Chapter 8

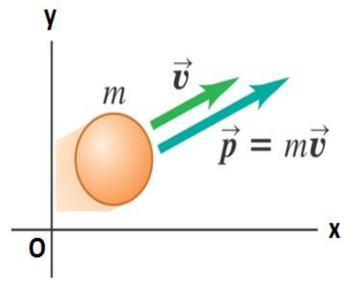
Momentum, Impulse and Collisions

Topic	Conceptual Problems (To be Discussed in Class)	In Class Problems	Assignment Problems
8.1 Momentum and Impulse **8.2** Conservation of Momentum	TYU-8.2	Example- 8.4	Exercise 8.5, 8.18, 8.33, 8.47,
8.3 Momentum Conservation and Collisions **8.4** Elastic Collisions	TYU-8.3	Example- 8.9	

Dr. Rajanikanta Parida
Associate Professor
Department of Physics, ITER,
Siksha 'O' Anusandhan Deemed to be University
rajanikantaparida@soa.ac.in

8.1 Momentum and Newton's second law

The momentum of a particle is the product of its mass and its velocity: $\vec{p} = m\,\vec{v}$



- Momentum **p** is a vector quantity. The momentum of the particle has the same direction as its velocity **v**.
- Momentum can be written from Newton's second law as:

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

- According to the equation we get:
  * A rapid change in momentum requires a large net force.
  * while a gradual change in momentum requires less net force.
  * This principle is used in the design of automobile safety devices such as air bags.

Example: What is the momentum of a 1000kg car going 25 m/s west?

Ans.: **p** = m**v** = (1000 kg)(25 m/s) = 25,000 kg.m/s **west**

Impulse and momentum

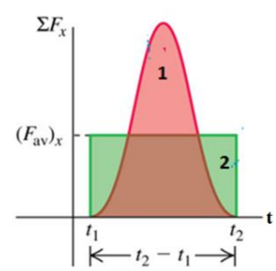
- Impulse of a force is the product of (force) & (time interval) during which it acts.

$$\vec{J} = \sum \vec{F}\,(t_2 - t_1) = \sum \vec{F}\,\Delta t \quad (\text{Assuming } \sum \vec{F} = \text{constant})$$

$$\Rightarrow \quad \vec{J} = \left(\frac{\Delta\vec{p}}{\Delta t}\right)\Delta t \quad (\text{According to Newton's 2nd law of motion})$$

$$\Rightarrow \quad \vec{J} = \Delta\vec{p} = \vec{p}_2 - \vec{p}_1 \quad \textbf{(Impulse–momentum theorem)}$$

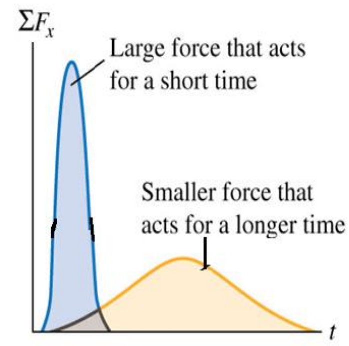
- **Impulse (= Change in momentum)** of particle during time interval equals net force acting on particle during interval
- Impulse is a vector. SI unit of impulse is N.s
- Area under the ΣF_x versus time curve, represents impulse of the particle.
- For a variable force, the impulse of the particle is represented by the curve **1**.



$$J_x = \int_{t_1}^{t_2} \sum F_x \, \Delta t = \text{Area of curve 1}.$$

- For a constant force, the impulse of the particle is represented by the curve **2**.

  Area = J_x = $(F_{av\text{-}x})$ $(t_2 - t_1)$ **Curve 2**



If the area under both curves is the same, then both forces deliver the same impulse.

In 1st case large force is applied for a short time

In 2nd case small force is applied for more time.

Distinction between momentum and kinetic energy.

Which will be easier to catch?

- A 0.50-kg ball moving at 4 m/s
- A 0.10-kg ball moving at 20 m/s

Ans.: p_{B1} = m v = (0.50-kg)(4 m/s) = 2.0 kg . m/s

p_{B2} = m v = (0.10-kg)(20 m/s) = 2.0 kg . m/s

Both the balls have the same magnitude of momentum,

$K_{B1} = \frac{1}{2}\ m v^2 = \frac{1}{2}\ (0.50\text{-kg})\ (4\ \text{m/s})^2 = 4.0\ \text{J}$

$K_{B2} = \frac{1}{2}\ m v^2 = \frac{1}{2}\ (0.10\text{-kg})(20\ \text{m/s})^2 = 20.0\ \text{J}$

Thus two balls have different values of kinetic energy.

