Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

A Level Clouds 出品

Time: 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. (a) Express $3\sin x - \cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

The temperature, θ°C, inside a building on a particular day, is modelled by the equation

$$\theta = 19 + 3\sin\left(\frac{\pi t}{12} + 4\right) - \cos\left(\frac{\pi t}{12} + 4\right), \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

(b) Using the answer to part (a),

(i) state the minimum value of θ predicted by this model,

(ii) find the value of t, to 2 decimal places, when this minimum occurs.

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. Find

(i) $\int \frac{e^{2x}}{(e^{2x}-1)^3}\,dx \qquad x \neq 0$

(2)

(ii) $\int \frac{5x}{4x^2+1}\,dx$

(2)

Question 2 continued

(Total for Question 2 is 4 marks)

3.

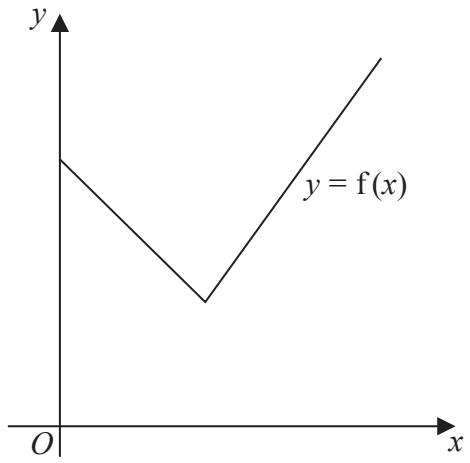

Figure 1

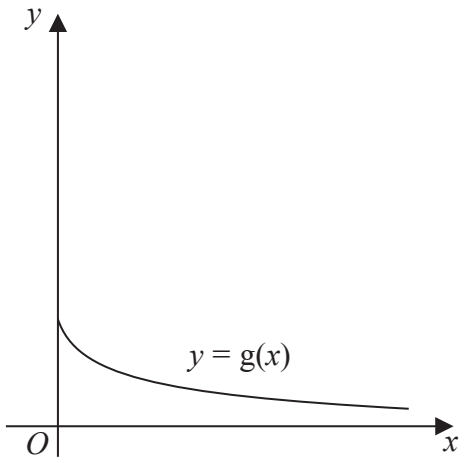

Figure 2

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

Figure 2 shows a sketch of part of the graph $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x + 9}{2x + 3}, \quad x \geqslant 0$$

(a) Find the value of fg(1)

(2)

(b) State the range of g

(2)

(c) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(4)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(d) state the range of possible values of k.

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

4. The mass, M kg, of a species of tree can be modelled by the equation

$$\log_{10} M = 1.93 \log_{10} r + 0.684$$

where r cm is the base radius of the tree.

The base radius of a particular tree of this species is 45 cm.

According to the model,

(a) find the mass of this tree, giving your answer to 2 significant figures.

(2)

(b) Show that the equation of the model can be written in the form

$$M = pr^q$$

giving the values of the constants p and q to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

Question 4 continued

(Total for Question 4 is 6 marks)

5. (i) Differentiate $y = 5x^2 \ln 3x, \quad x > 0$

(2)

(ii) Given that

$$y = \frac{x}{\sin x + \cos x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

(4)

Question 5 continued

(Total for Question 5 is 6 marks)

6. $$f(x) = \frac{x^2}{4} + \ln(2x), \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \frac{1}{2}e^{-\frac{1}{4}x^2}$$

(2)

The equation $f(x) = 0$ has a root near 0.5

(b) Starting with $x_1 = 0.5$ use the iterative formula

$$x_{n+1} = \frac{1}{2}e^{-\frac{1}{4}x_n^2}$$

to calculate the values of x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval, show that 0.473 is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

Question 6 continued

(Total for Question 6 is 7 marks)

7. (i) Find

$$\int \frac{4}{(5y - 7)^4} \, dy$$

(2)

(ii) Find, in simplest form,

$$\int (1 - 4\tan 3x)^2 \, dx$$

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 6 marks)

8. (a) Prove

$$\frac{\cos 3\theta}{2\sin\theta} + \frac{\sin 3\theta}{2\cos\theta} \equiv \cot 2\theta \qquad \theta \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $0 < x < \frac{\pi}{2}$

$$\frac{\cos 3x}{2\sin x} + \frac{\sin 3x}{2\cos x} = 5\cos 2x$$

giving your answers to 3 decimal places where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

9. The curve C has equation

$$x = 3\sec^2 2y \qquad x > 3 \qquad 0 < y < \frac{\pi}{4}$$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

(2)

(b) Hence show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{p}{qx\sqrt{x-3}}$$

where p is irrational and q is an integer, stating the values of p and q.

(3)

(c) Find the equation of the normal to C at the point where $y = \frac{\pi}{12}$, giving your answer in the form $y = mx + c$, giving m and c as exact irrational numbers.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

10.

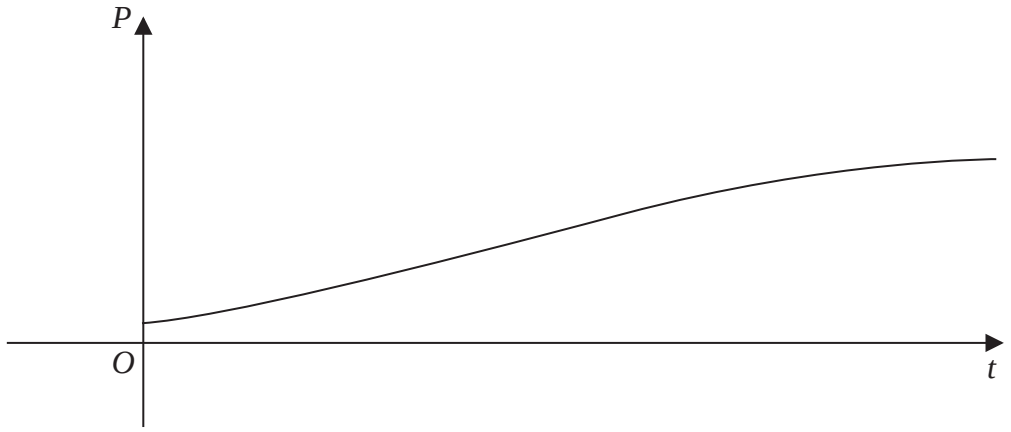

Figure 3

The population of a species of animal is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{9000e^{kt}}{3e^{kt} + 7}, \qquad t \geqslant 0$$

where k is a positive constant.

A sketch of the graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study,

(2)

(b) find the value for the upper limit of the population.

(1)

Given that $P = 2500$ when $t = 4$

(c) calculate the value of k, giving your answer to 3 decimal places.

(5)

Using this value for k,

(d) find, using $\frac{dP}{dt}$, the rate at which the population is increasing when $t = 10$

Give your answer to the nearest integer.

(3)

Question 10 continued

Question 10 continued

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Question 10 continued

Question 10 continued

(Total for Question 10 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS