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THE TEMPERATURE OF TREE TRUNKS—CALCULATED AND OBSERVED[1]

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ABSTRACT

A computer program based on Dusinberre's finite difference method was written to predict the diurnal temperature variations in a tree trunk. The program accounts for solar radiation, thermal radiation, re-radiation and forced convection. The trunk is heterogeneous and anisotropic. A comparison between observations and predictions is made.

THE TEMPERATURE of a tree trunk influences the physiological processes of a tree and the microclimate for insects, parasites, fungi, lichens, lizards, and other organisms. The diurnal and seasonal temperature variations of a tree trunk probably play a role in the growth of the trunk as exhibited by the annual ring. Trunk temperatures may be important in translocation of water and photosynthates. Finally a knowledge of trunk temperature should give an increased understanding of high and low temperature hardiness.

Air temperature, wind speed, and incident radiation determine the energy environment of a tree. During daylight, incident radiation consists of direct solar radiation, reflected solar radiation and long-wave thermal radiation. At night a tree continues to receive thermal radiation. The temperatures of the atmosphere, ground, and surrounding objects determine the amount of thermal radiation received. The thermal properties of a tree (absorptivity, specific heat, and conductivity) combined with geometrical factors govern the distribution of heat-energy. The surface temperature of the trunk will respond nearly instantaneously to the environmental factors, while the temperature at a considerable depth within the trunk will respond many minutes or hours later.

Forster (1953) presented an equation for predicting the temperature history of a tree in the open. Although his solution is very general, the thermal properties of the tree are assumed to be homogeneous. Thus bark, sapwood and heartwood are required to have the same thermal properties. Furthermore the mathematics are extremely complex. Herrington (1964) gives two methods for analyzing trees which are in radiative equilibrium with their surroundings. In other words the tree must be located at the center of a stand.

It is the purpose of the research reported here to demonstrate another method for computing the temperatures within tree trunks and to show comparison with observed temperatures.

METHOD—A tree trunk is assumed to be an infinite cylinder perpendicular to the surface of the earth, taken as an infinite plane. For purposes of computation, the circular cross section of a tree trunk is divided into 12 radial sectors and five concentric rings giving a total of 60 elements. Each element has a definite heat capacity. Also associated with each element are a number of conductances which govern the heat flow into the element. As in real trees the thermal properties of the computer tree are heterogeneous and anisotropic. A computer program using the finite difference method of Dusinberre (1961) was written for the IBM 7090 computer to predict temperature histories at the 60 locations within a tree trunk. This method uses conditions at time t to predict conditions at time t + Δt.

An understanding of the technique can be quickly acquired by considering a simple one-dimensional example in which there is no convection and no radiation. If heat is flowing through the insulated rod shown in Fig. 1 a very simple heat balance equation can be written for any of the interior elements. Such an equation relates the change in temperature of any given element to the heat lost or gained through conduction. Thus for element 4 the heat balance equation is

$$\frac{kA}{l}(T_3 - T_4) + \frac{kA}{l}(T_5 - T_4) = \frac{\rho c A l\,(T_4{}^1 - T_4)}{\Delta t} \qquad (1)$$

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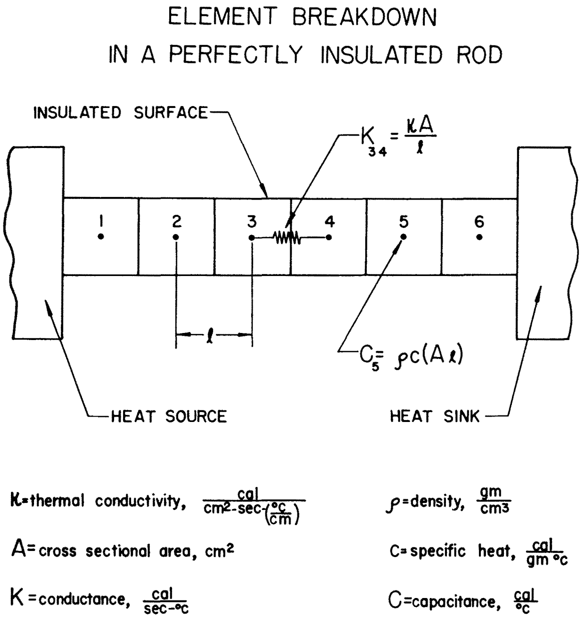


