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# Study of a two-dimension transient heat propagation in cylindrical coordinates by means of two finite difference methods

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**Abstract.** The analytical approach of unsteady conduction heat transfer under actual conditions represent a very difficult (if not insurmountable) problem due to the issues related to finding analytical solutions for the conduction heat transfer equation. Various techniques have been developed in order to overcome these difficulties, among which the alternate directions method and the decomposition method. Both of them are particularly suited for two-dimension heat propagation. The paper deals with both techniques in order to verify whether the results provided are in good accordance. The studied case consists of a long hollow cylinder, and considers that the time-dependent temperature field varies both in the radial and the axial directions. The implicit technique is used in both methods and involves the simultaneous solving of a set of equations for all of the nodes for each time step successively for each of the two directions. Gauss elimination is used to obtain the solution of the set, representing the nodal temperatures. After using the two techniques the results show a very good agreement, and since the decomposition is easier to use in terms of computer code and running time, this technique seems to be more recommendable.

## 1. Introduction

Transient heat conduction is particularly difficult to approach by means of analytical methods because it involves finding solutions for the partial differential equation describing the heat diffusion phenomenon, even in some of the simplest cases (one-dimension heat propagation in bodies with simple geometric shapes, constant thermo-physical properties of the heat transfer medium, and Dirichlet boundary conditions).

For example, the case of one-dimension heat conduction in long cylinders leads to laborious solutions involving the use of Bessel functions which are very difficult to manipulate when it comes to write the boundary conditions [1], [2].

Actual systems usually involve two- or three- dimension transient heat transfer with more complex boundary conditions, such as Neumann ones. Presently there are no analytical solutions for such cases and one must develop other approaches in order to find the temperature distribution at any moment. In such situations, numerical techniques (such as the finite difference approach) are being used [3 ... 5].

The finite difference method involves turning partial derivatives into finite differences and thus much more simple equations result, which are easy to manipulate. However, as soon as the heat conduction is two- or three dimensional, the approach becomes difficult.

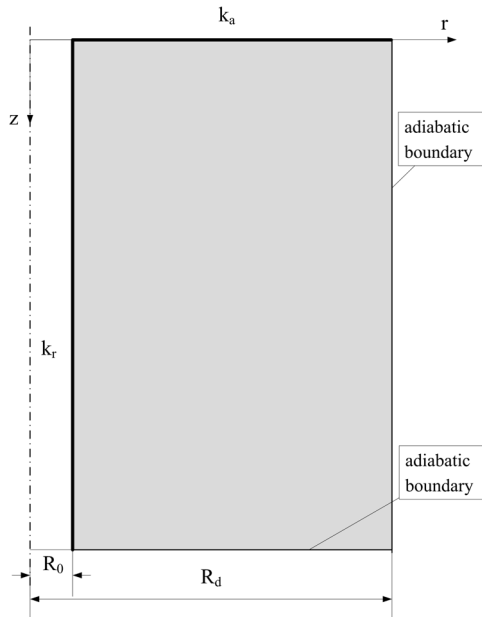


Finite difference schemes can be applied in two variations: explicit, respectively implicit, depending on how one considers the present time step. The explicit scheme involves using the three nodal temperatures in the finite difference equation at the past time step, which allows the calculation of the present time temperature in the current node. There is a limitation for the magnitude of the time step in this procedure, which is not allowed to exceed a maximum value imposed by the convergence and stability condition of the scheme. The implicit scheme does not involve such a constraint, but the price to be paid consists of the necessity to solve a set of linear algebraic equations corresponding to the total number of nodes, because the finite difference equations consider the three nodal temperatures at the present time step [6].

