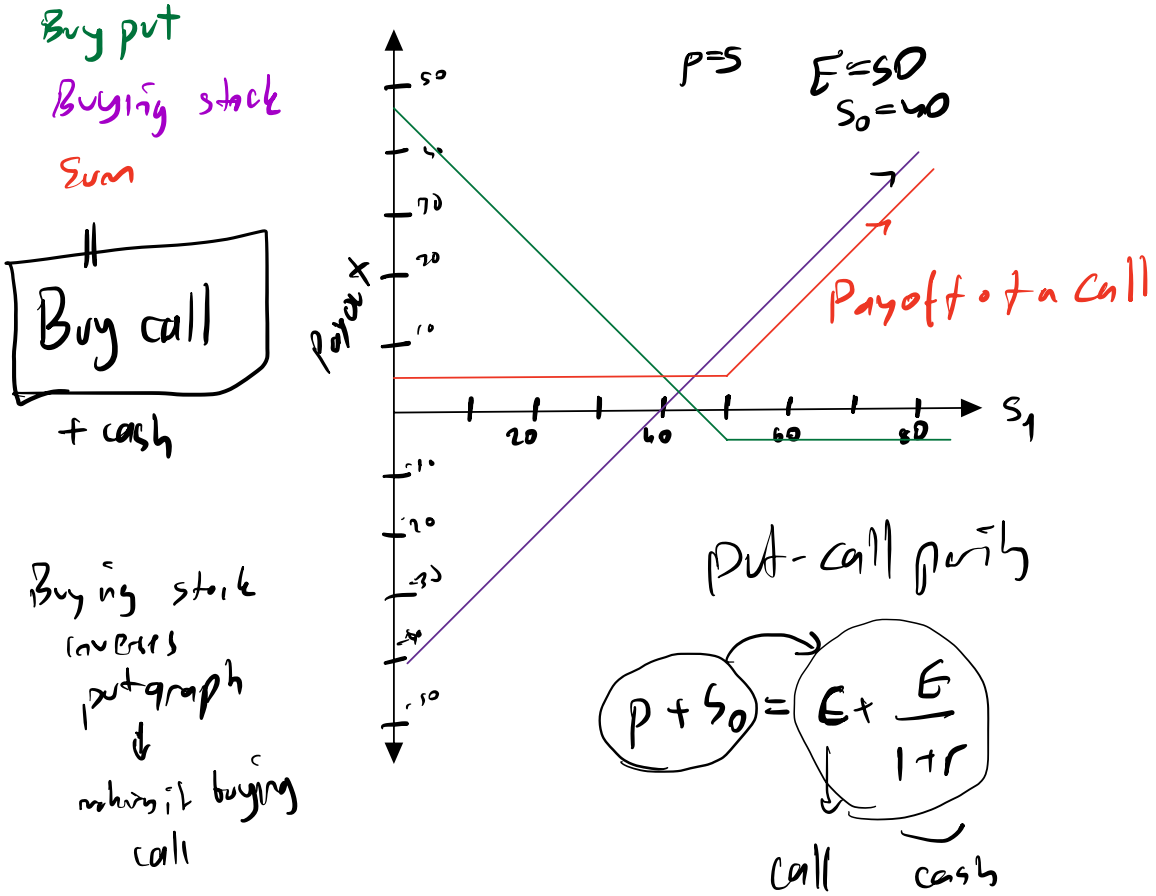
University of California, Los Angeles
Department of Statistics

Statistics C183/C283 Instructor: Nicolas Christou

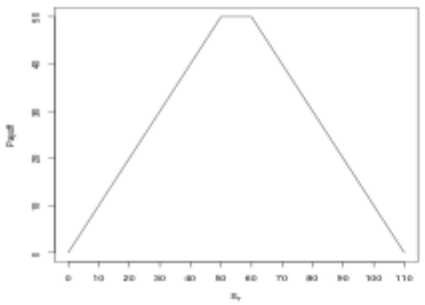
Homework 7

Exercise 1
Answer the following questions:

a. One of the investing strategies using European options is the "protective put," where the investor buys the put and buys the stock. What position in call options is equivalent to this strategy? Please explain and provide all the necessary plots to support your answer.



