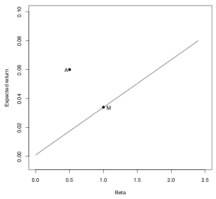
University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

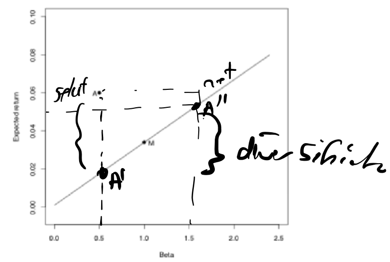
Homework 6

a. The following plot shows the expected return against beta of the market portfolio M and your portfolio A based on some model you chose.



It is given that $\bar{R}_A = 0.06, \beta_A = 0.5, \bar{R}_M = 0.034, \sigma^2_M = 0.015, R_f = 0.001$. The total risk of your portfolio is 0.03375. Compute the following components:
Return from selectivity
Return from net selectivity
Return from diversification

You are a portfolio manager and a client has given you a target risk $\beta_T = 0.25$. Compute the following components:
Return due to "investor's risk."
Return due to "manager's risk." This is the return due to the manager's choice of a different risk than the target.



Return from selectivity:

$R_{A'} = R_f + \frac{\bar{R}_m - R_f}{1} 0.5 = 0.0175 \Rightarrow$

$\bar{R}_A - \bar{R}_{A'} = 0.0425$

Return from net selectivity

$0.03375 = \beta_A^2 \sigma_m^2 \Rightarrow \beta_A = 1.5$

$\bar{R}_{A''} = 0.001 + \frac{0.034 - 0.001}{1} 1.5 = 0.0505$

$\bar{R}_A - \bar{R}_{A''} =$ net selectivity $= 0.0095$

Return from diversification:

$\bar{R}_{A''} - \bar{R}_{A'} = 0.0330$

Return due to investor's risk:

$R_T = 0.001 + \frac{0.034 - 0.001}{1} \times 0.25 = 0.0093$

$\bar{R}_T - R_f = 0.0083$

Return due to manager's risk:

$\bar{R}_{A'} - R_T = 0.0083$

b. A large pension fund wants to evaluate the performance of four portfolio managers for the last 5 years. During this time period the average annual return of the S&P500 was 14% with standard deviation 12%. The average annual risk free interest rate was 8%. The four portfolios gave the following data:

Portfolio	Average annual return (%)	Standard deviation (%)	Beta
A	16	19	1.2
B	22	16	1.9
C	10	10	0.8
D	15	13	1.3

For funds A and B, how much the return on B has to change to reverse the ranking using the Sharpe measure?

A: B:

$$\frac{16-8}{19}\cdot\cancel{\frac{1}{100}} < \frac{22-8}{16}\cdot\cancel{\frac{1}{100}}$$

B is better than A; so A should be better

$$\frac{16-8}{19} > \frac{R_B-8}{16} \Rightarrow$$

$16\,\frac{8}{19}+8 = 14.7368 > R_B$, R_B should decrease from 22 percent to 14.7368 percent $\Rightarrow$ %7.26 decrease

c. A large pension fund wants to evaluate the performance of four portfolio managers for the last 5 years. During this time period the average annual return of the S&P500 was 14% with standard deviation 12%. The average annual risk free interest rate was 8%. The four portfolios gave the following data:

Portfolio	Average annual return (%)	Standard deviation (%)	Beta
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Compute the Treynor performance measure for portfolio A and the Sharpe measure for portfolio B?

Treynor A:

$$\frac{16-8}{1.2} \times \frac{1}{100} = 0.0667$$

Sharpe B:

$$\frac{22-8}{16} \times \frac{1}{100} = 0{,}475 \times 10^{-2} = 0.0088$$

d. Refer to question (c). Compute the Jensen differential performance index for each portfolio and show them on the space expected return against beta.

