Calculate μ and Σ

1. $\mu = \sum_{i=1}^{I} x_i \Big/ I$

2. $I \cdot \Sigma = \sum_{i=1}^{I} (x_i - \mu)(x_i - \mu)^T$

$$\underset{D\times D}{\Sigma} = \sum_{i=1}^{I} \underbrace{(x_i - \hat{\mu})}_{D\times 1}\underbrace{(x_i - \hat{\mu})^T}_{1\times D} \Big/ I$$

I ↳ # of images of face/BG

$[\quad] = [\,]\cdot[\quad]$

Training

- X face $= \{x_1, \cdots x_I\}$ face
  $\Rightarrow \hat{\mu}$ face, $\hat{\Sigma}$ face

$$\mu_{face} = \sum^{I} \cdots \Big/ I$$
$$\Sigma_{face} = \sum^{I} \cdots \Big/ I$$

- X bg $= \{x_1, \cdots x_I\}$ bg
  $\Rightarrow \hat{\mu}$ bg, $\hat{\Sigma}$ bg

$$\mu_{bg} = \sum^{J} \cdots \Big/ J$$
$$\Sigma_{bg} = \sum^{J} \cdots \Big/ J$$

Testing

$\Theta = \{\mu, \Sigma\}$

Given a new testing image x^*, $Pr(y|x^*) = \frac{Pr(x^*|y)Pr(y)}{Pr(x^*)}$

Assume $Pr(y)$ is uniform, $Pr(y|x^*) \sim Pr(x^*|y)$

$Pr(y=1|x^*) \overset{?}{>} Pr(y=0|x^*)$

$$\Rightarrow \quad \frac{Pr(x^*|y=1)\cancel{Pr(y=1)}}{\cancel{Pr(x^*)}} > \frac{Pr(x^*|y=0)\cancel{Pr(y=0)}}{\cancel{Pr(x^*)}}$$

$$Pr(x^*|y=1) > Pr(x^*|y=0)$$

$$\Rightarrow$$

$$Norm_{x^*}[\mu_{face}, \Sigma_{face}] \overset{?}{>} Norm_{x^*}[\mu_{bg}, \Sigma_{bg}]$$

$$Norm_x(\mu, \Sigma) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$$

x^* : a test image (vector, $x^* \in R^{D\times 1}$)

μ : mean ($\mu \in R^{D\times 1}$)

Σ : variance ($\Sigma \in R^{D\times D}$)

$|\Sigma|$: determinant of Σ ($|\Sigma| \in R$)

Simplify the expression

$$Pr(x^*|y=1) \overset{?}{>} Pr(x^*|y=0)$$

$\Rightarrow$

$$\frac{1}{(\cancel{2\pi})^{D/2}\left(\prod_{D=1}^{D}(\sigma^2_{dd})_{y=1}\right)^{1/2}}\, e^{\sum_{d=1}^{D}\frac{(x_d-(\mu_d)_{y=1})^2}{2(\sigma^2_{dd})_{y=1}}} \overset{?}{>} \frac{1}{(\cancel{2\pi})^{D/2}\left(\prod_{D=1}^{D}(\sigma^2_{dd})_{y=0}\right)^{1/2}}\, e^{\sum_{d=1}^{D}\frac{(x_d-(\mu_d)_{y=0})^2}{2(\sigma^2_{dd})_{y=0}}}$$

take logrithm

$\Rightarrow$

$$\underbrace{-\cancel{\tfrac{1}{2}}\sum_{d=1}^{D}\lg(\sigma^2_{dd})_{y=1} - \sum_{d=1}^{D}\frac{(x_d-(\mu_d)_{y=1})^2}{\cancel{2}(\sigma^2_{dd})_{y=1}}}_{A} \overset{?}{>} \underbrace{-\cancel{\tfrac{1}{2}}\sum_{d=1}^{D}\lg(\sigma^2_{dd})_{y=0} - \sum_{d=1}^{D}\frac{(x_d-(\mu_d)_{y=0})^2}{\cancel{2}(\sigma^2_{dd})_{y=0}}}_{B}$$

$$-d_1 - y_1 \overset{?}{>} -d_0 - y_0$$

(in my codes)

$A > B \Rightarrow$ Face

$A < B \Rightarrow$ background.