CV201 HW4 HadadYakir BukraBarak

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January 2020

1 Filtering

1.1 Convolution

1.1.1 Problem 1

I prefer use f, g, h as matrices

$$\begin{aligned}
((f*g)*h)(x,y) &= \sum_{kl}(f*g)(k,l)h(x-k,y-l) \\
&= \sum_{kl}\big(\sum_{ij}f(i,j)g(k-i,l-j)\big)h(x-k,y-l) \\
&= \{k' = k-i,\ l' = l-j\} \\
&= \sum_{ij}\sum_{k'j'}f(i,j)g(k',l')h\big((x-i)-k',(y-j)-l'\big) \\
&= \sum_{ij}f(i,j)((g*h)(x-i,y-j)) \\
&= (f*(g*h))(x,y)
\end{aligned} \tag{1}$$

□

1.1.2 Problem 2

$$\begin{aligned}
(h*\delta)(i,j) &= \sum_{kl}h(i-k,j-l)\delta(k,l) \\
&= h(i-0,j-0)\delta(0,0) \\
&= h(i,j)
\end{aligned} \tag{2}$$

□

1.1.3 Computer Exercise 1

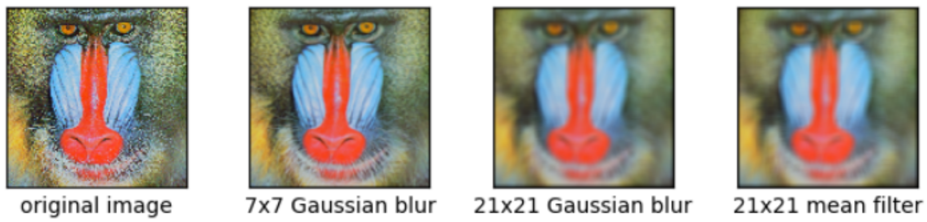

Figure 1: Computer exercise 1

1.1.4 Problem 3

H_i entry looks like below. $H_{ii} = h_{22}$ and the others h_{ij} entries are set respectively as mentioned below.
Every '$\cdots$' means place $0's$ and in case dimensions not fit for h_{ij} simply don't include those values in H_{ij}.

$$H_i = [\cdots \ h_{11} \ h_{12} \ h_{13} \ \underbrace{\cdots}_{\text{N-2}} \ \underbrace{h_{21} \ h_{22} \ h_{23}}_{\text{i-1 i i+1}} \ \underbrace{\cdots}_{\text{N-2}} \ h_{31} \ h_{32} \ h_{33} \ \cdots]$$

precisely:

$$H_{ij} = \begin{cases} h_{11} & j = i - N - 1 \\ h_{12} & j = i - N \\ h_{13} & j = i - N + 1 \\ h_{21} & j = i - 1 \\ h_{22} & j = i \\ h_{23} & j = i + 1 \\ h_{31} & j = i + N - 1 \\ h_{32} & j = i + N \\ h_{33} & j = i + N + 1 \\ 0 & else \end{cases}$$

1.1.5 Computer Exercise 2

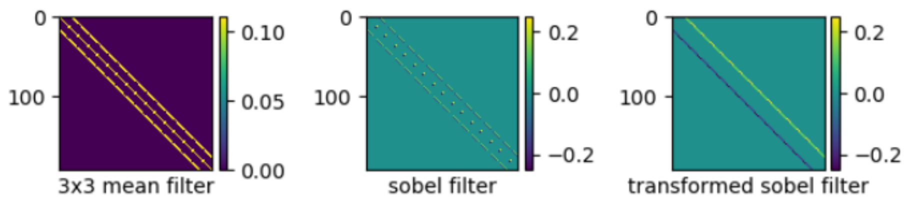

Figure 2: Computer exercise 2

1.1.6 Problem 4

From the definition of convolution it seems that this filter will replace every pixel (i, j) with pixel $(i+1, j+2)$ this is what I got to the image 10x10 that the value of every site $(i, j) = i + j$ image as :