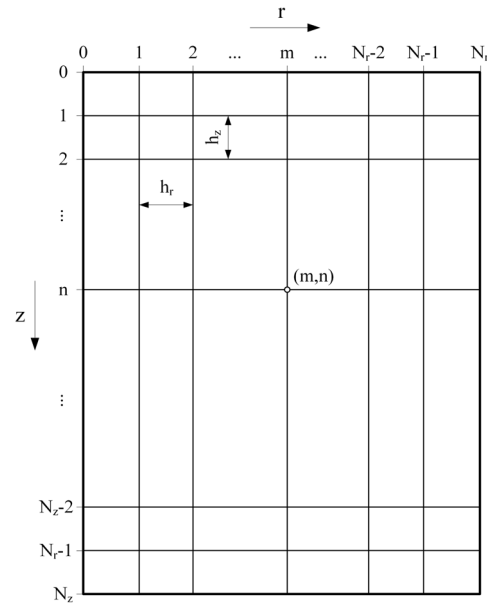
The purpose of the paper is to use two different finite difference approaches: the alternate directions, respectively the decomposition techniques, in order to solve the problem of the two-dimension transient conduction heat transfer in a long hollow cylinder and to compare the degree of agreement of the results.

## 2. Mathematical model

Let us consider a long hollow cylinder of length  $L$ , inner radius  $R_0$ , and outer radius  $R_d$ . The initial temperature of the cylinder is uniform and equal to  $T_L$ . Heat is transferred by convection from a fluid of temperature  $T_H > T_L$  flowing along the central channel of the cylinder (heat transfer coefficient:  $k_r$ ) and from another fluid of temperature  $T_a$  (heat transfer coefficient:  $k_a$ ) to the frontal surface  $z = 0$  (see figure 1). The boundaries at  $r = R_d$  and at the lower end ( $z = L$ ) are adiabatic.



**Figure 1.** Schematic of the cylindrical domain.



**Figure 2.** The mesh.

The phenomenon in the studied case is described by the transient heat conduction equation in cylindrical coordinates:

$$\frac{\partial \theta}{\partial \tau} = \frac{\lambda}{\rho c} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (1)$$

Due to the axial symmetry, the heat conduction is two-dimensional. In eq. (1), temperature  $T$  has been replaced by the dimensionless temperature defined as:

$$\theta = \frac{T - T_L}{T_H - T_L} \quad (2)$$

The boundary conditions are as follows:

– at  $r = R_0$ :

$$k_r (1 - \theta_0) = -\lambda \left( \frac{\partial \theta}{\partial r} \right) \Big|_{r=R_0} \quad (3)$$

– at  $z = 0$ :

$$k_z (\theta_a - \theta_0) = -\lambda \left( \frac{\partial \theta}{\partial z} \right) \Big|_{z=0} \quad (4)$$

– at  $r = R_d$ :

$$\left( \frac{\partial \theta}{\partial r} \right) \Big|_{r=R_d} = 0 \quad (5)$$

– at  $z = L$ :

$$\left( \frac{\partial \theta}{\partial z} \right) \Big|_{z=L} = 0 \quad (6)$$

where  $\lambda$  stands for the thermal conductivity of the material of the cylinder, and  $\theta_0$  represents the inner surface temperature of the cylinder.

### 3. Finite difference approach

In order to turn the heat transfer equation into finite difference equations, we have established a mesh consisting of  $N_r$  nodes in the radial direction (radial step:  $h_r$ ) and  $N_z$  nodes in the axial direction (axial step:  $h_z$ ) – see figure 2.

#### 3.1. The alternate directions technique

By using the finite difference operator  $D^2$  corresponding to the second order partial derivatives, the heat transfer equation (1) can be rewritten in the finite difference form for the time step  $\Delta\tau$ , as follows [6], [7]:

$$\frac{\theta_{m,n}^p - \theta_{m,n}^{p-1}}{\Delta\tau} = a \left( D_r^2 \theta_{m,n}^p + D_z^2 \theta_{m,n}^{p-1} \right) \quad (7)$$

where  $a$  stands for the thermal diffusivity of the cylinder material, and  $p$  for the present time step number.

The essence of the alternate directions technique (ADT) consists of splitting the time step  $\Delta\tau$  into two equal intervals of magnitude  $\Delta\tau / 2$ , thus resulting two corresponding phases:

– the intermediary phase: from the preceding time  $(p-1)\Delta\tau$  to the intermediary time  $(p-1/2)\Delta\tau$ . During this phase, one assumes an implicit approach in the radial direction, whereas the axial one is approached by the explicit technique. This means that in the finite difference equation (7),  $D_r^2$  is applied to the nodal temperatures in the radial direction (presently unknown) at the end of the phase  $(p-1/2)\Delta\tau$  while  $D_z^2$  applies to the nodal temperatures in the axial direction at the beginning  $(p-1)\Delta\tau$ , which are known because they have been already determined in the previous step of the procedure.

