

爱德思
Pure Mathematics 3
分类真题
2014-2022 册

A Level Clouds 出品

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Chapter 1

Functions and Graphs

4.

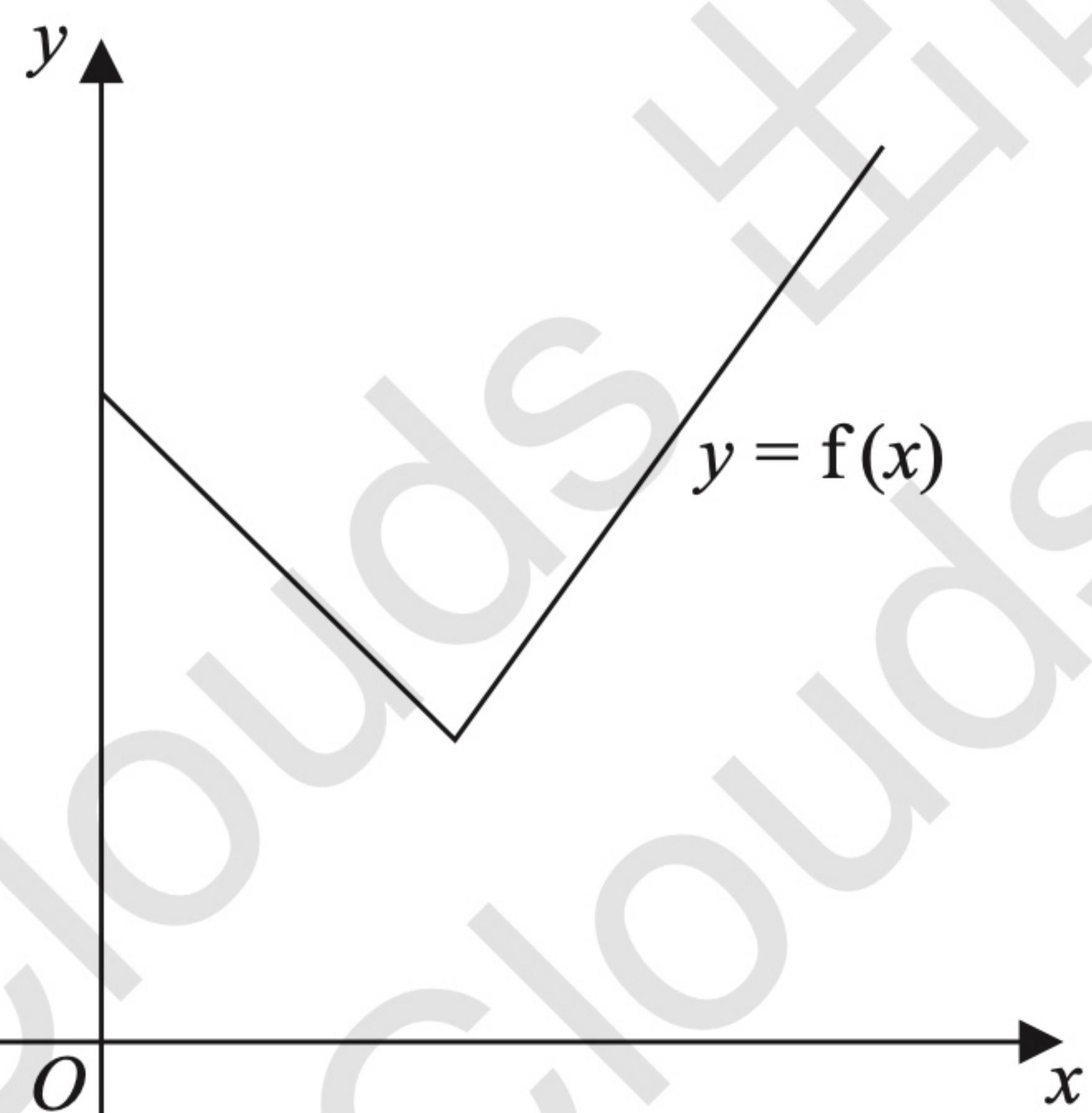
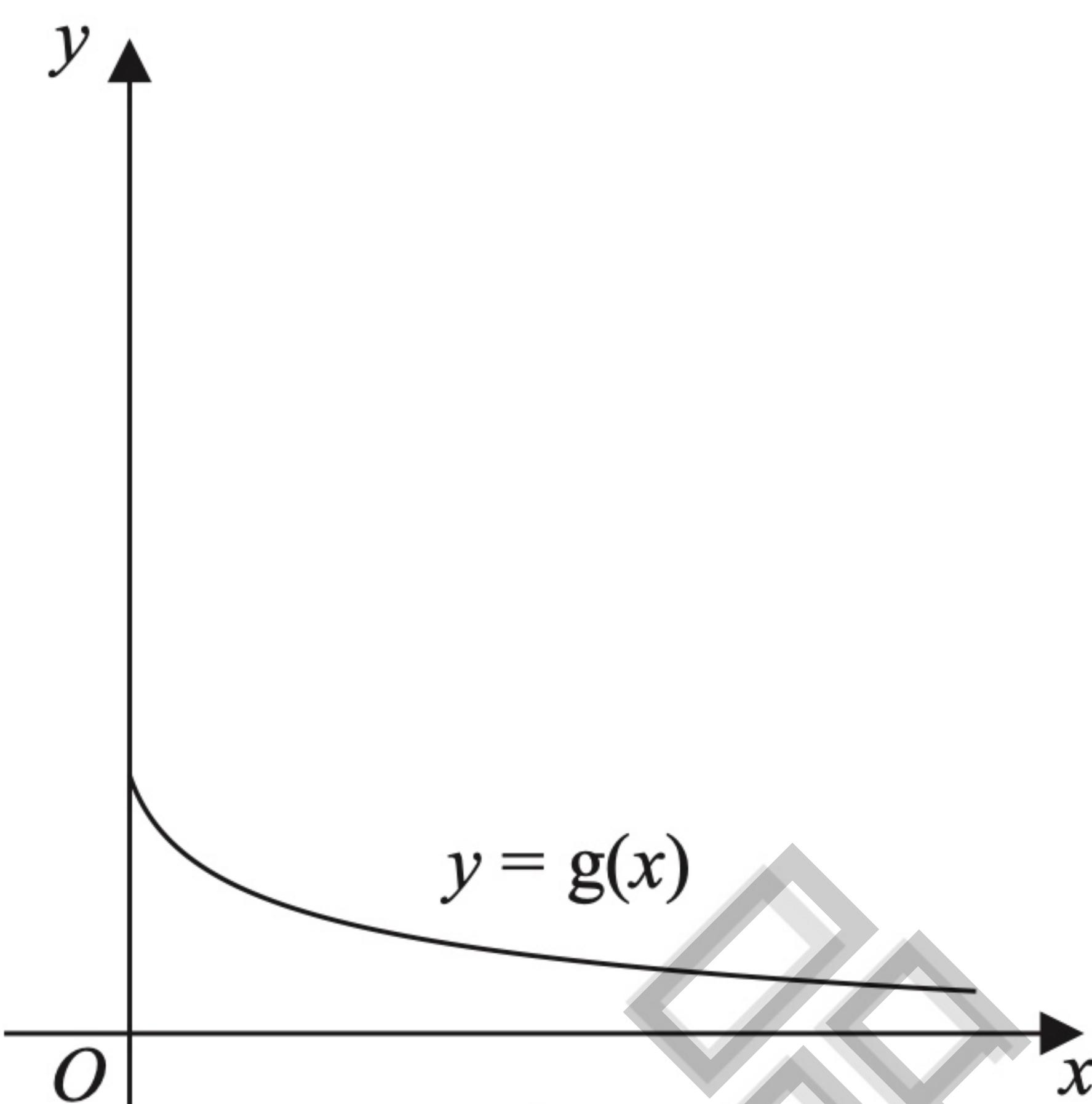
**Figure 1****Figure 2**