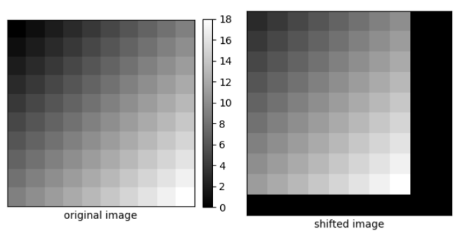

Figure 3: Optional Problem 4

1.1.7 Problem 5

part i
Using the property that $det(AB) = det(A)det(B)$ and that $det(A) = det(A^T)$ we an say that:

$$\begin{aligned} det(I) &= det(YY^T) \\ 1 &= det(Y)det(Y^T) \\ 1 &= det(Y)det(Y) \\ 1 &= det(Y)^2 \\ \pm 1 &= det(Y) \end{aligned} \tag{3}$$

□

part ii

$$\begin{aligned} YY^T &= I \\ (YY^T)^T &= I^T \\ Y^TY &= I \end{aligned} \tag{4}$$

□

part iii
$y_i^T y_i$ is row vector multiply column so its the scalars on the diagonal of YY^T, and we already know that this values are 1.
part iv
by definition $\|y_i\|_{l_2} = y_i y_i^T$ so $\|y_i\|_{l_2} = 1$

part v
Same as *part iii* $y_i^T y_j$ are scalars place but the diagonal on YY^T which means its equal to 0.
we know that $y_i^T yY_j = \|y_i\| \|y_j\| \, cos\theta = 0$, the size of y_i and y_j is bigger than 0 so $cos\theta = 0$ means that $\theta = \pm 90°$

part vi
We need to prove 3 conditions to show that the orthogonal matrices forms a matrix group:

(G_1) $I \in G$
I is orthogonal matrix by definition, $II^T = I$

(G_2) $A, B \in G \rightarrow AB \in G$
$AA^T = I$ and $BB^T = I$. we need to prove that $AB(AB)^T = I$

$$\begin{aligned} AB(AB)^T &= ABB^TA^T \\ &= AIA^T \\ &= AA^T \\ &= I \end{aligned} \tag{5}$$

□

(G_3) $A \in G,\ A^{-1}\ exist,\ \rightarrow\ A^{-1} \in G$
If $A \in G$ than obviously A^{-1} exist and its A^T and we saw in *part ii* that A^{-1} is orthogonal matrix too so $A^{-1} \in G$ □

The orthogonal matrices forms a matrix group.

1.1.8 Computer Exercise 3

From my code I've found that $K1$ is the only separable filter, with the singular value $s_1 = 1.1557$

1.1.9 Problem 6

K is inversible because all the lines are depend. For example W.L.O.G lets $K = [u_1\ u_2\ u_3] \begin{bmatrix} s_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = s_0 \begin{bmatrix} u_1v_1 & u_1v_2 & u_1v_3 \\ u_2v_1 & u_2v_2 & u_2v_3 \\ u_3v_1 & u_3v_2 & u_3v_3 \end{bmatrix}$
Its more visible this way, but in general every line is depend on the left right singular matrix(vector), we know that if there are 2 rows then the matrix is not invertible.

1.1.10 Problem 7

No, bilateral filter in nonlinear. let $g_I = \frac{1}{c}\sum_{x_i} f^r(|I(x)-I(x_i)|)f^d(|x-x_i|)I(x_i)$ be a bilateral filter. Suppose we have g_{I_1} and g_{I_2}, than the sum of those filters:

$$g_{I_1}+g_{I_2} = \frac{1}{c}\sum_{x_i} f^d(|x-x_i|)(f^r(|I_1(x)-I_1(x_i)|)I_1(x_i)+f^r(|I_1(x)-I_1(x_i)|)I_2(x_i))$$

and

$$g_{I_1+I_2} = \frac{1}{c}\sum_{x_i} f^r(|(I_1(x)+I_2(x))-(I_1(x_i)+I_2(x_i))|)f^d(|x-x_i|)(I_1(x_i)+I_2(x_i))$$

The only way to make this filter linear is if f^r linear which is not promised. Your example of the Gaussian show that is non linear function make this filter non linear either.

