Example: One term approximation- Cylinder

A large aluminum 5 cm diameter and initially at 200 °C is suddenly exposed to convection environment at 70 °C and h = 525 W/m².°C. Calculate the temperature at a radius of 1.25 cm and the heat loss per unit length 1 min after the cylinder is exposed to the environment.

$D = 2r_o = 5\,cm \quad r_o = 2.5\,cm$

$T_i = 200^\circ C$

$h = 525\ W/m^2 \cdot ^\circ C$

$T_\infty = 70^\circ C$

Al

$K = 215\ W/m \cdot ^\circ C$

$\rho = 900\ kg/m^3$

$C = 2700\ J/kg \cdot K$

$$\alpha = \frac{K}{\rho C} = \frac{215}{900 \times 2700} = 8.84 \times 10^{-5}\ m^2/s$$

$$F_o = \frac{\alpha t}{r_o^2} = \frac{8.84 \times 10^{-5} \times 60}{(0.025)^2} = 8.49$$

$$r^* = \frac{r}{r_o} = \frac{1.25}{2.5} = 0.5$$

$$Bi = \frac{h r_o}{K} = \frac{525 \times 0.025}{215} = 0.061$$

$$Bi^{-1} = 16.38$$

$F_o = 8.49$, $Bi^{-1} = 16.38$ From Fig 5 5.4 $\theta_o^* = 0.38$

$$\theta_o = \theta_o^* \theta_i = 0.38 (T_i - T_\infty) = 0.38 (200 - 70) = 49.4^\circ C$$

From Fig 5 5.5 $\begin{cases} Bi^{-1} = 16.38 \\ r^* = 0.5 \end{cases} \Rightarrow \frac{\theta}{\theta_o} = 0.98$

$$\theta = 0.98\, \theta_o = 0.98 \times 49.4 = 48.412$$

$$T - T_\infty = 48.412 \Rightarrow T = 70 + 48.412 = 118.4^\circ C$$