ECSE 526
Lab Report 1
MiniMax Algorithm with Alpha Beta Pruning

Yajiv Luckheenarain (260986423)

This project implements the Minimax algorithm with Alpha Beta pruning to create intelligent AI agents for board games. It focuses on heuristic score evaluation to enhance decision-making in complex game scenarios. By assigning heuristic values to game states, our AI agents navigate branching possibilities efficiently, reducing computational load. Demonstrated across various board games, this integration showcases their strategic thinking and decision-making capabilities. The project emphasizes the synergy between Minimax, Alpha Beta pruning, and heuristic evaluation, enabling AI agents to excel in board games, and improving gameplay.

I. Introduction

A Classic implementation of AI to win board games is known as the Minimax algorithm with Alpha-Beta pruning. We aim to build an AI that uses said algorithm and optimizes its performance through heuristic score evaluations of a game board's state. The game that this AI will be playing is a modified version of Connect 4. This game is played on a 7x7 grid where players take turns moving their pieces horizontally or vertically. The ultimate goal is to be the first to form a 2x2 square of four pieces. However, this game introduces an additional layer of complexity with an "impedance" field. This field affects the movement of an opponent's piece, contingent upon the number of pieces in its immediate neighbourhood.

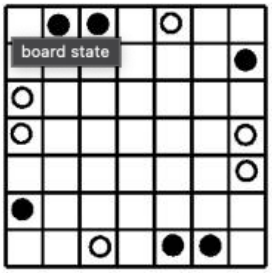

Fig. 1: Default Start Position of the Game

Number of opponent pieces in the surrounding 8 squares	Number of squares the piece can move
0	[1-3]
1	[1-2]
2	1
3 or more	0 (pinned)

Fig. 2: Maximum movement of pieces based on adjacent opponent pieces

II. Part I

A. Testing the Agent on various Game States

This modified version of Connect 4 is implemented using Python. The program reads a board and translates it into a grid containing "X" or "O" where "X" denotes a black piece and "O" denotes a white piece. This part of the report explores the behaviour of the agent on three different board state configurations. Fig.3 through Fig.5 denotes the default start state as well as those three board configurations as seen through my Python program.

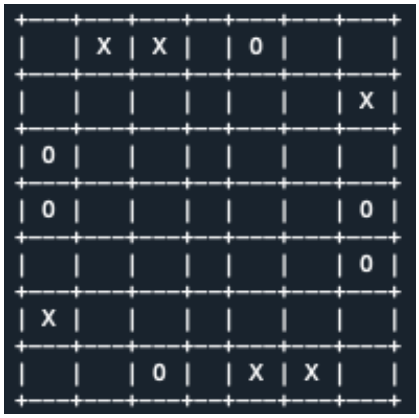

Fig. 3: Default Board State as seen in Python

Fig. 4: First Board Configuration To test Agent's Behaviour

For each starting configuration respectively, we assume it is white ("O") to play. The total number of states visited by the agent using MiniMax with and without Alpha-Beta pruning is shown in Table 1.