1.1.11 Computer Exercise 4

Using $\sigma_x = 0.5$ and $\sigma_y = 3$ gave me this result which is very similar to yours plot:

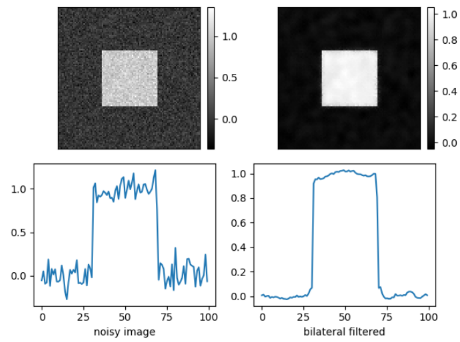

Figure 4: Optional Problem 4

1.1.12 Problem 8

$$
\begin{aligned}
G(x,y,\sigma) &= \frac{1}{2\pi\sigma^2} exp(-\frac{x^2+y^2}{\sigma^2}) \\
\frac{\partial G(x,y,\sigma)}{\partial x} &= \frac{-1}{2\pi\sigma^2}\frac{x}{\sigma^2} exp(-\frac{x^2+y^2}{\sigma^2}) \\
\frac{\partial^2 G(x,y,\sigma)}{\partial^2 x} &= -\frac{1}{2\pi\sigma^4} exp(-\frac{x^2+y^2}{\sigma^2}) \\
&+ \frac{1}{2\pi\sigma^2}\frac{x}{\sigma^2}\frac{x}{\sigma^2} exp(-\frac{x^2+y^2}{\sigma^2}) \\
&= \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right)\frac{1}{2\pi\sigma^2} exp(-\frac{x^2+y^2}{\sigma^2}) \\
&= \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) G(x,y,\sigma)
\end{aligned} \tag{6}
$$

$$
\frac{\partial^2 G(x,y,\sigma)}{\partial^2 y} = \left(\frac{y^2}{\sigma^4} - \frac{1}{\sigma^2}\right) G(x,y,\sigma) \tag{7}
$$

$$
\nabla^2 G(x,y,\sigma) = \left(\frac{x^2+y^2}{\sigma^4} - \frac{1}{\sigma^2}\right) G(x,y,\sigma)
$$

□

1.2 Problem 9

1.2.1 Computer Exercise 5

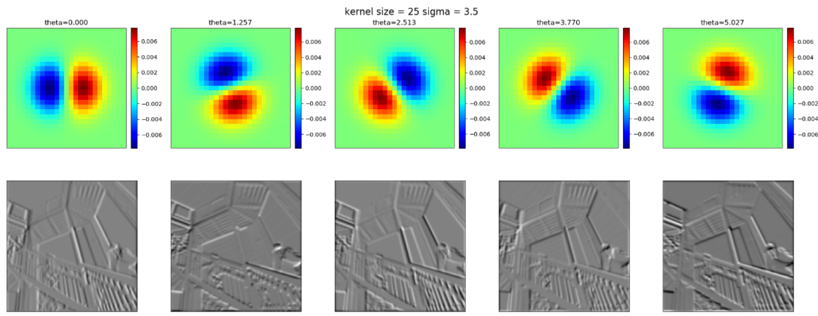

Figure 5: Computer exercise 5

1.2.2 Computer Exercise 6

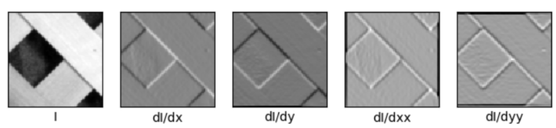

Figure 6: Computer exercise 6