Question Number	Scheme	Marks
1.	$\omega = \frac{10\pi}{60}$ (rad s^{-1})	B1
	$F = mg\mu$ (N)	B1
	$F = m \times 0.2\left(\frac{\pi}{6}\right)^2 = \frac{m\pi^2}{180}$	M1A1ft
	$mg\mu \geq \frac{m\pi^2}{180}$	dM1
	$\mu_{\min} = \frac{\pi^2}{180g}$, $(0.0056, \quad 0.00560)$	A1
		[6]
B1	Correct angular speed in radians per second, seen anywhere	
B1	Correct inequality or equation for Friction, seen or used anywhere	
M1	Attempt the equation of motion along the radius. Must only contain friction and resultant force (give BOD unless clearly not friction). Allow with their ω or just ω.	
A1ft	Correct equation. Follow through their ω	
dM1	Eliminate F and solve to find μ. Allow with an inequality or equation. Dependent on previous M1.	
A1	Correct answer, as shown or 2/3 sf decimal (0.00560). Must not be an inequality now.	

Special Case: If $F \geq mg\mu$ or $F < mg\mu$ used, leading to $\mu = \frac{\pi^2}{180g}$ award max B1B0 M1A1

M1A0

Question Number	Scheme	Marks
2	$0.5u = 1.5 \quad u = 3\text{ m s}^{-1}$	B1
	Work done against friction $= 0.7\times 0.5\cos 30g\times 0.6$	M1A1
	Initial EPE $= \dfrac{\lambda\times 0.6^2}{2\times 0.6}\left(= \dfrac{0.6\lambda}{2} = 0.3\lambda\right)$	B1
	$\dfrac{\lambda\times 0.6^2}{2\times 0.6} + \dfrac{1}{2}\times 0.5\times 9 = 0.7\times 0.5\cos 30g\times 0.6 + 0.5\times g\times 0.6\sin 30$	M1A1A1 Ft EPE and Work
	$\lambda = 3.340... = 3.3$ or 3.34	A1 [8]

B1	Correct value for u, seen explicitly or used.
M1	Attempt the work done against friction. Weight must be resolved (sin/cos interchange accepted.) Distance moved to be 0.6 m. Mass can be 0.5 or m
A1	Correct work done. Mass can be 0.5 or m Allow both of the above marks if the work done against friction is embedded in some incorrect work eg including other forces to form a resultant force.
B1	Correct initial EPE Need not be simplified. The work done and the EPE may not be shown explicitly. Check the equation if necessary.
M1	Attempt a complete work-energy equation. Must have an EPE, a GPE, a KE and a (dimensionally correct) work against friction term. The final KE may be included provided it becomes 0 here or later. EPE term must be of the form $\dfrac{k\lambda x^2}{l}$ $\quad k = \dfrac{1}{2}$, 1 or 2
A1ft **A1ft**	Deduct one per error. Follow through their EPE and work.
A1	Correct value of λ, 2 or 3 sf only.

