- **Chapter 6**: 1 (no C), 5, 7
- **Chapter 7**: 2, 5, 6, 21
- **Chapter 8**: 1, 10 (no (C)), 11

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6.1. An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 2^3 factorial design are run. The results are as follows:

A	B	C	Treatment Combination	Replicate I	Replicate II	Replicate III
−	−	−	(1)	22	31	25
+	−	−	a	32	43	29
−	+	−	b	35	34	50
+	+	−	ab	55	47	46
−	−	+	c	44	45	38
+	−	+	ac	40	37	36
−	+	+	bc	60	50	54
+	+	+	abc	39	41	47

(a) Estimate the factor effects. Which effects appear to be large?

(b) Use the analysis of variance to confirm your conclusions for part (a).

(c) Write down a regression model for predicting tool life (in hours) based on the results of this experiment.

(d) Analyze the residuals. Are there any obvious problems?

(e) On the basis of an analysis of main effect and interaction plots, what coded factor levels of A, B, and C would you recommend using?

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From the below do-by-hand we can see that Factor B, C and BC contribute the most to the Sum of Squares Total (SSTO). We can say that those effects appear to be large.

Effect Estimate Summary

Factor	Effect	SS	Percent Contribution	
A	0.33	0.67	< .01 %	
B	11.33	770.67	36.78%	*
AB	-1.67	16.67	0.80%	
C	6.83	280.17	13.37%	*
AC	-8.83	468.17	22.34%	*
BC	-2.83	48.17	2.30%	
ABC	-2.17	28.17	1.34%	

• We can see that Factor B, C, AC contribute the most to the SSTO. These appear to be large

AOVA for Problem 6.1

The GLM Procedure

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	1612.666667	230.380952	7.64	0.0004
Error	16	482.666667	30.166667		
Corrected Total	23	2095.333333			

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R-Square	Coeff Var	Root MSE	Y Mean
0.769647	13.45082	5.492419	40.83333

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	0.6666667	0.6666667	0.02	0.8837
B	1	770.6666667	770.6666667	25.55	0.0001
C	1	280.1666667	280.1666667	9.29	0.0077
AB	1	16.6666667	16.6666667	0.55	0.4681
AC	1	468.1666667	468.1666667	15.52	0.0012
BC	1	48.1666667	48.1666667	1.60	0.2245
ABC	1	28.1666667	28.1666667	0.93	0.3483

The results of the factor effect estimation are confirmed by the above SAS output. We can see that Factor A, AB, BC and ABC all have a P-Value larger than 0.05 which indicates that they are not significant. This is also reflected in our factor estimation. Additionally, Factor B, C, and AC have a P-value less than 0.05 which indicates that they are significant.

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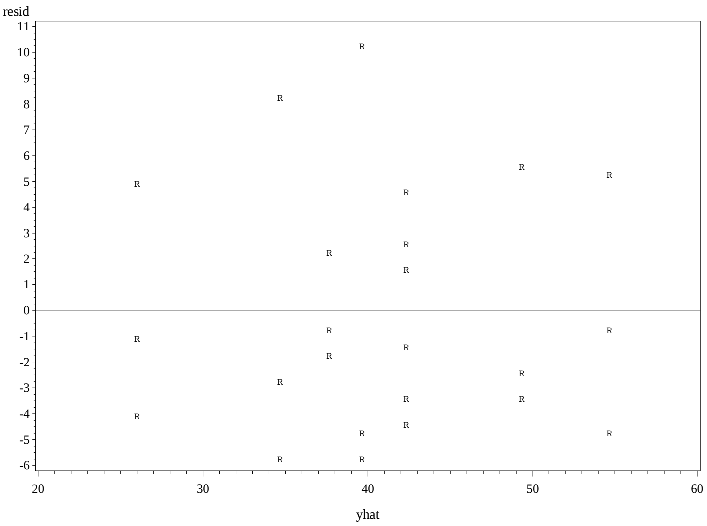
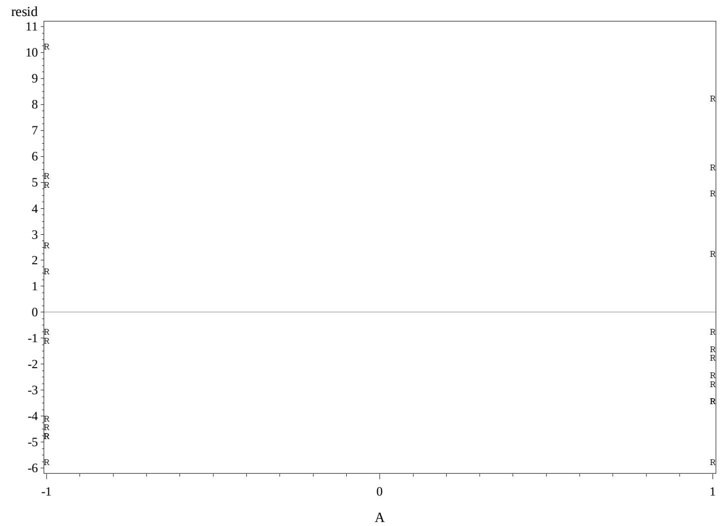
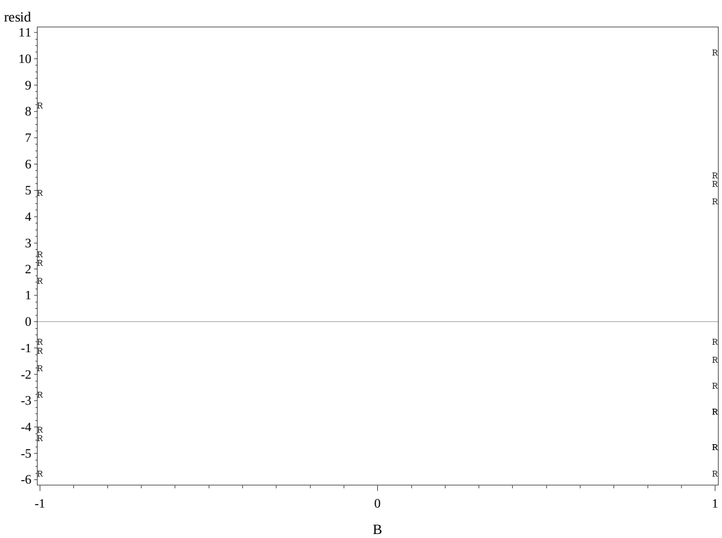
Residual Plots for Tool 2^3 Experiment



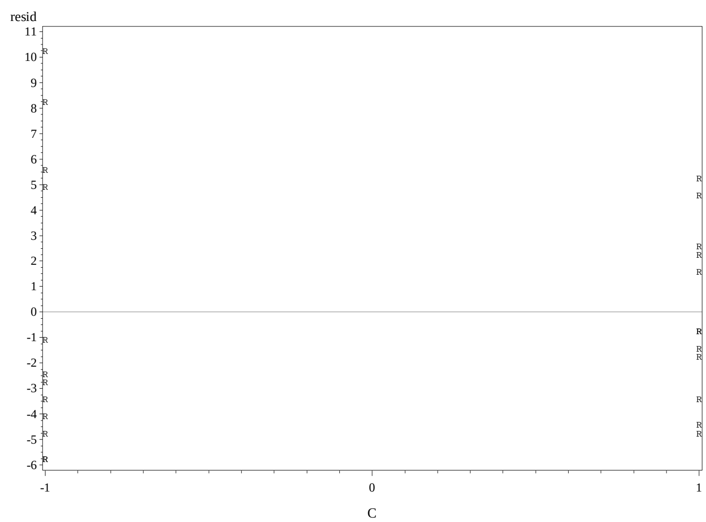
Residual Plots for Tool 2^3 Experiment



Residual Plots for Tool 2^3 Experiment



Residual Plots for Tool 2^3 Experiment



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Levene Test for Factor A

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
A	1	38293.4	38293.4	3.68	0.0683
Error	22	229146	10415.7		

Levene Test for Factor B

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
B	1	394.7	394.7	0.11	0.7398
Error	22	76784.7	3490.2		

Levene Test for Factor C

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
C	1	15742.3	15742.3	2.27	0.1458
Error	22	152312	6923.3		

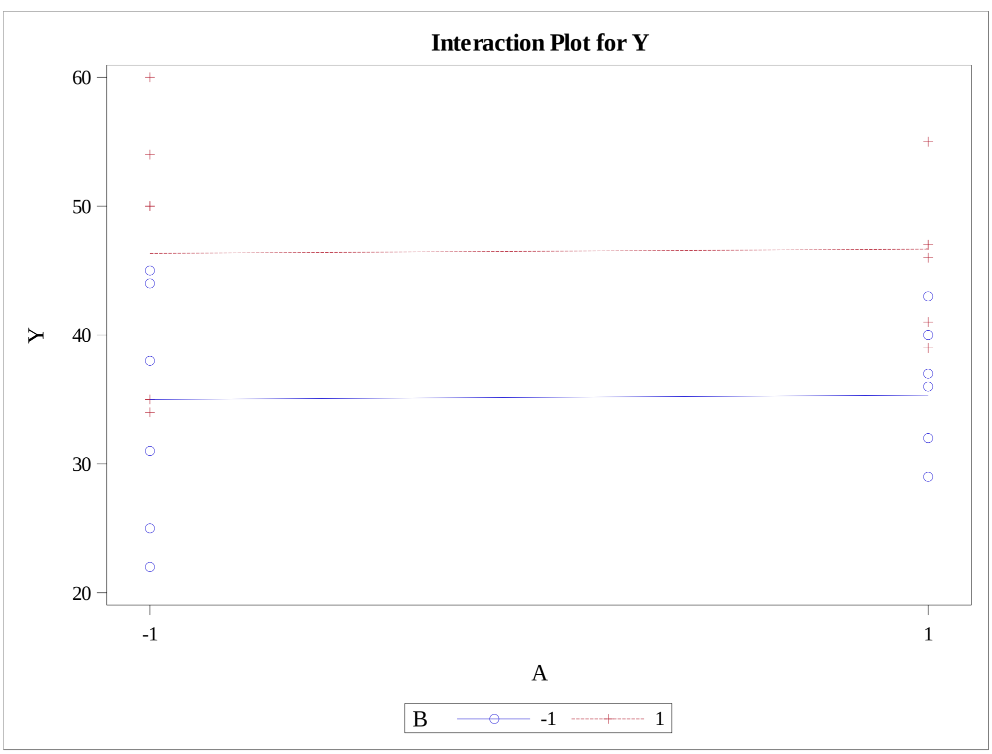
From the above SAS generated residual plots we can see that the residuals have constant variance. This is further supported by the above Levene Tests. Factor A has F Critical value of 3.68 with 1 and 22 degrees of freedom. This corresponds to a P-Value of 0.0683 which is larger than 0.05. Therefore, we conclude that Factor A has constant variance.

Factor B has F Critical value of 0.11 with 1 and 22 degrees of freedom. This corresponds to a P-Value of 0.7398 which is larger than 0.05. Therefore, we conclude that Factor B has constant variance.
Factor C has F Critical value of 2.27 with 1 and 22 degrees of freedom. This corresponds to a P-Value of 0.1458 which is larger than 0.05. Therefore, we conclude that Factor C has constant variance.

