

Theoretical Computer Science (M21276)

Part A/2: Grammars

(W1: Sept 25-29, 2023)

Question 1. Given the following grammar with the set of terminals $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the start symbol S , another non-terminal symbol D and the set of productions:

$$\begin{aligned} S &\rightarrow D \mid DS \\ D &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9. \end{aligned}$$

(i) Find the productions used for the steps of the following derivation:

$$S \Rightarrow DS \Rightarrow 7S \Rightarrow 7DS \Rightarrow 7DDS \Rightarrow 78DS \Rightarrow 780S \Rightarrow 780D \Rightarrow 7801$$

(ii) Write several strings generated by the grammar.

(iii) Find a leftmost derivation of the string 7801.

(iv) Find a rightmost derivation of the string 7801.

Question 2. In each case below find a language described by the grammars with the set of the terminals $\{a, b\}$, the non-terminal start symbol S and the productions:

(i) $S \rightarrow a \mid aS$

(ii) $S \rightarrow \Lambda \mid aSb$

(iii) $S \rightarrow aS \mid bS$

Question 3. Find a grammar (set of terminals, nonterminals, start symbol, production rules, ...) for each of the following languages:

(i) $\{bb, bbbb, bbbbbb, \dots\}$

(ii) $\{a, ba, bba, bbba, \dots\}$

(iii) $\{\Lambda, ab, abab, ababab, \dots\}$

(iv) $\{bb, bab, baab, baaab, \dots\}$

Question 4. If w is a string, let w^R denote the reverse of w . For example, $aabc$ is the reverse of $cbaa$. Find a grammar to describe the language $\{ww^R \mid w \in \Sigma^*\}$ for an alphabet $\Sigma = \{a, b, c\}$.

Question 5. Find a grammar for each of the following languages:

- (i) The set of binary numerals that represent odd natural numbers (leading zeros are allowed). Can you find a solution without leading zeros?
- (ii) The set of decimal numerals that represent odd natural numbers (leading 0 is allowed).

Question 6. Find a grammar for each of the following languages over the alphabet $\{a, b, c\}$:

- (i) $\{a^n b \mid n \geq 0\}$.
- (ii) $\{a^n b^m \mid n \geq 1 \text{ and } m \geq 1\}$.
- (iii) $\{a^n b c^n \mid n \geq 0\} \cup \{b^n a^m \mid n \geq 0 \text{ and } m \geq 0\}$.