Tree Power Notes

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ABSTRACT.

Key words:

Introduction

Literature review:

1 Assumptions

1. We assume all trees we consider are roughly the same. That means same height, same radius
2. With the previous assumption, we place all devices at the same height, and same depth

2 To do (Feb 7)

Our goal is to write Python scripts that would simulate the temperature distribution in the trunk of the tree and the ambient environment. We will use the heat model for both. See the following reference for more detail:

1. Within a tree stem: see [3]
2. Ambient temperature: see [5] and [6]

3 PDE model of temperature within a tree stem

3.1 Parametersto collect and verify. Nick

Notations:

1. T: temperature (K)
2. ρ: density (kg/m^3)

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3. c: specific heat (J/(kgK))

4. t: time (s)

5. k: thermal conductivity (W/(mK))

6. r: distance from center of the tree (m)

7. ϕ: azimuth angle, measured clockwise with south being $\phi = 0$ (radian/degree), heat [3] assumes $\partial k/\partial\phi = 0$

8. α: albedo of the surface

3.2 The heat equation in cylindircal coordinates

The basic heat equation for temperature distribution is

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \phi}\left(\frac{k}{r}\frac{\partial T}{\partial \phi}\right) \tag{1}$$ heat

We add source terms for the diffusion equation to obtain

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) - \frac{1}{\Delta r}[H + (1-\alpha)(S_{dir} + S_{dif}) + (IR_{in} - IR_{out})] \tag{2}$$

In the following, we explain the source terms:

1. Free convective heat loss/gain happens when the tree surface temp is different from the ambient temp. Forced convection happens when there is wind. We denote the heat loss/gain by H, and

$$H = h(T_{sfc} - T_{air}), \tag{3}$$

   with h (W/(m^2K)) being the convective heat transfer coefficient. Here

$$h = h_{free} - h_{forced}, \tag{4}$$

   more details in heat 3.

2. $S_{dir} + S_{dif}$ represents direct solar radiation, plus diffusion solar radiation.

3. $IR_{in} - IR_{out}$ represents wave radiation from and to the tree. More details see heat [3].

Our goal now is to solve this 2D differential equation with FD scheme.
https://pycav.readthedocs.io/en/latest/api/pde/crank_nicolson.html

3.3 FD scheme for the heat equation

A detailed discussion about polar heat equation is in 2017book [7]. However, the equation presented there was axi-symmetric, and our equation depends on the azimuth angle ϕ. Our goal is to modify the equation, therefore method presented. pg.251, section 3.5.6

Assume for now that $\rho,\ c, k$ are all constants

With constant coefficients, equation heat (3) is simplified to

$$\frac{\rho c}{k}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} \tag{5}$$ $\boxed{\texttt{simple\_h}}$

Discretize the above equation to get

$$\begin{aligned} T^1_{i,j} =& \frac{k\Delta t}{\rho c}\left[\frac{1}{(\Delta r)^2}\left(T^0_{i-1,j} + T^0_{i+1,j} - 2T^0_{i,j}\right) + \frac{1}{2r_i\Delta r}\left(T^0_{i+1,j} - T^0_{i-1,j}\right)\right. \\ &\left.+\frac{1}{r_i^2(\Delta\phi)^2}\left(T^0_{i,j-1} + T^0_{i,j+1} - 2T^0_{i,j}\right)\right] + T^0_{i,j} \end{aligned} \tag{6}$$

with centered difference in time. Will modify to be Crank-Nicolson

3.4 Implementation in Python and numerical results

Reference BAD FORMAT. FOR convience only

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