Part A, Problem #1:

- Given:
  - Binary classification: $L \in \{0,1\}$, $D \in \{0,1\}$
  - Loss matrix: λ_{ij} = loss when you decide i & truth is j
  - Priors: $P(L=0) = 0.65$, $P(L=1) = 0.35$
- Find: express the minimum expected risk classification rule in the form:

$$\frac{p(x|L=1)}{p(x|L=0)} \overset{?}{>} \gamma \text{, where } \gamma \text{ is a function of priors \& loss values}$$

∴ Risk of deciding $D=i$, given x:

$$\Longrightarrow R(D=i|x) = \lambda_{i0} \cdot P(L=0|x) + \lambda_{i1} \cdot P(L=1|x)$$

$$\therefore R(D=0|x) = \lambda_{00} \cdot P(L=0|x) + \lambda_{01} \cdot P(L=1|x)$$

$$\therefore R(D=1|x) = \lambda_{10} \cdot P(L=0|x) + \lambda_{11} \cdot P(L=1|x)$$

∴ Decision Rule: Choose $D=1$ when $R(D=1|x) < R(D=0|x)$

$$\Longrightarrow \lambda_{10} \cdot P(L=0|x) + \lambda_{11} \cdot P(L=1|x) < \lambda_{00} \cdot P(L=0|x) + \lambda_{01} \cdot P(L=1|x)$$

$$\Longrightarrow (\lambda_{10} - \lambda_{00}) \cdot P(L=0|x) < (\lambda_{01} - \lambda_{11}) \cdot P(L=1|x)$$

∴ Applying Bayes' theorem:

$$\Longrightarrow (\lambda_{10}-\lambda_{00}) \cdot \frac{P(x|L=0) \cdot P(L=0)}{P(x)} < (\lambda_{01} - \lambda_{11}) \cdot \frac{P(x|L=1) \cdot P(L=1)}{P(x)}$$

$$\Longrightarrow (\lambda_{10}-\lambda_{00}) \cdot P(x|L=0) \cdot P(L=0) < (\lambda_{01} - \lambda_{11}) \cdot P(x|L=1) \cdot P(L=1)$$

∴ Form Likelihood Ratio:

$$\Longrightarrow \frac{P(x|L=1)}{P(x|L=0)} > \frac{(\lambda_{10}-\lambda_{00}) \cdot P(L=0)}{(\lambda_{01}-\lambda_{11}) \cdot P(L=1)}$$

∴ Given $\lambda_{00}=0$, $\lambda_{11}=0$, $\lambda_{01}=1$, $\lambda_{10}=1$

$\gamma = 1.857$

$$\boxed{\therefore \frac{P(x|L=1)}{P(x|L=0)} > \gamma \text{ where } \gamma = \frac{(\lambda_{10}-\lambda_{00}) \cdot P(L=0)}{(\lambda_{01}-\lambda_{11}) \cdot P(L=1)} = 1.857}$$