assignment2_ theory.

1_1 $M_p = \int_{-\pi}^{\pi} K_\theta T_\theta T_\theta^T \, d\theta$

surface normal $\rightarrow$ $n^T T_\theta = 0 \quad \forall\theta$

product of surface normal and M_p:

$$M_p n = \int_{-\pi}^{\pi} K_\theta T_\theta \underbrace{T_\theta^T n}_{=0} \, d\theta$$

tangent and surface normal are orthonegal

$\rightarrow M_p n = \int_{-\pi}^{\pi} K_\theta T_\theta \cdot 0 \, d\theta = 0 \rightarrow M_p n = 0 \cdot n$

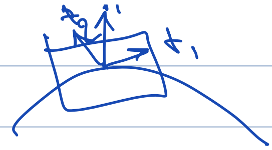
$\rightarrow$ "n" is eigenvector of the matrix M_p with eigenvalue 0.

1_2 eigenvector of M_p is aligne with maximum and minimum curvature.

$t_1, t_2 \Leftrightarrow$ unit vectors along the curvature at point p.

$M_p t_1 = K_1 t_1 \qquad M_p t_2 = K_2 t_2$

$\rightarrow K_1, K_2$ curvatures at point P. eigenvalues of matrix M_p.



$0 \to$ eigen value n (surface normal)
$k \to$ maximum curvature direction
$k_2' \to$ minimum curvature direction

(2-1) regular plane curve $\gamma : [0,1] \to R^2$ connect point A and B with $\gamma(0) = A$, $\gamma(1) = B$

$$S[\gamma(t)] = \int_0^1 \|\gamma'(t)\|_2 dt$$

$$y_h(t) = \gamma(t) + h v(t) \qquad v(0) = 0, \; v(1) = 0$$

arc length functional for y_h:

$$S[y_h] = \int_0^1 \|y_h'(t)\|_2 dt = \int_0^1 \|y(t) + h v(t)'\|_2 dt$$

$\hookrightarrow y_h'(t) = y'(t) + h v'(t)$

$$\Rightarrow S[y_h] = \int_0^1 \|y'(t) + h v'(t)\|_2 dt$$

$$dS[y](v) = \frac{d}{dh} S[y_h]\Big|_{h=0}$$

$\hookrightarrow$ gateaux derivative in direction v.

$$\frac{d}{dh} S[y_h] = \int_0^1 \frac{d}{dh} \|y'(t) + hv'(t)\|_2 \, dt$$

$$\frac{d}{dh} \|y'(t) + hv'(t)\|_2 = \frac{(y'(t) + hv'(t)) \cdot v'(t)}{\|y'(t) + hv'(t)\|_2}$$

$$\xrightarrow{h=0} \frac{d}{dt} \|y'(t) + hv'(t)\|_2 \Big|_{h=0} = \frac{y'(t) \cdot v'(t)}{\|y'(t)\|_2}$$

$$\rightarrow dS[y](v) = \int_0^1 \frac{y'(t) \cdot v'(t)}{\|y'(t)\|_2} \, dt = 0 \quad \forall v(t)$$

using boundry Conditions. $v(0)=0$, $v(1)=0$

$$\int_0^1 \frac{y'(t) \cdot v'(t)}{\|y'(t)\|_2} \, dt = -\int_0^1 \left(\frac{d}{dt} \left(\frac{y'(t)}{\|y'(t)\|_2} \right) \right) \cdot v(t) \, dt = 0$$

it has to go for all $v(t)$ so it has to follow

$$\frac{d}{dt} \left(\frac{y'(t)}{\|y'(t)\|_2} \right) = 0$$

$$\longrightarrow \frac{y(t)}{\|y'(t)\|_2} \rightarrow \text{constent} \rightarrow y'(t) = \text{Constant vector}$$

$\rightarrow$ curve $y(t)$ is a straight line.

2.2 $S(t) = \int_0^1 \|y'(x)\|_2 \, dx$

→ using (S) instead of (t), to make the curve move at a unit speed along the arc length.

→ $S[y(s)] = \int_0^L \|y'(s)\|_2 \, ds$ L: total length of the curve

parameterized by arc length, $\|y'(s)\|_2$ is always $= 1$ cause the rate of change with respect to arc length is constant for unit speed parameterization.

$$S[y(s)] = \int_0^L 1 \, ds = L$$

Gateaux derivative is used to find how a functional changes when we perturb the curve slightly in a particular direction.

in the original form it depended on the norm of the derivative $\|y'(t)\|_2$ along the curve it was different.

with arc length param:

$\|y'(s)\|_2 = 1$ → when calculating the Gateaux derivative, there is no dependence on the changes in speed or the magnitude of $y'(t)$ it just depends on the geometry of the curve rather than how it is parameterized.

2.3

arc length $S[y] = \int_0^1 \|y'(t)\|_2 dt$

$y_h(t) = y(t) + hv(t)$

$v(0) = 0 \quad , \quad v(1) = 0$

Gateaux derivative:

$$dS[y](v) = \frac{d}{dh} S[y+hv]\Big|_{h=0}$$

$\hookrightarrow S[y_h] = \int_0^1 \|y'(t) + hv'(t)\|_2 dt$

$$\frac{d}{dh} S[y_h] = \int_0^1 \frac{d}{dh} \|y'(t) + hv'(t)\|_2 \, dt$$

Chain rule $\longrightarrow$ $$\frac{d}{dh}\|y'(t)+hv'(t)\|_2 = \frac{(y'(t)+hv'(t)) \cdot v'(t)}{\|y'(t)+hv'(t)\|_2}$$

$\xrightarrow{h=0}$ $$\left.\frac{d}{dh}\|y'(t)+hv'(t)\|_2\right|_{h=0} = \frac{y'(t) \cdot v'(t)}{\|y'(t)\|_2}$$

$$\rightarrow dS[y](v) = \int_0^1 \frac{y'(t) \cdot v'(t)}{\|y'(t)\|_2} \, dt$$

y = shortest $\longrightarrow$ gateaux $= 0$

$$\int_0^1 \frac{y'(t) \cdot v'(t)}{\|y'(t)\|_2} \, dt = 0$$

integration by parts:

$$\int_0^1 u'(t) v(t) \, dt = [u(t) v(t)]_0^1 - \int_0^1 u(t) v'(t) dt$$

$$u(t) = \frac{y'(t)}{\|y'(t)\|_2}$$

$$\Rightarrow \int_0^1 \frac{y'(t)\cdot v'(t)}{\|y'(t)\|_2}\,dt = \left[\frac{y'(t)}{\|y'(t)\|_2}\cdot v(t)\right]_0^1 - \int_0^1 \frac{d}{dt}\left(\frac{y'(t)}{\|y'(t)\|_2}\right)\cdot v(t)\,dt$$

$$v(0)=v(1)=0 \rightarrow \left[\frac{y'(t)}{\|y'(t)\|_2}\cdot v(t)\right]_0^1 = 0$$

$$\rightarrow \int_0^1 \frac{y'(t)\cdot v'(t)}{\|y'(t)\|_2}\,dt = -\int_0^1 \frac{d}{dt}\left(\frac{y'(t)}{\|y'(t)\|_2}\right) v(t)\,dt = 0$$

it has to follow $\frac{d}{dt}\left(\frac{y'(t)}{\|y'(t)\|_2}\right) = 0$

$\frac{y'(t)}{\|y'(t)\|_2}$ constant $\rightarrow$ $y'(t)$ constant vector $\rightarrow$ curve straight line.