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Homework 4

# COMP206 MATHEMATICAL MODELING AND ALGORITHMIC THINKING

**Answer to Question 1.1**

$F(x) = -2x_1^2 - 2x_1x_2 - 2x_2^2 + x_1 + x_2 + 3 \; where \; f \;\in\; R^2$
$Finding\ minimum\ or\ maximum\ of\ that\ function,\ we\ need\ to\ calculate\ first\ derivatives$

$First\ derivative\ of\ function\ with\ respect\ to\ x_1:$

$$\frac{\partial F}{\partial x_1} \;=\; -4x_1 \;-\; 2x_2 \;+\; 1$$

$First\ derivative\ of\ function\ with\ respect\ to\ x_2:$

$$\frac{\partial F}{\partial x_2} \;=\; -\,4x_2 \;-\; 2x_1 \;+\; 1$$

$If\ we\ set\ them\ equal\ to\ 0\ and\ solve,\ we\ will\ got:$

$x_1 \;=\; \frac{1}{6}\ and\ x_2 \;=\; \frac{1}{6}\ that\ means\ (\frac{1}{6}, \frac{1}{6})\ as\ a\ stationary\ point.$

$After\ that\ we\ should\ find\ Hession\ matrix\ to\ see\ if\ it\ will\ give\ max\ or\ min\ of\ function;$

$Hession\ Matrix\ is\ : \begin{bmatrix} -4 & -2 \\ -2 & -4 \end{bmatrix}$

$To\ do\ so,\ We\ will\ calculate\ |D_1|\ and\ |D_2|$
$|D_1| \;=\; -4 \quad which\ is\ <\ 0\ (less\ then\ zero)$
$|D_2| \;=\; 12 \quad which\ is\ >\ 0\ (bigger\ then\ zero)$

$It\ is\ saying\ that\ it\ is\ negative\ definite.$
$That\ means\ our\ function\ has\ a\ maximum\ value\ at\ (\frac{1}{6}\ ,\ \frac{1}{6})$
$F \;=\; (\frac{1}{6}\ ,\ \frac{1}{6}) \;=\; \frac{19}{6}\ will\ be\ the\ maximum\ value\ of\ the\ function.$

$More\ work\ on\ this\ question:$

$While\ F\ function\ is\ in\ R^2,\ It\ may\ be\ 0\ as\ a\ minimum\ value\ with\ respect\ to\ the\ definition\ by\ square\ of\ real\ numbers.$
$So\ if\ we\ look\ for\ that\ one,$
$there\ may\ be\ a\ few\ solution\ for\ that\ such\ as\ (-1, 0).\ It\ will\ give\ 0\ that\ is\ minimum\ of\ function.$ (1)

**Answer to Question 1.2**

$F(x) = 2x^2 + xz + y^2 + 2yz + \frac{1}{2}z^2$

$Finding\ minimum\ or\ maximum\ of\ that\ function,\ we\ need\ to\ calculate\ first\ derivatives$

$First\ derivative\ of\ function\ with\ respect\ to\ x:$

$$\frac{\partial F}{\partial x} = 4x + z$$

$First\ derivative\ of\ function\ with\ respect\ to\ y:$

$$\frac{\partial F}{\partial y} = 2y + 2z$$

$First\ derivative\ of\ function\ with\ respect\ to\ z:$

$$\frac{\partial F}{\partial z} = x + 2y + z$$

$If\ we\ set\ them\ equal\ to\ 0\ and\ solve,\ we\ will\ got:$

$x = 0,\ y = 0\ and\ z = 0.\ That\ means\ (0,0,0)\ as\ a\ stationary\ point.$

$The\ point\ (0,0,0)\ is\ the\ point\ that\ we\ will\ check\ after\ looking\ Hession\ matrix$

$We\ should\ find\ Hession\ matrix\ to\ see\ if\ it\ will\ give\ max\ or\ min\ of\ function;$

$We\ can\ found\ hession\ matrix\ as\ H = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

$|D_1| = 4 > 0\ (Bigger\ than\ zero)$

$|D_2| = 8 > 0\ (Bigger\ than\ zero)$

$|D_3| = -10 < 0\ (Less\ than\ zero)$

$This\ is\ saying\ it\ is\ neither\ Negative\ Definite\ nor\ Positive\ Definite\ function.$

$As\ a\ conclusion,\ It\ has\ not\ maximum\ or\ minimum\ value\ at\ that\ point\ and\ also\ in\ anywhere\ while\ it\ is\ neither\ PD\ nor\ ND.$

(2)

**Answer to Question 2**

```
%We will apply steepest descent method to the function.

%Our function that we will apply steepest descent method.
f = @(x) 3*(x(1)).^2+6*(x(2)).^4;

%gradient of the function with respect to x(1) and x(2).
gradf = @(x)[6*x(1);24*(x(2).^3)];

iterNumber = 1000;

initial = [-3;5];


stepSize = 0.0001;


nextguess = [initial];

record = initial;

for i = 1: iterNumber
    nextguess = nextguess - stepSize*gradf(nextguess);
    record = [record,nextguess];
end


fig = figure;
myPlot = plot(record(1,:),record(2,:),"ro");
hold on;
plot(0,0,'b^');
hold off;
myPlot.Color = "magenta";

disp('Distance from minimum');
disp(norm([0,0]-nextguess));

saveas(fig,"recordingHW4", 'epsc');

% As it can be seen from the plot below, it approaches 0,0 in
%  each iteration.
```

