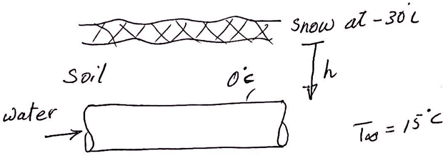
Example: A semi-infinite solid with constant surface temperature.

In areas where the air temperature remains below 0 ˚C for prolonged period, the freezing of water in underground pipes is a major concern. Fortunately, the soil remains relatively warm during those periods, and it takes weeks for the subfreezing temperature to reach the water in the ground. Thus, the soil effectively serves as an insulation to protect the water from the freezing atmospheric temperatures in winter.

The ground at a particular location is covered with snow pack at -30 ˚C for a continuous period of 90 days, the average soil properties at that location are k = 0.4 W/m.K and $\alpha = 0.15 \times 10^{-6}$ m^2/s. Assuming an initial uniform temperature of 15 ˚C for the ground, determine burial depth to prevent the water pipes from freezing.



$K = 0.4\ W/m.K$

$\alpha = 0.15 \times 10^{-6}\ m^2/s$

Case I Constant Surface temp

$$\frac{T(x,t) - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\frac{0-(-30)}{15-(-30)} = \frac{30}{45} = 0.67 = erf\left(\frac{x}{2\sqrt{\alpha t}}\right) = erf(\eta)$$

From table B.2 $\Rightarrow \eta = 0.68$

$$0.68 = \frac{x}{2\sqrt{\alpha t}} \Rightarrow x = 0.68 \times 2\sqrt{0.15 \times 10^{-6} \times 90 \times 24 \times 3600}$$

$$= 1.47 m \simeq 1.5 m$$