b. Design a portfolio using only call options and the underlying stock with the following payoff at expiration:



Stock price at expiration S_T	Long stock	Short call with $E_1 = 50$	Short call with $E_2 = 60$	Total
$S_1 < 50$	S_1	0	0	S_1
$50 < S_1 < 60$	S_1	$50 - S_1$	0	50
$S_1 > 60$	S_1	$50 - S_1$	$60 - S_1$	$110 - S_1$

Exercise 2
Answer the following questions:

a. Suppose data are collected for a certain stock:

Stock price	\$110
Call price (1-year expiration, $E = \$105$)	\$17
Put price (1-year expiration, $E = \$105$)	\$5
Risk-free interest rate	5% per year

Is there a mispricing of the call and put? If yes, can you exploit this mispricing to create arbitrage profit? Please provide the numerical example.

Check parity:

$p + S_0 = C + \frac{E}{1+r}$

$5 + 110 \neq 17 + \frac{105}{1.05} \Rightarrow 115 \neq 117$ → mispricing

Second portfolio is underpriced → borrow to buy put and the stock → sell the call

→ Borrow 115, +17 sell the call

→ Debt: $-115 + 17 = 98$ → interest → $98 \times 1.05 = 102.9$ → debt at exercise

$S_1 > 105$ → sell stock → $S_1 - 102.9 > 105 - 102.9$ (pay debt) → 2.1 min min profit

$S_1 < 105$ → exercise put, sell stock → $(105 - S_1 + S_1) - 102.9 = 2.1$ (pay debt) profit

b. The price of a European put option on stock A is \$4.0. The current price of the stock is $S_0 = \$46$, the exercise price of the put option is $E = \$51$, time to expiration is 1 month, and the risk-free interest rate for the one-month period is 0.005. Is there an opportunity for riskless profit? If there is, please explain the positions you need to hold with the corresponding payoffs.

Check ✗ Lower bound: Does not hold

$p \geq \frac{E}{1+r} - S_0 \Rightarrow 4 \geq 50.746 - 46$

$4 \not\geq 4.74$ ✗

Check upper bound: $4 \leq \frac{E}{1+0.005}$ ✓

Borrow 50 to buy S_0 and p: Debt = 50 \$

Debt → interest → $50 \times (1 + 0.005) = 50.25$ → debt at exercise and pay

$S_1 > 51$ → no exercise, sell stock → $S_1 - 50.25 > 51 - 50.25 = 0.75$ (min profit)

$S_1 < 51$ → exercise, sell stock → $51 - S_1 + S_1 - 50.25 = 0.75$ exact profit

Exercise 3

An investor sells a European call on a share for \$4. The stock price is \$47 and the exercise price is \$50. When does the investor make a profit? When will the option be exercised? Use R to draw a diagram showing the investors profit against the price of the stock at expiration.

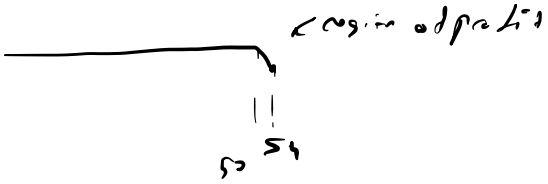
Exercised when stock price > exercise price