Checking for Interaction in 2^3 Tool Experiment between Factor A and Factor B

The GLM Procedure

Dependent Variable: Y



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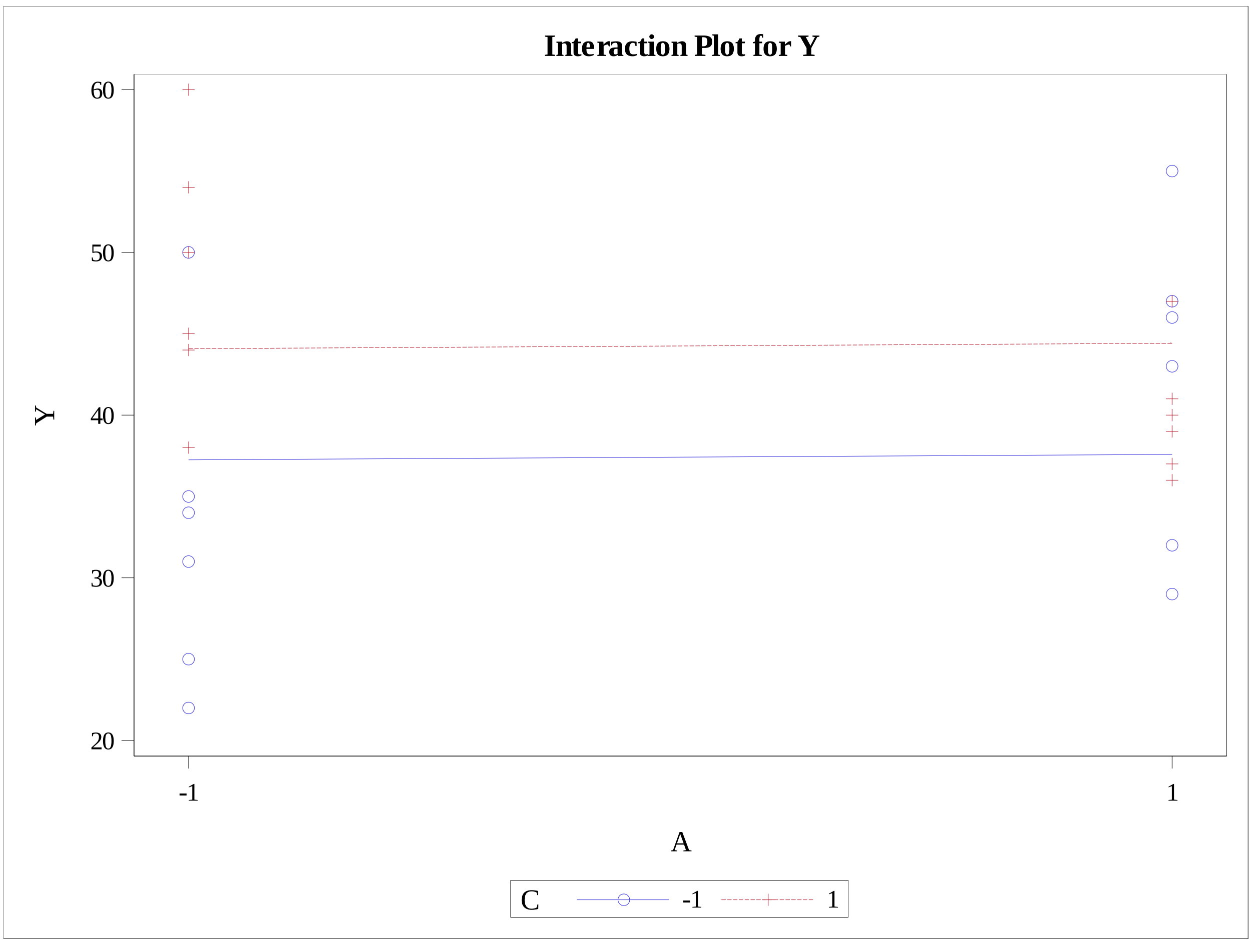
Checking for Interaction in 2^3 Tool Experiment between Factor A and Factor C

The GLM Procedure

Dependent Variable: Y



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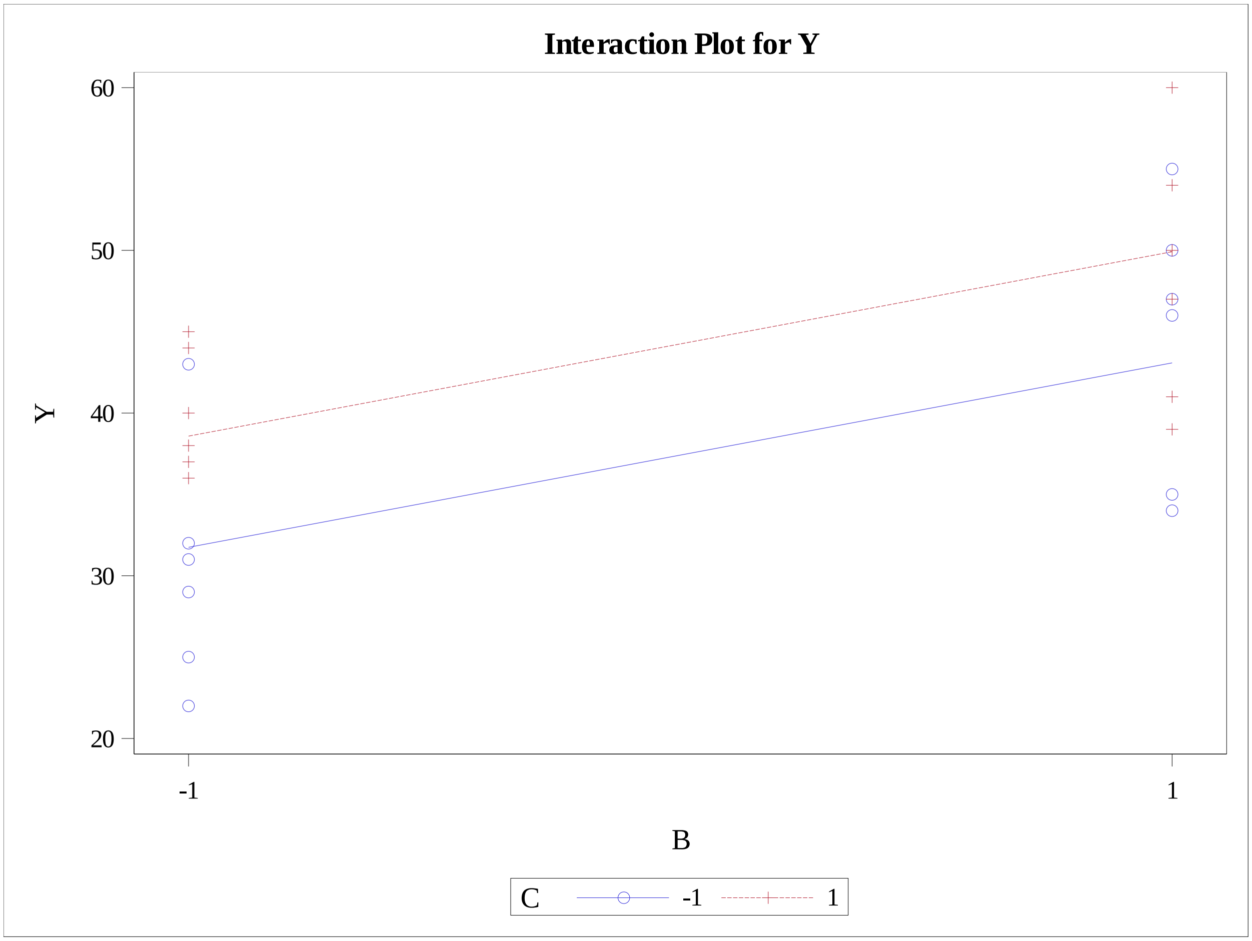
Checking for Interaction in 2^3 Tool Experiment between Factor B and Factor C

The GLM Procedure

Dependent Variable: Y



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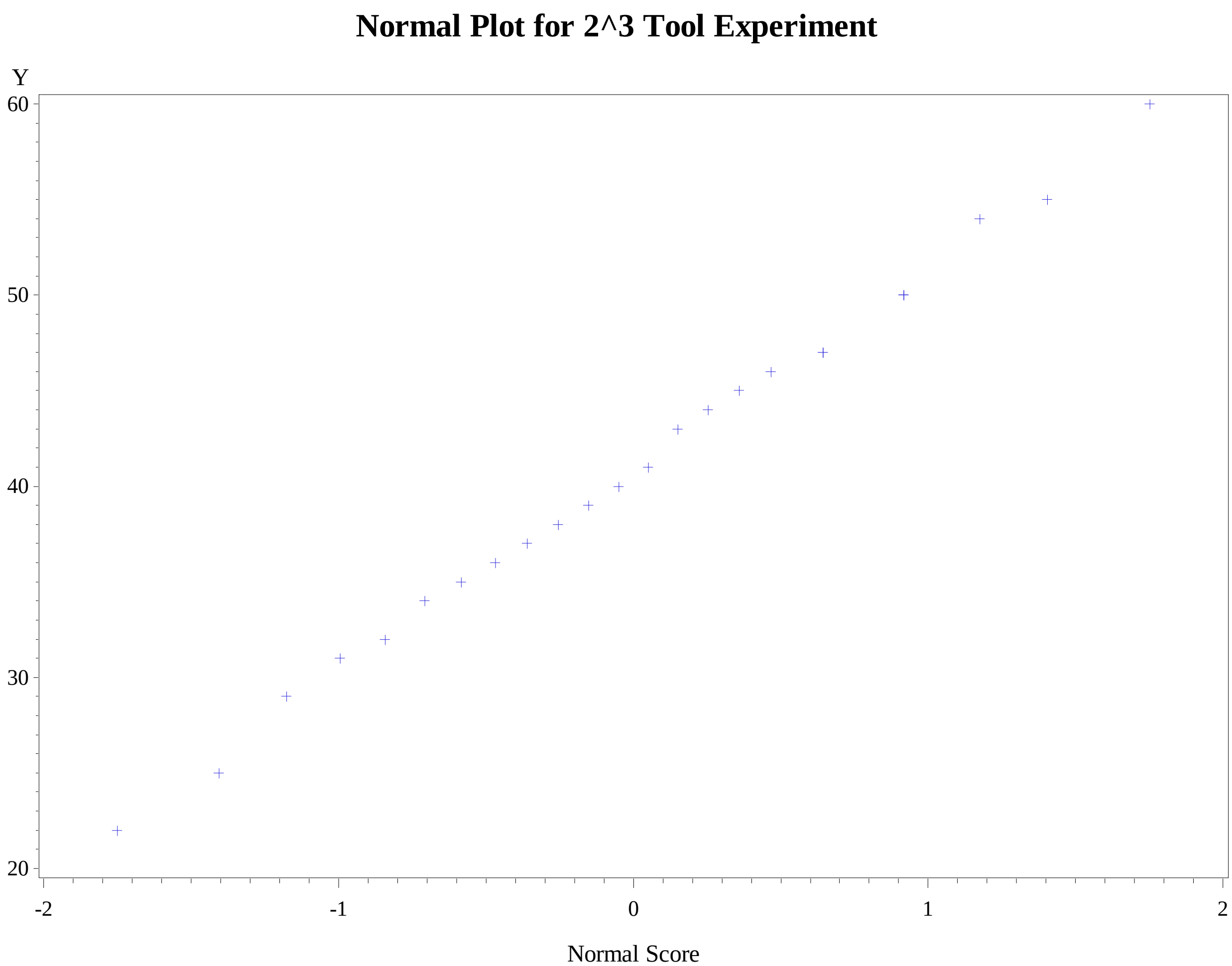
From the above interaction plots we can see that there is no significant interaction between Factor A and B, Factor A and C, or Factor B and C.



