

1a) Find the least squares estimator of $\hat{\beta}$

[,1]
[1,] -2.381630
[2,] 2.401792
[3,] 1.443504

1b) Compute the estimate s^2 of σ^2

[,1]
[1,] 125.5816

1c) Find the variance covariance matrix of $\hat{\beta}$

[,1] [,2] [,3]
[1,] 102.086172 4.931655 -12.72358
[2,] 4.931655 4.229413 -6.61544
[3,] -12.723579 -6.615440 10.65219

1d) Find the variance of the fitted values \hat{Y}

[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
[1,] 40.578424 2.649368 4.0777925 9.0653077 -27.866245 20.899684 36.157490 23.325611 16.694211
[2,] 2.649368 19.016398 22.8388298 15.1381062 29.964992 8.921667 3.447371 11.203954 12.400958
[3,] 4.077792 22.838830 101.1855691 -0.0980403 -1.593725 -25.715059 -13.498781 32.374959 6.010098
[4,] 9.065308 15.138106 -0.0980403 17.8627146 27.527664 19.719780 13.525001 8.075785 14.765325
[5,] -27.866245 29.964992 -1.5937249 27.5276641 91.183179 16.828731 -13.780362 -8.905707 12.223117
[6,] 20.899684 8.921667 -25.7150586 19.7197802 16.828731 32.229038 28.275506 6.679280 17.743014
[7,] 36.157490 3.447371 -13.4987812 13.5250013 -13.780362 28.275506 36.889373 16.734111 17.831934
[8,] 23.325611 11.203954 32.3749589 8.0757847 -8.905707 6.679280 16.734111 22.990450 13.103200
[9,] 16.694211 12.400958 6.0100984 14.7653251 12.223117 17.743014 17.831934 13.103200 14.809785

1e) Find the variance covariance matrix of the residuals

[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
[1,] 85.003220 -2.649368 -4.0777925 -9.0653077 27.866245 -20.899684 -36.157490 -23.325611 -16.694211
[2,] -2.649368 106.565246 -22.8388298 -15.1381062 -29.964992 -8.921667 -3.447371 -11.203954 -12.400958
[3,] -4.077792 -22.838830 24.3960745 0.0980403 1.593725 25.715059 13.498781 -32.374959 -6.010098
[4,] -9.065308 -15.138106 0.0980403 107.7189291 -27.527664 -19.719780 -13.525001 -8.075785 -14.765325
[5,] 27.866245 -29.964992 1.5937249 -27.5276641 34.398465 -16.828731 13.780362 8.905707 -12.223117
[6,] -20.899684 -8.921667 25.7150586 -19.7197802 -16.828731 93.352605 -28.275506 -6.679280 -17.743014
[7,] -36.157490 -3.447371 13.4987812 -13.5250013 13.780362 -28.275506 88.692271 -16.734111 -17.831934
[8,] -23.325611 -11.203954 -32.3749589 -8.0757847 8.905707 -6.679280 -16.734111 102.591193 -13.103200
[9,] -16.694211 -12.400958 -6.0100984 -14.7653251 -12.223117 -17.743014 -17.831934 -13.103200 110.771858

2a) Fit the quadratic model to the data, then conduct a test to determine if the incidence rate is curvilinearly related to the estimate rate.

Call:

lm(formula = Rate ~ Estimation + I(Estimation^2))

Residuals:

Min	1Q	Median	3Q	Max
-101356	-801	-61	207	104498

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-2.884e+02	8.049e+03	-0.036	0.972
Estimation	1.395e+00	3.651e+00	0.382	0.706
I(Estimation^2)	3.509e-05	9.724e-05	0.361	0.722

