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Pure Mathematics 4

分类真题

2014-2022 册

A Level Clouds 出品

目录

Chapter 1

Proof by Contradiction

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1. Given that n is an integer, use algebra, to prove by contradiction, that if n^3 is even then n is even.

(4)

Leave blank

3. Prove by contradiction that there is no greatest odd integer.

(2)

Leave blank

9. (ii) Given that $n \in \mathbb{N}$, prove by contradiction that if n^2 is a multiple of 3 then n is a multiple of 3

(5)

Leave blank

10. (a) A student's attempt to answer the question

"Prove by contradiction that if n^3 is even, then n is even"

is shown below. Line 5 of the proof is missing.

> Assume that there exists a number n such that n^3 is even, but n is odd.
>
> If n is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$
>
> So $n^3 = (2p + 1)^3$
>
> $= 8p^3 + 12p^2 + 6p + 1$
>
> $=$
>
> This contradicts our initial assumption, so if n^3 is even, then n is even.

Complete this proof by filling in line 5.

(1)

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.

(5)

Leave blank

6. Three **consecutive** terms in a sequence of real numbers are given by

$$k, \ 1 + 2k \ \text{ and } \ 3 + 3k$$

where k is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence.

(5)

Leave blank

8. Use proof by contradiction to prove that, for all positive real numbers x and y,

$$\frac{9x}{y} + \frac{y}{x} \geqslant 6$$

(4)

9. Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(4)

8. A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$

$$\Rightarrow (3x - y)(x + y) = 25$$

If $(3x - y) = 1$ and $(x + y) = 25$

$$\left.\begin{aligned} 3x - y &= 1 \\ x + y &= 25 \end{aligned}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5 \quad \text{Not integers}$$

Show the calculations and statements that are needed to complete the proof.

(4)

Chapter 2

Partial Fractions with Calculus