1. Hash Tables: Data Structure Invariants

(a)

- **Q:** Extend `is_ht` from above, adding code to check that every element in the hash table matches the chain it is located in, and that each chain is non-cyclic.
- **A:**

```c
bool is_ht(ht H) {
  if (H == NULL) return false;
  if (!(H->m > 0)) return false;
  if (!(H->n >= 0)) return false;
  //@assert H->m == \length(H->table);
  int nodecount = 0;
  for (int i = 0; i < H->m; i++)
  {
    // set p equal to a pointer to first node
    // of chain i in table, if any
    chain* p = H->table[i];
    while (p != NULL)
    {
      elem e = p->data;
      if ((e == NULL) || (abs(hash(elem_key(e)) % H->m) != i))
        return false;
      nodecount++;
      if (nodecount > H->n)
        return false;
      p = p->next;
    }
  }
  if (nodecount != H->n)
    return false;
  return true;
}
```

(b)

- **Q:** Give a simple postcondition for this function.
- **A:**

```c
/*@ensures \result == NULL
           || key_equal(k, elem_key(\result));
@*/
```

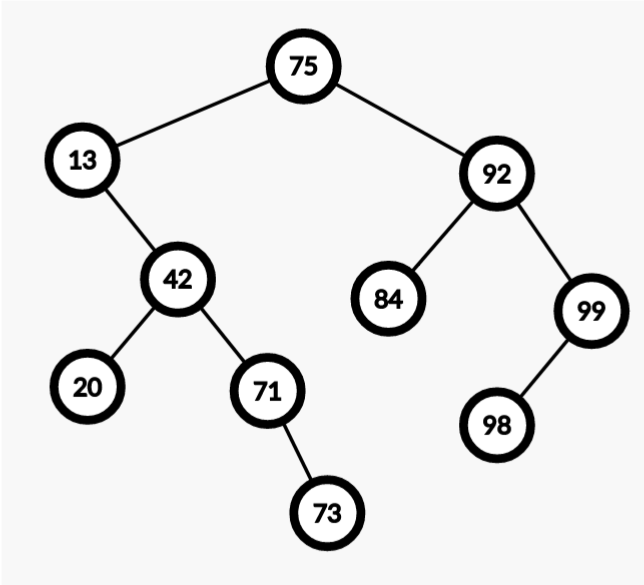
2. Binary Search Trees

(a)

- **Q:** Draw the binary search tree that results from inserting the following keys in the order given:

  `75 92 99 13 84 42 71 98 73 20`

- **A:**



(b)

- **Q:** How many different binary search trees can be constructed using the following five keys: `73, 28, 52, -9, 104` if they can inserted in any arbitrary order?

- **A:** 对于任意的二叉树结构，依据其中序遍历可以用这些数构建一颗合法的 BST，那么问题就转化为有多少种含 5 个节点的 二叉树。

  对于这个问题，已经被系统的研究过，答案即是卡特兰数（尽管它还有其他意义）。

$$Catalan(n) = \sum_{i=0}^{n-1} Catalan(i) \cdot Catalan(n-1-i)$$
$$Catalan(0) = 1$$

  可以理解为，含有 n 个节点的二叉树个数，可以由枚举合法的左右子树的所有节点数，由定义通过递归来计算出来。$Catalan(5) = 42$，一共可以构造 42 个合法的 BST。

(c)

- **Q:** Write an implementation of a new library function, `bst_height`, that returns the height of a binary search tree. The height of a binary search tree is defined as the maximum number of nodes as you follow a path from the root to a leaf. As a result, the height of an empty binary search tree is 0. Your function must include a **recursive** helper function `tree_height`.

- **A:**

```
int tree_height(tree* T)
//@requires is_ordered(T, NULL, NULL);
{
    if (T == NULL) return 0;
    int left_height = tree_height(T->left);
    int right_height = tree_height(T->right);
    if (left_height > right_height)
        return left_height + 1;
    else
        return right_height + 1;
}

int bst_height(bst B)
//@requires is_bst(B);
```

```
//@ensures is_bst(B);
{
        return tree_height(B->root);
}
```

(d)

- **Q:** Consider extending the BST library implementation with the following function which deletes an element from the tree with the given key.

```
void bst_delete(bst B, key k)
//@requires is_bst(B);
//@ensures is_bst(B);
{
        B->root = tree_delete(B->root, key k);
}
```

Complete the code for the recursive helper function `tree_delete` which is used by the `bst_delete` function. This function should return a pointer to the tree rooted at `T` once the key is deleted (if it is in the tree).

You will need to complete an additional helper function `largest_child` that removes and returns the largest child rooted at a given tree node T.

- **A:**

```
tree* tree_delete(tree* T, key k)
{
    if (T == NULL) {                   // key is not in the tree
        return NULL;
    }
    if (key_compare(k, elem_key(T->data)) < 0) {
       T->left = tree_delete(T->left, k);
       return T;
    } else if (key_compare(k, elem_key(T->data)) > 0) {
       T->right = tree_delete(T->right, k);
       return T;
    } else {      // key is in current tree node T
        if (T->left == NULL)         // node has only right child
            return T->right;
        else if (T->right == NULL)    // node has only left child
            return T->left;
        else {          // Node to be deleted has two children
            if (T->left->right == NULL) {
                // Replace the data in T with the data
                // in the left child.
                T->data = T->left->data;
                // Replace the left child with its left child.
                T->left = T->left->left;
               return T;
            }
            else {
                // Search for the largest child in the
                // left subtree of T and replace the data
                // in node T with this data after removing
```

```
                // the largest child in the left subtree.
                T->data = largest_child(T->left);
                return T;
            }
        }
    }
}

elem largest_child(tree* T)
//@requires T != NULL && T->right != NULL;
{
    if (T->right->right == NULL) {
        elem e = T->right->data;
        T->right = T->right->left;
        return e;
    }
    return largest_child(T->right);
}
```