Question Number	Scheme	Marks
3(a)	$\text{Vol} = (\pi)\int_{\frac{3}{5}r}^{r}(r^2 - x^2)\,\mathrm{d}x = (\pi)\left[r^2x - \frac{1}{3}x^3\right]_{\frac{3}{5}r}^{r}$	M1A1
	$= (\pi)\left(r^3 - \frac{1}{3}r^3 - \left(\frac{3}{5}r^3 - \frac{9}{125}r^3\right)\right)\ \left(= \frac{52}{375}(\pi)r^3\right)$	M1
	$(\pi)\int_{\frac{3}{5}r}^{r}x(r^2 - x^2)\,\mathrm{d}x = (\pi)\left[\frac{1}{2}r^2x^2 - \frac{1}{4}x^4\right]_{\frac{3}{5}r}^{r}$	M1A1
	$= (\pi)\left(\frac{1}{2}r^4 - \frac{1}{4}r^4 - \left(\frac{9}{50}r^4 - \frac{81}{2500}r^4\right)\right)\ \left(= \frac{64}{625}(\pi)r^4\right)$	M1
	$\bar{x} = \dfrac{\int xy^2\,\mathrm{d}x}{\int y^2\,\mathrm{d}x} = \dfrac{\frac{64}{625}}{\frac{52}{375}}r$	M1
	$= \frac{48}{65}r$ *	A1cso (8)
(b)	Bowl alone: Mass ratio 6^3 5^3 91	
	Dist from A: $\frac{3}{8}\times 6$ $\frac{3}{8}\times 5$ $\bar{y}$	
	$216\times\frac{3}{8}\times 6 - 125\times\frac{3}{8}\times 5 = 91\bar{y}$	M1A1A1
	$\bar{y} = 2.7651...$ $\left(\frac{2013}{728},\ 2\frac{557}{728}\right)$	A1
	Bowl and liquid: Mass ratio 5 2 7	
	Dist from A: 2.7651... $\frac{48}{13}$ $\bar{z}$	B1 (48/13)
	$7\bar{z} = 5\times 2.7651 + \frac{48}{13}\times 2$	M1A1ft
	$\bar{z} = 3.030... = 3.03$ cm	A1 (8) **[16]**
ALT	Find mass of whole hemisphere and part cut away in terms of M and use a single moments equation (see end)	

Question Number	Scheme	Marks
(a)	**Lamina scores 0/8.** **If no evidence of algebraic integration seen, only the last M mark is available.**	
M1	Attempt the volume integral, π and limits not needed (ignore any shown)	
A1	Correct integration, π and limits not needed (ignore any shown)	
dM1	Substitute the correct limits in their result. Evidence of substitution must be seen. Depends on previous M mark	
M1	Attempt $\int xy^2\,\mathrm{d}x$, π and limits not needed (ignore any shown)	
A1	Correct integration, π and limits not needed (ignore any shown)	
dM1	Substitute the correct limits in their result. Evidence of substitution must be seen. Depends on previous M mark	
M1	Use $\bar{x} = \dfrac{\int xy^2\mathrm{d}x}{\int y^2\mathrm{d}x}$ with their previous results (need not be simplified results). π in both or neither integral	
A1cso	Correct final **(given)** result obtained from fully correct working.	
(b)		
M1	Attempt a moments equation with the *difference* of two hemispheres. Dimensions for the hemispheres must be correct.	
A1	Correct masses or ratio of masses	
A1	Correct distances	
A1	Correct distance for the bowl – exact or decimal	
B1	For the correct distance of the c of m of the liquid from *A*	
M1	Attempt a moments equation – bowl and liquid added. Must attempt the distance for the liquid ie we are looking for a numerical distance , not just a letter and must have shown evidence of calculating the c of m of the bowl (M mark for this may have been lost)	
A1ft	Correct equation, follow through their distances (ie 48/13 and c of m of bowl)	
A1	Correct answer from correct working. Must be 3 sf	

Question Number	Scheme	Marks
ALT (b)	Vol of bowl $=\frac{2}{3}\pi\left(6^3-5^3\right)=\frac{2}{3}\pi\times 91$	
	$\frac{2}{3}\pi\rho\times 91=5M$	B1
	Mass ratio $\quad 6^3 \qquad\qquad 5^3$	
	$6^3\times\frac{5}{91}M \qquad 5^3\times\frac{5}{91}M \qquad 2M \qquad 7M$	M1A1A1
	Dist from A: $\frac{3}{8}\times 6 \qquad \frac{3}{8}\times 5 \qquad \frac{48}{13} \qquad \overline{y}$	B1(48/13)
	$6^3\times\frac{5}{91}M\times\frac{3}{8}\times 6-5^3\times\frac{5}{91}M\times\frac{3}{8}\times 5+2M\times\frac{48}{13}=7M\,\overline{y}$	M1A1ft
	$\overline{y}=3.030...=3.03$	A1 (8)
B1	For a correct equation connecting the mass of the bowl and $5M$. Award if $\frac{5}{91}M$ or $\frac{5}{91}$ is seen used correctly in at least one term in their equation. Enter as the first A mark on e-PEN	
M1	For attempting the mass ratio for the 4 parts needed including their "5/91"	
A1A1	Deduct one per error	
B1	For 48/13	
M1	Attempt a moments equation with 4 terms and correct signs. An attempt at the mass ratio of the parts based on the mass of the bowl being 5M must have been seen even if this attempt failed to qualify for the first M mark.	
A1ft	Correct equation, follow through their masses and distances (ie 48/13 and c of m of bowl)	
A1	Correct answer from correct working. Must be 3 sf	

