Coefficients of Friction f for Threaded Pairs

Source: H. A. Rothbart and T. H. Brown, Jr. Mechanical Design Handbook, 2nd ed., McGraw-Hill, New York, 2006.

Screw		Nut M	Nut Material				
Material	Steel	Bronze	Brass	Cast Iron			
Steel, dry	0.15-0.25	0.15-0.23	0.15-0.19	0.15-0.25			
Steel, machine oil	0.11-0.17	0.10-0.16	0.10-0.15	0.11 – 0.17			
Bronze	0.08-0.12	0.04-0.06		0.06-0.09			

#### Table 8-6

Thrust-Collar Friction Coefficients

Source: H. A. Rothbart and T. H. Brown, Jr., Mechanical Design Handbook, 2nd ed., McGraw-Hill, New York, 2006.

Combination	Running	Starting
Soft steel on cast iron	0.12	0.17
Hard steel on cast iron	0.09	0.15
Soft steel on bronze	0.08	0.10
Hard steel on bronze	0.06	0.08

common material pairs. Table 8-6 shows coefficients of starting and running friction for common material pairs.

#### 8-3 **Threaded Fasteners**

As you study the sections on threaded fasteners and their use, be alert to the stochastic and deterministic viewpoints. In most cases the threat is from overproof loading of fasteners, and this is best addressed by statistical methods. The threat from fatigue is lower, and deterministic methods can be adequate.

Figure 8-9 is a drawing of a standard hexagon-head bolt. Points of stress concentration are at the fillet, at the start of the threads (runout), and at the thread-root fillet in the plane of the nut when it is present. See Table A-29 for dimensions. The diameter of the washer face is the same as the width across the flats of the hexagon. The thread length of inch-series bolts, where d is the nominal diameter, is

$$L_T = \begin{cases} 2d + \frac{1}{4} \text{ in } & L \le 6 \text{ in} \\ 2d + \frac{1}{2} \text{ in } & L > 6 \text{ in} \end{cases}$$
 (8–13)

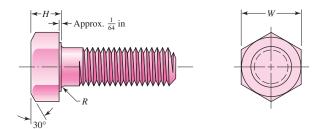
and for metric bolts is

$$L_T = \begin{cases} 2d + 6 & L \le 125 & d \le 48 \\ 2d + 12 & 125 < L \le 200 \\ 2d + 25 & L > 200 \end{cases}$$
 (8-14)

where the dimensions are in millimeters. The ideal bolt length is one in which only one or two threads project from the nut after it is tightened. Bolt holes may have burrs or sharp edges after drilling. These could bite into the fillet and increase stress concentration. Therefore, washers must always be used under the bolt head to prevent this. They should be of hardened steel and loaded onto the bolt so that the rounded edge of the stamped hole faces the washer face of the bolt. Sometimes it is necessary to use washers under the nut too.

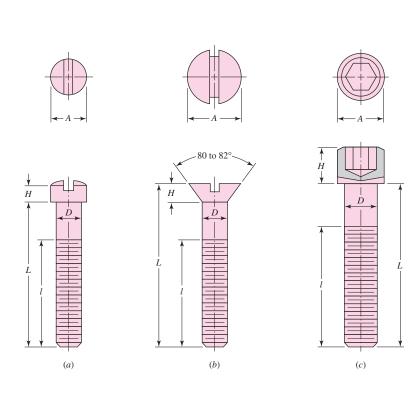
The purpose of a bolt is to clamp two or more parts together. The clamping load stretches or elongates the bolt; the load is obtained by twisting the nut until the bolt

Hexagon-head bolt; note the washer face, the fillet under the head, the start of threads, and the chamfer on both ends. Bolt lengths are always measured from below the head.



### Figure 8-10

Typical cap-screw heads:
(a) fillister head; (b) flat head;
(c) hexagonal socket head. Cap screws are also manufactured with hexagonal heads similar to the one shown in Fig. 8–9, as well as a variety of other head styles. This illustration uses one of the conventional methods of representing threads.



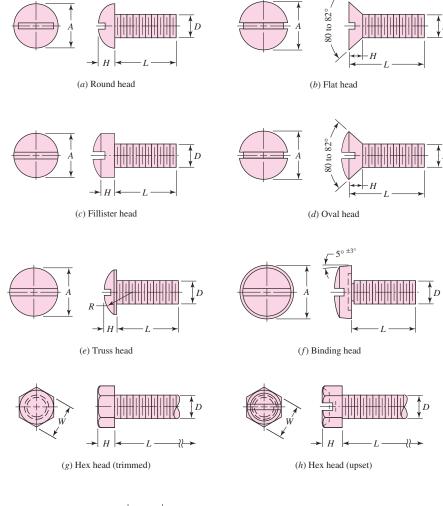
has elongated almost to the elastic limit. If the nut does not loosen, this bolt tension remains as the preload or clamping force. When tightening, the mechanic should, if possible, hold the bolt head stationary and twist the nut; in this way the bolt shank will not feel the thread-friction torque.

The head of a hexagon-head cap screw is slightly thinner than that of a hexagon-head bolt. Dimensions of hexagon-head cap screws are listed in Table A–30. Hexagon-head cap screws are used in the same applications as bolts and also in applications in which one of the clamped members is threaded. Three other common capscrew head styles are shown in Fig. 8–10.

A variety of machine-screw head styles are shown in Fig. 8–11. Inch-series machine screws are generally available in sizes from No. 0 to about  $\frac{3}{8}$  in.

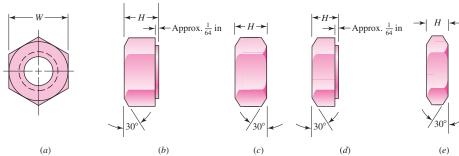
Several styles of hexagonal nuts are illustrated in Fig. 8–12; their dimensions are given in Table A–31. The material of the nut must be selected carefully to match that of the bolt. During tightening, the first thread of the nut tends to take the entire load; but yielding occurs, with some strengthening due to the cold work that takes place, and the load is eventually divided over about three nut threads. For this reason you should never reuse nuts; in fact, it can be dangerous to do so.

Types of heads used on machine screws.



## Figure 8-12

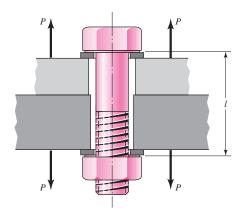
Hexagonal nuts: (a) end view, general; (b) washer-faced regular nut; (c) regular nut chamfered on both sides; (d) jam nut with washer face; (e) jam nut chamfered on both sides.