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Normal Tests for Tool 2^3 Experiment

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.921062	**Pr < W**	0.0617
Kolmogorov-Smirnov	**D**	0.182853	**Pr > D**	0.0372
Cramer-von Mises	**W-Sq**	0.115763	**Pr > W-Sq**	0.0678
Anderson-Darling	**A-Sq**	0.673781	**Pr > A-Sq**	0.0725

The above SAS generated Normal Plot appears linear so we can say that the residuals are normally distributed. This is confirmed by the Normal Tests which show that the Shapiro-Wilk P-Value is 0.0617 which is larger than 0.05. From this we can conclude that the residuals are normally distributed.

Since the residuals have constant variance and are normally distributed, we can say that all the model assumptions are met. We will continue with the analysis and perform multiple comparison using Tukey.

Tukey Multiple Comparison for Factor A

Alpha	0.05
Error Degrees of Freedom	16
Error Mean Square	30.16667
Critical Value of Studentized Range	2.99786
Minimum Significant Difference	4.7532

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Comparisons significant at the 0.05 level are indicated by ***.				
A Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
1 - -1	0.3333	-4.4198	5.0865	

Tukey Multiple Comparison for Factor B

Alpha	0.05
Error Degrees of Freedom	16
Error Mean Square	30.16667
Critical Value of Studentized Range	2.99786
Minimum Significant Difference	4.7532

Comparisons significant at the 0.05 level are indicated by ***.				
B Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
1 - -1	11.333	6.580	16.087	***

Tukey Multiple Comparison for Factor C

Alpha	0.05
Error Degrees of Freedom	16
Error Mean Square	30.16667

Critical Value of Studentized Range	2.99786
Minimum Significant Difference	4.7532

Comparisons significant at the 0.05 level are indicated by *.**				
C Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
1 - -1	6.833	2.080	11.587	***

From the above SAS generated Tukey results we can see that difference in level means for Factor B and C are significant when compared to the Tukey MSD of 4.7532

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6.5. A router is used to cut locating notches on a printed circuit board. The vibration level at the surface of the board as it is cut is considered to be a major source of dimensional variation in the notches. Two factors are thought to influence

vibration: bit size (A) and cutting speed (B). Two bit sizes ($\frac{1}{16}$ and $\frac{1}{8}$ in.) and two speeds (40 and 90 rpm) are selected, and four boards are cut at each set of conditions shown below. The response variable is vibration measured as the resultant vector of three accelerometers (x, y, and z) on each test circuit board.

			Replicate			
A	***B***	**Treatment Combination**	**I**	**II**	**III**	**IV**
−	−	(1)	18.2	18.9	12.9	14.4
+	−	*a*	27.2	24.0	22.4	22.5
−	+	*b*	15.9	14.5	15.1	14.2
+	+	*ab*	41.0	43.9	36.3	39.9

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AOVA for Problem 6.5

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	1638.111875	546.037292	91.36	<.0001
Error	12	71.722500	5.976875		
Corrected Total	15	1709.834375			

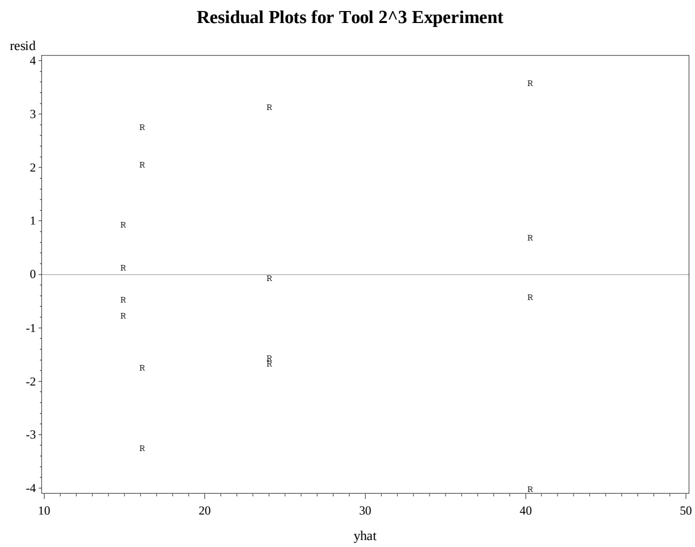
Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	1107.225625	1107.225625	185.25	<.0001
B	1	227.255625	227.255625	38.02	<.0001
AB	1	303.630625	303.630625	50.80	<.0001

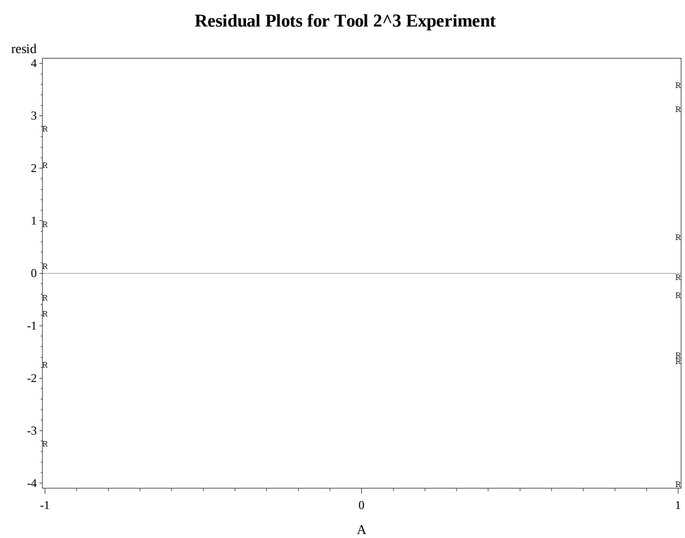
Effect Estimate Summary

Factor	Effect	SS	% of SSTO
A	16.64	1107.23	64.76 %
B	7.54	227.26	13.29 %
AB	20.14	303.63	17.76 %

All factors appear to be significant

From the above SAS generated ANOVA and the do-by-hand Effect Estimate Summary we can conclude that Factor A, B and AB are significant. Factor A has a F-Critical value of 185.25 and a P-Value of less than 0.001 with 1 and 12 degrees of freedom. Factor B has a F-Critical value of 38.02 with a P-Value of less than 0.001 with 1 and 12 degrees of freedom. Lastly, Factor AB has a F-Critical value of 303.63 with a P-Value of less than 0.001 with 1 and 12 degrees of freedom.





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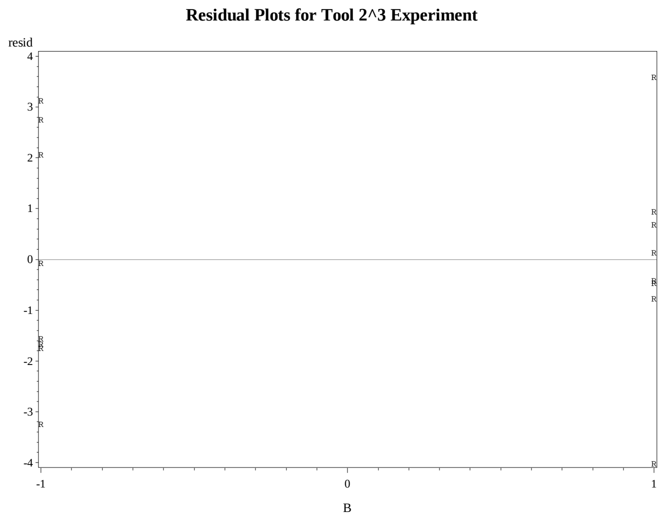

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
A	1	18424.8	18424.8	23.59	0.0003
Error	14	10936.0	781.1		

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
B	1	82728.3	82728.3	50.88	<.0001
Error	14	22764.1	1626.0		

From the above SAS generated residual plots and Levene Tests we can conclude that the residuals DO NOT have constant variance. Thus, we must perform a transformation on the data.

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Levene Test
Z = sqrt(y)

Levene's Test for Homogeneity of Z Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
A	1	1.0235	1.0235	21.96	0.0004
Error	14	0.6525	0.0466		

Levene's Test for Homogeneity of Z Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
B	1	6.6984	6.6984	51.73	<.0001
Error	14	1.8129	0.1295		

Levene Test
Z = 1/(y)

Levene's Test for Homogeneity of Z Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
A	1	1.45E-9	1.45E-9	0.47	0.5032
Error	14	4.3E-8	3.072E-9		

Levene's Test for Homogeneity of Z Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
B	1	3.006E-7	3.006E-7	11.40	0.0045
Error	14	3.692E-7	2.637E-8		

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Levene Test
Z = log(y)

Levene's Test for Homogeneity of Z Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
A	1	0.0133	0.0133	15.85	0.0014
Error	14	0.0117	0.000839		

Levene's Test for Homogeneity of Z Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
B	1	0.1462	0.1462	40.32	<.0001
Error	14	0.0508	0.00363		

From the above transformation and Levene Tests we can see that the transformation that best stabilizes our variance is the inverse transformation: 1/y. Using this transformation Factor A has constant variance, **however**, Factor B DOES NOT have constant variance.

Further investigation should be done to stabilize the residuals of Factor B. We will continue with our analysis using Z = 1/y. These results should be used with caution.

Another issue to note is that once the transformation is performed Factor B is NOT significant anymore. Factor A and Factor AB are still significant.

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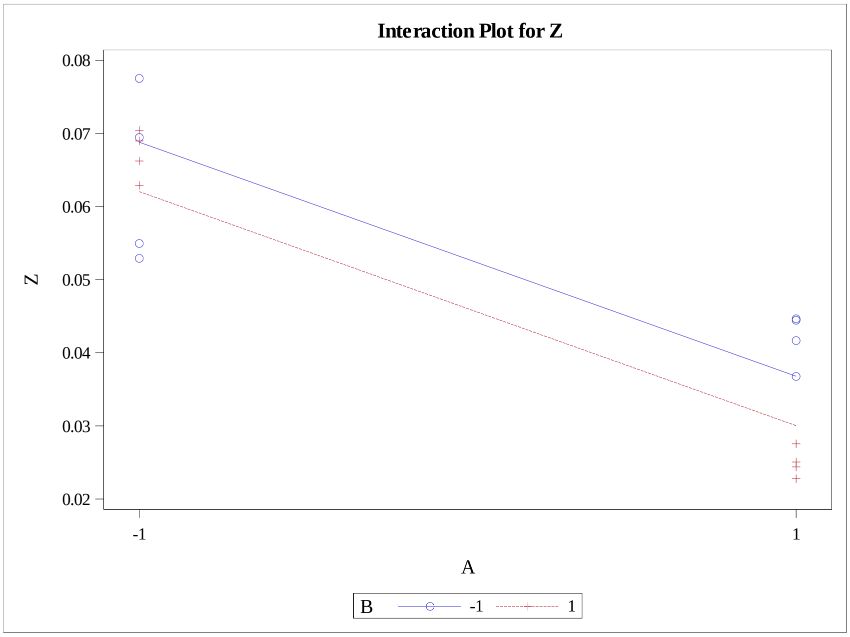
ANOVA for transformed data
Z = 1/(y)

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	0.00469383	0.00156461	37.38	<.0001
Error	12	0.00050224	0.00004185		
Corrected Total	15	0.00519606			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	0.00409684	0.00409684	97.89	<.0001
B	1	0.00018260	0.00018260	4.36	0.0587
AB	1	0.00041439	0.00041439	9.90	0.0084

Again, these results should be used with caution.

