Student Information

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Answer 1

a)

Since $\sum_x P\{X = x\} = 1$. By using this formula:

$$\begin{aligned} P(1) + P(2) + P(3) + P(4) + P(5) = 1 &\implies \tfrac{N}{1} + \tfrac{N}{2} + \tfrac{N}{3} + \tfrac{N}{4} + \tfrac{N}{5} = 1 \\ &\implies \tfrac{137N}{60} = 1 \implies N = 0.438 \end{aligned}$$

b)

Since $E(X) = \sum_x x.P(x)$, by using this formula:

$$E(X) = \textstyle\sum_{x=1}^{x=5} x.P(x) = 1.\tfrac{N}{1} + 2.\tfrac{N}{2} + 3.\tfrac{N}{3} + 4.\tfrac{N}{4} + 5.\tfrac{N}{5} = 5 * N = 5 * 0.438 = 2.190$$

c)

Since $Var(X) = E(x^2) + E(x)^2$, by using this formula:

$$[1 \cdot 0.438 + 2 \cdot 0.438 + 3 \cdot 0.438 + 4 \cdot 0.438 + 5 \cdot 0.438] - [2.190]^2 \cong 6.570 - 4.796 = 1.774$$

d)

Since $E(Y) = \sum_y x.P(y)$ by using this formula:

$$E(Y) = \textstyle\sum_{y=1}^{y=5} y.P(y) = \sum_{y=1}^{y=5} \frac{y^2}{15} = \frac{1}{15} + \frac{4}{15} + \frac{9}{15} + \frac{16}{15} + \frac{25}{15} = 3.667$$

$$\begin{aligned} E(XY) &= \textstyle\sum_x \sum_y x.y.P(x,y) \\ &= \textstyle\sum_{x=1}^{x=5} \sum_{y=1}^{y=5} x.y.P(x).P(y) \\ &= (1) \cdot (\tfrac{0.438}{1}) \cdot [1 \cdot 0.067 + 2 \cdot 0.267 + 3 \cdot 0.600 + 4 \cdot 1.067 + 5 \cdot 1.600] \\ &\quad + (2) \cdot (\tfrac{0.438}{2}) \cdot [1 \cdot 0.067 + 2 \cdot 0.267 + 3 \cdot 0.600 + 4 \cdot 1.067 + 5 \cdot 1.600] \\ &\quad + (3) \cdot (\tfrac{0.438}{3}) \cdot [1 \cdot 0.067 + 2 \cdot 0.267 + 3 \cdot 0.600 + 4 \cdot 1.067 + 5 \cdot 1.600] \\ &\quad + (4) \cdot (\tfrac{0.438}{4}) \cdot [1 \cdot 0.067 + 2 \cdot 0.267 + 3 \cdot 0.600 + 4 \cdot 1.067 + 5 \cdot 1.600] \\ &\quad + (5) \cdot (\tfrac{0.438}{5}) \cdot [1 \cdot 0.067 + 2 \cdot 0.267 + 3 \cdot 0.600 + 4 \cdot 1.067 + 5 \cdot 1.600] \end{aligned}$$

Which equals to $= 5 \cdot [1 \cdot 0.067 + 2 \cdot 0.267 + 3 \cdot 0.600 + 4 \cdot 1.067 + 5 \cdot 1.600] \cong 8.031$

$E(X).E(Y) = 2.190 \cdot 3.667 \cong 8.031$

By the formula $Cov(X,Y) = E(XY) - E(X)E(Y) = 8.031 - 8,031 = 0$
This means they are independent each other.

Answer 2

a)

Let p is the probability of success which is 0.95 given in the question. Following that, let q be the probability of failure which is equals to $(p = 1 - q)$ and, let $n =$ number of trials, which equals to 1000. To find the probability of at least one attempt is success in 1000 trial, we should find the probability of 0 success in 1000 trials and extract it from 1. The probability of 0 success in 1000 trial can be found with formula (q^n)

$$\begin{aligned}(q^n) &= (q)^{1000}\\ &= (1-p)^{1000}\\ &= 0.05\end{aligned}$$

Then,

$$\begin{aligned}(1-p)^{1000} &= 0.05\\ (1-p) &= \sqrt[1000]{0.05}\\ (1-p) &\cong 0.997\\ p &\cong 0.003\end{aligned}$$

b)

i)

We can use Binomial distribution in this question.

$$P(x) = P\{X = x\} = \binom{n}{x} p^x q^{n-x}$$

Where n, the number of trials, p, the probability of success. q, is the probability of failure.
Probability of an average player have to play more than 500 games to win twice against an IM can be converted into the probability of less than two wins in 500 games. To find this probability we can calculate the probabilities of 1 win in 500 games and 0 win in 500 games and sum them.
Where $p = 0.003$, $q = 0.997$, $n = 500$:

$$\begin{aligned}P\{X < 2\} &= P(0) + P(1)\\ &= \binom{500}{0}(0.003)^0(0.997)^{500} + \binom{500}{1}(0.003)^1(0.997)^{499}\\ &\cong 0.223 + 0.335\\ &\cong 0.558\end{aligned}$$

ii)

