Homework #3

Due date 11/05/2018

1. Suppose the following data are given below in vector form.

$$\mathbf{Y} = \begin{bmatrix} 50 \\ 73 \\ 32 \\ 121 \\ 156 \\ 98 \\ 62 \\ 51 \\ 80 \end{bmatrix} \text{ and } \mathbf{X} = \begin{bmatrix} 1 & 14 & 11 \\ 1 & 28 & 18 \\ 1 & 10 & 5 \\ 1 & 30 & 20 \\ 1 & 48 & 30 \\ 1 & 30 & 21 \\ 1 & 20 & 15 \\ 1 & 16 & 11 \\ 1 & 25 & 17 \end{bmatrix}$$

Find the following:

a) Find the least square estimator of $\hat{\boldsymbol{\beta}}$.

b) Compute the estimate s^2 of σ^2. (Hint: $s^2 = \dfrac{SSE}{n-(k+1)}$)

c) Find the variance-covariance matrix of $\hat{\boldsymbol{\beta}}$ (i.e., find $\text{cov}\left(\hat{\boldsymbol{\beta}}\right)$).

d) Find the variance of the fitted values ($\hat{\mathbf{Y}}$).
Hint: $\text{cov}\left(\hat{\mathbf{Y}}\right) = s^2\mathbf{H}$, $\mathbf{H} = \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$ is the hat matrix

e) Find the variance-covariance matrix of the residuals.
Hint: $\text{cov}\left(\text{residual}\right) = s^2\left(\mathbf{I} - \mathbf{H}\right)$

2. Page 208 # 42
3. Page 436 # 8.48