Theoretical Computer Science (M21276)

Part A/2: Grammars

(W1: Sept 25-29, 2023)

Question 1. Given the following grammar with the set of terminals $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the start symbol S, another non-terminal symbol D and the set of productions:

$$S \to D \mid DS$$
$$D \to 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9.$$

(i) Find the productions used for the steps of the following derivation:

$$S \Rightarrow DS \Rightarrow 7S \Rightarrow 7DS \Rightarrow 7DDS \Rightarrow 78DS \Rightarrow 780S \Rightarrow 780D \Rightarrow 7801$$

Answer: $S \to DS$; $D \to 7$; $S \to DS$; $S \to DS$; $D \to 8$; $D \to 0$; $S \to D$; $D \to 1$

(ii) Write several strings generated by the grammar.

(iii) Find a leftmost derivation of the string 7801.

Answer: $S \Rightarrow \underline{D}S \Rightarrow 7\underline{S} \Rightarrow 7\underline{D}S \Rightarrow 78\underline{S} \Rightarrow 78\underline{D}S \Rightarrow 780\underline{S} \Rightarrow 780\underline{D} \Rightarrow 7801$

(iv) Find a rightmost derivation of the string 7801.

Answer: $S \Rightarrow D\underline{S} \Rightarrow DD\underline{S} \Rightarrow DDD\underline{S} \Rightarrow DDD\underline{D} \Rightarrow DD\underline{D}1 \Rightarrow D\underline{D}01 \Rightarrow \underline{D}801 \Rightarrow 7801$

Question 2. In each case below find a language described by the grammars with the set of the terminals $\{a, b\}$, the non-terminal start symbol S and the productions:

(i) $S \to a \mid aS$

Answer: $\{a, aa, aaa, \dots\}$

(ii) $S \to \Lambda \mid aSb$

Answer: $\{\Lambda, ab, aabb, aaabbb, \dots\}$

(iii) $S \to aS \mid bS$

Answer: empty language, no string

Question 3. Find a grammar (set of terminals, nonterminals, start symbol, production rules, ...) for each of the following languages:

(i) $\{bb, bbbb, bbbbbb, \dots\}$

Answer: The set of terminals $\{b\}$, the only nonterminal start symbol S and the productions: $S \to bb \mid bbS$

(ii) $\{a, ba, bba, bbba, \dots\}$

Answer: The set of terminals $\{a, b\}$, the only nonterminal start symbol S and the productions: $S \to a \mid bS$

(iii) $\{\Lambda, ab, abab, ababab, \dots\}$

Answer: The set of terminals $\{a, b\}$, the only nonterminal start symbol S and the productions: $S \to \Lambda \mid abS$

(iv) $\{bb, bab, baab, baaab, \dots\}$

Answer: The set of terminals $\{a, b\}$, the set of nonterminal symbols $\{A, S\}$ (S is the start symbol) and the productions: $S \to bAb, A \to \Lambda \mid aA$

Question 4. If w is a string, let w^R denote the reverse of w. For example, $aabc$ is the reverse of $cbaa$. Find a grammar to describe the language $\{ww^R \mid w \in \Sigma^*\}$ for an alphabet $\Sigma = \{a, b, c\}$.

Answer: $T = \{a, b, c\}$. The only nonterminal start symbol S and the productions: $S \to \Lambda \mid aSa \mid bSb \mid cSc$

Question 5. Find a grammar for each of the following languages:

(i) The set of binary numerals that represent odd natural numbers (leading zeros are allowed). Can you find a solution without leading zeros?

Answer: The set of terminals $\{0, 1\}$, the nonterminal symbols $\{D, S\}$ (S is the start symbol) and the productions:

$$S \to D1 \mid 1$$
$$D \to 1 \mid D0 \mid D1.$$

(ii) The set of decimal numerals that represent odd natural numbers (leading 0 is allowed).

Answer: The set of terminals $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the nonterminal symbols $\{D, S\}$ (S is the start symbol) and the productions:

$$S \to D1 \mid D3 \mid D5 \mid D7 \mid D9$$
$$D \to \Lambda \mid D0 \mid D1 \mid D2 \mid D3 \mid D4 \mid D5 \mid D6 \mid D7 \mid D8 \mid D9$$

Question 6. Find a grammar for each of the following languages over the alphabet $\{a, b, c\}$:

(i) $\{a^n b \mid n \geq 0\}$.

Answer: The set of terminals $\{a, b\}$, the only nonterminal start symbol S and the productions: $S \to b \mid aS$

(ii) $\{a^n b^m \mid n \geq 1 \text{ and } m \geq 1\}$.

Answer: The set of terminals $\{a, b\}$, the nonterminal symbols $\{A, B, S\}$ (S is the start symbol) and the productions: $S \to AB$, $A \to a \mid aA$, $B \to b \mid bB$

(iii) $\{a^n b c^n \mid n \geq 0\} \cup \{b^n a^m \mid n \geq 0 \text{ and } m \geq 0\}$.

Answer: The set of terminals $T = \{a, b, c\}$, the nonterminal symbols $\{W, A, B, C, S\}$ (S is the start symbol) and the productions: $S \to W \mid C$, $W \to aWc \mid b$, $C \to BA$, $B \to \Lambda \mid bB$, $A \to \Lambda | aA$