Theoretical Computer Science (M21276)

Part A/1: Introduction to Languages

(W1: Sept 25-39, 2023)

Question 1.

(a) Write an example of an alphabet Σ of size 3.

(b) Write several strings made up from Σ.

(c) Write an example of a language L over Σ.

(d) What is Σ^*? Is your language L a subset of Σ^*.

Question 2. Consider two languages over the alphabet $\Sigma = \{a, b\}$, which are defined as $L = \{\Lambda, a, ab, bb\}$ and $M = \{a, b, ab, ba\}$.

(a) Find the following languages:

$L \cup M$, $L \cap M$, $L \cup \Sigma^*$, $L \cap \Sigma^*$, L^0, L^2, L^*.

(b) List all the stings of length at most 4 from M^* with exactly 3 $b's$.

(c) Give an example of a string from $L^* \setminus L^2$. Is your string also from $\Sigma^* \setminus L^2$? If yes, can you find one which is from $\Sigma^* \setminus L^2$, but not from $L^* \setminus L^2$?

Answer: (a) $L \cup M = \{\Lambda, a, ab, bb, b, ba\}$, $L \cap M = \{a, ab\}$, $L \cup \Sigma^* = \Sigma^*$, $L \cap \Sigma^* = L$, $L^0 = \{\Lambda\}$, $L^2 = \{\Lambda, a, ab, bb, aa, aab, abb, aba, abab, abbb, bba, bbab, bbbb\}$, $L^* = \ldots$ (b) bbb, $abbb$, $babb$, $bbab$, $bbba$, (c) An example of a string from $L^* \setminus L^2$: $bbabbb$. Yes, this string is also from $\Sigma^* \setminus L^2$. A string from $\Sigma^* \setminus L^2$, but not from $L^* \setminus L^2$ e.g. b, bbb

Question 3. Let $L = \{\Lambda, abb, b\}$ and $M = \{bba, ab, a\}$. Evaluate each of the following language expressions.

(a) $L \cdot M$

Answer: $L \cdot M = \{bba, ab, a, abbbba, abbab, abba, bbba, bab, ba\}$

(b) $M \cdot L$

Answer: $M \cdot L = \{bba, ab, a, bbaabb, bbab, ababb, abb, aabb\}$

(c) L^2

Answer: $L^2 = \{\Lambda, abb, b, abbabb, abbb, babb, bb\}$

Question 4. Use your wits to solve each of the following language equations for the unknown language.

(a) $\{\Lambda, a, ab\} \cdot L = \{b, ab, ba, aba, abb, abba\}$.

Answer: $L = \{b, ba\}$

(b) $L \cdot \{a, b\} = \{a, baa, b, bab\}$.

Answer: $L = \{\Lambda, ba\}$

(c) $\{a, aa, ab\} \cdot L = \{ab, aab, abb, aa, aaa, aba\}$.

Answer: $L = \{b, a\}$

(d) $L \cdot \{\Lambda, a\} = \{\Lambda, a, b, ab, ba, aba\}$

Answer: $L = \{\Lambda, b, ab\}$

Question 5. (hard) Let L and M be languages. Prove each of the following statements about the closure of languages.

(a) $\{\Lambda\}^* = \emptyset^* = \{\Lambda\}$.

Answer: $L^0 = \{\Lambda\}$ for any language, $\{\Lambda\}^n = \{\Lambda\}$ for all $n \geq 0$, $\emptyset^n = \{\Lambda\}$ for all $n > 0$

(b) $L^* = L^* \cdot L^* = (L^*)^*$.

Answer: Each set can contain only elements of the form L^n for any ≥ 0, and contains every element of this form.

(c) $\Lambda \in L$ if and only if $L^+ = L^*$.

Answer: $L^* = L^+ \cup L^0 = L^+ \cup \{\Lambda\}$; L^* is different only if L^+ doesn't contain Λ, which implies L doesn't contain Λ

(d) $L \cdot (M \cdot L)^* = (L \cdot M)^* \cdot L$.

Answer: These are two alternate ways of writing the following: $L \cup L \cdot M \cdot L \cup L \cdot M \cdot L \cdot M \cdot L \cup L \cdot M \cdot L \cdot M \cdot L \cdot M \cdot L \ldots$. More details can be found in the book James L. Hein: Discrete Structures, Logic, and Computability, Part 1.3

(e) $(L^* \cdot M^*)^* = (L^* \cup M^*)^* = (L \cup M)^*$.

Answer: All represent alternate ways of writing the set which contains every possible combination of elements from L and M.