## Theoretical Computer Science (M21276)

## Part A/7: Pushdown automata

(Oct 16-20, 2023)

In each question design a pushdown automaton that accepts the given language. Write also plan and discuss whether your automaton is deterministic or non-deterministic. Can you produce a context-free grammar for each language?

Question 1.  $L = \{ab^n cd^n \mid n \ge 0\}$  over the alphabet  $\Sigma = \{a, b, c, d\}$ .

Question 2.  $L = \{wcw^R \mid w \in \{a, b\}^*\}$  over the alphabet  $\Sigma = \{a, b, c\}$ .

**Question 3.** Any string from  $\{a,b\}^*$  with an odd number of b's over the alphabet  $\Sigma = \{a,b\}$ .

Think again about the same language: Do we really need a stack to recognise this language? Can you design a finite automaton that accepts that language?

Question 4.  $L = \{a^m b^m b^n a^n \mid m \ge 0, n \ge 0\}$  over the alphabet  $\Sigma = \{a, b\}$ .

Question 5.  $L = \{a^n b^{n+2} \mid n \ge 0\}$  over the alphabet  $\Sigma = \{a, b\}$ .

**Question 6.** Consider strings consisting entirely of left and right brackets. Such a string is called balanced if (a) reading from left to right the number of left brackets is always at least the number of right brackets, and (b) the total number of left brackets equals the total number of right brackets. For example, (()()) is balanced, but (() is not.

- (i) Can you produce a context-free grammar for balanced strings?
- (ii) Can you draw a PDA which will accept only balanced brackets?

**Question 7.** Any string with twice as many a's as b's.