HW 1 Complex Analysis Spring 2024

11$^{\text{th}}$ June, 2024

Exercise 1. *Describe using a drawing in* $\mathbb{C}$ *the following sets:*

(a) $|z - z_1| = |z - z_2|$ *for fixed* $z_1, z_2 \in \mathbb{C}$*;*

(b) $\frac{1}{z} = \overline{z}$*;*

(c) $\mathrm{Re}(z) > c,\ c \in \mathbb{R}$*;*

(d) $\mathrm{Re}(z) = 3$*;*

(e) $\mathrm{Re}(az + b) > 0,\ a, b \in \mathbb{C}$*;*

(f) $|z| = \mathrm{Re}(z) + 1$*;*

(g) $\mathrm{Re}(z) = c,\ c \in \mathbb{R}$.

Exercise 2. *Show that the Cauchy-Riemann Equations can be re-written in polar form as:*

$$u_r = r^{-1} v_\theta, r^{-1} u_\theta = -v_r. \tag{0.1}$$

Exercise 3. *Define the function* $f : \mathbb{C} \to \mathbb{C}$ *by letting* $f(x + iy) = \sqrt{|x||y|}$*. Show that* f *satisfies the Cauchy-Riemman Equations at* 0 *but* f *is not holomorphic in any neighborhood of* 0*.*

Exercise 4. *Determine the radius of convergence for the following power series:*

(a) $\sum_{n=1}^{\infty} (\log n)^2 z^n$*;*

(b) $\sum_{n=1}^{\infty} n! z^n$*;*

(c) $\sum_{n=1}^{\infty} \frac{n^2}{4^n + 3n} z^n$.

Exercise 5. *Define the function* $f : \mathbb{R} \to \mathbb{R}$ *by*

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ e^{-1/x^2}. & \text{otherwise} \end{cases} \tag{0.2}$$

Prove that f is infinitely differentiable in $\mathbb{R}$ and that $f^{(n)}(0) = 0$ for all $n \geq 1$. Conclude that f doesn't have a converging power series expansion around 0.