Since the momentum is the same for both balls, both require the same impulse to be brought to rest.

But stopping the 0.10-kg ball with your hand requires five times more work than stopping the 0.50-kg ball because the smaller ball has five times more kinetic energy.

8.2 Conservation of momentum:

For isolated system the total momentum is conserved.

For isolated system

$$\sum F = 0 \quad \Rightarrow \frac{dp}{dt} = 0 \quad \Rightarrow p = \text{constant}$$

8.3: Collisions

Inelastic Collision

In inelastic Collision the momentum remains conserved but the total kinetic energy after the collision is less than before the collision.

Momentum is conserved i.e $(P_A + P_B)_{\text{after collision}} = (P_A + P_B)_{\text{before collision}}$

Kinetic energy decreases i.e $(K_A + K_B)_{\text{after collision}} < (K_A + K_B)_{\text{before collision}}$

Completely Inelastic Collision

In completely inelastic collision, the colliding bodies stick together and move as one body after the collision.

From the conservation of momentum we have

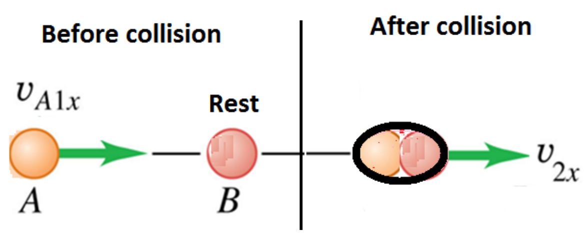
$$(\text{Total momentum})_{\text{before collision}} = (\text{Total momentum})_{\text{after collision}}$$

$$(p_A + p_B)_{\text{before collision}} = (p_A + p_B)_{\text{after collision}}$$

$$m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B) v_{2x}$$

Let the 2nd body is at rest. So, $v_{B1x} = 0$

So, $m_A v_{A1x} = (m_A + m_B) v_{2x}$



$$\Rightarrow v_{2x} = \left(\frac{m_A}{m_A + m_B}\right) v_{A1x}$$

Now, the kinetic energy are

$$(\text{Total K.E})_{\text{before collision}} = K_1 = \frac{1}{2} m_A v_{A1x}^2 + 0 = \frac{1}{2} m_A v_{A1x}^2$$

$$(\text{Total K.E})_{\text{after collision}} = K_2 = \frac{1}{2}(m_A + m_B) v_{2x}^2$$

$$\Rightarrow K_2 = \frac{1}{2}(m_A + m_B)\left[\left(\frac{m_A}{m_A + m_B}\right) v_{A1x}\right]^2 = \frac{1}{2}\left(\frac{m_A^2}{m_A + m_B}\right) v_{A1x}^2$$

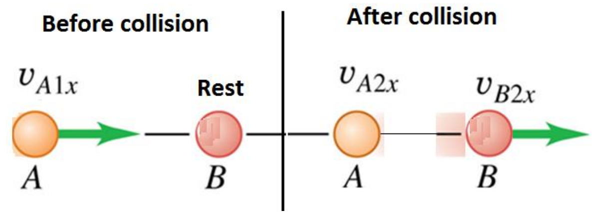
$$\Rightarrow K_2 = \left(\frac{m_A}{m_A + m_B}\right)\left(\frac{1}{2} m_A v_{A1x}^2\right) = \left(\frac{m_A}{m_A + m_B}\right) K_1$$

$$\Rightarrow \frac{K_2}{K_1} = \frac{m_A}{m_A + m_B} \quad \Rightarrow \frac{K_2}{K_1} < 1 \quad \left[\because m_A < (m_A + m_B)\right]$$

Thus the kinetic energy after a completely inelastic collision is always less than the before. So, the kinetic energy is always lost in a completely inelastic collision.