Fig. 5: Second Board Configuration To test Agent's Behaviour

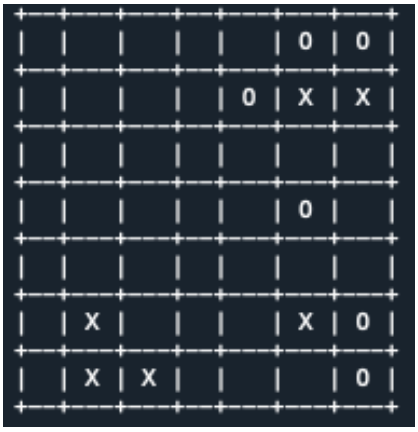

Fig. 6: Third Board Configuration To test Agent's Behaviour

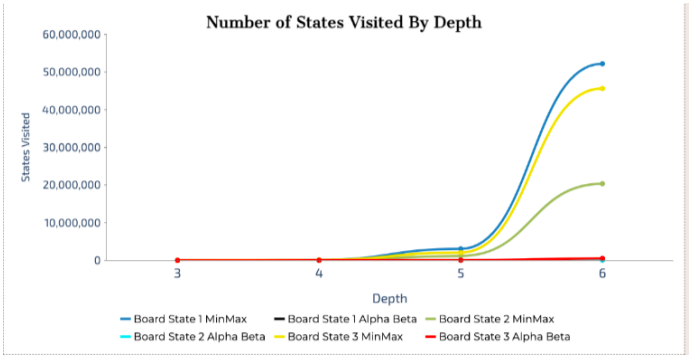

Fig. 7: Number of State vs Depth

B. Number of states visited and order of state visited

The number of states visited by the Minimax Algorithm without pruning is dependent on the order of the state visited, while the number of states visited by the MinMax algorithm without alpha-beta pruning isn't. Table 2 shows the number of states visited in Board 1 and Board 2 with a depth of 3 and 4 for both algorithms which compares the original number of states visited from Table 1 and the new number of states visited when the list of original possible moves are explored in a randomized order.

It is obvious that for each of the different configurations involving the MinMax algorithm, the number of states is the same. This is a direct consequence of the fact that the MinMax algorithm generates every possible subtree for a given game state regardless of the order of the sub tree it explores first.

TABLE I: Number of States Visited

	Depth 3	Depth 4	Depth 5	Depth 6
Board State 1 MinMax	7378	113634	3048194	52217982
Board State 1 Alpha Beta	1213	5844	49539	154814
Board State 2 MinMax	3812	56768	1159669	20357335
Board State 2 Alpha Beta	695	3568	21531	73012
Board State 3 MinMax	5420	109671	2063479	45639612
Board State 3 Alpha Beta	872	10860	25783	509045

On the other hand, the number of states visited using the Alpha Beta Pruning algorithm shows discrepancies. Sometimes, the number of states visited is slightly larger and sometimes it is slightly smaller. That is because this pruning algorithm doesn't explore some subtrees based on the evaluation of other subtrees it has already explored. Randomizing the order of the moves being explored will sometimes make the algorithm explore subtrees that allows it to prune more efficiently (less states explored), while sometimes it may delay the exploration of a subtree that would've pruned some subtrees that have already been explored (more states explored).

It is important to note that re-enacting these tests will yield different numbers of states visited by the Alpha Beta pruning algorithm as the order is randomized.

C. Agent vs Agent

For the third board configuration in Fig.6, if two agents were to play each other on that board with a depth cutoff of 6, white ("O") moves in a way to delay its defeat as can be seen in the following game log, while black ("X") moves in a way that tries to go around white's motions and inevitably win. As this game was played with a depth of 6, it is computationally expensive which is why only two game states are shown. However, it becomes clear that white's first move tries to stop the 2x2 square being formed by "X" in the bottom left of the board, while "X" moves in a way to build its square closer to the bottom right of the board as to avoid "O" blocking motion.

```
  1,  2,  3,  4,  5,  6,  7
+--+---+---+--+---+---+---+
|  |   |   |  |   | O | O |
+--+---+---+--+---+---+---+
|  |   |   |  | O | X | X |
+--+---+---+--+---+---+---+
|  |   |   |  |   |   |   |
+--+---+---+--+---+---+---+
|  |   |   |  |   | O |   |
+--+---+---+--+---+---+---+
|  |   |   |  |   |   |   |
+--+---+---+--+---+---+---+
|  | X |   |  |   | X | O |
+--+---+---+--+---+---+---+
|  | X | X |  |   |   | O |
+--+---+---+--+---+---+---+

visited 872581 states
```

```
Player1 Plays: 64W2

visited 1747827 states
Player2 Plays: 26E3
  1,  2,  3,  4,  5,  6,  7
+--+---+---+---+---+---+---+
|  |   |   |   |   | O | O |
+--+---+---+---+---+---+---+
|  |   |   |   | O | X | X |
+--+---+---+---+---+---+---+
|  |   |   |   |   |   |   |
+--+---+---+---+---+---+---+
|  |   |   | O |   |   |   |
+--+---+---+---+---+---+---+
|  |   |   |   |   |   |   |
+--+---+---+---+---+---+---+
|  |   |   |   | X | X | O |
+--+---+---+---+---+---+---+
|  | X | X |   |   |   | O |
+--+---+---+---+---+---+---+
 33.599999999999945
```

TABLE II: Number of States Visited

	Depth 3	Depth 4
Board 1 MinMax Original	7378	113634
Board 1 Alpha Beta Original	1213	5844
Board 1 MinMax Randomized	7378	113634
Board 1 Alpha Beta Randomized	1227	5874
Board 2 MinMax Original	3812	56768
Board 2 Alpha Beta Original	695	3568
Board 2 MinMax Randomized	3812	56768
Board 2 Alpha Beta Randomized	687	3720

III. Part II

In the previous part of the assignment, the board evaluation function associated a +1 to a winning board state, a -1 to a losing board state, and a 0 for every other possible board state. This function is overly simplistic this part of the assignment explores a heuristic approach to evaluate non-terminal nodes and give them a score that aims to steer the agent's move toward a winning position.

