Project 4

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Please answer the following questions assuming the single index model holds. Use your project data in the period 01-Jan-2014 to 01-Jan-2019.

1. Compute estimates for α_i , β_i, $\sigma^2_{e_i}$, i = 1:30 by regressing each stock's return on the S&P500.

```
a_all <- read.csv("stockData.csv", sep=",", header=TRUE)
a <- a_all[1:60,]
r <- (a[-1,4:ncol(a)]-a[-nrow(a),4:ncol(a)])/a[-nrow(a),4:ncol(a)]
r_m <- (a[-1,3]-a[-nrow(a),3])/a[-nrow(a),3]
n_stocks = 30
mean_Rm = mean(r_m)
var_Rm <- var(r_m)
stdev_Rm <- var_Rm^.5

mean_Ri = colMeans(r)

betas = rep(0,n_stocks)
alphas = rep(0,n_stocks)
var_es = rep(0,n_stocks)
var_betas = rep(0,n_stocks)
```

Fit Coefficients

```
for (i in 1:n_stocks){
  fit <- lm(r[,i] ~ r_m)
  betas[i] = fit$coefficients[2]
  alphas[i] = fit$coefficients[1]
  var_es[i] = sum(fit$residuals^2)/ (nrow(r) - 2)
  var_betas[i] = vcov(fit)[2,2]
}

#find beta_i
print("Betas are:")
```

```
## [1] "Betas are:"
```

```
print(betas)
```

```
##  [1] 1.2929549 1.1593250 2.0248862 1.0718453 1.2582033 0.9442843 1.0393931
##  [8] 1.0530930 1.0025766 0.4342799 0.4776197 1.0354303 1.6022329 0.6063894
## [15] 1.0001717 2.2070856 0.4415925 0.7638836 0.3725548 0.3437258 0.4144253
## [22] 0.5458005 0.8674676 0.5747603 0.9214674 0.7875896 0.9919519 1.1254699
## [29] 0.8974680 0.9648233
```

```r
print("Number of negative betas")
```

```
## [1] "Number of negative betas"
```

```r
print(sum(which(betas < 0)))
```

```
## [1] 0
```

```r
#find alpha_i
print("Alphas are:")
```

```
## [1] "Alphas are:"
```

```r
print(alphas)
```

```
##  [1]  8.655393e-03  1.667838e-02  3.383866e-02  1.181871e-02  9.787943e-03
##  [6]  1.609522e-02  8.128193e-03  1.012012e-02  1.934431e-03  1.388844e-02
## [11]  5.867074e-03  3.144260e-03  1.750136e-02  1.420789e-02  6.910948e-03
## [16]  8.588360e-04  1.230233e-02 -3.223398e-03  5.838137e-03  7.126095e-03
## [21]  4.799198e-03  4.797339e-03  8.091456e-03  3.196362e-05  3.180270e-03
## [26]  5.679208e-03  5.579035e-03  1.433911e-02  4.663745e-03  4.477371e-03
```

