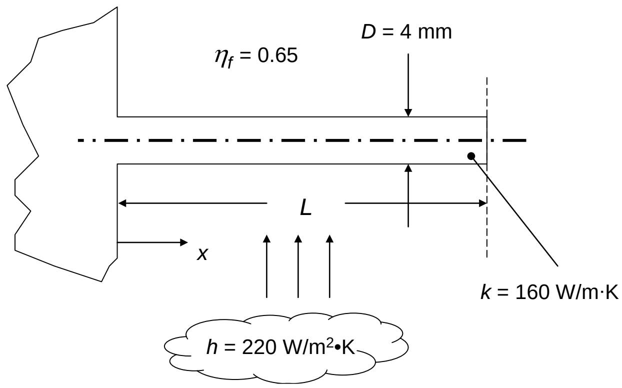
PROBLEM 3.105

KNOWN: Thermal conductivity and diameter of a pin fin. Value of the heat transfer coefficient and fin efficiency.

FIND: (a) Length of fin, (b) Fin effectiveness.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Negligible radiation heat transfer, (3) Constant properties, (4) Convection from fin tip.

PROPERTIES: *Given*, Aluminum Alloy: k = 160 W/m·K.

ANALYSIS: For an active fin tip, the efficiency may be expressed in terms of the corrected fin length as:

$$\eta_f = \frac{\tanh(mL_c)}{mL_c}$$

where $m = \sqrt{hP/kA_c} = \sqrt{4h/kD} = \sqrt{4 \times 220\text{W/m}^2 \cdot \text{K}/\left(160\text{W/m}\cdot\text{K} \times 4 \times 10^{-3}\text{m}\right)} = 37.1\text{m}^{-1}$

Hence, $\eta_f = 0.65 = \dfrac{\tanh(37.1\text{m}^{-1} \times L_c)}{37.1\text{m}^{-1} \times L_c}$ which may be solved by trial-and-error (or by using *IHT*) to yield L_c = 0.0362 m = 36.2 mm. The fin length is therefore, $L = L_c - D/4$ = 0.0362 m – 0.004m/4 = 0.0352 m = 35.2 mm. <

The fin effectiveness is:

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{M\tanh(mL_c)}{hA_{c,b}\theta_b} = \frac{\sqrt{hPkA_{c,b}}\tanh(mL_c)}{hA_{c,b}} = \frac{2}{\sqrt{hD/k}}\tanh(mL_c)$$

$$= \frac{2}{\sqrt{\dfrac{220\text{W/m}^2\cdot\text{K}\times 4\times10^{-3}\text{m}}{160\text{W/m}\cdot\text{K}}}}\tanh(37.1\text{m}^{-1}\times 36.2\times10^{-3}\text{m}) = 23.5$$ <

COMMENTS: The values of the fin effectiveness and fin efficiency are independent of the base or fluid temperatures.