Chapter 12

Fluid Mechanics

Topic	Conceptual Problems (To be Discussed in Class)	In Class Problems	Assignment Problems
12.4 Fluid Flow 12.5 Bernoulli's Equation	TYU- 12.5	Example- 12.9	Exercise 12.41, 12.43, 12.44

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12.4 Fluid Flow

Flow line: The path of an individual particle in a moving fluid is called a **flow line.**

Steady flow: If the overall flow pattern does not change with time, the flow is called **steady flow.** In steady flow, every element passing through a given point follows the same flow line. In this case the "map" of the fluid velocities at various points in space remains constant, although the velocity of a particular particle may change in both magnitude and direction during its motion.

Streamline: A **streamline** is a curve whose tangent at any point is in the direction of the fluid velocity at that point. When the flow pattern changes with time, the streamlines do not coincide with the flow lines. For steady-flow situations, the flow lines and streamlines are identical.

Flow tube: The flow lines passing through the edge of an imaginary element of area, form a tube called a **flow tube.** In steady flow no fluid can cross the side walls of a flow tube; the fluids in different flow tubes cannot mix.

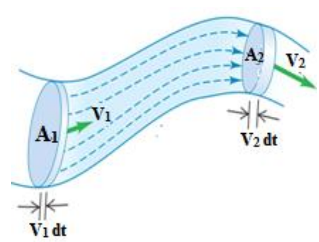
Laminar flow: In **laminar flow,** the adjacent layers of fluid slide smoothly past each other and the flow is steady. (A *lamina* is a thin sheet.)

Turbulent flow: At sufficiently high flow rates, or when boundary surfaces cause abrupt changes in velocity, the flow can become irregular and chaotic. This is called **turbulent flow** (Fig. 12.20). In turbulent flow there is no steady-state pattern; the flow pattern changes continuously.

Continuity Equation:

The mass of a moving fluid doesn't change as it flows. This leads to an important quantitative relationship called the **continuity equation.**

Let us consider an incompressible fluid flowing through a non-uniform pipe.



The distance (ds_1) moved by the fluid during a small time interval dt, at A_1 is, $ds_1 = v_1\,dt$

Volume flow rate (the rate at which volume crosses a section of the tube) at A_1 is

$$\text{Volume flow rate at } A_1 = \frac{dV_1}{dt} = \frac{A_1\, v_1\, dt}{dt} = A_1\, v_1$$

Mass flow rate (the mass flow rate is the mass of fluid flow per unit time) at A_1 is:

$$\text{Mass flow rate at } A_1 = \frac{dm_1}{dt} = \frac{\rho\, dV_1}{dt} = \rho\, A_1\, v_1$$

Now, mass flow rate at $A_2 = \frac{dm_2}{dt} = \frac{\rho\, dV_2}{dt}\ \frac{\rho\, A_2\, v_2\, dt}{dt} = \rho\, A_2\, v_2$

In steady flow the mass flow rate in the tube is constant i.e $dm_1 = dm_2$

$$\frac{dm_1}{dt} = \frac{dm_2}{dt}$$

$\Rightarrow \quad \rho\,(A_1 v_1) = \rho\,(A_2 v_2)$

$\Rightarrow \quad A_1 v_1 = A_2 v_2$

$\Rightarrow \quad A\, v = \text{constant}$

This is known as continuity equation for incompressible fluid. This is also known as the conservation of mass equation.

According to the continuity equation, the speed of fluid flow can vary along the paths of the fluid. Smaller the area of cross-section, larger will be the speed of the fluid and vive versa.

For compressible fluid, the density of fluid at both the ends of the tube are not same, i.e

$\rho_1 \neq \rho_2$

But the mass flow rate will be same. So,

$$\frac{dm_1}{dt} = \frac{dm_2}{dt}$$

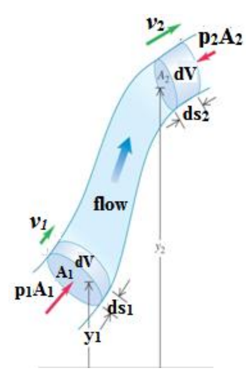
$\rho_1\,(A_1 v_1) = \rho_2\,(A_2 v_2)$

$\Rightarrow \quad \rho\, A\, v = \text{constant.}$

This is known as continuity equation for compressible fluid.

