ECE 247 HW5

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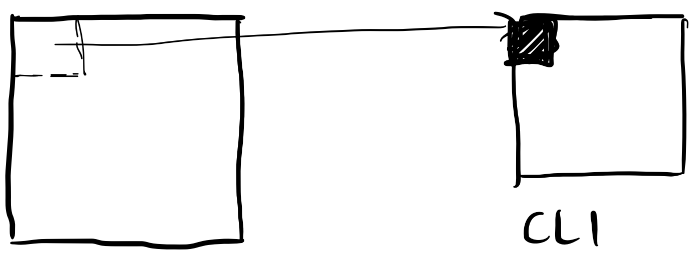
Q1:

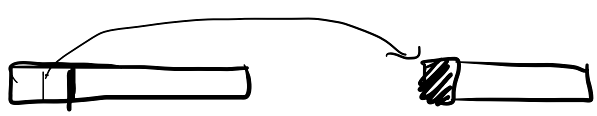
a) Receptive field of a neuron in CL_1 is $m_1 \times m_1$ or m_1.

Suppose $m_1 = 2$,

2D:



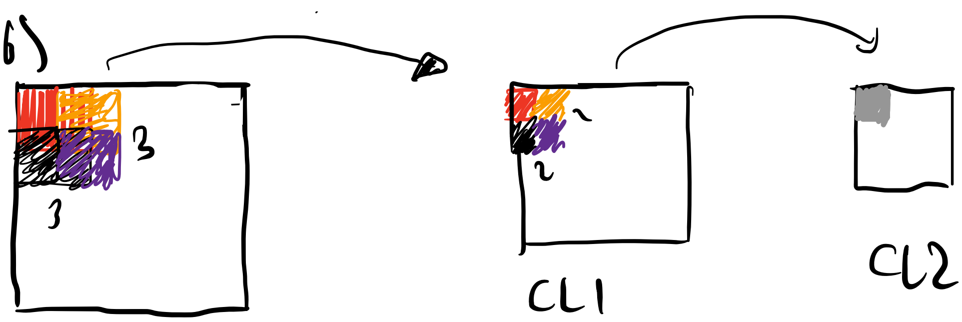
1D:



Each neuron in CL_1 is linked to $m_1 \times m_1$ patch in input. Therefore, RF is $m_1 \times m_1$

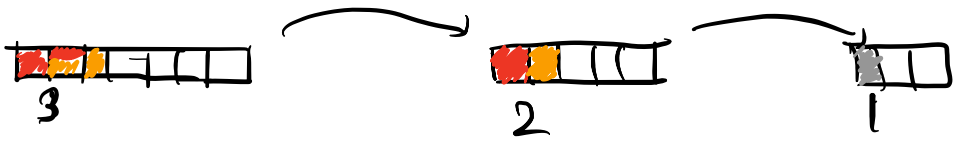
b)

2D

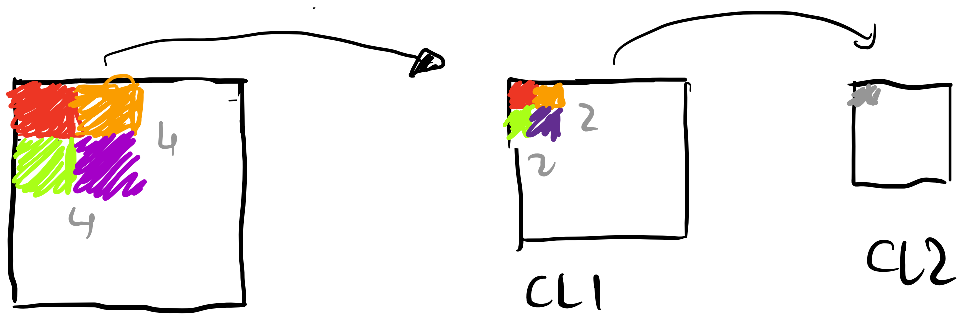


When $m_1 = m_2 = 2$, RF of a neuron in CL_2 is 3×3 or 3 which is $m_1 + m_2 - 1$ since we deduct the overlap. Therefore, receptive field of each neuron in CL_2 is $m_1 + m_2 - 1 \times m_1 + m_2 - 1$ or $m_1 + m_2 - 1$ when $stride_1 = stride_2 = 1$.

1D:



c)



It is not trivial to find receptive field, therefore one should consider a single layer.

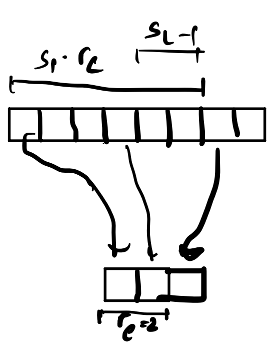
Let r_e denote the number of features in feature map CL_e which contribute generate one feature in CL_e. L is the last featuremap. $r_L = 1$

k_e : kernel size (M_e in our case)

s_e : stride

$r_{L-1} = k_L$: we found that in a part.

Suppose we know, r_e and we want to compute r_{e-1}.