8.4 Elastic collisions

Let body A is moving along x-direction and body B is at rest



i.e $v_{B1x} = 0$

In an elastic collision, the total momentum of the system remains constant:

Total momentum before collision = Total momentum after collision

$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$

$m_B v_{B2x} = m_A (v_{A1x} - v_{A2x})$ -------(1)

In an elastic collision, the total kinetic energy of the system is the same after the collision as before.

$$(\text{Total K.E})_{\text{before collision}} = (\text{Total K.E})_{\text{after collision}}$$

$$\frac{1}{2} m_A v_{A1x}^2 + \frac{1}{2} m_B (0)^2 = \frac{1}{2} m_A v_{A2x}^2 + \frac{1}{2} m_B v_{B2x}^2$$

$$\frac{1}{2} m_A v_{A1x}^2 = \frac{1}{2} m_A v_{A2x}^2 + \frac{1}{2} m_B v_{B2x}^2$$

$$m_A v_{A1x}^2 = m_A v_{A2x}^2 + m_B v_{B2x}^2$$

$$m_B v_{B2x}^2 = m_A \left(v_{A1x}^2 - v_{A2x}^2\right)$$

$$m_B v_{B2x}^2 = m_A (v_{A1x} - v_{A2x})(v_{A1x} + v_{A2x})$$

$$m_B v_{B2x}^2 = m_B v_{B2x} (v_{A1x} + v_{A2x}) \quad (\text{using eqn } 1)$$

$$v_{B2x} = v_{A1x} + v_{A2x} \quad -----(2)$$

From eq^n (1) we get

$m_B v_{B2x} = m_A (v_{A1x} - v_{A2x})$

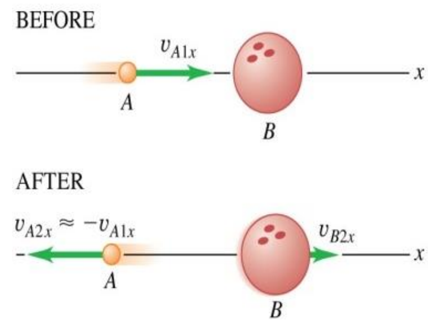
$m_B (v_{A1x} + v_{A2x}) = m_A (v_{A1x} - v_{A2x})$ using eq^n (2)

$$v_{A2x} = \left(\frac{m_A - m_B}{m_A + m_B}\right) v_{A1x} \quad -----(3)$$

Putting this value in eq^n (2) we get,

$$v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right) v_{A1x} \quad --(4)$$

Case – I: If $m_A << m_B$



Putting this condition eqn (3) and (4) we get ,

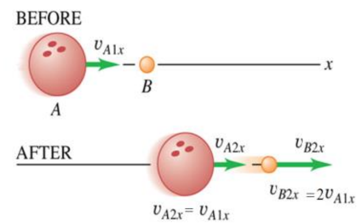
$$v_{A2x} = \left(\frac{m_A - m_B}{m_A + m_B}\right) v_{A1x} \quad \Rightarrow \quad v_{A2x} = \left(\frac{-m_B}{m_B}\right) v_{A1x}$$

$$\Rightarrow \quad v_{A2x} = -v_{A1x}$$

$$\text{Again, } v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right) v_{A1x} \quad \Rightarrow \quad v_{B2x} = 0$$

Ball ‘A’ re after returns back after the collision with a velocity equal to its original value but in the opposite direction .

Case – II: If $m_A >> m_B$



Putting this condition eq^n (3) and (4) we get ,

$$v_{A2x} = \left(\frac{m_A - m_B}{m_A + m_B}\right) v_{A1x} \quad \Rightarrow \quad v_{A2x} = \left(\frac{m_A}{m_A}\right) v_{A1x}$$

$$\Rightarrow \quad v_{A2x} = v_{A1x}$$

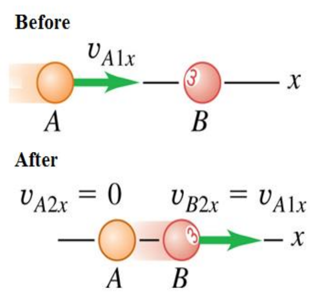
Again,

$$v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right) v_{A1x} \quad \Rightarrow \quad v_{B2x} = \left(\frac{2m_A}{m_A}\right) v_{A1x}$$

$$\Rightarrow \quad v_{B2x} = 2v_{A1x}$$

Ball A’s velocity remain unchanged after the collision . Ball B’s velocity will be doubled after the collision.