Question Number	Scheme	Marks
4(a)	$0.5v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\sin 2x$	M1
	$\int 0.5v\,\mathrm{d}v = \int -\sin 2x\,\mathrm{d}x$	DM1
	$0.25v^2 = \dfrac{1}{2}\cos 2x\,(+c)$	A1
	$v^2 = 2\cos 2x + c$	
	$x = 0, v = 2 \Rightarrow 4 = 2 + c$	DM1
	$v^2 = 2\cos 2x + 2\left(= 4\cos^2 x\right)$	A1
	$v = 2\cos x$ *	A1*
		(6)
ALT	*Using definite integration*	
	$0.5v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\sin 2x$	M1
	$\int_2^v 0.5v\,\mathrm{d}v = \int_0^x -\sin 2x\,\mathrm{d}x \quad \left(\text{or } \int_0^x \sin 2x\,\mathrm{d}x\right)$	DM1
	$\left[0.25v^2\right]_2^v = \left[\dfrac{1}{2}\cos 2x\right]_0^x \quad \left(\text{or } \left[-\dfrac{1}{2}\cos 2x\right]_x^0\right)$	A1
	$0.25\left(v^2 - 4\right) = \dfrac{1}{2}\cos 2x - \dfrac{1}{2}$	DM1A1
	$v = 2\cos x$ *	A*
		(6)

Question Number	Scheme	Marks
(b)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 2\cos x$	M1
	$\int \sec x \mathrm{d}x = \int 2\mathrm{d}t$	
	$\ln\lvert\sec x + \tan x\rvert = 2t + k$	DM1
	$t = 0, x = 0 \ \ \ln 1 = 2(0) + k \Rightarrow k = 0$	A1
	$t = \dfrac{1}{2}\ln\lvert\sec x + \tan x\rvert = \dfrac{1}{2}\ln\left(\sec\dfrac{\pi}{4} + \tan\dfrac{\pi}{4}\right)$	DM1
	$t = \dfrac{1}{2}\ln\left(\sqrt{2}+1\right)$ *	A1*
		(5)
ALT	*Using definite integration*	
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 2\cos x$	M1
	$\int_0^{\frac{\pi}{4}} \sec x \mathrm{d}x = \int_0^t 2\mathrm{d}t$	
	$\left[\ln\lvert\sec x + \tan x\rvert\right]_0^{\frac{\pi}{4}} = \left[2t\right]_0^t$	DM1A1
	$2t = \ln\left(\sec\dfrac{\pi}{4} + \tan\dfrac{\pi}{4}\right)$	DM1
	. $t = \dfrac{1}{2}\ln\left(\sqrt{2}+1\right)$ ..*	A1*
		(5)
		[11]

