Student Information

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Answer 1

a)

To determine the size of the Monte Carlo simulation with 0.99 probability. The answer differs true value no more than 0.02, we can use normal approximation and use the Central limit theorem to ensure our estimate is within the desired range with the specific confidence.
Let $\hat{p}$ be the estimate of the probability that the total weight exceeds 250 tons.

- $z_{\frac{\alpha}{2}} \cong 2.576$, which is the z-value for the 99% confidence level
- σ, the standard derivation of the proportion
- E, is the margin of error (0.02)

To find margin of error for a proportion we can use the formula:

$$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Replace z with the z-value found (2.576):

$$0.02 = 2.576\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Solving for n:

$$n = \left(\frac{2.576}{0.02}\right)^2 \hat{p}(1-\hat{p})$$

Since we don't know the $\hat{p}$, we can use worst-case scenario, where the $\sigma^2 = \hat{p}(1-\hat{p})$ is maximized. To achieve this $\hat{p}$ must be equal to 0.5. Put the value to the equation:

$$n = \left(\frac{2.576}{0.02}\right)^2 (0.5)^2 n = \left(\frac{2.576 \cdot 0.5}{0.02}\right)^2 n = \left(\frac{1.288}{0.02}\right)^2 n = (64.4)^2 n \cong 4147.36$$

By using our calculations, the sample size should be at least 4148 to ensure that with 99% probability

b)

1. Expected value for the weight of an automobile:

- The weight of each automobile is found by Gamma distribution with parameters $\alpha = 120$ and $\lambda = 0.1$
- The expected value $E[W_A]$ for Gamma distribution is:

$$E[W_A] = \frac{\alpha}{\lambda} = \frac{120}{0.1} = 1200\text{kg}$$

2. Expected value for the weight of a truck:

- The weight of each truck is found by Gamma distribution with parameters $\alpha = 14$ and $\lambda = 0.001$
- The expected value $E[W_T]$ for Gamma distribution is:

$$E[W_T] = \frac{\alpha}{\lambda} = \frac{14}{0.001} = 14000\text{kg}$$

3. Expected value for the total weight of all automobiles that pass over the bridge on a day:

- The number of automobiles is found by Poisson random variable with $\lambda_A = 60$
- Let N_A be the number of automobiles passed per day
- The expected value of the total weight is $E[N_A] \cdot E[W_A]$

$$E[\text{Total weight of automobiles}] = \lambda_A \cdot EW_A = 60 \cdot 1200 = 7200\text{kg}$$

4. Expected value for the total weight of all trucks that pass over the bridge on a day:

- The number of trucks is found by Poisson random variable with $\lambda_T = 12$
- Let N_T be the number of automobiles passed per day
- The expected value of the total weight is $E[N_T] \cdot E[W_T]$

$$E[\text{Total weight of automobiles}] = \lambda_T \cdot EW_T = 12 \cdot 14000 = 168000\text{kg}$$

Answer 2

```
N = 164;
lambdaA = 60;
lambdaT = 12;
alphaA = 120;
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```
    betaA = 0.1;
    alphaT = 14;
    betaT = 0.001;
    weight_threshold = 250000;

    TotalWeight = zeros(N, 1);

    for k = 1:N
      numA = poissrnd(lambdaA);
      numT = poissrnd(lambdaT);

    auto_weights = gamrnd(alphaA, 1/betaA, numA, 1);
    total_auto_weight = sum(auto_weights);
    truck_weights = gamrnd(alphaT, 1/betaT, numT, 1);
    total_truck_weight = sum(truck_weights);
    weight = total_auto_weight + total_truck_weight;
    TotalWeight(k) = weight;
end

p_est = sum(TotalWeight > weight_threshold) / N;
expectedWeight = mean(TotalWeight);
stdWeight = std(TotalWeight);

fprintf('Estimated probability = %f\n', p_est);
fprintf('Expected weight = %f\n', expectedWeight);
fprintf('Standard deviation = %f\n', stdWeight);
```

Results:

```
Estimated probability = 0.384146
Expected weight = 237818.581904
Standard deviation = 49640.619406
```

```
    new_lambdaT = lambdaT;
    while true
    collapse_count = 0;
    for k = 1:N
        numA = poissrnd(lambdaA);
        numT = poissrnd(new_lambdaT);
        auto_weights = gamrnd(alphaA, 1/betaA, numA, 1);
        truck_weights = gamrnd(alphaT, 1/betaT, numT, 1);
        total_weight = sum(auto_weights) + sum(truck_weights);
        if total_weight > weight_threshold
```

```
            collapse_count = collapse_count + 1;
        end
    end
    if (collapse_count / N) < 0.1
        break;
    end
    new_lambdaT = new_lambdaT - 0.1;
end

fprintf('New T to keep collapse probability < 0.1: %f\n', new_lambdaT);
```

```
New λT to keep collapse probability < 0.1: 8.000000
```