

Homework 4

Solved by Y.Y

Exercise 1

Prove equation (2) for the prediction sum of squares (PRESS) in the paper "The Adjustment of Beta Forecasts", by Robert C. Klemkosky and John D. Martin, *The Journal of Finance*, Vol. 30, No. 4 (Sep., 1975), handout #23. Use a numerical example from your project data to show the decomposition of PRESS. Please submit the R code and the value of PRESS and the three components.

Equation:

$$\frac{1}{n} \sum (A_j - P_j)^2 = (\bar{A} - \bar{P})^2 + (1 - \beta_1)^2 s_p^2 + (1 - R^2) s_A^2$$

$$\frac{1}{n} \sum (A_j - P_j)^2 = \frac{1}{n} \sum \underbrace{(A_j - \bar{A})}_x + \underbrace{(P_j - \bar{P})}_y + \underbrace{(\bar{A} - \bar{P})}_z^2$$

$$= \frac{1}{n} \sum (x^2) + (y^2) + (z^2) + \underline{(xy)} + \underline{(xz)} + \underline{(yx)} + \underline{(yz)} + \underline{(zx)} + \underline{(zy)}$$

$$= \frac{1}{n} \sum (x^2) + (y^2) + (z^2) + 2(xy) + 2(xz) + 2(yz)$$

$$= \frac{1}{n} \sum (A_j - \bar{A})^2 + \frac{1}{n} \sum (P_j - \bar{P})^2 + (\bar{A} - \bar{P})^2 - \frac{2}{n} \sum (A_j - \bar{A})(P_j - \bar{P})$$

$$+ \frac{2}{n} \sum (A_j - \bar{A})(\bar{A} - \bar{P}) - \frac{2}{n} \sum (P_j - \bar{P})(\bar{A} - \bar{P})$$

so,

$$\frac{2}{n} \sum (A_j - \bar{A})(\bar{A} - \bar{P}) = 2(\bar{A} - \bar{P}) \left(\frac{\sum A_j}{n} - \frac{\sum \bar{A}}{n} \right) = 2(\bar{A} - \bar{P}) (\bar{A} - \cancel{\frac{\sum \bar{A}}{n}})$$

then,

$$= \frac{1}{n} \sum (A_j - \bar{A})^2 + \frac{1}{n} \sum (P_j - \bar{P})^2 + (\bar{A} - \bar{P})^2 - \frac{2}{n} \sum (A_j - \bar{A})(P_j - \bar{P})$$

$$= s_A^2 + s_p^2 + (\bar{A} - \bar{P})^2 - \frac{2}{n} \sum (A_j - \bar{A})(P_j - \bar{P})$$

$$\beta_1 = \frac{\sum (A_j - \bar{A})(P_j - \bar{P})}{s_p^2/n} \Rightarrow$$

$$= s_A^2 + s_p^2 + (\bar{A} - \bar{P})^2 - 2\beta_1 s_p^2$$

$$R^2 = \frac{SSR}{SST} = \frac{\beta_1^2 \sum (P_j - \bar{P})^2}{\sum (A_j - \bar{A})^2} = \beta_1^2 \frac{S_P^2}{S_A^2}$$

$$= S_A^2 + S_P^2 + (\bar{A} - \bar{P})^2 - 2\beta_1 S_P^2 + (1 - R^2)S_A^2 - (1 - R^2)S_A^2$$

$$= S_A^2 + S_P^2 - 2\beta_1 S_P^2 - (1 - \beta_1^2 \frac{S_P^2}{S_A^2})S_A^2 + (\bar{A} - \bar{P})^2 + (1 - R^2)S_A^2$$

$$= \cancel{S_A^2} - \cancel{S_A^2} + S_P^2 - 2\beta_1 S_P^2 + \beta_1^2 S_P^2 + (\bar{A} - \bar{P})^2 + (1 - R^2)S_A^2$$

$$= (1 - \beta_1^2)S_P^2 + (\bar{A} - \bar{P})^2 + (1 - R^2)S_A^2 \quad \checkmark$$

R code attached to
back.