Checking for Interaction in 2^2 Cutting Experiment between Factor A and Factor B
Z = 1/(y)



We can see that there is no significant interaction between our main factors A and B

Normal Tests for 2^2 Cutting Experiment
Z = 1/(y)

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.953102	**Pr < W**	0.5404
Kolmogorov-Smirnov	**D**	0.159473	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.059551	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.376864	**Pr > A-Sq**	>0.2500

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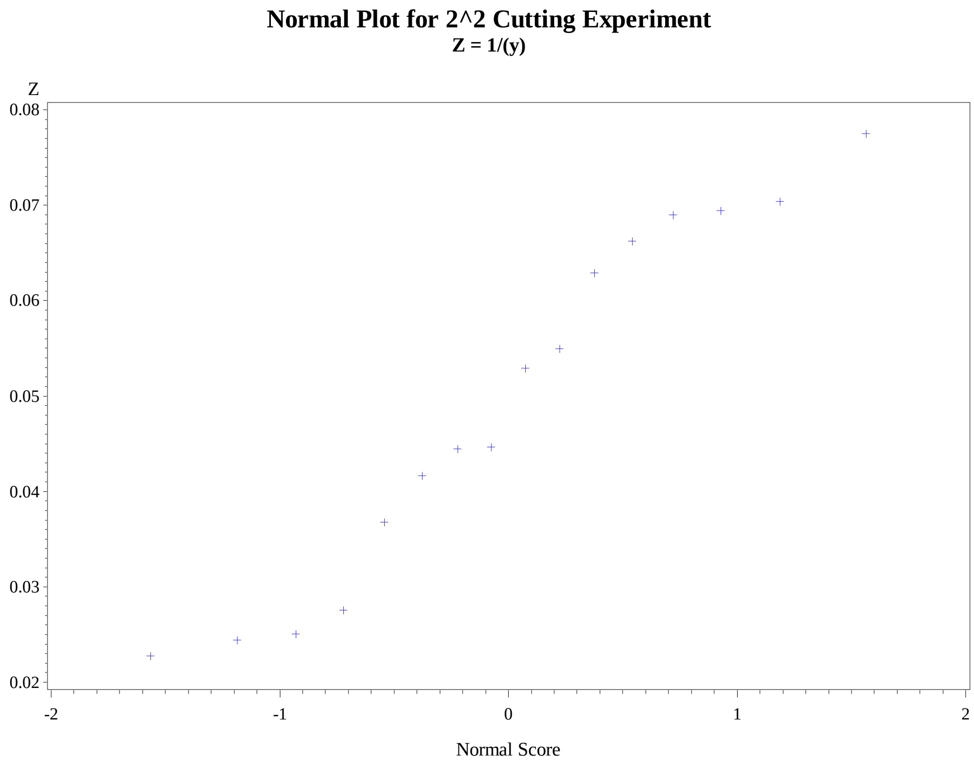

From the above SAS generated plot and Shapiro-Wilk test we can conclude that the residuals are normally distributed.

We can conclude that the variance for Factor A is constant and the residuals are normally distributed. However, Factor B is not significant and does not have constant variance. Again, these results should be used with caution. We will now perform multiple comparison for Factor A and B using Tukey.

Tukey Multiple Comparison for Factor A in 2^2 Cutting Experiment
Z = 1/(y)

Alpha	0.05
Error Degrees of Freedom	12
Error Mean Square	0.000042
Critical Value of Studentized Range	3.08118
Minimum Significant Difference	0.007

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Comparisons significant at the 0.05 level are indicated by ***.				
A Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
1 - -1	-0.032003	-0.039051	-0.024956	***

Tukey Multiple Comparison for Factor B in 2^2 Cutting Experiment
Z = 1/(y)

Alpha	0.05
Error Degrees of Freedom	12
Error Mean Square	0.000042
Critical Value of Studentized Range	3.08118
Minimum Significant Difference	0.007

Comparisons significant at the 0.05 level are indicated by ***.				
B Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
1 - -1	-0.006756	-0.013804	0.000291	

We can see that the difference in level means for Factor A is significant when compared to the Tukey MSD value of 0.07

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6.7. An experiment was performed to improve the yield of a chemical process. Four factors were selected, and two replicates of a completely randomized experiment were run. The results are shown in the following table:

Treatment Combination	Replicate I	Replicate II	Treatment Combination	Replicate I	Replicate II
(1)	90	93	*d*	98	95
a	74	78	*ad*	72	76
b	81	85	*bd*	87	83
ab	83	80	*abd*	85	86
c	77	78	*cd*	99	90
ac	81	80	*acd*	79	75
bc	88	82	*bcd*	87	84
abc	73	70	*abcd*	80	80

(a) Estimate the factor effects.

(b) Prepare an analysis of variance table and determine which factors are important in explaining yield.

(c) Write down a regression model for predicting yield, assuming that all four factors were varied over the range from -1 to $+1$ (in coded units).

(d) Plot the residuals versus the predicted yield and on a normal probability scale. Does the residual analysis appear satisfactory?

(e) Two three-factor interactions, *ABC* and *ABD*, apparently have large effects. Draw a cube plot in the factors *A*, *B*, and *C* with the average yields shown at each corner. Repeat using the factors *A*, *B*, and *D*. Do these two plots aid in data interpretation? Where would you recommend that the process be run with respect to the four variables?

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Effect Estimate

Factor	Effect	SS	% of SSTO	
A	-9.0625	657.03	40.37	*
B	-1.3125	13.78	0.84	
C	-2.6875	57.78	3.55	*
D	3.9375	124.03	7.62	*
AB	4.0625	132.03	8.11	*
AC	0.6875	3.78	0.23	
BC	-0.5625	2.53	0.15	
ABC	-5.1875	215.28	13.23	*
AD	-2.1875	38.28	2.35	*
BD	-0.1875	0.28	0.02	
ABD	4.6875	175.78	10.80	*
CD	1.6875	22.78	1.40	*
ACD	-0.9375	7.03	0.43	
BCD	-0.9375	7.03	0.43	
ABCD	2.4375	47.53	2.92	*

Factor A, C, D, AB, ABC, AD, ABD, CD and ABCD appear to be significant

AOVA for Problem 6.7 Chemical Experiment

The GLM Procedure

Dependent Variable: Y

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Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	15	1504.968750	100.331250	13.10	<.0001
Error	16	122.500000	7.656250		
Corrected Total	31	1627.468750			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	657.0312500	657.0312500	85.82	<.0001
B	1	13.7812500	13.7812500	1.80	0.1984
C	1	57.7812500	57.7812500	7.55	0.0143
D	1	124.0312500	124.0312500	16.20	0.0010
AB	1	132.0312500	132.0312500	17.24	0.0007
AC	1	3.7812500	3.7812500	0.49	0.4923
BC	1	2.5312500	2.5312500	0.33	0.5733
ABC	1	215.2812500	215.2812500	28.12	<.0001
AD	1	38.2812500	38.2812500	5.00	0.0399
BD	1	0.2812500	0.2812500	0.04	0.8504
ABD	1	175.7812500	175.7812500	22.96	0.0002
CD	1	22.7812500	22.7812500	2.98	0.1038
ACD	1	7.0312500	7.0312500	0.92	0.3522
BCD	1	7.0312500	7.0312500	0.92	0.3522
ABCD	1	47.5312500	47.5312500	6.21	0.0241

From the above SAS generated ANOVA table, we can see that Factors A< C, D, AB, ABC, AD, ABD, and ABCD are all significant when compared to a F-Value of 4.49 with 1 and 16 degrees of freedom. This is different from the Estimated Factor Effect table. For the Estimated Factor Effect table, we said that CD

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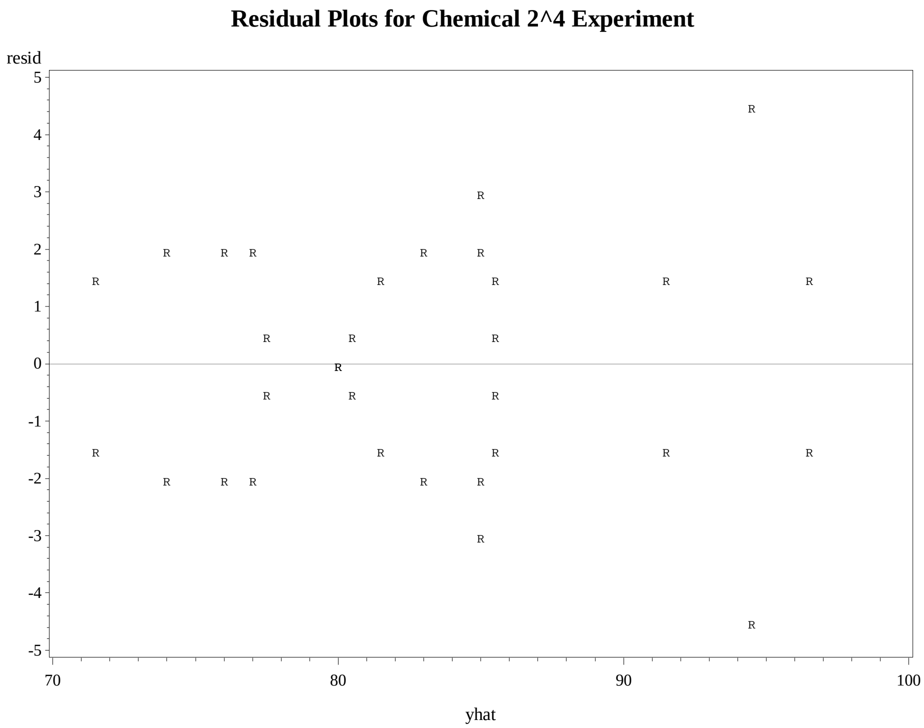
might be significant because it's contribution to SSTO was larger than 1%. However; from the ANOVA table we can see that its contribution is not significant.