Jensen dif perf. index: $\bar{R}_p = R_f + \frac{(\bar{R}_m - R_f)}{1}\beta_p$

Portfolio A:

$R_A' = 0{,}08 + (0{,}14 - 0{,}08)\,1.2 = 0.152$ $\quad \bar{R}_A - \bar{R}_A' \overset{16}{=} 0.008$

$R_B' = 0{,}08 + (0{,}14 - 0{,}08)\,1.9 = 0.194$ $\quad \bar{R}_B - \bar{R}_B' \overset{22}{=} 0.0260$

$R_C' = 0{,}08 + (0{,}14 - 0{,}08)\,0{,}8 = 0.128$ $\quad \bar{R}_C - \bar{R}_C' \overset{10}{=} -0.0280$

$R_D' = 0{,}08 + (0{,}14 - 0{,}08)\,1.3 = 0.158$ $\quad \bar{R}_D - \bar{R}_D' \overset{15}{=} -0.008$

e. Consider the multi-index model as discussed in class. Derived the covariance between stocks that belong in the same industry and the covariance between stocks that belong in different industries. Please show the details.

Same industry

$$\sigma_{ik} = \text{cov}[\alpha_i + \beta_i I_j + \epsilon_i, \alpha_k + \beta_k I_j + \epsilon_k]$$

$$= \beta_i \beta_k \sigma_j^2 = \boxed{\beta_i \beta_j [b_j \sigma_m^2 + \sigma_{e_j}^2]}$$

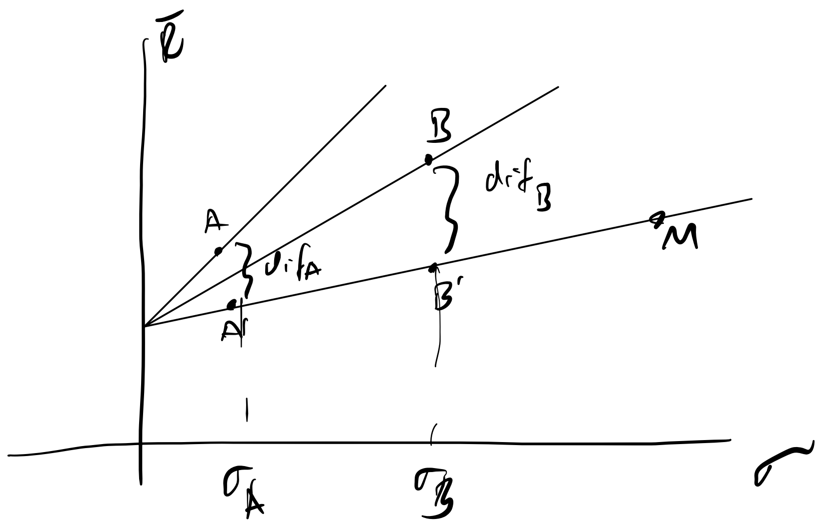
Different industry

$$\sigma_{ik} = \text{cov}[\alpha_i + \beta_i I_j + \epsilon_i, \alpha_k + \beta_k I_e + \epsilon_k]$$

$$= \beta_i \beta_k \text{cov}(I_j, I_e)$$

$$\boxed{= \beta_i \beta_k b_j b_e \sigma_m^2}$$

f. Consider the following two measures of portfolio performance: The Sharpe ratio and the differential excess return. Show graphically a situation of two portfolios A and B that are ranked as $A > B$ using the Sharpe ratio but at the same time $B > A$ using the differential excess return. Please explain why $A > B$ and $B > A$ for the respective measures of performance mentioned above.