– the final phase: from  $(p-1/2)\Delta\tau$  to  $p\Delta\tau$ . The approach is implicit for the axial direction, respectively explicit for the radial one, for which the intermediate temperatures have been determined for the previous phase. The result consists of the final temperatures for the current time step  $p$ .

The operator  $D^2$  for the two directions can be written respectively:

– in the radial direction:

$$D_r^2\theta_{m,n} = \frac{1}{h_r^2} \left( \frac{c_m-1}{c_m} \theta_{m-1,n} - 2\theta_{m,n} + \frac{c_m+1}{c_m} \theta_{m+1,n} \right) \quad (8)$$

where:

$$c_m = 2 \left( N_r \frac{R_0}{R_d - R_0} + m \right) \quad (9)$$

– in the axial direction:

$$D_z^2\theta_{m,n} = \frac{1}{h_z^2} (\theta_{m,n-1} - 2\theta_{m,n} + \theta_{m,n+1}) \quad (10)$$

Now, we can write the finite difference equation (7) for the two phases of the procedure:

– the intermediate phase (superscript  $i$  for the unknown temperatures):

$$\frac{\theta_{m,n}^i - \theta_{m,n}^{p-1}}{\frac{\Delta\tau}{2}} = a \left( \underbrace{D_r^2\theta_{m,n}^i}_{\text{implicit}} + \underbrace{D_z^2\theta_{m,n}^{p-1}}_{\text{explicit}} \right) \quad (11)$$

– the final phase (superscript  $p$  for the unknown temperatures):

$$\frac{\theta_{m,n}^p - \theta_{m,n}^i}{\frac{\Delta\tau}{2}} = a \left( \underbrace{D_r^2\theta_{m,n}^i}_{\text{explicit}} + \underbrace{D_z^2\theta_{m,n}^p}_{\text{implicit}} \right) \quad (12)$$

One can notice that in each of the two phases, there are three nodal temperatures in the implicit operator ( $\theta_{m,n}$  also appears in the left hand side of the equation) that are unknown and need to be determined. The three temperatures from the explicit operator are known since they have been determined in the previous phase. As a matter of fact, the ADT derives from the Crank-Nicolson scheme in one direction [8].

After a series of manipulations, and considering the stability and convergence criteria  $\alpha_r$  and  $\alpha_z$ :

$$\alpha_r = \frac{a\Delta\tau}{2h_r^2} \quad (13)$$

$$\alpha_z = \frac{a\Delta\tau}{2h_z^2} \quad (14)$$

equations (11) and (12) become respectively:

$$-\theta_{m-1,n}^i + \sigma_{r,m} \theta_{m,n}^i - \gamma_{r,m} \theta_{m+1,n}^i = \beta_{r,m} \quad (15)$$

$$-\theta_{m,n-1}^p + \sigma_z \theta_{m,n}^p - \theta_{m,n+1}^p = \beta_{z,n} \quad (16)$$

where:

$$\sigma_{r,m} = \frac{c_m}{c_m - 1} \left( 2 + \frac{1}{\alpha_r} \right) \quad (17)$$

$$\gamma_{r,m} = \frac{c_m + 1}{c_m - 1} \quad (18)$$

$$\beta_{r,m} = \frac{c_m}{c_m - 1} \frac{\alpha_z}{\alpha_r} \left[ \theta_{m,n-1}^{p-1} - \left( 2 - \frac{1}{\alpha_z} \right) \theta_{m,n}^{p-1} + \theta_{m,n+1}^{p-1} \right] \quad (19)$$

$$\sigma_z = 2 + \frac{1}{\alpha_z} \quad (20)$$

$$\beta_{z,n} = \frac{\alpha_r}{\alpha_z} \left[ \frac{c_m - 1}{c_m} \theta_{m-1,n}^i - \left( 2 - \frac{1}{\alpha_r} \right) \theta_{m,n}^i + \frac{c_m + 1}{c_m} \theta_{m+1,n}^i \right] \quad (21)$$

Equations (15) and (16), written for the central node ( $m, n$ ) with  $m$  varying between 1 and  $N_r - 1$  and  $n$  varying between 1 and  $N_z - 1$ , are used to determine the temperatures at the end of the initial phase, and at the end of the final phase, respectively. The procedure involves finding the solution of two sets of  $N_r - 1$  respectively  $N_z - 1$  linear algebraic equations, and can be performed by the Gauss elimination technique. The temperatures on the boundaries of the domain result from the boundary conditions written in the finite difference form.

