CS577
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Rui Yin
HW1

1. For example, consider $A[1,2,3,4] = [1,5,2,4]$. Because the lines 6-8 in STRANGHTEN is removed, the procedure will begin from line 9 to line 11, i.e., STRANGHTEN($A[1,2]$), STRANGHTEN($A[3,4]$), and STRANGHTEN($A[2,3]$). In the last one, the item 5 and 2 will be swapped, and in the former two, nothing happens, so the final output is $A[1,2,3,4] = [1,2,5,4]$, which is unsorted.

2. For example, consider $A[1,2,3,4] = [1,5,4,2]$. After executing lines 6-8, $A[1,2,3,4] = [1,4,5,2]$. After executing STRANGHTEN($A[1,2]$), nothing happens; then STRANGHTEN($A[2,3]$) is executed before STRANGHTEN($A[3,4]$). After executing STRANGHTEN($A[2,3]$), nothing happens, and after executing STRANGHTEN($A[3,4]$), $A[1,2,3,4] = [1,4,2,5]$, which is unsorted.

3. Base case 1, in which the size of the input array is two. Obviously, it will be sorted after executing WIREDSORT (the lines 2-4 in STRANGHTEN ensure for this).
Base case 2, in which the size of the input array is four. It will be divided into two arrays of size two, each one will be sorted. That is to say, $A[1] < A[2]$ and $A[3] < A[4]$. Then, $A[2,3]$ will be swapped and then the following three lines ensure that $A[1] < A[2] < A[3] < A[4]$.
Induction hypothesis: any array of size 2^k can be sorted by WIREDSORT.
Induction step: Then, for the case in which an array of size 2^{k+1}. To make the following derivation more clearly, denote:

$$I_1 = A\left[1 \dots \frac{n}{4}\right]$$

$$I_2 = A\left[\frac{n}{4} + 1 \dots \frac{n}{2}\right]$$

$$I_3 = A\left[\frac{n}{2} + 1 \dots \frac{3}{4}n\right]$$

$$I_4 = A\left[\frac{3}{4}n + 1 \dots n\right]$$

Because of our induction hypothesis, $A\left[1 \dots \frac{n}{2}\right]$ and $A\left[\frac{n}{2} + 1 \dots n\right]$ is sorted. That is to say:

(i) All items in I_1 is smaller than that in I_2. So, all items in I_1 CANNOT appear at $A\left[\frac{3}{4}n + 1 \dots n\right]$;

(ii) All items in I_3 is smaller than that in I_4. So, all items in I_4 CANNOT appear at $A\left[1 \dots \frac{n}{4}\right]$.

Then I_2 is swapped with I_3. After that, STRANGHTEN function rearranges $[I_1, I_3]$, $[I_2, I_4]$. After that, the smallest and largest part ($A\left[1 \dots \frac{n}{4}\right], and\ A\left[\frac{3}{4}n + 1 \dots n\right]$) are fixed, and the remaining part $A\left[\frac{n}{4} + 1 \dots \frac{3}{4}n\right]$ is rearranged at last. So an array of size 2^{k+1} can be also sorted.

In conclusion, WIREDSORT can sort any input array of size a power of two.

4. Suppose that $n = 2^k$. Denote the number of comparisons made by STRANGHTEN on an array of size *n* is $f_S(n)$, note that when n=2, only one comparison is needed. So we can obtain that:

$$f_S(n) = 3f_S\left(\frac{n}{2}\right) = 3^2 f_S\left(\frac{n}{2^2}\right) = \cdots = 3^{k-1} f_S\left(\frac{n}{2^{k-1}}\right) = 3^{k-1} = \Omega(n)$$

Denote the number of comparisons made by WIREDSORT on an array of size *n* is $f_W(n)$. Then,

$$f_W(n) = 2f_W\left(\frac{n}{2}\right) + f_S(n) = 4f_W\left(\frac{n}{4}\right) + \left[2f_S\left(\frac{n}{2}\right) + f_S(n)\right] = \cdots$$

$$= 2^{k-1} f_S\left(\frac{n}{2^{k-1}}\right) + \cdots + 2f_S\left(\frac{n}{2}\right) + f_S(n)$$

$$= \sum_{i=0}^{k-1} 2^i 3^{k-1-i} = 3^{k-1} \sum_{i=0}^{k-1} \left(\frac{2}{3}\right)^i$$

As *n,k* goes up to infinity, $f_W(n) = 3^k = O(n)$.