Homework #1

Due date 9/18/2018

NOTE: You may form a study group to work on homework problems. However, each student must submit solutions to the homework problems in your own words, demonstrating your own understanding of the problems.

1. Assume $y_1, y_2, \ldots, y_n$ are uncorrelated random variables with $E(y_i) = \beta_0 + \beta_1 x_i$ and $Var(y_i) = \sigma^2$, and that $\hat{\beta}_0$ and $\hat{\beta}_1$ are the least square estimates of β_0 and β_1. Show each of the following:

   a. $E(\hat{\beta}_1^2) = \beta_1^2 + \sigma^2 / \sum (x_i - \bar{x})^2$.

   b. $E\left[\sum (y_i - \bar{y})^2\right] = (n-1)\sigma^2 + \beta_1^2 \sum (x_i - \bar{x})^2$.

   c. $\hat{\sigma}^2 = \dfrac{\sum \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right)^2}{n-2}$ is an unbiased estimator of σ^2.

2. Use technology (Excel, R, Minitab, ..) to find each of the following for the LungCap data.
   a. Compute the least-square regression line for predicting lungcap (y) from age (x).
   b. Compute the residual standard deviation $\hat{\sigma}$ or s_ε.
   c. Compute the standard errors of $\hat{\beta}_0$ and $\hat{\beta}_1$.
   d. Determine whether the assumptions of the linear model are satisfied. If they are not, specify which assumptions are violated.
   e. How good is the model? [Hint. Find C.V.]