2. Construct the 30x30 variance covariance matrix based on the single index model.

```r
#Compute covariance matrix using single index model
covariance_matrix = matrix(0,n_stocks,n_stocks)
for (i in 1:n_stocks)
{
  for(j in 1:n_stocks){
    if(i == j)
    {
      covariance_matrix[i,j] = betas[i] * betas[i] * var_Rm + var_es[i]
    }else{
      covariance_matrix[i,j] = betas[i] * betas[j] * var_Rm
    }
  }
}
print("Single Index Model cov_mat")
```

```
## [1] "Single Index Model cov_mat"
```

```r
print(covariance_matrix)
```

```
##               [,1]         [,2]         [,3]         [,4]         [,5]
##  [1,] 0.0058854120 0.0017800635 0.0031090731 0.0016457446 0.0019318843
##  [2,] 0.0017800635 0.0035026910 0.0027877431 0.0014756530 0.0017322195
##  [3,] 0.0031090731 0.0027877431 0.0160154517 0.0025773873 0.0030255084
##  [4,] 0.0016457446 0.0014756530 0.0025773873 0.0044761947 0.0016015108
##  [5,] 0.0019318843 0.0017322195 0.0030255084 0.0016015108 0.0042591949
##  [6,] 0.0014498834 0.0013000346 0.0022706507 0.0012019373 0.0014109139
##  [7,] 0.0015959165 0.0014309748 0.0024993519 0.0013229972 0.0015530220
##  [8,] 0.0016169517 0.0014498360 0.0025322950 0.0013404351 0.0015734918
##  [9,] 0.0015393872 0.0013802879 0.0024108218 0.0012761350 0.0014980120
## [10,] 0.0006668068 0.0005978908 0.0010442808 0.0005527755 0.0006488846
## [11,] 0.0007333521 0.0006575584 0.0011484967 0.0006079408 0.0007136413
## [12,] 0.0015898317 0.0014255189 0.0024898226 0.0013179530 0.0015471008
## [13,] 0.0024601180 0.0022058591 0.0038527710 0.0020394107 0.0023939958
## [14,] 0.0009310691 0.0008348410 0.0014581399 0.0007718460 0.0009060442
```

```
## [15,] 0.0015356947 0.0013769771 0.0024050391 0.0012730740 0.0014944188
## [16,] 0.0033888277 0.0030385844 0.0053072157 0.0028093007 0.0032977439
## [17,] 0.0006780348 0.0006079583 0.0010618648 0.0005620834 0.0006598108
## [18,] 0.0011728905 0.0010516696 0.0018368544 0.0009723133 0.0011413660
## [19,] 0.0005720322 0.0005129113 0.0008958549 0.0004742083 0.0005566573
## [20,] 0.0005277673 0.0004732213 0.0008265321 0.0004375133 0.0005135821
## [21,] 0.0006363214 0.0005705561 0.0009965378 0.0005275034 0.0006192186
## [22,] 0.0008380389 0.0007514257 0.0013124460 0.0006947250 0.0008155144
## [23,] 0.0013319366 0.0011942779 0.0020859352 0.0011041608 0.0012961373
## [24,] 0.0008825047 0.0007912958 0.0013820836 0.0007315867 0.0008587851
## [25,] 0.0014148496 0.0012686216 0.0022157846 0.0011728947 0.0013768218
## [26,] 0.0012092895 0.0010843066 0.0018938586 0.0010024877 0.0011767866
## [27,] 0.0015230736 0.0013656604 0.0023852733 0.0012626112 0.0014821370
## [28,] 0.0017280814 0.0015494802 0.0027063344 0.0014325604 0.0016816346
## [29,] 0.0013780002 0.0012355806 0.0021580749 0.0011423469 0.0013409627
## [30,] 0.0014814196 0.0013283114 0.0023200393 0.0012280805 0.0014416025
##               [,6]         [,7]         [,8]         [,9]        [,10]
##  [1,] 0.0014498834 0.0015959165 0.0016169517 0.0015393872 0.0006668068
##  [2,] 0.0013000346 0.0014309748 0.0014498360 0.0013802879 0.0005978908
##  [3,] 0.0022706507 0.0024993519 0.0025322950 0.0024108218 0.0010442808
##  [4,] 0.0012019373 0.0013229972 0.0013404351 0.0012761350 0.0005527755
##  [5,] 0.0014109139 