Figure 1 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

Figure 2 shows a sketch of part of the graph $y = g(x)$, where

$$g(x) = \frac{x+9}{2x+3}, \quad x \geq 0$$

(a) Find the value of $fg(1)$

(2)

(b) State the range of g

(2)

(c) Find $g^{-1}(x)$ and state its domain.

(4)

Given that the equation $f(x) = k$, where k is a constant, has exactly two roots,

(d) state the range of possible values of k .

(3)

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7. The function f is defined by

$$f:x \mapsto \frac{3x - 5}{x + 1}, \quad x \in \mathbb{R}, x \neq -1$$

- (a) Find an expression for $f^{-1}(x)$

(3)

- (b) Show that

$$ff(x) = \frac{x + a}{x - 1}, \quad x \in \mathbb{R}, x \neq -1, x \neq 1$$

where a is an integer to be determined.

(4)

- The function g is defined by

$$g:x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leq x \leq 5$$

- (c) Find the value of $fg(2)$

(2)

- (d) Find the range of g

(3)

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3. The function g is defined by

$$g : x \mapsto |8 - 2x|, \quad x \in \mathbb{R}, \quad x \geq 0$$

- (a) Sketch the graph with equation $y = g(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(3)

- (b) Solve the equation

$$|8 - 2x| = x + 5$$

(3)

The function f is defined by

$$f : x \mapsto x^2 - 3x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 4$$

- (c) Find $fg(5)$.

(2)

- (d) Find the range of f . You must make your method clear.

(4)

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6. Given that a and b are constants and that $a > b > 0$

- (a) on separate diagrams, sketch the graph with equation

(i) $y = |x - a|$

(ii) $y = |x - a| - b$

Show on each sketch the coordinates of each point at which the graph crosses or meets the x -axis and the y -axis.

(5)

- (b) Hence or otherwise find the complete set of values of x for which

$$|x - a| - b < \frac{1}{2}x$$

giving your answer in terms of a and b .

(4)

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4. Given that

$$f(x) = \frac{4}{3x+5}, \quad x > 0$$

$$g(x) = \frac{1}{x}, \quad x > 0$$

- (a) state the range of f ,
- (b) find $f^{-1}(x)$,
- (c) find $fg(x)$.
- (d) Show that the equation $fg(x) = gf(x)$ has no real solutions.

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(2)

(3)

(1)

(4)