University of California, Los Angeles
Department of Statistics

Statistics C183/C283 Instructor: Nicolas Christou

Homework 9

Solved by XY

Exercise 1:
Suppose a stock has annual expected return and standard deviation $\mu = 0.20$ and $\sigma = 0.25$. The current price of the stock is $s = \$50$. Suppose that $\Delta t = 1\text{week}$.

a. Find the distribution of the return of the stock during Δt.

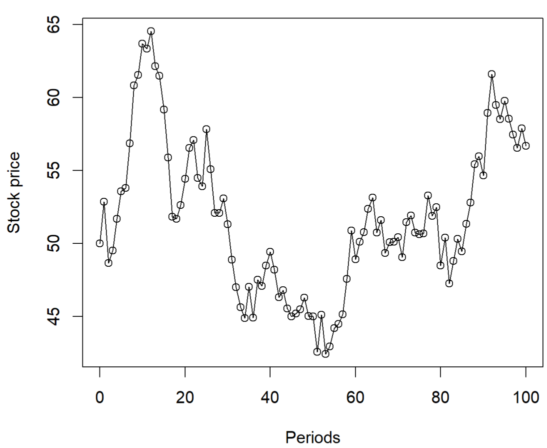
$S_0 = 50\$$, $\sigma = 0.25$, $\mu = 0.20$, $\Delta t = \frac{1}{52}$

$$\frac{\Delta S}{S} = \mu \Delta t + \sigma \epsilon \sqrt{\Delta t}$$

$$= \underbrace{0.20 \frac{1}{52}}_{\mu_s} + \underbrace{0.25 \epsilon \sqrt{\frac{1}{52}}}_{\sigma_s}$$

$$\frac{\Delta S}{S} \sim N(0.0038461\,54,\ 0.034\,66876)$$

b. Simulate the path of the stock from now until 1 year from now (52 weeks). Submit the random samples and the plot of the price of the stock against time.




```
s
 [1] 50.00000 52.85870 48.67820 49.53319 51.68112 53.56325 53.81262 56.85002
 [9] 60.83114 61.54311 63.68502 63.32750 64.52732 62.13444 61.48310 59.15835
[17] 55.88416 51.83352 51.68144 52.62516 54.42807 56.55561 57.08684 54.48446
[25] 53.91166 57.83958 55.08857 52.07864 52.10034 53.08755 51.32888 48.88308
[33] 47.01276 45.62442 44.90435 47.03328 44.93293 47.52371 47.09782 48.48435
[41] 49.42574 48.19847 46.33651 46.81864 45.54952 45.02066 45.20684 45.50129
[49] 46.29345 45.03861 44.99940 42.58138 45.13674 42.44864 42.95138 44.19869
[57] 44.48381 45.16371 47.57901 50.90379 48.92345 50.11827 50.77839 52.37504
[65] 53.14899 50.74373 51.59622 49.34272 50.09172 50.11931 50.42046 49.05366
[73] 51.46120 51.91975 50.75942 50.64049 50.68052 53.29616 51.88377 52.49013
[81] 48.47967 50.39188 47.26826 48.81958 50.31002 49.45252 51.33500 52.80611
[89] 55.41963 55.96636 54.66866 58.94550 61.59728 59.47744 58.52101 59.78159
[97] 58.55063 57.45634 56.52885 57.89599 56.69732
```

Exercise 2:
Suppose that a stock price has an expected return of $\mu = 0.16$ per year and standard deviation $\sigma = 0.30$ per year. Suppose at the end of a certain day the price of the stock is $s = \$50$. Find:

a. The expected stock price at the end of the next day.

$\mu = 0.16 \quad \Delta t = \frac{1}{365}, \quad S = 50, \quad \sigma = 0.30$

$\frac{\Delta S}{S} = \mu \Delta t + \sigma \epsilon \sqrt{\Delta t} \qquad E(S + \Delta S) = S + \mu S \Delta t = 50.0219$

$\Delta S = \mu S \Delta t + \sigma S \epsilon \sqrt{\Delta t}$

$S + \Delta S = S + \mu S \Delta t + \sigma S \epsilon \sqrt{\Delta t}$

b. The standard deviation of the stock price at the end of the next day.