Used historic data to compute
unadjusted betas and forecast data
to actual values.

Exercise 2

Answer the following questions:

- a. The betas of 30 stocks were obtained using simple regression in two successive periods: 2008-12-31 to 2013-01-31 (period 1) and 2013-02-28 to 2017-03-31 (period 2). There are 49 months in each period. Suppose we use the unadjusted betas in the first period as predictions of the betas in the second period. We can then compute the prediction sum of squares (PRESS) to evaluate the performance of these unadjusted betas. The following information is obtained from these data:

$\sum_{i=1}^{30} P_i = 32.44349$	Sum of the betas in period 1.
$\sum_{i=1}^{30} A_i = 32.26206$	Sum of the betas in period 2.
$\sum_{i=1}^{30} P_i^2 = 51.70104$	Sum of the squared betas in period 1.
$\sum_{i=1}^{30} A_i^2 = 48.43207$	Sum of the squared betas in period 2.
$\sum_{i=1}^{30} (A_i - \bar{A})(\hat{A}_i - \bar{A}) = 4.299189$	\hat{A}_i are the fitted values of the simple regression of A on P .
$var(\beta_1) = 0.4558062$	Variance of the betas in period 1.
$var(\beta_2) = 0.3235921$	Variance of the betas in period 2.
$cov(\beta_1, \beta_2) = 0.1700069$	Covariance between the betas in the two periods.

Find the value of the prediction sum of squares (PRESS) using its decomposition presented in the paper "The Adjustment of Beta Forecasts", by Robert C. Klemkosky and John D. Martin, *The Journal of Finance*, Vol. 30, No. 4 (Sep., 1975).

$$= \overbrace{(1-\beta_1)^2 s_P^2}^{0.1177419} + \overbrace{(\bar{A} - \bar{P})^2}^{3.6574110^{-5}} + \overbrace{(1-R^2)s_A^2}^{0.380066}$$

$$\bar{A} = \frac{32.26206}{30} = 1.075402 \quad \bar{P} = \frac{32.44349}{30} = 1.08144967$$

$$s_P^2 = \frac{\sum (P_i - \bar{P})^2}{30} = \frac{\sum P_i^2 - 30\bar{P}^2}{30} = 5538341185$$

$$s_A^2 = \frac{\sum (A_i^2) - 30\bar{A}^2}{30} = 0.4579129717$$

$$\beta_{\text{PRESS}} = \frac{\text{cov}(\beta_1, \beta_2)}{\text{var}(\beta_1)} = 0.371980$$

$$R^2 = \beta_{\text{PRESS}}^2 \frac{s_P^2}{s_A^2} = 0.1682557791$$

$$\text{PRESS} = 0.5946474686$$

- b. Refer to question (a). Suppose now we use the Vasicek adjustment procedure to adjust the betas in period 1 in order to be better predictions for betas in period 2. Which one of the three components of PRESS do you expect to decrease using the adjustment betas? Explain.

$(1 - \hat{\beta}_{press})^2 \sigma^2$ will decrease, since predicted betas will be more similar to actual betas, so β_{press} will be nearer 1, reducing the term above.

$$0.03 + 0.70 + 0.16 = 0.89 \Rightarrow 0.079$$

Exercise 3

Suppose the single index model holds. Also, short sales are allowed and there is a risk free rate $R_f = 0.002$. For 3 stocks the following were obtained based on monthly returns for a period of 5 years:

Stock i	α	β	σ_e^2
1	0.01	1.08	0.003
2	0.04	0.80	0.006
3	0.08	1.22	0.001

The expected return and variance of the market are $\bar{R}_m = 0.10$ and $\sigma_m^2 = 0.002$ for the same period.