12.5 Bernoulli's Equation

Bernoulli's equation states that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow. It is similar to work-kinetic energy theorem.



$$\frac{\text{Work done}}{\text{Volume}} = \frac{\text{Change in K.E}}{\text{Volume}} + \frac{\text{Change in P.E}}{\text{Volume}}$$

Let us consider a fluid element entered into the tube through 1st end in time dt.
Distance moved by the fluid at end '1' in the time interval *dt*, is
$ds_1 = v_1\, dt$
Distance moved by the fluid at end '2' in the same time interval dt, is
$ds_2 = v_2\, dt$
The fluid is incompressible; hence by the continuity equation, the volume of fluid dV passing any crosses section during time dt is the same.
So, $dV = A_1\, ds_1 = A_2\, ds_2$

A_1= Area of cross-section at end '1'

A_2= Area of cross-section at end '2'

Force on the cross section at end '1' is

$F_1 = p_1 A_1$

Let dW_1 = work done on the fluid element during dt at end '1' and

dW_2 = work done on the fluid element during dt at the end '2'

Now the net work done (dW) on the element by the surrounding fluid during this displacement is

$dW = dW_1 + dW_2$

$$\Rightarrow \quad dW = p_1 A_1 ds_1 + \left(-p_2 A_2 ds_2\right)$$
$$\Rightarrow \quad dW = p_1 dV - p_2 dV$$
$$\Rightarrow \quad dW = \left(p_1 - p_2\right) dV$$

The second term has a negative sign because the force at end '2' opposes the displacement of the fluid.

The change in kinetic energy (dK) due to the motion of the fluid element is

$dK = dK_2 - dK_1$

$$\Rightarrow \quad dK = \frac{1}{2} m\, v_2^2 - \frac{1}{2} m\, v_1^2$$

$$\Rightarrow \quad dK = \frac{1}{2}(\rho\, dV) v_2^2 - \frac{1}{2}(\rho\, dV) v_1^2$$

$$\Rightarrow \quad dK = \frac{1}{2}\rho\, dV\left(v_2^2 - v_1^2\right)$$

The change in potential energy (dU) due to height of the fluid element is

$dU = dU_2 - dU_1$

$$\Rightarrow \quad dU = mgy_2 - mgy_1$$
$$\Rightarrow \quad dU = mg\left(y_2 - y_1\right)$$
$$\Rightarrow \quad dU = \rho\ dV\, g\left(y_2 - y_1\right)$$

We know that the work done is the total mechanical energy. So,

$$dW = dU + dK$$

$$\Rightarrow \quad (p_1 - p_2) dV = \rho\, dV\, g\, (y_2 - y_1) + \frac{1}{2}\rho\, dV\, (v_2^2 - v_1^2)$$

$$\Rightarrow \quad (p_1 - p_2) = \rho\ g\, (y_2 - y_1) + \frac{1}{2}\rho\ (v_2^2 - v_1^2)$$

$$\Rightarrow \quad p_1 + \rho\ g\, y_1 + \frac{1}{2}\rho\, v_1^2 = p_2 + \rho\ g\, y_2 + \frac{1}{2}\rho\, v_2^2$$

$$\Rightarrow \quad p + \rho\, g\, y + \frac{1}{2}\rho\, v^2 = \text{constant}$$

This is Bernoulli's equation

Case-I Horizontal Flow of Fluid

For horizontal flow, we have, y = constant.
So, Bernoulli's equation will be

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

From this equation it is clear that, the pressure of fast moving fluid decreases and vice versa.

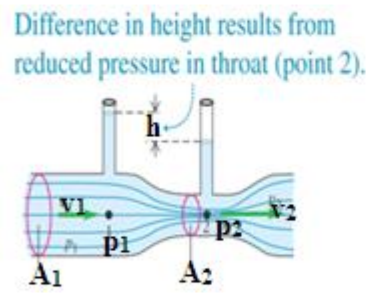
Case-II Fluid not Flowing

Here, $v_1 = v_2 = 0$
So, Bernoulli's equation will be

$$p + \rho g y = \text{constant}$$

Venturi meter: (Example - 12.9):

Figure shows a Venturi meter device, used to measure flow speed (v_1) in a pipe in terms of the cross-sectional areas A_1 and A_2 and the difference in height 'h' of the liquid levels in the two vertical tubes.
The flow is steady, and we assume the fluid is incompressible and has negligible internal friction. Hence we can use Bernoulli's equation. We apply that equation to the wide part (point 1) and narrow part (point 2, the throat) of the pipe.



Points 1 and 2 have the same vertical coordinate $y_1 = y_2$
Applying Bournoulli's equation to points 1 and 2:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho\left[(A_1/A_2)v_1\right]^2 \qquad [\because A_1 v_1 = A_2 v_2]$$

$$p_1 - p_2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

But $p_1 - p_2 = \rho g h$

Thus $\rho g h = \frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$

$$v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$$

Thus, A_1, A_2 and h we can determine the speed of the fluid.

Again, as $A_1 > A_2$, so, $v_2 > v_1$ (according to the continuity equation)

=> $p_1 > p_2$ (according to the Bernoulli's equation for horizontal flow)

This pressure differences ($p_1 - p_2$), produce a net force to the right that makes the fluid speed up as it enters the throat.

Test Your Understanding of Section 12.5

Which is the most accurate statement of Bernoulli's principle?

(i) Fast-moving air causes lower pressure;
(ii) lower pressure causes fast-moving air;
(iii) both (i) and (ii) are equally

Answer: (ii)
Newton's second law tells us that a body accelerates (its velocity changes) in response to a net force. In fluid flow, a pressure difference between two points means that fluid particles moving between those two points experience a force, and this force causes the fluid particles to accelerate and change speed.