A. The Rationale

The new evaluation function is an imitation of the heuristic thought process a player would use when trying to win this game. In this case, the heuristics were modelled after my own thought process. As I played this game, I realized that pieces along the edges of the board are less mobile than centrepieces. Moreover, as the goal of this game is to form a 2x2 square of likewise pieces, I generally prioritize getting my pieces together while avoiding opponent pieces that could get in the way of my potential squares.

I modelled this thought process through a linear combination of a proximity score, center score, and edge score. Each of these scores is scaled by a corresponding scale factor. It is through playing against my agent that I have modified these values until I deemed its performance to be at an appropriate skill level. Here are the values given to the proximity score, center score and edge score respectively; 1.2, 2.1 and -1.5 the reason being that while it is positive to have your own pieces in proximity with themselves given that it gives an offensive edge I have deemed it less important than the center score given that it gives access to most parts of the board and gives an easier time to achieve objectives. Finally, for the edge score, I perceive it as a bad position given that movement is hard and therefore it can be blocked more conveniently than centrepieces.

B. Average States Visited

In this section, we explore the number of states visited by our algorithm using our improved evaluation function. The following table shows the number of states visited for a depth cutoff of 3, and 4 for the board states given in Fig.4, and Fig.5.

TABLE III: Number of States Visited with the new evaluation Function

	Depth 3	Depth 4
Board 1 MinMax	7378	113634
Board 1 Alpha Beta	3125	36422
Board 2 MinMax	3812	56768
Board 2 Alpha Beta	1537	15375

Comparing Table 3 with Table.1, we can clearly see that the number of states visited with the new evaluation function is higher than the original evaluation function. This shows that the increased performance of this evaluation function comes with a tradeoff that a higher number of states need to be visited. Fig.8 illustrates this higher number of states visited.

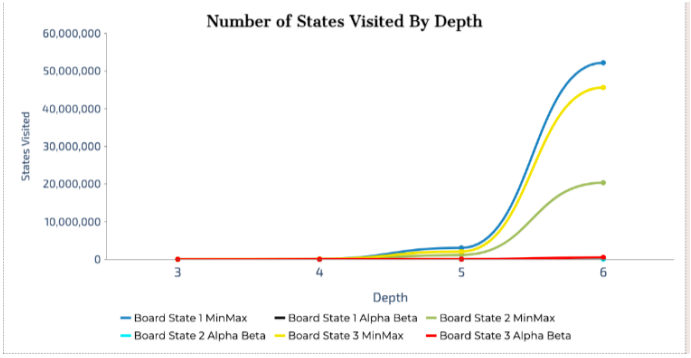

Fig. 8: Number of State vs Depth with new evaluation function

C. Computational TradeOffs

In terms of time complexity and space complexity, Table.1 shows that the number of states visited exponentially increases with the depth that the MiniMax and Alpha Beta Algorithm are allowed to explore in the tree of game states. Consequentially, this results in an exponential increase in time and space complexity to compute the "next best move" as we increase the depth that the agent is allowed to explore.

However, if the evaluation function is complex enough that it can correctly yield board scores, it will significantly reduce both the time and space used to compute the next best

move when using Alpha Beta pruning. That is because, for every depth of the tree that the algorithm builds, it will more efficiently prune sub-trees because of the better evaluation that the evaluation function yields.

D. Agent vs Agent Log

With a depth cutoff of 4, this section presents a game log between two agents playing on the boards shown in Fig.4 and Fig.6.

