Question Number	Scheme	Marks
1	$(3-2x)^{-4} = 3^{-4}\left(1-\frac{2}{3}x\right)^{-4}$ $\qquad 3^{-4}$ or $\frac{1}{81}$	B1
	$=\frac{1}{81}\times\underline{\left(1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2+\ldots\right)}$	M1A1
	$=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$	A1
		(4 marks)
	Alternative: $(3-2x)^{-4} = 3^{-4}+(-4)(3)^{-5}(-2x)+\frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2+\ldots$	B1 M1 A1
	$=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$	A1
		(4 marks)

B1 For taking out a factor of 3^{-4}

Evidence would be seeing either 3^{-4} or $\frac{1}{81}$ before the bracket.

M1 For the form of the binomial expansion with $n=-4$ and a term of (kx)

To score M1 it is sufficient to see just the second and third term with the correct coefficient multiplied by the correct power of x. Condone sign slips. Look for $\ldots+(-4)(kx)+\frac{(-4)(-5)}{2!}(kx)^2\ldots$

A1 Any (unsimplified) form of the binomial expansion. Ignore the factor before the bracket.

The bracketing must be correct but it is acceptable for them to recover from "missing" brackets for full marks.

Look for $1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2+$ or $1+\frac{8}{3}x+\frac{40}{9}x^2+$

A1 cao $=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$. Ignore any further terms.

Alternative

B1 For seeing either 3^{-4} or $\frac{1}{81}$ as the first term

M1 It is sufficient to see the second and third term (unsimplified or simplified) condoning missing brackets.

ie. Look for $\ldots+(-4)(3)^{-5}(kx)+\frac{(-4)(-5)}{2}(3)^{-6}(kx)^2$

A1 Any (un simplified) form of the binomial expansion. $\ldots+(-4)(3)^{-5}(-2x)+\frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2$

A1 Must now be simplified cao

Question	Scheme	Marks
2	Assume the sequence is geometric	B1
	So $(r =)\dfrac{1+2k}{k} = \dfrac{3+3k}{1+2k}$	M1
	$\Rightarrow (1+2k)^2 = k(3+3k) \Rightarrow k^2+k+1=0$	A1
	But $k^2+k+1=\left(k+\dfrac{1}{2}\right)^2-\dfrac{1}{4}+1 \geqslant \dfrac{3}{4}>0$ (since $\left(k+\dfrac{1}{2}\right)^2 \geqslant 0$ for all (real) k)	dM1
	This is a contradiction and hence the original assumption is not true. The sequence is not geometric.	A1
		(5)
		(5 marks)

Notes:

(a)

B1: States an appropriate assumption to set up the contradiction.

M1: Uses the assumption to set up an equation in k only.

Allow equivalent work e.g. $kr = 1+2k,\ kr^2 = 3+3k \Rightarrow 3+3k = k\left(\dfrac{1+2k}{k}\right)^2$

Allow use of $\neq$ for $=$ e.g. $\sqrt{\dfrac{3+3k}{k}} \neq \dfrac{1+2k}{k}$

This may be implied by e.g. $\dfrac{1+2k}{k}$ is not the same as $\dfrac{3+3k}{1+2k}$.

A1: Reaches a correct quadratic equation in k, need not be all on one side, but terms in k and k^2 should be collected. Allow use of $\neq$ for $=$ e.g. $k^2+k+1 \neq 0$

dM1: Completes the square, considers the discriminant or other valid means used to reach a point where a contradiction can be deduced. E.g. as scheme, or $b^2-4ac = 1^2-4(1)(1) = -3 < 0$ may be used. Accept use of calculator to give roots $k = \dfrac{-1 \pm \mathrm{i}\sqrt{3}}{2}$ so k is not real, which contradicts k being a member of the real sequence.

Depends on the previous M.

A1: Correct work leading to a contradiction with deduction of a contradiction made and conclusion given. **This mark is available even if B0 is given at the start. So 01111 is possible.**

If they are using the discriminant or calculator route then there is no need to mention "real" as long as they conclude that e.g. the geometric sequence is not possible. This can score both the dM1 and A1.

Note that it is possible to answer Q3 using integration by parts (either way round) BUT it is very demanding and candidates are unlikely to get very far and will gain no marks.
If they reach $Ax + B\ln x + C\ln(x-4), A, B, C \neq 0$ send to review.