Again we can use binomial distribution with the same logic. We should calculate same probabilities with different numbers.
Where $p = 0.0001$, $q = 0.9999$, $n = 10000$:

$$\begin{aligned} P\{X < 2\} &= P(0) + P(1) \\ &= \binom{10000}{0}(0.0001)^0(0.9999)^{10000} + \binom{10000}{1}(0.0001)^1(0.9999)^{9999} \\ &\cong 0.368 + 0.368 \\ &\cong 0.736 \end{aligned}$$

c)

We can use Poisson distribution in this question. By given in the question, let p, the probability of not feeling sick is 0.98. Let q, probability of feeling sick is 0.02. To find the probability of you not feeling sick for at least 360 days, we can calculate the probability of feeling sick at most 6 days in 366 days. Since $q = 0.02 \leq 0.05$ and $n = 366 \geq 30$ we can take λ as np in binomial distribution. We can calculate by using Poisson distribution table.
The formula is:

$$P\{x \leq 6\} = F(6) = 0.378$$

Where, $\lambda = 7.32 \cong 7.5$, $F(x)$ is the cumulative distribution function (cdf).

Answer 3

a)

By using Octave with codes:

octave: 1 > poisscdf(6,7.32)

the result is,

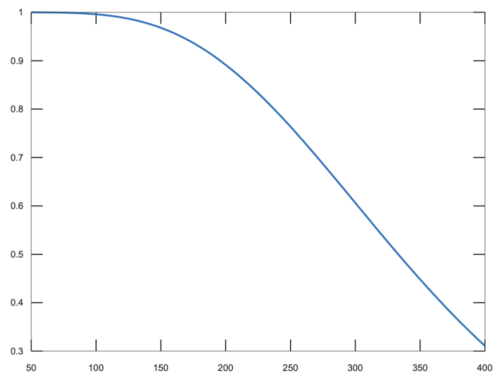
ans = 0.4032

The Octave result is more accurate and it is higher because while calculating the probability with Poission table we round $\lambda = 7.32$ to 7.5, as a result of this there is an error margin in the first calculation.

b)

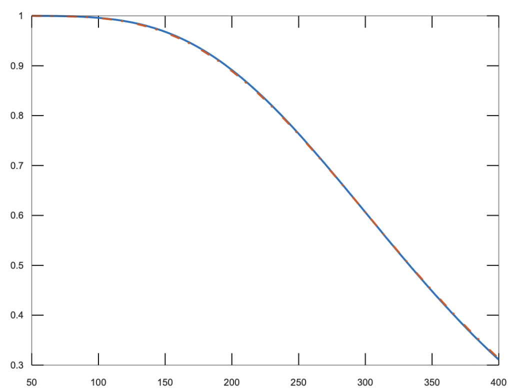
By using Octave:

```
octave:1> p = 0.02;
octave:2> ns = 50:400;
octave:3> bino_prob = binocdf(6,ns,p);
octave:4> lam = ns*p;
octave:5> pois_prob = poisscdf(6,lam);
octave:6> close all;
octave:7> plot(ns,bino_prob,'linewidth', 2);
```



$q = 0.02$, Binomial Distribution

```
octave:48>  plot(ns,pois_prob, '-.','linewidth', 2);
```

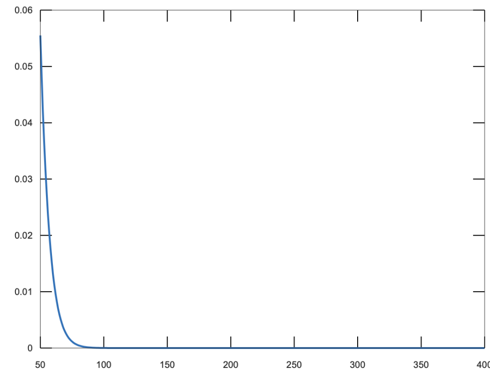


$q = 0.02$, Poisson Distribution

c)

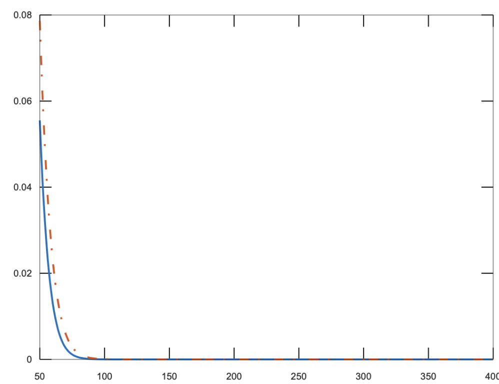
By changing p to 0.22 we obtain following graphs:

```
octave:12> p = 0.22;
octave:13> ns = 50:400;
octave:14> bino_prob = binocdf(6,ns,p);
octave:15> lam = ns*p;
octave:16> pois_prob = poisscdf(6,lam);
octave:17> close all;
octave:18> plot(ns,bino_prob,'linewidth', 2);
```



$q = 0.22$, Binomial Distribution

```
octave:23> plot(ns,pois_prob,'-.' , 'linewidth', 2);
```



$q = 0.22$, Poisson Distribution

In both questions we used q which is failure probability. When we compare $q = 0.02$ and $q = 0.22$ we can see that in the lower probabilities binomial distribution is more accurate with Poisson distribution, when q gets larger the difference between Binomial distribution and Poisson distribution gets larger.