The Game taking place on the board shown in Fig.4:

```
1,  2,  3,  4,  5,  6,  7
+---+---+--+--+---+---+---+
|   | O |  |  | X | X | X |
+---+---+--+--+---+---+---+
|   |   |  |  | X | O | X |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   |   |
+---+---+--+--+---+---+---+
| O |   |  |  |   |   |   |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   |   |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   | O |
+---+---+--+--+---+---+---+
| O | O |  |  |   |   | X |
+---+---+--+--+---+---+---+

visited 33239 states
Player1 Plays: 21S3

visited 4633 states
Player2 Plays: 52S1
  1,  2,  3,  4,  5,  6,  7
+---+---+--+--+---+---+---+
|   |   |  |  | X | X | X |
+---+---+--+--+---+---+---+
|   |   |  |  |   | O | X |
+---+---+--+--+---+---+---+
|   |   |  |  | X |   |   |
+---+---+--+--+---+---+---+
| O | O |  |  |   |   |   |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   |   |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   | O |
+---+---+--+--+---+---+---+
| O | O |  |  |   |   | X |
+---+---+--+--+---+---+---+
 -162.36000000000024
visited 34412 states
Player1 Plays: 14S2

visited 7953 states
Player2 Plays: 53N1
  1,  2,  3,  4,  5,  6,  7
+---+---+--+--+---+---+---+
|   |   |  |  | X | X | X |
+---+---+--+--+---+---+---+
|   |   |  |  | X | O | X |
+---+---+--+--+---+---+---+
|   | O |  |  |   |   |   |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   |   |
+---+---+--+--+---+---+---+
| O |   |  |  |   |   | O |
+---+---+--+--+---+---+---+
| O | O |  |  |   |   | X |
+---+---+--+--+---+---+---+
 -258.4199999999998
visited 31995 states
Player1 Plays: 24S2

  1,  2,  3,  4,  5,  6,  7
+---+---+--+--+---+---+---+
|   |   |  |  | X | X | X |
+---+---+--+--+---+---+---+
|   |   |  |  | X | O | X |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   |   |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   |   |
+---+---+--+--+---+---+---+
|   |   |  |  |   |   |   |
+---+---+--+--+---+---+---+
| O | O |  |  |   |   | O |
+---+---+--+--+---+---+---+
| O | O |  |  |   |   | X |
+---+---+--+--+---+---+---+

O wins!
inf
```

