Row Reduction & Echelon Forms

True or False: The reduced echelon form of a matrix is unique.

Row Reduction & Echelon Forms

True or False: If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.

Row Reduction & Echelon Forms

True or False: The pivot positions in a matrix depend on whether interchanges are used in the row reduction process.

Row Reduction & Echelon Forms

True or False: A general solution of a system is an explicit description of all solutions of the system.

Row Reduction & Echelon Forms

True or False: Whenever a system has free variables, the solution set contains more than one solution.

Vector Equations

True or False: Asking whether the linear system corresponding to an augmented matrix $[\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{b}]$ has a solution amounts to asking whether $\mathbf{b}$ is in $\mathrm{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

Vector Equations

True or False: Any ordered list of five real numbers is a vector in $\mathbb{R}^5$.

Matrix Equations

True or False: If the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then $\mathbf{b}$ is in the set spanned by the columns of A.

Matrix Equations

True or False: Any linear combination of vectors can always be written in the form $A\mathbf{x}$ for a suitable matrix A and vector $\mathbf{x}$.

Matrix Equations

True or False: If the coefficient matrix A has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent.

Matrix Equations

True or False: If A is an $m \times n$ matrix whose columns do not span $\mathbb{R}^n$, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^n$.

Solution Sets of Linear Equations

True or False: A homogenous system of equations can be inconsistent.

Solution Sets of Linear Equations

True or False: If x is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry of x is nonzero.

Solution Sets of Linear Equations

True or False: A equation $A\mathbf{x} = \mathbf{b}$ is homogenous if the zero vector is a solution.

Solution Sets of Linear Equations

True or False: If $A\mathbf{x} = \mathbf{b}$ is consistent, then the solution set of $A\mathbf{x} = \mathbf{b}$ is obtained by translating the solution set of $A\mathbf{x} = \mathbf{0}$.

Linear Independence

True or False: If $\mathbf{u}$ and $\mathbf{v}$ are linearly independent and $\mathbf{w}$ is in $\mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent.

Linear Independence

True or False: If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

Linear Independence

True or False: If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more than n vectors.

Linear Transformations

True or False: The range of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the rows of A.

Linear Transformations

True or False: Every matrix transformation is a linear transformation.

Linear Transformations

True or False: A linear transformation preserves the operations of vector addition and scalar multiplication.

Linear Transformations

True or False: A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ always maps the zero vector in $\mathbb{R}^n$ to the zero vector in $\mathbb{R}^m$.

Matrix of a Linear Transformation

True or False: If A is a 4×3 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^3$ onto $\mathbb{R}^4$.

Matrix of a Linear Transformation

True or False: The columns of the standard matrix for a linear transformation T from $\mathbb{R}^n$ to $\mathbb{R}^m$ are the images of the columns of the $n \times n$ identity matrix under T.

Matrix of a Linear Transformation

True or False: A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each vector in $\mathbb{R}^n$ maps to a unique vector in $\mathbb{R}^m$.

Matrix Operations

True or False: The first row of AB is the first row of A multiplied on the right by B.

Matrix Operations

True or False: If A and B are 3×3 matrices and $B = [\, \mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3 \quad]$, then

$$AB = [\, A\mathbf{b}_1 + A\mathbf{b}_2 + A\mathbf{b}_3 \,]$$

Matrix Operations

True or False: For any square matrix A, $(A^2)^T = (A^T)^2$.

Matrix Operations

True or False: The transpose of the sum of two matrices equals the sum of their transposes.

Matrix Inverses

True or False: If A is invertible, then any sequence of elementary row operations that transforms A to the identity matrix I also transforms A^{-1} to I.

Matrix Inverses

True or False: Any product of $n \times n$ invertible matrices is itself invertible, and its inverse is the product of its factors' inverses in the same order.

Matrix Inverses

True or False: If A is an $n \times n$ matrix and $A\mathbf{x} = \mathbf{e_j}$ is consistent for every $j \in \{1, 2, \ldots, n\}$, then A is invertible. (Here e_j denotes the standard basis vector, i.e., the jth column of the identity matrix.)

Matrix Inverses

True or False: If A can be row-reduced to the identity matrix, then A must be invertible.

