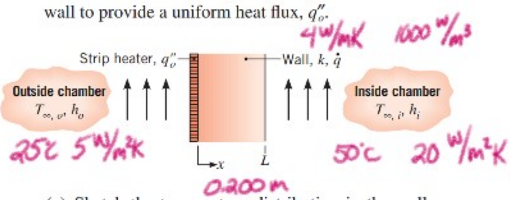
HW3.2

Sunday, September 24, 2023 5:49 PM

Chapter 3:
Problems: ~~64~~, ~~66~~, 86, 99 and 103

3.64 The air *inside* a chamber at $T_{\infty,i} = 50°C$ is heated convectively with $h_i = 20\ W/m^2 \cdot K$ by a 200-mm-thick wall having a thermal conductivity of 4 W/m · K and a uniform heat generation of 1000 W/m^3. To prevent any heat generated within the wall from being lost to the *outside* of the chamber at $T_{\infty,o} = 25°C$ with $h_o = 5\ W/m^2 \cdot K$, a very thin electrical strip heater is placed on the outer wall to provide a uniform heat flux, q''_o.



(a) Sketch the temperature distribution in the wall on $T - x$ coordinates for the condition where no heat generated within the wall is lost to the *outside* of the chamber.

(b) What are the temperatures at the wall boundaries, $T(0)$ and $T(L)$, for the conditions of part (a)?

(c) Determine the value of q''_o that must be supplied by the strip heater so that all heat generated within the wall is transferred to the *inside* of the chamber.

(d) If the heat generation in the wall were switched off while the heat flux to the strip heater remained constant, what would be the steady-state temperature, $T(0)$, of the outer wall surface?

$$\dot{q} L \cancel{A} = h \cancel{A} (T - T_{\infty i})$$

$$\frac{(1000\ W/m^3)(0.2m)}{(20\ W/m^2K)} + 50°C = T_L = 60°C$$

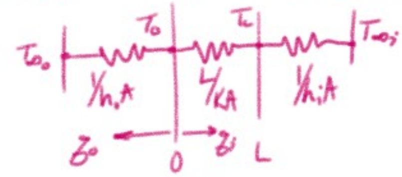
$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s \qquad (3.47)$$

USE KNOWN HEAT GEN & T_L (FROM CV) TO BACK CALC T_0 ...

$$T(0) = \frac{(1000)(0.2)^2}{2(4)}\left(1 - \cancel{\frac{(0)^2}{(0.2)^2}}\right) + (60) = 65°C$$

$$q''_o \cancel{A} = h_o \cancel{A}(T_0 - T_{\infty o}) = (5)(65 - 25) = 200\ W/m^2$$

NOW: $\dot{q} = 0$



$$q = \frac{\Delta T}{R}$$

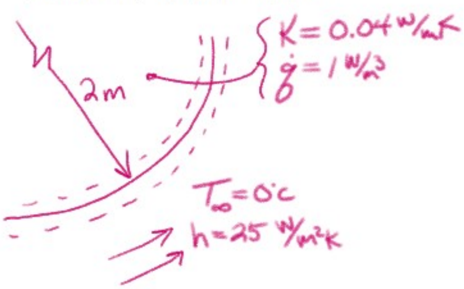
$$q''_o A = q_o + q_i = \frac{T_0 - T_{\infty o}}{\frac{1}{h_o A}} + \frac{T_0 - T_{\infty i}}{\frac{L}{kA} + \frac{1}{h_i A}}$$

?

$$200 = \frac{T_0 - 25}{\frac{1}{5}} + \frac{T_0 - 50}{\frac{0.2}{4} - \frac{1}{20}} = 5T_0 - 125 + \frac{T_0 - 50}{0}$$

$T_0 = 65°C$

3.66 Large, cylindrical bales of hay used to feed livestock in the winter months are $D = 2$ m in diameter and are stored end-to-end in long rows. Microbial energy generation occurs in the hay and can be excessive if the farmer bales the hay in a too-wet condition. Assuming the thermal conductivity of baled hay to be $k = 0.04$ W/m · K, determine the maximum steady-state hay temperature for dry hay ($\dot{q} = 1$ W/m^3), moist hay ($\dot{q} = 10$ W/m^3), and wet hay ($\dot{q} = 100$ W/m^3). Ambient conditions are $T_\infty = 0°C$ and $h = 25$ W/m^2 · K.



$$T(r) = \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s \qquad (3.58)$$

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h} = (273K) + \frac{(1\ W/m^3)(\cancel{2}m)}{\cancel{2}(25\ W/m^2K)} = 273.04K$$

T_{MAX} @ center for symmetric cylinder:

$$T(0m) = \frac{(1\ W/m^3)(2m)^2}{\cancel{4}(0.04\ W/mK)}\left(1 - \frac{(0m)^2}{(2m)^2}\right) + (273.04K) = 298.04K = 25.04°C \text{ DRY}$$

$$250 + 273.4 = 523.4K = 250.4°C \text{ MOIST}$$

$$2500 + 277 = 2777K = 2504°C \text{ WET}$$

SEEMS HIGH?