## 8-4 **Joints—Fastener Stiffness**

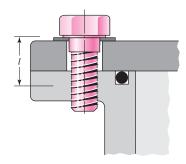
When a connection is desired that can be disassembled without destructive methods and that is strong enough to resist external tensile loads, moment loads, and shear loads, or a combination of these, then the simple bolted joint using hardened-steel washers is a good solution. Such a joint can also be dangerous unless it is properly designed and assembled by a *trained* mechanic.

A bolted connection loaded in tension by the forces *P*. Note the use of two washers. Note how the threads extend into the body of the connection. This is usual and is desired. *l* is the grip of the connection.



### Figure 8-14

Section of cylindrical pressure vessel. Hexagon-head cap screws are used to fasten the cylinder head to the body. Note the use of an O-ring seal. *l* is the effective grip of the connection (see Table 8–7).



A section through a tension-loaded bolted joint is illustrated in Fig. 8–13. Notice the clearance space provided by the bolt holes. Notice, too, how the bolt threads extend into the body of the connection.

As noted previously, the purpose of the bolt is to clamp the two, or more, parts together. Twisting the nut stretches the bolt to produce the clamping force. This clamping force is called the *pretension* or *bolt preload*. It exists in the connection after the nut has been properly tightened no matter whether the external tensile load *P* is exerted or not.

Of course, since the members are being clamped together, the clamping force that produces tension in the bolt induces compression in the members.

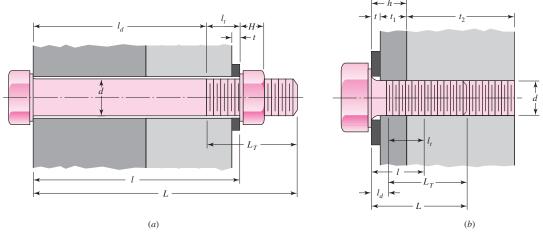
Figure 8–14 shows another tension-loaded connection. This joint uses cap screws threaded into one of the members. An alternative approach to this problem (of not using a nut) would be to use studs. A stud is a rod threaded on both ends. The stud is screwed into the lower member first; then the top member is positioned and fastened down with hardened washers and nuts. The studs are regarded as permanent, and so the joint can be disassembled merely by removing the nut and washer. Thus the threaded part of the lower member is not damaged by reusing the threads.

The *spring rate* is a limit as expressed in Eq. (4–1). For an elastic member such as a bolt, as we learned in Eq. (4–2), it is the ratio between the force applied to the member and the deflection produced by that force. We can use Eq. (4–4) and the results of Prob. 4–1 to find the stiffness constant of a fastener in any bolted connection.

The *grip l* of a connection is the total thickness of the clamped material. In Fig. 8-13 the grip is the sum of the thicknesses of both members and both washers. In Fig. 8-14 the effective grip is given in Table 8-7.

The stiffness of the portion of a bolt or screw within the clamped zone will generally consist of two parts, that of the unthreaded shank portion and that of the

### Suggested Procedure for Finding Fastener Stiffness



Given fastener diameter d and pitch p in mm or number of threads per inch

Washer thickness: t from Table A-32 or A-33

Nut thickness [Fig. (a) only]: H from Table A-31

Grip length:

For Fig. (a): l = thickness of all material squeezed

between face of bolt and face of nut

For Fig. (b): 
$$l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \ge d \end{cases}$$

Fastener length (round up using Table A–17\*):

For Fig. (a): L > l + H

For Fig. (*b*): L > h + 1.5d

Threaded length  $L_T$ : Inch series:

$$L_T = \begin{cases} 2d + \frac{1}{4} \text{ in,} & L \le 6 \text{ in} \\ 2d + \frac{1}{2} \text{ in,} & L > 6 \text{ in} \end{cases}$$

Metric series:

$$L_T = \begin{cases} 2d + 6 \text{ mm}, & L \le 125 \text{ mm}, d \le 48 \text{ mm} \\ 2d + 12 \text{ mm}, & 125 < L \le 200 \text{ mm} \\ 2d + 25 \text{ mm}, & L > 200 \text{ mm} \end{cases}$$

Length of unthreaded portion in grip:  $l_d = L - L_T$ 

Length of threaded portion in grip:  $l_t = l - l_d$ Area of unthreaded portion:  $A_d = \pi d^2/4$ 

Area of threaded portion:  $A_t$  from Table 8–1 or 8–2

Fastener stiffness:  $k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$ 

<sup>\*</sup>Bolts and cap screws may not be available in all the preferred lengths listed in Table A–17. Large fasteners may not be available in fractional inches or in millimeter lengths ending in a nonzero digit. Check with your bolt supplier for availability.

threaded portion. Thus the stiffness constant of the bolt is equivalent to the stiffnesses of two springs in series. Using the results of Prob. 4–1, we find

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$
 or  $k = \frac{k_1 k_2}{k_1 + k_2}$  (8–15)

for two springs in series. From Eq. (4–4), the spring rates of the threaded and unthreaded portions of the bolt in the clamped zone are, respectively,

$$k_t = \frac{A_t E}{l_t} \qquad k_d = \frac{A_d E}{l_d} \tag{8-16}$$

where  $A_t$  = tensile-stress area (Tables 8–1, 8–2)

 $l_t$  = length of threaded portion of grip

 $A_d$  = major-diameter area of fastener

 $l_d$  = length of unthreaded portion in grip

Substituting these stiffnesses in Eq. (8–15) gives

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} \tag{8-17}$$

where  $k_b$  is the estimated effective stiffness of the bolt or cap screw in the clamped zone. For short fasteners, the one in Fig. 8–14, for example, the unthreaded area is small and so the first of the expressions in Eq. (8–16) can be used to find  $k_b$ . For long fasteners, the threaded area is relatively small, and so the second expression in Eq. (8–16) can be used. Table 8–7 is useful.

# 8-5 Joints—Member Stiffness

In the previous section, we determined the stiffness of the fastener in the clamped zone. In this section, we wish to study the stiffnesses of the members in the clamped zone. Both of these stiffnesses must be known in order to learn what happens when the assembled connection is subjected to an external tensile loading.

There may be more than two members included in the grip of the fastener. All together these act like compressive springs in series, and hence the total spring rate of the members is

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_i}$$
 (8–18)

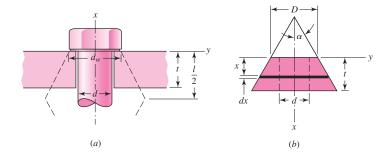
If one of the members is a soft gasket, its stiffness relative to the other members is usually so small that for all practical purposes the others can be neglected and only the gasket stiffness used.

