Theoretical Computer Science (M21276)

Part A/3: Regular Languages

(Oct 2-6, 2023)

Question 1. Find the language corresponding to each of the following regular expressions over the alphabet $\{a, b, c\}$.

(i) $a + b$

(ii) $a + bc$

(iii) $ab + ba$

(iv) ab^*

(v) $a + b^*$

(vi) $ab^* + c$

(vii) $ab^* + bc^*$

(viii) $a^*bc^* + ac$

Question 2. Find all strings in $L((a + b)^*b(a + ab)^*)$ of length less than four.

Question 3. Find a regular expression to describe each of the following languages over the alphabet $\{a, b, c\}$:

(i) $\{a, b, c\}$

(ii) $\{aa, ab, ac\}$

(iii) $\{\Lambda, bb, bbbb, bbbbbb, \dots\}$

(iv) $\{a, b, ab, ba, abb, baa, \dots, ab^n, ba^n, \dots\}$

(v) $\{a, aaa, aaaaa, \dots, a^{2n+1}, \dots\}$

(vi) $\{\Lambda, a, abb, abbbb, \dots, ab^{2n}, \dots\}$

(vii) $\{\Lambda, a, b, c, aa, bb, cc, \dots, a^n, b^n, c^n, \dots\}$

(viii) $\{\Lambda, a, b, ca, bc, cca, bcc, \dots, c^n a, bc^n, \dots\}$

(ix) $\{a^n b^m, n \geq 4, m \leq 3\}$

(x) $\{a^n b^m, n \geq 3, m \text{ is even}\}$

(xi) $\{a^n b^m, (n + m) \text{ is even }\}$

Question 4. Find a regular expression for each of the following languages over the alphabet $\{a, b\}$.

(i) All strings with even length.

(ii) All strings whose length is a multiple of 3.

(iii) All strings containing the substring aba

(iv) All strings with an odd number of a's.

(v) All strings except those with the substring aa.

(vi) All strings except those with the substring aaa.

Question 5.[hard] Prove the following equalities of regular expressions.

(i) $b + ab^* + aa^*b + aa^*ab^* = a^*(b + ab^*)$

(ii) $a^*(b + ab^*) = b + aa^*b^*$

(iii) $a(a + b)^* + aa(a + b)^* + aaa(a + b)^* = a(a + b)^*$

Question 6.[hard] Prove that for any regular expression R holds $R^* = R^*R^*$.