Question Number	Scheme		Marks
3	$\dfrac{A}{x}+\dfrac{B}{x-4}=-\dfrac{2}{x}+\dfrac{14}{x-4}$	For an attempt to find partial fractions of the form $\dfrac{A}{x}+\dfrac{B}{x-4}$ where A and B are numeric and non-zero	M1
		Correct fractions $-\dfrac{2}{x}+\dfrac{14}{x-4}$	A1
	$\dfrac{3x^2+8}{x^2-4x}=3+\mathrm{f}(x)$ Where $\mathrm{f}(x)=\dfrac{A}{x}+\dfrac{B}{x-4}$ with numeric A and B or the letters "A" and "B" **or** Where $\mathrm{f}(x)=\dfrac{Cx+D}{x^2-4x}$ with numeric C and D with C, D not both zero		B1
	This mark is for integrating at least 2 terms of the form $\dfrac{\alpha}{x\pm k}$ to obtain $\beta\ln(x\pm k)$ where k may be zero Allow e.g. $\ln(x\pm k)$, $\ln(k\pm x)$, $\ln\lvert x\pm k\rvert$, also allow $\ln x \pm k$ for this mark		M1
	For $\displaystyle\int 3-\frac{2}{x}+\frac{14}{x-4}\,\mathrm{d}x \rightarrow 3x-2\ln\lvert x\rvert+14\ln\lvert x-4\rvert$ following through on their coefficients requires modulus signs and/or brackets around the $x-4$ unless they are implied by later work. E.g. allow $3x-2\ln x+14\ln(x-4)$		A1ft
	$=9-2\ln 3-3-14\ln 3=\ldots$ Evidence of the use of both limits 3 and 1 and subtracts the right way round and reaches an expression of the form $P+Q\ln R$, where P, Q and R are rational and non-zero and $R>0$ **Dependent on the previous method mark**		**d**M1
	$=6-16\ln 3$ Accept equivalents e.g. $6-8\ln 9, 6+16\ln\left(\dfrac{1}{3}\right), 6-\ln 3^{16}, 6+\ln\dfrac{1}{43046721}, 6-\ln 43046721$ $6+\ln\dfrac{1}{9\times 3^{14}}, 6-\ln\left(9\times 3^{14}\right)$ etc.		A1
			(7)
			[7 marks]

Special Case:

Some students know to use PF but fail to see it is an improper fraction and the solution will look similar to this:

$$\frac{3x^2+8}{x^2-4x}=\frac{14}{x-4}-\frac{2}{x}$$

$$\int_1^3 \frac{3x^2+8}{x^2-4x}\,\mathrm{d}x=\int_1^3 \frac{14}{x-4}-\frac{2}{x}\,\mathrm{d}x=\Big[14\ln\lvert x-4\rvert-2\ln\lvert x\rvert\Big]_{(x=1)}^{(x=3)}$$

$$=14\ln 1-2\ln 3-(14\ln 3-2\ln 1)=-16\ln 3$$

These students can potentially score **M1 A1 B0 M1 A0 dM0 A0** for 3 out of 7

Question Number	Scheme	Marks
4 (a)	For correct parameter $t=\frac{\pi}{4}$ or x coordinate at P $x=4$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}=\frac{-4\sin 2t}{3\sec^2 t}$	M1 A1
	Equation of tangent $y-0=-\frac{2}{3}\left(x-4\right)\Rightarrow y=-\frac{2}{3}x+\frac{8}{3}$	dM1 A1
		(5)
(b)	$k=1-\sqrt{3}$	B1
		(1)
(c)	$-1\leqslant f\leqslant 2$	M1 A1
		(2) **(8 marks)**

(a)

B1: Correct parameter or x coordinate at P.

The correct x coordinate $x=4$ (can even be scored following $x=45°$)

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ using $\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$. Condone poor differentiation.

A1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-4\sin 2t}{3\sec^2 t}$. You may see other forms for this following use of identities.

If you see $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-4\sin 2t}{3\sec^2 \underline{\underline{x}}}$ then only allow if they subsequently use this as $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-4\sin 2t}{3\sec^2 t}$

Other possible correct gradients are $\frac{-8\sin t\cos t}{3\sec^2 t}$ and $\frac{-8\sin t\cos^3 t}{3}$

ISW after you see a correct answer. Quite often we see candidates incorrectly adapting a correct answer.