### 3.2. The decomposition technique

This technique, as described in [7], involves the use of a one-dimension implicit finite difference scheme during a time step  $\Delta\tau$  in a successive manner, as if the two-dimension heat propagation had been decomposed in a radial propagation followed by an axial one (the order is arbitrary). The process can be imagined like firstly inserting thermal insulating barriers (separated by a space step) parallel to the direction of propagation thus preventing heat from diffusing in the normal direction, and subsequently repeating the procedure for the other direction of propagation.

Formally, the same finite difference equations govern the process as in the previous case – equations (15) and (16). The only differences reside in the expressions of the stability and convergence criteria  $\alpha_r$  and  $\alpha_z$  which are twice the values in equations (13) and (14) respectively because of the fact that in decomposition the time step is twice the time step for the ADT, and in the expression of the right hand term of equations (15) and (16):

$$\beta_{r,m} = \frac{1}{\alpha_r} \frac{c_m}{c_m - 1} \theta_{m,n}^{p-1} \quad (22)$$

$$\beta_z = \frac{1}{\alpha_z} \theta_{m,n}^{p-1} \quad (23)$$

One can notice that since the explicit term is no more present,  $\beta_{z,n}$  does not depend on the axial position of the node  $n$ , which is why in this case subscript  $n$  is no longer necessary.

For the reasons above, decomposition, as compared with ADT seems to be easier to apply due to the fact that the finite difference equations are simpler (the right hand term is less complicated).

The finite difference equations for the boundary conditions are similar in both methods, since they only rely upon the temperatures in the first three nodes (the boundary node included).

#### 4. Results and discussion

Both methods (ADT and decomposition) have been applied to a 2 meters long hollow cylinder of inner radius  $R_0 = 28$  mm and outer radius  $R_d = 2000$  mm. The temperatures were:  $T_H = 100^\circ\text{C}$ ,  $T_L = 12^\circ\text{C}$ ,  $T_a = 25^\circ\text{C}$ . The considered thermo-physical properties of the cylinder material are:  $\lambda = 0.25$  W/mK,  $a = 0.18 \cdot 10^{-6}$  m<sup>2</sup>s<sup>-1</sup>. The convection heat transfer coefficients are:  $k_r = 1000$  W/m<sup>2</sup>K and  $k_a = 10$  W/m<sup>2</sup>K.

The grid attached to the cylindrical domain had  $N_r = 20$  and  $N_z = 20$  nodes respectively.

For each of the two methods we have written a computer code the output of which were the nodal temperatures at the end of each time step of the procedure.

After running the computer codes, the first step was to compare the results in order to establish whether or not the agreement between the two methods is good. Figures 3 and 4 represent the output screens at the end of the computing process for a total time of 240 hours. By looking at the two screen captures one can obviously see that both procedures lead to practically identical results. Only minor discrepancies appear in some of the nodes. This can be explained by the fact that dimensionless temperatures have been displayed with four decimal digits, which means that the last digit had been rounded. For instance, the temperature in node (6,10) is 0.1013 for ADT (figure 3), respectively 0.1014 for decomposition (figure 4). The actual values are 0.101348, and 0.1013556 respectively. One must notice that, due to the limited horizontal number of digits available, in the output screen, temperatures in the radial direction have only been displayed for the even number of the node; this is why the mentioned value appears in the fourth column (column zero corresponds to the temperature of the inner surface of the cylinder).

