Appearance of Quasinormal modes in Quantum Gravity

Project report

Yamamoto Sho

April 26, 2024

Contents

1 Abstract **1**

2 Introduction **1**

3 Background **2**

3.0.1 AdS/CFT correspondence 2

3.0.2 The CFT side of Maldacena's case 2

3.0.3 The AdS side of Maldacena's case 2

3.0.4 Maldacena's correspondence 3

3.1 Loop Quantum Gravity and Immirzi parameter 4

4 Appearance of QNMs in QG **4**

4.1 Appearance of QNMs in string theory 4

5 Role of QNMs in LQG **4**

6 Conclusion **5**

1 Abstract

Quasinormal modes (QNMs) refer to the eigenmodes of dissipative harmonic oscillators. These modes appear canonically when studying gravitational waves (GWs) generated by black holes (BHs). In this paper, we review the appearance of QNMs in various quantum gravity (QG), namely AdS/CFT correspondence in string theory, and the Immirzi parameter in loop quantum gravity (LQG).

2 Introduction

In a typical BH-BH merger event, the ringdown, which is referred to as the event from which the two BHs merged into one the merged BH settled down from the perturbation due to the collision forms a stable non-oscillating BH. It is this ringdown that produces a QNM signal, which is characterized by a complex frequency $\omega \in \mathbb{C}$.

QNMs serve a wide variety of uses in studying modern physics, with constraining modified gravity and QG being the prominent research direction[1][2]. This review paper begins by briefly introducing the physical picture of string theory, namely the AdS/CFT correspondence, and the Immirzi parameter appearing in quantized area in LQG, see (**??**) Followed by the precise description in which QNMs appear in the two aforementioned theories, see section (4), and concluding with the promising future direction for theoretical research in QG through studying QNMs.

3 Background

3.0.1 AdS/CFT correspondence

In this section, we review the concept of AdS/CFT correspondence, in particular, we briefly give insights to the correspondence with Maldacena's case[3] while omitting some details, in particular the consideration of supersymmetry (SUSY) that goes into the correspondence. For a review of SUSY, one may refer to [4].

It is also beneficial to rephrase the correspondence in detail. It states that, in Maldacena's study

$$\begin{array}{c} \text{Type IIB string theory on } \mathrm{AdS}_5 \times S^5 \equiv 4D \text{ super-Yang-Mills (SYM) theory, with} \\ \text{(or the low energy limit supergravity)} \quad \mathcal{N} = 4 \text{ SUSY, and gauge group SU(N).} \end{array} \tag{1}$$

The equivalence here means in particular that in the end we have the relationship

$$Z_{4D}[J] \equiv \int \mathrm{d}\phi \, e^{iS + \int i\mathrm{d}^4 x J \mathcal{O}} = e^{iS_{cl}}, \tag{2}$$

with Z_{4D} denoting the partition (or generating) function of the R.H.S. of (1), while S_{cl} denoting the classical action on the L.H.S..

3.0.2 The CFT side of Maldacena's case

The CFT side, i.e. R.H.S. of (1), while neglecting SUSY, can be interpreted simply as the SU(N) YM theory in 4-dimensional space, in particular, N = 3 is the familiar Quantum Chromodynamics (QCD). The physical interpretation for this is thus as intuitive as a theory describing N-color charged particles.

It is also worth mentioning one general feature of SU(N) YM theories are non-perturbative. In particular, QCD is indeed non-perturbative, and that the dual AdS side is otherwise, which motivates for more study into the correspondence, as it can give more insight into non-perturbative QFT.

3.0.3 The AdS side of Maldacena's case

The AdS side, i.e. L.H.S. of (**??**), is essentially type IIB string theory on $\mathrm{AdS}_5 \times S^5$. some tricks considered by Maldacena, one may in fact physically interpret this as gravity in 5D, which again the SUSY is ignored for providing physical intuition.