0.0015530220 0.0015734918 0.0014980120 0.0006488846
##  [6,] 0.0066383364 0.0011655464 0.0011809091 0.0011242613 0.0004869893
##  [7,] 0.0011655464 0.0035859853 0.0012998509 0.0012374975 0.0005360391
##  [8,] 0.0011809091 0.0012998509 0.0051112551 0.0012538086 0.0005431045
##  [9,] 0.0011242613 0.0012374975 0.0012538086 0.0037599297 0.0005170520
## [10,] 0.0004869893 0.0005360391 0.0005431045 0.0005170520 0.0029073827
## [11,] 0.0005355893 0.0005895342 0.0005973046 0.0005686522 0.0002463195
## [12,] 0.0011611025 0.0012780494 0.0012948949 0.0012327793 0.0005339954
## [13,] 0.0017966991 0.0019776636 0.0020037305 0.0019076124 0.0008263086
## [14,] 0.0006799881 0.0007484769 0.0007583423 0.0007219650 0.0003127291
## [15,] 0.0011215646 0.0012345292 0.0012508011 0.0011908007 0.0005158117
## [16,] 0.0024749640 0.0027242438 0.0027601511 0.0026277477 0.0011382452
## [17,] 0.0004951895 0.0005450652 0.0005522495 0.0005257583 0.0002277395
## [18,] 0.0008565977 0.0009428747 0.0009553024 0.0009094769 0.0003939525
## [19,] 0.0004177725 0.0004598508 0.0004659119 0.0004435623 0.0001921351
## [20,] 0.0003854445 0.0004242667 0.0004298588 0.0004092387 0.0001772674
## [21,] 0.0004647249 0.0005115322 0.0005182746 0.0004934131 0.0002137287
## [22,] 0.0006120454 0.0006736909 0.0006825706 0.0006498279 0.0002814819
## [23,] 0.0009727539 0.0010707302 0.0010848431 0.0010328036 0.0004473731
## [24,] 0.0006445201 0.0007094365 0.0007187873 0.0006843074 0.0002964172
## [25,] 0.0010333078 0.0011373831 0.0011523746 0.0010970956 0.0004752221
## [26,] 0.0008831810 0.0009721354 0.0009849488 0.0009377012 0.0004061782
## [27,] 0.0011123470 0.0012243833 0.0012405214 0.0011810141 0.0005115726
## [28,] 0.0012620705 0.0013891869 0.0014074973 0.0013399802 0.0005804309
## [29,] 0.0010063955 0.0011077602 0.0011223612 0.0010685220 0.0004628450
## [30,] 0.0010819258 0.0011908980 0.0012065948 0.0011487149 0.0004975817
##              [,11]        [,12]        [,13]        [,14]        [,15]
##  [1,] 0.0007333521 0.0015898317 0.0024601180 0.0009310691 0.0015356947
##  [2,] 0.0006575584 0.0014255189 0.0022058591 0.0008348410 0.0013769771
##  [3,] 0.0011484967 0.0024898226 0.0038527710 0.0014581399 0.0024050391
##  [4,] 0.0006079408 0.0013179530 0.0020394107 0.0007718460 0.0012730740
##  [5,] 0.0007136413 0.0015471008 0.0023939958 0.0009060442 0.0014944188
##  [6,] 0.0005355893 0.0011611025 0.0017966991 0.0006799881 0.0011215646
```

```
##  [7,] 0.0005895342 0.0012780494 0.0019776636 0.0007484769 0.0012345292
##  [8,] 0.0005973046 0.0012948949 0.0020037305 0.0007583423 0.0012508011
##  [9,] 0.0005686522 0.0012327793 0.0019076124 0.0007219650 0.0011908007
## [10,] 0.0002463195 0.0005339954 0.0008263086 0.0003127291 0.0005158117
## [11,] 0.0024364925 0.0005872865 0.0009087717 0.0003439385 0.0005672882
## [12,] 0.0005872865 0.0035097659 0.0019701234 0.0007456232 0.0012298223
## [13,] 0.0009087717 0.0019701234 0.0067873051 0.0011537832 0.0019030366
## [14,] 0.0003439385 0.0007456232 0.0011537832 0.0153421403 0.0007202332
## [15,] 0.0005672882 0.0012298223 0.0019030366 0.0007202332 0.0026260377
## [16,] 0.0012518386 0.0027138571 0.0041994436 0.0015893434 0.0026214446
## [17,] 0.0002504672 0.0005429870 0.0008402224 0.0003179950 0.0005244972