Residual standard error: 31900 on 21 degrees of freedom

Multiple R-squared: 0.4591, Adjusted R-squared: 0.4076

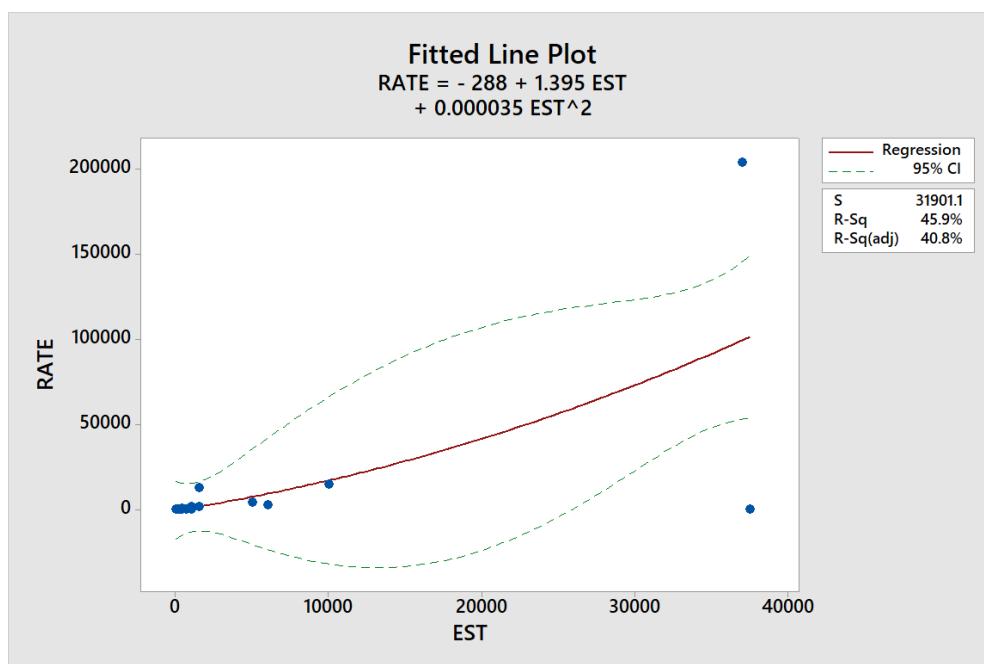
F-statistic: 8.912 on 2 and 21 DF, p-value: 0.001577

The regression equation is

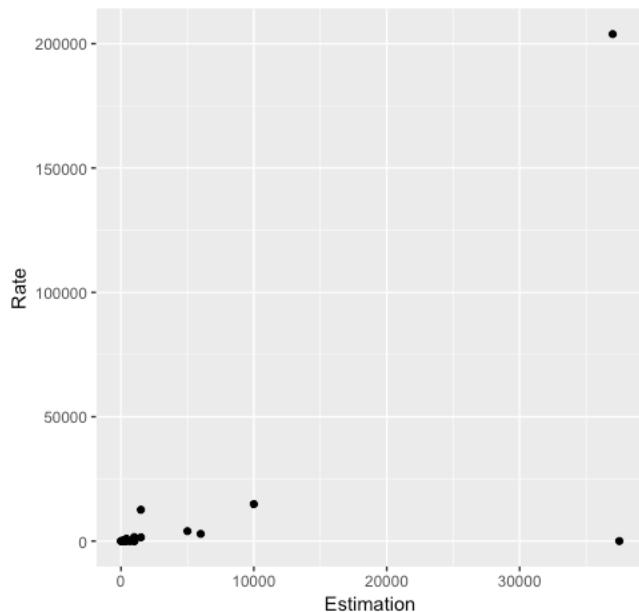
$$\text{RATE} = -288 + 1.395 \text{ EST} + 0.000035 \text{ EST}^2$$

We test at $\alpha = .05$ $H_0: \beta_2 = 0$ vs $H_A: \beta_2 \neq 0$

Our p value is $\frac{0.722}{2} = 0.361 > .05$. We fail to reject our null hypothesis and conclude that the data is not curvilinearly related.



2b) Construct a scatterplot for the data. Locate the data point for Botulism on the graph. What do you observe?



The point for Botulism is located in the right bottom corner of our scatter plot. The data appears to be curvilinear except for that particular point. This point might be an outlier and might be affecting the R-Squared and Adj.-R Squared

2c) Repeat part a, but omit the data point for Botulism from the analysis. Has the fit of the model improved?

