 7?

c) a number greater than 3 d) a number smaller than 9

Solution:

There are 6 possible outcomes. Hence, n(S) = 6

a) Only one of these outcomes is 6. Hence, n(E) = 1

Example:

A die is thrown once. What is the probability that the number appearing on the upper face will be

a) 6? b) 7?
c) a number greater than 3 d) a number smaller than 9

Solution:

There are 6 possible outcomes. Hence, n(S) = 6

a) Only one of these outcomes is 6. Hence, n(E) = 1

Therefore, P(E) = $n(E)$

$n(S) = \frac{1}{6}$

b) 7 do not appear on the experiment. Hence, n(E) = 0

Therefore, P(E) = $\frac{n(E)}{n(S)} = 0$

c) 4, 5, 6 are the required elements. Hence, n(E) = 3

Therefore, P(E) = $n(E)$

$n(S) = \frac{3}{6} = \frac{1}{2}$

d) All possible outcomes are smaller than 9. Hence n(E) = 6

Therefore, P(E) = $\frac{nE)}{n(E)} = \frac{6}{6} = 1$

Example:

A card is selected at random from a pack of 52 playing cards. What is the probability that it is:

a) A red card d) an even number
b) A queen e) the 7 of hearts
c) A red “Ace”

Solution:

A standard deck of playing cards consists of 52 cards in each of the four suits of Spades, Hearts, Diamonds, and clubs. Each suit contains 13 cards: Ace, 2, 3, ….10, Jack, Queen, and King.

As each card is equally likely to be drawn from the pack,

a) There are 26 red cards in the pack, so:

P(red) = $\frac{26}{52} = \frac{1}{2}$

b) There are 4 Queens in the pack, so:

P(queens) = $\frac{4}{52} = \frac{1}{13}$

There are 2 red Aces in the pack, so: P(red Ace) = $\frac{2}{52} = \frac{1}{26}$

c) There are 20 cards that have even numbers in the pack, so:

P(even number) = $\frac{20}{52} = \frac{5}{13}$

d) There is only one 7 of hearts in the pack, so:

P(7 of hearts) = $\frac{1}{52}$

Example:

Toss a coin twice (or toss two coins once at a time).

a) Find the possible out comes

b) Find the probability of getting exactly two heads.

c) Find the probability of getting at least one tail.

Solution:

a) The possible outcomes are: HH, HT, TH, TT.

Therefore, the total number of possible outcomes, n(S) = 4

b) The favorable outcome for the event "exactly two heads" is HH.

Hence, the total number favorable outcomes = 1

Therefore, P(exactly two heads) = $\frac{1}{4}$

c) The favorable outcomes for the event "at least one tail" are HT, TH, TT.

Hence, the total number favorable outcomes = 3

Therefore, P(at least one tail) = $\frac{3}{4}$

Example:

Determine the probability that the sum 7 appear in a single toss of a pair of fair dice?

Solution:

Each of the six faces of one die can be associated with each of the six faces of the other die, so that the total number of cases that can arise are all equally likely is 6× 6 = 36. These can be denoted by (1,1), (2,1),….,(6,6)

There are six ways of obtaining the sum 7, denoted by (1,6), (2,5), (3,4), (5,2), (4,3), and (6,1).

Thus p(sum 7) = $\frac{6}{36}$

= $\frac{1}{6}$

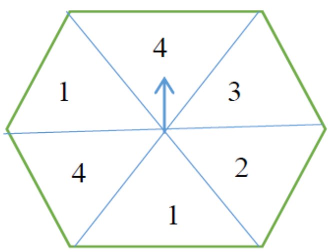
Exercise 8.3

1) A spinner is numbered as shown below in the diagram. Each score is equally likely to occur. What is the probability of scoring

a) 1 c) 3

b) 2 d) 4

e) a number less than 5?



2) A letter is chosen at random from the word" ETHIOPIA". Find the probability that it will be:

a) the letter I?

c) the letter B?

Answer

1 a. p (€) = $\frac{n\,(E)}{n\,(s)} = \frac{2}{6} = \frac{1}{3}$

b.p (E) = $\frac{n\,(E)}{n\,(s)} = \frac{1}{6}$

c. p(€) = $\frac{n\,(E)}{n\,(s)} = \frac{1}{6}$

d. p (E) = $\frac{n\,(E)}{n\,(s)} = \frac{1}{6}$

e. p (E) =$\frac{n\,(E)}{n\,(s)} = \frac{6}{6}$ = 1

2. a. n(letters I) = 2 and n (s) = 8

P(E) = $\frac{n\,(E)}{n(s)} = \frac{2}{8} = \frac{1}{4}$

b. N (letters B) = 0 , p (E) = 0

Definition:

The probability of an event is the ratio of the number of successful outcomes in the event to the total number of possible outcomes in the experiment.

