4- A solid sphere with diameter of 100 mm, initially at uniform temperature of $T_i = 40\ ^{\circ}C$, is placed in a flowing fluid at $T_\infty = 20\ ^{\circ}C$. The sphere properties are $k = 2$ W/m.K, $\rho = 2500$ kg/m^3, and $C_p = 750$ J/kg. K. The convection heat transfer coefficient is $h = 20$ W/m^2. K. At what time does the sphere's surface temperature reach 25°C?

$$Bi = \frac{h r_o}{3k} = \frac{20 \times 0.05}{3 \times 2} = 0.167 > 0.1$$, Thus LHC method is not valid.

one term approximation:

$$Bi = \frac{h r_o}{k} = \frac{20 \times 0.05}{2} = 0.5$$

From table 5.1 $\longrightarrow$ $\zeta_1 = 1.1656$ rad

$C_1 = 1.1441$

$$\text{eq: 5.53a} \Longrightarrow \theta^* = C_1 \exp(-\zeta_1^2 F_o) \frac{1}{\zeta_1 r^*} \sin(\zeta_1 r^*)$$

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} = \frac{25 - 20}{40 - 20} = 0.25$$

$$r^* = \frac{r}{r_o} = \frac{0.05}{0.05} = 1$$

$$F_o = -\frac{1}{\zeta_1^2} \ln\left[\frac{\theta^* \zeta_1 r^*}{C_1 \sin(\zeta_1 r^*)}\right]$$

$$= -\frac{1}{1.1656^2} \ln\left[\frac{0.25 \times 1.1656 \times 1}{1.1441 \sin\left(1.1656 \times 1 \times \frac{180}{\pi}\right)}\right]$$

$$= 0.9445$$

$$F_o = \frac{\alpha t}{r_o^2} \Longrightarrow t = \frac{F_o r_o^2}{\alpha} = \frac{0.9445 \times (0.05)^2}{1.067 \times 10^{-6}} = 2213.00\ s$$

$$\alpha = \frac{k}{\rho C} = \frac{2}{2500 \times 750} = 1.067 \times 10^{-6}\ m^2/s$$