Advanced C++ : HW 4 - Problem 1

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Problem 1: Vega of a Digital Option

Compute analytically the Vega of a digital call and put option in the Black-Scholes model. For a short digital call position with strike K and expiration T, under what condition is the Vega positive? Answer the same for a short digital put position.

Solution

The Vega for a digital option (call or put) is derived as:

$$\frac{\partial V}{\partial \sigma}(t) = \phi \cdot e^{-r(T-t)} \mathcal{N}'(d_2) \left(\frac{\ln(F/K)}{\sigma^2 \sqrt{T-t}} + \frac{1}{2}\sqrt{T-t} \right),$$

where $\phi = 1$ for a call and $\phi = -1$ for a put. For a **short** position, the Vega sign depends on the term:

$$A = \frac{\ln(F/K)}{\sigma^2 \sqrt{T-t}} + \frac{1}{2}\sqrt{T-t}.$$

Conditions for Positive Vega

- **Short Digital Call** ($\phi = 1$)**:**

$$Vega > 0 \quad \text{when} \quad A > 0 \quad \Rightarrow \quad \boxed{F > Ke^{-\frac{1}{2}\sigma^2(T-t)}}$$

- **Short Digital Put** ($\phi = -1$)**:**

$$Vega > 0 \quad \text{when} \quad A < 0 \quad \Rightarrow \quad \boxed{F < Ke^{-\frac{1}{2}\sigma^2(T-t)}}$$

Explanation

Since $\mathcal{N}'(d_2) > 0$, the sign of Vega is determined by A. The inequalities simplify to the forward price F relative to $Ke^{-\frac{1}{2}\sigma^2(T-t)}$.