Call:

```
lm(formula = Rate_Subset ~ Estimation_Subset + I(Estimation_Subset^2))
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Residuals:

Min	1Q	Median	3Q	Max
-2742.0	-726.9	-686.6	-81.4	11666.8

Coefficients:

		Estimate	Std. Error	t value	Pr(> t)
(Intercept)	7.350e+02	6.959e+02		1.056	0.303
Estimation_Subset	-8.096e-02	3.167e-01		-0.256	0.801
I(Estimation_Subset^2)	1.505e-04	8.683e-06		17.336	1.62e-13 ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

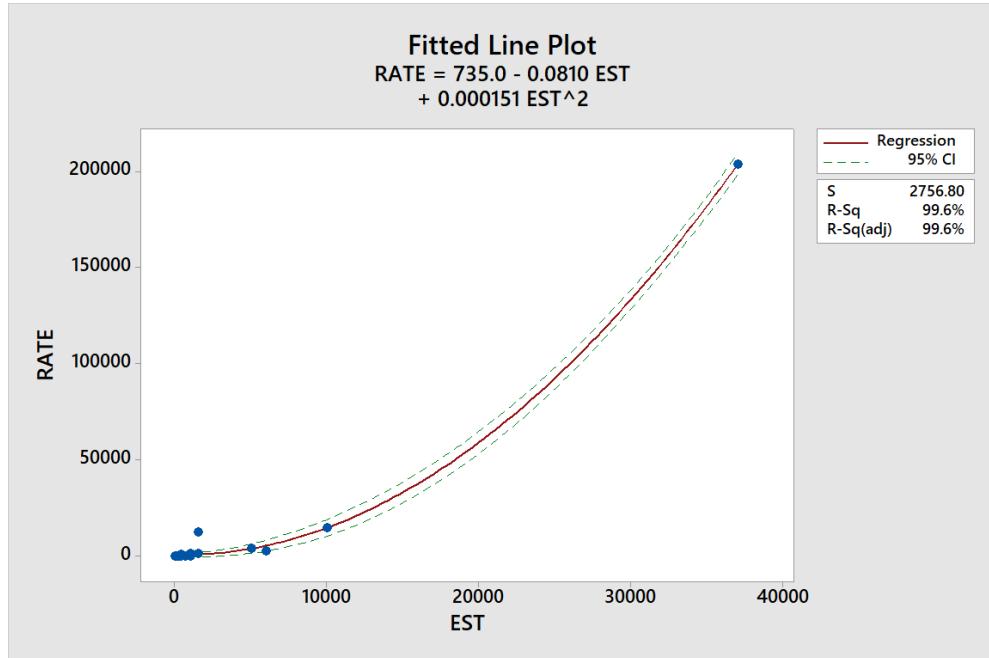
Residual standard error: 2757 on 20 degrees of freedom
 Multiple R-squared: 0.9961, Adjusted R-squared: 0.9958
 F-statistic: 2582 on 2 and 20 DF, p-value: < 2.2e-16

The regression equation is

RATE = 735.0 - 0.0810 EST + 0.000151 EST²

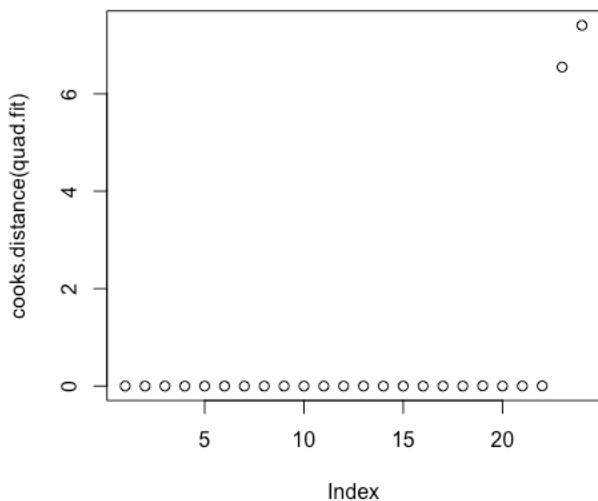
We test at $\alpha = .05$ $H_0: \beta_2 = 0$ vs $H_A: \beta_2 \neq 0$

Our p value is $\frac{1.62(10^{-13})}{2} < .05$ We reject our null hypothesis and conclude that the data is curvilinearly related. The fit of the model has improved. Prior to removing the Botulism data point our Adj. R Squared was 0.4076. After removing the data point our Adj. R Squared increased to 0.9958



3) Identify any outliers in the INFECTION dataset. Are any of these outliers influential? If so, how should the researchers proceed?

	Rate	Estimation	Cook.Distance	F.50thpercentile
1	0.25	300.0	3.620496e-07	0.8148716
2	1.00	1000.0	2.172167e-05	0.8148716
3	1.75	691.0	8.459856e-06	0.8148716
4	2.00	200.0	2.163351e-09	0.8148716
5	3.00	17.5	1.679587e-06	0.8148716
6	5.00	0.8	2.031311e-06	0.8148716
7	9.00	1000.0	2.141802e-05	0.8148716
8	10.00	150.0	1.718607e-07	0.8148716
9	22.00	326.5	4.479051e-07	0.8148716
10	23.00	146.5	2.490307e-07	0.8148716
11	39.00	370.0	7.438326e-07	0.8148716
12	98.00	400.0	6.157626e-07	0.8148716
13	119.00	225.0	1.786763e-07	0.8148716
14	152.00	200.0	5.496425e-07	0.8148716
15	179.00	200.0	7.507440e-07	0.8148716
16	936.00	400.0	8.569887e-06	0.8148716
17	1514.00	1500.0	2.296657e-06	0.8148716
18	1627.00	1000.0	3.935263e-06	0.8148716
19	2926.00	6000.0	5.696970e-03	0.8148716
20	4019.00	5000.0	1.075258e-03	0.8148716
21	12619.00	1500.0	1.945861e-03	0.8148716
22	14889.00	10000.0	4.563122e-03	0.8148716
23	203864.00	37000.0	6.548829e+00	0.8148716
24	15.00	37500.0	7.405753e+00	0.8148716



From Cooks Distance it seems that data point 23 and 24 are influential outliers. The researchers should investigate why these outliers occurred.