Thus, P(Event) =$\frac{number\ of\ times\ the\ event\ occor}{total;number\ of\ ovservations}$

Example:

If record show that getting 120 tails out of 10,000 trial of throwing a coin, then the probability of getting another tail is given by:

P(Event) =$\frac{number\ of\ times\ the\ event\ occor}{total;number\ of\ ovservations}$

=$\frac{120}{10000}$

= 0.012

8.3. Applications on Business, Climate, Road Transport, Accidents and Drug Effects

Example:

At a car park there are 52 vehicles, 26 of which are cars, 8 are vans and
the remaining Lorries. If every vehicle is equally likely to leave, find the probability of

a) A van leaving first.

b) A lorry leaving first.

Solution:

a) Let E be the event of van leaving first. Then n(E) = 8

P(van leaving first) = P(E) = $\frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

b) Let A be the vent of a lorry leaving first. Then

n(A) = 52 – 26−8 = 18

P(lorry leaving first) = P(A) = $\frac{n(E)}{n(S)} = \frac{18}{52} = \frac{9}{26}$

Example:

A factory has 2 machines A and B producing 300 and 720 bulbs per day respectively. A produces 1% defective and B produces 1.5% defective.

A bulb is chosen at random at the end of a day and found defective. What is the probability that

a) Machine A produces a defective bulb

b) Machine B produces a defective bulb.

Solution:

a) Number of defective bulbs produced by machine

A = 1% × 300 = 3

Therefore, the probability of the chosen bulb is defective is

P(defective bulb) = $\frac{3}{300}$ = 0.01

b) Number of defective bulbs produced by machine

B = 1.5% × 720 = 108

Therefore, the probability of the chosen bulb is defective is

P(defective bulb) $\frac{= 108}{720}$= 0.15

REVIEW EXERCISE FOR UNIT 8

1) Write true if the statement is correct and false if it is not.

a) The probability of an event that is sure to occur is $\frac{1}{2}$.

b) If three coins are thrown at a time, then n(S) = 8

c) If the set of all possible outcomes is an event, the probability of an event is 1.

d) The range of the values of the probability of an event is between 0 and 1.

2) Answer the following questions.

a) What is a probability experiment?

b) Define sample space.

c) What is the difference between an outcome and an event?

d) What are equally likely events?

e) If an event cannot happen, what value is assigned to its probability?

3) Which of the following statements describe a certain event?

a) Roll a die once and the event of getting a number less than 7.

b) Toss a coin twice and the event of getting two tails.

c) Out of a family of 4girls the event of selecting two boys.

4) Suppose you write the days of the week on identical pieces of paper. Mix them and select one at a time. What is the probability that the day you select will have

a) The letter r in it.

b) The letter d in it.

c) The letter b in it.

5) A number is chosen out of the numbers 1 through 30. What is the probability that it is:

a) A multiple of 6?

b) A perfect square?

6) A coin has two sides, heads and tails

a) Surafel is going to toss a coin. What is the probability that Surafel will get heads? Write your answe as a fraction and decimal.

7) If a die is rolled one time, find these probabilities

a) Of getting a 4

b) Of getting an even number

c) Of getting a number greater than 4

d) Of getting a number less than 7

8) Suppose a pair of dice is thrown.

a) Set up a sample space.

b) What is the probability that

i) The sum of the scores is 7?

ii) Both numbers are the same?

iii) The first number is odd and the second number is even.

iv)A sum is greater than 9

9) There are 18 tickets marked with numbers 1 to 18. What is the probability of selecting a ticket having the following property?

a) Even number

b) Number divisible by 3

c) Prime number

d) Number divisible by 4

10) There are 20 pens in a box, 13 of the pens are blue. 7 of the pens are black.

If one pen is chosen random, what is the probability of getting

a) Blue pen?

b) Red pen?

c) Both color?

11) A spinner is marked with the letters A, B, C, and D, so that eachletter is equally likely to be obtained. The spinner is spun twice.

a) List the 16 possible outcomes.

b) What is the probability that letter is equally likely to be obtained. The spinner is spun twice.

a) List the 16 possible outcomes.
b) What is the probability that
i) A is obtained twice
ii) A is obtained at least once.
iii) both letters are the same.
iv) the letter B is not obtained at all

12) In a class there are 12 boys and 14 girls. The teacher calls on a

student at random to read a theorem stated on a text book. What is the probability that the first person called a girl?