$dif_B > dif_A \Rightarrow R_B - R_{B'} > R_A - R_{A'}$

But

Shape A > Shape B

$$\frac{R_A - R_f}{\sigma_A} > \frac{R_B - R_f}{\sigma_B}$$

Example in the class

since $\sigma_B > \sigma_A$ and R_B is not big enough to compensate

g. Consider the multigroup model. In class we used an example of two stocks and two industries and then we extended the result to the general case. Use the same steps to develop the system of equations when there are are three stocks and three industries to find the elements of the equation $\mathbf{A}\mathbf{\Phi} = \mathbf{C}$. You can begin with $\bar{R}_1 - R_f = z_1\sigma_1^2 + z_2\sigma_{12} + z_3\sigma_{13} + z_4\sigma_{14} + z_5\sigma_{15} + z_6\sigma_{16} + z_7\sigma_{17} + z_8\sigma_{18} + z_9\sigma_{19}$.

$$\begin{bmatrix} 1+\frac{3\rho_{11}}{1-\rho_{11}} & \frac{3\rho_{12}}{1-\rho_{11}} & \frac{3\rho_{13}}{1-\rho_{11}} \\ \frac{3\rho_{21}}{1-\rho_{22}} & 1+\frac{3\rho_{22}}{1-\rho_{22}} & \frac{3\rho_{23}}{1-\rho_{22}} \\ \frac{3\rho_{31}}{1-\rho_{33}} & \frac{3\rho_{32}}{1-\rho_{33}} & 1+\frac{3\rho_{33}}{1-\rho_{33}} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{3} \frac{\bar{R}_i - R_f}{\sigma_i(1-\rho_{11})} \\ \sum_{i=4}^{6} \frac{\bar{R}_i - R_f}{\sigma_i(1-\rho_{22})} \\ \sum_{i=7}^{9} \frac{\bar{R}_i - R_f}{\sigma_i(1-\rho_{33})} \end{bmatrix}$$

Derivation:

$$\bar{R}_1 - R_f = z_1\sigma_1^2 - z_1\rho_{11}\sigma_1^2 + z_1\rho_{11}\sigma_1^2 + z_2\rho_{12}\sigma_1\sigma_2 + \dots + z_9\rho_{13}\sigma_1\sigma_9$$

$$\bar{R}_1 - R_f = z_1\sigma_1^2(1-\rho_{11}) + \sigma_1\sum_{g=1}^{3}\rho_{1g}\Phi_g$$

$$z_1 = \frac{1}{\sigma_1(1-\rho_{11})}\left[\frac{\bar{R}_1 - R_f}{\sigma_1} - \sum_{g=1}^{3}\rho_{1g}\Phi_g\right]$$

$$z_2 = \frac{1}{\sigma_2(1-\rho_{11})}\left[\frac{\bar{R}_2 - R_f}{\sigma_2} - \sum_{g=1}^{3}\rho_{1g}\Phi_g\right]$$

$$z_3 = \frac{1}{\sigma_3(1-\rho_{11})}\left[\frac{\bar{R}_3 - R_f}{\sigma_3} - \sum_{g=1}^{3}\rho_{1g}\Phi_g\right]$$

Multiply z_i with σ_i and sum

$$\underline{\sigma_1 z_1 + \sigma_2 z_2 + \sigma_3 z_3} = \frac{1}{1-\rho_{11}}\left[\sum_{i=1}^{3}\frac{\bar{R}_i - R_f}{\sigma_i} - 3\sum_{g=1}^{3}\rho_{1g}\Phi_g\right]$$

Φ_1

$$(1-\rho_{11})\,\Phi_1 + 3\sum_{g=1}^{3}\rho_{1g}\Phi_g = \sum_{i=1}^{3}\frac{\bar{R}_i - R_f}{\sigma_i}$$

Also for other groups:

$$(1-\rho_{22})\,\Phi_2 + 3\sum_{g=1}^{3}\rho_{2g}\Phi_g = \sum_{i=4}^{6}\frac{\bar{R}_i - R_f}{\sigma_i}$$

$$(1-\rho_{33})\,\Phi_3 + 3\sum_{g=1}^{3}\rho_{3g}\Phi_g = \sum_{i=7}^{9}\frac{\bar{R}_i - R_f}{\sigma_i}$$

$$\Downarrow$$

$$\Phi_1 + \frac{3}{(1-\rho_{11})}\sum_{g=1}^{3}\rho_{1g}\Phi_g = \sum_{i=1}^{3}\frac{\bar{R}_i - R_f}{\sigma_i\,(1-\rho_{11})}$$

