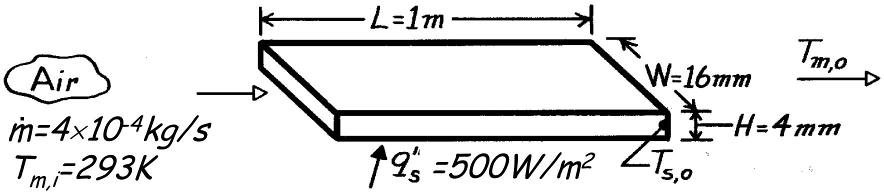
PROBLEM 8.61

KNOWN: Flow rate and inlet temperature of air passing through a rectangular duct of prescribed dimensions and surface heat flux.

FIND: Air and duct surface temperatures at outlet.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface heat flux, (3) Constant properties, (4) Atmospheric pressure, (3) Fully developed conditions at duct exit, (6) Ideal gas with negligible viscous dissipation and pressure variation.

PROPERTIES: *Table A-4*, Air $\left(\bar{T}_m \approx 300\text{K, 1 atm}\right)$: c_p = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: For this uniform heat flux condition, the heat rate is

$$q = q_s'' A_s = q_s''\left[2(L\times W)+2(L\times H)\right]$$

$$q = 500\ \text{W/m}^2\left[2(1\text{m}\times 0.016\text{m})+2(1\text{m}\times 0.004\text{m})\right] = 20\ \text{W}.$$

From an overall energy balance

$$T_{m,o} = T_{m,i} + \frac{q}{\dot{m}\,c_p} = 293\text{K} + \frac{20\ \text{W}}{4\times 10^{-4}\ \text{kg/s}\times 1007\ \text{J/kg}\cdot\text{K}} = 343\ \text{K}. \qquad <$$

The surface temperature at the outlet may be determined from Newton's law of cooling, where

$$T_{s,o} = T_{m,o} + q''/h.$$

From Eqs. 8.66 and 8.1

$$D_h = \frac{4\ A_c}{P} = \frac{4(0.016\text{m}\times 0.004\text{m})}{2(0.016\text{m}+0.004\text{m})} = 0.0064\ \text{m}$$

$$Re_D = \frac{\rho\, u_m\, D_h}{\mu} = \frac{\dot{m}\, D_h}{A_c\mu} = \frac{4\times 10^{-4}\text{kg/s}(0.0064\text{m})}{64\times 10^{-6}\text{m}^2\left(184.6\times 10^{-7}\ \text{N}\cdot\text{s/m}^2\right)} = 2170.$$

Hence the flow is laminar, and from Table 8.1

$$h = \frac{k}{D_h}5.33 = \frac{0.0263\ \text{W/m}\cdot\text{K}}{0.0064\ \text{m}}5.33 = 22\ \text{W/m}^2\cdot\text{K}$$

$$T_{s,o} = 343\ \text{K} + \frac{500\ \text{W/m}^2}{22\ \text{W/m}^2\cdot\text{K}} = 365\ \text{K}. \qquad <$$

COMMENTS: The calculations should be repeated with properties evaluated at $\bar{T}_m$ = 318 K. The change in $T_{m,o}$ would be negligible, and $T_{s,o}$ would decrease slightly.