It is also possible for candidates to find $\frac{\mathrm{d}y}{\mathrm{d}t}$ at $t="\frac{\pi}{4}"$, $\frac{\mathrm{d}x}{\mathrm{d}t}$ at $t="\frac{\pi}{4}"$ and divide the two values.

dM1: Attempts to find the equation of the tangent at $t=\frac{\pi}{4}$

It is dependent upon the previous M and use of $\left(4,0\right)$ and $t=\frac{\pi}{4}$

Allow substitution into their adapted $\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: $y=-\frac{2}{3}x+\frac{8}{3}$

(b)

B1: $k=1-\sqrt{3}$

(c)

M1: Either end correct. Allow strict inequalities here. E.g f < 2 scores M1 but f > 2 is M0

A1: $-1\leqslant f\leqslant 2$ o.e such as $\left[-1,2\right]$ and $-1\leqslant y\leqslant 2$

Question	Scheme	Marks
5(a)	$16x^3 - 9kx^2y + 8y^3 = 875$	
	$(8)y^3 \to (8\times)3y^2\dfrac{\mathrm{d}y}{\mathrm{d}x}$	B1
	$-9kx^2y \to ...kxy \pm ... - 9kx^2\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$48x^2 - 18kxy - 9kx^2\dfrac{\mathrm{d}y}{\mathrm{d}x} + 24y^2\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}(24y^2 - 9kx^2) = 18kxy - 48x^2 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = ...$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6kxy - 16x^2}{8y^2 - 3kx^2}$ *	A1*
		(4)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0, x = \dfrac{5}{2} \Rightarrow \dfrac{6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2}{8y^2 - 3k\left(\frac{5}{2}\right)^2} = 0$ or $x = \dfrac{5}{2} \Rightarrow 16\left(\dfrac{5}{2}\right)^3 - 9k\left(\dfrac{5}{2}\right)^2 y + 8y^3 = 875$	M1
	$15ky - 100 = 0$ or $250 - \dfrac{225}{4}ky + 8y^3 = 875$	A1
	E.g. $16\left(\dfrac{5}{2}\right)^3 - 9k\left(\dfrac{5}{2}\right)^2\left(\dfrac{20}{3k}\right) + 8\left(\dfrac{20}{3k}\right)^3 = 875 \Rightarrow k^3 = ... \left(= \dfrac{64}{27}\right) \Rightarrow k = ...$	M1
	$k = \dfrac{4}{3}$	A1
		(4)
		(8 marks)

Notes

(a)

B1: For $y^3 \to 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow if seen in aside working without the 8.

M1: Correct attempt at implicit differentiation on the $-9kx^2y$. Look for $-9kx^2y \to ...kxy \pm ... -9kx^2\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Collects both of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms together, collects non $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms the other side of the equation, factorises and divides to achieve $\frac{\mathrm{d}y}{\mathrm{d}x} = ...$ Must have two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms, one from the attempt at differentiating $-9kx^2y$ and one from the attempt at differentiating y^3, but condone if an extra $\frac{\mathrm{d}y}{\mathrm{d}x} = ...$ term has been included.

A1*: Achieves $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$ with no errors

(b)

M1: Uses the information to produce one equation in k and y, e.g. sets the $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to 0 and substitutes $x = \frac{5}{2}$, or substitutes $x = \frac{5}{2}$ into the given equation. Allow one slip substituting.

A1: A correct equation without fraction and with simplified coefficients, so $15ky - 100 = 0$ oe or $250 - \frac{225}{4}ky + 8y^3 = 875$ oe

M1: For a complete method to find k so solves the equations simultaneously to achieve a value for k. May find y first e.g substitutes their $k = \frac{20}{3y}$ into the original equation, solves to find y and substitutes this back into $k = \frac{20}{3y}$ to find k via $250 - 375 + 8y^3 = 875 \Rightarrow y = 5 \Rightarrow k = \frac{20}{3\times 5} = ...$

A1 $k = \frac{4}{3}$

Alt:

If they do not substitute $x = \frac{5}{2}$ initially then score

M1: Uses numerator of $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to 0 to find y in terms of x and k and substitute into original equation (allowing one slip)

A1: Correct equation:

$$6kxy - 16x^2 = 0 \Rightarrow y = \frac{8x^2}{3kx} \Rightarrow 16x^3 - 9kx^2\left(\frac{8x^2}{3kx}\right) + 8\left(\frac{8x^2}{3kx}\right)^3 = 875 \text{ oe}$$

M1: Substitutes $x = \frac{5}{2}$ and solves to find k

A1: $k = \frac{4}{3}$

Question Number	Scheme	Marks
6	$x = 2\sin\theta \Rightarrow \dfrac{dx}{d\theta} = 2\cos\theta$	B1
	$\displaystyle\int \frac{1}{\left(4-x^2\right)^{3/2}}dx = \int \frac{1}{\left(4-4\sin^2\theta\right)^{3/2}} 2\cos\theta(d\theta)$	M1
	$\displaystyle= \int \frac{1}{4}\sec^2\theta(d\theta)\ OR\ \int \frac{1}{4}\times\frac{1}{\cos^2\theta}(d\theta)$	M1
	$= \dfrac{1}{4}\tan\theta$	dM1A1
	Uses limits 0 and $\dfrac{\pi}{3}$ in their integrated expression $= \left[\dfrac{1}{4}\tan\theta\right]_0^{\frac{\pi}{3}} = \dfrac{\sqrt{3}}{4}$	M1A1
		(7 marks)