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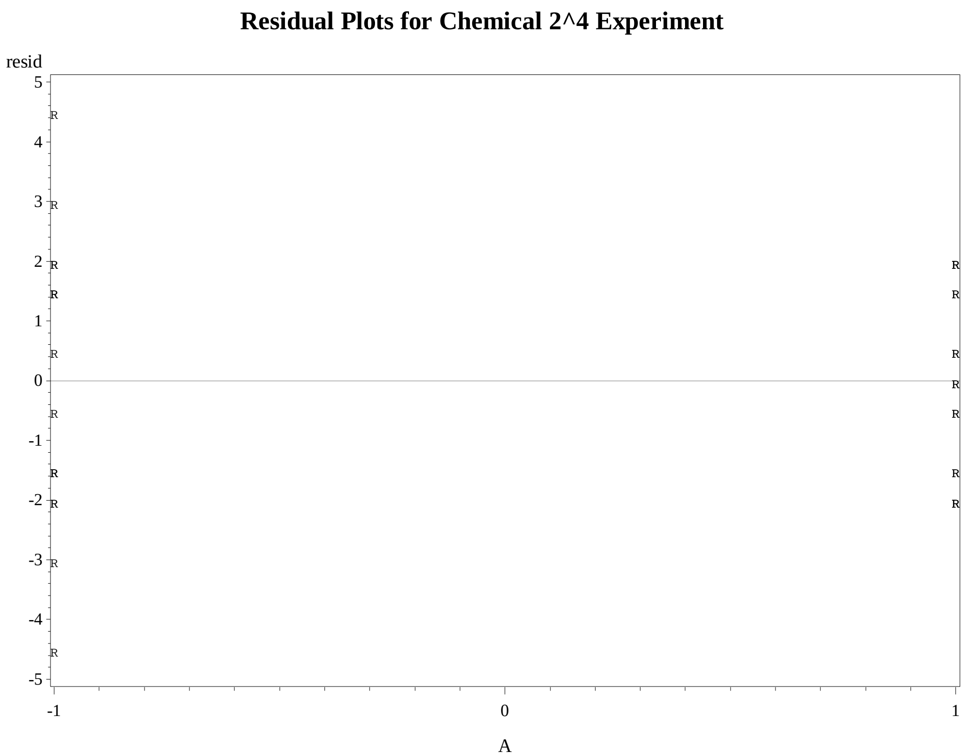

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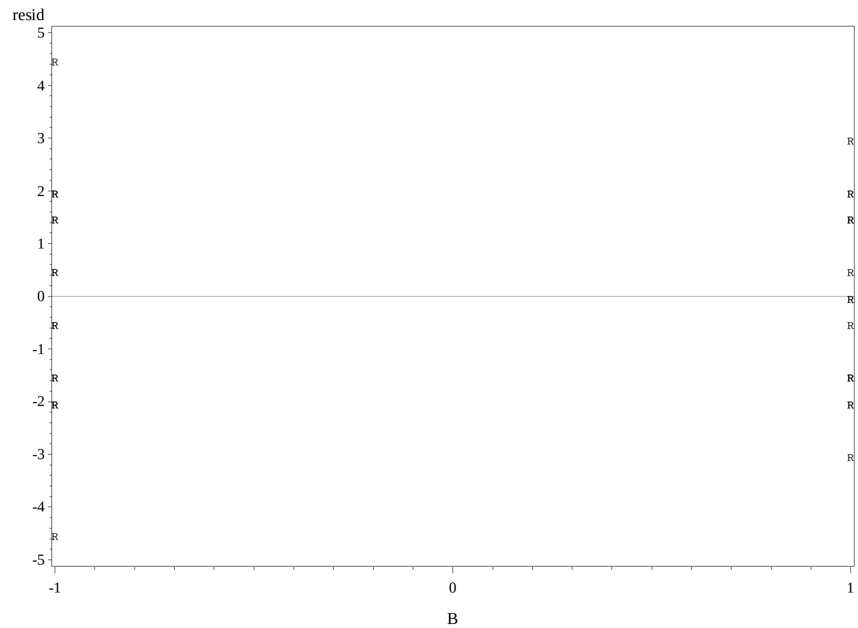
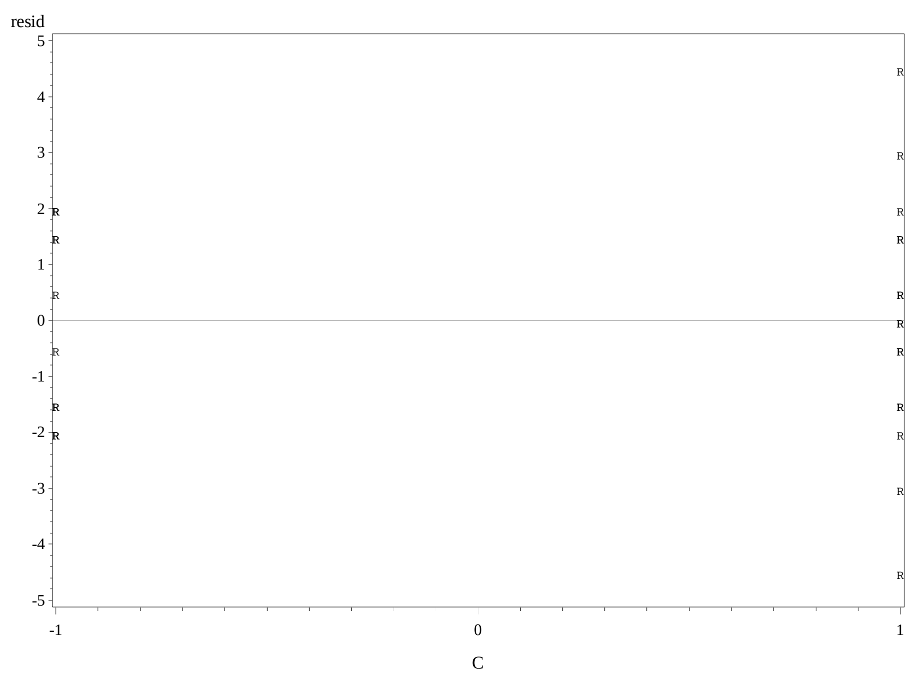
Residual Plots for Chemical 2^4 Experiment



Residual Plots for Chemical 2^4 Experiment



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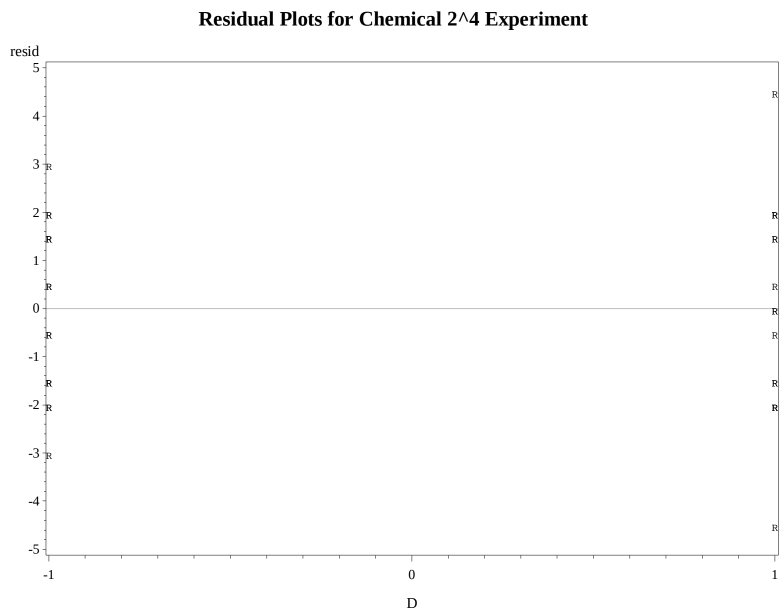

Levene Test for Factor A

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
A	1	3537.2	3537.2	2.74	0.1084
Error	30	38761.3	1292.0		

Levene Test for Factor B

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
B	1	24541.5	24541.5	7.66	0.0096
Error	30	96165.5	3205.5		

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Levene Test for Factor C

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
C	1	152.3	152.3	0.03	0.8615
Error	30	147494	4916.5		

Levene Test for Factor D

Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
D	1	4384.3	4384.3	1.26	0.2706
Error	30	104408	3480.3		

From the above SAS generated plots and Levene Tests we can see that Factor A, C and D all have a P-Value larger than 0.05 with 1 and 30 degrees of freedom. From this we can say that Factors A, C, and D have constant variance.

Note that Factor B DOES NOT have constant variance. However; Factor B is not significant in our model so the non-homogeneity of the variance for Factor B will not impact our analysis.

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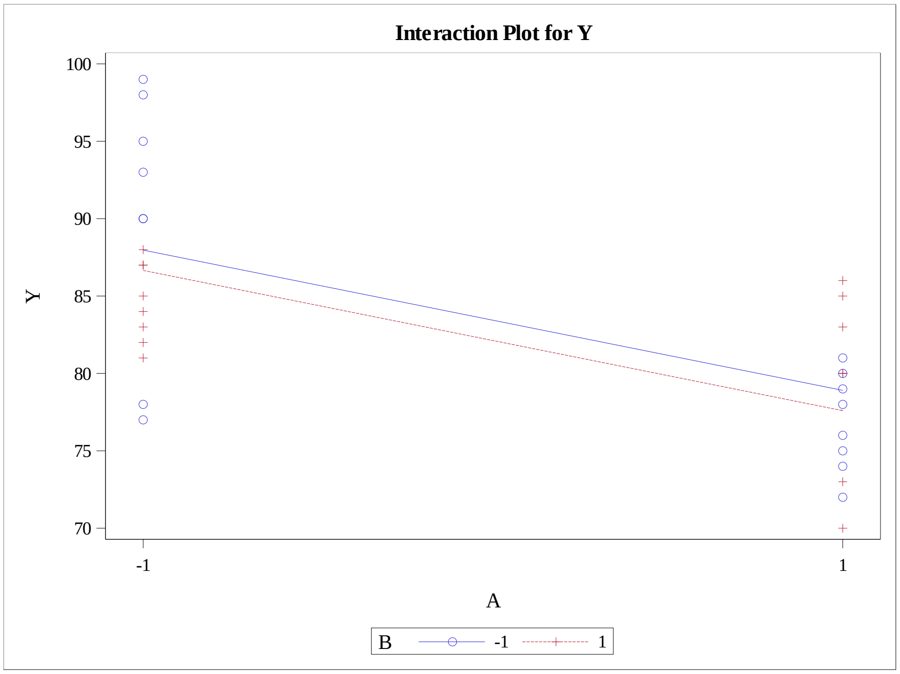
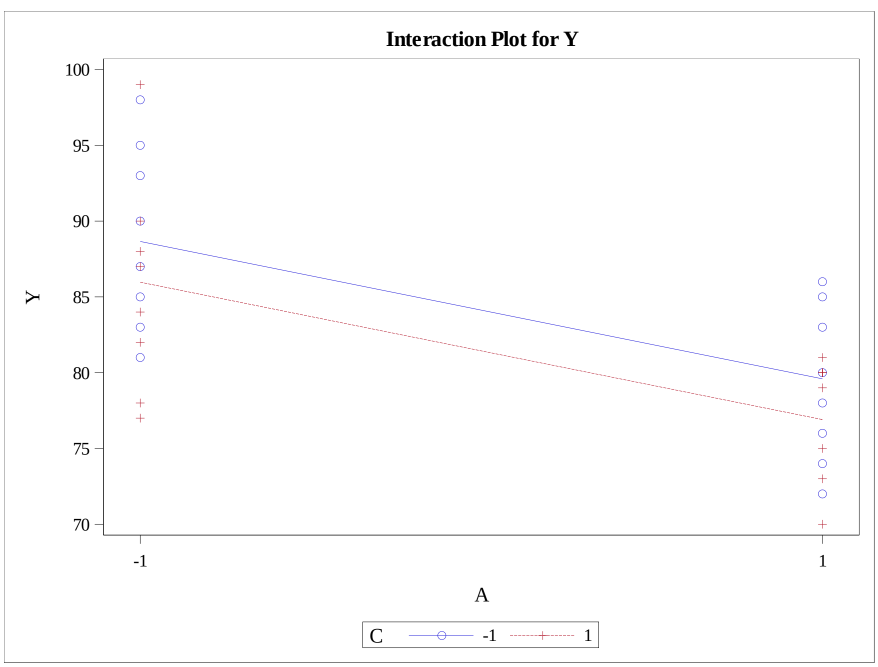
Checking for Interaction in Chemical 2^4 Experiment between Factor A and Factor B



