Final Project: MECH 6371

Computational Thermal and Fluid Science

Group 3

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1. Introduction

A MATLAB subroutine was developed as an initial part of this project to evaluate the Reynold's stresses in a 3-dimensional channel flow using the mixing length model. This subroutine needs an input of the three velocity components which are assumed to be time averaged. Prandtl's mixing length model was used to evaluate the eddy viscosity based on the absolute value of the gradient of the streamwise velocity component. The strain rate tensor was then evaluated later and directly used to evaluate the Reynold's turbulent stress components. A time-averaged DNS dataset was also used as a sample velocity field input to validate this subroutine.

A user input for the consideration of damping near the wall was also added to the program in order to compute the stress components in the presence of *Van Driest* damping. In this case, the mean streamwise velocity was used to compute y+ and the dependence of the mixing length in the wall normal direction. The eddy viscosity as a function of the wall normal coordinate was then computed. The respective Reynold's stresses were also computed using this eddy viscosity.

As a final part of the project, an attempt was made to develop an explicit nondimensional 3D Reynold's Averaged Navier-Stokes (RANS) solver using the Poisson pressure correction method. The subroutine to compute the turbulent Reynold's stresses was then integrated with the RANS solver. The advective and viscous terms were computed in a separate subroutine. The pressure is computed imposing the continuity equation and solving the resulting Poisson's equation using relevant boundary conditions. The last step included computing the velocity components at the next timestep using discretized momentum equations after which the divergence criteria was then verified.

The rest of this report describes in detail the methodology and setup, numerical schemes used, respective results and discussions.

The description of the mixing length for the primary objective of the project is detailed in sections 2.1-2.2 (methodology) and section 2.3.3 (code description).

2. Methodology and setup

2.1 Mixing length model

According to the Boussinesq hypothesis, the turbulent stresses are related to the rate of mean strain by the turbulent or *eddy* viscosity. Using mass conservation and incompressibility,

$$Strain\ rate\ tensor = S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad (1)$$

$$Reynold's\ stress\ tensor = -\rho\overline{u'_\iota u'_J} = 2\mu_T S_{ij} - \frac{2}{3}\delta_{ij}\left(\mu_T \frac{\partial u_k}{\partial x_k} + \rho\bar{k}\right) \qquad (2)$$

Using mass conservation,

$$-\rho\overline{u'_\iota u'_J} = 2\mu_T S_{ij} - \frac{2}{3}\delta_{ij}\left(\rho\bar{k}\right) \qquad (3)$$

$$Turbulent\ Kinetic\ Energy = \bar{k} = \frac{1}{2}\overline{u'_\iota u'_\iota} \qquad (4)$$

According to the Prandtl's mixing length model, the eddy viscosity is defined as:

$$\mu_T = \rho l^2 \left|\frac{\partial u}{\partial y}\right|;\ \ l = mixing\ length \qquad (5)$$

where u is the streamwise velocity and y is the wall normal direction. It is important to note that for the sake of simplicity, all the parameters including the three velocity components and pressure are formulated to be at the center of each cell, i.e. the first node in the domain is not at the wall but rather at $\frac{\Delta x}{2}, \frac{\Delta y}{2}, \frac{\Delta z}{2}$ distance away from the domain limit. The input velocity is assumed to follow this coordinate system for simplicity. The velocity gradient is calculated using central difference scheme at the inner points and forward/backward difference scheme at the nodes near the lower/upper wall respectively using no slip boundary condition. This helps us to exploit the no slip boundary condition in getting the gradients at the end points for the wall normal direction. Thus, we get a matrix of the eddy viscosity at all nodes in the domain.

The mixing length l is evaluated using the Prandtl's mixing length model, where damping is constant in the center region of and decays to linearly at each wall. mixing length model can be defined as the following:

$$\ell = \kappa d \qquad for\ d < 0.2\delta$$
$$\ell = 0.089(\delta) \qquad for\ d > 0.2\delta$$

$$\kappa = 0.41 \qquad d = distance\ from\ top\ or\ bottom\ wall$$

The strain rate tensor calculation involved evaluating the gradients in all three directions. For the spanwise and streamwise gradients, the central difference scheme was used everywhere including at the first and last nodes using periodic boundary conditions. For the wall normal gradients, a no slip boundary condition was used at the wall using forward/backward scheme and central difference at the inner nodes.

It is important to note that the Reynold's stress calculation involves an estimate of the TKE term in the equation (3) for the diagonal elements. This parameter is initialized to have a default value of 1 but because of the averaged velocity input, the diagonal elements of the Reynold's stress are anyway zero. Hence the stress components are calculated as follows:

$$\tau_{ij} = -\rho\overline{u'_i u'_j} = 2\mu_T S_{ij}; \qquad (i \neq j) \qquad (6)$$

The sample averaged DNS velocity dataset is a time averaged result of a flat channel simulation at Re_b=2800, where Re_b is the Reynolds number based on the bulk velocity and the half height of the channel.

2.2 Damping near the wall

The *Van Driest* damping induces a variation of the mixing length value as a function of the wall normal distance from a wall. The mixing length is formulated as follows:

Near the wall,

$$l_i(y) = \kappa\, y\left(1 - e^{-\frac{y+}{A+}}\right) \qquad (7)$$

In the outer region,

$$l_o(y) = C_1\delta = Constant \qquad (8)$$

where, $A^+ = 26$, $C_1 = 0.089$, $\kappa = 0.41$, $\delta = Channel\ half\ height$ and the $y^+$is estimated as follows:

$$y^+ = y\sqrt{\frac{1}{Re}\frac{\partial u}{\partial y}} \qquad (9)$$

The distinction of the inner and outer region has been formulated when $l_i(y) > l_o(y)$. Here, the wall normal gradient is calculated in the same way explained in section 2.1. Using this, we thus get the estimate of the mixing length at all wall normal nodes in the domain. It is important to note that the damping criteria has been considered for both – lower and upper walls. In the MATLAB script, a user input parameter named *damping* is created and is assigned a value of 1 if damping is supposed to be considered. Else it is set to 0. The rest of the calculations of the strain rates and gradients are similar to section 2.1.

2.3 3D Explicit RANS solver

Seen below are the primary equations for the 3D RANS Solver. As detailed below, code was developed to model these equations and generate a steady state solution. Figure 1 shows an example of the flow environment, with a fixed top and bottom wall, an open inlet and outlet, as well as an infinite (modeled as periodic) spanwise direction. The region is symmetric about the centerline of the domain. Seen in figure 2 is a flow chart for the RANS solver code. It is important to note that several other functions were also created for this code, but they are used to calculate simple items such as a gradient and divergence with consideration to the applied boundary conditions, and thus were not included in the flow chart. All custom functions for the 3D RANS solver have been included in the ZIP folder for this project. The main functions and their purposes are also detailed below.

