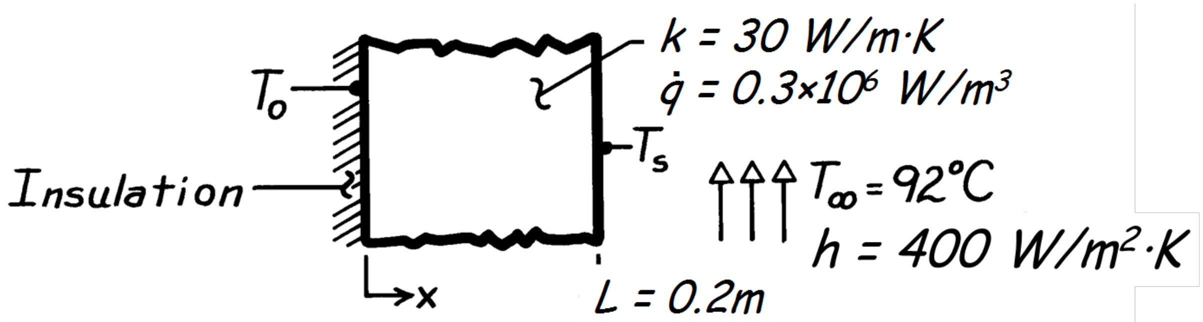
PROBLEM 3.65

KNOWN: Plane wall with internal heat generation which is insulated at the inner surface and subjected to a convection process at the outer surface.

FIND: Maximum temperature in the wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform volumetric heat generation, (3) Inner surface is adiabatic.

ANALYSIS: The temperature at the inner surface is given by Eq. 3.48 and is the maximum temperature within the wall,

$$T_o = \dot{q}L^2/2k + T_s.$$

The outer surface temperature follows from Eq. 3.51,

$$T_s = T_\infty + \dot{q}L/h$$

$$T_s = 92^\circ C + 0.4\times 10^6 \frac{W}{m^3} \times 0.2m/400 W/m^2\cdot K = 92^\circ C + 200^\circ C = 292^\circ C.$$

It follows that

$$T_o = 0.4\times 10^6 W/m^3 \times (0.2m)^2 / 2\times 30 W/m\cdot K + 292^\circ C$$

$$T_o = 267^\circ C + 292^\circ C = 559^\circ C.$$ <

COMMENTS: The heat flux leaving the wall can be determined from knowledge of h, T_s and T_∞ using Newton's law of cooling.

$$q''_{conv} = h(T_s - T_\infty) = 400 W/m^2\cdot K(292-92)^\circ C = 80 kW/m^2.$$

This same result can be determined from an energy balance on the entire wall, which has the form

$$\dot{E}_g - \dot{E}_{out} = 0$$

where

$$\dot{E}_g = \dot{q}AL \qquad \text{and} \qquad \dot{E}_{out} = q''_{conv}\cdot A.$$

Hence,

$$q''_{conv} = \dot{q}L = 0.4\times 10^6 W/m^3 \times 0.2m = 80 kW/m^2.$$