In conclusion, both methods provide practically the same accuracy of the results, which are in very good agreement. Although alternate direction procedures are characterized by a higher accuracy because errors tend to annul each other in the two phases of a time step, decomposition seems to be (at least from the standpoint of the particular case in study) as accurate as ADT. From the point of view of the computer running time necessary to obtain the result, decomposition is definitely much faster than ADT. For the example discussed, the running time for a process time step of 60 seconds was of about 6 minutes for decomposition, whereas ADT took about 9 minutes to display the output.

```

C:\Users\j\\Desktop\ACME20-1\QBASIC.EXE
ACME.DA
delta tau = ? 60
total time = [hours]? 240
=====
time step # = 14400 dt = 60 total time = 240 hours
=====
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.9981 .2754 .1766 .1446 .1313 .1255 .1231 .1221 .1217 .1216 .1216
.9984 .3338 .1931 .1410 .1185 .1084 .1041 .1024 .1017 .1015 .1015
.9985 .3651 .2029 .1363 .1061 .0924 .0865 .0841 .0832 .0829 .0828
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If you wish to save your data in a file, hit << y >>.
If not, hit << n >>.
Enter filename (max. 8 chars)? DA_240h
Data are saved in file DA_240h

Press any key to continue
  
```

**Figure 3.** ADT output screen for a process time of 240 hours, and a time step of 60 seconds.

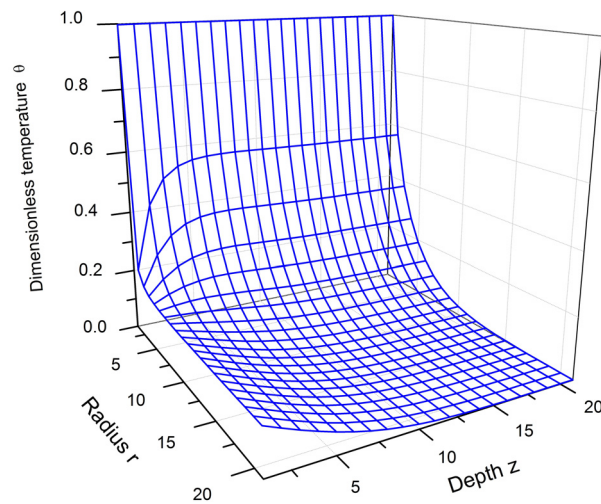
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.9986 .3825 .1904 .0942 .0441 .0192 .0077 .0029 .0011 .0005 .0003
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If you wish to save your data in a file, hit << y >>.
If not, hit << n >>.
Enter filename (max. 8 chars)? DC_240h
Data are saved in file DC_240h

Press any key to continue
  
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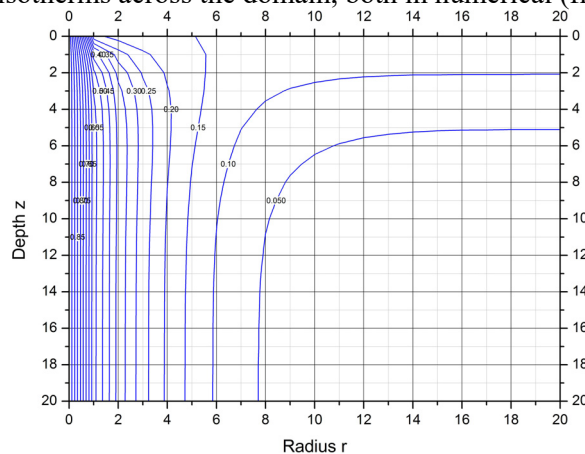
**Figure 4.** Decomposition output screen for a process time of 240 hours, and a time step of 60 seconds.



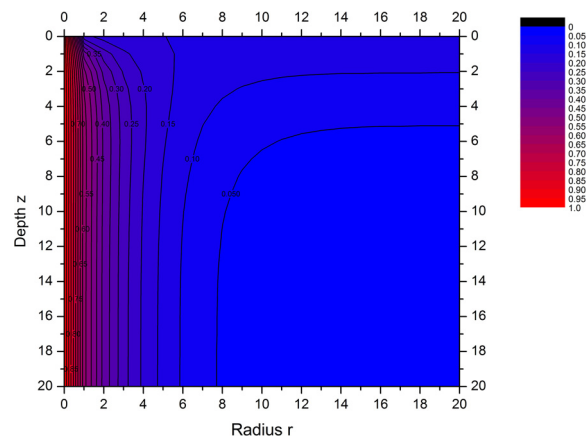
**Figure 5.** 3-D plot of the temperature field in the cylindrical domain – decomposition.