B1 States either $\dfrac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta\, d\theta$. Condone $x' = 2\cos\theta$

M1 Attempt to produce integral in just θ by substituting $x = 2\sin\theta$ and using $dx = \pm A\cos\theta\ (d\theta)$
You may condone a missing $d\theta$

M1 Uses $1-\sin^2\theta = \cos^2\theta$ and simplifies integral to $\displaystyle\int C\sec^2\theta(\,d\theta)$ or $\displaystyle\int \frac{C}{\cos^2\theta}(\,d\theta)$

Again you may condone a missing $d\theta$

dM1 Dependent upon previous M1 for $\displaystyle\int \sec^2\theta \to \tan\theta$

A1 $\dfrac{1}{4}\tan\theta(+c)$. No requirement for the $+c$

M1 Changes limits in x to limits in θ of 0 and $\dfrac{\pi}{3}$, then subtracts their integrated expression either way around. The subtraction of 0 can be implied if f (0) =0. If the candidate changes the limits to 0 and 60 (degrees) it scores M0, A0. Alternatively they could attempt to change their integrated expression in θ back to a function in x and use the original limits. Such a method would require seeing either $\cos\theta = \sqrt{1-\dfrac{x^2}{4}}$ or $\tan\theta = \dfrac{\frac{x}{2}}{\sqrt{1-\frac{x^2}{4}}}$

A1 $\dfrac{\sqrt{3}}{4}$.

Question Number	Scheme	Marks
7.	$A(1, a, 5)$, $B(b, -1, 3)$, $l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$	
(a)	**Either** at point $A: \lambda = 1$ **or** at point $B: \lambda = 3$	M1
	leading to either $a = -3$ **or** $b = 5$	A1
	leading to both $a = -3$ **and** $b = 5$	A1
		[3]
(b)	Attempts $\pm\left[('5\mathbf{i}' - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i}\ '-3\mathbf{j}' + 5\mathbf{k})\right]$ subtraction either way round	M1
	$\overrightarrow{AB} = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ o.e. subtraction correct way round	A1
		[2]
(c)	Way 1: $(\overrightarrow{AC}) = \begin{pmatrix} 3 \\ "0" \\ -3 \end{pmatrix}$ or $(\overrightarrow{CA}) = \begin{pmatrix} -3 \\ "0" \\ 3 \end{pmatrix}$ Way 2: $AB = 2\sqrt{6},\ AC = 3\sqrt{2},\ BC = \sqrt{6}$	M1
	Way 1: $\cos C\hat{A}B = \dfrac{\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2}\,.\sqrt{(3)^2 + (0)^2 + (-3)^2}}$ Way 2: $\cos C\hat{A}B = \dfrac{24 + 18 - 6}{2\sqrt{24}\sqrt{18}}$ Or right angled triangle and $\cos C\hat{A}B = \dfrac{\sqrt{3}}{2}$ o.e.	dM1
	Way 1: $\cos C\hat{A}B = \dfrac{12 + 0 + 6}{\sqrt{24}.\sqrt{18}} = \dfrac{\sqrt{3}}{2}$ (o.e.) $\Rightarrow C\hat{A}B = 30^\circ$ * Way 2: so $C\hat{A}B = 30^\circ$	A1 * cso
		[3]
(d)	Area $CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ$	M1
	$= 3\sqrt{3}$ (or $k = 3$)	A1
		[2]
(e)	$\left(\overrightarrow{OD_1}\right) = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ **or** $= \begin{pmatrix} 1 \\ a \\ 5 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$ or $= \begin{pmatrix} b \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$; = or…	M1; oe
	$\begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$	A1
	$\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ **or** $= \begin{pmatrix} 1 \\ a \\ 5 \end{pmatrix} - 2\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$ or $= \begin{pmatrix} b \\ -1 \\ 3 \end{pmatrix} - 3\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$;	M1; oe
	$= \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	A1
		[4]
	See notes for a common approach to part (e) using the length of *AD*	**14**

Notes

Throughout – allow vectors to be written as a row, with commas, as this is another convention.

(a) **M1**: Finds, or implies, correct value of λ for at least one of the two given points

A1: **At least one** of a **or** b correct

A1: **Both** a **and** b correct

(b) **M1**: Subtracts the position vector of A from that of B or the position vector of B from that of A. Allow any notation. Even allow coordinates to be subtracted. Follow through their a and b for this method mark.