Characterizing Matrix Inverses

True or False: Let A be an $n \times n$ matrix. If there is an $n \times n$ matrix D such that $AD = I$, then $DA = I$.

Characterizing Matrix Inverses

True or False: Let A be an $n \times n$ matrix. If the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$, then the row-reduced echelon form of A is I.

Characterizing Matrix Inverses

True or False: Let A be an $n \times n$ matrix. If the columns of A are linearly independent, then the columns of A span $\mathbb{R}^n$.

Characterizing Matrix Inverses

True or False: Let A be an $n \times n$ matrix. If the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for every $b \in \mathbb{R}^n$, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is not one-to-one.

Characterizing Matrix Inverses

True or False: Let A be an $n \times n$ matrix. If there is a $b \in \mathbb{R}^n$ such that the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then the solution is unique.

Partitioned (Block) Matrices

True or False: Let A be an $k \times \ell$ matrix, and as usual let I_n denote the $n \times n$ identity matrix. Then the matrix

$$\begin{bmatrix} I_k & A \\ 0 & I_\ell \end{bmatrix}$$

is invertible.

Partitioned (Block) Matrices

True or False: The columns of the following matrix span $\mathbb{R}^5$

$$\left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 1 & 2 \\ \hline 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Determinants

True or False: If A is an invertible matrix, then $\det A$ is the product of its diagonal entries.

Determinants

True or False: The cofactor expansion of $\det A$ down a column is the negative of the cofactor expansion along the corresponding row.

Determinants

True or False: For any $n \times n$ matrix A

$$\det A^T = (-1)\det A\,.$$

Determinants

True or False: If two successive column interchanges are made to an $n \times n$ matrix A, then the determinant remains unchanged.

Determinants

True or False: If $\det A = 0$, then either a row or column is all zeroes, or there are two rows (or two columns) which are the same.

Determinants

True or False: If A is a 2×2 matrix with zero determinant, then one column of A is a multiple of the other.

Determinants

True or False: For any $n \times n$ matrix A

$$(\det A)(\det A^{-1}) = 1$$

Vector Spaces

True or False: A vector space is also a subspace.

Vector Spaces

True or False: $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

Vector Spaces

True or False: A subset H of a vector space V is a subsapce of V if the following conditions are satisfied:

1. the zero vector of V is in H;
2. $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u}+\mathbf{v}$ are in H; and
3. c is a scalar, and $c\mathbf{u}$ is in H.

Vector Spaces

True or False: The range of a linear transformation is a vector space.

Vector Spaces

True or False: $\operatorname{Nul} A$ is the kernal of the mapping $\mathbf{x} \mapsto A\mathbf{x}$. space.

Vector Spaces

True or False: A linearly independent set in a subspace H is a basis for H.

Vector Spaces

True or False: If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.

Vector Spaces

True or False: A basis is a linearly independent set that is as large as possible.

Vector Spaces

True or False: If B is an echelon form of a matrix A, then the pivot columns of B form a basis for $\mathrm{Col}\, A$.

Vector Spaces

True or False: If $\mathcal{B}$ is the standard basis for $\mathbb{R}^n$, then the $\mathcal{B}$-coordinate vector of some $\mathbf{x} \in \mathbb{R}^n$ is $\mathbf{x}$ itself.

Vector Spaces

True or False: The correspondence $[\mathbf{x}]_{\mathcal{B}} \mapsto \mathbf{x}$ is called the *coordinate mapping*.

Vector Spaces

True or False: A plane in $\mathbb{R}^3$ is always isomorphic to $\mathbb{R}^2$.

Eigenvalues & Eigenvectors

True or False: If $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ, then $\mathbf{x}$ is an eigenvector of A.

Eigenvalues & Eigenvectors

True or False: If $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent eigenvectors, then they correspond to distinct eigenvalues.

Eigenvalues & Eigenvectors

True or False: A steady-state vector for a stochastic matrix is actually an eigenvector.

Eigenvalues & Eigenvectors

True or False: An eigenspace of A is a null space of a certain matrix.

Eigenvalues & Eigenvectors

True or False: A row replacement operation on A does not change the eigenvalues.