- a. Suppose that the optimum portfolio consists of 30% of stock 1, 50% of stock 2, and 20% of stock 3. What is the β of this portfolio.

$$\beta_p = \frac{3}{10} 1.08 + \frac{5}{10} 0.8 + \frac{2}{10} 1.22 = \boxed{0.968}$$

- b. Suppose that you are a portfolio manager and you have \$500000 to invest in this optimum portfolio on behalf of a client. In addition this client wants to invest another \$300000 by borrowing this amount at the risk free rate $R_f = 0.002$. What is the expected return and standard deviation of this portfolio. Show it on the expected return standard deviation space.

$$\bar{R}_p = \frac{5+3}{5} \bar{R}_G - \frac{3}{5} R_f \quad \alpha_p = \frac{3}{10} 0.01 + \frac{5}{10} 0.04 + \frac{2}{10} 0.08 = 0.039$$

$$\bar{R}_G = \alpha_p + \beta_p \bar{R}_m = 0.039 + 0.968 \cdot 0.1 = 0.1358$$

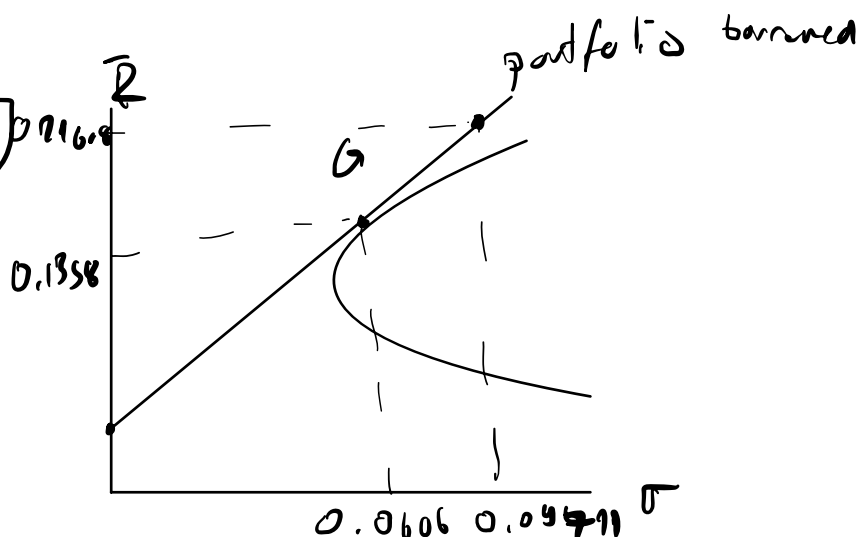
$$\bar{R}_p = \frac{8}{5} 0.1358 - \frac{3}{5} 0.002 = \boxed{0.21608}$$

$$\sigma_G^2 = \sum_{i=1}^n \alpha_i^2 \sigma_{e_i}^2 + \beta_p^2 \sigma_m^2 = 0.00169 + (0.968)^2 (0.002) = 0.003684048$$

$$\sigma_m = 0.0606$$

$$\sigma_p = \frac{8}{5} \sigma_G$$

$$\boxed{\beta_p = 0.971141745}$$



c. What is the covariance between the portfolio of part (a) and the market?

$$\begin{aligned}
 R_p &= \sum x_i R_i \Rightarrow \text{cov}(\sum x_i R_i, R_m) = \text{cov}(\alpha_p + \beta_p R_m, R_m) \\
 &= \beta_p \sigma_m^2 \\
 &= 0.001936
 \end{aligned}$$

d. If the client wants to allocate 60% of his initial funds in the optimum portfolio and the remaining 40% in the risk free asset, what would be the expected return and standard deviation of this position?

$$R_p = 0.6 R_G + 0.4 R_f$$

$$\bar{R}_p = 0.08228$$

$$\sigma_p = 0.6 \sigma_G = 0.364178154$$

e. What is the covariance between stock 1 and the market?