## [18,] 0.0004332677 0.0009392798 0.0014534488 0.0005500799 0.0009072954
## [19,] 0.0002113096 0.0004580975 0.0007088636 0.0002682802 0.0004424983
## [20,] 0.0001949581 0.0004226491 0.0006540105 0.0002475202 0.0004082570
## [21,] 0.0002350582 0.0005095819 0.0007885310 0.0002984316 0.0004922296
## [22,] 0.0003095730 0.0006711223 0.0010384999 0.0003930361 0.0006482692
## [23,] 0.0004920196 0.0010666478 0.0016505392 0.0006246717 0.0010303263
## [24,] 0.0003259987 0.0007067316 0.0010936020 0.0004138904 0.0006826660
## [25,] 0.0005226478 0.0011330466 0.0017532851 0.0006635575 0.0010944640
## [26,] 0.0004467136 0.0009684290 0.0014985546 0.0005671508 0.0009354520
## [27,] 0.0005626259 0.0012197151 0.0018873966 0.0007143140 0.0011781813
## [28,] 0.0006383561 0.0013838904 0.0021414428 0.0008104616 0.0013367661
## [29,] 0.0005090355 0.0011035366 0.0017076212 0.0006462753 0.0010659589
## [30,] 0.0005472389 0.0011863575 0.0018357788 0.0006947785 0.0011459596
##             [,16]        [,17]        [,18]        [,19]        [,20]
##  [1,] 0.0033888277 0.0006780348 0.0011728905 0.0005720322 0.0005277673
##  [2,] 0.0030385844 0.0006079583 0.0010516696 0.0005129113 0.0004732213
##  [3,] 0.0053072157 0.0010618648 0.0018368544 0.0008958549 0.0008265321
##  [4,] 0.0028093007 0.0005620834 0.0009723133 0.0004742083 0.0004375133
##  [5,] 0.0032977439 0.0006598108 0.0011413660 0.0005566573 0.0005135821
##  [6,] 0.0024749640 0.0004951895 0.0008565977 0.0004177725 0.0003854445
##  [7,] 0.0027242438 0.0005450652 0.0009428747 0.0004598508 0.0004242667
##  [8,] 0.0027601511 0.0005522495 0.0009553024 0.0004659119 0.0004298588
##  [9,] 0.0026277477 0.0005257583 0.0009094769 0.0004435623 0.0004092387
## [10,] 0.0011382452 0.0002277395 0.0003939525 0.0001921351 0.0001772674
## [11,] 0.0012518386 0.0002504672 0.0004332677 0.0002113096 0.0001949581
## [12,] 0.0027138571 0.0005429870 0.0009392798 0.0004580975 0.0004226491
## [13,] 0.0041994436 0.0008402224 0.0014534488 0.0007088636 0.0006540105
## [14,] 0.0015893434 0.0003179950 0.0005500799 0.0002682802 0.0002475202
## [15,] 0.0026214446 0.0005244972 0.0009072954 0.0004424983 0.0004082570
## [16,] 0.0118019090 0.0011574115 0.0020021347 0.0009764640 0.0009009035
## [17,] 0.0011574115 0.0017659698 0.0004005860 0.0001953704 0.0001802523
## [18,] 0.0020021347 0.0004005860 0.0019780049 0.0003379592 0.0003118073
## [19,] 0.0009764640 0.0001953704 0.0003379592 0.0027693928 0.0001520720
## [20,] 0.0009009035 0.0001802523 0.0003118073 0.0001520720 0.0015759637
## [21,] 0.0010862064 0.0002173276 0.0003759416 0.0001833510 0.0001691630
## [22,] 0.0014305400 0.0002862217 0.0004951172 0.0002414743 0.0002227886
## [23,] 0.0022736278 0.0004549062 0.0007869142 0.0003837871 0.0003540889
## [24,] 0.0015064435 0.0003014084 0.0005213878 0.0002542868 0.0002346096
## [25,] 0.0024151610 0.0004832241 0.0008358996 0.0004076778 0.0003761310
## [26,] 0.0020642681 0.0004130176 0.0007144537 0.0003484473 0.0003214838
## [27,] 0.0025999004 0.0005201867 0.0008998388 0.0004388617 0.0004049018
## [28,] 0.0029498505 0.0005902045 0.0010209583 0.0004979331 0.0004594021
## [29,] 0.0023522587 0.0004706386 0.0008141287 0.0003970599 0.0003663347
```

```
