Advanced C++ HW3

Yalid Rahman

February 2025

Problem 1: Knockout Forward Contract

Inputs for a forward contract:

- Strike price: K
- Expiration: T
- Knockout barrier: H at time T_1
- Nonzero dividend yield q and interest rate r

The contract gets **killed** if $S(T_1) < H$.

We are given the expectation relation:

$$\mathbb{E}_{T_1}[S(T)] = e^{(r-q)(T-T_1)}S(T_1) \tag{1}$$

From equation (16), the contract payoff is:

$$V(T) = (S(T) - K)\Theta(S(T_1) - H) \tag{2}$$

The time-t value of the contract is given by discounting the expectation:

$$V(t) = e^{-r(T-t)}\mathbb{E}_t[(S(T) - K)\Theta(S(T_1) - H)] \tag{3}$$

Using the expectation relation:

$$\mathbb{E}_{T_1}[(S(T) - K)] = e^{(r-q)(T-T_1)}S(T_1) - K \tag{4}$$

Substituting:

$$V(t) = e^{-r(T-t)}\mathbb{E}_t\left[(e^{(r-q)(T-T_1)}S(T_1) - K)\Theta(S(T_1) - H)\right] \tag{5}$$

From equation (17):

$$V(t) = e^{-r(T-t)}\mathbb{E}_t\left[(S(T_1) - H)^+ + (H-K)\Theta(S(T_1) - H)\right] \tag{6}$$

where:

- $C(t, S; H, T_1)$ is the price of a European call with strike H and expiration T_1.
- $D(t, S; H, T_1)$ is the price of a digital call with strike H and expiration T_1.

Rewriting in terms of these options:

$$V(t) = e^{-r(T-t)}\left[e^{(r-q)(T-T_1)}C(t, S; H, T_1) + (H-K)D(t, S; H, T_1)\right] \tag{7}$$

The time-t value of the knockout forward contract is:

$$V(t) = e^{-r(T-t)}\left[e^{(r-q)(T-T_1)}C(t, S; H, T_1) + (H-K)D(t, S; H, T_1)\right] \tag{8}$$