$$\Phi_2 + \frac{3}{(1-\rho_{22})}\sum_{g=1}^{3}\rho_{1g}\Phi_g = \sum_{i=4}^{6}\frac{\bar{R}_i - R_f}{\sigma_i\,(1-\rho_{22})}$$

$$\Phi_3 + \frac{3}{(1-\rho_{33})}\sum_{g=1}^{3}\rho_{1g}\Phi_g = \sum_{i=7}^{9}\frac{\bar{R}_i - R_f}{\sigma_i\,(1-\rho_{33})}$$

$$\Downarrow$$

$$\begin{bmatrix} 1+\frac{3\rho_{11}}{1-\rho_{11}} & \frac{3\rho_{12}}{1-\rho_{11}} & \frac{3\rho_{13}}{1-\rho_{11}} \\ \frac{3\rho_{21}}{1-\rho_{22}} & 1+\frac{3\rho_{22}}{1-\rho_{22}} & \frac{3\rho_{23}}{1-\rho_{22}} \\ \frac{3\rho_{31}}{1-\rho_{33}} & \frac{3\rho_{32}}{1-\rho_{33}} & 1+\frac{3\rho_{33}}{1-\rho_{33}} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{3}\frac{\bar{R}_i - R_f}{\sigma_i(1-\rho_{11})} \\ \sum_{i=4}^{6}\frac{\bar{R}_i - R_f}{\sigma_i(1-\rho_{22})} \\ \sum_{i=7}^{9}\frac{\bar{R}_i - R_f}{\sigma_i(1-\rho_{33})} \end{bmatrix}$$

h. For this problem assume that the single index model holds, $R_f = 0.002$ and short sales are not allowed. Using monthly returns from the past 5 years the following calculations were performed in order to determine the cut-off point C^*. For the same period we also estimate that $\bar{R}_m = 0.10, \sigma_m^2 = 0.125$.

Stock i	β_i	$\sigma_{\epsilon i}^2$	$\frac{\bar{R}_i - R_f}{\beta_i}$	C_i
1	0.80	0.02	0.28	0.188
2	0.82	0.01	0.25	0.224
3	0.90	0.03	0.22	0.223
4	0.91	0.02	0.20	0.221
5	0.94	0.01	0.17	0.220
6	0.97	0.02	0.16	0.202
7	0.97	0.03	0.13	0.157
8	1.10	0.03	0.12	0.128
9	1.12	0.01	0.10	0.126
10	1.15	0.04	0.08	0.101
11	1.17	0.05	0.05	0.062
12	1.20	0.06	0.03	0.049

1. Find the cut-off point C^*.
2. Find the composition of the optimum portfolio.
3. Find the β of the optimum portfolio.
4. Suppose a new stock has $\alpha = -0.025, \beta = 0.8$, and $\sigma_\epsilon^2 = 0.08$. Will you include this stock in your optimum portfolio?

1. $C^* = 0.224$

2) $z_1 = \frac{\beta_1}{\sigma_{\epsilon_1}^2}\left(\frac{\bar{R}_1 - R_f}{\beta_1} - C^*\right) = 2.2400$

$z_2 = \frac{\beta_2}{\sigma_{\epsilon_2}^2}\left(\frac{\bar{R}_2 - R_f}{\beta_2} - C^*\right) = 2.1320$

$$\boxed{\begin{aligned} X_1 &= 0.5124 \\ X_2 &= 0.4876 \end{aligned}}$$

3) $\sum x_i \beta_i = \beta_p = \boxed{0.8098} = X_1\beta_1 + X_2\beta_2$

4) $\bar{R}_{new} = \alpha + \beta\bar{R}_m = -0.025 + 0.8(0.1) = 0.0550$

$\frac{\bar{R}_{new} - R_f}{\beta_{new}} = \frac{0.0550 - 0.002}{0.8} = \boxed{0.0613 < C^* \text{ will not be included}}$