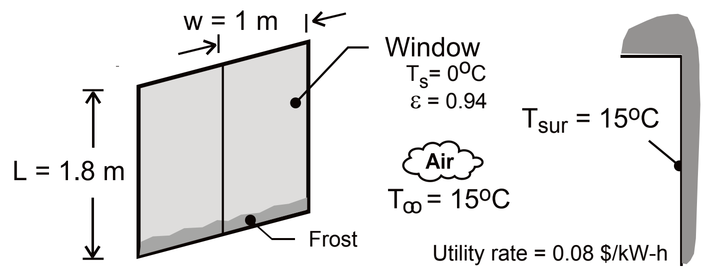
PROBLEM 9.15

KNOWN: During a winter day, the window of a patio door with a height of 1.8 m and width of 1.0 m shows a frost line near its base.

FIND: (a) Explain why the window would show a frost layer at the base of the window, rather than at the top, and (b) Estimate the rate of heat loss through the window due to free convection and radiation. If the room has electric baseboard heating, estimate the daily cost of the window heat loss for this condition based upon the utility rate of 0.18 \$/kW·h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Window has a uniform temperature, (3) Ambient air is quiescent, and (4) Room walls are isothermal and large compared to the window.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 280$ K, 1 atm): $\nu = 14.11 \times 10^{-6}$ m^2/s, k = 0.0247 W/m·K, $\alpha = 1.986 \times 10^{-5}$ m^2/s, Pr = 0.710.

ANALYSIS: (a) For these winter conditions, a frost line could appear and it would be at the bottom of the window. The boundary layer is thinnest at the top of the window, and hence the heat flux from the warmer room is greater than compared to that at the bottom portion of the window where the boundary layer is thicker. Also, the air in the room may be stratified and cooler near the floor compared to near the ceiling.

(b) The rate of heat loss from the room to the window having a uniform temperature $T_s = 0°C$ by convection and radiation is

$$q_{loss} = q_{cv} + q_{rad} \quad (1)$$

$$q_{loss} = A_s\left[\bar{h}_L\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right] \quad (2)$$

The average convection coefficient is estimated from the Churchill-Chu correlation, Eq. 9.26, using properties evaluated at $T_f = (T_s + T_\infty)/2$.

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \quad (3)$$

$$Ra_L = g\beta T\left(T_\infty - T_s\right)L^3/\nu\alpha \quad (4)$$

Substituting numerical values in the correlation expressions, find

$$Ra_L = 1.084\times10^{10} \qquad \overline{Nu}_L = 258.9 \qquad \bar{h}_L = 3.6\ W/m^2\cdot K$$

Continued …

PROBLEM 9.15 (Cont.)

Using Eq. (2), the rate of heat loss with $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$K^4 is

$$q_{loss} = (1 \times 1.8)\,\mathrm{m}^2 \left[3.6\ \mathrm{W/m^2 \cdot K}\,(15\ \mathrm{K}) + 0.940\,\sigma\left(288^4 - 273^4\right)\mathrm{K}^4\right]$$

$$q_{loss} = (96.1 + 127.1)\,\mathrm{W} = 223\ \mathrm{W}$$

The daily cost of the window heat loss for the given utility rate is

cost = q_{loss} × (utility rate) × 24 hours

cost = 223 W × (10^{-3} kW/W) × 0.18 \$/kW·h × 24 h

cost = 0.96 \$/day <

COMMENTS: Note that the heat loss by radiation is 30% larger than by free convection.