

Theoretical Computer Science (M21276)

Part A/1: Introduction to Languages

(W1: Sept 25-39, 2023)

Question 1.

- (a) Write an example of an alphabet Σ of size 3.
- (b) Write several strings made up from Σ .
- (c) Write an example of a language L over Σ .
- (d) What is Σ^* ? Is your language L a subset of Σ^* .

Question 2. Consider two languages over the alphabet $\Sigma = \{a, b\}$, which are defined as $L = \{\Lambda, a, ab, bb\}$ and $M = \{a, b, ab, ba\}$.

- (a) Find the following languages:
 $L \cup M$, $L \cap M$, $L \cup \Sigma^*$, $L \cap \Sigma^*$, L^0 , L^2 , L^* .
- (b) List all the strings of length at most 4 from M^* with exactly 3 b 's.
- (c) Give an example of a string from $L^* \setminus L^2$. Is your string also from $\Sigma^* \setminus L^2$? If yes, can you find one which is from $\Sigma^* \setminus L^2$, but not from $L^* \setminus L^2$?

Question 3. Let $L = \{\Lambda, abb, b\}$ and $M = \{bba, ab, a\}$. Evaluate each of the following language expressions.

- (a) $L \cdot M$
- (b) $M \cdot L$
- (c) L^2

Question 4. Use your wits to solve each of the following language equations for the unknown language.

- (a) $\{\Lambda, a, ab\} \cdot L = \{b, ab, ba, aba, abb, abba\}$.
- (b) $L \cdot \{a, b\} = \{a, baa, b, bab\}$.
- (c) $\{a, aa, ab\} \cdot L = \{ab, aab, abb, aa, aaa, aba\}$.
- (d) $L \cdot \{\Lambda, a\} = \{\Lambda, a, b, ab, ba, aba\}$

Question 5. (hard) Let L and M be languages. Prove each of the following statements about the closure of languages.

(a) $\{\Lambda\}^* = \emptyset^* = \{\Lambda\}$.

(b) $L^* = L^* \cdot L^* = (L^*)^*$.

(c) $\Lambda \in L$ if and only if $L^+ = L^*$.

(d) $L \cdot (M \cdot L)^* = (L \cdot M)^* \cdot L$.

(e) $(L^* \cdot M^*)^* = (L^* \cup M^*)^* = (L \cup M)^*$.