Example Problems: 12.9

A Venturimeter, used to measure flow speed in a pipe. Derive an expression for the flow speed in terms of the cross-sectional areas A_1 and A_2 and the difference in height h of the liquid levels in the two vertical tubes.
Solution: Page-9

Assignment Problems: 12.41, 12.43, 12.44

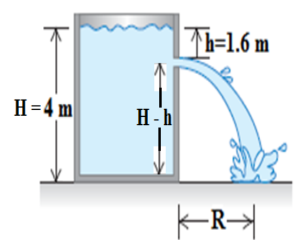
Exercise Problems: 12.41

Water stands at a depth of 4 m in a tank, whose sides are vertical. A hole is made at one of the walls, 1.6 m below the water level. Calculate the: (a) velocity of efflux (b) range of emerging water stream. (g = 9.8 m/s^2).

Solution:

Speed of efflux = $v = \sqrt{2\,g\,h}$

$$\Rightarrow \quad v = \sqrt{2\left(9.8\ \text{m/s}^2\right)(1.6\ \text{m})} = 5.6\ \text{m/s}$$



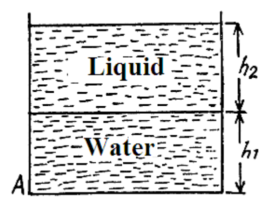
The time taken by any portion of the water to reach the ground is

$$t = \sqrt{\frac{2H'}{g}} = \sqrt{\frac{2(H-h)}{g}} = \sqrt{\frac{2(4\ \text{m} - 1.6\ \text{m})}{9.8\ \text{m/s}^2}} = 0.6999\ \text{s}$$

The time taken by any portion of the water to reach the ground is

$$R = v\,t = (5.6\ \text{m/s})(0.6999\ \text{s}) = 3.9194\ \text{m}$$

Exercise Problems: 12.43



A wide vessel with a small hole at the bottom is filled with water (density 1000 kg/m^3) for a height of 2.4m, and then a liquid (density 800 kg/m^3) for a height 1.2 m. Calculate the speed with which water will be flowing out (g = 9.8 m/s^2).

Solution:

Pressure due to water level = $h_1\rho_1 g$
Pressure due to liquid = $h_2\rho_2 g$.
So, net pressure = $h_1\rho_1 g + h_2\rho_2 g$.
From Bernoulli's theorem, this pressure energy will be converted into kinetic energy while flowing through the whole 'A'

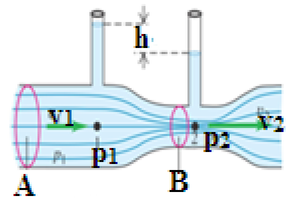
$$\rho_1 g\, h_1 + \rho_2 g\, h_2 = \frac{1}{2}\rho_1 v^2$$

$$v = \sqrt{\frac{2\left(\rho_1 h_1 + \rho_2 h_2\right) g}{\rho_1}} = \sqrt{\frac{2\left[\left(1000\ \text{kg/m}^3\right)(2.4\text{m}) + \left(800\ \text{kg/m}^3\right)(1.2\text{m})\right]\left(9.8\text{m/s}^2\right)}{\left(1000\ \text{kg/m}^3\right)}} = 8.1152\text{m/s}$$

Exercise Problems: 12.44

A pipe is running full of water. At a point A, its diameter is 55 cm, from where it tapers to 15 cm diameter at B. The pressure difference between A and B is 98 cm of water column. Find the rate of flow of water through the pipe.

Solution:



$d_1 = 55\text{cm} = 55\text{x}10^{-2}\text{m}$, $d_2 = 15\ \text{cm} = 15\text{x}10^{-2}\text{m}$, $h = 98\text{cm} = 98\text{x}10^{-2}\text{m}$

$$A_1 = \frac{\pi d_1^2}{4} \qquad A_2 = \frac{\pi d_2^2}{4}$$

$$\frac{A_1}{A_2} = \frac{\pi d_1^2/4}{\pi d_2^2/4} = \frac{d_1^2}{d_2^2} = \frac{\left(55\text{x}10^{-2}\text{m}\right)^2}{\left(15\text{x}10^{-2}\text{m}\right)^2} = 13.4444$$

$$v_1 = \sqrt{\frac{2g\,h}{\left(A_1/A_2\right)^2 - 1}} = \sqrt{\frac{2\left(9.8\text{ms}^{-2}\right)\left(98\text{x}10^{-2}\text{m}\right)}{(13.4444)^2 - 1}} = \sqrt{\frac{19.208}{179.7531}} = 0.3269\text{ms}^{-1}$$

$$\frac{dV}{dt} = \frac{A_1\ v_1\ dt}{dt} = A_1\ v_1 = \left(\frac{\pi d_1^2}{4}\right)v_1 = \frac{\pi d_1^2 v_1}{4} = \frac{\pi\left(55\text{x}10^{-2}\text{m}\right)^2\left(0.32727\text{ms}^{-1}\right)}{4} = 7.7762\text{x}10^{-2}\text{m}^3/\text{s}$$