Case – III: If $m_A = m_B = m$



Putting this condition eq^n (3) and (4) we get ,

$$v_{A2x} = \left(\frac{m_A - m_B}{m_A + m_B}\right) v_{A1x} \quad \Rightarrow \quad v_{A2x} = 0$$

$$v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right) v_{A1x} \quad \Rightarrow \quad v_{B2x} = \left(\frac{2m}{2m}\right) v_{A1x}$$

$$\Rightarrow \quad v_{B2x} = v_{A1x}$$

Thus the body that was moving stops dead; it gives all its momentum and kinetic energy to the body that was at rest.

Elastic Collisions and Relative Velocity

We know that

$$v_{B2x} = v_{A1x} + v_{A2x} \quad \Rightarrow \quad v_{A1x} = v_{B2x} - v_{A2x}$$

Here $(v_{B2x} - v_{A2x})$ is the velocity of B relative to A after the collision.

But v_{A1x} is the negative of the velocity of B relative to A before the collision.

The relative velocity has the same magnitude, but opposite sign, before and after the collision.

The sign changes because A and B are approaching each other before the collision but moving apart after the collision.

This means that if B is moving before the collision, then we have,

$(v_{B2x} - v_{A2x}) = -(v_{B1x} - v_{A1x})$

Test Your Understanding: TYU-8.2, TYU-8.3

Test Your Understanding 8.2:

A spring-loaded toy sits at rest on a horizontal, frictionless surface. When the spring releases, the toy breaks into three equal-mass pieces, A, B, and C, which slide along the surface. Piece A moves off in the negative x-direction, while piece B moves off in the negative y-direction.

(a) What are the signs of the velocity components of piece C?

(b) Which of the three pieces is moving the fastest?

Answers: (a) $v_{C2x} > 0$, $v_{C2y} > 0$, (b) piece C

Piece A moves off in the negative x-direction, so its y-component velocity is zero ($v_{A2y} = 0$)

Piece B moves off in the negative y-direction, so its x-component velocity is zero ($v_{B2x} = 0$)

We are given that

$m_A = m_B = m_C$,

$v_{A2x} < 0$, $v_{A2y} = 0$, and

$v_{B2x} = 0$, and $v_{B2y} < 0$.

External horizontal forces (F) = 0

So the x- and y-components of the total momentum of the system are conserved.

Total momentum along x- direction (before release) = Total momentum along x- direction (After release)

- ⇨ $0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$
- ⇨ $0 = m_A v_{A2x} + 0 + m_C v_{C2x}$
- ⇨ $m_C v_{C2x} = -m_A v_{A2x}$
- ⇨ $v_{C2x} = -v_{A2x}$ (since $m_A = m_C$)
- ⇨ is opposite to v_{A2x} i.e along +ve x-direction

Again for y- direction we have

Total momentum along y- direction (before release) = Total momentum along y- direction (After release)

⇨ $0 = m_A v_{A2y} + m_B v_{B2y} + m_C v_{C2y}$

⇨ $0 = 0 + m_B v_{B2y} + m_C v_{C2y}$

⇨ $m_C v_{C2y} = - m_B v_{B2y}$

⇨ $v_{C2y} = - v_{A2y}$ (since $m_B = m_C$)

⇨ is opposite to v_{A2y} i.e along +ve y-direction

From the above it is clear that

v_{C2x} (= $-v_{A2x}$) > 0 and v_{C2y} (= $-v_{B2y}$) > 0,

So the velocity components of piece C are both positive.

The speed of Piece is

$$v_C = \sqrt{v_{C2x}^2 + v_{C2y}^2} = \sqrt{v_{A2x}^2 + v_{B2y}^2}$$

This is greater than the speed of either piece A or piece B.

Test Your Understanding8.3:

For each situation, state whether the collision is elastic or inelastic. If it is inelastic, state whether it is completely inelastic.

(a) You drop a ball from your hand. It collides with the floor and bounces back up so that it just reaches your hand.