A1: Need correct answer : so $\overrightarrow{AB} = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ or $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$ or (4, 2, -2) This is not ft.

(c) Way 1:

M1: Subtracts the position vector of A from that of C or the position vector of C from that of A. Allow any notation. Even allow coordinates to be subtracted. Follow through their a for this method mark.

dM1: Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$.

A1*: Correctly proves that $C\hat{A}B = 30^\circ$. This is a printed answer. Must have used $\left(\overrightarrow{AB} \text{ with } \overrightarrow{AC}\right)$ or $\left(\overrightarrow{BA} \text{ with } \overrightarrow{CA}\right)$ for this mark and must not have changed a negative to a positive to falsely give the answer, that would result in M1M1A0

Do not need to see $\frac{\sqrt{3}}{2}$ but should see equivalent value. Allow $\frac{\pi}{6}$ as final answer.

Way 2:

M1: Finds lengths of AB, AC and BC

dM1: Uses cosine rule or trig of right angled triangle, either sin, cos or tan

A1: Correct proof that angle = 30 degrees

(d) **M1**: Applies $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sin 30^\circ$ **-** must try to use their vectors (b – a) and (c – a) or state formula and try to use it. Could use vector product. Must not be using $\frac{1}{2}\left|\overrightarrow{OB}\right|\left|\overrightarrow{OC}\right|\sin 30^\circ$

A1: $3\sqrt{3}$ cao – must be exact and in this form (see question)

(e) **M1**: Realises that AD is twice the length of AB and uses **complete method** to find one of the points. Then uses one of the three possible starting points on the line (A, B, or the point with position vector $-\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$) to reach D. See one of the equations in the mark scheme and ft their a or b.

So accept $\left(\overrightarrow{OD_1}\right) = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or $= \begin{pmatrix} 1 \\ a \\ 5 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or $= \begin{pmatrix} b \\ -1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

A1: Accept (9, 1, 1) or $9\mathbf{i} + \mathbf{j} + \mathbf{k}$ or $\begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ cao

M1: Realises that AD is twice the length of AB but is now in the opposite direction so uses one of the three possible starting points to reach D. See one of the equations in the mark scheme and ft their a or b.

	So accept $(\overrightarrow{OD_2}) = \begin{pmatrix}-1\\-4\\6\end{pmatrix} - 3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ **or** $= \begin{pmatrix}1\\a\\5\end{pmatrix} - 2\begin{pmatrix}4\\2\\-2\end{pmatrix}$ or $= \begin{pmatrix}b\\-1\\3\end{pmatrix} - 3\begin{pmatrix}4\\2\\-2\end{pmatrix}$ **A1**: Accept (-7, -7, 9) or -7**i** -7**j** + 9**k** or $\begin{pmatrix}-7\\-7\\9\end{pmatrix}$ cao **NB** Many long methods still contain unknown variables *x*, *y* and *z* or *λ*. These are not complete methods so usually earn M0A0M0A0 on part (e) PTO.		
(e)	**Mark scheme for a common approach to part (e) using the length of *AD* is given below:** $(2\lambda-2)^2+(\lambda-1)^2+(\text{"1"}-\lambda)^2=\text{"96"}$ then obtain $\lambda^2-2\lambda-15=0$ so $\lambda=$, then substitute value of λ to find coordinates. May make a slip in algebra expanding brackets or collecting terms (even if results in two term quadratic) This may be simplified to $\sqrt{6}(\lambda-1)=4\sqrt{6}$ or to $\sqrt{6}(1-\lambda)=4\sqrt{6}$ NB $6(1-\lambda)^2=4\sqrt{6}$ is M0 as one side has dimension (length)^2 and the other is length $\begin{pmatrix}9\\1\\1\end{pmatrix}$ (from λ=5) Substitute other value of λ. May make a slip in algebra $=\begin{pmatrix}-7\\-7\\9\end{pmatrix}$ (from λ=-3) **Special case – uses AD is half AB instead of double AB** $(2\lambda-2)^2+(\lambda-1)^2+(\text{"1"}-\lambda)^2=\text{"6"}$ then obtain $\lambda^2-2\lambda=0$ so $\lambda=$, then substitute value of λ to find coordinates $\begin{pmatrix}-1\\-4\\6\end{pmatrix}$ (from λ=0) Substitute other value of λ $=\begin{pmatrix}3\\-2\\4\end{pmatrix}$ (from λ=2) For this solution score M1A0M1A0 i.e. 2/4	M1 A1 M1 A1	**[4]**