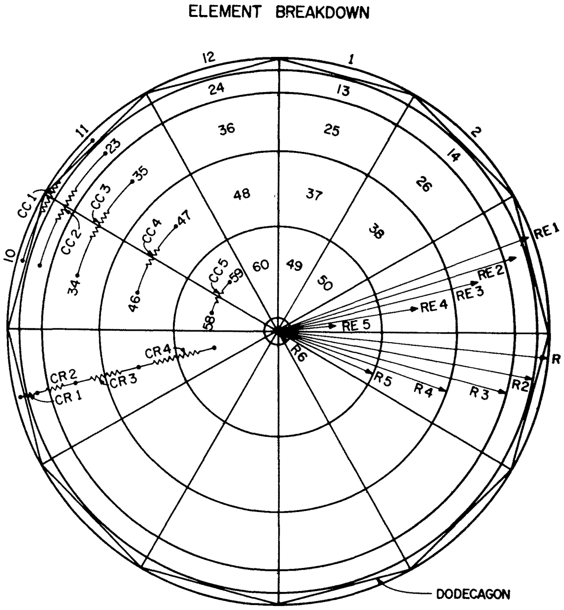


Fig. 1, 2. Element breakdown for a perfectly insulated rod and element breakdown for an infinite cylinder.

where k is the thermal conductivity, A is the cross section area of the rod, l is the distance between the centers of the elements, ρ is density, c is specific heat, T is present temperature, T^1 is future temperature and Δt is the time increment.

The product ρcAl is the heat capacity of an element and has units of calories per degree centigrade. Henceforth it will be given the symbol C. This capacitance is assumed to be concentrated at the center of each element.

The result obtained when a conductivity is multiplied by an area and divided by a length is called a conductance, K. Conductance has units of calories per degree C per second. Thus equation (1) could be rewritten:

$$K_{34}(T_3 - T_4) + K_{54}(T_5 - T_4) = \frac{C_4(T_4^1 - T_4)}{\Delta t} \qquad (2)$$

The symbol K_{34} means "the conductance from element 3 to element 4."

Next, the equation is solved for T_4^1

$$T_4^1 = \frac{\Delta t K_{34}}{C_4} T_3 + \frac{\Delta t K_{54}}{C_4} T_5 + \left[1 - \frac{(K_{34} + K_{54})\,\Delta t}{C_4}\right] T_4 \qquad (3)$$

and reduced to

$$T_4^1 = F_{34}\,T_3 + F_{54}\,T_5 + F_{44}\,T_4 \qquad (4)$$

A similar equation exists for all of the elements in the rod. Once the future temperatures of all elements have been calculated the future temperature becomes present temperature and the process is repeated.

The equations for calculating the future temperatures of the interior elements in Fig. 2 are only slightly more complicated than equation (4). For example the equation for element 21 is

$$T_{21}^1 = F_a T_{20} + F_b T_{22} + F_c T_9 + F_d T_{33} + F_e T_{21} \qquad (5)$$

The subscripts on the F's are written as letters instead of numbers to make the equation more compact. Thus F_a is really "F from 20 to 21" and F_b is "F from 22 to 21."

The equations for the surface elements are similar to those for the interior elements, except that they contain four additional terms to account for the heat losses or gains due to solar radiation, infrared radiation, convection, and re-radiation at the surface. As one would expect, the boundary is the source of many difficulties. For example, the time increment, Δt, must be kept very small or the re-radiation term will result in wildly divergent solutions. In the problems discussed in this paper Δt is 0.005 hr. Thus the machine solves equations like the one above 4800 times for each element for each 24-hr period. Since there are 60 elements, a total of 288,000 solutions must be found.

The area terms used in the basic heat balance equation need special consideration when one is working with polar coordinates. To calculate the radial conductances an average area of the heat flow path was used. Real trees are heterogeneous. Sapwood, heartwood and bark have different thermal properties. Thus the thermal conductivity may not be constant when moving from the center of one element to the center of the next. The finite difference method, however, does not require that the thermal properties of adjacent elements be the same. In the work reported here the thermal properties were assumed to be functions of radial position but not of circumferential position. The procedure used to calculate the radial conductances, CR1, CR2, etc., is described below. See Fig. 2.

The conductance from the center of an element to the border of the adjacent element is calculated. Next, the conductance from the border of the adjacent element to its center is calculated. Then the two conductances are combined in the same way that one would combine two resistances in series.