Incompressible Navier Stokes $$\partial_t(\rho\bar{u}_i) + \partial_{x_j}\left(\rho\bar{u}_i\bar{u}_j\right) = \partial_{x_i}\bar{P} + \partial_{x_j}(\bar{\tau}_{ij} - \rho\overline{u_i'u_j'}) \quad (10)$$

Reynolds Stress Model $$-\rho\,\overline{u_j'u_j'} = 2\mu_T S_{ij} - \frac{2}{3}\delta_{ij}\left(\mu_T\partial_{x_k}u_k + \rho\bar{k}\right) \quad (11)$$

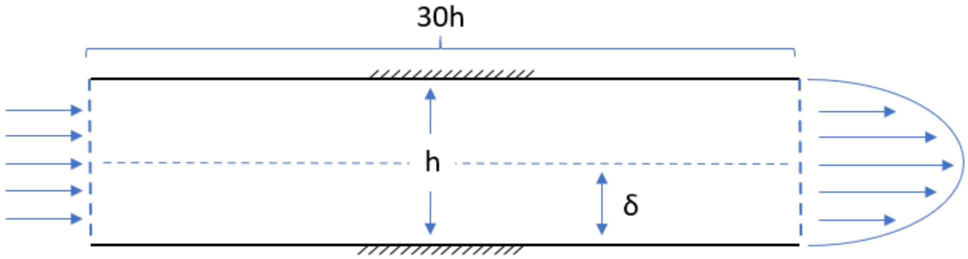

Figure 1: Sample setup for the simulation

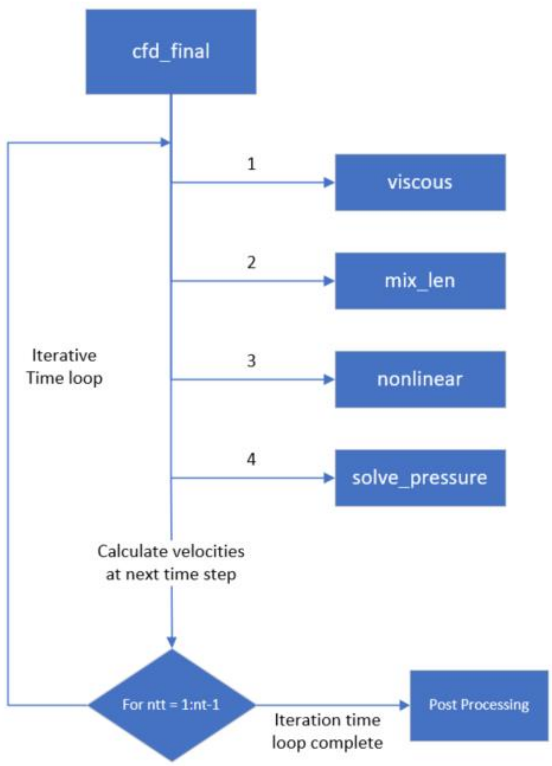

Figure 2: 3D Rans Solver Flow Chart

2.3.1 cfd_final

Variable Nomenclature

Table 1: Primary variables used in the RANS solver

Variable Name	Size	Description
n1,n2,n3	--	Number of nodes in spanwise, vertical, and streamwise directions
alx1,alx2, alx3	--	Length of region in spanwise, vertical, and streamwise directions
dx,dy,dz	--	Distance between each node in the spanwise, vertical, and streamwise directions
dt		Time increment for the respective time step
Re	--	Reynolds number for the given flow
TI	--	Turbulent intensity
Tmax	--	Maximum non dimensional time
u,v,w	(n1,n2,n3)	Spanwise, vertical, and streamwse velocity component for each cell
x,y,z	n1 , n2, n3	Coordinates in the spanwise, streamwise, and vertical directions
ke	(n1,n2,n3)	Calculated turbulent kinetic energy at each cell
damping	--	Defines the type of turbulent model (0 = constant mixing length) (1 = mixing length with damping near the wall)
H_ij		
H_uu	(n1,n2,n3)	Diagonal shear stress component in the spanwise direction **(lam + turb)**
H_uv	(n1,n2,n3)	Spanwise shear stress component normal to the vertical direction **(lam + turb)**
H_uw	(n1,n2,n3)	Spanwise shear stress component normal to the streamwise direction **(lam + turb)**
H_vv	(n1,n2,n3)	Diagonal shear stress component in the normal direction **(lam + turb)**
H_vw	(n1,n2,n3)	Vertical shear stress component normal to the streamwise direction **(lam + turb)**
H_ww	(n1,n2,n3)	Diagonal shear stress component in the streamwise direction **(lam + turb)**
Tau_ij		
tau_uu	(n1,n2,n3)	**Turbulent** Diagonal shear stress component in the spanwise direction
tau_uv	(n1,n2,n3)	**Turbulent** Spanwise shear stress component normal to the vertical direction
tau_uw	(n1,n2,n3)	**Turbulent** Spanwise shear stress component normal to the streamwise direction
tau_vv	(n1,n2,n3)	**Turbulent** Diagonal shear stress component in the normal direction
tau_vw	(n1,n2,n3)	**Turbulent** Vertical shear stress component normal to the streamwise direction

tau_ww	(n1,n2,n3)	**Turbulent** Diagonal shear stress component in the streamwise direction
H_i		
H_u	(n1,n2,n3)	Summation of nonlinear and shear stress components in the spanwise direction
H_v	(n1,n2,n3)	Summation of nonlinear and shear stress components in the vertical direction
H_w	(n1,n2,n3)	Summation of nonlinear and shear stress components in the streamwise direction
Pressures		
p	(n1,n2,n3)	Corrected pressure at each cell in the volume
px	(n1,n2,n3)	Calculated pressure gradient in the spanwise direction
py	(n1,n2,n3)	Calculated pressure gradient in the vertical direction
pz	(n1,n2,n3)	Calculated pressure gradient in the streamwise direction

Description

The function **cfd_final** is the primary script for running the 3D explicit RANS solver. All starting values are initialized here, and all functions are called here. Following the call of the function, plotting is also performed. Variables that are needed are first initialized here to define the domain, the boundary conditions, method of turbulent modeling, and initial conditions. The first time step is calculated based upon the initial conditions of the flow. The time loop is initialized to run for a specific number of iterations based upon the final time step, and the first time step. For the entire code, u,v,w (1,2,3 or i,j,k) correspond to the spanwise, vertical, and streamwise components respectively. For simplicity, the code used nodes with all values referred to at the center of the cell and did not use a staggered grid.

For each iteration of the loop, the components necessary to calculate the velocity at the next time are found. All major components are calculated in separate functions.

The stresses are first found. The function **viscous** is called to calculate the components of the Reynolds Shear stress tensor. The symmetric components (12,13,23) are calculated and held as symmetric. The turbulent shear stress components are calculated using the function **mix_len.** The turbulent kinetic energy is calculated from the returned diagonal components of the turbulent stress and is sent to **mix_len** on every iteration. For the first time step, the turbulent kinetic energy is assumed to be zero.

It is important to note that while the project listed a mixing length as an input for the **mix_len** function, this code does not include that. This is because the function calculates both versions of the damping, and thus instead of the mixing length as an input, the type of damping to be performed is instead set. Based upon this choice of damping, the results can reflect either the normal project assignment, or the extra credit portion. Further information on the damping variable can be seen in section 3.3.