If there is no gasket, the stiffness of the members is rather difficult to obtain, except by experimentation, because the compression region spreads out between the bolt head and the nut and hence the area is not uniform. There are, however, some cases in which this area can be determined.

Ito<sup>2</sup> has used ultrasonic techniques to determine the pressure distribution at the member interface. The results show that the pressure stays high out to about 1.5 bolt radii.

<sup>&</sup>lt;sup>2</sup>Y. Ito, J. Toyoda, and S. Nagata, "Interface Pressure Distribution in a Bolt-Flange Assembly," ASME paper no. 77-WA/DE-11, 1977.

Compression of a member with the equivalent elastic properties represented by a frustum of a hollow cone. Here, *l* represents the grip length.



The pressure, however, falls off farther away from the bolt. Thus Ito suggests the use of Rotscher's pressure-cone method for stiffness calculations with a variable cone angle. This method is quite complicated, and so here we choose to use a simpler approach using a fixed cone angle.

Figure 8–15 illustrates the general cone geometry using a half-apex angle  $\alpha$ . An angle  $\alpha=45^{\circ}$  has been used, but Little³ reports that this overestimates the clamping stiffness. When loading is restricted to a washer-face annulus (hardened steel, cast iron, or aluminum), the proper apex angle is smaller. Osgood⁴ reports a range of  $25^{\circ} \leq \alpha \leq 33^{\circ}$  for most combinations. In this book we shall use  $\alpha=30^{\circ}$  except in cases in which the material is insufficient to allow the frusta to exist.

Referring now to Fig. 8–15b, the contraction of an element of the cone of thickness dx subjected to a compressive force P is, from Eq. (4–3),

$$d\delta = \frac{P \, dx}{E \, A} \tag{a}$$

The area of the element is

$$A = \pi \left(r_o^2 - r_i^2\right) = \pi \left[\left(x \tan \alpha + \frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right]$$

$$= \pi \left(x \tan \alpha + \frac{D+d}{2}\right) \left(x \tan \alpha + \frac{D-d}{2}\right)$$
(b)

Substituting this in Eq. (a) and integrating gives a total contraction of

$$\delta = \frac{P}{\pi E} \int_0^t \frac{dx}{[x \tan \alpha + (D+d)/2][x \tan \alpha + (D-d)/2]}$$
 (c)

Using a table of integrals, we find the result to be

$$\delta = \frac{P}{\pi E d \tan \alpha} \ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}$$
 (d)

Thus the spring rate or stiffness of this frustum is

$$k = \frac{P}{\delta} = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}}$$
(8–19)

<sup>&</sup>lt;sup>3</sup>R. E. Little, "Bolted Joints: How Much Give?" *Machine Design*, Nov. 9, 1967.

<sup>&</sup>lt;sup>4</sup>C. C. Osgood, "Saving Weight on Bolted Joints," Machine Design, Oct. 25, 1979.

With  $\alpha = 30^{\circ}$ , this becomes

$$k = \frac{0.5774\pi Ed}{\ln\frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}}$$
(8–20)

Equation (8–20), or (8–19), must be solved separately for each frustum in the joint. Then individual stiffnesses are assembled to obtain  $k_m$  using Eq. (8–18).

If the members of the joint have the same Young's modulus E with symmetrical frusta back to back, then they act as two identical springs in series. From Eq. (8–18) we learn that  $k_m = k/2$ . Using the grip as l = 2t and  $d_w$  as the diameter of the washer face, from Eq. (8–19) we find the spring rate of the members to be

$$k_{m} = \frac{\pi E d \tan \alpha}{2 \ln \frac{(l \tan \alpha + d_{w} - d) (d_{w} + d)}{(l \tan \alpha + d_{w} + d) (d_{w} - d)}}$$
(8–21)

The diameter of the washer face is about 50 percent greater than the fastener diameter for standard hexagon-head bolts and cap screws. Thus we can simplify Eq. (8–21) by letting  $d_w = 1.5d$ . If we also use  $\alpha = 30^{\circ}$ , then Eq. (8–21) can be written as

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\frac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)}$$
(8–22)

It is easy to program the numbered equations in this section, and you should do so. The time spent in programming will save many hours of formula plugging.

To see how good Eq. (8–21) is, solve it for  $k_m/Ed$ :

$$\frac{k_m}{Ed} = \frac{\pi \tan \alpha}{2 \ln \left[ \frac{(l \tan \alpha + d_w - d) (d_w + d)}{(l \tan \alpha + d_w + d) (d_w - d)} \right]}$$

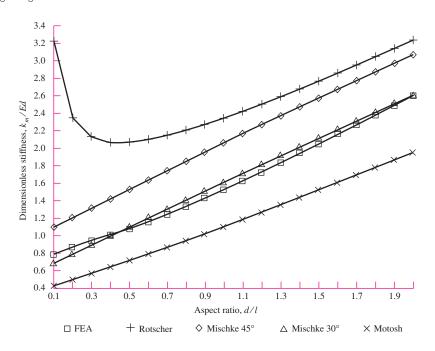
Earlier in the section use of  $\alpha=30^\circ$  was recommended for hardened steel, cast iron, or aluminum members. Wileman, Choudury, and Green<sup>5</sup> conducted a finite element study of this problem. The results, which are depicted in Fig. 8–16, agree with the  $\alpha=30^\circ$  recommendation, coinciding exactly at the aspect ratio d/l=0.4. Additionally, they offered an exponential curve-fit of the form

$$\frac{k_m}{E_d} = A \exp(Bd/l) \tag{8-23}$$

with constants A and B defined in Table 8–8. Equation (8–23) offers a simple calculation for member stiffness  $k_m$ . However, it is very important to note that the *entire joint* must be made up of the *same material*. For departure from these conditions, Eq. (8–20) remains the basis for approaching the problem.

<sup>&</sup>lt;sup>5</sup>J. Wileman, M. Choudury, and I. Green, "Computation of Member Stiffness in Bolted Connections," *Trans. ASME, J. Mech. Design*, vol. 113, December 1991, pp. 432–437.

The dimensionless plot of stiffness versus aspect ratio of the members of a bolted joint, showing the relative accuracy of methods of Rotscher, Mischke, and Motosh, compared to a finite-element analysis (FEA) conducted by Wileman, Choudury, and Green.



#### Table 8-8

Stiffness Parameters of Various Member Materials<sup>†</sup>

†Source: J. Wileman, M. Choudury, and I. Green, "Computation of Member Stiffness in Bolted Connections," *Trans. ASME, J. Mech. Design*, vol. 113, December 1991, pp. 432–437.

