Quasinormal modes and its relation to Quantum Gravity

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Background

AdS/CFT correspondence

Idea: Theory A $\equiv$ Theory B, in particular the partition function Z

Example: The Maldecena case: Type IIB string theory (more on next) on $\mathrm{AdS}_5 \times S^5 \equiv$ 4D super-Yang-Mills (SYM) theory, with $\mathcal{N} = 4$ supersymmetry, and gauge group SU(N) [1]

Precisely, $\mathrm{Z}_{4D}[J] \equiv \int d\phi \; e^{iS + \int id^4xJ\mathcal{O}} = e^{iS_{cl}}$.
Motivation: QFT where perturbation theory fails, the duality counter works, and vice versa.

Takeaway: Leading hint on what QG is, a powerful tool to study strong coupling SYM theory.

String theory in a minute, maybe.

1. point (worldline) to string (worldsheet) in the high-energy limit.
   1. reparametrization of $(\tau, x^\mu) \to (\sigma)$
   2. (well) define string action
   3. Quantization (if going quantum eventually)
   4. partition function
2. Constraints:
   1. closed strings
   2. open strings $\implies$ boundary conditions (b.c.) at both ends.
      1. Neumann b.c.: endpoint of the string are constrained by $\partial x^\mu = 0$
      2. Dirichlet b.c.: endpoint of the string is fixed by $x^I = c^I$
      3. pictorial interpretation: D-branes
3. Strings with N D-branes stacked $\implies$ various fields from QFT (or beyond).

AdS/CFT: Maldecena's Example

CFT side: QCD with N color charges + SUSY

AdS side: Type IIB string theory on $\mathrm{AdS}_5 \times S^5$, after taking $\frac{\delta S_{\mathrm{IIB}}}{\delta g^{\mu\nu}} = 0$, and implementing N stacked D3 branes, we get

$$ds^2 = \left(1 + \frac{L^4}{y^4}\right)^{-1/2} \eta_{ij} dx^i dx^j + \left(1 + \frac{L^4}{y^4}\right)^{1/2} (dy^2 + y^2 d\Omega_5^2), \tag{1}$$

with L being the radius of the D3-brane via $L^4 = 4\pi g_s N(\alpha')^2$. x^i, y^μ denote coordinate $\parallel$ & $\perp$ to the D branes respectively. Then take a change of variable $u \equiv L^2/y$, and $u \to \infty$, we get

$$ds^2 = L^2[u^2 \eta_{ij} dx^i dx^j + \frac{du^2}{u^2} + d\Omega_5^2] \tag{2}$$

1st 2 terms is the AdS^5 spacetime, and the 3rd term is the extra dimensions 5-sphere.

Maldacena's Example continued

1. rescale the previous metric
2. insert to non-linear sigma model (an action).

Now, identify $g_s = g_{YM}^2, L^4 = 4\pi g_s N(\alpha'^2$, and the two theory agrees, i.e. the correspondence.

More generally, $\mathrm{Z}_{4D}[J] \equiv \int d\phi\ e^{iS+\int id^4xJ\mathcal{O}} = e^{iS_{cl}}$.

Even more generally, Holographic principle by 'Hooft[2].

The picture

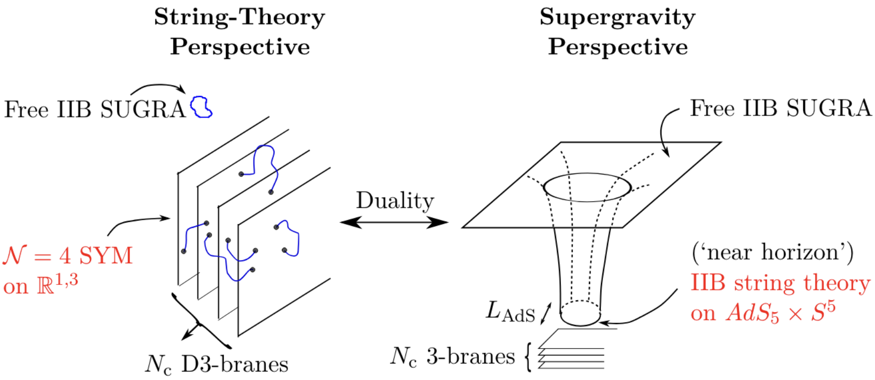

Figure: AdS/CFT visualised for the Maldecena's case. Extracted from [3]

Background review: QNM

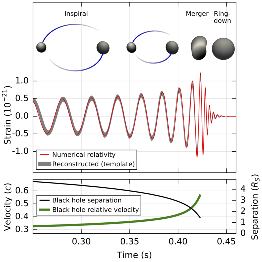

Figure: The famous 1st GW signal. Extracted from [4].

QNM is essentially the normal mode for a damped harmonic oscillator. It takes the form of:

$$\zeta \approx e^{-\omega_2 t} \cos \omega_1 t \tag{3}$$

where ω_1 is the frequency of oscillating amplitude, and ω_2 is the decay frequency for the GWs produced during the ringdown phase of the BH merger.[5].

AdS/CFT correlation to QNMs

It is well-known that the retarded Green's function of a wave equation is the normal modes of the wave functions under certain b.c.

It turns out QNMs of asymptotically AdS black holes coincide with the poles of the retarded Green's functions in the dual CFT[6].

Take a simple example, $(2+1)$ asymptotically AdS BH studied by Birmingham, Sachs and Solodukhin[7], then the QNMs are given by

$$\omega = \pm q - 4\pi T i(n+1), \tag{4}$$

where T is the Hawking temperature (or the temperature equilibrium in the dual CFT.

LQG: Immirzi Paramter

Another QG Loop quantum Gravity (LQG), essentially performs densitized "square-root" of the spatial metric h_{ij} under (3+1) ADM formalism $\tilde{E}_i^a = \sqrt{\det(h)}E_i^a$[8]. and the two are related by

$$\det(q)q^{ab} = \tilde{E}_i^a \tilde{E}_j^b \delta^{ij} \tag{5}$$

Proceeding quantization, in particular of area, we introduce the Area operator, which is a crucial step in recovering the Bekenstein Entropy Formula as well:

$$A = 8\pi\gamma l_P^2 \sqrt{j(j+1)} \tag{6}$$

where $j(j+1)$ arises from considering the eigenvalue of the angular momentum operator $\mathbf{J}^2$, and γ is the Immirzi parameter So far there are different values for γ

and thus the ambiguity for its actual value, and this is why Dreyer [9] proposed looking into QNM to fix this parameter.

Holographic Duality: AdS/CFT correspondence

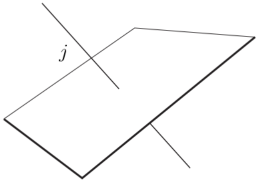

Figure: area opeartor visualized. Extracted from [9].

Conclusion

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1. Our understanding of BH is still little compared to other astronomical objects, and for physicists, BH is essentially a very powerful tool in providing insight between quantum and gravity.
2. There are numerous paradoxes produced by BH: (a) BH information paradox, (b) BH firewall, (c) Naked Singularity and (d) the final parsec problem.
3. study of BH motivated the current forefront study of fundamental physics, namely the strong motivation for the study of string theory for AdS/CFT duality.

THANK YOU!

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QnA

QnA

Reason for looking into QNM for the Immirzi parameter is as follow

Numerical result by Nollert[?] found that for the limiting behavior of the QNM frequencies as:

$$M\omega = 0.04371235 + \frac{i}{4}(n + \frac{1}{2}) \tag{7}$$

And Hod[?] observed that this is exactly $\frac{\ln 3}{8\pi}$[9].