Math 3240 (Roby) **Quiz #6 (Take Home)** DUE: 30 November 2010

PLEASE WRITE YOUR NAME ON THE BACK OF THIS SHEET AT THE BOTTOM, NOT ON THE FRONT.

This quiz is worth twenty points total.

1. Evaluate the following Legendre symbols: (a) $\left(\frac{71}{73}\right)$; (b) $\left(\frac{461}{773}\right)$; (c) $\left(\frac{1234}{4567}\right)$

2. Without assuming anything except the main and supplementary laws of quadratic reciprocity, show that

$$\left(\frac{-3}{p}\right) = \begin{cases} 1 & \text{if } p \equiv 1 \pmod 6 \\ -1 & \text{if } p \equiv 5 \pmod 6 \end{cases}$$

3. Use the result above to show that there are infinitely many primes of the form $6k+1$.

4. How many solutions does each of the following equations have in the given mod? (You don't need to find them explicitly, but explain your reasoning.)

   (a) $x^2 \equiv 11 \pmod{35}$

   (b) $3x^2 + 6x + 5 \equiv 0 \pmod{89}$

   (c) $x^2 + y^2 \equiv 23 \pmod{93}$

   (d) $x^2 \equiv 2 \pmod{1007}$