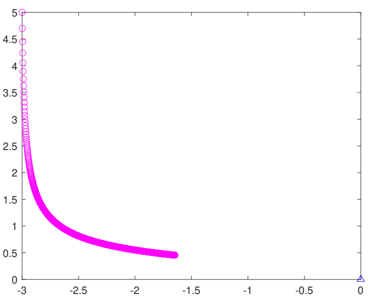


Figure 1: Plot for Q2

**Answer to Question 3**

$$Newton's\ method\ for\ max - min\ problem\ is:$$
$$x_{n+1} \ = \ x_n \ - \ \frac{f'(x_n)}{f''(x_n)}$$

$$Answer\ to\ Question\ 3.1$$
$$*\,For\ f(x) = x^2$$

$$First\ derivative\ of\ f\ \ f'(x) = 2x$$
$$Second\ derivative\ of\ f\ \ f''(x) = 2$$
$$I\ choose\ starting\ point\ x = \ 10\ so\ x_1\ is\ 1.\ We\ can\ continue\ to\ formula$$
$$x_2 \ = \ 1 \ - \ \frac{2*(1)}{2} \ => \ x_2 = 0$$
$$x_3 \ = \ 0 \ - \ \frac{2*(0)}{2} \ => \ x_3 = 0\ ...$$
$$So,\ it\ always\ going\ to\ zero\ after\ second\ step.$$
$$Thus,\ while\ second\ derivative\ of\ function\ is\ bigger\ than\ 0\ (which\ means\ convex),$$
$$and\ function\ goes\ 0,\ It\ has\ minimum\ value\ at\ 0:\ \ f(0) = 0$$

$$Answer\ to\ Question\ 3.2$$
$$*\,For\ f(x) = x^3$$

$$First\ derivative\ of\ f\ \ f'(x) = 3x^2$$
$$Second\ derivative\ of\ f\ \ f''(x) = 6x$$
$$I\ choose\ starting\ point\ x = \ 1\ so\ x_1\ is\ 0.\ We\ can\ continue\ to\ formula$$
$$x_2 \ = \ 1 \ - \ \frac{3*(1)^2}{6*(1)} \ => \ x_2 = \frac{1}{2}$$
$$x_3 \ = \ \frac{1}{2} \ - \ \frac{3*(\frac{1}{2})^2}{6*\frac{1}{2}} \ => \ x_3 = 1/4$$
$$x_4 \ = \ \frac{1}{4} \ - \ \frac{3*(\frac{1}{4})^2}{6*\frac{1}{4}} \ => \ x_4 = 1/8$$
$$x_5 \ = \ \frac{1}{8} \ - \ \frac{3*(\frac{1}{8})^2}{6*\frac{1}{8}} \ => \ x_5 = 1/16...$$

$$So,\ it\ always\ going\ to\ zero\ after\ a\ lot\ of\ steps.\ It\ is\ approaching\ 0.$$
$$But\ here\ 0\ point\ is\ a\ critical(saddle)\ point\ so\ we\ need\ to\ check\ it.$$
$$If\ we\ look\ for\ the\ second\ derivative\ of\ function:\ \ 6x$$
$$Thus,\ we\ can\ say\ that\ from\ right\ hand\ side\ of\ 0\ point,\ it's\ bigger\ than\ zero(convex)\ but$$
$$from\ left\ hand\ side,\ it\ is\ negative(concave).$$

$$Moreover,\ we\ can\ say\ that\ it\ is\ neither\ convex\ nor\ concave\ that\ means\ it\ has\ no\ min\ or\ max$$
$$value\ while\ its\ domain\ is\ in\ R \tag{3}$$

**Answer to Question 3.3**

*Answer to Question* 3.3

$* \, For \; f(x) = \sqrt{x}$

$First \; derivative \; of \; f \;\; f'(x) = \dfrac{1}{2\sqrt{x}}$

$Second \; derivative \; of \; f \;\; f''(x) = -\dfrac{1}{4x^{(3/2)}}$

$I \; choose \; starting \; point \; x = \; 1 \; so \; x_1 \; is \; 1. \; We \; can \; continue \; to \; formula$

$x_2 = \; 1 \; - \; \dfrac{\frac{1}{2\sqrt{1}}}{-\frac{1}{4*1^{(3/2)}}} \; => \; x_2 \; = \; 3$

$x_3 = \; 3 - \; \dfrac{\frac{1}{2\sqrt{3}}}{-\frac{1}{4*3^{(3/2)}}} \; => which \; is \; bigger \; than \; 3 \; ...$

$So, \; always \; it \; is \; going \; bigger.$
$This \; means, \; it \; goes \; infinity \; so \; it \; has \; no \; min \; or \; max \; value.$

$And \; from \; this \; formula, \; we \; could \; not \; find \; any \; min \; value.$
$But, \; we \; now \; that \; domain \; of \; \sqrt{x} \; is \; bigger \; than \; or \; equal \; to \; 0.$

$Thus, \; if \; we \; want \; to \; find \; minimum \; value \; of \; the \; function, \; we \; need \; to \; look \; at \; its \; domain.$
$While \; it \; starts \; from \; point \; (0,0), \; this \; is \; the \; minimum \; value \; because \; every \; time \; it \; goes \; bigger.$
$So, \; the \; minimum \; value \; should \; be \; at \; x = 0 \; which \; is \; f(0) = 0$

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