To demonstrate it mathematically, we begin by considering N coincident D3-brane on type IIB string theory on $\mathrm{AdS}_5 \times S^5$ and in Maldacena's case, the branes are placed at the origin, and encoded hidden in the extra dimensions that are not visible to us. It is then possible to obtain the solution for this system, with equations for the D-branes, and with the solution of the spacetime metric as

$$\mathrm{d}s^2 = \left(1 + \frac{L^4}{y^4}\right)^{-1/2} \eta_{ij}\mathrm{d}x^i\mathrm{d}x^j + \left(1 + \frac{L^4}{y^4}\right)^{1/2} (\mathrm{d}y^2 + y^2\mathrm{d}\Omega_5^2), \tag{3}$$

where L is the radius of the D3-brane, with the relation $L^4 = 4\pi g_s N(\alpha')^2$, while x^i, y^μ denote the coordinates both parallel and perpendicular to the brane respectively. To have a better understanding this geometry, we first make the change of variable $u \equiv L^2/y$, and take the limit $u \to \infty$, resulting

$$\mathrm{d}s^2 = L^2\left(\frac{1}{u^2}\eta_{ij}\mathrm{d}x^i\mathrm{d}x^j + \frac{\mathrm{d}u^2}{u^2} + \mathrm{d}\Omega_5^2\right). \tag{4}$$

First, the interpretation of this limit is simply zooming into the brane, and the first two terms correspond to the geometry AdS_5 as $\eta_{ij} = \mathrm{diag}(-1, 1, 1, 1)$. The latter term $\mathrm{d}\Omega_5^2$ denotes the 5-sphere and thus the geometry S^5 that are hidden to us.

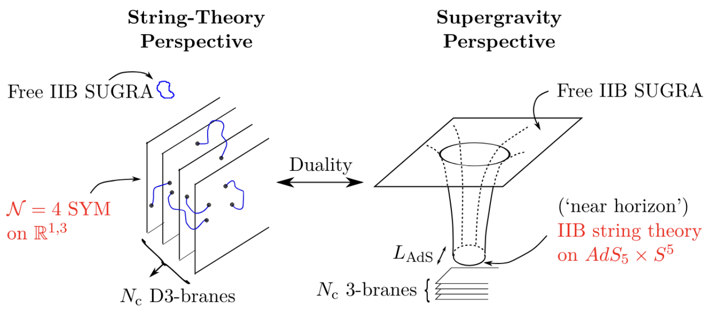

Figure 1: Sketch of the AdS/CFT duality. On the CFT side, it is the $\mathcal{N} = 4$ SYM, or interpreted in string theory as D-3 branes stack that produce the SYM gauge fields. On the other hand, it is the low-energy limit SUGRA perspective, where the spacetime metric of the form $\mathrm{AdS}_5 \times S^5$ that is dual to the other. Modified from [5].

3.0.4 Maldacena's correspondence

The Maldacena's correspondence states that if we identify the coupling constants of the two theories discussed previously, namely $g^2_{\mathrm{YM}} = 4\pi g_s, \quad g^2_{YM}N_c = L^2/\alpha'$, where α' is the universal Regge slope, $g_s, N_c,$ and g_{YM} string coupling constant, number of colour charges and YM coupling constant, Maldacena showed that

$$Z_{\mathrm{CFT}}[J] \equiv e^{iS_{cl}}. \tag{5}$$

In other words, the generating function of the CFT side is equivalent to the exponent of the action of the dual AdS gravity theory. This remarkable result motivated others to look for more such correspondences in different settings. Later t'Hooft conjectured, motivated by the holographic principle[6], that any D+1 dimensional gravity is dual to its D dimensional CFT[7].

3.1 Loop Quantum Gravity and Immirzi parameter

LQG is a candidate for QG, with its formalism based on 3+1 decomposition, or so-called ADM formalism, of general relativity(GR)[8]. It was Ashtekar[9] further had the inspiration to decompose the spatial metric as composition of two densitized triad

$$\tilde{E}^a_i = \sqrt{\det(h)}E^a_i, \tag{6}$$

with this and the spatial metric related by $\det(q)q^{ab} = \tilde{E}^a_i\tilde{E}^b_j\delta^{ij}$, where the indices (a,b) denote the spin once the spin *connection* is defined from this theory which is constructed similarly to the Christoeffel connection for GR.

Proceeding quantization under this formalism, the quantized area of this theory can be obtained, which is given by

$$A(j) = 8\pi l_P^2\gamma\sqrt{j(j+1)}, \tag{7}$$

where l_P is the Planck length and γ the Immirzi parameter, while the positive half-integers j denotes the spin representation of the given surface[10].

4 Appearance of QNMs in QG

In this section, we mention the appearance of QNMs in certain QG theories, namely string theory and LQG.

4.1 Appearance of QNMs in string theory

It was shown by Birmingham, Sach and Solodukhin that for the AdS/CFT correspondence, with AdS_{2+1} at the near-horizon geometry as the AdS side dual to the corresponding CFT, the QNMs of the gravity side is exactly the poles of the retarded correlation function of the dual CFT[11].

