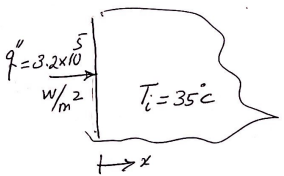
Example: A semi-infinite problem with constant heat flux

A large block of steel k = 45 W/m.°C, $\alpha = 1.4 \times 10^{-5}$ m²/s is initially at uniform temperature of 35 °C. The surface is exposed to a heat flux a) by suddenly raising the surface temperature to 250 °C and b) through constant heat flux of 3.2×10^5 W/m². Calculate the temperature at depth of 2.5 cm after 30 s for both these cases.



Case 1

$$\frac{x}{2\sqrt{\alpha t}} = \frac{0.025}{2\sqrt{1.4\times10^{-5}\times 30}} = 0.61$$

Table B.2 $\quad erf\left(\frac{x}{2\sqrt{\alpha t}}\right) = erf(0.61) = 0.61164$

eq: 5.60 $\Rightarrow \quad \frac{T(x,t) - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$

$$\frac{T - 250}{35 - 250} = 0.61164 \Rightarrow T = 118.5\,°C$$

Case 2 Constant heat flux

eq: 5.62

$$T(x,t) - T_i = \frac{2q''_0 \left(\frac{\alpha t}{\pi}\right)^{1/2}}{K} \exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{q''_0 x}{K} \times \left(1 - erf\left(\frac{x}{2\sqrt{\alpha t}}\right)\right)$$

$$T_{(2.5cm,\ t=30s)} = 79.3\,°C$$