Question Number	Scheme		Marks
8 (a)	$\frac{\mathrm{d}h}{\mathrm{d}t} \propto \sqrt{h}$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = k\sqrt{h}$ o.e.	$\frac{\mathrm{d}h}{\mathrm{d}t} \propto -\sqrt{h}$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = -k\sqrt{h}$	M1
	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = kh^{\frac{1}{2}} \Rightarrow\right\} \int \frac{1}{h^{\frac{1}{2}}}\,\mathrm{d}h = \int k\,\mathrm{d}t \Rightarrow \ldots$	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = -kh^{\frac{1}{2}} \Rightarrow\right\} \int \frac{1}{h^{\frac{1}{2}}}\,\mathrm{d}h = \int -k\,\mathrm{d}t \Rightarrow \ldots$	M1
	$\int h^{-\frac{1}{2}}\,\mathrm{d}h = \int k\,\mathrm{d}t \Rightarrow \frac{h^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\ \{+c\}$ or $2h^{\frac{1}{2}} = kt\ \{+c\}$	$\int h^{-\frac{1}{2}}\,\mathrm{d}h = \int -k\,\mathrm{d}t \Rightarrow \frac{h^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = -kt\ \{+c\}$ or $2h^{\frac{1}{2}} = -kt\ \{+c\}$	A1
	$\{t = 0,\ h = 225 \Rightarrow\}$ $2\sqrt{225} = k(0) + c\ \{\Rightarrow c = 30\}$	$\{t = 0,\ h = 225 \Rightarrow\}$ $2\sqrt{225} = -k(0) + c\ \{\Rightarrow c = 30\}$	M1
	$t = 125,\ h = 100$ $\Rightarrow 2\sqrt{100} = k(125) + 30$ $\Rightarrow 20 = k(125) + 30 \Rightarrow k = -0.08$	$t = 125,\ h = 100$ $\Rightarrow 2\sqrt{100} = -k(125) + 30$ $\Rightarrow 20 = -k(125) + 30 \Rightarrow k = 0.08$	dM1
	$\Rightarrow 2h^{\frac{1}{2}} = -0.08t + 30$ $\Rightarrow h^{\frac{1}{2}} = -0.04t + 15$ CSO	$\Rightarrow 2h^{\frac{1}{2}} = -0.08t + 30$ $\Rightarrow h^{\frac{1}{2}} = -0.04t + 15$ CSO	A1*
	$a = 375$ or $0 \le t \le 375$		B1
			[7]
(b)	$\{h = 50 \Rightarrow\}\ 50 = (15 - 0.04t)^2 \Rightarrow t = (198.2233047\ldots)$		M1
	Time = $198.2233047\ldots - 125$		dM1
	$= 73.2233047\ldots = 73$ (minutes)		A1
			[3]
			10 marks

(a) **Note that k can be set as positive (above left) or negative (above right) at the start of (a)**

M1: Converts the given information into maths. Do not award if k has been given a pre defined value.

Condone $\mathrm{d}h = \pm k\sqrt{h}\ \mathrm{d}t$ and allow versions such as $\frac{\mathrm{d}t}{\mathrm{d}h} = \pm\frac{k}{\sqrt{h}}$

M1: Separates the variables for their differential equation (which in the form $\frac{\mathrm{d}h}{\mathrm{d}t} = \mathrm{f}(h)$) and attempts to integrate at least one side. Condone lack of integral signs. One side must be of the correct form

A1: Correct integration. with or without a constant of integration.

Watch for $\frac{h^{\frac{1}{2}}}{2} = \pm kt\ \{+c\}$ which leads to the correct answer. This scores A0.

Follow through on their k if this has been assigned a value. This would occur when $k = 1$ for instance.

M1: Substitutes $t = 0,\ h = 225$ into a changed equation and finds c

Can be scored when k has been set to 1, for example, so these solutions generally score 011100-

dM1: Substitutes $t = 125,\ h = 100$ and their value of 'c' into their changed equation and find a value for k

A1*: CSO Proceeds without errors to the given answer. A penultimate line of $h^{\frac{1}{2}} = -0.04t + 15$ or equivalent should be seen.

B1: States $a = 375$ or $0 \le t \le 375$ Condone if any units are attached to this value

(b)

M1: Substitutes $h = 50$ into the given equation and rearranges to find $t = ...$

May be scored from a 3TQ

dM1: ...and then subtracts 125 from their value for t. This can be implied by $t_{h=50} - 125$

A1: awrt 73 (minutes) following correct work .