Material Used	Poisson Ratio	Elastic GPa	Modulus Mpsi	A	В
Steel	0.291	207	30.0	0.787 15	0.628 73
Aluminum	0.334	71	10.3	0.796 70	0.638 16
Copper	0.326	119	17.3	0.795 68	0.635 53
Gray cast iron	0.211	100	14.5	0.778 71	0.616 16
General expression				0.789 52	0.629 14

### **EXAMPLE 8-2**

As shown in Fig. 8–17*a*, two plates are clamped by washer-faced  $\frac{1}{2}$  in-20 UNF ×  $1\frac{1}{2}$  in SAE grade 5 bolts each with a standard  $\frac{1}{2}$  N steel plain washer.

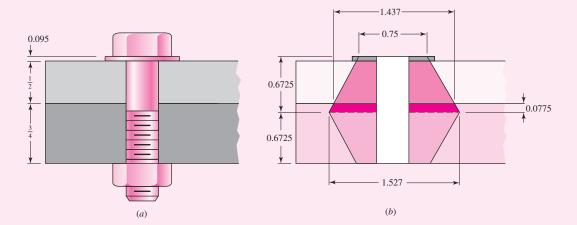
- (a) Determine the member spring rate  $k_m$  if the top plate is steel and the bottom plate is gray cast iron.
- (b) Using the method of conical frusta, determine the member spring rate  $k_m$  if both plates are steel.
- (c) Using Eq. (8–23), determine the member spring rate  $k_m$  if both plates are steel. Compare the results with part (b).
- (d) Determine the bolt spring rate  $k_b$ .

Solution

From Table A-32, the thickness of a standard  $\frac{1}{2}$  N plain washer is 0.095 in. (a) As shown in Fig. 8-17b, the frusta extend halfway into the joint the distance

$$\frac{1}{2}(0.5 + 0.75 + 0.095) = 0.6725$$
 in

Dimensions in inches.



The distance between the joint line and the dotted frusta line is 0.6725 - 0.5 - 0.095 = 0.0775 in. Thus, the top frusta consist of the steel washer, steel plate, and 0.0775 in of the cast iron. Since the washer and top plate are both steel with  $E = 30(10^6)$  psi, they can be considered a single frustum of 0.595 in thick. The outer diameter of the frustum of the steel member at the joint interface is 0.75 + 2(0.595) tan  $30^\circ = 1.437$  in. The outer diameter at the midpoint of the entire joint is 0.75 + 2(0.6725) tan  $30^\circ = 1.527$  in. Using Eq. (8–20), the spring rate of the steel is

$$k_1 = \frac{0.5774\pi(30)(10^6)0.5}{\ln\left\{\frac{[1.155(0.595) + 0.75 - 0.5](0.75 + 0.5)}{[1.155(0.595) + 0.75 + 0.5](0.75 - 0.5)}\right\}} = 30.80(10^6) \text{ lbf/in}$$

For the upper cast-iron frustum

$$k_2 = \frac{0.5774\pi(14.5)(10^6)0.5}{\ln\left\{\frac{[1.155(0.0775) + 1.437 - 0.5](1.437 + 0.5)}{[1.155(0.0775) + 1.437 + 0.5](1.437 - 0.5)}\right\}} = 285.5(10^6) \text{ lbf/in}$$

For the lower cast-iron frustum

$$k_3 = \frac{0.5774\pi(14.5)(10^6)0.5}{\ln\left\{\frac{[1.155(0.6725) + 0.75 - 0.5](0.75 + 0.5)}{[1.155(0.6725) + 0.75 + 0.5](0.75 - 0.5)}\right\}} = 14.15(10^6) \text{ lbf/in}$$

The three frusta are in series, so from Eq. (8-18)

$$\frac{1}{k_m} = \frac{1}{30.80(10^6)} + \frac{1}{285.5(10^6)} + \frac{1}{14.15(10^6)}$$

Answer

This results in  $k_m = 9.378 (10^6)$  lbf/in.

(b) If the entire joint is steel, Eq. (8–22) with l = 2(0.6725) = 1.345 in gives

Answer

$$k_m = \frac{0.5774\pi(30.0)(10^6)0.5}{2\ln\left\{5\left[\frac{0.5774(1.345) + 0.5(0.5)}{0.5774(1.345) + 2.5(0.5)}\right]\right\}} = 14.64(10^6) \text{ lbf/in.}$$

(c) From Table 8–8,  $A = 0.787 \, 15$ ,  $B = 0.628 \, 73$ . Equation (8–23) gives

Answer

 $k_m = 30(10^6)(0.5)(0.787 \ 15) \exp[0.628 \ 73(0.5)/1.345] = 14.92(10^6) \ lbf/in$ 

For this case, the difference between the results for Eqs. (8–22) and (8–23) is less than 2 percent.

(d) Following the procedure of Table 8–7, the threaded length of a 0.5-in bolt is  $L_T = 2(0.5) + 0.25 = 1.25$  in. The length of the unthreaded portion is  $l_d = 1.5 - 1.25 = 0.25$  in. The length of the unthreaded portion in grip is  $l_t = 1.345 - 0.25 = 1.095$  in. The major diameter area is  $A_d = (\pi/4)(0.5^2) = 0.196$  3 in. From Table 8–2, the tensile-stress area is  $A_t = 0.159$  9 in. From Eq. (8–17)

Answer

$$k_b = \frac{0.1963(0.1599)30(10^6)}{0.1963(1.095) + 0.1599(0.25)} = 3.69(10^6) \text{ lbf/in}$$

## 8-6 Bolt Strength

In the specification standards for bolts, the strength is specified by stating SAE or ASTM minimum quantities, the *minimum proof strength*, or *minimum proof load*, and the *minimum tensile strength*. The *proof load* is the maximum load (force) that a bolt can withstand without acquiring a permanent set. The *proof strength* is the quotient of the proof load and the tensile-stress area. The proof strength thus corresponds roughly to the proportional limit and corresponds to 0.0001-in permanent set in the fastener (first measurable deviation from elastic behavior). Tables 8–9, 8–10, and 8–11 provide *minimum* strength specifications for steel bolts. The values of the mean proof strength, the mean tensile strength, and the corresponding standard deviations are not part of the specification codes, so it is the designer's responsibility to obtain these values, perhaps by laboratory testing, if designing to a reliability specification.

The SAE specifications are found in Table 8–9. The bolt grades are numbered according to the tensile strengths, with decimals used for variations at the same strength level. Bolts and screws are available in all grades listed. Studs are available in grades 1, 2, 4, 5, 8, and 8.1. Grade 8.1 is not listed.

