Tree Power Notes

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ABSTRACT.

Key words:

Introduction

Literature review:

1 Assumptions

1. We assume all trees we consider are roughly the same. That means same height, same radius
2. With the previous assumption, we place all devices at the same height, and same depth

2 To do (March)

1. get heat program running in Python3 (eliminate all syntax error)
2. find appropriate parameters for testing
3. think of ways to validate computation (analytical solution, similar and simpler problem)
4. write a documentation (i.e., this file??)

3 PDE model of temperature within a tree stem

3.1 Parameters to collect and verify. Nick

Notations:

1. T: temperature (K)
2. ρ: density (kg/m^3)

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3. c: specific heat (J/(kgK))
4. t: time (s)
5. k: thermal conductivity (W/(mK))
6. r: distance from center of the tree (m)
7. ϕ: azimuth angle, measured clockwise with south being $\phi = 0$ (radian/degree), [3] assumes $\partial k/\partial\phi = 0$
8. α: albedo of the surface

3.2 The heat equation in cylindrical coordinates

The standard heat equation for temperature distribution in cylindrical coordinates is

$$\rho c\frac{\partial T}{\partial t} = \nabla\cdot(k\nabla T) = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial\phi}\left(\frac{k}{r}\frac{\partial T}{\partial\phi}\right) \tag{1}$$

heat

We add source terms for the diffusion equation to obtain

$$\rho c\frac{\partial T}{\partial t} = \nabla\cdot(k\nabla T) - \frac{1}{\Delta r}[H + (1-\alpha)(S_{dir} + S_{dif}) + (IR_{in} - IR_{out})] \tag{2}$$

In the following, we explain the source terms:

1. Free convective heat loss/gain happens when the tree surface temp is different from the ambient temp. Forced convection happens when there is wind. We denote the heat loss/gain by H, and
$$H = h(T_{sfc} - T_{air}), \tag{3}$$
with h (W/(m^2K)) being the convective heat transfer coefficient. Here
$$h = h_{free} - h_{forced}, \tag{4}$$
more details in 3.
2. $S_{dir} + S_{dif}$ represents direct solar radiation, plus diffusion solar radiation.
3. $IR_{in} - IR_{out}$ represents wave radiation from and to the tree. More details see [3].

In this work, we solve the heat equation to model the temperature in tree trunks, for the purpose of energy harvesting. We modify above source terms to fit the parameters for trees in Washington state.

https://pycav.readthedocs.io/en/latest/api/pde/crank_nicolson.html

3.3 FD scheme for the heat equation

We solve the above equation numerically with the Finite-Difference (FD) method, as the FD method is simple to implement, and robust.

A detailed discussion on axi-symmetric heat equation is in [7]. We model the heat equation to depend on the azimuth angle ϕ, as trees in Washington state experience different sunlight and therefore growth in different direction. pg.251, section 3.5.6

For simplicity of discussion, we assume ρ, c, k are all constants for now. Then the constant coefficient equation (3) simplifies to

$$\frac{\rho c}{k}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} \qquad (5)$$

simple_heat

With centered FD discretization in time and space, we have

- $\dfrac{\partial T}{\partial t} \approx \dfrac{T_{i,j}^1 - T_{i,j}^0}{\Delta t}$
- $\dfrac{\partial T}{\partial r} \approx \dfrac{T_{i+1,j}^0 - T_{i-1,j}^0}{2\Delta r}$
- $\dfrac{\partial^2 T}{\partial r^2} \approx \dfrac{T_{i-1,j}^0 + T_{i+1,j}^0 - 2T_{i,j}^0}{(\Delta r)^2}$
- $\dfrac{\partial^2 T}{\partial \phi^2} \approx \dfrac{T_{i,j-1}^0 + T_{i,j+1}^0 - 2T_{i,j}^0}{(\Delta \phi)^2}$

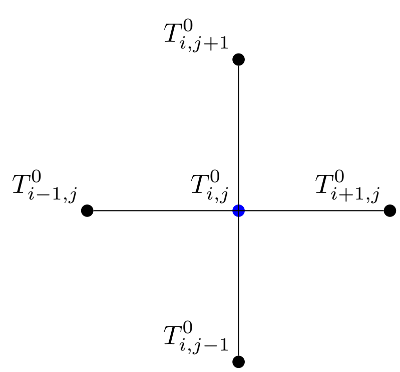
Here $T_{i,j}^n \approx T(r_i, \phi_j, t_n)$ denotes the numerical solution at the point (r_i, ϕ_j, t_n). Then the above *continuous differential equation* becomes the following *discrete difference equation*:

$$\begin{aligned} T_{i,j}^1 =& \frac{k\Delta t}{\rho c}\left[\frac{1}{(\Delta r)^2}\left(T_{i-1,j}^0 + T_{i+1,j}^0 - 2T_{i,j}^0\right) + \frac{1}{2r_i\Delta r}\left(T_{i+1,j}^0 - T_{i-1,j}^0\right)\right. \\ &\left. + \frac{1}{r_i^2(\Delta\phi)^2}\left(T_{i,j-1}^0 + T_{i,j+1}^0 - 2T_{i,j}^0\right)\right] + T_{i,j}^0, \end{aligned} \qquad (6)$$

with $k_1 = \dfrac{k\Delta t}{\rho c(\Delta r)^2}$, $k_2 = \dfrac{k\Delta t}{2\rho c r_i \Delta r}$, and $k_3 = \dfrac{k\Delta t}{\rho c r_i^2(\Delta\phi)^2}$, above equation can be implemented as

$$T_{i,j}^1 = k_1\left(T_{i-1,j}^0 + T_{i+1,j}^0 - 2T_{i,j}^0\right) + k_2\left(T_{i+1,j}^0 - T_{i-1,j}^0\right) + k_3\left(T_{i,j-1}^0 + T_{i,j+1}^0 - 2T_{i,j}^0\right) + T_{i,j}^0, \qquad (7)$$

where we obtain $T_{i,j}^1$ each time step with T values in several locations from previous step.



Space information at each time step.

Image one more axis normal to the paper, pointing outwards above the blue dot. That would be $T_{i,j}^1$.

with centered difference in time. Will modify to be Crank-Nicolson

3.4 Implementation in Python and numerical results

For vectorization, we rewrite the above equation as

$$T^1_{i,j} = (k_1 - k_2)T^0_{i-1,j} + (k_1 + k_2)T^0_{i+1,j} + (1 - 2k_1 - 2k_3)T^0_{i,j} + +k_3T^0_{i,j-1} + k_3T^0_{i,j+1} \tag{8}$$

Read Chapter 9 of Randy LeVeque.

Reference BAD FORMAT. FOR convience only

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