Do not need units but withhold if they state a different unit. E.g 73 seconds

Question Number	Scheme	Marks
9(a)	$y^2=\left(x(\sin x+\cos x)\right)^2=x^2(\sin x+\cos x)^2$	
	$=x^2(\sin^2 x+\cos^2 x+2\sin x\cos x)$	M1
	$=x^2\left(1+\sin 2x\right)$	A1
	$V=\int_0^{\frac{\pi}{4}}\pi y^2dx=\int_0^{\frac{\pi}{4}}\pi x^2(1+\sin 2x)dx$	A1*
		(3)
(b)	$V=\int_0^{\frac{\pi}{4}}\pi x^2(1+\sin 2x)dx=\int_0^{\frac{\pi}{4}}(\pi x^2+\pi x^2\sin 2x)dx$	
	$=\int_0^{\frac{\pi}{4}}\pi x^2\mathrm{d}x+\int_0^{\frac{\pi}{4}}\pi x^2\sin 2x\mathrm{d}x$	
	$\int_0^{\frac{\pi}{4}}\pi x^2\mathrm{d}x=\left[\pi\frac{x^3}{3}\right]_0^{\frac{\pi}{4}}=\pi\frac{\left(\frac{\pi}{4}\right)^3}{3}$ OR $\int_0^{\frac{\pi}{4}}x^2\mathrm{d}x=\left[\frac{x^3}{3}\right]_0^{\frac{\pi}{4}}=\frac{\left(\frac{\pi}{4}\right)^3}{3}$	M1A1
	$\cancel{\pi}\int x^2\sin 2x\,\mathrm{d}x=\cancel{\pi}\left(\pm Bx^2\cos 2x\pm C\int x\cos 2x\,\mathrm{d}x\right)$	M1
	$=\cancel{\pi}\left(-x^2\frac{\cos 2x}{2}+\int x\cos 2x\,\mathrm{d}x\right)$	A1
	$=\cancel{\pi}\left(\pm Bx^2\cos 2x\pm Cx\sin 2x\pm\int D\sin 2x\mathrm{d}x\right)$	dM1
	$=\cancel{\pi}\left(-x^2\frac{\cos 2x}{2}+x\frac{\sin 2x}{2}+\frac{\cos 2x}{4}\right)$	A1
	$\cancel{\pi}\int_{x=0}^{x=\frac{\pi}{4}}x^2\sin 2x\mathrm{d}x=\cancel{\pi}\left[\pm Bx^2\cos 2x\pm Cx\sin 2x\pm D\cos 2x\right]_0^{\frac{\pi}{4}}=\cancel{\pi}\left(\frac{\pi}{8}-\frac{1}{4}\right)$	ddM1
	$V=\int_0^{\frac{\pi}{4}}\pi x^2(1+\sin 2x)dx=\int_0^{\frac{\pi}{4}}\pi x^2\mathrm{d}x+\int_0^{\frac{\pi}{4}}\pi x^2\sin 2x\mathrm{d}x$	
	$=\left(\frac{\pi^4}{192}\right)+\left(\frac{\pi^2}{8}-\frac{\pi}{4}\right)$ oe	A1,A1
		(9) **(12 marks)**

Notes for Question 9

(a)

M1 For squaring y AND attempting to multiply out the bracket. The minimum requirement is that $y^2 = x^2(\sin^2 x + \cos^2 x + ...)$. There is no need to include 'π' for this mark.

A1 Using $\sin^2 x + \cos^2 x = 1$ **and** $2\sin x\cos x = \sin 2x$ to achieve $y^2 = x^2(1+\sin 2x)$

There is no need to include 'π' for this mark.

You may accept $\sin^2 x + 2\sin x\cos x + \cos^2 x = 1 + \sin 2x$

A1* It must be stated or implied that $V = \int_0^{\frac{\pi}{4}} \pi y^2 \mathrm{d}x$.

It may be implied by replacing y^2 by $(x(\sin x + \cos x))^2$

A correct proof must follow involving all that is required for the previous M1A1

The limits could just appear in the final line without any explanation. Note that this is a given answer

(b)

M1 For splitting the given integral into a sum **and** integrating x^2 or πx^2 to Ax^3.

There is no need for limits at this stage

A1 $\int_0^{\frac{\pi}{4}} x^2 \mathrm{d}x = \left[\frac{x^3}{3}\right]_0^{\frac{\pi}{4}} = \frac{\left(\frac{\pi}{4}\right)^3}{3}$. There is no need to simplify this. Accept $\int_0^{\frac{\pi}{4}} \pi x^2 \mathrm{d}x = \pi\left[\frac{x^3}{3}\right]_0^{\frac{\pi}{4}} = \pi\frac{\left(\frac{\pi}{4}\right)^3}{3}$

M1 For integrating $\int \pi x^2 \sin 2x \,\mathrm{d}x$ or $\int x^2 \sin 2x \,\mathrm{d}x$ by parts. The integration must be the correct way around. There is no need for limits. If the rule is quoted it must be correct, a version of which appears in the formula booklet.