ASTM specifications are listed in Table 8–10. ASTM threads are shorter because ASTM deals mostly with structures; structural connections are generally loaded in shear, and the decreased thread length provides more shank area.

Specifications for metric fasteners are given in Table 8–11.

It is worth noting that all specification-grade bolts made in this country bear a manufacturer's mark or logo, in addition to the grade marking, on the bolt head. Such marks confirm that the bolt meets or exceeds specifications. If such marks are missing, the bolt may be imported; for imported bolts there is no obligation to meet specifications.

Bolts in fatigue axial loading fail at the fillet under the head, at the thread runout, and at the first thread engaged in the nut. If the bolt has a standard shoulder under the head, it has a value of  $K_f$  from 2.1 to 2.3, and this shoulder fillet is protected

**Table 8-9**SAE Specifications for Steel Bolts

SAE Grade No.	Size Range Inclusive, in	Minimum Proof Strength,* kpsi	Minimum Tensile Strength,* kpsi	Minimum Yield Strength,* kpsi	Material	Head Marking
1	$\frac{1}{4} - 1\frac{1}{2}$	33	60	36	Low or medium carbon	
2	$\frac{1}{4} - \frac{3}{4}$	55	74	57	Low or medium carbon	
	$\frac{7}{8}$ - 1 $\frac{1}{2}$	33	60	36		
4	$\frac{1}{4} - 1\frac{1}{2}$	65	115	100	Medium carbon, cold-drawn	
5	$\frac{1}{4}$ – 1	85	120	92	Medium carbon, Q&T	
	$1\frac{1}{8} - 1\frac{1}{2}$	74	105	81		
5.2	$\frac{1}{4}$ -1	85	120	92	Low-carbon martensite, Q&T	
7	$\frac{1}{4}$ – $1\frac{1}{2}$	105	133	115	Medium-carbon alloy, Q&T	
8	$\frac{1}{4} - 1\frac{1}{2}$	120	150	130	Medium-carbon alloy, Q&T	
8.2	$\frac{1}{4}$ – 1	120	150	130	Low-carbon martensite, Q&T	

<sup>\*</sup>Minimum strengths are strengths exceeded by 99 percent of fasteners.

from scratching or scoring by a washer. If the thread runout has a  $15^{\circ}$  or less half-cone angle, the stress is higher at the first engaged thread in the nut. Bolts are sized by examining the loading at the plane of the washer face of the nut. This is the weakest part of the bolt *if and only if* the conditions above are satisfied (washer protection of the shoulder fillet and thread runout  $\leq 15^{\circ}$ ). Inattention to this requirement has led to a record of 15 percent fastener fatigue failure under the head, 20 percent at thread runout, and 65 percent where the designer is focusing attention. It does little good to concentrate on the plane of the nut washer face if it is not the weakest location.

Nuts are graded so that they can be mated with their corresponding grade of bolt. The purpose of the nut is to have its threads deflect to distribute the load of the bolt more evenly to the nut. The nut's properties are controlled in order to accomplish this. The grade of the nut should be the grade of the bolt.

**Table 8-10** ASTM Specifications for Steel Bolts

ASTM Desig- nation No.	Size Range, Inclusive, in	Minimum Proof Strength,* kpsi	Minimum Tensile Strength,* kpsi	Minimum Yield Strength,* kpsi	Material	Head Marking
A307	$\frac{1}{4} - 1\frac{1}{2}$	33	60	36	Low carbon	
A325,	$\frac{1}{2}$ - 1	85	120	92	Medium carbon, Q&T	
type 1	$1\frac{1}{8} - 1\frac{1}{2}$	74	105	81		(A325)
A325, type 2	$\frac{1}{2}$ – 1	85	120	92	Low-carbon, martensite, Q&T	
type 2	$1\frac{1}{8} - 1\frac{1}{2}$	74	105	81	Qui	A325
A325,	$\frac{1}{2}$ -1	85	120	92	Weathering steel,	
type 3	$1\frac{1}{8} - 1\frac{1}{2}$	74	105	81	Q&T	(A325)
A354, grade BC	$\frac{1}{4}$ - $2\frac{1}{2}$	105	125	109	Alloy steel, Q&T	
grade BC	$2\frac{3}{4}$ -4	95	115	99		BC
A354, grade BD	<del>1</del> / <sub>4</sub> -4	120	150	130	Alloy steel, Q&T	
A449	$\frac{1}{4}$ – 1	85	120	92	Medium-carbon, Q&T	
	$1\frac{1}{8} - 1\frac{1}{2}$	74	105	81		
	$1\frac{3}{4}$ – 3	55	90	58		
A490, type 1	$\frac{1}{2}$ - 1 $\frac{1}{2}$	120	150	130	Alloy steel, Q&T	A490
A490, type 3	$\frac{1}{2}$ - $1\frac{1}{2}$	120	150	130	Weathering steel, Q&T	<u>A490</u>

<sup>\*</sup>Minimum strengths are strengths exceeded by 99 percent of fasteners.

**Table 8–11**Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs\*

Property Class	Size Range, Inclusive	Minimum Proof Strength,† MPa	Minimum Tensile Strength,† MPa	Minimum Yield Strength,† MPa	Material	Head Marking
4.6	M5-M36	225	400	240	Low or medium carbon	4.6
4.8	M1.6-M16	310	420	340	Low or medium carbon	4.8
5.8	M5-M24	380	520	420	Low or medium carbon	5.8
8.8	M16-M36	600	830	660	Medium carbon, Q&T	8.8
9.8	M1.6-M16	650	900	720	Medium carbon, Q&T	9.8
10.9	M5-M36	830	1040	940	Low-carbon martensite, Q&T	10.9
12.9	M1.6-M36	970	1220	1100	Alloy, Q&T	12.9

<sup>\*</sup>The thread length for bolts and cap screws is

$$L_T = \begin{cases} 2d + 6 & L \le 125 \\ 2d + 12 & 125 < L \le 200 \\ 2d + 25 & L > 200 \end{cases}$$

where L is the bolt length. The thread length for structural bolts is slightly shorter than given above.

## 8–7 **Tension Joints—The External Load**

Let us now consider what happens when an external tensile load P, as in Fig. 8–13, is applied to a bolted connection. It is to be assumed, of course, that the clamping force, which we will call the *preload*  $F_i$ , has been correctly applied by tightening the nut *before* P is applied. The nomenclature used is:

$$F_i$$
 = preload

 $P_{\text{total}}$  = Total external tensile load applied to the joint

<sup>†</sup>Minimum strengths are strengths exceeded by 99 percent of fasteners.