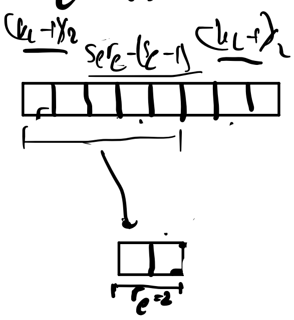
$k_e = 1$

$s_e = 3$

$s_\ell r_\ell$ will cover all features that contribute, however, it will be covered $s_\ell - 1$ more.
Therefore, formula will be

$$r_{\ell-1} = s_\ell r_\ell - (s_\ell - 1)$$

Suppose $k_\ell > 1$,



$k_\ell : 5$

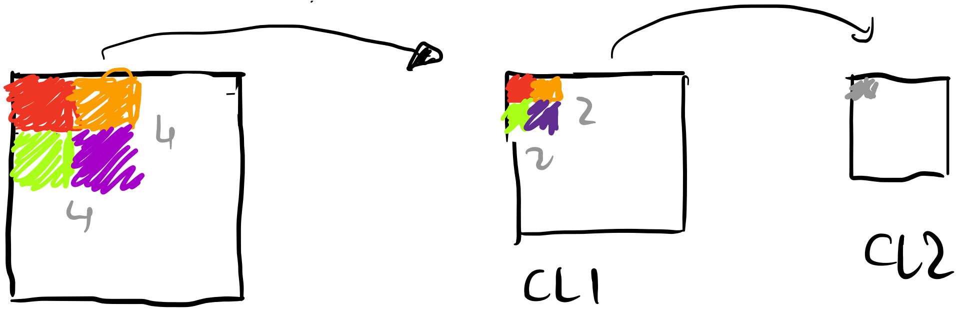
$s_\ell : 3$

Therefore, we will add $k_\ell - 1$ features to cover all.

$$r_{\ell-1} = s_\ell r_\ell - (s_\ell - 1) + k_\ell - 1$$
$$= s_\ell r_\ell + (k_\ell - s_\ell)$$

or in our case

$$= s_\ell r_\ell + (m_\ell - s_\ell)$$



$s_1 = 2$
$m_1 = 2$

$m_2 = 2$
$s_2 = ? = x$

$r_2 = 1$

$$r_1 = s_2 \cancel{\cdot 1} + m_2 - \cancel{s_2} = \boxed{m_2} \Rightarrow r_{L-1} = m_L = 2 ✓$$

$$r_0 = s_1 r_1 + m_1 - s_1 = 2 * 2 + 2 - 2 = 4 ✓$$

Therefore, receptive field of CL_2 is

$r_2 = 1$

$r_1 = m_2$

$r_0 = \boxed{s_1 m_2 + m_1 - s_1}$ or $\boxed{(s_1 m_2 + m_1 - s_1 \times s_1 m_2 + m_1 - s_1)}$

Same formula can be used for any layer, you just need to assign that layer to be L.

Therefore, receptive field of CL_1 is

$r_1 = r_L = 1$

$r_0 = r_{L-1} = m_\ell = \boxed{m_1}$ or $\boxed{(m_1 \times m_1)}$

Note that receptive field of CL_1 is independent of stride, therefore unchanged.

However, receptive field of CL_2 can change.

when $s_1 = 1 \Rightarrow RF_1 = m_2 + m_1 - 1 > 0$

when $s_1 = 2 \Rightarrow RF_2 \quad 2m_2 + m_1 - 2 > 0$

For minimum m_2, m_1 $\quad RF_1 = 1$

$RF_2 = 2m_2 + m_1 - 2 = 1$

For $m_2 = 2, m_1 = 1$ $\quad RF_1 = 2$

$RF_2 = 4 + 1 - 2 = 3 > 2$

For $n_2 = 1, n_1 = 2$ $\quad RF_1 = 1$

$$RF_1 = 1 \quad \| \quad RF_2 = 1$$

In result, CL_2 receptive field gets bigger as s_1 grows.

d) The recurrence relation is

$$r_{e-1} = s_e n_e + n_e - s_e \qquad r_L = 1$$

$$r_{L-1} = n_L$$

which has the solution,

$$r_0 = \sum_{e=1}^{L} \left((n_e - 1) \prod_{i=1}^{e-1} s_i \right) + 1$$

This was solved in practice problems, so I have only written the answer.

Note that we are asked for the k^{th} layer. therefore assign layer L as k^{th} layer. Solution becomes

$$r_0 = \sum_{e=1}^{K} \left((n_e - 1) \prod_{i=1}^{e-1} s_i \right) + 1$$

Solution makes sense when $s_i = 1$,

$$r_0 = m_k + m_{k-1} + \dots + m_1 - k + 1$$

when $k = 2$

$r_0 = m_2 - m_1 - 1$ which I found in part b.

Therefore, receptive layer of k^{th} layer is

$$\left(\sum_{e=1}^{k} \left((m_e - 1) \prod_{i=1}^{e-1} s_i \right) + 1 \times \sum_{e=1}^{k} \left((m_e - 1) \prod_{i=1}^{e-1} s_i \right) + 1 \right)$$

or

$$\sum_{e=1}^{k} \left((m_e - 1) \prod_{i=1}^{e-1} s_i \right) + 1$$

e) a) Increasing filter size m_i

b) Adding more layers

c) Increasing stride of filters except stride of last layer.

$$r_0 = \sum_{e=1}^{k} \left((m_e - 1) \prod_{i=1}^{e-1} s_i \right) + 1$$

a) $(m_e - 1) \uparrow$

b) $k \uparrow$

c) $s_i \uparrow$