Tree Power Notes

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ABSTRACT.

Key words:

Introduction

Literature review:

1 Assumptions

With these assumptions, the DE problem reduces to 1D.

1. We assume all trees we consider are roughly the same. That means same height, same radius
2. With the previous assumption, we place all devices at the same height, and same depth

2 To do (Feb 7)

Our goal is to write Python scripts that would simulate the temperature distribution in the trunk of the tree and the ambient environment. We will use the heat model for both. See the following reference for more detail:

1. Within a tree stem: see [3]heat
2. Ambient temperature: see [5]airtemp and [6]forestgreen

2.1 FD model of temperature within a tree stem

Notations:

1. T: temperature (K)
2. ρ: density (kg/m^3)
3. c: specific heat (J/(kgK))

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4. t: time (s)
5. k: thermal conductivity (W/(mK))
6. r: distance from center of the tree (m)
7. ϕ: azimuth angle, measured clockwise with south being $\phi = 0$ (radian/degree), [3] assumes $\partial k/\partial \phi = 0$
8. α: albedo of the surface

The basic heat equation for temperature distribution is

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \phi}\left(\frac{k}{r}\frac{\partial T}{\partial \phi}\right) \tag{1}$$

We add source terms for the diffusion equation to obtain

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) - \frac{1}{\Delta r}[H + (1-\alpha)(S_{dir} + S_{dif}) + (IR_{in} - IR_{out})] \tag{2}$$

In the following, we explain the source terms:

1. Free convective heat loss/gain happens when the tree surface temp is different from the ambient temp. Forced convection happens when there is wind. We denote the heat loss/gain by H, and
$$H = h(T_{sfc} - T_{air}), \tag{3}$$
with h (W/(m^2K)) being the convective heat transfer coefficient. Here
$$h = h_{free} - h_{forced}, \tag{4}$$
more details in [3].
2. $S_{dir} + S_{dif}$ represents direct solar radiation, plus diffusion solar radiation.
3. $IR_{in} - IR_{out}$ represents wave radiation from and to the tree. More details see [3].

Our goal now is to solve this 2D differential equation with FD scheme.
ref, see section 3.6: https://link.springer.com/content/pdf/10.1007%2F978-3-319-55456-3.pdf and
https://pycav.readthedocs.io/en/latest/api/pde/crank_nicolson.html

2.2 FD model of temperature in the forest

To do.

3 To do (Jan 31)

1. (DISCUSSION ITEM) Understand the hardware more: which equations to use, heat? thermoelectric? both? DONE. use heat equation. Goal is to model the temp in the tree and in the environment. At a same height. see sapwood 4.

2. (ACTION ITEM) Write code (Matlab/Python) to solve equations (equations see next section). ref: http://www.claudiobellei.com/2016/11/10/crank-nicolson/

   http://www.claudiobellei.com/2016/10/15/explicit-parabolic/

   1D code. Keep code in github repo: https://github.com/yajuna/treeRemote

3. Verify the assumptions.

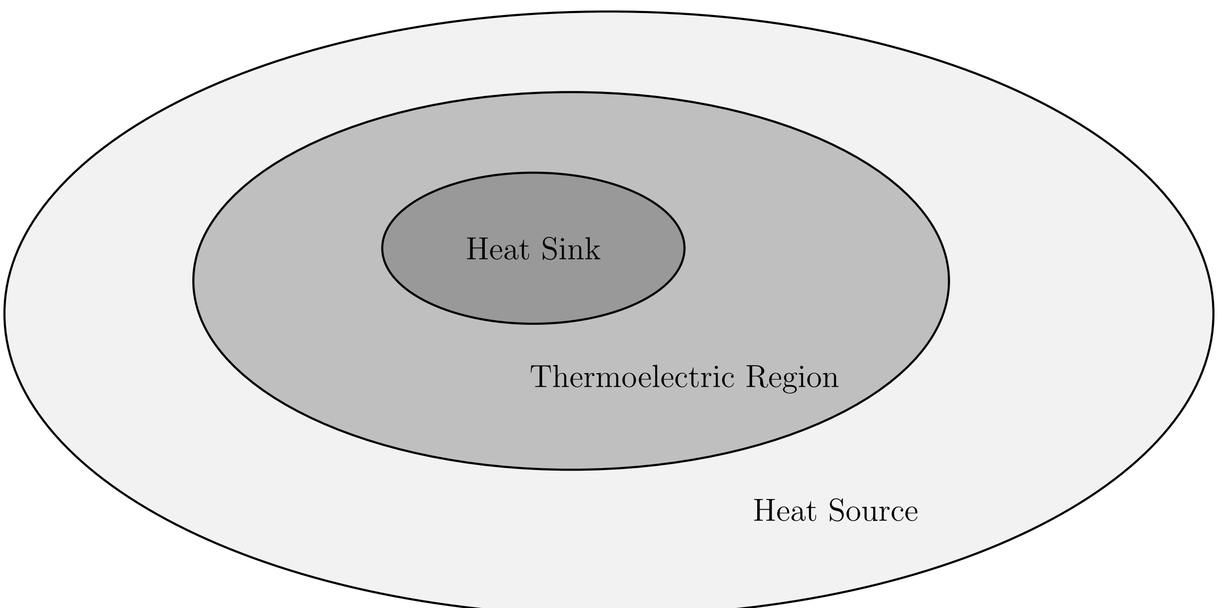
   (a) With vertical drilling and data collection, find the best height. Call $h_0$see sapwood 4