As mentioned earlier, wood is anisotropic. Thus the conductivity in the circumferential direction will be different from that in the radial direction. This phenomenon can easily be accounted for when the circumferential conductances, CC1, CC2, etc., are calculated.

The specific values of the parameters mentioned above were evaluated from many sources. For example, thermal conductivity was evaluated from moisture content using MacLean's equation (1941) for the thermal conductivity of green wood. From Martin (1963) it appeared that conductivity of bark could be safely approximated by McLean's equation. Reasonable values of moisture content were estimated from Gibbs (1958) and MacLean (1933). In the study reported at the end of this paper, moisture content and specific gravity actually were measured for an aspen tree. Data on the anisotropicity and specific heat of wood were obtained from Kollmann (1951).

An analysis of a tree during very cold weather presented an additional challenge. When a pure substance changes from a liquid to a solid state it remains at a constant temperature, the melting point, until all the latent heat of fusion is removed. (For example, to freeze 1 cc of water, 80 calories must be removed.) Thus any method for calculating temperatures during cooling must require temperature to remain constant while the latent heat is removed. Once the latent heat is removed the temperature must be allowed to continue falling. In order to handle this problem one might keep two "sets of books," one for energy and the other for temperature. This is undesirable since the problem is already quite complicated. A method for telling the computer at what point all latent heat has been removed is required. Dusinberre's method of "excess degrees" is a convenient solution.

If the latent heat is divided by the specific heat a number with units of degrees C results. This number represents the number of degrees temperature would fall below the melting point during cooling before an amount of energy equal to the latent heat of fusion is removed. The computer is programmed to hold temperature at the melting point while the "excess degrees" (degrees below the melting point) are being used up. At this time all latent heat has been removed and temperature is allowed to sink below the melting point. During thawing the process is run backwards.

From the point of view of botany, the model implied by the above discussion is rather crude. For example, Levitt (1956) shows that many plants have two melting points. Furthermore, no definite temperature at which all moisture freezes has been observed. On the other hand, the added complexity resulting from a more sophisticated model does not appear to be justified at this time.

Radiation—The tree trunk is irradiated with direct beam sunlight from a definite direction at any given time and by hemispherical thermal radiation streaming downward from the atmosphere and upward from the ground. Scattered skylight is not included in the computation. The thermal radiation, unlike solar radiation, is present day and night.

The intensity of solar radiation received by a plane perpendicular to the sun's rays at the earth's surface was calculated using the equation

$$S = S_0 \tau^{\sec z} \qquad (6)$$

Where S_0 is the solar constant (2.0 cal cm^{-2} min^{-1}), τ is the transmission coefficient of the atmosphere and z is the zenith angle. The amount of solar radiation received by different parts of the tree was assumed to be that received by the different faces of the dodecagon in Fig. 2. This is obtained by multiplying the value of S by the cosine of the angle between the outward normal of each surface and the line pointing toward the sun.

Two approaches were used to get information about the motion of the sun into the program. Simple polynomial curves were fitted to plots of azimuth and zenith angle as functions of time of day. This method proved quite tedious and inflexible. Consequently, the computer was programmed to solve the spherical triangle, sun-zenith-pole, for every 15 sec. The only input data necessary for this operation are declination and latitude.

The thermal radiation from the atmosphere and ground is a little stronger during midday than it is during the night. This diurnal variation was taken into consideration in the computer program. Since the time variation of the thermal radiation is relatively small, it is usually reasonable to keep it constant during the computation.

Convection—From the standpoint of heat transfer, the most difficult problem was the selection of a convection coefficient, h_c. When there is some wind, no matter how slight, h_c varies continuously along the circumference of the tree. Consequently, completely calm days were assumed in the first problems. The computer could then calculate h_c for itself using standard equations for natural convection. Calm days, however, are very rare. To preserve the spirit of realism, an analysis including forced convection became a necessity. Values of h_c for each of the 12 faces of the dodecagon were calculated by extrapolating experimental data reported by Giedt (1951). Several assumptions about the wind appeared at this point: constant speed and constant direction. For many parts of the world the second assumption is quite reasonable. The first assumption, constant speed, is rather artificial. Fortunately, both assumptions could be eliminated by a more powerful program.