The function **nonlinear** calculates the nonlinear components of the Navier Stokes equations and returns a summation of the stress components and the nonlinear components. These H components are then used to calculate the pressure correction, and pressure gradients at each cell in accordance with the Poisson Equation. This pressure correction is necessary to achieve a divergence free flow.

Due to the boundary conditions imposed in the pressure correction function, there is a loss of energy at the walls of the fluid, and no pressure gradient to impose a flow. To continue to drive the flow, a force is applied in the streamwise direction based upon the calculated shear stress components at the wall. Without this imposed force, the flow would die out and not continue.

The velocity components are then calculated using the Navier Stokes equation with the force included in the streamwise component. The divergence of the flow is then checked to make sure it is within tolerances. If the flow is outside the tolerances, the time loop iteration is canceled. Otherwise, the loop continues. Following the time loop, the pressure and velocity components are plotted at different time steps.

2.3.2 Viscous

This function calculated the Reynolds shear stress components for the flow. The diagonal components are first calculated using the first order central differencing scheme. In the spanwise and streamwise direction, periodic boundary conditions are applied. However, in the vertical direction, the wall must be considered. At the top and bottom cells which are closest to the wall, the central differencing scheme cannot be used. Instead, a forward and backward differencing was used at the bottom and top walls respectively to model the no slip condition. An example of this modeling is shown below in figure 3 and equation 13. At the wall, the velocity is known to be zero, and the velocity gradient at the cell is calculated.

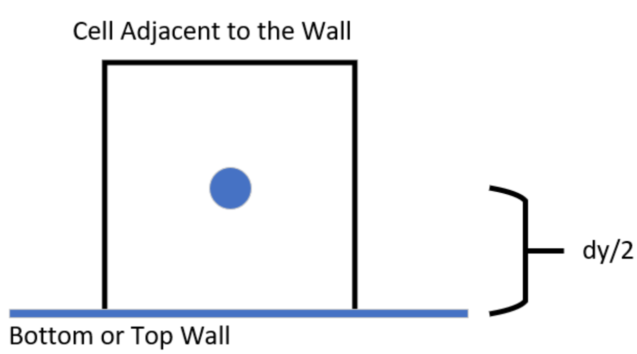

Figure 3 : Example of the boundary condition at the wall for terms calculated from velocity changes in the y direction

$$\frac{\partial u}{\partial y}\Big|_{j=1} = \frac{u_{i,1,k} - 0}{dy/2} \qquad (13)$$

The symmetric components of the stress tensor must also be calculated. The entire tensor for the Reynolds shear stress is shown. Each component of the symmetric portion was calculated individually, and then summed together. The boundary conditions at the top and bottom wall were again used for the necessary derivatives (any derivative in the vertical, y direction), while the periodic boundary conditions were used for the remaining components.

$$S_{ij} = \frac{1}{2}\left(\nabla \mathrm{u} + \nabla \mathrm{u}^{\mathrm{T}}\right) = \frac{1}{2}\begin{bmatrix} 2\,\partial_x u & \partial_x v + \partial_y u & \partial_x w + \partial_z u \\ & 2\partial_y v & \partial_z v + \partial_y w \\ & & 2\partial_z w \end{bmatrix}$$

2.3.3 mix_len

This function is the primary submission of the project outside of the extra credit 3D RANS solver. Its primary purpose is to take the 3 velocity components at each iteration and return the calculated turbulent shear stress components. The function receives 5 different variables, which are the three velocity components, the turbulent kinetic energy from the last iteration, and the type of damping that is being performed. The theory behind the model used for the turbulent stress and mixing length modeling is discussed in **Section 2.1.**

Depending upon the input of the damping variable, the function will perform two separate types of modeling, which are described below. The damping value should be either 0 (constant mixing length) or 1 (varying mixing length with damping near the wall). Because the damping variable defines the type of damping in the function, no mixing length value is set as an input. Instead, the mixing length for the case of damping equals 0 is set as a constant, which can be changed within the function.

Damping = 0 (Main Project Deliverable)

With damping equal to zero, the main project deliverable is satisfied. Here, with damping = 0, the mixing length is set to a constant value. For a standard no damping model, the mixing length must still decay to 0 at the walls, and thus a linear trend is used by 0.217*y. Using this mixing length trend, the turbulent viscosity is then found at every point in the normal direction using Prandtl's mixing length model (**Section 2.1**). The Reynolds shear stress components (S_ij) are then found for at each node. As mentioned previously, periodic boundary conditions were applied in the spanwise and streamwise directions, while a forward and backward differencing as seen in figure 3 was applied in the normal direction. Lastly, the summation of the components as defined in the Reynolds stress approximation is calculated and returned to **cfd_final**. It is important to note that the divergence is assumed to be zero is this function, and thus the divergence in the stress approximation cancels.

Damping = 1 (Extra Credit Deliverable)

A damping value equal to 1 satisfies the first extra credit portion of the project. In this scenario, damping now must be considered at the wall, and through if statements, the code includes further calculations for the mixing length. Instead of setting a constant mixing length value, the Van Driest damping is applied near the wall. Far away from the wall, the mixing length is set to constant. The Van Driest mixing length is only applied where the value is found to be below the outer layer mixing length (Detailed in section 2.2). The Turbulent viscosity was then found through this damped mixing length. Lastly, the stresses were calculated in the same manner as for the constant mixing length case, and then sent back to **cfd_final**.

2.3.4 Nonlinear

The nonlinear components of the Navier Stokes equation are necessary for finding the three velocity components at each node. This function sums the nonlinear components and the gradient of the stress components. These summed components are labeled H_u, H_v, and H_w for each direction. The function first calculates all of the components for the spanwise direction and combines them. It then does the same for the vertical and streamwise directions.

First, the gradients of the stresses are calculated. Because the stresses are not 0 at the wall, the method used to calculate the stresses directly as shown in the **viscous** function cannot be used. Instead, a forward and backward differencing function can be employed to rely on the points further away from the wall. This allows a standard function to be used, which is called **gradient1**. The gradient function that is created by MATLAB was not used because it flips the x and y directions. A figure of the employed forward and backward differencing when calculated the gradient of the stresses is shown below. Once the gradients of the stresses were returned, the appropriate boundary conditions were applied.

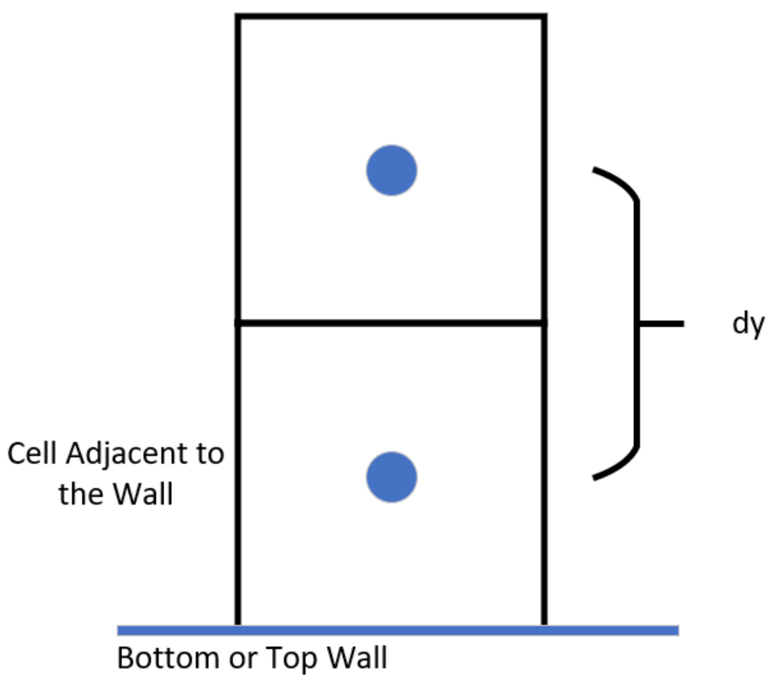

Figure 4 : Example of the boundary condition at the wall for forward and backward differencing.

