

Theoretical Computer Science (M21276)

Part A/6: Regular grammars & beyond

(Oct 9-13, 2023)

Question 1.

- What is the difference between regular and context-free grammar?
- Can you give an example of a language which is context-free but not regular?
- How can we prove that a given language is not regular?
- What is the pumping lemma?
- What is the pigeonhole principle? Why within ANY group of 13 people, there must be at least two who have their birthdays in the same months.

Question 2. Consider the following grammar with A , B nonterminals, S the initial nonterminal; and 0, 1 terminals

$$S \rightarrow 0 \mid 0A$$

$$A \rightarrow 1B$$

$$B \rightarrow 0 \mid 0A$$

- (i) What type is this grammar? Give the name of the smallest Chomsky class to which the language belongs to.
- (ii) Which strings belong to the language generated by this grammar? Find three strings from the language and describe the structure of all strings belonging to the language.
- (iii) If it is possible, find a regular expression for the language.
- (iv) If it is possible, construct a DFA which recognises the language. Does your DFA have a minimum number of states?

Question 3. Consider the following grammar:

$$S \rightarrow 0A \mid 1A$$

$$A \rightarrow 1BB$$

$$B \rightarrow 11 \mid 01$$

with the nonterminals S (initial), A , B and a set of terminals $\{0, 1\}$.

- (i) What type is this grammar? Is the language finite? Give the name of the smallest Chomsky class to which the language belongs to.

- (ii) What is the structure of the strings from the language?
- (iii) If the language is regular, can you find a regular expression for the language?

Question 4. For each of the following languages over the alphabet $\{a, b\}$, state whether it is regular or not. If not, give a reason why is not regular. If the language is regular, can you construct an NFA which recognise it?

- (i) $\{(ab)^n \mid n > 100\}$
- (ii) $\{(ab)^n \mid n < 100\}$
- (iii) $\{a^n b^n \mid n < 100\}$
- (iv) $\{a^n b^n a^n \mid n > 1\}$

Question 5. Find a context-free grammar for the following languages over $\Sigma = \{a, b\}$

- (i) $L = \{ab(bbaa)^n bba(ba)^n \mid n \geq 0\}$
- (ii) $L = \{a^n b^m \mid 0 \leq n \leq m + 3\}$
- (iii) $L = \{a^n b^m \mid 0 \leq 2n \leq m \leq 3n\}$

Question 6. Consider the following grammar:

$$\begin{aligned}
 S &\rightarrow SABC \mid ABC \\
 AB &\rightarrow BA \\
 BA &\rightarrow AB \\
 BC &\rightarrow CB \\
 CB &\rightarrow BC \\
 AC &\rightarrow CA \\
 CA &\rightarrow AC \\
 A &\rightarrow a \\
 B &\rightarrow b \\
 C &\rightarrow c
 \end{aligned}$$

with the nonterminals S (initial), A , B , C which generates the language L over the alphabet $\{a, b, c\}$.

- (i) Find three strings from L .
- (ii) Is $abac$ in the language? What is the structure of the strings from the language?
- (iii) Can you find a regular expression for L ?

- (iv) What type is the grammar? Give the name of the smallest Chomsky class to which the language L belongs to.

Question 7. Consider the following grammar over the alphabet $\Sigma = \{0, 1\}$, and the set of non-terminal $N = \{S \text{ (initial)}, A, B\}$:

$$\begin{aligned} S &\rightarrow 0AB \\ B &\rightarrow SA \mid 01 \\ A1 &\rightarrow SB1 \\ 1B &\rightarrow 0 \\ A0 &\rightarrow S0B \end{aligned}$$

- (i) What type is this grammar?
(ii) Is the language empty?

Question 8. A monotonic grammar with the restriction “any production rule is allowed as long as there are no rules for making strings shorter, except one $S \rightarrow \Lambda$ ” also describes a context-sensitive language. Can you rewrite the rule of the type $XY \rightarrow YX$ using only context-sensitive production rules?

Question 9. Let L and M be context-free languages. Can you decide whether the languages $L \cup M$, $L \cdot M$, L^* are context-free?