

## Introduction

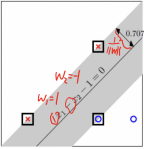
In this assignment, we implement and apply the linear support vector machine onto the given dataset to obtain the  $(w, b)$ . One linear separable data set ('X\_LinearSeparable.txt' and 'Y\_LinearSeparable.txt') is provided, where dataset X (20 x 2) contains 20 training samples. Each column in X is a sample with feature dimension of 2. Y (20 x 1) contains the ground truth binary labels of all samples. We also visualize the result by plotting the training data, support vectors, the decision boundary line and the two lines along the support vectors.

## Method

The method I used in this assignment is based on what I learned in class. I implement the linear support vector machine by following the instructions shown in the lecture slides. I use quadratic programming to solve the SVM. I mainly use for loops and if-else statements to implement my algorithm.

### Support Vector Machine

- support vectors: samples on boundary locate the large-margin hyperplane
- If  $\gamma_n(w^T x_n + b) = 1$  then sample  $n$  is a support vector.
- Other samples are not needed.
- SVM: learn the large-margin hyperplane with the help from only SV.
- $(margin(w, b) \approx \frac{1}{\|w\|_2})$
- Decision boundary:  $w^T x + b = 0$
- Line equations along the SVs:  $w^T x_{sv} + b = 1$   
 $w^T x_{sv} + b = -1$



### Algorithm: SVM by a QP Solver

Linear hard-margin SVM:

- ①  $H = \begin{bmatrix} 0 & 0 \\ 0 & I_{n \times n} \end{bmatrix}$ ,  $\phi = 0_{n \times 1}$ ,  $A = \begin{bmatrix} -x_1 & \dots & -x_n \\ x_1 & \dots & x_n \end{bmatrix} \in \mathbb{R}^{n \times (2n)}$ ,  $c = -\frac{1}{2} \mathbf{1}_{n \times 1}$
- ②  $q \leftarrow \text{quadprog}(H, \phi, A, c)$ ,  $q = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} b \\ w_1 \\ \vdots \\ w_n \end{bmatrix} \approx \begin{bmatrix} b \\ w \end{bmatrix}$
- ③ return  $y_{svm}(x) = \text{sign}(w^T x + b)$

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- Linear: decision bdn is hyperplane.
- hard-margin: linear separable, no mistakes,  $\forall n, \gamma_n(w^T x_n + b) \geq 1$

## Experiments

q is the (w,b) we want:

```
10 print(q)
```

```
[ 0.10204082 28.57142857 -27.75510204]
```

b = q[0] = 0.10204082

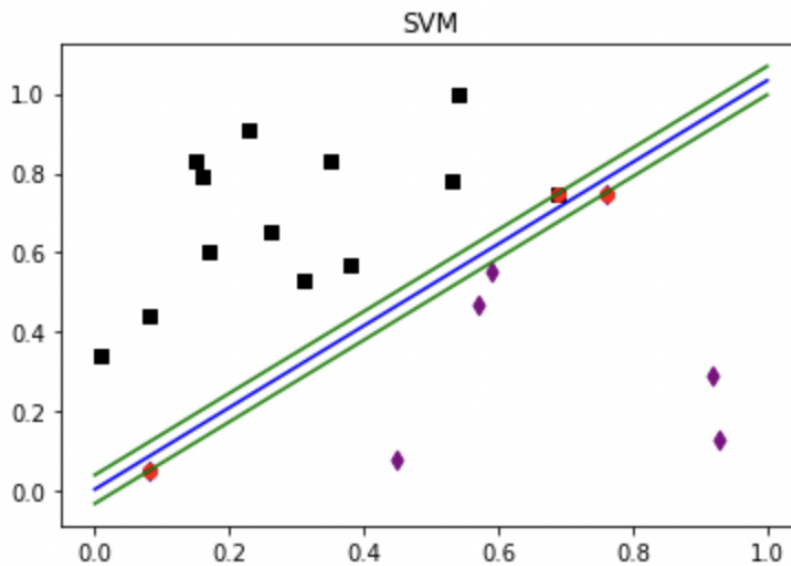
w = [q[1],q[2]] = [28.57142857, -27.75510204]

support vectors:

```
8 print(svs);
```

```
[(0.76, 0.75), (0.08, 0.05), (0.69, 0.75)]
```

SVM image:



The largest margin is the area between two green lines.

The red points are the support vectors.

The two lines along the support vectors are in blue.

The decision boundary line is the blue line.

The purple points are the data with ground truth y = 1.

The black points are the data with ground truth y = -1.

## Discussions

The program works as expected. The SVM image shows that our decision boundary line separates the training data in the way we want. Three red points are the support vectors we want. The qp solver I use in this assignment is a very powerful tool. It's a bit hard to find out how to use it at the beginning. But once I figure it out, it works perfectly. I don't need to worry about all the complicated computations.