The QNMs in this setting, with the Banados-Teitelboim-Zanelli (BTZ) BH as the metric solution[12], we obtain

$$\omega = \pm k - 4\pi Ti(n+1), \tag{8}$$

where k denotes momentum, T the Hawking the temperature of BTZ BH and n the modes.

There are more on-going search for the appearance of QNMs in different AdS/CFT settings, and it is also possible that one can constraint the string landscape with observational QNMs through AdS/CFT correspondence of 4D gravity dual to 3D CFT.

5 Role of QNMs in LQG

Rather than QNMs appearing LQG, QNMs provide a way to fix Immirzi parameter in the theory, given there are no conclusive ways to obtain a consistent Immirzi parameter theoretically. To do so, we may refer to numerical result of QNMs, and match it with the theory[13].

For example, taking the result from Nollert[14], we have the numerical QNM value as

$$M\omega = 0.04371235 + \frac{i}{4}(n + \frac{1}{2}), \tag{9}$$

where M is the mass of the merged BH, this result was later observed that the constant real part is equal to $\ln 3/8\pi$.

We can then fix the Immirzi parameter by computing the entire surface of the BH through the (7), with the surface of the Schwarzschild BH related to M by $A = 16\pi M^2$. With these systems of equations, and consideration of the Bekenstei-Hawking equation, in turn then fixes the Immirzi parameter as

$$\gamma = \frac{\ln 3}{2\pi\sqrt{2}}. \tag{10}$$

6 Conclusion

To conclude, QNMs are closely related to QG as naturally the dynamics of QNMs are encoded by the dynamics of BHs, which requires QG to fully describe its nature. We have reviewed where this is observed in the theory, namely QNMs are related to the poles in the retarded green the function of the CFT theory that is dual to the QNMs generated by the BH solution in the AdS gravity via the AdS/CFT correspondence. However, we still lack enough research on more cases to validate and generalise this observation in the theory. We also reviewed the numerical results of QNM serve to fix the Immirzi parameter in LQG, though once again more verifications are needed to justify the procedure, as there could be degeneracies from both the theory and the numerical results.

References

[1] Sebastian H Völkel, Nicola Franchini, Enrico Barausse, and Emanuele Berti. Constraining modifications of black hole perturbation potentials near the light ring with quasinormal modes. *Physical Review D*, 106(12):124036, 2022.

[2] Dhruba Jyoti Gogoi and Supakchai Ponglertsakul. Constraints on quasinormal modes from black hole shadows in regular non-minimal einstein yang-mills gravity. *arXiv preprint arXiv:2402.06186*, 2024.

[3] Juan Maldacena. The large-n limit of superconformal field theories and supergravity. *International journal of theoretical physics*, 38(4):1113–1133, 1999.

[4] Martin F Sohnius. Introducing supersymmetry. *Physics reports*, 128(2-3):39–204, 1985.

[5] A Samberg. *A Holographic View on Screening in Strongly Coupled Plasmas with Chemical Potential*. PhD thesis, diploma thesis, Heidelberg University, 2012.

[6] Leonard Susskind. The world as a hologram. *Journal of Mathematical Physics*, 36(11):6377–6396, 1995.

[7] Gerard't Hooft. The holographic principle. In *Basics and Highlights in Fundamental Physics*, pages 72–100. World Scientific, 2001.

[8] Alejandro Corichi and Dario Núñez. Introduction to the adm formalism. *arXiv preprint arXiv:2210.10103*, 2022.

[9] Abhay Ashtekar and Eugenio Bianchi. A short review of loop quantum gravity. *Reports on Progress in Physics*, 84(4):042001, 2021.

[10] Abhay Ashtekar and Jerzy Lewandowski. Quantum theory of geometry: I. area operators. *Classical and Quantum Gravity*, 14(1A):A55, 1997.

[11] Danny Birmingham, Ivo Sachs, and Sergey N Solodukhin. Conformal field theory interpretation of black hole quasinormal modes. *Physical review letters*, 88(15):151301, 2002.

[12] Thanasis Karakasis, Eleftherios Papantonopoulos, Zi-Yu Tang, and Bin Wang. Black holes of (2+ 1)-dimensional f (r) gravity coupled to a scalar field. *Physical Review D*, 103(6):064063, 2021.

[13] Olaf Dreyer. Quasinormal modes, the area spectrum, and black hole entropy. *Physical Review Letters*, 90(8):081301, 2003.

[14] Hans-Peter Nollert. Quasinormal modes: the characteristicsound'of black holes and neutron stars. *Classical and Quantum Gravity*, 16(12):R159, 1999.