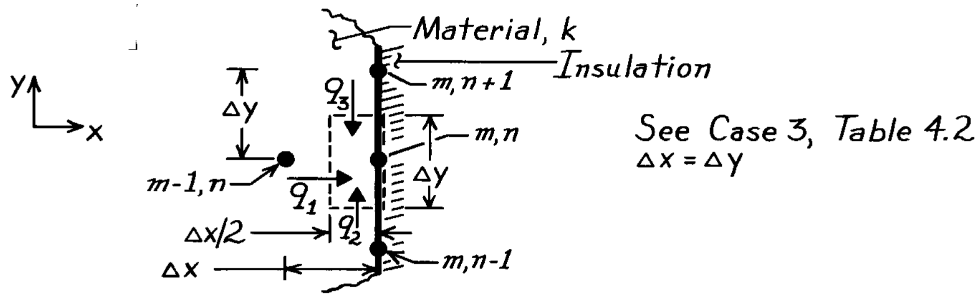
PROBLEM 4.34

KNOWN: Plane surface of two-dimensional system.

FIND: The finite-difference equation for nodal point on this boundary when (a) insulated; compare result with Eq. 4.42, and when (b) subjected to a constant heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional, steady-state conduction with no generation, (2) Constant properties, (3) Boundary is adiabatic.

ANALYSIS: (a) Performing an energy balance on the control volume, $(\Delta x/2)\cdot\Delta y$, and using the conduction rate equation, it follows that

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_3 = 0 \tag{1,2}$$

$$k(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\left[\frac{\Delta x}{2}\cdot 1\right]\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + k\left[\frac{\Delta x}{2}\cdot 1\right]\frac{T_{m,n+1} - T_{m,n}}{\Delta y} = 0. \tag{3}$$

Note that there is no heat rate across the control volume surface at the insulated boundary. Recognizing that $\Delta x = \Delta y$, the above expression reduces to the form

$$2T_{m-1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} = 0. \tag{4}$$ <

The Eq. 4.42 of Table 4.2 considers the same configuration but with the boundary subjected to a convection process. That is,

$$\left(2T_{m-1,n} + T_{m,n-1} + T_{m,n+1}\right) + \frac{2h\Delta x}{k}T_\infty - 2\left[\frac{h\Delta x}{k} + 2\right]T_{m,n} = 0. \tag{5}$$

Note that, if the boundary is insulated, h = 0 and Eq. 4.42 reduces to Eq. (4).

(b) If the surface is exposed to a constant heat flux, q_o'', the energy balance has the form

$q_1 + q_2 + q_3 + q_o'' \cdot \Delta y = 0$ and the finite difference equation becomes

$$2T_{m-1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} = -\frac{2q_o''\Delta x}{k}.$$ <

COMMENTS: Equation (4) can be obtained by using the "interior node" finite-difference equation, Eq. 4.29, where the insulated boundary is treated as a symmetry plane as shown below.