While the gradient of the stresses can be calculated using a standard function, the calculation of the nonlinear components was performed directly because of the combination of the two velocity components. Due to the velocity being 0 at the wall, the differencing method used in the y direction and described in the **viscous** function was again used. After all the components were calculated in the spanwise direction, they were summed together. These steps were repeated for the vertical and streamwise direction, then sent back to the function **cfd_final**.

2.3.5 solve_pressure

The nonlinear, viscous shear stress, and turbulent shear stress terms are first calculated from the initial velocity profiles. The pressure must then be recalculated from these values to ensure a divergence free flow. Seen below in equation 14 is the Poisson pressure correction equation, which was used to find the pressure gradients for every point in the domain. H corresponds to the sum of the three Navier Stokes components (nonlinear, viscous stresses, and turbulent stresses) and is shown in equation 15.

$$\frac{\delta}{\delta \mathrm{x_i}}\left(\frac{\delta p^n}{\delta x_i}\right) = \frac{\delta H_i}{\delta x_i} \qquad (14)$$

$$H_i = -\frac{\delta(\rho u_i u_j)}{\delta x_j} + \left(\frac{\delta \tau_{ij}}{\delta x_j}\right)_{lam} + \left(\frac{\delta \tau_{ij}}{\delta x_j}\right)_{turb} = -\frac{\delta(\rho u_i u_j)}{\delta x_j} + \left(\frac{\delta \tau_{ij}}{\delta x_j}\right)_{lam} + \frac{\delta\left(-\rho \overline{u_i' u_j'}\right)}{\delta x_j} \quad (15)$$

Defining the Poisson equation in a 3-Dimensional field yields a system of equations that are dependent upon x, y, and z. The discretized equation for each pressure value at a node inside the volume is shown in equation 16 below. The equation shown is the second order central differencing method, which calculates the pressure at point (i,j,k) using the pressure values on either side, in all three coordinate directions. These values are not known to start however and lead to the formation of an *A_matrix* to contain all these values, and a vector containing the known values of the divergence of H. This can then be solved in MATLAB as a system of equations.

$$\frac{p_{i+1}-2p_i+p_{i-1}}{dx^2} + \frac{p_{j+1}-2p_j+p_{j-1}}{dy^2} + \frac{p_{k+1}-2p_k+p_{k-1}}{dz^2} = \frac{\delta H_i}{\delta x_i} \quad (16)$$

While this equation applies for a large portion of the flow volume, certain considerations must be made at the walls. First, the streamwise and spanwise walls must consider periodic boundary conditions. For example, at k = 1, (i.e. the first xy plane of the flow) k – 1 = 0 is not known. Therefore, the pressure will rely upon the pressure value at the outlet. The same also holds for the spanwise walls where i = 1, and i = n1. Along with these periodic boundary conditions, the inclusion of the top and bottom solid walls of the channel must also be factored into the equations. However, a central differencing scheme cannot be used since j-1 at the top wall, and j + 1 at the top wall are not known. Instead it is known that the pressure gradient at the top and bottom walls must be zero. To enforce this, the node directly adjacent to the top and bottom walls can be set equal to the next node in the normal direction. This is shown in the figure below, and applies to both walls.

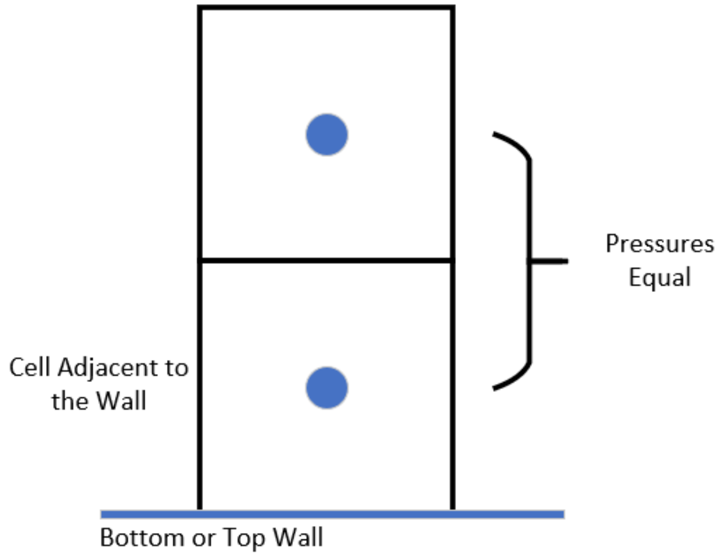

Figure 5 : Example of the pressure relation for the top and bottom walls.

Bottom Wall

$$p_j = p_{j+1}$$

Top Wall (Backward Differencing)

$$p_j = p_{j-1}$$

Lastly, the pressure equations can all be combined, but cannot be solved accurately in this state. Since all pressures are unknown, the equations cannot determine a relative size of the pressures, and some pressure values in the set of equations must be defined. For this code, the outlet was defined to have a nondimensional pressure of 1 (i.e. all pressure values of i,j at k = n3 were set to 1). Therefore, the periodic boundary condition in the spanwise direction only applied for k = 1, while the value at k = n3 were set to constant. Seen below is a sample plane in the A matrix for the pressures. The b,c, and d values correspond to x,y,z second order approximations (Equation 16), while the 1 and –1 components relate to the assignment of pressure as discussed above at the top and bottom rows.

K = 1												K = 2												K = 3											
J = 1				J = 2				J = 3				J = 1				J = 2				J = 3				J = 1				J = 2				J = 3			
i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3	i = 4
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Figure 6 : Sample of the A matrix for an internal xy plane of the flow.

3. Results

3.1 Mixing length model

Figure (7) shows the mixing length *(l)* along the vertical direction. The figure on the left reports the case when no damping (No Van Driest damping) at the wall is applied. The figure on the right shows instead *l(y)* when the Van-Driest damping at the wall is applied. The effect of the damping can be observed in the region close to the wall where the derivative of *l* in the wall normal direction obtained from the two model are different.

