END SEMESTER EXAMINATION, DECEMBER-2015
CALCULUS-I (MTH-1001)

Programme: B. Tech **Semester: 1st**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Use limit laws and Squeeze Theorem to evaluate the limit of a function and analyse it and demonstrate the existence of limit and continuity of functions by ε, δ approach. CO-1	(L3) (L3) (L4)	1(a), 1(b) 1(c)	2 2 2
Compute slope of tangent lines using numerical data and compute derivatives by different techniques such as product and quotient rules, logarithmic differentiation and implicit differentiation. CO-2	(L3)	2(a)	2
Interpret derivative as rate of change to find tangents and velocities and comply rates of change in time and distance problem, Newton's law of cooling problem, radioactive decay problem, profit loss problem, marginal revenue problem. CO-3	(L3) (L3)	2(b) 2(c)	2 2
Applications of differentiation to the Mean Value Theorems and to study maximum and minimum values of a function. CO-4	(L2) (L3) (L3)	3(a) 3(b) 3(c)	2 2 2
Apply L' Hospital's rule to evaluate limits of functions in indeterminate forms. Sketch curves of functions with the knowledge of domain, intercepts, maxima and minima, concavity.CO-5	(L3) (L3) (L3) (L3)	4(a) 4(b) 4(c) 5(c)	2 2 2 2
Compute indefinite integrals using techniques of integration. Apply The Net Change Theorem to find the mass of a segment of a rod, change in volume of water in a reservoir, change in concentration of the product of a chemical reaction, change of the population in a given time interval CO-6	(L3) (L3) (L3) (L3) (L3)	5(a) 5(b) 6(a) 6(b) 6(c)	2 2 2 2 2
Apply the concept of integration to find volume, work done, surface area and average value of an integral.CO-7	(L3) (L3) (L3) (L3)	7(a) 7(b) 7(c) 8(c)	2 2 2 2
Use mid-point, trapezoidal and Simpson's rule to compute approximate value of integrals and compute improper integrals and study their convergence.CO-8	(L3) (L3)	8(a) 8(b)	2 2
Apply the concepts of integration to Physics and Engineering CO-9	(L2) (L3) (L3)	9(a) 9(b) 9(c)	2 2 2
Apply the principles of calculus to study and calculate areas, arc lengths etc. of parametric and polar curves.CO-10	(L1) (L3)	10(a) 10(b)	2 2
Analyze infinite series and sequences and discuss their convergences using comparison test, root test and ratio test CO-11	(L4)	10(c)	2

Answer all questions. Each question carries equal mark.
Answer all three bits of particular questions in one place only.

1. (a) Compute $\lim_{x\to 4} f(x)$ If $4x-9 \le f(x) \le x^2-4x+7$ for $x \ge 0$. 2

   (b) Apply ε,δ definition of limit to prove that $\lim_{x\to -2}(3x+5)=-1$. 2

   (c) Examine the continuity of the function 2

$$f(x)=\begin{cases}\dfrac{1}{x+2} & \text{if } x\neq -2\\ 1 & \text{if } x=-2\end{cases} \text{ at a=-2}$$

2. (a) Use $g(x)=xf(x)$, where $f(3)=4$ and $f'(3)=-2$, to find the tangent line to g(x) at point $x=3$. 2

   (b) Compute the linear approximation of the function $g(x)=\sqrt[3]{1+x}$. Use it to approximate $\sqrt[3]{1.1}$. 2

   (c) A bottle of soda pop at room temperature 22°C is placed in a refrigerator where the temperature is 7°C. After half an hour the soda pop has cooled to 16°C. Compute the temperature of the soda pop after another half an hour. 2

3. (a) Identify the intervals on which $f(x)=2x^3-3x^2-12x$ is increasing. 2

   (b) Show that the function $f(x)=x^3-x^2-6x+2$ satisfies Rolle's theorem on the interval [0,3]. 2

   (c) Use the concept of derivatives to find local maximum and minimum values for the function $h(x)=5x^3-3x^5$. 2

4. (a) Use L'Hospital rule to calculate $\lim_{x\to\infty} x^3e^{-x^2}$. 2

   (b) Apply Newton's method to compute $\sqrt[6]{2}$ correct to two decimal places. 2

   (c) Sketch the curve $y=x^3+x$. 2

5. (a) Compute the derivative of the function $g(r) = \int_0^r \sqrt{x^2+4}\,dx$. 2

(b) If f is continuous and $\int_0^4 f(x)dx = 10$, compute $\int_0^2 f(2x)dx$. 2

(c) Show that $2 \le \int_{-1}^{1} \sqrt{1+x^2}\,dx \le 2\sqrt{2}$. 2

6. (a) Compute average value of the function $g(x) = \sqrt[3]{x}$ on the interval [1, 8]. 2

(b) Compute the work done in moving a particle from $x=1$ to $x=2$ if a force of $\cos(\pi x/3)$ Newton acts on the particle when it is located at a distance of x meters from origin. 2

(c) Use the method of cylindrical shells to find the volume generated by rotating a region bounded by the curves $y = x^3, y = 8, x = 0$ about x-axis. 2

7. (a) Compute the integral $\int_1^3 r^3 \ln r\,dr$. 2

(b) Apply Simpson's rule to approximate the integral $\int_1^4 \sqrt{\ln x}\,dx$ with n=2. 2

(c) Show that the integral $\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}dx$ is divergent. 2

8. (a) Estimate M_x and M_y and the center of mass of the system with masses $m_1 = 6$, $m_2 = 5$ and $m_3 = 10$ which are located at the points $p_1(1,5)$, $p_2(3,-2)$ and $p_3(-2,-1)$ respectively. 2

(b) Compute the length of the curve $y = 2x-5$, $-1 \le x \le 3$. 2

(c) Show that the centroid of the region bounded by the curves $y = x^2$, $x = y^2$ is (9/20,9/20). 2

9. (a) Compute the area of the region that is bounded by the curve $r = e^{\theta/2}$ lies in the sector $\pi \leq \theta \leq 2\pi$. 2

(b) Convert the parametric curve $x = \frac{1}{2}\cos\theta, y = 2\sin\theta$, $0 \leq \theta \leq \pi$ to its Cartesian form. 2

(c) Compute the equation of the tangent line to the curve $x = 1 + \sqrt{t}$, $y = e^{t^2}$ at $(2,e)$. 2

10 (a) List the first four partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ correct to three decimal places. 2

(b) Write the Maclaurian series expansion of the function $f(x) = \ln(1 + x)$. 2

(c) Examine the series $\sum_{n=1}^{\infty} \frac{n}{5^n}$ for absolute convergence and conditional convergence. 2

End of Questions