Checking for Interaction in Chemical 2^4 Experiment between Factor A and Factor C



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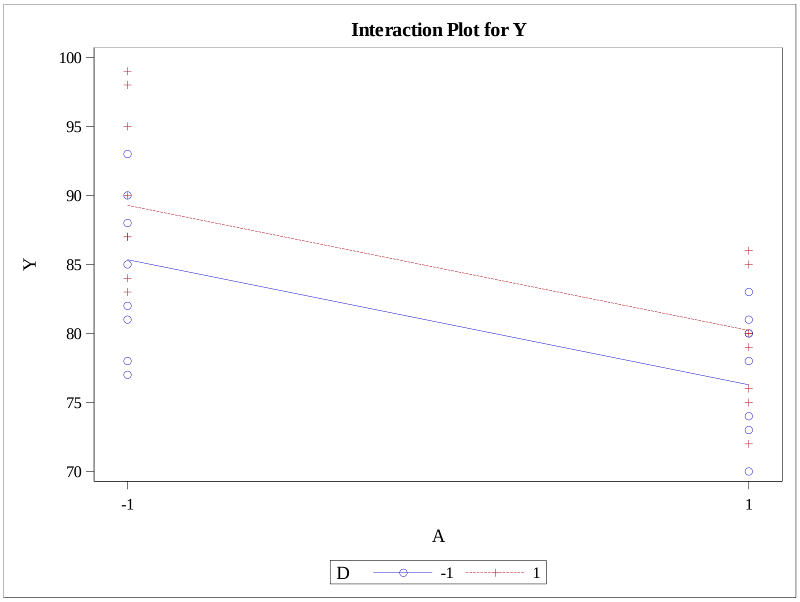
Checking for Interaction in Chemical 2^4 Experiment between Factor A and Factor D



Checking for Interaction in Chemical 2^4 Experiment between Factor B and Factor C

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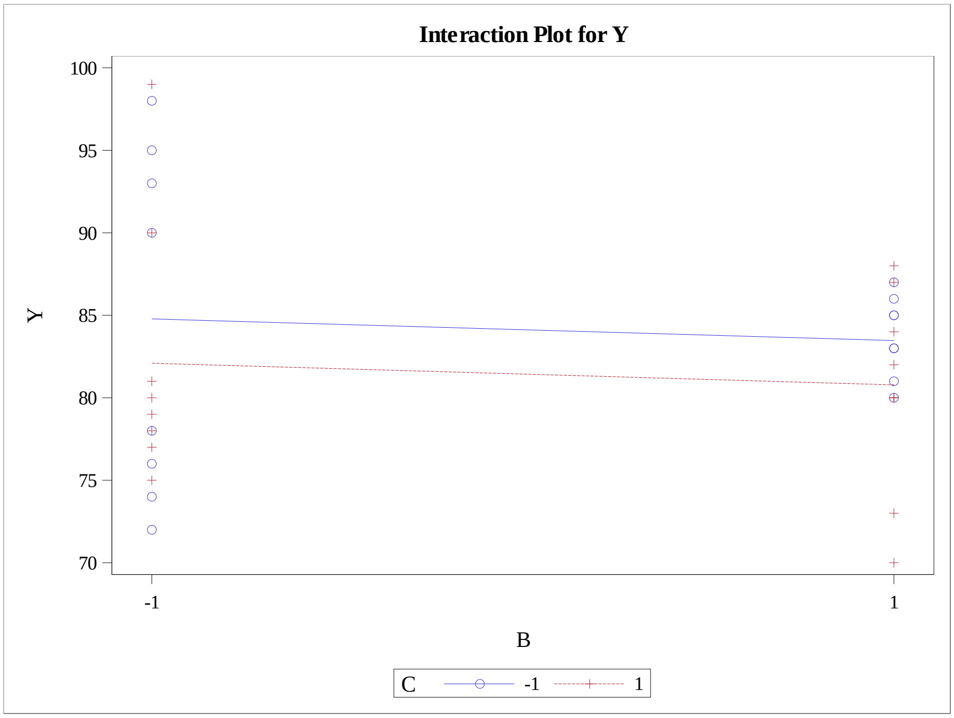
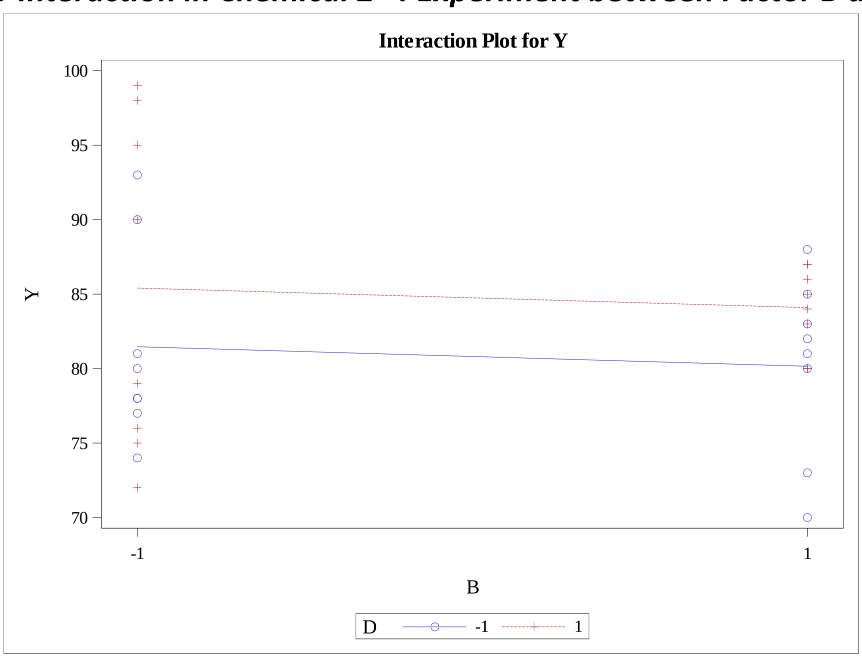


Checking for Interaction in Chemical 2^4 Experiment between Factor B and Factor D



Checking for Interaction in Chemical 2^4 Experiment between Factor C and Factor D

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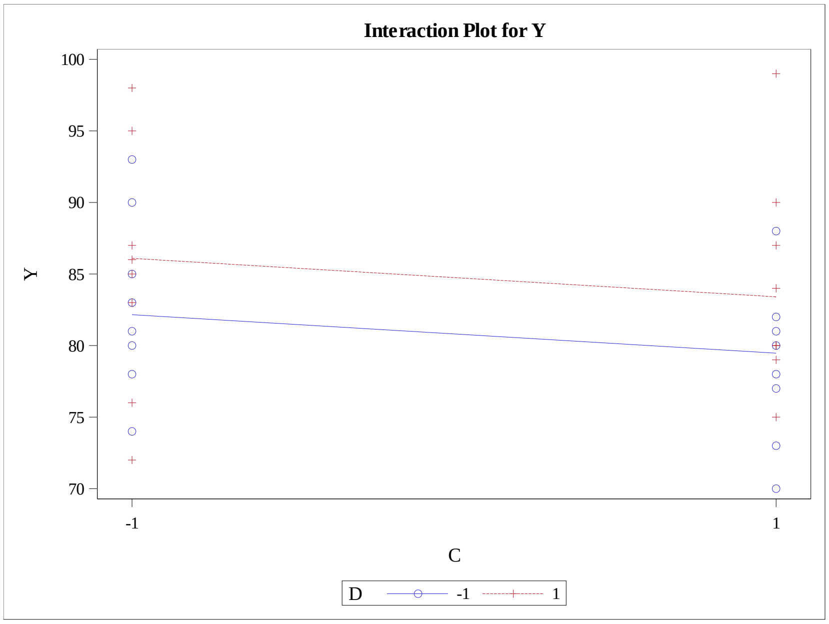
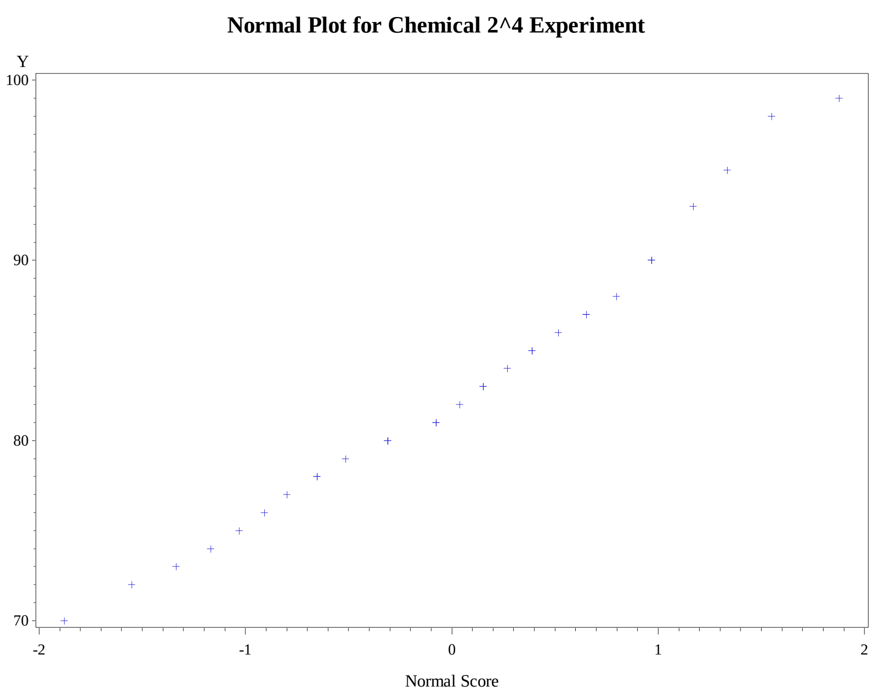


From the above SAS generated interaction plots we can see that there is no significant interaction between our main factors A, B, C and D



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Normal Tests for Chemical 2^4 Experiment

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.961348	**Pr < W**	0.2989
Kolmogorov-Smirnov	**D**	0.149749	**Pr > D**	0.0677
Cramer-von Mises	**W-Sq**	0.103525	**Pr > W-Sq**	0.0974
Anderson-Darling	**A-Sq**	0.630437	**Pr > A-Sq**	0.0936

The above Shaprio-Wilk test has a P-Value of 0.2989 which is large than 0.05 and the above SAS generated normality plot looks linear. From these we can conclude that the residuals are normally distributed.

Since all significant main factors have constant variance and are normally distributed, we can say that the model assumptions are met. We will continue with multiple comparison using Tukey.

