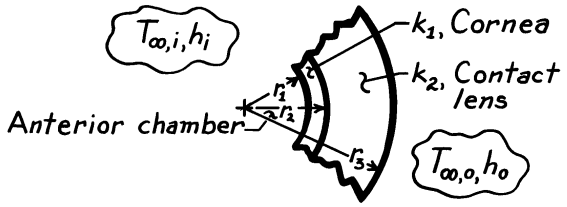
PROBLEM 3.57

KNOWN: Representation of the eye with a contact lens as a composite spherical system subjected to convection processes at the boundaries.

FIND: (a) Thermal circuits with and without contact lens in place, (b) Heat loss from anterior chamber for both cases, and (c) Implications of the heat loss calculations.

SCHEMATIC:

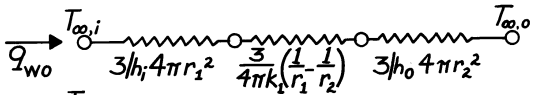


r_1=10.2mm k_1=0.35 W/m·K
r_2=12.7mm k_2=0.80 W/m·K
r_3=16.5mm
$T_{\infty,i}$=37°C h_i=12 W/m^2·K
$T_{\infty,o}$=21°C h_o=6 W/m^2·K

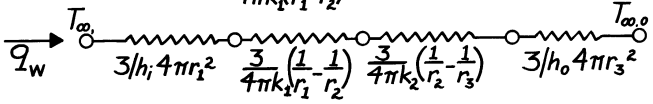
ASSUMPTIONS: (1) Steady-state conditions, (2) Eye is represented as 1/3 sphere, (3) Convection coefficient, h_o, unchanged with or without lens present, (4) Negligible contact resistance.

ANALYSIS: (a) Using Eqs. 3.9 and 3.41 to express the resistance terms, the thermal circuits are:

Without lens:



With lens:



(b) The heat losses for both cases can be determined as $q = (T_{\infty,i} - T_{\infty,o})/R_t$, where R_t is the thermal resistance from the above circuits.

Without lens: $$R_{t,wo} = \frac{3}{12\text{W/m}^2\cdot\text{K}4\pi\left(10.2\times10^{-3}\text{m}\right)^2} + \frac{3}{4\pi\times0.35\ \text{W/m}\cdot\text{K}}\left[\frac{1}{10.2}-\frac{1}{12.7}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\ \text{W/m}^2\cdot\text{K}4\pi\left(12.7\times10^{-3}\text{m}\right)^2} = 191.2\ \text{K/W}+13.2\ \text{K/W}+246.7\ \text{K/W}=451.1\ \text{K/W}$$

With lens: $$R_{t,w} = 191.2\ \text{K/W}+13.2\ \text{K/W}+\frac{3}{4\pi\times0.80\ \text{W/m}\cdot\text{K}}\left[\frac{1}{12.7}-\frac{1}{16.5}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\text{W/m}^2\cdot\text{K}4\pi\left(16.5\times10^{-3}\text{m}\right)^2} = 191.2\ \text{K/W}+13.2\ \text{K/W}+5.41\ \text{K/W}+146.2\ \text{K/W}=356.0\ \text{K/W}$$

Hence the heat loss rates from the anterior chamber are

Without lens: $q_{wo} = (37-21)^\circ\text{C}/451.1\ \text{K/W}=35.5\text{mW}$ <

With lens: $q_w = (37-21)^\circ\text{C}/356.0\ \text{K/W}=44.9\text{mW}$ <

(c) The heat loss from the anterior chamber increases by approximately 20% when the contact lens is in place, implying that the outer radius, r_3, is less than the critical radius.