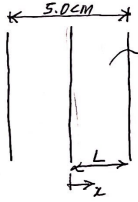
Example: One term approximation- Plate

A large plate of aluminum 5 cm thick and initially at 200°C is suddenly exposed to convection surface environment at 70°C and h = 525 W/m²·°C. Calculate the temperature at a depth of 1.25 cm from one of the faces after 1 min the plate has been exposed to convection. How much energy has been removed from the plate at this time per unit area?



Al

$K = 215\ W/m.K$

$\rho = 2700\ kg/m^3$

$c = 900\ J/kg.K$

$\alpha = \frac{K}{\rho c}$

$$Bi = \frac{hL}{K} = \frac{525 \times 0.025}{215} = 0.061$$

From table 5.1 $\begin{cases} C_1 = 1.0098 \\ \zeta_1 = 0.2425 \end{cases}$

$$Fo = \frac{\alpha t}{L^2} = \frac{8.84 \times 10^{-5}}{(0.025)^2} \times 60 = 8.48$$

$$\theta_0^* = C_1 \exp(-\zeta_1^2 Fo) = 1.0098 \exp(-0.2425 \times 8.48) = 0.6133$$

$$x^* = \frac{x}{L} = \frac{1.25}{2.5} = 0.5$$

$$\theta^* = \theta_0^* \cos(\zeta_1 x^*) = 0.6133 \cos(0.2425 \times 0.5) = 0.6133$$

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} \Rightarrow T = T_\infty + \theta^*(T_i - T_\infty)$$

$$= 70 + 0.6133(200 - 70) = 149.7^\circ C$$

$$\frac{Q}{Q_o} = 1 - \frac{\sin\zeta_1}{\zeta_1}\theta_0^* = 1 - \frac{\sin(0.2425)}{0.2425} \times 0.6133$$

$$= 0.393$$

$$\frac{Q/A}{Q_o/A} = 0.393 \Rightarrow \frac{Q}{A} = 0.393 \times Q_o/A = 6.48 \times 10^6\ J/m^2$$

$$Q_o/A = \frac{\rho c V \theta_i}{A} = \rho c \times 2L\theta_i = 2700 \times 900 \times 2 \times 0.025 \times (200 - 70)$$

$$= 15.8 \times 10^6\ J/m^2$$