P = external tensile load per bolt

 $P_b$  = portion of P taken by bolt

 $P_m$  = portion of P taken by members

 $F_b = P_b + F_i = \text{resultant bolt load}$ 

 $F_m = P_m - F_i = \text{resultant load on members}$ 

C = fraction of external load P carried by bolt

1 - C = fraction of external load P carried by members

N = Number of bolts in the joint

If N bolts equally share the total external load, then

$$P = P_{\text{total}}/N \tag{a}$$

The load P is tension, and it causes the connection to stretch, or elongate, through some distance  $\delta$ . We can relate this elongation to the stiffnesses by recalling that k is the force divided by the deflection. Thus

$$\delta = \frac{P_b}{k_b}$$
 and  $\delta = \frac{P_m}{k_m}$  (b)

or

$$P_m = P_b \frac{k_m}{k_b} \tag{c}$$

Since  $P = P_b + P_m$ , we have

$$P_b = \frac{k_b P}{k_b + k_m} = CP \tag{d}$$

and

$$P_m = P - P_b = (1 - C)P$$
 (e)

where

$$C = \frac{k_b}{k_b + k_m} \tag{f}$$

is called the stiffness constant of the joint. The resultant bolt load is

$$F_b = P_b + F_i = CP + F_i$$
  $F_m < 0$  (8-24)

and the resultant load on the connected members is

$$F_m = P_m - F_i = (1 - C)P - F_i$$
  $F_m < 0$  (8–25)

Of course, these results are valid only as long as some clamping load remains in the members; this is indicated by the qualifier in the equations.

Table 8–12 is included to provide some information on the relative values of the stiffnesses encountered. The grip contains only two members, both of steel, and no

Computation of Bolt and Member Stiffnesses. Steel members clamped using a  $\frac{1}{2}$  in-13 NC steel bolt.  $C = \frac{k_b}{k_b + k_m}$ 

Stiffnesses, M lbf/in								
Bolt Grip, in	k <sub>b</sub>	k <sub>m</sub>	С	1 – C				
2	2.57	12.69	0.168	0.832				
3	1.79	11.33	0.136	0.864				
4	1.37	10.63	0.114	0.886				

washers. The ratios C and 1-C are the coefficients of P in Eqs. (8–24) and (8–25), respectively. They describe the proportion of the external load taken by the bolt and by the members, respectively. In all cases, the members take over 80 percent of the external load. Think how important this is when fatigue loading is present. Note also that making the grip longer causes the members to take an even greater percentage of the external load.

## 8–8 Relating Bolt Torque to Bolt Tension

Having learned that a high preload is very desirable in important bolted connections, we must next consider means of ensuring that the preload is actually developed when the parts are assembled.

If the overall length of the bolt can actually be measured with a micrometer when it is assembled, the bolt elongation due to the preload  $F_i$  can be computed using the formula  $\delta = F_i l/(AE)$ . Then the nut is simply tightened until the bolt elongates through the distance  $\delta$ . This ensures that the desired preload has been attained.

The elongation of a screw cannot usually be measured, because the threaded end is often in a blind hole. It is also impractical in many cases to measure bolt elongation. In such cases the wrench torque required to develop the specified preload must be estimated. Then torque wrenching, pneumatic-impact wrenching, or the turn-of-the-nut method may be used.

The torque wrench has a built-in dial that indicates the proper torque.

With impact wrenching, the air pressure is adjusted so that the wrench stalls when the proper torque is obtained, or in some wrenches, the air automatically shuts off at the desired torque.

The turn-of-the-nut method requires that we first define the meaning of snug-tight. The *snug-tight* condition is the tightness attained by a few impacts of an impact wrench, or the full effort of a person using an ordinary wrench. When the snug-tight condition is attained, all additional turning develops useful tension in the bolt. The turn-of-the-nut method requires that you compute the fractional number of turns necessary to develop the required preload from the snug-tight condition. For example, for heavy hexagonal structural bolts, the turn-of-the-nut specification states that the nut should be turned a minimum of 180° from the snug-tight condition under optimum conditions. Note that this is also about the correct rotation for the wheel nuts of a passenger car. Problems 8–15 to 8–17 illustrate the method further.

Although the coefficients of friction may vary widely, we can obtain a good estimate of the torque required to produce a given preload by combining Eqs. (8–5) and (8–6):

$$T = \frac{F_i d_m}{2} \left( \frac{l + \pi f d_m \sec \alpha}{\pi d_m - f l \sec \alpha} \right) + \frac{F_i f_c d_c}{2}$$
 (a)

Distribution of Preload  $F_i$  for 20 Tests of Unlubricated Bolts Torqued to 90 N  $\cdot$  m

23.6,	27.6,	28.0,	29.4,	30.3,	30.7,	32.9,	33.8,	33.8,	33.8,
34.7,	35.6,	35.6,	37.4,	37.8,	37.8,	39.2,	40.0,	40.5,	42.7

Mean value  $\bar{F}_i = 34.3$  kN. Standard deviation,  $\hat{\sigma} = 4.91$  kN.

where  $d_m$  is the average of the major and minor diameters. Since  $\tan \lambda = l/\pi d_m$ , we divide the numerator and denominator of the first term by  $\pi d_m$  and get

$$T = \frac{F_i d_m}{2} \left( \frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha} \right) + \frac{F_i f_c d_c}{2}$$
 (b)

The diameter of the washer face of a hexagonal nut is the same as the width across flats and equal to  $1\frac{1}{2}$  times the nominal size. Therefore the mean collar diameter is  $d_c = (d+1.5d)/2 = 1.25d$ . Equation (b) can now be arranged to give

$$T = \left[ \left( \frac{d_m}{2d} \right) \left( \frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha} \right) + 0.625 f_c \right] F_i d \tag{c}$$

We now define a torque coefficient K as the term in brackets, and so

$$K = \left(\frac{d_m}{2d}\right) \left(\frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha}\right) + 0.625 f_c \tag{8-26}$$

Equation (c) can now be written

$$T = KF_i d (8-27)$$

The coefficient of friction depends upon the surface smoothness, accuracy, and degree of lubrication. On the average, both f and  $f_c$  are about 0.15. The interesting fact about Eq. (8–26) is that  $K \doteq 0.20$  for  $f = f_c = 0.15$  no matter what size bolts are employed and no matter whether the threads are coarse or fine.