Alpha	0.05
Error Degrees of Freedom	16
Error Mean Square	7.65625
Critical Value of Studentized Range	2.99786
Minimum Significant Difference	2.0738

Tukey Multiple Comparison for Factor A

Comparisons significant at the 0.05 level are indicated by ***.				
A Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
-1 - 1	9.0625	6.9887	11.1363	***

Tukey Multiple Comparison for Factor B

Comparisons significant at the 0.05 level are indicated by ***.				
B Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
-1 - 1	1.3125	-0.7613	3.3863	

Tukey Multiple Comparison for Factor C

Comparisons significant at the 0.05 level are indicated by ***.				
C Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
-1 - 1	2.6875	0.6137	4.7613	***

Tukey Multiple Comparison for Factor D

Comparisons significant at the 0.05 level are indicated by ***.				
D Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
1 - -1	3.9375	1.8637	6.0113	***

We can see that the difference between level means for Factor A, C and D is significant at the 0.05 level with the Tukey MSD equal to 2.9738 with a critical value of 3.0 with 2 and 16 degrees of freedom.

7.2. Consider the experiment described in Problem 6.5. Analyze this experiment assuming that each one of the four replicates represents a block.

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A	B	Treatment Combination	Replicate I	II	III	IV
−	−	(1)	18.2	18.9	12.9	14.4
+	−	a	27.2	24.0	22.4	22.5
−	+	b	15.9	14.5	15.1	14.2
+	+	ab	41.0	43.9	36.3	39.9

runs	A	B	AB	Block1	Block2	Block3	Block4	Total
(1)	−	−	+	18.2	18.9	12.9	14.4	64.40
A	+	−	−	27.2	24	22.4	22.5	96.10
B	−	+	−	15.9	14.5	15.1	14.2	59.70
AB	+	+	+	41	43.9	36.3	39.9	161.10
total				102.3	101.3	86.7	91	381.30

$$SS_{block} = \frac{1}{4}(102.3^2 + 101.3^2 + 86.7^2 + 91^2) - \frac{381.30^2}{16} = 44.36$$

AOVA for Problem 7.2

X = Block

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	1682.473750	280.412292	92.24	<.0001
Error	9	27.360625	3.040069		
Corrected Total	15	1709.834375			

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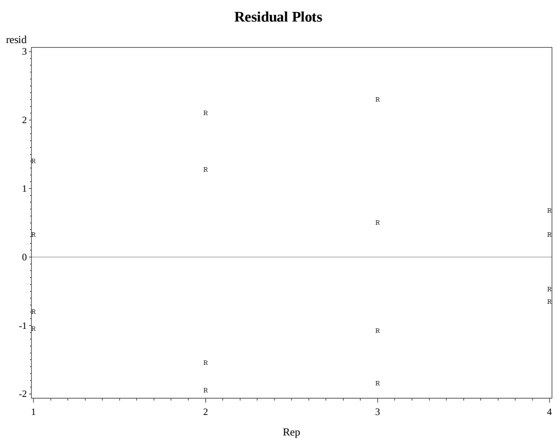
Source	DF	Type III SS	Mean Square	F Value	Pr > F
A	1	1107.225625	1107.225625	364.21	<.0001
B	1	227.255625	227.255625	74.75	<.0001
X	3	44.361875	14.787292	4.86	0.0280
AB	1	303.630625	303.630625	99.88	<.0001

From the above SAS generated ANOVA we can see that Factor A, B and AB are still significant after we block on replication. Additionally, the block effect is significant.

We will check Homogeneity of Variance for the Block Effect since that is a new significant factor



Levene's Test for Homogeneity of Y Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
X	3	3944.2	1314.7	0.09	0.9643
Error	12	175777	14648.1		

We can see that the Homogeneity of Variance for the Block effect is satisfied. The Levene test has a P-Value of 0.9643 with 3 and 12 degrees of freedom. Residual analysis and multiple comparison will be the same as in question 6.5 above.

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7.5. Consider the data from the first replicate of Problem 6.7. Construct a design with two blocks of eight observations each with *ABCD* confounded. Analyze the data.

Treatment Combination	Replicate I	Replicate II	Treatment Combination	Replicate I	Replicate II
(1)	90	93	*d*	98	95
a	74	78	*ad*	72	76
b	81	85	*bd*	87	83
ab	83	80	*abd*	85	86
c	77	78	*cd*	99	90
ac	81	80	*acd*	79	75
bc	88	82	*bcd*	87	84
abc	73	70	*abcd*	80	80

Using the contrast chart from question 6.7 we can assign block based on the + or – value of ABCD.

Block 1	Block 2
(1)	A
AB	B
AC	C
AD	D
BC	ABC
BD	ABD
CD	ACD
ABCD	BCD

Using this design ABCD is confounded with Block effect

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Contrast		SS	% of SSTO = 959.75	
A:	-80	400	41.67	*
B:	-6	2.25	0.23	
AB:	36	81	8.4	*
C:	-6	2.25	0.23	
AC:	4	1	0.01	
BC:	-10	6.25	0.65	
ABC:	-48	144	15	*
D:	40	100	10.4	*
AD:	-30	56.25	5.9	*
BD:	-12	9	0.94	
ABD:	38	90.25	9.4	*
CD:	12	9	0.94	
ACD:	-2	0.25	< .01	
BCD:	-16	16	1.7	*
Block (ABCD)		42.25	4.4	*

Testing for importance HW 7.5

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	15	959.7500000	63.9833333	.	.
Error	0	0.0000000	.		
Corrected Total	15	959.7500000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Rep	0	0.0000000	.	.	.
Block(Rep)	1	42.2500000	42.2500000	.	.
A	1	400.0000000	400.0000000	.	.
B	1	2.2500000	2.2500000	.	.
A*B	1	81.0000000	81.0000000	.	.
C	1	2.2500000	2.2500000	.	.
A*C	1	1.0000000	1.0000000	.	.

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Source	DF	Type I SS	Mean Square	F Value	Pr > F
B*C	1	6.2500000	6.2500000	.	.
A*B*C	1	144.0000000	144.0000000	.	.
D	1	100.0000000	100.0000000	.	.
A*D	1	56.2500000	56.2500000	.	.
B*D	1	9.0000000	9.0000000	.	.
A*B*D	1	90.2500000	90.2500000	.	.
C*D	1	9.0000000	9.0000000	.	.
A*C*D	1	0.2500000	0.2500000	.	.
B*C*D	1	16.0000000	16.0000000	.	.
A*B*C*D	0	0.0000000	.	.	.

ANOVA with Significant Factors HW 7.5

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	10	934.2500000	93.4250000	18.32	0.0025
Error	5	25.5000000	5.1000000		
Corrected Total	15	959.7500000			

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Rep	0	0.0000000	.	.	.
Block(Rep)	1	42.2500000	42.2500000	8.28	0.0347
A	1	400.0000000	400.0000000	78.43	0.0003
B	1	2.2500000	2.2500000	0.44	0.5360
C	1	2.2500000	2.2500000	0.44	0.5360
AB	1	81.0000000	81.0000000	15.88	0.0105

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Source	DF	Type III SS	Mean Square	F Value	Pr > F
ABC	1	144.0000000	144.0000000	28.24	0.0032
D	1	100.0000000	100.0000000	19.61	0.0068
AD	1	56.2500000	56.2500000	11.03	0.0210
ABD	1	90.2500000	90.2500000	17.70	0.0084
BCD	1	16.0000000	16.0000000	3.14	0.1367

We can see that A, AB, ABC, D, AD, ABD, and the block effect are significant with P-Values larger than $\alpha = 0.05$ and a F-Value for 5.99 with 1 and 16 degrees of freedom.

Residual analysis is the same as for question 6.7 above

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7.6. Repeat Problem 7.5 assuming that four blocks are required. Confound *ABD* and *ABC* (and consequently *CD*) with blocks.

Let $x_1 \rightarrow A, x_2 \rightarrow B, x_3 \rightarrow C, x_4 \rightarrow D$, then $L_1 = x_1 + x_2 + x_4, L_2 = x_1 + x_2 + x_3$

Run	L1	L1(mod2)	L2	L2(mod2)
(1)	0	0	0	0
A	1	1	1	1
B	1	1	1	1
AB	2	0	2	0
C	0	0	1	1
AC	1	1	2	0
BC	1	1	2	0
ABC	2	0	3	1
D	1	1	0	0
AD	2	0	1	1
BD	2	0	1	1
ABD	3	1	2	0
CD	1	1	1	1
ACD	2	0	2	0
BCD	2	0	2	0
ABCD	3	1	3	1

BLOCK 1
(1) = 90
AB = 83
ACD = 79
BCD = 87
Total = 339

BLOCK 2
C = 77
ABC = 73
AD = 72
BD = 87
Total = 309

BLOCK 3
AC = 81
BC = 88
D = 98
ABD = 85
Total = 352

BLOCK 4
A = 74
B = 81
CD = 99
ABCD = 80
Total = 334

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$$SS_{block} = \frac{1}{4}(339^2 + 309^2 + 352^2 + 334^2) - \frac{1334^2}{16} = 243.25$$

Contrast		SS
A:	-80	400
B:	-6	2.25
AB:	36	81
C:	-6	2.25
AC:	4	1
BC:	-10	6.25
ABC:	-48	144
D:	40	100
AD:	-30	56.25
BD:	-12	9
ABD:	38	90.25
CD:	12	9
ACD:	-2	0.25
BCD:	-16	16
ABCD:	26	42.25

With ABD ABC CD Confanded

Contrast		SS	%
A:	-80	400	41.67
B:	-6	2.25	0.23
AB:	36	81	8.4
C:	-6	2.25	0.23
AC:	4	1	0.01
BC:	-10	6.25	0.65
D:	40	100	10.4
AD:	-30	56.25	5.9
BD:	-12	9	0.94
ACD:	-2	0.25	<.01
BCD:	-16	16	1.7
ABCD:	26	42.25	4.4
Block:		243.25	25.35

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Testing for importance HW 7.6

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Rep	0	0.0000000	.	.	.
Block(Rep)	3	243.2500000	81.0833333	.	.
A	1	400.0000000	400.0000000	.	.
B	1	2.2500000	2.2500000	.	.
A*B	1	81.0000000	81.0000000	.	.
C	1	2.2500000	2.2500000	.	.
A*C	1	1.0000000	1.0000000	.	.
B*C	1	6.2500000	6.2500000	.	.
A*B*C	0	0.0000000	.	.	.
D	1	100.0000000	100.0000000	.	.
A*D	1	56.2500000	56.2500000	.	.
B*D	1	9.0000000	9.0000000	.	.
A*B*D	0	0.0000000	.	.	.
C*D	0	0.0000000	.	.	.
A*C*D	1	0.2500000	0.2500000	.	.
B*C*D	1	16.0000000	16.0000000	.	.
A*B*C*D	1	42.2500000	42.2500000	.	.

