Math 2210Q (Roby) **Practice Quiz #1 Solutions** [Today's Date]

PLEASE WRITE YOUR NAME AT THE BOTTOM OF THE BACK OF THIS SHEET, NOT ON THE FRONT.

1. Mark each of the following as **True** or **False**. You may give reasoning to support your answer, which may give you partial credit. **To show a statement is false, a specific numerical counterexample is generally best!**

   (a) The vector $2\mathbf{v}_1 + \sqrt{5}\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$.
   True. It equals $2 \cdot \mathbf{v_1} + 0 \cdot \mathbf{v}_2 + \sqrt{5} \cdot \mathbf{v_3}$

   (b) Asking whether the linear system corresponding to an augmented matrix $[\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{b}]$ has a solution amounts to asking whether $\mathbf{b}$ is in Span$\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$. True, by writing out the definitions of span and noticing that the single vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{b}$ translate into the corresponding system of simultaneous equations.

   (c) Whenever a system has free variables, the solution set contains more than one solution.
   False. An inconsisent system can have free variables. For example, if we consider the matrix in #3 below to be the augmented matrix of a system, then x_3 is a free variable, but the bottom row indicates that $0 = 1$, so the system is inconsistent and there are no solutions.

2. Find the general solution of the system whose augmented matrix is as follows:

$$\begin{pmatrix} 0 & 1 & -6 & 5 \\ 1 & -2 & 7 & -6 \end{pmatrix}$$

Row reduce the system to get:

$$\begin{pmatrix} 0 & 1 & -6 & 5 \\ 1 & -2 & 7 & -6 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & -6 & 5 \\ 1 & 0 & -5 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -5 & 4 \\ 0 & 1 & -6 & 5 \end{pmatrix}$$

Hence the general solution is

$$\begin{aligned} x_1 &= 5x_3 + 4 \\ x_2 &= 6x_3 + 5 \\ x_3 &\text{ is free.} \end{aligned}$$

3. Is the following matrix in reduced echelon form, echelon form, or neither? Explain!

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This matrix is in reduced echelon form, and the pivots are indicated in red above. You can check that it satisfies the three conditions of echelon form and the additional two for reduced echelon from.