

MID-SEMESTER EXAMINATION, OCTOBER-2018

CALCULUS-I (MTH - 1001)

Semester: 1st Semester
Full marks: 30

Branch: ALL (Except M.E.)
Time: 2 Hours

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Use limit laws to evaluate the limit of a function and demonstrate the existence of limit and continuity of functions .	L1,L1,L3	1.a,b,c	2,2,2
Compute slope of tangent lines and derivatives by different techniques of functions and solve various physical and Engineering problems .	L1,L3,L3 L1,L1,L3	2.a,b,c 3.a,b,c	2,2,2 2,2,2
Discuss the Mean Value Theorems and study maximum and minimum values of a function as well as apply L' Hospital's rule to evaluate limits of functions and sketch curves of functions	L3,L3,L3	4.a,b,c	2,2,2
Compute indefinite integrals using techniques of integration and apply it to physical and Engineering problems	L1,L3	5.a,b	2,2
Apply the concept of integration to find volume, work done, surface area and average value of an integral and study numerical integration using different methods.	L3	5.c	2

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1.(a) Find $\lim_{x \rightarrow -6} \frac{2x+12}{|x+6|}$, if it exists. If the limit does not exist, explain why. 2

(b) Find $\lim_{x \rightarrow 3^+} (x^2 - 9)$, if it exists and also find the vertical asymptote of the function $f(x) = \ln(x^2 - 9)$. 2

- (c) If a rock is thrown upward on the planet Mars with a velocity of 10 m/s , its height in meters t seconds later is given by $h(t) = 10t - 1.86t^2$ then compute the average velocity for the time period beginning when $t = 1.5$ and lasting i) 0.5 seconds, ii) 0.1 seconds, iii) 0.05 seconds. 2

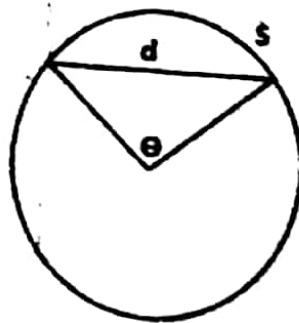
- 2(a) Find the derivative of the function $f(t) = \frac{1-2t}{3+t}$ using the definition of derivative and also state the domain of both the function and its derivative. 2

- (b) Compute $\frac{d^{99}}{dx^{99}}(\sin x)$ by finding the first few derivatives and observing the pattern that occurs. 2

- (c) Use the ϵ, δ definition of limit to prove the statement $\lim_{x \rightarrow a} c = c$. 2

- 3.(a) A freshly brewed cup of coffee has temperature 95°C in a 20°C room. When its temperature is 70°C , it is cooling at a rate of 1°C per minute. Find the time when this occurs. 2

- (b) The following figure shows a circular arc of length s and a chord of length d , both subtended by a central angle θ . Find $\lim_{\theta \rightarrow 0} \frac{s}{d}$ 2



- (c) Use implicit differentiation to compute an equation of the tangent line to the curve $x^2 + xy + y^2 = 3$ at the point $(1, 1)$. 2

on ~~$\sin(x+y) = 2x$~~
 $x^{2/3} + y^{2/3} = 4, (-3\sqrt{3}, 1)$
 on $x^3 + y^3 = 6xy, (3, 3)$

4 (a)

Compute $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{1 - \sin x}$ using L'Hospital's rule.

2

(b)

Apply closed interval method to determine the absolute maximum and absolute minimum values of the function $f(x) = x^3 - 3x + 1$ in the interval $[0, 3]$.

2

(c)

Use Newton's method to approximate the root of the equation $(x - 2)^2 = \ln x$ in the interval $[1, 2]$ correct to 3 decimal places.

2

5.(a) Find the area of the region enclosed by the curves $y = 12 - x^2$, $y = x^2 - 6$.

2

(b)

Use the Fundamental Theorem of Calculus to find the derivative of the function $g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1} du$.

2

(c) Compute the upper and lower sums for $f(x) = 1 + x^2$, $-1 \leq x \leq 1$, with $n = 4$.

2

$\lim_{n \rightarrow 1} \frac{2+4n}{2} = 2$ $\lim_{n \rightarrow -2} (\frac{1}{2}n+3) = 2$
 $\lim_{n \rightarrow 0} \sqrt{n} = 0$ $\lim_{n \rightarrow 3} (4n-5) = 7$
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9. The half-life of cerium-137 is 30 years. Suppose we have a 100-mg sample.
- Find the mass that remains after t years
 - How much of the sample remains after 100 years?
 - After how long will only 1 mg. remain?
10. When a cold drink is taken from a refrigerator, it's temp. is 5°C . After 25 minutes in a 20°C room it's temp. has increased to 10°C .
- What is the temp. of the drink after 50 minutes?
 - When will it's temp. be 15°C ?
11. Find $\frac{dy}{dx}$ if $x^y = y^x$ (b), $y = (\tan x)^{\ln x}$
- Find f' & $\lim_{x \rightarrow 0} f$ if $f(x) = \ln \ln \ln x$
12. Where does the normal line to the ellipse $x^2 - xy + y^2 = 3$ at the pt. $(-1, 1)$ intersect the ellipse a second time?
13. Find all pts on the curve $xy^2 + xy = 2$ where the slope of the tangent line is -1 .
14. If $h(x) = \sqrt{4 + 3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.
15. A semicircle with diameter PA sits on an isosceles $\triangle PQR$ to form a region shaped like a two dimensional ice-cream cone, as shown in the figure. If $A(\theta)$ is the area of the semicircle and $B(\theta)$ is the area of the \triangle , find $\lim_{\theta \rightarrow 0} \frac{A(\theta)}{B(\theta)}$
16. Show that the func $f(x) = |x-6|$ is not differentiable at 6 . Find a formula for f' and sketch the graph
17. $\lim_{n \rightarrow \infty} (x + \sqrt{x^2 + 2x})$