CENG 222

Statistical Methods for Computer Engineering

Spring 2023-2024

Homework IV

Due date: 03/06/2024, Monday, 23:59

Introduction

In this homework, there are two questions related to Monte Carlo simulations. The first is a classical question that requires written calculations, while the second is a programming question. For the first question, **show your work** and the steps of your calculations. Give an explanation about what numbers mean in those steps. Also, include the code for the programming question in your report as appendix.

Questions

In a small village, several vehicles use a bridge to cross over a river. Everyday several automobiles and trucks use this bridge. The number of automobiles that pass over the bridge on a day is a Poisson random variable with $\lambda_A = 60$ and the number of trucks that pass over the bridge on a day is a Poisson random variable with $\lambda_T = 12$.

The weight of each automobile is a Gamma distributed random variable in kilograms with $\alpha = 120$ and $\lambda = 0.1$. The weight of each truck is a Gamma distributed random variable in kilograms with $\alpha = 14$ and $\lambda = 0.001$.

The bridge is not safe and it would collapse if the total weight of all the vehicles that pass over the bridge on a day is more than 250 tons. You are asked to estimate the probability of this by conducting a Monte Carlo simulation study.

Q1. (**15 pts.**)

(a) Use Normal approximation to determine the size of your Monte Carlo simulation so that with 0.99 probability, your answer should differ from the true value by no more than 0.02. *(5pts)*

(b) Answer the following questions; *(2.5pts each)*

- What is the expected value for the weight of an automobile?

- What is the expected value for the weight of a truck?

- What is the expected value for the total weights of all automobiles that pass over the bridge on a day?

- What is the expected value for the total weights of all trucks that pass over the bridge on a day?

Q2. (**85 pts.**)

Implement a Matlab/Octave code that does this simulation. As the size of the simulation use your answer in part **a** of Q1. In your report provide the following;

- Estimate the probability that the bridge would collapse (the total weight of all the vehicles that pass over the bridge on a day is more than 250 tons).

- Estimate the total weight, X, of all the vehicles that pass over the bridge on a day.

- Estimate $Std(X)$ and comment on the accuracy of your estimator of X.

- Assume we can propose a limit to the number of trucks. What would be the new Poisson variable λ_T so that the probability of bridge's collapse is less than 0.1. Provide the highest limit possible.

For this question, along the standard report template you are also given a template "hw4.m". You need to fill out necessary parts of this template file. Please provide a screenshot of the output of **fprintf** lines of the template file (three lines at the bottom) in your report.

You can use Example 5.9 from the book to sample from the Poisson distribution and Example 5.11 to sample from the Gamma distribution.

Specifications

- You are expected to write your answers in LaTeX format. You can use the given template.

- Please do not skip the calculation steps. Show every step of your work.

- You have a total of 2 late days for this homework. For each day you have submitted late, you will lose 25 points. If you submit your homework at least 2 days later than the deadline, you will get zero.

- Cheating is forbidden. The violators will be punished according to the department regulations.

- Provide your code as an appendix in your report.

- Follow the course page on ODTUClass for any updates and clarifications. Please ask your questions on ODTUClass instead of e-mailing if they do not contain some part of the solution. If they contain, you can send an email to "mduymus@ceng.metu.edu.tr".

Submission

Submissions will be done via ODTUClass. You are expected to submit a **single** PDF file named "hw4.pdf".