## [30,] 0.0025287966 0.0005059602 0.0008752294 0.0004268594 0.0003938283
##              [,21]        [,22]        [,23]        [,24]        [,25]
##  [1,] 0.0006363214 0.0008380389 0.0013319366 0.0008825047 0.0014148496
##  [2,] 0.0005705561 0.0007514257 0.0011942779 0.0007912958 0.0012686216
##  [3,] 0.0009965378 0.0013124460 0.0020859352 0.0013820836 0.0022157846
##  [4,] 0.0005275034 0.0006947250 0.0011041608 0.0007315867 0.0011728947
##  [5,] 0.0006192186 0.0008155144 0.0012961373 0.0008587851 0.0013768218
##  [6,] 0.0004647249 0.0006120454 0.0009727539 0.0006445201 0.0010333078
##  [7,] 0.0005115322 0.0006736909 0.0010707302 0.0007094365 0.0011373831
##  [8,] 0.0005182746 0.0006825706 0.0010848431 0.0007187873 0.0011523746
##  [9,] 0.0004934131 0.0006498279 0.0010328036 0.0006843074 0.0010970956
## [10,] 0.0002137287 0.0002814819 0.0004473731 0.0002964172 0.0004752221
## [11,] 0.0002350582 0.0003095730 0.0004920196 0.0003259987 0.0005226478
## [12,] 0.0005095819 0.0006711223 0.0010666478 0.0007067316 0.0011330466
## [13,] 0.0007885310 0.0010384999 0.0016505392 0.0010936020 0.0017532851
## [14,] 0.0002984316 0.0003930361 0.0006246717 0.0004138904 0.0006635575
## [15,] 0.0004922296 0.0006482692 0.0010303263 0.0006826660 0.0010944640
## [16,] 0.0010862064 0.0014305400 0.0022736278 0.0015064435 0.0024151610
## [17,] 0.0002173276 0.0002862217 0.0004549062 0.0003014084 0.0004832241
## [18,] 0.0003759416 0.0004951172 0.0007869142 0.0005213878 0.0008358996
## [19,] 0.0001833510 0.0002414743 0.0003837871 0.0002542868 0.0004076778
## [20,] 0.0001691630 0.0002227886 0.0003540889 0.0002346096 0.0003761310
## [21,] 0.0013032131 0.0002686130 0.0004269199 0.0002828654 0.0004534957
## [22,] 0.0002686130 0.0016482478 0.0005622560 0.0003725354 0.0005972564
## [23,] 0.0004269199 0.0005622560 0.0026340792 0.0005920889 0.0009492491
## [24,] 0.0002828654 0.0003725354 0.0005920889 0.0033699736 0.0006289464
## [25,] 0.0004534957 0.0005972564 0.0009492491 0.0006289464 0.0080005591
## [26,] 0.0003876084 0.0005104824 0.0008113349 0.0005375683 0.0008618405
## [27,] 0.0004881842 0.0006429415 0.0010218586 0.0006770555 0.0010854692
## [28,] 0.0005538945 0.0007294822 0.0011594021 0.0007681881 0.0012315749
## [29,] 0.0004416845 0.0005817010 0.0009245261 0.0006125657 0.0009820778
## [30,] 0.0004748331 0.0006253579 0.0009939121 0.0006585389 0.0010557831
##              [,26]        [,27]        [,28]        [,29]        [,30]
##  [1,] 0.0012092895 0.0015230736 0.0017280814 0.0013780002 0.0014814196
##  [2,] 0.0010843066 0.0013656604 0.0015494802 0.0012355806 0.0013283114
##  [3,] 0.0018938586 0.0023852733 0.0027063344 0.0021580749 0.0023200393
##  [4,] 0.0010024877 0.0012626112 0.0014325604 0.0011423469 0.0012280805
##  [5,] 0.0011767866 0.0014821370 0.0016816346 0.0013409627 0.0014416025
##  [6,] 0.0008831810 0.0011123470 0.0012620705 0.0010063955 0.0010819258
##  [7,] 0.0009721354 0.0012243833 0.0013891869 0.0011077602 0.0011908980
##  [8,] 0.0009849488 0.0012405214 0.0014074973 0.0011223612 0.0012065948
##  [9,] 0.0009377012 0.0011810141 0.0013399802 0.0010685220 0.0011487149
## [10,] 0.0004061782 0.0005115726 0.0005804309 0.0004628450 0.0004975817
## [11,] 0.0004467136 0.0005626259 0.0006383561 0.0005090355 0.0005472389
## [12,] 0.0009684290 0.0012197151 