The Game taking place on the board shown in Fig.6:

```
  1,  2,  3,  4,  5,  6,  7
+--+---+---+--+---+---+---+
|  |   |   |  |   | O | O |
+--+---+---+--+---+---+---+
|  |   |   |  | O | X | X |
+--+---+---+--+---+---+---+
|  |   |   |  |   |   |   |
+--+---+---+--+---+---+---+
|  |   |   |  |   | O |   |
+--+---+---+--+---+---+---+
|  |   |   |  |   |   |   |
+--+---+---+--+---+---+---+
|  | X |   |  |   | X | O |
+--+---+---+--+---+---+---+
|  | X | X |  |   |   | O |
+--+---+---+--+---+---+---+

visited 19810 states
Player1 Plays: 64S1
```

```
visited 17358 states
Player2 Plays: 26E3
  1,  2,  3,  4,  5,  6,  7
+---+---+---+---+---+---+---+
|   |   |   |   |   | O | O |
+---+---+---+---+---+---+---+
|   |   |   |   | O | X | X |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   | O |   |
+---+---+---+---+---+---+---+
|   |   |   |   | X | X | O |
+---+---+---+---+---+---+---+
|   | X | X |   |   |   | O |
+---+---+---+---+---+---+---+
 42.839999999999975
visited 11100 states
Player1 Plays: 77W2

visited 9874 states
Player2 Plays: 27N1
  1,  2,  3,  4,  5,  6,  7
+---+---+---+---+---+---+---+
|   |   |   |   |   | O | O |
+---+---+---+---+---+---+---+
|   |   |   |   | O | X | X |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   | O |   |
+---+---+---+---+---+---+---+
|   | X |   |   | X | X | O |
+---+---+---+---+---+---+---+
|   |   | X |   | O |   |   |
+---+---+---+---+---+---+---+
 27.059999999999967
visited 16656 states
Player1 Plays: 52N1

visited 14190 states
Player2 Plays: 26E2
  1,  2,  3,  4,  5,  6,  7
+---+---+---+---+---+---+---+
|   |   |   |   | O | O | O |
+---+---+---+---+---+---+---+
|   |   |   |   |   | X | X |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   | O |   |
+---+---+---+---+---+---+---+
|   |   |   | X | X | X | O |
+---+---+---+---+---+---+---+
|   |   | X |   | O |   |   |
+---+---+---+---+---+---+---+
 46.49999999999999
visited 8085 states
Player1 Plays: 76S1

visited 7415 states
Player2 Plays: 37N2
  1,  2,  3,  4,  5,  6,  7
+---+---+---+---+---+---+---+
|   |   |   |   | O | O | O |
+---+---+---+---+---+---+---+
|   |   |   |   |   | X | X |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   | X |   |   | O |   |
+---+---+---+---+---+---+---+
|   |   |   | X | X | X |   |
+---+---+---+---+---+---+---+
|   |   |   |   | O |   | O |
+---+---+---+---+---+---+---+
 33.3
visited 10442 states
Player1 Plays: 77N1

visited 10547 states
Player2 Plays: 35N2
  1,  2,  3,  4,  5,  6,  7
+---+---+---+---+---+---+---+
|   |   |   |   | O | O | O |
+---+---+---+---+---+---+---+
|   |   |   |   |   | X | X |
+---+---+---+---+---+---+---+
|   |   | X |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+
|   |   |   |   |   | O |   |
+---+---+---+---+---+---+---+
|   |   |   | X | X | X | O |
+---+---+---+---+---+---+---+
|   |   |   |   | O |   |   |
+---+---+---+---+---+---+---+
 52.55999999999994
visited 10915 states
Player1 Plays: 76N2

visited 17456 states
Player2 Plays: 33E2
  1,  2,  3,  4,  5,  6,  7
+---+---+---+---+---+---+---+
|   |   |   |   | O | O | O |
+---+---+---+---+---+---+---+
```

```
|  |  |  |  |  |  | X | X |
+--+--+--+---+---+---+---+
|  |  |  |  | X |  |  |  |
+--+--+--+---+---+---+---+
|  |  |  |  |  |  |  | O |
+--+--+--+---+---+---+---+
|  |  |  |  |  |  | O |  |
+--+--+--+---+---+---+---+
|  |  |  | X | X | X |  |  |
+--+--+--+---+---+---+---+
|  |  |  |  | O |  |  |  |
+--+--+--+---+---+---+---+
 41.03999999999996
visited 12952 states
Player1 Plays: 65W1

visited 7059 states
Player2 Plays: 72S1
  1,  2,  3,  4,  5,  6,  7
+--+--+--+---+---+---+---+
|  |  |  |  |  | O | O | O |
+--+--+--+---+---+---+---+
|  |  |  |  |  |  | X |  |
+--+--+--+---+---+---+---+
|  |  |  |  | X |  |  | X |
+--+--+--+---+---+---+---+
|  |  |  |  |  |  |  | O |
+--+--+--+---+---+---+---+
|  |  |  |  | O |  |  |  |
+--+--+--+---+---+---+---+
|  |  |  | X | X | X |  |  |
+--+--+--+---+---+---+---+
|  |  |  |  | O |  |  |  |
+--+--+--+---+---+---+---+
 52.73999999999995
visited 7481 states
Player1 Plays: 74S1

visited 6277 states
Player2 Plays: 53W2
  1,  2,  3,  4,  5,  6,  7
+--+--+---+---+---+---+---+
|  |  |  |  | O | O | O |
+--+--+---+---+---+---+---+
|  |  |  |  |  | X |  |
+--+--+---+---+---+---+---+
|  |  | X |  |  |  | X |
+--+--+---+---+---+---+---+
|  |  |  |  |  |  |  |
+--+--+---+---+---+---+---+
|  |  |  |  | O |  | O |
+--+--+---+---+---+---+---+
|  |  |  | X | X | X |  |
+--+--+---+---+---+---+---+
|  |  |  |  | O |  |  |
+--+--+---+---+---+---+---+
 74.58000000000007
visited 9583 states
Player1 Plays: 75N1

visited 12311 states
Player2 Plays: 33E2
  1,  2,  3,  4,  5,  6,  7
+--+--+--+---+---+---+---+
|  |  |  |  | O | O | O |
+--+--+--+---+---+---+---+
|  |  |  |  |  | X |  |
+--+--+--+---+---+---+---+
|  |  |  |  | X |  | X |
+--+--+--+---+---+---+---+
|  |  |  |  |  |  | O |
+--+--+--+---+---+---+---+
|  |  |  |  | O |  |  |
+--+--+--+---+---+---+---+
|  |  |  | X | X | X |  |
+--+--+--+---+---+---+---+
|  |  |  |  | O |  |  |
+--+--+--+---+---+---+---+
 52.73999999999995
visited 7460 states
Player1 Plays: 74S1
```