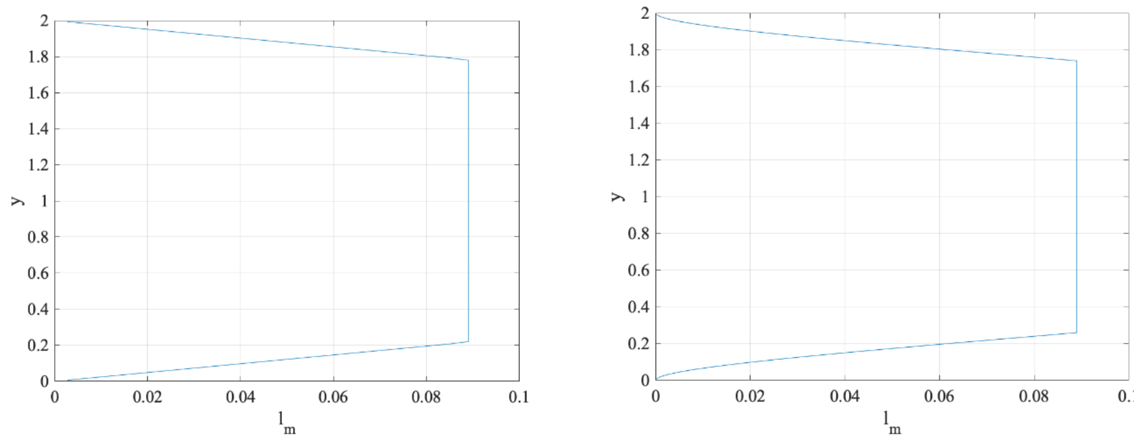

Figure 7 : No damping mixing length profile with a linear trend (left) and a damping model using the Van Driest mixing length

A validation of the mixing length model for the evaluation of the Reynolds stress tensor was carried out. A Direct numerical simulation at Re_b=2800 was used as reference. The file '*q_database_tavg.mat*' stores the time averaged field from the DNS. Figure 8 shows the time averaged velocity profile from the DNS and was used as input for the validation process. From the same DNS database, the time-averaged Reynolds stresses were computed and used as reference for mixing-length model.

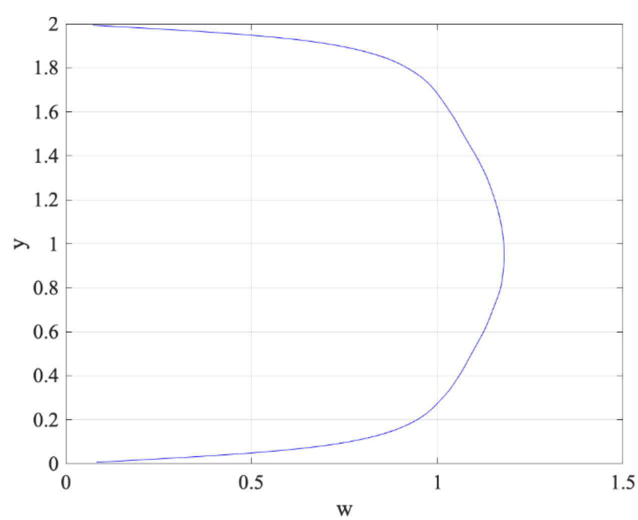

Figure 8 : Time averaged velocity profile sample for testing the mixing length model.

The validation was performed using the program '*mixlen_test_v3.m*'. The inputs are the time-averaged field, the Reynolds number (*Re*) and the index *'damping'* (see table 1) to choose whether apply the damping at the wall or not.

Figure (9) shows the time-averaged component $\boldsymbol{\tau_{vw}}$ of the Reynolds stresses. The black line refers to $\boldsymbol{\tau_{vw}}$ computed with the mixing length model while in red the reference from the DNS is reported. Without the damping at the wall the Reynolds stresses computed with the Prandtl's mixing length model are much higher than the reference case (Figure 9, left). On the contrary, when the Van-Driest damping law at the wall is applied, a much better agreement is obtained (Figure 9, right)

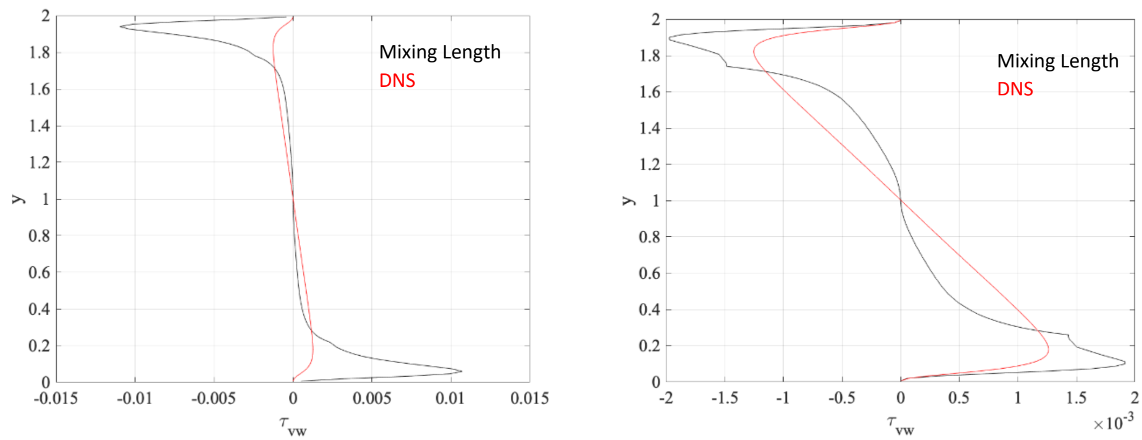

Figure 9: Distribution of $\boldsymbol{\tau_{vw}}$ *using the no damping mixing length model (left) and the Van Driest mixing length damping (right).*

3.2 3D RANS Evaluation

Three simulations were performed to evaluate the 3D RANS solver. The domain used for the simulation had the following dimensions: 6δ x 2δ x δ in the streamwise, vertical and spanwise direction respectively, with a grid of 5 x 80 x 40 nodes. Two Reynolds number were simulated: Re_b=1000 (Laminar and turbulent case) and Re_b=100000 (turbulent only). A uniform velocity streamwise profile was imposed as the initial velocity condition.

The following figures show the mean streamwise velocity profile, the turbulent and the total shear stress when damping and no-damping at the wall for the mixing length model were applied.

Although the program appears to reach a reasonable solution, the divergence consistently increased and exceeded the tolerance at around the 20th iteration. In an analysis of the plots, it was concluded that somewhere in the code, there is a slow loss of energy that is preventing the center region of the flow from increasing in magnitude to make up for the no slip condition at the walls. There is a component that is not evaluated properly, leading to a loss of a divergence free field on each time step.

The results from the 3D RANS solver are shown below for the 3 cases. The first case was a simple laminar flow at Re = 1000. The following two cases were at Re = 1000, and 100000 where turbulence was modeled both with or without damping. A 2D plot of the flow profile is also shown for the laminar case for completeness.

Laminar Re = 1000

As shown in the figures below, the velocity profile shows an expected trend of being uniform in the streamwise direction, and symmetric in the normal direction. This is due to the application of the wall boundary conditions, and the periodic boundary conditions. However, it can be seen in the mean streamwise velocity profile that the center velocity is uncharacteristically low. This supports the aforementioned conclusion that there is an error in the code where the energy is dissipated.

