QP Simple Example

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```
library(quadprog)
```

To illustrate the solution method of a quadratic programming problem with linear equality and inequality constraints, consider the following problem:

$$
\min_x \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}^T x + \frac{1}{2} x^T \Sigma \ x
$$

such that

$$
\begin{aligned}
\sum x_i &= 1 \\
x_1 &<= 0.5 \\
x_2 + x_3 &>= 0.8
\end{aligned}
$$

The constraints are written in matrix notation, taking good care to separate the equality and inequality constraints:

$$
\begin{aligned}
A_e^T x &= b_e \\
A_n^T x &>= b_n
\end{aligned}
$$

with

$$
\begin{aligned}
A_e = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} & \quad b_e = 1 \\
A_n = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} & \quad b_n = \begin{bmatrix} -0.5 \\ 0.8 \end{bmatrix}
\end{aligned}
$$

Data

To stay in the domain of portfolio optimization, Σ is a covariance matrix, constructed from the correlation matrix and a column vector of standard deviation.

```
Cor <- matrix(c(1, 0.5, 0.2, 0.5, 1, 0.7, 0.2, 0.7, 1),3,3)
sigma <- matrix(c(1, 2, 1.5), nrow=3)
Sigma <- sigma %*% t(sigma) * Cor
```

$$\sigma = \begin{bmatrix} 1 \\ 2 \\ 1.5 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 1 & 0.3 \\ 1 & 4 & 2.1 \\ 0.3 & 2.1 & 2.25 \end{bmatrix}$$

QP

The QP solver is invoked as follows. Note that the A matrix must contain first the rows corresponding to the equality constraints, then the rows corresponding to the inequality constraints. Parameter `meq` indicates the number of equality constraints.

```
b <- rbind(b_e, b_n)
Amat <- cbind(A_e, A_n)
sol = solve.QP(Sigma, c(1,2,3), Amat, b, meq=1)
```

```
library(kableExtra)
names(sol$solution) <- seq(3)
kable(sol$solution, "latex", booktabs=T, row.names = TRUE) %>%
  kable_styling(full_width = F, position = "center")
```

	x
1	0.1316348
2	-0.4692144
3	1.3375796