```{r}
#Question 1

rm(list=ls())
load("/Users/kartikayjain/Downloads/returns.Jan.19.2024.RData")
options(width=80)
MCD=returns[,"MCD"]
library(rugarch)

loglik = function(theta) {
  -sum(log(ddist(distribution="std",MCD,mu=theta[1], sigma=theta[2],shape=theta[3])))
}
start=c(mean(MCD), sd(MCD), 2.1)
options(digits=3)
fit_std = optim(start, loglik, method="L-BFGS-B", lower=c(-0.01,0.001,2.1),
                upper=c(0.01,0.1,20), hessian=TRUE)

cat("MLE of the mean is ", signif(fit_std$par[1], digits=3))
cat("\n")
cat("MLE of the sd is ", signif(fit_std$par[2], digits=3))
cat("\n")
cat("MLE of the df is ", signif(fit_std$par[3], digits=3))
cat("\n")


neg_logL_t = fit_std$value
AIC = 2*neg_logL_t + 2*length(fit_std$par)
BIC= 2*neg_logL_t + log(length(MCD))*length(fit_std$par)
cat("AIC value is ",AIC)
cat("\n")
cat("BIC value is ",BIC)




```

```
MLE of the mean is  0.000654
MLE of the sd is  0.012
MLE of the df is  3.13
AIC value is  -17687
BIC value is  -17669
```

```r
#Question 2
loglik_s = function(x) {
  -sum(log(ddist(distribution="sstd",MCD,mu=x[1], sigma=x[2],shape=x[3],skew=x[4])))
}
start=c(mean(MCD),sd(MCD),3,1)
fit_s = optim(start, loglik_s,method="L-BFGS-B",
                  lower=c(-0.1,0.001,2.1,0.25),
                  upper=c(0.1,1,20,4), hessian=TRUE,
                  control=list(maxit=1000,tmax=100))

cat("MLE of the mean is ", signif(fit_s$par[1], digits=3))
cat("\n")
cat("MLE of the sd is ", signif(fit_s$par[2], digits=3))
cat("\n")
cat("MLE of the df is ", signif(fit_s$par[3], digits=3))
cat("\n")
cat("MLE of the skew is ", signif(fit_s$par[4], digits=3))
cat("\n")
neg_logL_s = fit_s$value
AIC_s = 2*neg_logL_s + 2*length(fit_s$par)
BIC_s= 2*neg_logL_s + log(length(MCD))*length(fit_s$par)
cat("AIC for sstd value is ",AIC_s)
cat("\n")
cat("BIC for sstd value is ",BIC_s)
cat("\n")
cat("AIC difference is ",signif(AIC_s-AIC,3))
cat("\n")
cat("BIC difference is ",signif(BIC_s-BIC,3))
cat("\n")
#Since AIC difference is less than 0, AIC will select the skewed t distribution
#Since BIC difference is >0, BIC will select the symmetric t distribution
standard_e=sqrt(diag(solve(fit_s$hessian)))
signif(standard_e,3)
conf_i=matrix(nrow=4,ncol=2)
for (i in 1:4) {
  conf_i[i,]=fit_s$par[i]+c(-1,1)*standard_e[i]*qnorm(0.995)
}
signif(conf_i,3)
#Confidence interval for the skeweness coefficient includes 1
```

```
MLE of the mean is  0.00047
MLE of the sd is  0.0119
MLE of the df is  3.18
MLE of the skew is  0.959
AIC for sstd value is  -17688
BIC for sstd value is  -17664
AIC difference is  -0.601
BIC difference is  5.33
[1] 0.000208 0.000494 0.197000 0.025000
          [,1]    [,2]
[1,] -6.49e-05 0.00101
[2,]  1.06e-02 0.01310
[3,]  2.67e+00 3.68000
[4,]  8.94e-01 1.02000
```

```r
#Question 3

signif(2*(neg_logL_t-neg_logL_s),3) > signif(qchisq(0.95,1),3)

#Since it is false we don't reject H0 at alpha=0.05

```

```
[1] FALSE
```

```{r}
#Question 4

library(moments)

cat("Skewness is ", skewness(MCD))
cat("\n")
cat("Kurtosis is ", kurtosis(MCD))


#Skewness coefficient is less than 0 so it is a negatively skewed distribution
#Kurtosis is 37.1 which means the tails in the distribution are very heavy relative to the normal distribution.

#We use the formula for MOM estimate of v as v=6/kurtosis + 4

df_MOM = 6/kurtosis(MCD) + 4
df_MOM[1]

#Hence, the MOM estimate is higher than the MLE estimate of the df.
```

```
Skewness is  -0.143
Kurtosis is  37.1 MCD
4.16
```