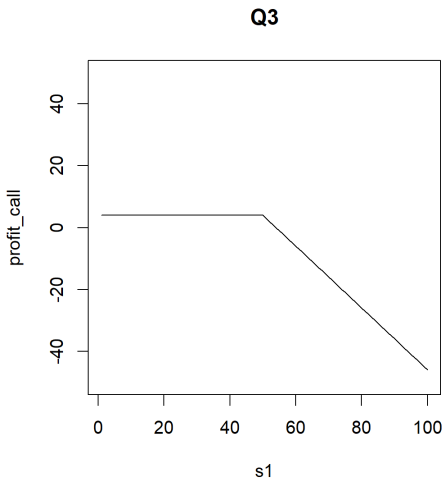
$S_1 > 50$

Makes profit if $S_1 < S_4$

profit = $C = 4$ \$ $S_1 < 50$

profit = $E - S_1 + 4$ or $S_1 < S_4$





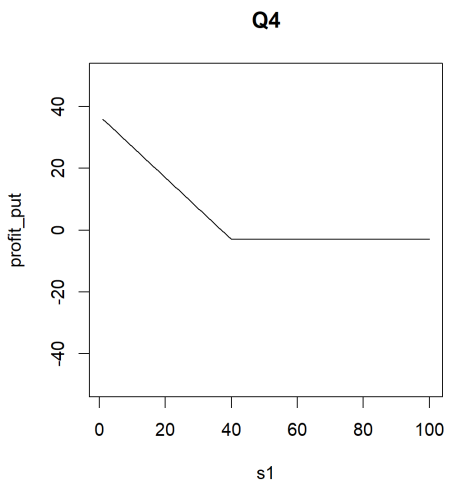
Exercise 4

An investor buys a European put on a share for \$3. The stock price is \$42 and the exercise price is \$40. When does the investor make a profit? When will the option be exercised? Use `R` to draw a diagram showing the investors profit against the price of the stock at expiration.

Exercised when $S_1 < E$ or $S_1 < 40$

Makes profit if $S_1 < 37$

profit $= 37 - S_1 = 40 - S_1 - 3$



Exercise 5

You want to purchase 2 puts and 1 call. The call option costs \$5 and the put option costs \$6. The exercise price for the call or the put is \$50. Use `R` to plot the profit against the stock price at the expiration date:

a. For the 2 puts.

b. For the call.

c. For the combination of the 2 puts and 1 call.

a) $S_1 < E \rightarrow$ put 1, 2 : $E - S_1 - 6 = 44 - S_1$ (x2)

$= 88 - 2S_1$

$S_1 > E \rightarrow -2 * 6 = -12$

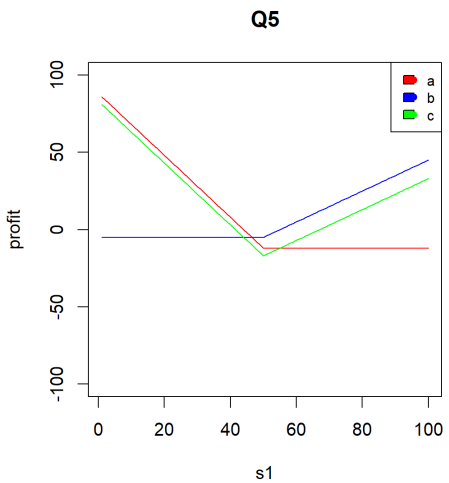
b) $S_1 > E \rightarrow$ call : $S_1 - E - 5 = S_1 - 55$

$S_1 < E \rightarrow -5 = -5$

c) Sum

$S_1 > E$: $S_1 - 67$

$S_1 < E$: $88 - 2S_1 - 5 = 83 - 2S_1$



Exercise 6

Consider the following strategy: You write 2 call options (each one with $E = \$45, C = \$5$) and you buy 1 call option (with $E = \$40, C = \$8$). Both buying and selling call options have the same expiration date. Use `R` to plot the profit against the stock price at the expiration date for this strategy.

2 call options : $S_1 > 45 \rightarrow \quad 45 - S_1 + 5 = 50 - S_1 \quad (\times 2) \Rightarrow 100 - 2S_1$

$S_1 < 45 \rightarrow \quad 5 \times 2 = 10$

1 call option : $S_1 > 40 \rightarrow \quad S_1 - 40 - 8 = S_1 - 48$

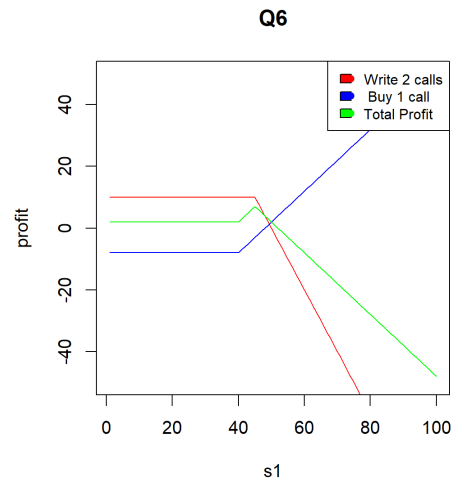
$S_1 < 40 \rightarrow \quad -8$

Sum

$S_1 < 40 \quad = -8$

$40 < S_1 < 45 \quad = S_1 - 48 + 10 = S_1 - 38$

$45 > S_1 \quad = \quad 100 - 2S_1 + S_1 - 46 = 52 - S_1$



Exercise 7

By rearranging the put call parity equation $p + S_0 = c + Ee^{-rt}$ give an example in R to show the payoff and profit using the following investing strategies:

a. Long put long stock.

b. Short put short stock.

c. Long call short stock.