3.86 Radioactive wastes ($k_{rw} = 20$ W/m · K) are stored in a spherical, stainless steel ($k_{ss} = 15$ W/m · K) container of inner and outer radii equal to $r_i = 0.5$ m and $r_o = 0.6$ m. Heat is generated volumetrically within the wastes at a uniform rate of $\dot{q} = 10^5$ W/m^3, and the outer surface of the container is exposed to a water flow for which $h = 1000$ W/m^2 · K and $T_\infty = 25°C$.

$$\cancel{\frac{\Delta E}{\Delta t}}^{0} = \cancel{E_{IN}}^{0} - E_{OUT} + E_g$$

STEADY STATE → $\Delta E = 0$

$$E_g = E_{OUT}$$

$$\dot{q}V = q_{CONV}$$

WHY r_i IF CALC @ OUTER?

$$\dot{q}\,\tfrac{4}{3}\pi r^3 = h 4\pi r^2 (T - T_\infty)$$

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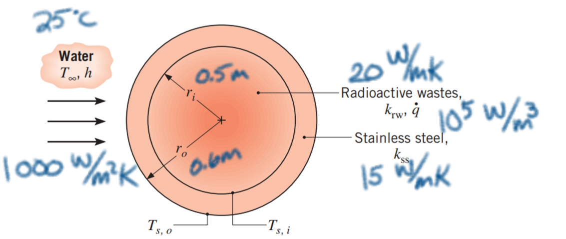


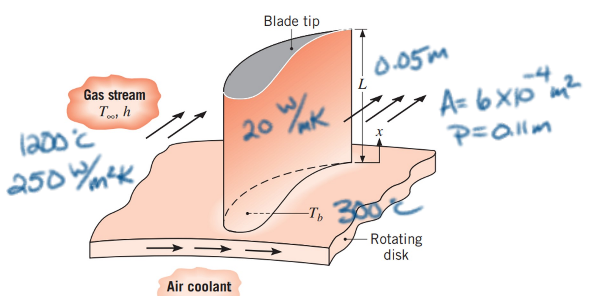
(a) Evaluate the steady-state outer surface temperature, $T_{s,o}$.

(b) Evaluate the steady-state inner surface temperature, $T_{s,i}$.

(c) Obtain an expression for the temperature distribution, $T(r)$, in the radioactive wastes. Express your result in terms of r_i, $T_{s,i}$, k_{rw}, and $\dot{q}$. Evaluate the temperature at $r = 0$.

(d) A proposed extension of the foregoing design involves storing waste materials having the same thermal conductivity but twice the heat generation ($\dot{q} = 2\times10^5$ W/m^3) in a stainless steel container of equivalent inner radius (r_i = 0.5 m). Safety considerations dictate that the maximum system temperature not exceed 475°C and that the container wall thickness be no less than t = 0.04 m and preferably at or close to the original design (t = 0.1 m). Assess the effect of varying the outside convection coefficient to a maximum achievable value of h = 5000 W/m$^2\cdot$K (by increasing the water velocity) and the container wall thickness. Is the proposed extension feasible? If so, recommend suitable operating and design conditions for h and t, respectively.

3.99 Turbine blades mounted to a rotating disc in a gas turbine engine are exposed to a gas stream that is at T_∞ = 1200°C and maintains a convection coefficient of h = 250 W/m$^2\cdot$K over the blade.



The blades, which are fabricated from Inconel, $k \approx 20$ W/m$\cdot$K, have a length of L = 50 mm. The blade profile has a uniform cross-sectional area of $A_c = 6\times10^{-4}$ m^2 and a perimeter of P = 110 mm. A proposed blade-cooling scheme, which involves routing air through the supporting disc, is able to maintain the base of each blade at a temperature of T_b = 300°C.

(a) If the maximum allowable blade temperature is 1050°C and the blade tip may be assumed to be adiabatic, is the proposed cooling scheme satisfactory?

(b) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?

$\dot{q}V = \dot{q}_{conv}$ WHY r_i IF CALC @ OUTER?

$\dot{q}\frac{4}{3}\pi r_i^3 = h4\pi r_o^2(T_{so}-T_\infty)$

$T_{so} = \frac{\dot{q}\,4/3\pi r_i^3}{h4\pi r_o^2} + T_\infty = \frac{(10^5\ W/m^3)(0.5m)^3}{(1000\ W/m^2K)3(0.6m)^2} + (298K) = 309.6K$

366°C

$$q_r = \frac{4\pi k(T_{s,1}-T_{s,2})}{(1/r_1)-(1/r_2)} \qquad (3.40)$$

$\cancel{\Delta E_{st}} = E_{in} - E_{out} + \cancel{E_g}$

$q_{conv} = hA\Delta T$

$q_{cond} = q_{conv}$

$\frac{4\pi k_s(T_{si}-T_{so})}{\frac{1}{r_i}-\frac{1}{r_o}} = h4\pi r_o^2(T_{so}-T_\infty)$

$T_{si} = \frac{hr_o^2}{k_{ss}}\left(\frac{1}{r_i}-\frac{1}{r_o}\right)(T_{so}-T_\infty)+T_{so}$

$T_{si} = \frac{(1000)(0.6)^2}{(15)}\left(\frac{1}{0.5}-\frac{1}{0.6}\right)(366-25)+(366) = 129.4°C$

HOW DO I MAKE HEAT EQ?

3.103 A brass rod 100 mm long and 5 mm in diameter extends horizontally from a casting at 200°C. The rod is in an air environment with $T_\infty = 20°C$ and $h = 30$ $W/m^2 \cdot K$. What is the temperature of the rod 25, 50, and 100 mm from the casting?