13) There are seven cups of coffee on a table. Three of them contain

sugar. What is your chance of choosing a cup with sugar in it?

Answer

1. a. False b. True c. True d. False
2. b. a sample space is a collection or a set of possible outcomes of random experiment.
3. A.
4. Sample space = { Monday , Tuesday ,Wednesday , Thursday , Friday ,Saturday , Sunday }

   n(s) = 50

   a. n(E) = 3

   P (€) = $\frac{n\,(E)}{n\,(s)} = \frac{3}{50}$

   b. n (E) = 8

   p (E) = $\frac{n\,(E)}{n\,(s)} = \frac{8}{50}$

   c. n (E) = 0

   p (E) = $\frac{n\,(E)}{n\,(s)} = \frac{0}{50} = 0$

5. S = (1,2,3,4,5,6,7,8,9,10,11,12,13 …..30}

n(s) = 30

a. E = (6,12,18,24,30}

P(a Multiple of 6) = $\frac{n\,(E)}{n\,(s)} = \frac{5}{30} = \frac{1}{6}$

b. E = {1,4,9,16,25} , n(E) = 5

P(a perfect square) = $\frac{n\,(E)}{n\,(s)} = \frac{5}{30} = \frac{1}{6}$

6. a. S = (H,T) , n (S) = 2

E = {H} , n(E) = 1

P(getting heads) = $\frac{n\,(E)}{n\,(s)} = \frac{1}{2} = 0.5$

b. p(E) = $\frac{1}{4}$

7. S = {1,2,3,4,5,6, } , n(S) = 6

a. E = {4} , n(E) = 1

P(E) = $\frac{n(E)}{n(S)} = \frac{1}{6}$

b.E = {2,4,6 } , n(E) = 3

p (E) = $\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

c. E = {5,6} , n (E) = 2

P(E) = $\frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

d. E = {1,2,3,4,5,6} , n(E) = 6

P(E) =$\frac{n(E)}{n(S)} = \frac{6}{6} = 1$

8.a. S = ((1,1) ,(1,2) ,(1,3) ,(1,4) ,(1,5),(1,6)

(2,1) ,(2,2) ,(2,3) ,(2,4) ,(2.5),(2,6)

(3,1) ,(3,2) ,(3,3),(3,4),(3,5) ,(3,6)

(4,1),(4,2), (4,3) ,(4,4) ,(4,5) ,(4,6)

(5,1) ,(5,2) ,(5,3) ,(5,3) ,(5,4),(5,5),(5,6)

(6,1) ,(6,2) ,(6,3) ,(6,4) ,(6,5),(6,6)

n(S) = 36

b. i. e = {(1,6) ,(2,5) ,(3,4),(5,2) ,(6,1) n (E) = 6

P(E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

ii. E = {(1,2) ,(1,4) ,(1, 6),(3,2) ,(3,4) ,(3,6) ,(5,2) ,(5,4) ,(5,6) } n(E) = 9

p(E) =$\frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

iii. E = { (4,6) ,(5,5),(5,6),(6,4) ,(6,5) ,(6,6) }

n(E) = 6

P(E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

9.S = (1,2,3,4,5------- 18} n(E) = 6

a. E = {2,4,6,8,10,12,14,16,18} , n(E) = 9

P(E) = $\frac{n(E)}{n(S)} = \frac{9}{18} = \frac{1}{2}$

b. E = {3,6,9,12,15,18} , n(E) = 6

P(E) = $\frac{n(E)}{n(S)} = \frac{6}{18} = \frac{1}{3}$

c. E = {2,3,5,7,11,13,17 } , n(E) = 7

P(E) = $\frac{n(E)}{n(S)} = \frac{7}{18}$

d. E = {4,8,12,16} , n(E) = 4

P(E) = $\frac{n(E)}{n(S)} = \frac{4}{8} = \frac{2}{9}$

10.n(S) = 20

a. n(E) = 13

P(E) = $\frac{n(E)}{n(S)} = \frac{13}{20}$

b. n(E) = 0

P(E) = $\frac{n(E)}{n(S)} = \frac{0}{18} = 0$

c. n(E) = 20

P(E) = $\frac{n(E)}{n(S)} = \frac{20}{20} = 1$

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11. a. n(S) = 16

b. i. p(E) = $\frac{n(E)}{n(S)} = \frac{1}{16}$

ii. P(E) = $\frac{n(E)}{n(S)} = \frac{7}{16}$

iii. P(E) = $\frac{n(E)}{n(S)} = \frac{9}{16}$

12. P(E) = $\frac{n(E)}{n(S)} = \frac{7}{13}$

13. $\frac{3}{7}$