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Assignment 3

COMP206

Answer to Question 1.1

$$
\begin{aligned}
&While\ f \in R\\
&F(x) = -2x_1^2 - 2x_1x_2 + 2x_2^2 + x_1 + x_2 + 3\\
&So\ that\ we\ can\ find\ the\ derivative\ as\ below\\
&x_1' \ = \ -4x_1 - 2x_2 + 1\\
&x_2' \ = \ -2x_1 - 4x_2 + 1\\
&x_1'' \ = \ -4\\
&x_2'' \ = \ -4\\
&x_1'x_2' \ = \ -2\\
&x_1'x_2' \ = \ -2 \qquad (1)\\
&We\ can\ find\ our\ hession\ matrix\ as\ : \begin{bmatrix} -4 & -2 \\ -2 & -4 \end{bmatrix}\\
&|D_1| \ = \ -4 \ < 0\\
&|D_2| \ = \ 16 - 4 \ = 12 \ > 0\\
&Thus,\ by\ definition\ that\ says\ |D_1|\ is\ less\ then\ 0\ and\ |D_2|\ is\ bigger\ than\ 0\\
&It\ means\ it\ is\ neither\ positive\ nor\ negative\ definite.\\
&So,\ it\ is\ not\ convex.\\
&Also\ we\ can\ find\ this\ solution\ with\ other\ test\ but\ it\ will\ give\ the\ same\ result.
\end{aligned}
$$

Answer to Question 1.2

$$F(x,y,z) \;=\; 2x^2 \;+xz \;+y^2 \;+2yz \;+1/2\;z^2$$

$$\begin{aligned}
&\frac{\partial^2 F}{\partial\, x^2} \;=\; 4\; \frac{\partial^2 F}{\partial\, x\, \partial y} \;=\; 0\; \frac{\partial^2 F}{\partial\, x\, \partial z} \;=\; 1\\
&\frac{\partial^2 F}{\partial\, y\, \partial x} \;=\; 0\; \frac{\partial^2 F}{\partial\, y^2} \;=\; 2\; \frac{\partial^2 F}{\partial\, y\, \partial z} \;=\; 2\\
&\frac{\partial^2 F}{\partial\, z\, \partial x} \;=\; 1\; \frac{\partial^2 F}{\partial\, z\, \partial y} \;=\; 2\; \frac{\partial^2 F}{\partial\, z^2} \;=\; 1\\
\\
&|D_1| \;=\; 4\\
&|D_2| \;=\; 8\\
&|D_3| \;=\; -10\\
\\
&Our\ Hession\ matrix\ will\ be\ \;=\; \begin{bmatrix} 4 & 0 & 1\\ 0 & 2 & 2\\ 1 & 2 & 1 \end{bmatrix}\\
&It\ is\ neither\ positive\ nor\ negative\ definite\ while\ |D_1| > 0,\ |D_2| > 0,\\
&but\ the\ last\ one\ |D_3| < 0\ thus\ it\ is\ not\ convex
\end{aligned} \tag{2}$$

Answer to Answer To Question 3

$$\begin{aligned}
&We\ can\ conclude\ while\ f\ is\ convex\ that\ is:\\
&f(\lambda y + (1-\lambda)x) \leq \lambda f(y) + (1-\lambda) f(x),\\
&\forall x \in [0,1], x, y \in domain\ of\ f\ function\ while\ \lambda\ is\ between\ 0\ and\ 1\\
\\
&So\ we\ can\ say\ that\ f(x + \lambda(y-x)) \leq f(x) + \lambda(f(y) - f(x))\\
\\
&So\ as\ \lambda\ goes\ 0\ ,\ we\ find:\ f(y) - f(x) \geq \frac{f(x+\lambda(y-x)) - f(x)}{\lambda}\\
&As\ conclusion\ we\ find\ that\ :\ f(y) - f(x) \geq \bigtriangledown f^T(x)(y-x)\\
&Thus\ if\ f\ is\ convex\ we\ can\ say\ that\ our\ proposition\ is\ true.
\end{aligned}$$

Answer to Answer To Question 4

$If\ we\ say\ that\ f\ is\ convex,\ Then\ we\ can\ include\ function\ g\ as:$

$g(y) = f(y) - \triangledown f(x)^T(y-x)$

$g\ is\ convex\ by\ the\ definition\ so\ that\ we\ can\ write:$

$\triangledown g(y) = \triangledown f(y) - \triangledown f(x)$

And

$\triangledown^2 g(y) = \triangledown^2 f(y)$

$for\ all\ y \in R^n$

$Thus\ \triangledown g(x)\ =\ 0$

$Thus,\ it\ implies\ \triangledown^2 g(x)\ is\ positive\ semi\ definite$

$In\ addition\ to\ that\ I\ found\ Taylor\ Theorem\ as\ below$

$f(y) = f(x) + \triangledown f(x)^T(y-x) + \dfrac{1}{2}(y-x)^T\triangledown^2 f(x+t(y-x))(y-x)$

$for\ some\ t\ is\ in\ 0\ and\ 1.\ Since\ f\ is\ positive\ semi\ definite\ the\ quadratic\ equation\ will\ be\ always$
$non\ negative\ Thus\ We\ will\ get$

$f(y) \geq f(x) + \triangledown f(x)^T(y-x)$

$It\ is\ what\ we\ want\ to\ prove$

(3)