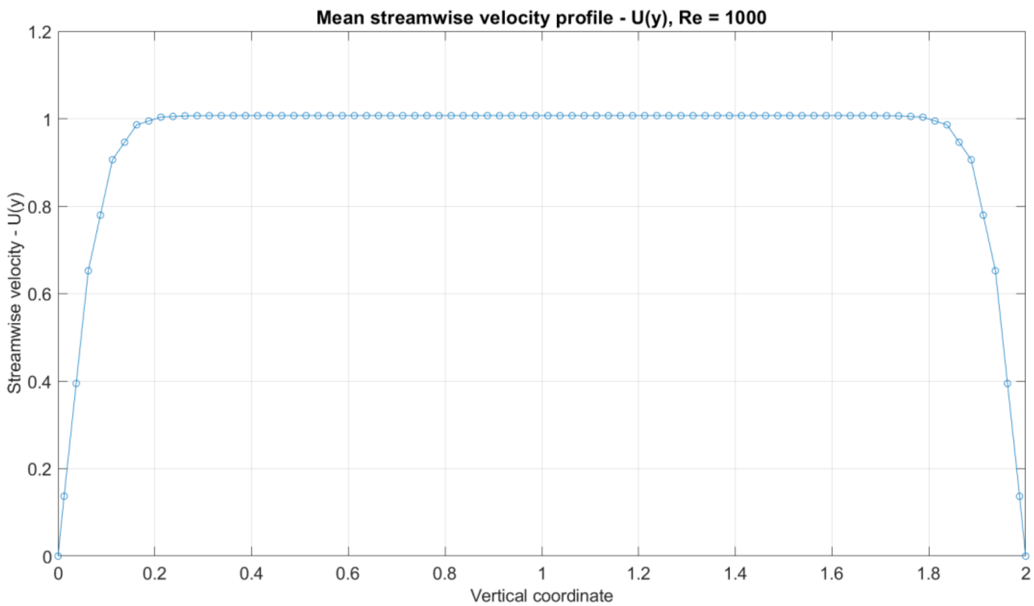

Figure 11: Mean streamwise velocity profile for RE = 1000 (Laminar)

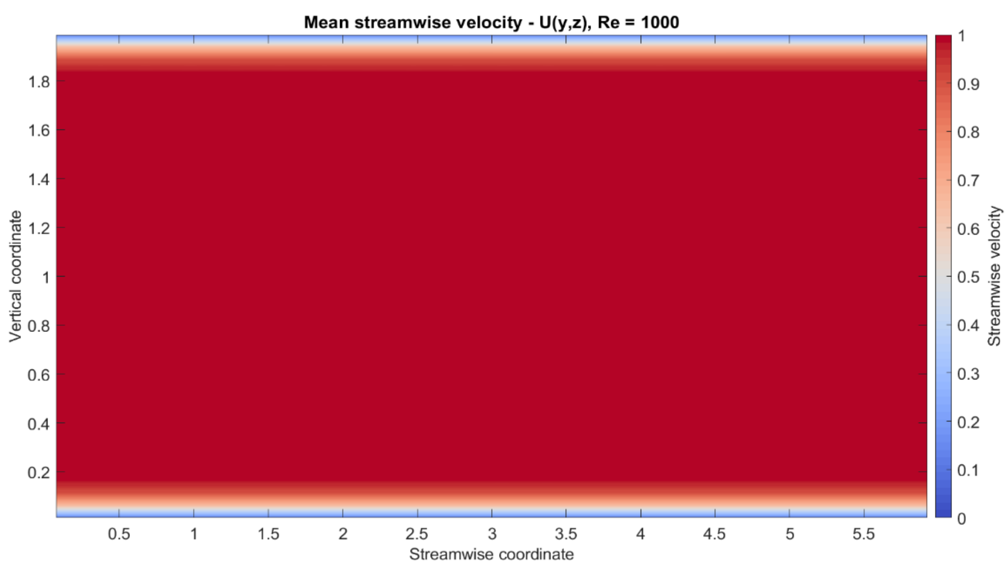

Figure 10: 2D Normal/Streamwise velocity profile for RE = 1000 (Laminar)

Turbulent Re = 1000

For the case of turbulent flow, both cases of damping were evaluated for the mean streamwise velocity profile, mean turbulent shear profile, and the mean total shear stress. In the case Van Driest damping, less significant changes were seen in all three profiles. This corresponds the effects of the damping, which reduce the turbulent viscosity at the wall, and thus reduce the trends seen in the profiles as compared with a regular mixing length. This also supports the conclusions that were drawn from figure 9 (DNS Comparison). This result is also seen in the stress profiles for the given Reynolds number. The turbulent shear stress is larger at the wall and decreases as you move away from the wall. Without the damping, the turbulent shear stress is greater as opposed to the profile with damping.

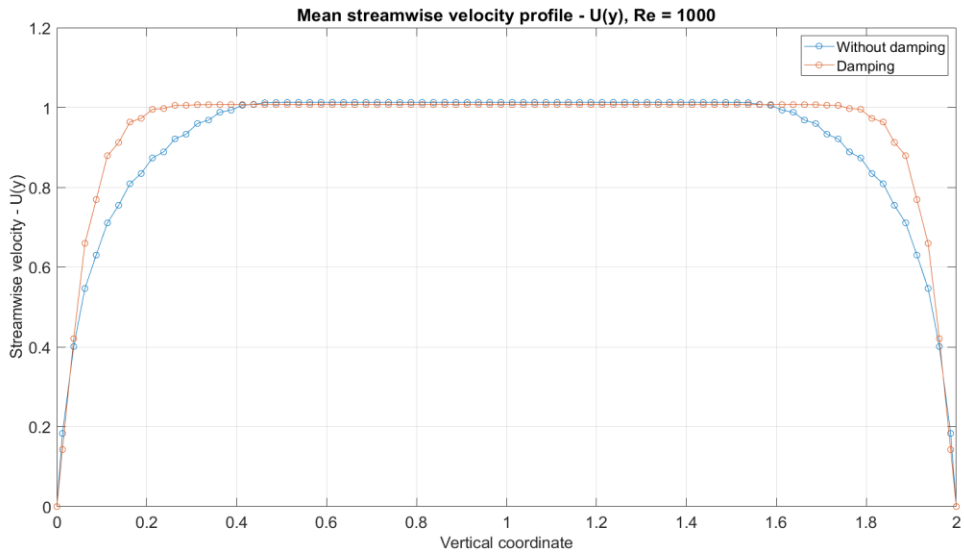

Figure 12: Mean streamwise velocity profile for Re = 1000 (Turbulent) with both damping and no damping