(a)	*Indefinite integration*	
M1	Equation of motion, with acceleration in the form $v\dfrac{dv}{dx}$. Condone sign error.	
DM1	Separate variables to prepare for integration. Depends on the M mark above.	
A1	Correct integration. Constant not needed.	
DM1	Substitute $x = 0,\ v = 2$ to find the constant. Depends on both M marks above.	
A1	A correct result for v^2	
A1*	**Given** result reached through use of double angle formula. (Formula need not be shown.).	
ALT	*Definite integration*	
M1	Equation of motion, with acceleration in the form $v\dfrac{dv}{dx}$. Condone sign error.	
DM1	Separate variables, to prepare for integration. Limits not needed for this mark. Depends on the M mark above.	
A1	Correct integration – limits not needed	
DM1	Correct substitution of correct limits in their integrated function. Limits must be "paired" correctly. Depends on both previous M marks in (a) (Formula need not be shown.).	
A1	Correct expression which can yield v^2	
A1*	**Given** result reached through use of double angle formula. (Formula need not be shown.).	
(b)		
M1	Use of $v = \dfrac{dx}{dt}$	
DM1	Correct separation of variables and attempt integration (integral is in the formula book). Depends on first M of (b) Modulus signs may be missing.	
A1	Correct integration and use limits to find correct value for constant.	
DM1	Substitute $x = \dfrac{\pi}{4}$ and solve for t. Depends on both previous M marks in (b)	
A1*	Given result reached from fully correct working. (Modulus signs may be missing throughout.).	

ALT	*Definite integration*	
M1	Use of $v = \frac{dx}{dt}$	
DM1	Correct separation of variables and attempt integration. Limits not needed. Depends on first M of (b). Modulus signs may be missing.	
A1	Correct integration including correct limits.	
DM1	Substitute their limits and solve for t. Depends on both previous M marks in (b)	
A1*	Given result reached from fully correct working. (Modulus signs may be missing throughout.).	

Question Number	Scheme	Marks
5(a)	$\frac{1}{2}m(8ag)+mg(8a)=\frac{1}{2}mv^2+mg(8a\cos\theta)$	M1A1A1
	$\left(v^2=24ga-16ga\cos\theta\right)$	
	$T+mg\cos\theta=\frac{mv^2}{8a}$	M1A1
	$T+mg\cos\theta=\frac{m\left(24ga-16ga\cos\theta\right)}{8a}$	DM1
	$T+mg\cos\theta=3mg-2mg\cos\theta$	
	$T=3mg-3mg\cos\theta=3mg(1-\cos\theta)$ *	A1*
		(7)
(b)	At B $v_B{}^2=24ga$	B1
	$T_1=\frac{m(24ag)}{8a}=3mg$ or $T_2=\frac{m(24ag)}{3a}=8mg$	B1
	$\Delta T=5mg$	B1
		(3)
(c)	$\frac{1}{2}mv_1{}^2=\frac{1}{2}m(8ag)+mg(11a)$	M1
	$v_1{}^2=30ag$	
	After impact $v_2{}^2=20ag$	A1
	$\frac{1}{2}m(20ag)-mg(3a)=\frac{1}{2}mv_2{}^2+mg(8a\cos\alpha)$	M1A1
	$\left(v_2{}^2=14ga-16ga\cos\alpha\right)$	
	$mg\cos\alpha=\frac{m\left(14ga-16ga\cos\alpha\right)}{8a}$	M1A1
	$mg\cos\alpha=\frac{7mg}{4}-2mg\cos\alpha$	
	$\cos\alpha=\frac{7}{12}$ *	A1*

Question Number	Scheme	Marks
		(7)
		[17]

(a)

M1 Attempt at energy equation at a general point. Must be dimensionally correct and contain two KE terms and a change in GPE.

A1, A1 Correct unsimplified equation. -1 each error.

M1 Attempt to resolve radially. Acceleration can be in either circular form.

A1 Correct equation. Must be $\frac{mv^2}{r}$

DM1 Eliminate v to produce equation in T, m, g, θ. Dependent of the previous 2 M marks.

A1* Reach given result with no errors seen.

(b)

B1 $v_B{}^2 = 24ga$. Correct expression for speed (or speed squared) at B. This mark will **not** be implied by a correct tension if they simply use the final result in (a).

B1 Correct expression for Tension at B, for either radius. Can be found using the result from (a).

B1 Correct expression for change in tension.

(c)

M1 Attempt at energy equation at wall. Must include 2 KE terms and a change in GPE.

A1 Correct speed (or speed squared, or KE) after impact.

M1 Attempt at Energy equation to α. Must include 2 KE terms and a change in GPE.