$$R_1 = \alpha_1 + \beta_1 R_m$$

$$\text{cov}(R_1, R_m) = \beta_1 \sigma_m^2 = 0.00216$$

Exercise 4

Answer the following questions:

- a. Suppose the single index model holds and the investor holds an equally weighted portfolio. Using data from the market and 20 stocks we have obtained the following: $\bar{R}_m = 0.10$, $\sigma_m^2 = 0.20$, $\beta_p = 0.9$, and the average residual risk of these 20 stocks is equal to 0.05. What is the variance of the portfolio of these 20 stocks?

$$\begin{aligned}\sigma_p^2 &= \sum_{i=1}^n \alpha_i^2 \sigma_{\epsilon_i}^2 + \beta_p^2 \sigma_m^2 = \\ &= \frac{1}{n} \bar{\sigma}_{\epsilon}^2 + \beta_p^2 \sigma_m^2 = \frac{1}{20} 0.05 + (0.9)^2 (0.20) \\ &= 0.1645\end{aligned}$$

- b. For 15 stocks we computed the betas for the periods 2009-13 and 2014-2018. The results are shown below. Note: **beta1** are the betas for the period 2009-13, and **beta2** are the betas for the period 2013-18. The last column are the variances of the betas ($\sigma_{\beta_i}^2$) for the period 2014-18.

Stock	beta2	beta1	var(betas)
C	2.5513904	1.3007678	0.15441843
KEY	0.6925806	0.6097833	0.09365164
WFC	1.3693579	0.3199561	0.08151656
JPM	1.1405373	1.7509958	0.04390123
HE	0.5642004	0.3998354	0.03604710
EIX	0.7178932	0.8312986	0.01089619
LUV	1.1163844	0.9741340	0.04095247
AMR	1.3936788	4.9807786	0.28777332
AMGN	0.4647108	0.7520735	0.04315126
GILD	0.3950172	0.8265294	0.03191687
HIT	1.1809416	0.7343903	0.05492128
IMO	0.9749884	0.2739596	0.03149144
MRO	1.1796907	0.5272593	0.03534135
HES	1.0657046	0.5036067	0.06206313
YPF	0.8229743	1.2438814	0.06601110

You are also given:

```
> cor(beta2, beta1)
0.2744995
> mean(beta2)
1.042003
> mean(beta1)
1.068617
> sd(beta2)
0.5225564
> sd(beta1)
1.154281
```

Find the adjusted beta for stock C for the period 2019-23 using Blume's technique.

$$\sigma_{\beta_2 \beta_1} = \rho_{\beta_2 \beta_1} \sigma_{\beta_2} \sigma_{\beta_1} = 0.165571767$$

$$\beta_{\text{regress}} = \frac{\text{cov}(\beta_2, \beta_1)}{\text{var}(\beta_1)} = 0.1242691082$$

$$\alpha_{\text{regress}} = \bar{\beta}_2 - \beta_{\text{regress}} \bar{\beta}_1 = 0.9092069187$$

$$\beta_{C, \text{adjusted}} = \alpha_{\text{regress}} + \beta_{\text{regress}} \cdot \beta_{21C} = \frac{1.226265778}{(2.55139)}$$

c. Assume that the single index model holds. The characteristics of two stocks A and B are the following:

Stock	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}_e^2$
A	0.0082	0.79	0.027
B	0.0099	1.12	0.006

In addition, $\sum_{t=1}^{60} (R_{mt} - \bar{R}_m)^2 = 0.13$ and $\sigma_m^2 = 0.0022$

Consider the simple regression of R_A on R_m . Compute the coefficient of determination R^2 .