(b) You drop a different ball from your hand and let it collide with the ground. This ball bounces back up to half the height from which it was dropped.

(c) You drop a ball of clay from your hand. When it collides with the ground, it stops.

Answers:

(a) elastic, (b) inelastic, (c) completely inelastic

In each case gravitational potential energy is converted to kinetic energy as the ball falls, and the collision is between the ball and the ground.

In (a) all of the initial energy is converted back to gravitational potential energy, so no kinetic energy is lost in the bounce and the collision is elastic.

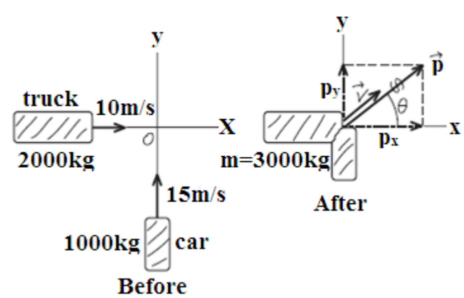
In (b) there is less gravitational potential energy at the end than at the beginning, so some kinetic energy was lost in the bounce. Hence the collision is inelastic.

In (c) the ball loses all the kinetic energy it has to give, the ball and the ground stick together, and the collision is completely inelastic.

Example Problems

Example 8.4 :

A marksman holds a rifle of mass m_R=3.0 kg loosely, so it can recoil freely. He fires a bullet of mass m_B=5.0 kg horizontally with a velocity relative to the ground of v_{Bx}=300m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum and kinetic energy of the bullet and rifle?



Solution:

Conservation of the x-component of total momentum gives

$$0 = m_B v_{B2} + m_R v_{R2}$$

$$v_{Rx} = -\frac{m_B}{m_R} v_{Bx} = -\frac{(0.005 \text{ kg})(300 \text{ m/s})}{3.0 \text{ kg}} = -0.5 \text{ m/s}$$

The negative sign means that the recoil is in the direction opposite to that of the bullet.

The final momenta and kinetic energies are

$p_{Bx} = m_B v_{Bx} = (0.00500 \text{ kg})(300 \text{ m/s}) = 1.50 \text{ kg.m/s}$

$$K_B = \frac{1}{2} m_B v_{Bx}^2 = \frac{1}{2}(0.005\text{kg})(300\text{m/s})^2 = 225\text{J}$$

$p_{Rx} = m_R v_{Rx} = (3.00 \text{ kg})(-0.50 \text{ m/s}) = -1.50 \text{ kg.m/s}$

$$K_R = \frac{1}{2} m_R v_{Rx}^2 = \frac{1}{2}(3.0\text{kg})(-0.5\text{m/s})^2 = 0.375\text{J}$$

Example 8.9:

A 1000-kg car travelling north at 15m/s collides with a 2000-kg truck travelling east at 10m/s. The occupants, wearing seat belts, are uninjured, but the two vehicles move away from the impact point as one. The insurance adjustor asks you to find the velocity of the wreckage just after impact. What is your answer?

Solution:

The components of are the components of are

$P_x = p_{Cx} + p_{Tx} = m_C v_{Cx} + m_T v_{Tx} = = (1000 \text{ kg})(0) + (2000 \text{ kg})(10 \text{ m/s}) = 2 \times 10^4 \text{ kg.m/s}$

$P_y = p_{Cy} + p_{Ty} = m_C v_{Cy} + m_T v_{Ty} = = (1000 \text{ kg})(15\text{m/s}) + (2000 \text{ kg})(0) = 1.5 \times 10^4 \text{ kg.m/s}$

The magnitude of P is

$$P = \sqrt{(2 \times 10^4\text{kg.m/s})^2 + (1.5 \times 10^4\text{kg.m/s})^2} = 2.5 \times 10^4\text{kg.m/s}$$

The direction P is given by

$$\tan\theta = \frac{P_y}{P_x} = \frac{1.5 \times 10^4\text{kg.m/s}}{2 \times 10^4\text{kg.m/s}} = 0.75 \quad \Rightarrow \quad \theta = 37^0$$

Again, P = M V

$$V = \frac{P}{M} = \frac{2.5 \times 10^4\text{kg.m/s}}{3000\text{kg}} = 8.3 \text{ m/s}$$