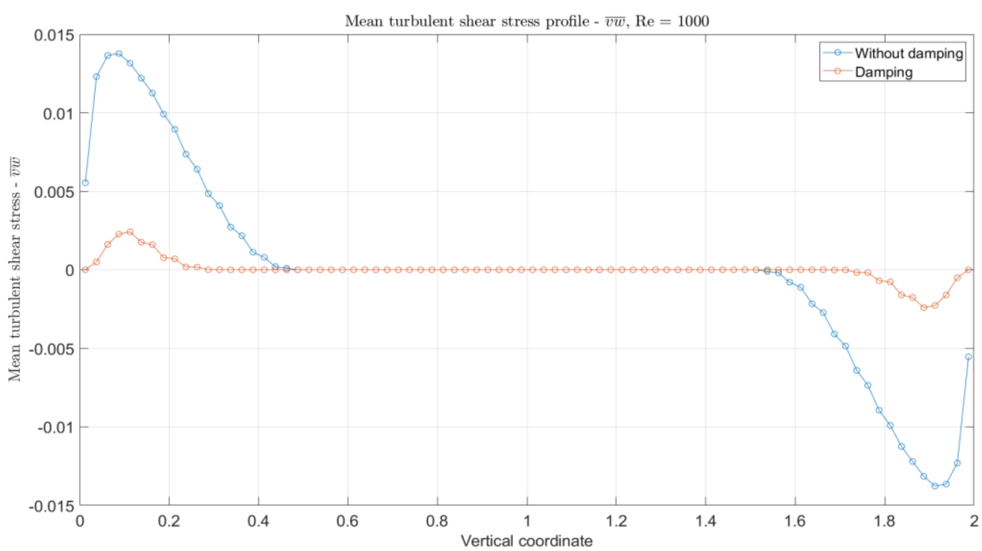

Figure 13: Mean turbulent shear stress profile for Re = 1000 (turbulent) with both damping and no damping

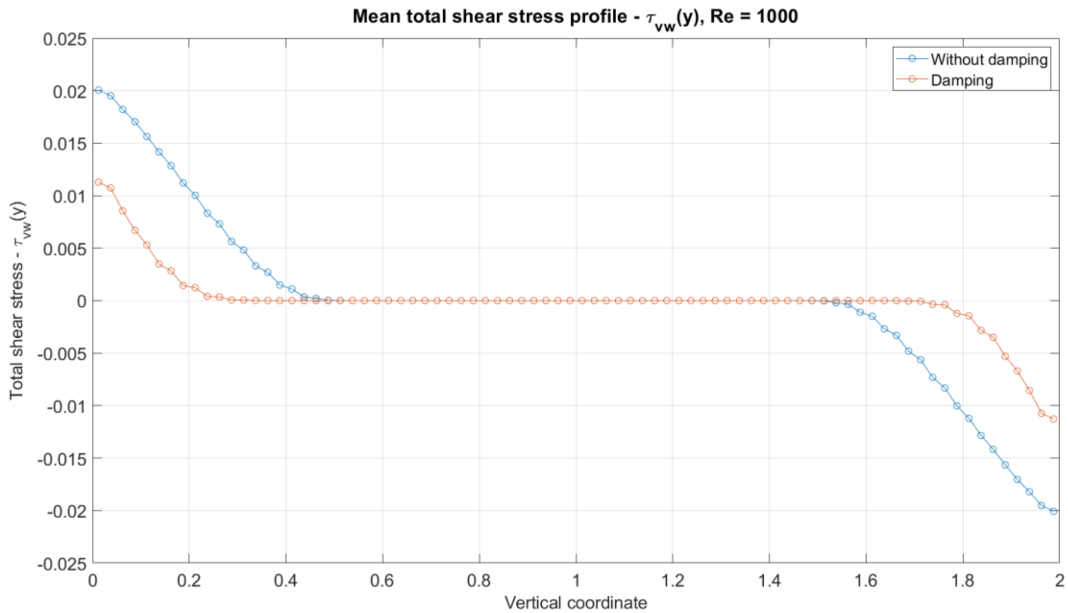

Figure 14: Mean total shear stress profile for Re = 1000 (Turbulent) with both damping and no damping

Turbulent 100000

For the case of the Reynolds number equal to 100000, the velocity profiles show similar trends compared to other flow cases, but with closer results between damping and no damping (figure 15). This is due to the high Reynolds number, which decreases the effects of the viscous shear stresses near the walls. These viscous stresses are very low in the momentum equation when compared to the turbulent Reynolds stresses, thereby defining turbulence as the dominant flow characteristic near the wall. Figures 16 through 18 support this conclusion, where the turbulent stresses are much higher than the viscous stresses.

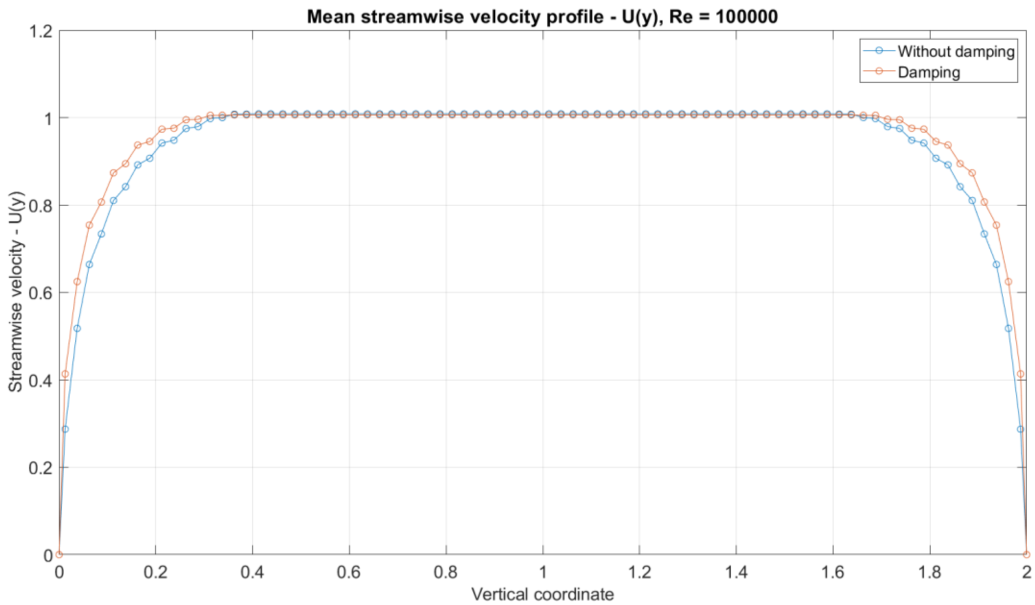

Figure 16: Mean streamwise velocity profile for Re = 100000 (Turbulent) for both damping and no damping