$$R^2 = \frac{SSR}{SST} = \frac{\beta_A^2 \sum (R_{mt} - \bar{R}_m)^2}{(n-2) \sigma_{\epsilon_A}^2 + \beta_A^2 \sum (R_{mt} - \bar{R}_m)^2}$$

$$= 0.0492571031$$

(0.1) $\frac{SSR}{SSR}$

hw4_q1

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```
#Compute PRESS
a_all <- read.csv("stockData.csv", sep=",", header=TRUE)
# Use 5 year data to train
a <- a_all[1:60,]
#Convert adjusted close prices into returns:
r <- (a[-1,4:ncol(a)]-a[-nrow(a),4:ncol(a)])/a[-nrow(a),4:ncol(a)] # return of stocks
r_m <- (a[-1,3]-a[-nrow(a),3])/a[-nrow(a),3] # return of market

#use test data to compute actual predictions
a_test <- a_all[61:nrow(a_all),]

r_test <- (a_test[-1,4:ncol(a_test)]-a_test[-nrow(a_test),4:ncol(a_test)])/a_test[-nrow(a_test),4:ncol(a_test)]
r_m_test <- (a_test[-1,3]-a_test[-nrow(a_test),3])/a_test[-nrow(a_test),3] # return of market

n_stocks = 30

#Compute unadjusted historical betas
mean_Rm = mean(r_m)
var_Rm <- var(r_m)
stdev_Rm <- var_Rm^.5

mean_Ri = colMeans(r)

betas = rep(0,n_stocks)
alphas = rep(0,n_stocks)
var_es = rep(0,n_stocks)
var_betas = rep(0,n_stocks)

for (i in 1:n_stocks){
  fit <- lm(r[,i] ~ r_m)
  betas[i] = fit$coefficients[2]
  alphas[i] = fit$coefficients[1]
  var_es[i] = sum(fit$residuals^2)/ (nrow(r) - 2)
  var_betas[i] = vcov(fit)[2,2]
}

#Compute actual betas
mean_Rm_test= mean(r_m_test)
var_Rm_test <- var(r_m_test)
stdev_Rm_test <- var_Rm_test^.5

mean_Ri_test = colMeans(r)
```

```

betas_test = rep(0,n_stocks)
alphas_test = rep(0,n_stocks)
var_es_test = rep(0,n_stocks)
var_betas_test = rep(0,n_stocks)

for (i in 1:n_stocks){
  fit <- lm(r_test[,i] ~ r_m_test)
  betas_test[i] = fit$coefficients[2]
  alphas_test[i] = fit$coefficients[1]
  var_es_test[i] = sum(fit$residuals^2)/ (nrow(r_test) - 2)
  var_betas_test[i] = vcov(fit)[2,2]
}

```

Compute PRESS

```
PRESS_direct = sum((betas - betas_test)^2)/(n_stocks)
```

```

fit <- lm(betas_test ~ betas)
beta_regress = fit$coefficients[2]
r2 = summary(fit)$r.squared

```

```

term1 = (mean(betas_test) - mean(betas))^2
Sa2 <- (29/30)*var(betas_test)
Sp2 <- (29/30)*var(betas)
term2 = ((1-beta_regress)^2)*Sp2
term3 = (1-r2)*Sa2

```

```
print("BIAS term:")
```

```
## [1] "BIAS term:"
```

```
print(term1)
```

```
## [1] 0.002072871
```

```
print("Inefficiency")
```

```
## [1] "Inefficiency"
```

```
print(term2)
```

```
##      betas
```

```
## 0.1124162
```

```
print("Random error")
```

```
## [1] "Random error"
```

```
print(term3)
```

```
## [1] 0.1767718
```

```
PRESS_indirect = term1 + term2 + term3
```

```
print("Press calculated directly")
```

```
## [1] "Press calculated directly"
```

```
print(PRESS_direct)
```

```
## [1] 0.2912608
```

```
print("Press calculated with decomposition")
```

```
## [1] "Press calculated with decomposition"
```

```
print(PRESS_indirect)
```

```
##      betas
```

```
## 0.2912608
```