A1 Correct energy equation.

M1 Attempt at radial equation. If T included, it must be set to zero before this mark is awarded. Condone use of $3a$ for this mark?

A1 Correct equation in $\cos\alpha$ only oe.

A1* Solve to reach given result.

Question number	Scheme	Marks
6(a)	$\frac{\lambda(D-l)}{l}=mg$	M1A1
	$\frac{\lambda(2l)^2}{2l}=mg\times 3l$	M1A1A1
	$D=\frac{5l}{3}$ *	A1*
		(6)
(b)	$mg-T=m\ddot{x}$ or $T-mg=m\ddot{x}$	M1
	$mg-\frac{3mg}{2l}(\frac{2l}{3}+x)=m\ddot{x}$ or $\frac{3mg}{2l}(\frac{2l}{3}-x)-mg=m\ddot{x}$	dM1A1
	$-\frac{3g}{2l}x=\ddot{x}$ hence SHM	A1
	period = $\frac{2\pi}{\omega}=\frac{2\pi}{\sqrt{\frac{3g}{2l}}}$ $\{\omega=\sqrt{\frac{3g}{2l}}\}$	M1
	$=2\pi\sqrt{\frac{2l}{3g}}$ *	A1*
		(6)
(c)	$-\frac{2l}{3}=\frac{4l}{3}\cos\sqrt{\frac{3g}{2l}}t$	M1A1A1A1
	$t=\frac{2\pi}{3}\sqrt{\frac{2l}{3g}}$	A1
	OR	
	Complete method $t=\frac{1}{4}2\pi\sqrt{\frac{2l}{3g}}+t_1$ where $\frac{2l}{3}=\frac{4l}{3}\sin\sqrt{\frac{3g}{2l}}t_1$	M1A1A1A1
	$t=\frac{2\pi}{3}\sqrt{\frac{2l}{3g}}$ oe	A1
	OR	
	Complete method $t=\frac{1}{2}2\pi\sqrt{\frac{2l}{3g}}-t_1$ where $\frac{2l}{3}=\frac{4l}{3}\cos\sqrt{\frac{3g}{2l}}t_1$	M1A1A1A1
	$t=\frac{2\pi}{3}\sqrt{\frac{2l}{3g}}$ or equivalent exact form.	A1
		(5)
		(17)
	Notes	
(a)		
M1	Use Hooke's law in *D* and equate to *mg*	
A1	Correct equation	

M1	Energy equation with correct no. of terms. EPE of the form $\frac{\lambda x^2}{kl}, k \neq 1$
A1	Equation with at most one error
A1	Correct equation
A1*	Given answer correctly obtained
(b)	
M1	Equation of motion in a *general* position, allow *a* for acceleration, correct no. of terms, condone sign errors
dM1	Use Hooke's Law to sub for the tension with extension measured from the equilibrium position and allow *a* for acceleration
A1	Correct unsimplified equation, allow *a* for acceleration
A1	Correct SHM equation and conclusion. Must use $\ddot{x}$ for acceleration **and** conclude SHM.
M1	Use of $\frac{2\pi}{\omega}$ where ω has come from an attempt at using N2L at a general point.
A1*	Obtain the given answer for the period. Must follow from fully correct working, including N2L. At least one line of working must be seen between $\ddot{x} = -\omega^2 x$ and reaching the given answer. Eg • period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3g}{2l}}} = 2\pi\sqrt{\frac{2l}{3g}}$ • $\omega = \sqrt{\frac{3g}{2l}}$, period = $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2l}{3g}}$
(c)	
M1	Complete method to find the required time. Do not ISW. For example, If the sine approach is used, it must include $\frac{1}{4}T$ + their *t* value for M1. If the cos approach is used with $+\frac{2l}{3}$, it must include $\frac{1}{2}T$ – their *t* value for M1. The correct ω must be used. For the method, condone any multiple of *l* for the amplitude.
A1	Equation with at most two errors
A1	Equation with at most one error
A1	Correct equation
A1	Cao