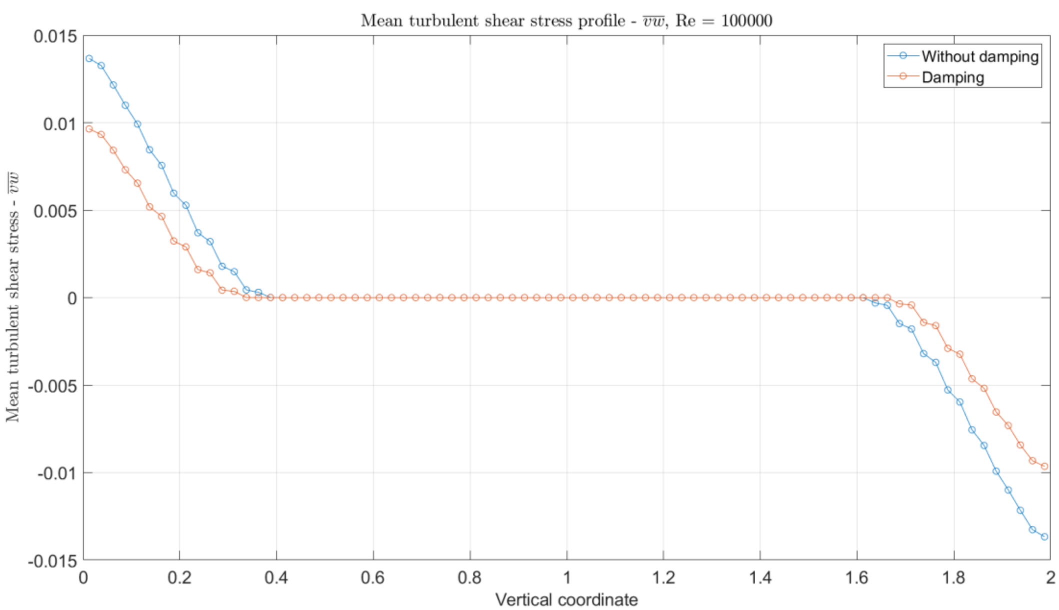

Figure 15: Mean turbulent shear stress profile for Re = 100000 (Turbulent) with both damping and no damping

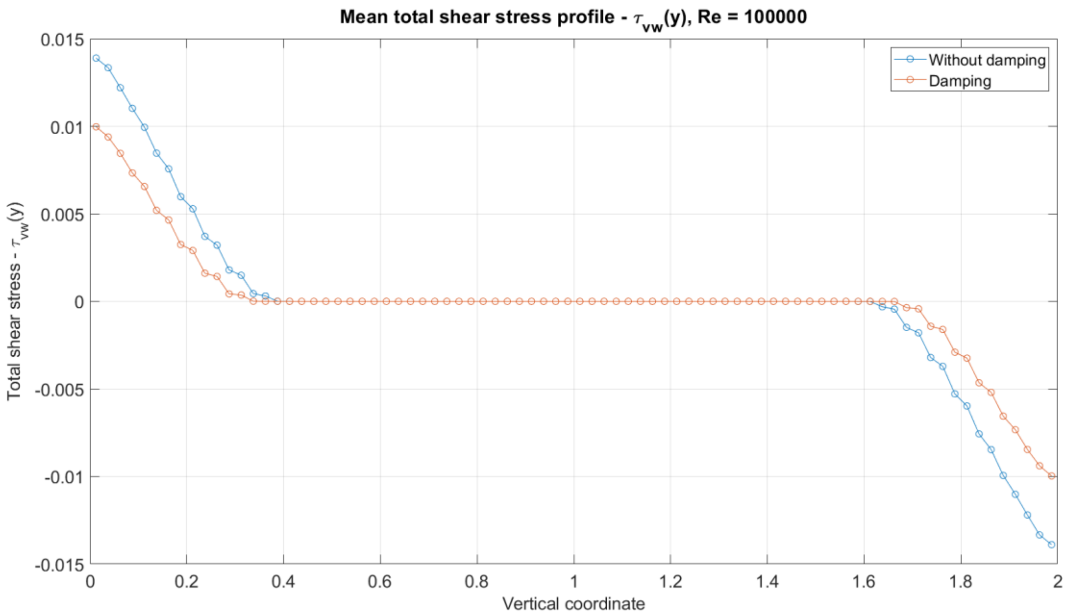

Figure 18: Mean total shear stress profile for Re = 100000 (Turbulent) both damping and no damping

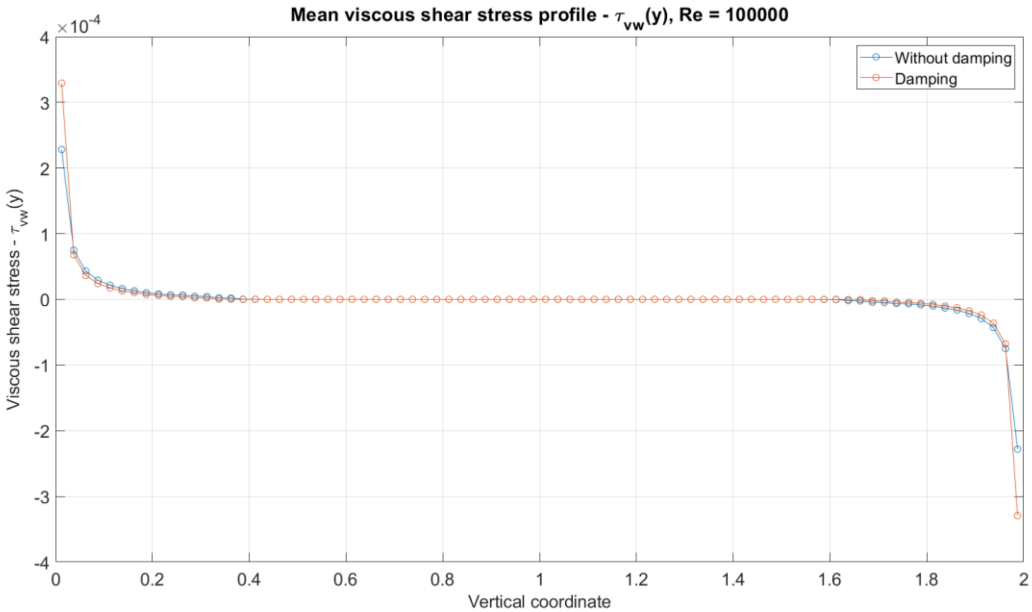

Figure 17: Mean viscous shear stress profile for Re = 100000 (Turbulent) with both damping and no damping.

4. Conclusions

The formulation of a mixing length model successfully showed the effects the mixing length distribution has on the flow field. The mixing length model with no damping was found to be ineffective in modeling the viscous sublayer, whereas the Van Driest model came much closer to resembling a DNS solution. This shows that the modeling of the mixing length with exponential damping near the wall will yield much better results in the entire flow domain.

The 3D RANS solver was also found to be effective at approximating results for turbulent and laminar profiles, despite errors that presented a loss of energy in the flow. These errors resulted in a loss of stability once divergence was found to be above the tolerances of the program. Even with these issues, the effects of damping at the wall were shown in both velocity profiles and shear stress profiles. This resulted in accurate depictions of how the mixing length affects the flow field.

With the successes and challenges faced during the coding of the 3D RANS solver, this project was successful in introducing the concept of how 3D flow fields can be modeled numerically. Detailed below are some challenges that were faced and overcome by the time during this project.

Primary Challenges Faced During this Project

- Solving the Poisson equation
- Modeling the coefficient matrix of the Poisson equation
- Implementing proper boundary conditions in the normal direction
- Calculation of the stress components

Disclaimer – This code takes approximately 20-25 minutes to run with a computer containing 22 Gb of memory in MATLAB 2018b. Please contact the team if any issues arise while attempting to run the program.

In case there are any issues with the zip folder contents, a box folder link has also been provided, and contains all files related to the project.

Link - https://utdallas.box.com/s/74u6bxcyeyhzzb083i9flvaiaqxvyqlq