Since both methods discussed above provide practically the same results, we have chosen to use only decomposition due to its faster computer running time. Figure 5 shows the temperature field in 3-D at the end of the process (240 hours).

For a richer information about the temperature distribution, figures 6 and 7 show the position of the isotherms across the domain, both in numerical (figure 6) and contour color fill options (figure 7).



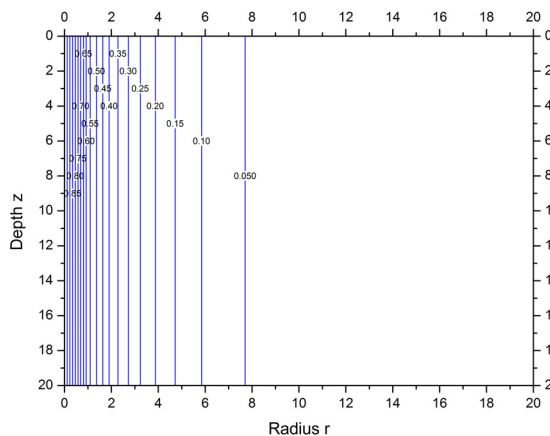
**Figure 6.** Isotherms – contour lines.



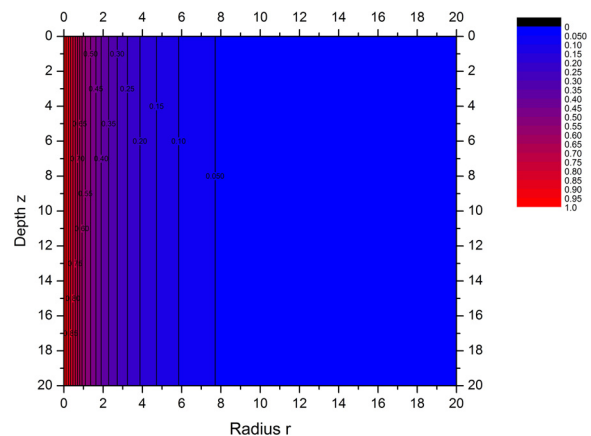
**Figure 7.** Isotherms – contour color fill.

The relatively low values of the temperatures for high values of the radius  $r$  and of the depth  $z$  can be explained by noticing that the heat transfer characteristics of the cylinder material are poor, this substance being a clay soil. We have used these properties because the present paper is part of a wider theoretical approach which aims at studying the charge/discharge characteristics of a single pipe borehole seasonal ground heat storage, intended to be used in conjunction with a heat pump for residential heating purposes. In the model presented in this paper we only consider the charging phase, further work being dedicated to approaching the discharge process by using as start data the temperature distribution at the end of the charging phase.





**Figure 8.** One-dimension heat transfer.  
Isotherms – contour lines.



**Figure 9.** One-dimension heat transfer.  
Isotherms – contour color fill.

For the sake of comparison, figures 8 and 9 show the isotherms (contour lines – figure 8 and contour color fill – figure 9) that result when considering a one-dimension conduction heat transfer (all boundaries except the left hand one are adiabatic). At the bottom of the domain – as expected – the isotherms in the two-dimension heat conduction are identical to those from the one-dimension propagation (compare figures 6 and 7 with figures 8 and 9 respectively), the important depth with respect to the upper boundary turning the two-dimension conduction into a one-direction one. Near the surface, the influence of the convection heat transfer is important and therefore the isotherms are strongly curved in the upper region of the domain.

## 5. Conclusions

Both finite difference methods (ADT and decomposition) show the same accuracy. Decomposition has two advantages: i) it directly uses the one-dimension transient conduction transfer for each phase of the numerical treatment for a time step and thus it has a physical meaning, making it easy to understand and interpret the numerical results provided, and ii) due to the simplicity of the finite equations in terms of the complexity of the relations for the right hand term, decomposition requires less computer running time, making the computing process faster than ADT, which operates with more complicated finite difference equations. For this reason, decomposition is recommendable for use in modeling two-dimension transient heat conduction problems.

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