CIE

Further Pure Mathematics 2

分类真题

2020-2022 册

A Level Clouds 出品

目录

Chapter 1

Hyperbolic Functions

Q1: 9231/21/S22

2 **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh 2x = 2\sinh^2 x + 1.$$ [3]

(b) Find the set of values of k for which $\cosh 2x = k\sinh x$ has two distinct real roots. [5]

Chapter 2

Matrices

Q1: 9231/21/S20

8 **(a)** Find the values of a for which the system of equations

$$\begin{aligned} 3x + y + z &= 0, \\ ax + 6y - z &= 0, \\ ay - 2z &= 0, \end{aligned}$$

does not have a unique solution. [3]

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The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

(b) Use the characteristic equation of **A** to find the inverse of $\mathbf{A}^2$. [4]

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(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [7]

Q2: 9231/23/S20

3 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 7 \\ 0 & 6 & 0 \\ 7 & 7 & 5 \end{pmatrix}.$$

(a) Find the eigenvalues of $\mathbf{A}$. [4]

(b) Use the characteristic equation of **A** to find $\mathbf{A}^{-1}$. [4]

Q3: 9231/21/W20

3 **(a)** Show that the system of equations

$$\begin{aligned} x-2y-4z &= 1, \\ x-2y+kz &= 1, \\ -x+2y+2z &= 1, \end{aligned}$$

where k is a constant, does not have a unique solution. [2]

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(b) Given that $k=-4$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

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(c) Given instead that $k=-2$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

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(d) For the case where $k\neq-2$ and $k\neq-4$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

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Q4: 9231/21/W20

7 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

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(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

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