A COMPUTER EXPERIMENT—During the early stages of this study no field work was contemplated. Nevertheless it seemed advisable to try the program on a real tree. Could the program predict temperatures reported in the literature? Experiments of Mix (1916), Harvey (1923) and Eggert (1944) were considered. Concern for the problem of conduction errors in thermoelectric thermometry made the work of Eggert most suitable for a correlation study.

During January, February, and March of 19 4 Eggert measured the cambial temperatures in apple and peach trees. Specifically, he hoped to ascertain whether or not a coat of white paint would sufficiently lower temperatures to prevent frequent thawing and refreezing. In the upper part of Fig. 3 the cross-hatched area represents the range of maximum daily cambial temperatures observed on the south side of an unpainted tree over a six-week period. The solid lines represent the temperature histories of certain elements as predicted by the computer program. Element 19 represents the cambium. The agreement is gratifying when the large number of estimates and assumptions are considered: (1) moisture content, (2) specific gravity, (3) air temperature (except at one point), (4) absorptivity of apple bark, (5) atmospheric transmission, (6) infrared radiation, (7) arbitrary freezing point of -1 C. Furthermore, the upper boundary of the cross-hatched area includes data taken as late as early March. The movement of the sun and the temperature of the air in the program are for late January. It is worth noting that the south side of a tree may well receive a more intense solar radiation during the midday hours of January than at any other time of the year. This rather interesting effect is discussed in detail by Krenn (1933) and is in agreement with the calculations made by the computer.

The lower part of Fig. 3 shows the results for the identical problem assuming that white paint had been applied to the tree. Data for the absorptivity of a painted surface were obtained from Sieber (1941). The unnatural looking flat spot on curve No. 7 is inherent in the finite difference method since an element remains at the freezing point until all the moisture contained is either frozen or melted. In an actual tree the phase boundary would move slowly but continuously through a given region. In our analysis the boundary moves by jumps. More sophisticated techniques could be added to the computer program to eliminate this difficulty. There appears to be no reason why finite

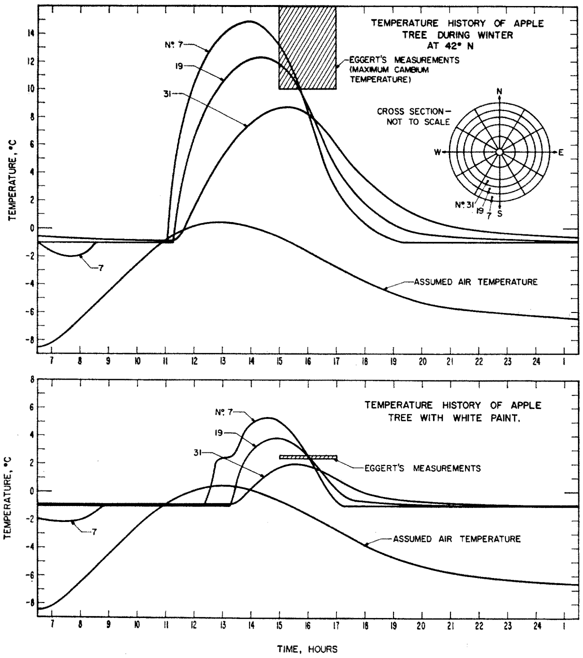


Fig. 3. Measured and predicted temperatures in an apple tree in winter.

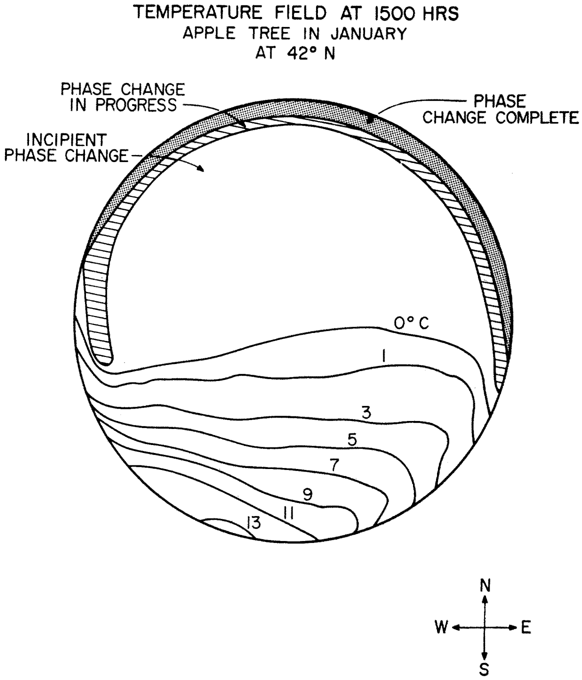


Fig. 4. Calculated isothermal lines in an apple tree during cold weather.

difference techniques cannot be applied to the heat transfer problems of cryobiology.

