Math 2210Q (Roby) **Practice Midterm #1** Fall 2016

SHOW ALL YOUR WORK! Make sure you give reasons to support your answers. If you have any questions, do not hesitate to ask! For this exam **no calculators are to be used.**

1. Go back over your old homework and quizzes to review and make sure you understand any problem on which you lost points.

2. Compute all solutions to the following linear system by reducing its associated augmented matrix to Reduced Row-Echelon Form.

$$\begin{array}{rcrcrcrl} x_1 & + & & & x_3 & & & = 2 \\ x_1 & + & 2x_2 & + & 5x_3 & + & 2x_4 & = 0 \\ 2x_1 & + & x_2 & + & 4x_3 & & & = 2 \\ x_1 & + & x_2 & + & 3x_3 & + & x_4 & = 1 \end{array}$$

3. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$. Write each of the following vectors as a linear combinations of $\mathbf{v}_1$ and $\mathbf{v}_2$, or show that this cannot be done.

   (a) $\begin{bmatrix} -2 \\ 14 \\ 9 \end{bmatrix}$ (b) $\begin{bmatrix} 10 \\ 4 \\ 1 \end{bmatrix}$

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the transformation defined by $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + y \\ x - 3y \\ x + xy \end{bmatrix}$. Is T a **linear** transformation or not? Give a complete and careful explanation.

5. Perform the indicated matrix operations, where $A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 & 4 \\ 2 & 1 & 0 \end{bmatrix}$, $C = [\,2 \quad 4 \quad -1\,]$, and $D = [\,-2 \quad -3\,]$. If any operation is not possible, explain why.

   a) A^2 b) $B^T A$ c) DAB d) BC e) D^3

6. Let R be the linear transformation on two-dimensional real vectors that multiplies the vector components by 2. Let S be the linear tranformation which projects vectors onto the first coordinate (i.e., the x-axis). Let T be the linear transformation which rotates a vector by $\pi/2$ radians.

   a) Write a matrix for the composite linear tranformation which performs R, then S, then T (in that order)

   b) use your answer to the prevous part to compute the image of the vector $\mathbf{u} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ under this composite tranformation.

   c) Is the composite tranformation injective (one-to-one)? You may give your answer in clear English if you feel that no calculation is necessary.

7. Prove or Disprove and Salvage if possible:

   (a) If A and B are matrices such that both products AB and BA are defined, then A and B must be square matrices of the same size.

   (b) If A, B, and C are matrices such that $AB = AC$, then $B = C$.

   (c) If T is a linear transformation, then $T(\mathbf{0}) = \mathbf{0}$.

   (d) If two vectors in $\mathbb{R}^n$ are linearly dependent, then they both lie on the same line through the origin.

   (e) If A is **any** $k \times k$ matrix, then the block matrix $\begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$ is invertible.

8. Compute the inverse of each of the following matrices (if they exist). Show your work. (If you use a special formula, state the formula before applying it.) If an inverse does not exist, explain clearly why.

   a) $\begin{bmatrix} 1 & 6 \\ -1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 4 & 1 & 6 \\ 1 & 2 & 1 & 2 \\ -1 & -1 & -1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 \end{bmatrix}$

9. Suppose a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ has the property that $T(u) = T(v)$ for some pair of distinct vectors u and v in $\mathbb{R}^n$. Can T map $\mathbb{R}^n$ onto $\mathbb{R}^n$? Why or why not?

10. Show that if a product of two $n \times n$ matrices AB is invertible, then so is A.