Theoretical Computer Science (M21276)

Part Essential from Mathematics (Review)

(before we start...)

Question 1. We are given the sets $A = \{3, 8, 7, 16, 10, 15\}$ and $B = \{1, 2, 8, 10, 15\}$.

- Is it true that $B \subset A$?

  Answer: no

- Which elements belong to the set $A \cup B$?

  Answer: $\{1, 2, 3, 7, 8, 10, 15, 16\}$

- Which elements belong to the set $A \cap B$?

  Answer: $\{8, 10, 15\}$

- Which elements belong to the set $A \setminus B$?

  Answer: $\{3, 7, 16\}$

- What is the meaning and the value of $|A|$?

  Answer: the number of element in A, $|A| = 6$

- Find a subset of B which will be a subset of A as well.

  Answer: any subset of the following set: $\{8, 10\}$

- How many different subsets can you create from the set $\{1, 2, 8\}$? And how many from the set B?

  Answer: From the set $\{1, 2, 8\}$ we can create 8 subsets, from the set B: $2^5 = 32$.

Question 2.

- Are the sets $\{1, 2, 3, 5\}$ and $\{5, 1, 3, 2\}$ different?

  Answer: no, they are the same

- Are the set $\{2, 4, 6, \ldots\}$ and $\{2k - 2, k \in \mathbb{N}\}$ different?

  Answer: yes, -2 is not in the first set

Question 3. If $f(x) = 2x^2 + 3x + 1$, what is $f(2)$?
Answer: 15

Question 4. Write down the formula for $1 + 2 + 3 + 4 + \ldots + n$.

Answer: $n(n+1)/2$

Question 5. Given $T(n) = T(n-1) + n$, and $T(1) = 0$, write down the value of (i) $T(3)$, (ii) $T(n)$.
Answer: (i) $T(3) = 5$, (ii) $T(n) = n(n+1)/2 - 1$

Question 6. What is 4! ?
Answer: $4 \cdot 3 \cdot 2 \cdot 1 = 24$

Question 7. What is the value of: $\sum_{i=1}^{4} i^2$
Answer: $1 + 4 + 9 + 16 = 30$

Question 8. What is the value of:

$$1 + 2 + 4 + 8 + 16 + \cdots + 2^{n-1}?$$

Answer: $2^n - 1$

Question 9. What is the meaning of **logarithm**? What is the value (i) $\log_2 8$, (ii) $\log_2 32$, (iii) $\log_2 2^8$?
Answer: (i) 3, (ii) 5, (iii) 8

Question 10. What is the value (i) $\log_2 64 - \log_2 4$, (ii) $\log_2 32 + \log_2 4$?
Answer: (i) $\log_2 \frac{64}{4} = \log_2 16 = 4$, (ii) $\log_2 32 \cdot 4 = \log_2 128 = 7$

Question 11. Give a value for n, so that 2^{10} is very close to 10^n.
Answer: $2^{10} = 1024$ which is approx. equals to $1000 = 10^3$, so $n = 3$

Question 12. If $T(n) = Kn^2$, and $T(100) = 0.5 \times 10^{-3}$, what is the value of K?
Answer: $T(100) = K \cdot 100^2 = 0.5 \times 10^{-3}$, so $K = 0.5 \times 10^{-7}$

Question 13.[hard] If $T(n) = Kn \log_{10} n$, and $T(100) = 0.5 \times 10^{-3}$, what is the value of n when $T(n)$ is close to 60?
Answer: $T(100) = K \cdot 100 \log_{10} 100 = 0.5 \times 10^{-3}$. So, $K = 0.5 \times 10^{-5}/2 = 0.25 \cdot 10^{-5}$. When $T(n) = 60 = 0.25 \times 10^{-5} \times n \log_{10} n$, $n \log n = 24 \times 10^6 = 4 \times 6 \times 10^6 \approx 3.6 \times 10^6 \times (\log_{10} 3.6 + 6)$. So, $n = 3.6 \times 10^6$.