Figure 4 is a plot of the isothermal lines in the tree at 1500 hr. Note that the assumptions made imply that parts of the north side are completely frozen. This would hardly ever happen except after many days of extreme cold.

A LOG EXPERIMENT—After the apple tree analysis was completed it appeared clear that a certain amount of field work would add realism to the study. In particular the large diurnal variations in temperatures predicted by the computer required confirmation by direct observation. Thus the ideal tree for an experiment would be located in an area sheltered from the wind but with a clear view of the sun. Since the computer program does not contain any temperature modification caused by translocation, a test case could involve a log and not a live tree. A freshly cut 8-ft lodgepole pine log, approximately 7 inches in diameter and with the bark undisturbed, was mounted vertically in a broad open field adjacent to the National Bureau of Standards laboratory at Boulder, Colo.

Diurnal temperature variations at 13 locations on and within the log were recorded on two occasions. The first results were discarded when Dr. Kenneth Knoerr of Duke University pointed out that, despite the use of fine thermocouple wires entering the log from the north side, the data were probably being contaminated by conduction errors. To eliminate all possibility of conduction errors a section of the log approximately 8 inches long was removed from the midsection and taken to the shop where 13 three-inch-deep holes parallel to the axis of the log were drilled (see Fig. 5). The fine thermocouple wires were placed in the holes and brought to the surface of the log through shallow grooves in the end plane. The 8-inch midsection of the log was then glued back in its original position in the trunk. The log now contained 13 thermocouples at various radial distances each of which ran 3 inches parallel to the axis of the log before being brought to the surface. An ice bath was used as a reference temperature for the copper-constantan thermocouples. Voltages were read from a Honeywell portable potentiometer.

The data taken on 28 and 29 October, 1964 are shown in Fig. 5. The temperature history of this log is undoubtedly very similar to that of trees in the open and on the south edge of stands.

TEMPERATURES IN AN ASPEN—Even a fresh log with bark is not a tree. An additional experiment seemed in order. On 1 April, 1965, seventeen 30-gauge copper-constantan thermocouple wires were inserted into an aspen tree near Black Hawk, Colo., at elevation 9300 ft. In order to avoid conduction errors, the holes for the wires were made with a 12-inch-long No. 51 drill (approximately 1/16 inch) held almost parallel to the axis of the tree. As the drill went into the tree it became bent because it is easier for the drill to move with the grain than across it. At the conclusion of the experiment the thermocouples were cut off at the point where they entered the holes. The tree was cut down and the section containing the