   (b) With horizontal drilling and data collection, find the best depth. Call r_0

   With the assumption that we have found the "best" height and radius (best: highest voltage, most activities, depending on the device??), we simplify the problem into a 1D problem.

4. Find appropriate parameters in DE. For now, we take simple ones. ISSUE: if real parameters are small or have varying magnitudes, might cause unexpected numerical errorsWaiting for confirmation from Nick.

4 Differential Equations



We modify the equations from [thermoelectric 1], to radial equations. Heat source and thermoelectric material in Fig.2[thermoelectric 1] is now along the radius r. (ASK ABOUT HARDWARE. Fig.1, and Fig.2. from ref)

Heat sink and resource for region R are governed by the heat equation

$$\rho_R \, C_{vR} \, \frac{\partial T}{\partial t} = \nabla \cdot (k_R \nabla T), \tag{5}$$

heat

here, T is the temperature, t time, ρ_R the density for region R, C_{vR} specific heat, k_R is the thermal conductivity.

The middle layer is the thermoelectric region, governed by

$$\rho_m \, C_{vm} \, \frac{\partial T}{\partial t} = \sigma \mathbf{E} \cdot \mathbf{E} - \sigma \cdot \alpha \mathbf{E} \cdot \nabla T + \nabla \cdot [(k_m + \sigma \alpha^2 T)\nabla T - \sigma \alpha T \mathbf{E}], \tag{6} \quad \boxed{\texttt{elec}}$$

$$\frac{\partial \rho}{\partial t} = \nabla \cdot (-\sigma \mathbf{E} + \sigma \alpha \nabla T). \tag{7} \quad \boxed{\texttt{chargede}}$$

Here $\mathbf{E}$ is the electric field, ρ the charge density, σ the electric conductivity, and α the Seebeck coefficient (with temp dependence $\alpha = \alpha(T)$. Need curve fitting to decide).

DO NOT KNOW ANY PARAMETERS

4.1 Reduction to 1D problem

Based on our assumptions, we reduce spatial dependence to only on radius r:

$$\rho_R \, C_{vR} \, \frac{\partial T}{\partial t} = \frac{\partial}{\partial r}\big(k_R \frac{\partial T}{\partial r}\big), \tag{8} \quad \boxed{\texttt{heat1d}}$$

$$\rho_m \, C_{vm} \, \frac{\partial T}{\partial t} = \sigma E^2 - \sigma \alpha E \cdot \frac{\partial T}{\partial r} + \frac{\partial}{\partial r}[(k_m + \sigma \alpha^2 T)\frac{\partial T}{\partial r} - \sigma \alpha T E], \tag{9} \quad \boxed{\texttt{elec1d}}$$

$$\epsilon \frac{\partial E}{\partial t} = J_0 - \sigma E + \sigma \alpha \frac{\partial T}{\partial r}. \tag{10} \quad \boxed{\texttt{chargede}}$$

Here ($\overset{\texttt{chargedensity1d}}{10}$) is the result of integrating ($\overset{\texttt{chargedensity}}{7}$), and J_0 a constant.

4.2 Radial boundary conditions

From outside inwards, the boundary conditions are:

- Outside: tree bark has T_{amb}.
- Between Heat Source and Thermoelectric region, a voltage V_0 is generated, heat flux and temperature??
- section A. in Yan paper

Reference BAD FORMAT. FOR convience only

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2. MIT notes online mit
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