Blake and Kurtz have published results of numerous tests of the torquing of bolts. By subjecting their data to a statistical analysis, we can learn something about the distribution of the torque coefficients and the resulting preload. Blake and Kurtz determined the preload in quantities of unlubricated and lubricated bolts of size  $\frac{1}{2}$  in-20 UNF when torqued to 800 lbf  $\cdot$  in. This corresponds roughly to an M12  $\times$  1.25 bolt torqued to 90 N  $\cdot$  m. The statistical analyses of these two groups of bolts, converted to SI units, are displayed in Tables 8–13 and 8–14.

We first note that both groups have about the same mean preload, 34 kN. The unlubricated bolts have a standard deviation of 4.9 kN and a COV of about 0.15. The lubricated bolts have a standard deviation of 3 kN and a COV of about 0.9.

The means obtained from the two samples are nearly identical, approximately 34 kN; using Eq. (8–27), we find, for both samples, K = 0.208.

Bowman Distribution, a large manufacturer of fasteners, recommends the values shown in Table 8–15. In this book we shall use these values and use K=0.2 when the bolt condition is not stated.

<sup>&</sup>lt;sup>6</sup>J. C. Blake and H. J. Kurtz, "The Uncertainties of Measuring Fastener Preload," *Machine Design*, vol. 37, Sept. 30, 1965, pp. 128–131.

Distribution of Preload  $F_i$ for 10 Tests of Lubricated Bolts Torqued to 90 N  $\cdot$  m

30.3,	32.5,	32.5,	32.9,	32.9,	33.8,	34.3,	34.7,	37.4,	40.5

Mean value,  $\bar{F}_i = 34.18$  kN. Standard deviation,  $\hat{\sigma} = 2.88$  kN.

#### **Table 8-15**

Torque Factors *K* for Use with Eq. (8–27)

<b>Bolt Condition</b>	K
Nonplated, black finish	0.30
Zinc-plated	0.20
Lubricated	0.18
Cadmium-plated	0.16
With Bowman Anti-Seize	0.12
With Bowman-Grip nuts	0.09

### **EXAMPLE 8-3**

A  $\frac{3}{4}$  in-16 UNF  $\times$   $2\frac{1}{2}$  in SAE grade 5 bolt is subjected to a load P of 6 kip in a tension joint. The initial bolt tension is  $F_i = 25$  kip. The bolt and joint stiffnesses are  $k_b = 6.50$  and  $k_m = 13.8$  Mlbf/in, respectively.

(a) Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.

(b) Specify the torque necessary to develop the preload, using Eq. (8-27).

(c) Specify the torque necessary to develop the preload, using Eq. (8–26) with f = $f_c = 0.15$ .

Solution

From Table 8–2,  $A_t = 0.373 \text{ in}^2$ .

(a) The preload stress is

Answer

$$\sigma_i = \frac{F_i}{A_i} = \frac{25}{0.373} = 67.02 \text{ kpsi}$$

The stiffness constant is

$$C = \frac{k_b}{k_b + k_m} = \frac{6.5}{6.5 + 13.8} = 0.320$$

From Eq. (8–24), the stress under the service load is

$$\sigma_b = \frac{F_b}{A_t} = \frac{CP + F_i}{A_t} = C\frac{P}{A_t} + \sigma_i$$

$$= 0.320\frac{6}{0.373} + 67.02 = 72.17 \text{ kpsi}$$

Answer

From Table 8–9, the SAE minimum proof strength of the bolt is  $S_p = 85$  kpsi. The preload and service load stresses are respectively 21 and 15 percent less than the proof strength.

(b) From Eq. (8-27), the torque necessary to achieve the preload is

Answer

$$T = KF_i d = 0.2(25)(10^3)(0.75) = 3750 \text{ lbf} \cdot \text{in}$$

(c) The minor diameter can be determined from the minor area in Table 8–2. Thus  $d_r=\sqrt{4A_r/\pi}=\sqrt{4(0.351)/\pi}=0.6685$  in. Thus, the mean diameter is  $d_m=(0.75+0.6685)/2=0.7093$  in. The lead angle is

$$\lambda = \tan^{-1} \frac{l}{\pi d_m} = \tan^{-1} \frac{1}{\pi d_m N} = \tan^{-1} \frac{1}{\pi (0.7093)(16)} = 1.6066^{\circ}$$

For  $\alpha = 30^{\circ}$ , Eq. (8–26) gives

$$T = \left\{ \left[ \frac{0.7093}{2(0.75)} \right] \left[ \frac{\tan 1.6066^{\circ} + 0.15(\sec 30^{\circ})}{1 - 0.15(\tan 1.6066^{\circ})(\sec 30^{\circ})} \right] + 0.625(0.15) \right\} 25(10^{3})(0.75)$$

$$= 3551 \text{ lbf} \cdot \text{in}$$

which is 5.3 percent less than the value found in part (b).

## 8-9 Statically Loaded Tension Joint with Preload

Equations (8–24) and (8–25) represent the forces in a bolted joint with preload. The tensile stress in the bolt can be found as in Ex. 8–3 as

$$\sigma_b = \frac{F_b}{A_c} = \frac{CP + F_i}{A_c} \tag{a}$$

Thus, the yielding factor of safety guarding against the static stress exceeding the proof strength is

$$n_p = \frac{S_p}{\sigma_b} = \frac{S_p}{(CP + F_i)/A_t} \tag{b}$$

or

$$n_p = \frac{S_p A_t}{CP + F_t} \tag{8-28}$$

Since it is common to load a bolt close to the proof strength, the yielding factor of safety is often not much greater than unity. Another indicator of yielding that is sometimes used is a *load factor*, which is applied only to the load P as a guard against overloading. Applying such a load factor to the load P in Eq. (a), and equating it to the proof strength gives

$$\frac{Cn_LP + F_i}{A_t} = S_p \tag{c}$$

Solving for the load factor gives

$$n_L = \frac{S_p A_t - F_i}{CP} \tag{8-29}$$

It is also essential for a safe joint that the external load be smaller than that needed to cause the joint to separate. If separation does occur, then the entire external load

will be imposed on the bolt. Let  $P_0$  be the value of the external load that would cause joint separation. At separation,  $F_m = 0$  in Eq. (8–25), and so

$$(1 - C)P_0 - F_i = 0 (d)$$

Let the factor of safety against joint separation be

$$n_0 = \frac{P_0}{P} \tag{e}$$

Substituting  $P_0 = n_0 P$  in Eq. (d), we find

$$n_0 = \frac{F_i}{P(1 - C)} \tag{8-30}$$

as a load factor guarding against joint separation.