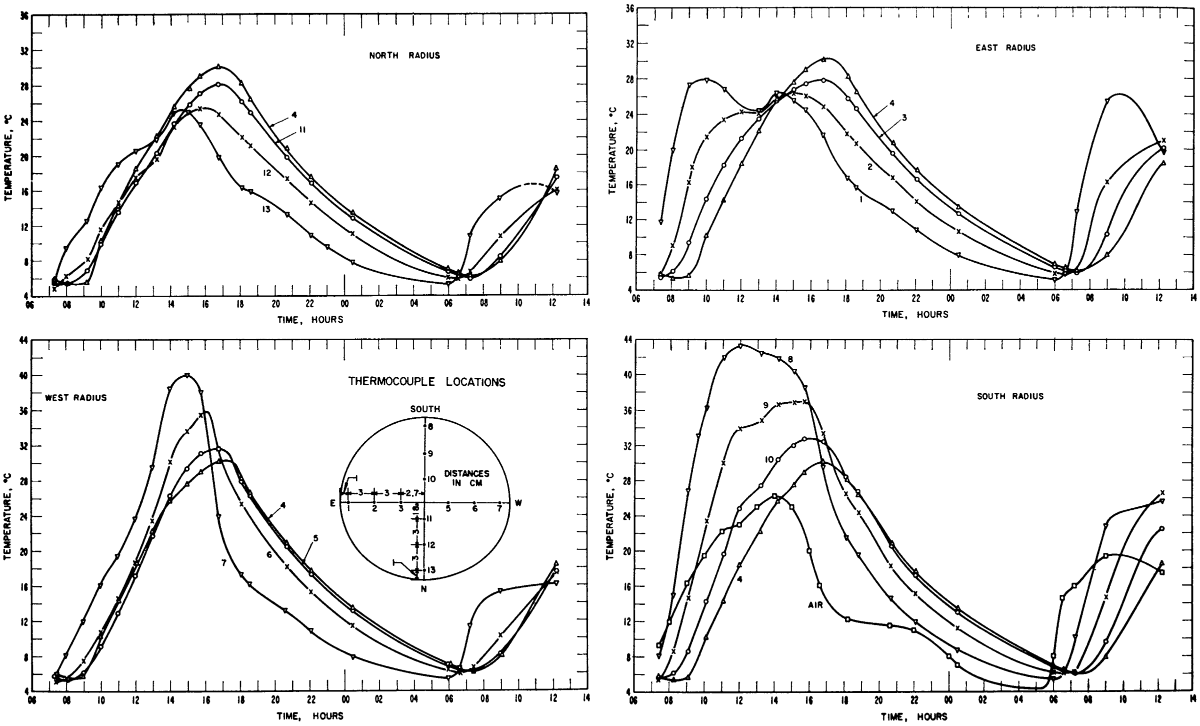
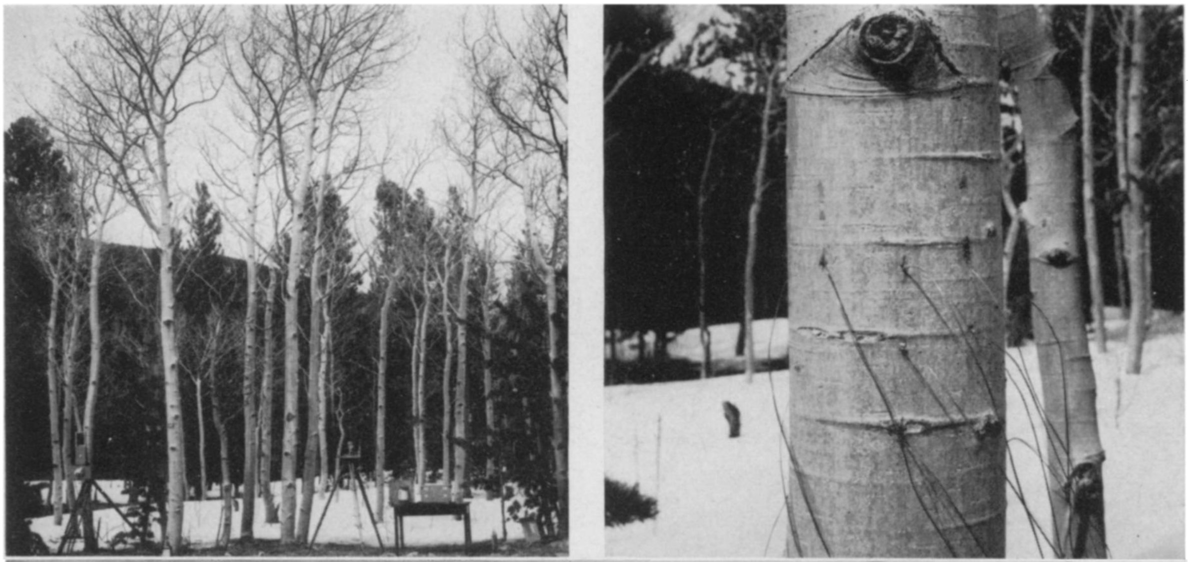


Fig. 5. Temperature histories in vertical lodgepole pine log; October 28 and 29, 1964.