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ANOVA with Significant Factors HW 7.6

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	10	927.2500000	92.7250000	14.27	0.0045
Error	5	32.5000000	6.5000000		
Corrected Total	15	959.7500000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Rep	0	0.0000000	.	.	.
Block(Rep)	3	243.2500000	81.0833333	12.47	0.0093
A	1	400.0000000	400.0000000	61.54	0.0005
B	1	2.2500000	2.2500000	0.35	0.5819
C	1	2.2500000	2.2500000	0.35	0.5819
D	1	100.0000000	100.0000000	15.38	0.0112
AB	1	81.0000000	81.0000000	12.46	0.0167
AD	1	56.2500000	56.2500000	8.65	0.0322
ABCD	1	42.2500000	42.2500000	6.50	0.0513

We can see that A, D, AB, AD, ABCD, and Block Effect are all significant when compared to a F-Value with 1 and 5 degrees of freedom for factors A, D, AB, AD, ABCD and a F-Value with 3 and 5 degrees of freedom for the block effect.

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7.21. Consider the 2^6 design in eight blocks of eight runs each with *ABCD*, *ACE*, and *ABEF* as the independent effects chosen to be confounded with blocks. Generate the design. Find the other effects confounded with blocks.

See attached do by hand for linear combination technique for deciding blocks

Block 1	Block 2	Block 3	Block 4	Block 5	Block 6	Block 7	Block 8
(1)	a	b	c	d	e	f	ab
acf	ef	ce	af	ae	ad	ac	cd
ade	de	df	be	bf	bc	bd	ef
bce	bcd	acd	abd	abc	abf	abe	ace
abcd	abce	aef	def	acdf	acf	cdf	adf
cdef	abdf	abcf	acde	bcde	abcde	adef	bcf
abef	bef	bcdef	bcdf	cef	acef	abcdf	abcdef
bdf	acdef	abde	abcef	abdef	bdef	bcef	bde

(ABCD)(ACE) = BDE
(ABCD)(ABEF) = CDEF
(ACE)(ABEF) = BCF
(ABCD)(BDE) = ACE
(ABCD)(CDEF) = ABEF
(ABCD)(BCF) = ADF
(ACE)(BDE) = ABCD
(ACE)(CDEF) = ADF
(ACE)(BCF) = ABEF
(ABEF)(BDE) = ADF
(ABEF)(CDEF) = ABCD
(ABEF)(BCF) = ACE

So, confounded factors are
ABCD, ACE, ABEF, BDE, CDEF, BCF, ADF

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8.1. Suppose that in the chemical process development experiment described in Problem 6.7, it was only possible to run a one-half fraction of the 2^4 design. Construct the design and perform the statistical analysis, using the data from replicate I.

Treatment Combination	Replicate I	Replicate II	Treatment Combination	Replicate I	Replicate II
(1)	90	93	*d*	98	95
a	74	78	*ad*	72	76
b	81	85	*bd*	87	83
ab	83	80	*abd*	85	86
c	77	78	*cd*	99	90
ac	81	80	*acd*	79	75
bc	88	82	*bcd*	87	84
abc	73	70	*abcd*	80	80

RUN	A	B	C	D = ABC	TRT	VALUE
1	-	-	-	-	(1)	90
2	+	-	-	+	AD	72
3	-	+	-	+	BD	87
4	-	-	+	+	CD	99
5	+	+	-	-	AB	83
6	+	-	+	-	AC	81
7	-	+	+	-	BC	88
8	+	+	+	+	ABCD	80

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Factor	Effect	SS	PERCENTAGE
A	-12	288	64
B	-1	2	0.44
C	4	32	7
D	-1	2	0.44
AB	6	72	16.1
AC	-1	2	0.45
AD	-5	50	11.16
TOTAL		448	

SOURCE	DF	SS	MS	F*	F 1,1 (0.05)
A	1	288	288	144	161.4
B	1	2	2	1	161.4
C	1	32	32	16	161.4
D	1	2	2	1	161.4
AB	1	72	72	36	161.4
AD	1	50	50	25	161.4
ERROR	1	2	2		
TOTAL	7	448			

From the above ANOVA table we can see that when we run the experiment as a fractional factorial none of our factors are significant.

Our design is a 2_{III}^{4-1} with the below Aliased structure

I = ABCD

A = BCD   B = ACD   C = ABD   D = ABC

AB = CD   AC = BD   AD = BC

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8.10. An article by J. J. Pignatiello Jr. and J. S. Ramberg in the *Journal of Quality Technology* (Vol. 17, 1985, pp. 198–206) describes the use of a replicated fractional factorial to investigate the effect of five factors on the free height of leaf springs used in an automotive application. The factors are A = furnace temperature, B = heating time, C = transfer time, D = hold down time, and E = quench oil temperature. The data are shown in Table P8.1

■ **TABLE P8.1**
Leaf Spring Experiment

A	B	C	D	E	Free Height		
−	−	−	−	−	7.78	7.78	7.81
+	−	−	+	−	8.15	8.18	7.88
−	+	−	+	−	7.50	7.56	7.50
+	+	−	−	−	7.59	7.56	7.75
−	−	+	+	−	7.54	8.00	7.88
+	−	+	−	−	7.69	8.09	8.06
−	+	+	−	−	7.56	7.52	7.44
+	+	+	+	−	7.56	7.81	7.69
−	−	−	−	+	7.50	7.25	7.12
+	−	−	+	+	7.88	7.88	7.44
−	+	−	+	+	7.50	7.56	7.50
+	+	−	−	+	7.63	7.75	7.56
−	−	+	+	+	7.32	7.44	7.44
+	−	+	−	+	7.56	7.69	7.62
−	+	+	−	+	7.18	7.18	7.25
+	+	+	+	+	7.81	7.50	7.59

(a) Write out the alias structure for this design. What is the resolution of this design?

(b) Analyze the data. What factors influence the mean free height?

(c) Calculate the range and standard deviation of the free height for each run. Is there any indication that any of these factors affects variability in the free height?

(d) Analyze the residuals from this experiment, and comment on your findings.

(e) Is this the best possible design for five factors in 16 runs? Specifically, can you find a fractional design for five factors in 16 runs with a higher resolution than this one?

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Generator is ABCD. Refer to the below table to see that ABCD has all + value

A	B	C	D	E	ABCD	AB	AC	AD	AE	BE	CE	DE
-	-	-	-	-	+	+	+	+	+	+	+	+
+	-	-	+	-	+	-	-	+	-	+	+	-
-	+	-	+	-	+	-	+	-	+	-	+	-
+	+	-	-	-	+	+	-	-	-	-	+	+
-	-	+	+	-	+	+	-	-	+	+	-	-
+	-	+	-	-	+	-	+	-	-	+	-	+
-	+	+	-	-	+	-	-	+	+	-	-	+
+	+	+	+	-	+	+	+	+	-	-	-	-
-	-	-	-	+	+	+	+	+	-	-	-	-
+	-	-	+	+	+	-	-	+	+	-	-	+
-	+	-	+	+	+	-	+	-	-	+	-	+
+	+	-	-	+	+	+	-	-	+	+	-	-
-	-	+	+	+	+	+	-	-	-	-	+	+
+	-	+	-	+	+	-	+	-	+	-	+	-
-	+	+	-	+	+	-	-	+	-	+	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+

Alias Structure

I = ABCD

A = BCD   B = ACD   C = ABD   D = ABC   E = ABCDE   AB = CD   AC = BD   AD = BC
AE = BCDE   BE = ACDE   CE = ABDE   DE = ABCE

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Estimating Factor Effect

Parameter	Estimate	Standard Error	t Value	Pr > |t|
A	0.24208333	0.04006907	6.04	<.0001
B	-0.16375000	0.04006907	-4.09	0.0002
C	-0.04958333	0.04006907	-1.24	0.2242
D	0.09125000	0.04006907	2.28	0.0290
E	-0.23875000	0.04006907	-5.96	<.0001
AB	-0.02958333	0.04006907	-0.74	0.4652
AC	0.00125000	0.04006907	0.03	0.9753
AD	-0.02291667	0.04006907	-0.57	0.5710
AE	0.06375000	0.04006907	1.59	0.1206
BE	0.15291667	0.04006907	3.82	0.0005
CE	-0.03291667	0.04006907	-0.82	0.4169
DE	0.03958333	0.04006907	0.99	0.3300

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ANOVA with Significant Factors

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	1.98964167	0.49741042	23.74	<.0001
Error	43	0.90113958	0.02095673		
Corrected Total	47	2.89078125			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	0.70325208	0.70325208	33.56	<.0001
B	1	0.32176875	0.32176875	15.35	0.0003
E	1	0.68401875	0.68401875	32.64	<.0001
BE	1	0.28060208	0.28060208	13.39	0.0007

We can see that factors A, B, E and BE are significant when compared to a F-Value with 1 and 43 degrees of freedom and an alpha level of $\alpha = 0.05$

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8.11. An article in *Industrial and Engineering Chemistry* ("More on Planning Experiments to Increase Research Efficiency," 1970, pp. 60–65) uses a 2^{5-2} design to investigate the effect of A = condensation temperature, B = amount of material 1, C = solvent volume, D = condensation time, and E = amount of material 2 on yield. The results obtained are as follows:

$$e = 23.2 \quad ad = 16.9 \quad cd = 23.8 \quad bde = 16.8$$
$$ab = 15.5 \quad bc = 16.2 \quad ace = 23.4 \quad abcde = 18.1$$

(a) Verify that the design generators used were $I = ACE$ and $I = BDE$.

(b) Write down the complete defining relation and the aliases for this design.

(c) Estimate the main effects.

(d) Prepare an analysis of variance table. Verify that the AB and AD interactions are available to use as error.

(e) Plot the residuals versus the fitted values. Also construct a normal probability plot of the residuals. Comment on the results.

RUN	A	B	C	D = BE	E = AC	TREATMENT	AB	AD
1	-	-	-	-	+	E = 23.2	+	+
2	+	-	-	+	-	AD = 16.9	-	+
3	-	+	-	+	+	BDE = 16.8	-	-
4	-	-	+	+	-	CD = 23.8	+	-
5	+	+	-	-	-	AB = 15.5	+	-
6	+	-	+	-	+	ACE = 23.4	-	-
7	-	+	+	-	-	BC = 16.2	-	+
8	+	+	+	+	+	ABCDE = 18.1	+	+

<u>Complete Defining Relationship and Aliases Structure</u>

I = ACE = BDE = ABCD

A = CE = ABDE = BCD	B = ABCE = DE = ACD	C = AE = BCDE = ABD	E = AC = BD = ABCDE
AB = CD = BCE = ADE	AD = BC = CDE = ABE	AD = BC = CDE = ABE	I = ACE = BDE = ABCD

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Main Effect Estimate

Effect A = -1.525

Effect B = -5.175

Effect C = 2.275

Effect D = -0.675

Effect E = 2.275

Effect AB = 1.825

Effect = AD = -1.25

Sums of Squares

SSA = 4.65

SSB = 53.56

SSC = 10.35

SSD = 0.911

SSE = 10.35

SSTO = 89.74

Percent Contribution

A = 5.18

B = 59.68

C = 11.53

E = 11.53

AB = 7.40

AD = 3.48

We can use AB and AD as error since they contribute less to the SSTO

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ANOVA

SOURCE	DF	SS	MS	F*	F 1,2 (.05)
A	1	4.65	4.65	3.22	18.51
B	1	53.56	53.56	0.94	18.51
C	1	10.35	10.35	10.81	18.51
D	1	0.91	0.91	2.09	18.51
E	1	10.35	10.35	0.18	18.51
ERROR	2	9.91	4.96	2.09	18.51
TOTAL	7	89.74			

We can see that none of the main factors are significant.