0.0013838904 0.0011035366 0.0011863575
## [13,] 0.0014985546 0.0018873966 0.0021414428 0.0017076212 0.0018357788
## [14,] 0.0005671508 0.0007143140 0.0008104616 0.0006462753 0.0006947785
## [15,] 0.0009354520 0.0011781813 0.0013367661 0.0010659589 0.0011459596
## [16,] 0.0020642681 0.0025999004 0.0029498505 0.0023522587 0.0025287966
## [17,] 0.0004130176 0.0005201867 0.0005902045 0.0004706386 0.0005059602
## [18,] 0.0007144537 0.0008998388 0.0010209583 0.0008141287 0.0008752294
## [19,] 0.0003484473 0.0004388617 0.0004979331 0.0003970599 0.0004268594
## [20,] 0.0003214838 0.0004049018 0.0004594021 0.0003663347 0.0003938283
## [21,] 0.0003876084 0.0004881842 0.0005538945 0.0004416845 0.0004748331
```

```
## [22,] 0.0005104824 0.0006429415 0.0007294822 0.0005817010 0.0006253579
## [23,] 0.0008113349 0.0010218586 0.0011594021 0.0009245261 0.0009939121
## [24,] 0.0005375683 0.0006770555 0.0007681881 0.0006125657 0.0006585389
## [25,] 0.0008618405 0.0010854692 0.0012315749 0.0009820778 0.0010557831
## [26,] 0.0021395003 0.0009277640 0.0010526423 0.0008393940 0.0009023909
## [27,] 0.0009277640 0.0074378960 0.0013257799 0.0010571984 0.0011365415
## [28,] 0.0010526423 0.0013257799 0.0239558495 0.0011994987 0.0012895216
## [29,] 0.0008393940 0.0010571984 0.0011994987 0.0022198648 0.0010282854
## [30,] 0.0009023909 0.0011365415 0.0012895216 0.0010282854 0.0069683514
```

3. Answer the same question as in project 2, part (e) using the new inputs from (1) above. Draw the frontier on the same plot as in project 2. Now you will have two frontiers, one using the historical variance covariance matrix (project 2) and one using the variance covariance matrix with inputs from the single index model.

```
#Project 2 part e replicated:

n_stocks = 30
means <- colMeans(r)
covmat <- cov(r)
cormat <- cor(r)
variances <- diag(covmat)
stdev <- diag(covmat)^.5
inv_covmat <- solve(covmat)
ones = rep(1,n_stocks)

A = as.numeric(t(means) %*% inv_covmat %*% ones)
B = as.numeric(t(means) %*% inv_covmat %*% means)
C = as.numeric(t(ones)  %*% inv_covmat %*% ones)
D = B*C - A^2

E <- seq(-0.2,0.2,.001)

sigmas <- sqrt(seq(1/C,0.03,.0001))
upper_part <- (A + sqrt(D*(C*sigmas^2 - 1)))*(1/C)
lower_part <- (A - sqrt(D*(C*sigmas^2 - 1)))*(1/C)

plot(sigmas, upper_part, lwd=5,type = "l",ylab = 'E', xlab = expression(sigma),ylim = c(-0.15,0.15),mai
lines(sigmas,lower_part, lwd=5,type = "l")

#Finding efficient frontier using the single index model:

mean_Ri = colMeans(r)
inv_covmat_single_index = solve(covariance_matrix)
ones = rep(1,n_stocks)

A = as.numeric(t(mean_Ri) %*% inv_covmat_single_index %*% ones)
B = as.numeric(t(mean_Ri) %*% inv_covmat_single_index %*% mean_Ri)
C = as.numeric(t(ones)  %*% inv_covmat_single_index %*% ones)
D = B*C - A^2

E <- seq(-0.2,0.2,.001)
sigmas <- sqrt(seq(1/C,0.03,.0001))
upper_part <- (A + sqrt(D*(C*sigmas^2 - 1)))*(1/C)
lower_part <- (A - sqrt(D*(C*sigmas^2 - 1)))*(1/C)
```

```
lines(sigmas, upper_part, lwd=5,type = "l",col = "green")
lines(sigmas,lower_part, lwd=5,type = "l",col = "green")
legend("topright",
       legend=c("Historical Var-Cov", "Single index model var-cov"),
       col=c("black", "green"),
       pch = 19,
       fill =c("black", "green"),
       cex=0.8)
```