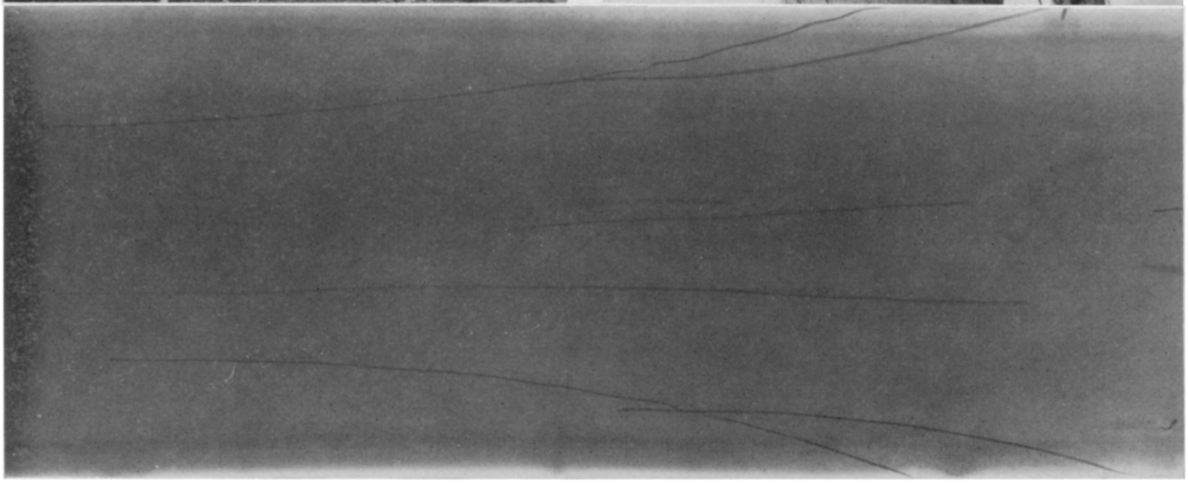


Fig. 6a–7.—Fig. 6a. Above left. South edge of stand where aspen experiment was performed.—Fig. 6b. Above right. Close-up of tree showing thermocouple wires.—Fig. 7. X-ray of tree showing paths of thermocouple wire. View from west side. Long curved wires are from north and south side. The short wires at the right are in the cambium.

wires was taken to St. Anthony's Hospital, Denver, where the enclosed X-ray, Fig. 7, was taken. The curvature of the wires is even more dramatic than had been expected. An adjacent section of the log was cut into little pieces and used to determine moisture content and specific gravity.

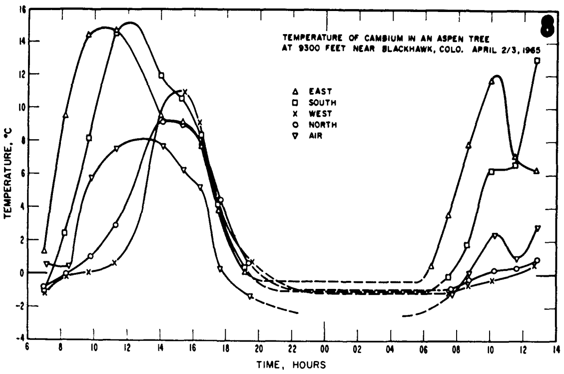
The general setup can be seen from Fig. 6a. The tree used in the experiment is the one with the initials carved on it. The flask and the bottle contained distilled water and ice for the reference temperature.

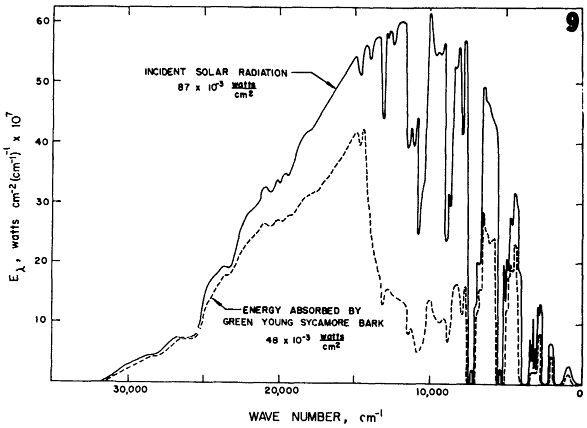
The Honeywell potentiometer and a box containing a stepping switch can be seen on the table. A view of the south side taken at close range can be seen in Fig. 6b.

The tree was located at the south edge of a stand and is thus not surrounded by snow as are some of the other trees visible in the picture. The Eppley pyrheliometer and the Gier and Dunkle radiometer shown in the picture were used to make spot checks. Note the tripod mounts which allow the instruments to be turned to any azimuth and elevation. A 1500-w generator was used to run the electric drill and the radiometer.

The temperature histories for the cambium at the four cardinal points can be seen in Fig. 8. The weather during this period was typical: cold, clear and calm in the early mornings; cool, cloudy and windy in the afternoons. Spot checks of wind velocity were made with a Byram anemometer. Velocities of 10 mph were frequently observed and very few readings of less than 3 mph occurred.

