Pearson Edexcel International Advanced Level

考前模拟卷 – **A Level Clouds**出品

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics
**International Advanced Subsidiary/Advanced Level
Pure Mathematics P2**

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
● Use **black** ink or ball-point pen.
● If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
● **Fill in the boxes** at the top of this page with your name,
  centre number and candidate number.
● Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
● Answer the questions in the spaces provided
  – *there may be more space than you need.*
● You should show sufficient working to make your methods clear. Answers without working
  may not gain full credit.
● Inexact answers should be given to three significant figures unless otherwise stated.

Information
● A booklet 'Mathematical Formulae and Statistical Tables' is provided.
● There are 10 questions in this question paper. The total mark for this paper is 75.
● The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice
● Read each question carefully before you start to answer it.
● Try to answer every question.
● Check your answers if you have time at the end.
● If you change your mind about an answer, cross it out and put your new answer
  and any working underneath.

Turn over ▶

Pearson

1. $\qquad f(x) = ax^3 + 3x^2 - 8x + 2 \qquad$ where a is a constant

Given that when $f(x)$ is divided by $(x - 2)$ the remainder is 3, find the value of a.

(3)

Question 1 continued

(Total for Question 1 is 3 marks)

2. **(i)** Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b\log_2 3$, where a and b are constants to be found.

(4)

(ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2\log_5(y + 1) - 1$$

(5)

Question 2 continued

(Total for Question 2 is 9 marks)

3. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{ax}{2}\right)^5$$

where a is a positive constant. Give each term in its simplest form.

(4)

Given that, in the expansion, the coefficient of x is equal to the coefficient of x^3,

(b) find the exact value of a in its simplest form.

(3)

Question 3 continued

(Total for Question 3 is 7 marks)

4. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.

(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. (i) The circle C_1 has equation

$$x^2 + y^2 + 10x - 12y = k \qquad \text{where } k \text{ is a constant}$$

(a) Find the coordinates of the centre of C_1

(2)

(b) State the possible range in values for k.

(2)

(ii) The point $P(p, 0)$, the point $Q(-2, 10)$ and the point $R(8, -14)$ lie on a different circle, C_2

Given that

- p is a positive constant
- QR is a diameter of C_2

find the exact value of p.

(4)

Question 5 continued

(Total for Question 5 is 8 marks)

6. A student was asked to prove by exhaustion that

if n is an integer then $2n^2 + n + 1$ is **not** divisible by 3

The start of the student's proof is shown in the box below.

Consider the case when $n = 3k$
$$2n^2 + n + 1 = 18k^2 + 3k + 1 = 3(6k^2 + k) + 1$$

which is not divisible by 3

Complete this proof.

(4)

(Total for Question 6 is 4 marks)

7. (i) An arithmetic series has first term a and common difference d.

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a + (n-1)d\}$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) u_5

(1)

(b) $\displaystyle\sum_{n=1}^{59} u_n$

(3)

Question 7 continued

Question 7 continued

Question 7 continued

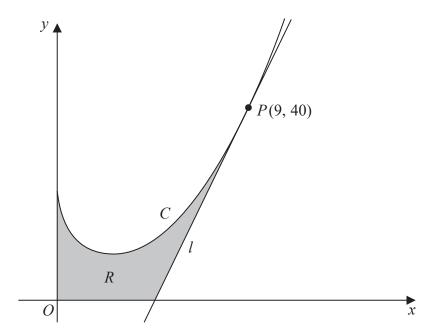**(Total for Question 7 is 7 marks)**

8.

Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \qquad x \geqslant 0$$

(a) Find, using calculus, the range of values of x for which y is increasing.

(4)

The point P lies on C and has coordinates (9, 40).

The line l is the tangent to C at the point P.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the line l, the x-axis and the y-axis.

(b) Find, using calculus, the exact area of R.

(8)

Question 8 continued

Question 8 continued

(Total for Question 8 is 12 marks)

9. (i) Showing each step in your reasoning, prove that

$$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$$

(3)

(ii) Solve, for $0 \leqslant \theta < 360°$,

$$3\sin \theta = \tan \theta$$

giving your answers in degrees to 1 decimal place, as appropriate.

(6)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 9 marks)

10. (a) Given $0 < a < 1$, sketch the curve with equation

$$y = a^x$$

showing the coordinates of the point at which the curve crosses the y-axis.

(2)

x	2	2.5	3	3.5	4
y	4.25	6.427	9.125	12.34	16.06

The table above shows corresponding values of x and y for $y = x^2 + \left(\dfrac{1}{2}\right)^x$

The values of y are given to 4 significant figures as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\displaystyle\int_{2}^{4}\left(x^2 + \left(\dfrac{1}{2}\right)^x\right)dx$

(3)

Using your answer to part (b) and making your method clear, estimate

(c) $\displaystyle\int_{2}^{4}\left(x(x-3) + \left(\dfrac{1}{2}\right)^x\right)dx$

(2)

Question 10 continued

Question 10 continued

TOTAL FOR PAPER IS 75 MARKS