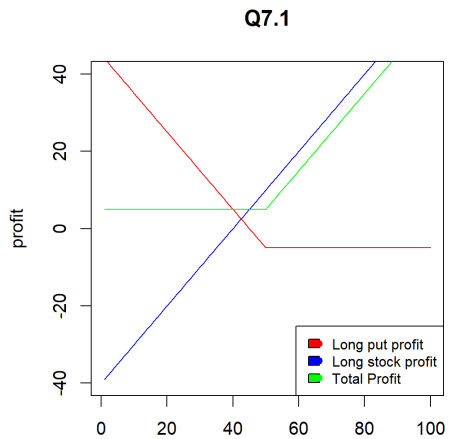
d. Short call long stock.

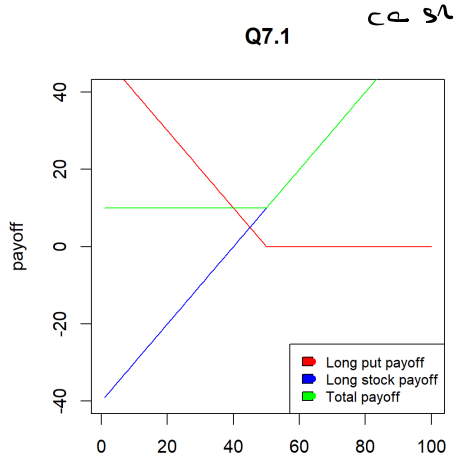
a) long put long stock = $(p + S_0) \Rightarrow (c + Ee^{-rt})$ $\Rightarrow$ buy call

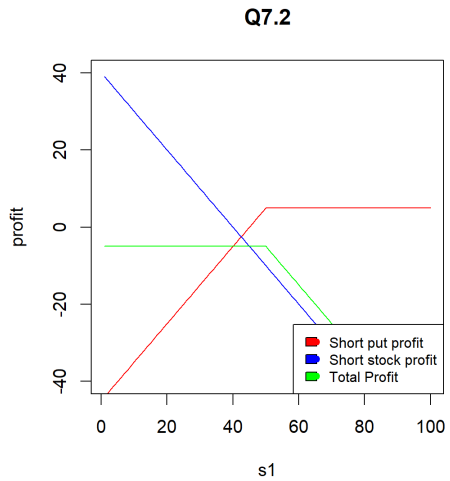
b) short put short stock = $p + S_0 \Rightarrow c + Ee^{-rt}$ cash $\Rightarrow$ sell call

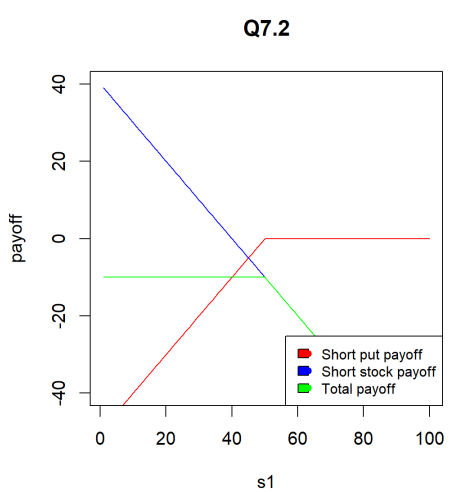
c) long call short stock = $S_0 - c \Rightarrow -p + Ee^{-rt}$ $\Rightarrow$ buy put