Accept for this mark expressions of the form $\int x^2 \sin 2x \,\mathrm{d}x = \pm Bx^2\cos 2x \pm \int Cx\cos 2x\,\mathrm{d}x$

A1 $\int x^2 \sin 2x \,\mathrm{d}x = -x^2\frac{\cos 2x}{2} + \int x\cos 2x\,\mathrm{d}x$ OR $\int \pi x^2 \sin 2x \,\mathrm{d}x = -\pi x^2\frac{\cos 2x}{2} + \int \pi x\cos 2x\,\mathrm{d}x$

dM1 A second application by parts, the correct way around. No need for limits. See the previous M1 for how to award. It is dependent upon this having been awarded.

Look for $\int x^2 \sin 2x \,\mathrm{d}x = \pm Bx^2\cos 2x \pm Cx\sin 2x \pm \int D\sin 2x\,\mathrm{d}x$

A1 A fully correct answer to the integral of $\int x^2 \sin 2x \,\mathrm{d}x = -x^2\frac{\cos 2x}{2} + x\frac{\sin 2x}{2} + \frac{\cos 2x}{4}$

ddM1 For substituting in both limits and subtracting. The two M's for int by parts must have been scored.

A1 Either of $\left(\frac{\pi^4}{192}\right)$ linked to first M or $\left(\frac{\pi^2}{8} - \frac{\pi}{4}\right)$ linked to ddM. Accept in the form $\pi\left(\frac{\pi^3}{192} + ...\right)$

A1 Correct answer and correct solution only. Accept exact equivalents $V = \pi\left(\frac{\pi^3}{192} + \frac{\pi}{8} - \frac{1}{4}\right)$

Alt way (2)- Where candidate does not split up first.

(b)	$V=\int_0^{\frac{\pi}{4}}\cancel{\pi}x^2(1+\sin 2x)dx=\cancel{\pi}x^2\left(x\pm A\cos 2x\right)-\int_0^{\frac{\pi}{4}}B\cancel{\pi}x\left(x\pm A\cos 2x\right)dx$	2nd M1
	$=\cancel{\pi}x^2\left(x-\frac{\cos 2x}{2}\right)-\int_0^{\frac{\pi}{4}}2\cancel{\pi}x\left(x-\frac{\cos 2x}{2}\right)dx$	A1
	$=\cancel{\pi}\left(x^2\left(x\pm A\cos 2x\right)-Bx\left(Cx^2\pm D\sin 2x\right)\pm\int_0^{\frac{\pi}{4}}Ex^2\pm F\sin 2x\,dx\right)$	3rd dM1
	$=\cancel{\pi}\left(x^2\left(x-\frac{\cos 2x}{2}\right)-2x\left(\frac{x^2}{2}-\frac{\sin 2x}{4}\right)+\int_0^{\frac{\pi}{4}}2\left(\frac{x^2}{2}-\frac{\sin 2x}{4}\right)dx\right)$	A1
	$=\cancel{\pi}\left(x^2\left(x-\frac{\cos 2x}{2}\right)-2x\left(\frac{x^2}{2}-\frac{\sin 2x}{4}\right)+\mathbf{2}\left(\frac{\mathbf{x^3}}{\mathbf{6}}+\frac{\cos 2x}{8}\right)\right)$	**1st M1A1**
	$V=\cancel{\pi}\left[x^2\left(x-\frac{\cos 2x}{2}\right)-2x\left(\frac{x^2}{2}-\frac{\sin 2x}{4}\right)+2\left(\frac{x^3}{6}+\frac{\cos 2x}{8}\right)\right]_0^{\frac{\pi}{4}}$	ddM1
	$=\frac{\pi^4}{192},+\frac{\pi^2}{8}-\frac{\pi}{4}$	A1, A1
		(9)

1ST M1,A1 Seen after two (not necessarily) correct applications of integration by parts, it is for integrating the x^2 term

2nd M1A1 It is for the first attempt at an application of integration by parts on $\int x^2(1+\sin 2x)dx$

Look for $x^2\left(x\pm A\cos 2x\right)-\int_0^{\frac{\pi}{4}}Bx\left(x\pm A\cos 2x\right)dx$ for the method

3rd dM1A1 It is for a further attempt at an application of integration by parts the correct way around. It is dependent upon the first method having been awarded.

Look for $=x^2\left(x\pm A\cos 2x\right)-Bx\left(Cx^2\pm D\sin 2x\right)\pm\int_0^{\frac{\pi}{4}}Ex^2\pm F\sin 2x\,dx$