Figure 8–18 is the stress-strain diagram of a good-quality bolt material. Notice that there is no clearly defined yield point and that the diagram progresses smoothly up to fracture, which corresponds to the tensile strength. This means that no matter how much preload is given the bolt, it will retain its load-carrying capacity. This is what keeps the bolt tight and determines the joint strength. The pretension is the "muscle" of the joint, and its magnitude is determined by the bolt strength. If the full bolt strength is not used in developing the pretension, then money is wasted and the joint is weaker.

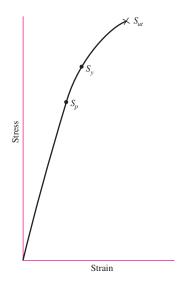
Good-quality bolts can be preloaded into the plastic range to develop more strength. Some of the bolt torque used in tightening produces torsion, which increases the principal tensile stress. However, this torsion is held only by the friction of the bolt head and nut; in time it relaxes and lowers the bolt tension slightly. Thus, as a rule, a bolt will either fracture during tightening, or not at all.

Above all, do not rely too much on wrench torque; it is not a good indicator of preload. Actual bolt elongation should be used whenever possible—especially with fatigue loading. In fact, if high reliability is a requirement of the design, then preload should always be determined by bolt elongation.

Russell, Burdsall & Ward Inc. (RB&W) recommendations for preload are 60 kpsi for SAE grade 5 bolts for nonpermanent connections, and that A325 bolts (equivalent to SAE grade 5) used in structural applications be tightened to proof load or beyond

## Figure 8-18

Typical stress-strain diagram for bolt materials showing proof strength  $S_p$ , yield strength  $S_y$ , and ultimate tensile strength  $S_{ut}$ .



(85 kpsi up to a diameter of 1 in). Bowman recommends a preload of 75 percent of proof load, which is about the same as the RB&W recommendations for reused bolts. In view of these guidelines, it is recommended for both static and fatigue loading that the following be used for preload:

$$F_i = \begin{cases} 0.75F_p & \text{for nonpermanent connections, reused fasteners} \\ 0.90F_p & \text{for permanent connections} \end{cases}$$
 (8–31)

where  $F_p$  is the proof load, obtained from the equation

$$F_p = A_t S_p \tag{8-32}$$

Here  $S_p$  is the proof strength obtained from Tables 8–9 to 8–11. For other materials, an approximate value is  $S_p = 0.85S_y$ . Be very careful not to use a soft material in a threaded fastener. For high-strength steel bolts used as structural steel connectors, if advanced tightening methods are used, tighten to yield.

You can see that the RB&W recommendations on preload are in line with what we have encountered in this chapter. The purposes of development were to give the reader the perspective to appreciate Eqs. (8–31) and a methodology with which to handle cases more specifically than the recommendations.

#### **EXAMPLE 8-4**

Figure 8–19 is a cross section of a grade 25 cast-iron pressure vessel. A total of N bolts are to be used to resist a separating force of 36 kip.

- (a) Determine  $k_b$ ,  $k_m$ , and C.
- (b) Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.
- (c) With the number of bolts obtained in part (b), determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.

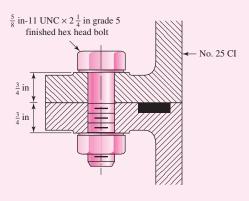
Solution

(a) The grip is l=1.50 in. From Table A–31, the nut thickness is  $\frac{35}{64}$  in. Adding two threads beyond the nut of  $\frac{2}{11}$  in gives a bolt length of

$$L = \frac{35}{64} + 1.50 + \frac{2}{11} = 2.229$$
 in

From Table A–17 the next fraction size bolt is  $L=2\frac{1}{4}$  in. From Eq. (8–13), the thread length is  $L_T=2(0.625)+0.25=1.50$  in. Thus, the length of the unthreaded portion

| Figure 8-19



<sup>&</sup>lt;sup>7</sup>Russell, Burdsall & Ward Inc., *Helpful Hints for Fastener Design and Application*, Mentor, Ohio, 1965, p. 42.

<sup>&</sup>lt;sup>8</sup>Bowman Distribution–Barnes Group, Fastener Facts, Cleveland, 1985, p. 90.

in the grip is  $l_d=2.25-1.50=0.75$  in. The threaded length in the grip is  $l_t=l-l_d=0.75$  in. From Table 8–2,  $A_t=0.226$  in<sup>2</sup>. The major-diameter area is  $A_d=\pi(0.625)^2/4=0.3068$  in<sup>2</sup>. The bolt stiffness is then

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.3068(0.226)(30)}{0.3068(0.75) + 0.226(0.75)}$$
  
= 5.21 Mlbf/in

Answer

From Table A–24, for no. 25 cast iron we will use E=14 Mpsi. The stiffness of the members, from Eq. (8–22), is

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\frac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(14)(0.625)}{2\ln\left[5\frac{0.5774(1.5) + 0.5(0.625)}{0.5774(1.5) + 2.5(0.625)}\right]}$$
  
= 8.95 Mlbf/in

Answer

If you are using Eq. (8–23), from Table 8–8, A = 0.77871 and B = 0.61616, and

$$k_m = EdA \exp(Bd/l)$$
  
= 14(0.625)(0.778 71) exp[0.616 16(0.625)/1.5]  
= 8.81 Mlbf/in

which is only 1.6 percent lower than the previous result.

From the first calculation for  $k_m$ , the stiffness constant C is

Answer

$$C = \frac{k_b}{k_b + k_m} = \frac{5.21}{5.21 + 8.95} = 0.368$$

(b) From Table 8–9,  $S_p = 85$  kpsi. Then, using Eqs. (8–31) and (8–32), we find the recommended preload to be

$$F_i = 0.75 A_t S_p = 0.75(0.226)(85) = 14.4 \text{ kip}$$

For N bolts, Eq. (8–29) can be written

$$n_L = \frac{S_p A_t - F_i}{C(P_{\text{total}}/N)} \tag{1}$$

or

$$N = \frac{Cn_L P_{\text{total}}}{S_p A_t - F_i} = \frac{0.368(2)(36)}{85(0.226) - 14.4} = 5.52$$

Answer

Six bolts should be used to provide the specified load factor.

(c) With six bolts, the load factor actually realized is

Answer

$$n_L = \frac{85(0.226) - 14.4}{0.368(36/6)} = 2.18$$

From Eq. (8-28), the yielding factor of safety is

Answer

$$n_p = \frac{S_p A_t}{C(P_{\text{total}}/N) + F_i} = \frac{85(0.226)}{0.368(36/6) + 14.4} = 1.16$$

From Eq. (8-30), the load factor guarding against joint separation is

Answer

$$n_0 = \frac{F_i}{(P_{\text{total}}/N)(1-C)} = \frac{14.4}{(36/6)(1-0.368)} = 3.80$$