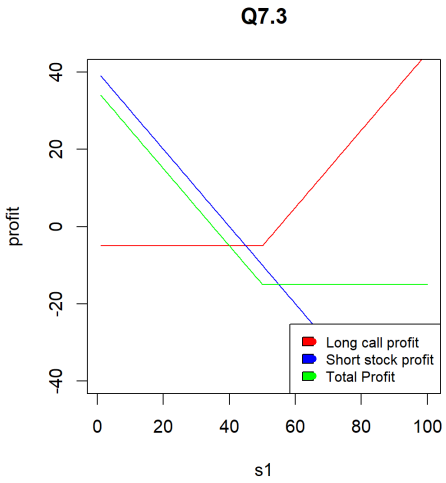
d) short call long stock = $c - S_0 \Rightarrow p + Ee^{-rt}$ $\Rightarrow$ sell put

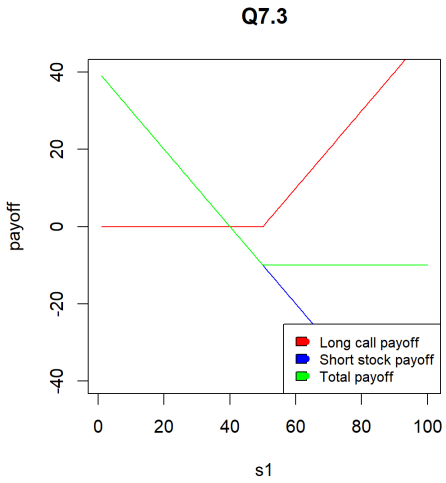


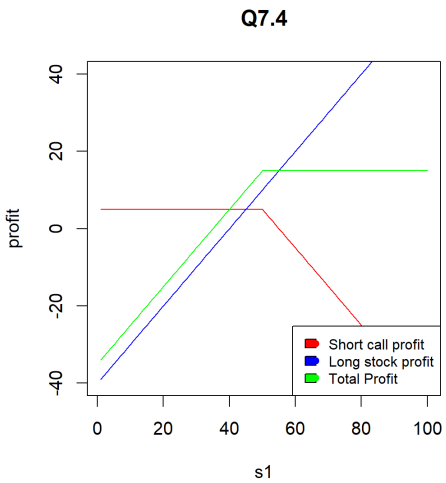


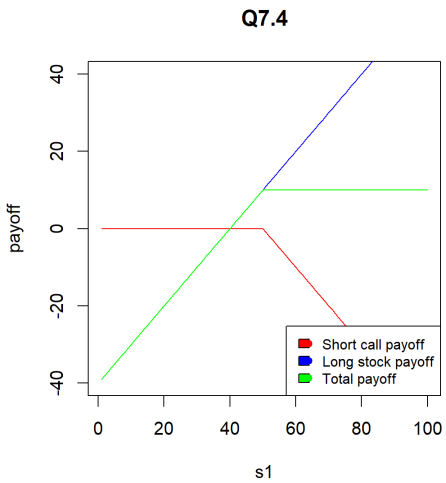





Q7.3
profit
s1
Long call profit
Short stock profit
Total Profit


Q7.3
payoff
s1
Long call payoff
Short stock payoff
Total payoff


Q7.4
profit
s1
Short call profit
Long stock profit
Total Profit


Q7.4
payoff
s1
Short call payoff
Long stock payoff
Total payoff

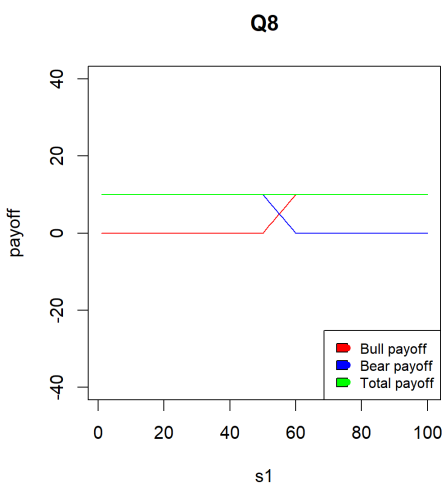
Exercise 8

Consider the box spread strategy: It is a combination of a bull call spread and a bear put spread.
Bull call spread: Buy one call with exercise $E_1 = \$50$ and sell one call with exercise $E_2 = \$60$.
Bear put spread: Buy one put with exercise $E_2 = \$60$ and sell one put with exercise $E_1 = \$50$.

a. Complete the table that shows the payoffs for all the positions above.

b. Construct the diagram in R that shows the payoff for the bull call spread, for the bear put spread, and the total.