$\sigma_{S + \Delta S} = \sigma S \sqrt{\Delta t} = 0.785136$

Exercise 3:
A stock price follows the lognormal distribution. Its current price is \$38, its annual expected return is $\mu = 0.16$, and its annual standard deviation is $\sigma = 0.35$.

a. What is the probability that a European call option on this stock with an exercise price of 40 and expiration date 6 months from now will be exercised?

Exercised if $S > 40 \Rightarrow P(S > 40) = P(\ln S > \ln 40)$

use Ito

$$\ln S_T \sim N(\ln S + (\mu - \frac{\sigma^2}{2})(T-t), \sigma\sqrt{T-t})$$

$$\ln S_{T0,5} \sim N(3.68696116, 0.247487)$$

$$P(\ln S > \ln 40) = P(Z > \frac{\ln 40 - \mu}{\sigma}) = P(Z > 0.0077510)$$

$$\boxed{= 0.4969078}$$

a. What is the probability that a European put on this stock with an exercise price of 40 and expiration date 6 months from now will be exercised?

Exercised if $S < 40 \Rightarrow P(S < 40)$

$$1 - P(S > 40) = P(S < 40) = 1 - 0.4919078$$

$$\boxed{= 0.50342}$$

Exercise 4:
Using the lognormal distribution result of the price of a stock at time T show that:

$$P\left(Se^{(\mu-\frac{\sigma^2}{2})(T-t)-1.96\sigma\sqrt{T-t}} \le S_T \le Se^{(\mu-\frac{\sigma^2}{2})(T-t)+1.96\sigma\sqrt{T-t}}\right) = 0.95.$$

$$\ln S_T \sim N(\underbrace{\ln S + (\mu - \tfrac{\sigma^2}{2})(T-t)}_{\mu_{S_T}}, \underbrace{\sigma\sqrt{T-t}}_{\sigma_{S_T}})$$

$$P(\mu_{S_T} - 1.96\sigma^* < \ln S_T < \mu^* + 1.96\sigma^*) = 0.95$$

plugging in and taking e power

$$P(Se^{(\mu-\frac{\sigma^2}{2})(T-t)-1.96\sigma\sqrt{T-t}} \le S_T \le Se^{(\mu-\frac{\sigma^2}{2})(T-t)+1.96\sigma\sqrt{T-t}}) = 0.95$$

Suppose the current price of a stock is s = \$40, and the annual expected return and standard deviation $\mu = 0.10$, $\sigma = 0.15$. Find:

a. A 95% confidence interval for the price of the stock in 2 months.

plug in values $\Rightarrow$ $(36.0046, 45.7731)$

b. The expected price of the stock in 2 months.

$$E[S_T] = Se^{\mu(T-t)} = 40e^{0.1(\frac{1}{6})} = \boxed{40.672}$$

c. The standard deviation of the price of the stock in 2 months.

$$\text{var}(S_T) = S^2 e^{2\mu(T-t)}[e^{\sigma^2(T-t)} - 1]$$

$$= 40^2 e^{2(0.1)(\frac{1}{6})}[e^{(0.15)^2\frac{1}{6}} - 1]$$

$$= 6.21502$$

$$\sigma_{S_T} = \sqrt{6.21502} = \boxed{2.49299}$$

Exercise 5:
Using the lognormal property of stock prices estimate the annual volatility of `APPLE` (ticker is `AAPL`) using the adjusted daily close prices for the period `01-March-2023` to `26-May-2023`. Save the data in a csv file and then read the data in `R` as follows:

```
s1 <- read.csv("AAPL.csv", sep=",", header=TRUE)
```

$\bar{u} = -0.002965706$

$s = 0.01302358$

$\Downarrow$

$\hat{\sigma} = \sqrt{252}\, s \Rightarrow 0.2067429$

$\Downarrow$

$\boxed{\%\ 20.7}$

Code

```
s1 <- read.csv("stockData.csv", sep=",", header=TRUE)
s1 <- s1[,3]
a <- s1[-(length(s1))]/s1[-1]
u <- log(a)
m <- mean(u)
s <- sqrt(var(u))
sigma_hat = sqrt(252)*s
```