The absorptivity of young sycamore bark was used in the computer analysis. The grayish color and the chlorophyll content make sycamore a logical substitute for aspen bark. Figure 9 shows





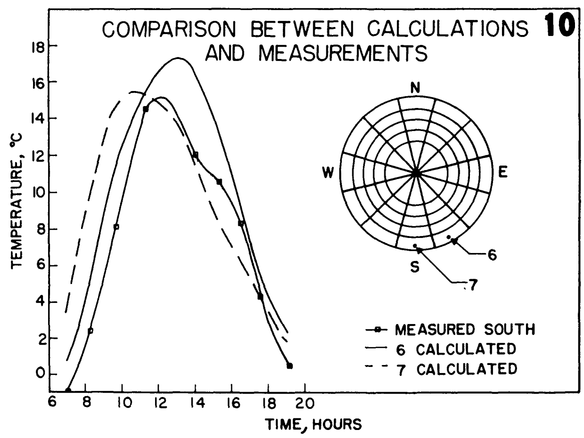


Fig. 8–10.—Fig. 8. Temperature histories of cambium at the four cardinal points.—Fig. 9. Upper curve shows incident solar energy as a function of frequency. Lower curve shows energy absorbed. The ratio of the areas under the curves is the absorptivity, 0.56.—Fig. 10. Comparison between measured temperature of the cambium on the south side and predicted temperatures at two locations.

a plot of the energy absorbed by this bark as a function of the frequency of incident radiation. This curve is based on data obtained by means of spectrophotometers of the National Bureau of Standards, Washington, D. C.

Figure 10 shows a comparison between the temperature histories predicted by the computer program and the measured temperature of the cambium on the south side.

Figure 11 shows the isothermal lines as calculated by the computer program for the environmental conditions and material properties of this experiment.

At 1 00 hr the lowest temperatures in the tree are on the leeward and windward side. Fahnestock and Hare (unpublished) noted a similar phenomenon in forest fires: highest temperatures occur on the leeward side.

An inspection of Fig. 10 shows that the results of the computer analysis are reasonable but not perfect. The difficulties are caused by approximations and assumptions put into the model. For example, h_c varies with location on the tree but not with time. Few trees, especially in the mountains, are circular in cross-section. Not one circular tree could be found in the stand shown in Fig. 6a. Different sides of the tree receive different amounts of infrared radiation and different amounts of reflected solar radiation.

Discussion—The finite difference approach to heat transfer within tree trunks has been shown to be realistic and powerful. Unfortunately the computer is only as good as the assumptions and data put into it. For example, the role of translocation has not been settled to the satisfaction of everyone, in spite of the negative findings of Herrington (1964). The effect of bark roughness on convection and radiation is yet to be understood. More data on the radial distribution of moisture in tree stems should be published.

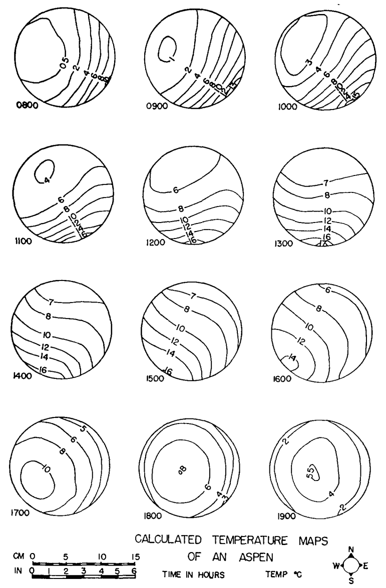


Fig. 11. Calculated maps of the isothermal lines in an aspen at latitude 40°N and at elevation 9300 ft (2800 m) on 2 April. Wind was assumed constant at 6 mph from the west. The time of day appears below and to the left of each map. Temperatures are in degrees C. All even integral values are shown. Occasionally odd integral values as well as certain fractional values are shown for completeness.

Data on the spectral properties of bark are badly needed.

With the present state of the art, it is possible to predict the relative importance of any given factor: convection, radiation, conductivity, etc. It is possible to show the influence of latitude and season. It is also possible to predict the approximate temperature histories of different locations in a tree. But it will be impossible to give highly accurate predictions until the research outlined above has been completed. When that day comes, the methods outlined in this paper will be useful in fields as widely divergent as arboriculture and paleoclimatology.

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