	Payoff buy call, 50	Payoff sell call, 60	Payoff bull	Payoff buy put 60	Payoff sell put 50	Payoff bear	Total
$S_1 < 50$	0	0	0	$60 - S_1$	$S_1 - 50$	10	10
$50 \le S_1 < 60$	$S_1 - 50$	0	$S_1 - 50$	$60 - S_1$	0	$60 - S_1$	10
$60 < S_1$	$S_1 - 50$	$60 - S_1$	10	0	0	0	10



Exercise 9

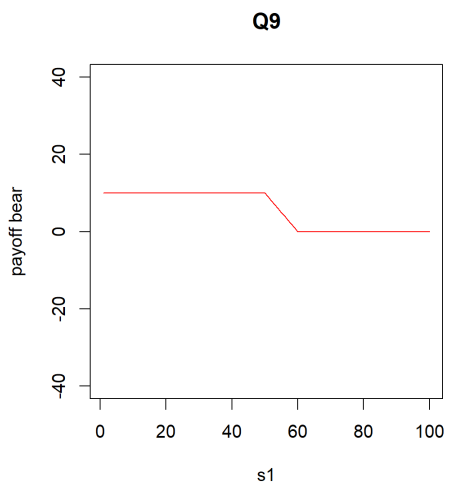
Consider creating a bear spread using puts: Sell one put with exercise E_1 and buy one put with exercise price E_2, with $E_2 > E_1$. Complete the table that shows the payoff and profit for each position and the total and use a numerical example in R to show the diagram for each position and the total.

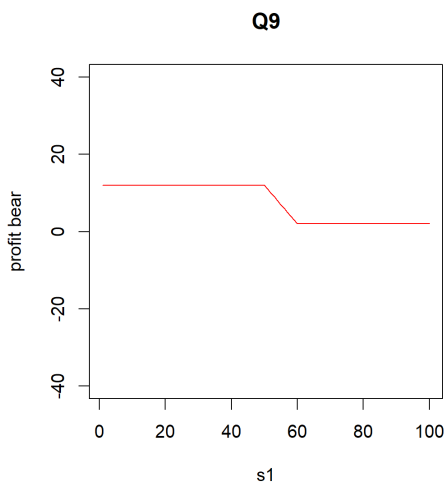
$E_2 = 60$, $E_1 = 50$ $\quad P_2 = 5$, $P_1 = 3$

	Payoff buy put 60	Payoff sell put 50	Payoff bear
$S_1 < 50$	$60 - S_1$	$S_1 - 50$	10
$50 \le S_1 < 60$	$60 - S_1$	0	$60 - S_1$
$60 < S_1$	0	0	0

	Profit buy put 60	Profit sell put 50	Profit bear
$S_1 < 50$	$60 - S_1 - 3$	$S_1 - 50 + 5$	$10 + 2$
$50 \le S_1 < 60$	$60 - S_1 - 3$	$+5$	$60 - S_1 + 2$
$60 < S_1$	-3	$+5$	2

$P_2 - P_1$ can be second if $P_1 \neq P_2$





Exercise 10

Consider creating a bear spread using calls: Sell one call with exercise E_1 and buy one call with exercise price E_2, with $E_2 > E_1$. Complete the table that shows the payoff and profit for each position and the total and use a numerical example in R to show the diagram for each position and the total.

$E_2 = 60$, $E_1 = 50$ $\quad c_1 = 5$ $\quad c_2 = 3$ $\Rightarrow$ some profit can be made if $c_1 > c_2$

	Payoff sell call 50	Payoff buy call 60	Payoff bear
$S_1 < 50$	0	0	0
$50 \le S_1 < 60$	$50 - S_1$	0	$50 - S_1$
$60 < S_1$	$50 - S_1$	$S_1 - 60$	-10

	Profit sell call 50	Profit buy call 60	Profit bear
$S_1 < 50$	5	-3	2
$50 \le S_1 < 60$	$50 - S_1 + 5$	-3	$50 - S_1 + 2$
$60 < S_1$	$50 - S_1 + 5$	$S_1 - 60 - 3$	$-10 + 2$

$\rightarrow$ $c_1 - c_2$ can be earned