Student Information

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Answer 1

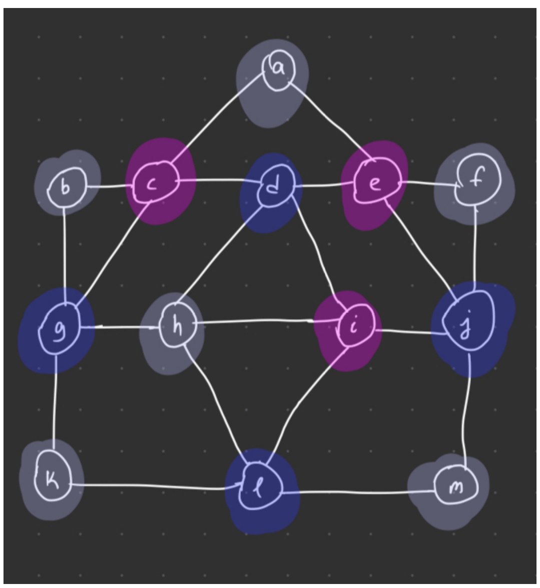
a)Yes since $|v_0| = 0$ and it is connected there exists a Eularian Circuit by using Theorem 1 in pp.696, all vertices have even number of degree.

b) No. Since $|v_0| = 0$ and graph is connected, all Eularian Paths are Euilarian Circuits.

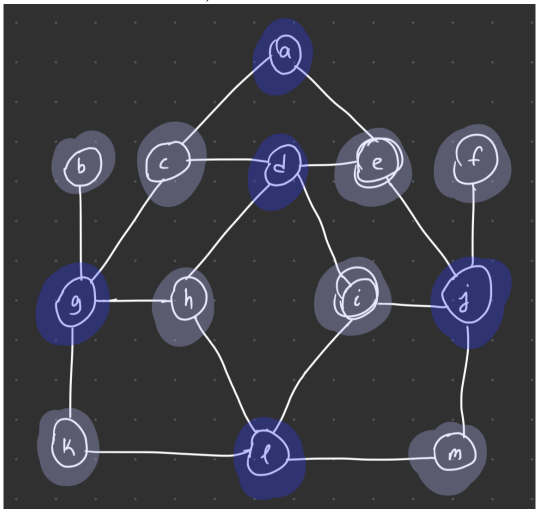
c) No there is not. Since try to use Ore's Theorem, choose vertex f, m for a pair. $deg(f)+deg(m) = 4$, there are 13 vertices in total. $4 \leq 13$ so there is not a hamilton circuit.

d)Yes there is. For example, $a - c - b - g - k - l - m - j - f - e - d - h - i$ is a Hamiltonian path goes every vertex exactly once and not a circuit since first and last vertex is not equal.

e) Minimum number of colors required to color G is $X(G) = 3$



f) No it is not since $X(G) = 3$, for bipartite graphs $X(G)$ must be equal to 2. Minimum 3 edges ($b-c$, $e-f$, $h-i$) should be deleted from the set G to make it bipartite.



g) No it does not have. If we take subgraph of 4 nodes as d, h, i, l and call that S_1 if we add an edge between $d-l$ S_1 will be a complete subgraph of G.

Answer 2

it is isomorphic,G and H both have 8 vertices 16 edges and each vertex have degree 4. We can define a function F which is a 1-to-1 and onto from G to H.
$F(a) = a', F(b) = g', F(c) = e', F(d) = c', F(e) = h', F(f) = b', F(g) = f', F(h) = d'$
Matrix representation for each graph is:

$$A(G) = \begin{array}{cc} & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}, \quad A(H) = \begin{array}{cc} & \begin{array}{cccccccc} a' & g' & e' & c' & h' & b' & f' & d' \end{array} \\ \begin{array}{c} a' \\ g' \\ e' \\ c' \\ h' \\ b' \\ f' \\ d' \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Since $A(H) = G(H)$ they are isomorphic.

Answer 3

a) for $n \geq 3$ C_n, C represents cycle graphs, if n is even $X(G) = 2$ and graph is bipartite since we can color it by using only 2 colors, we can separate that into 2 parts.
Else if n is odd $X(G) = 3$ and graph is not bipartite because we cannot separate it to 2 parts.

b) for $n \geq 1$ Q_n, Q represents cube graphs, $\forall n \geq 1$, 2 color is enough to color a cubic graph since every vertex has n neighbors and neighbors are not connected as an example, we can color $Black - White - Black$. $X(G) = 2$ Hence, graph is bipartite.

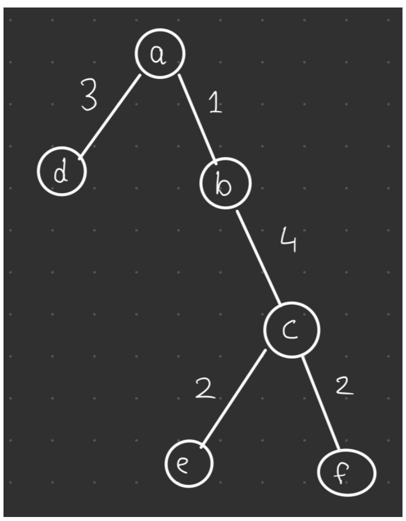
Answer 4

a) For Prim's Algorithm we should choose minimum weighted edge and successively add the tree edges of minimum weight that are incident to a vertex already in tree.

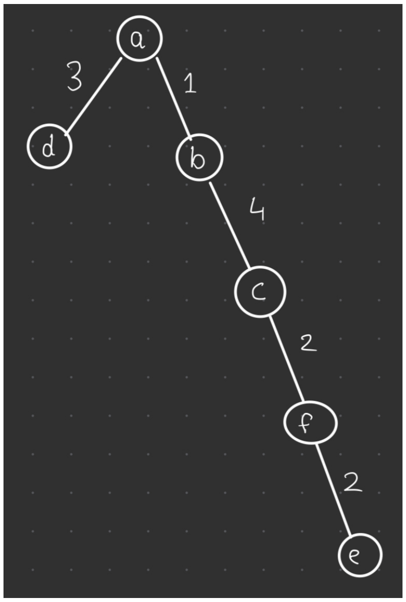
1) $a - b : weight$ 1
2) $a - d : weight$ 3
3) $b - c : weight$ 4
4) $c - e : weight$ 2
5) $c - f : weight$ 2

We should stop since there are 6 vertex = n and we added 5 edge which is n-1.

b) Minimum spanning tree is in the picture below.



c) No it is not. For example, while adding 5^{th} edge we could add $e - f$ instead of $c - e$ and it will result a different minimum spanning tree as shown in the picture.



Answer 5

a) A full binary tree contains $\sum_{h=0}^{h} 2^h$ nodes in total where h = height of the tree.
$\sum_{h=0}^{h} 2^h = 2^0 + 2^1 + 2^2 + \cdots + 2^h = 2^{h+1} - 1$. To increase height by 1 we should add 2 vertices for each leaf node. The count of leaf nodes for each level is $\sum_{h=0}^{h} 2^h - \sum_{h=0}^{h-1} 2^h$ for $h \geq 1$ and initial condition for leaf node count $h = 0 \implies 1$
$\sum_{h=0}^{h} 2^h - \sum_{h=0}^{h-1} 2^h = (2^{h+1} - 1) - (2^h - 1) = 2.2^h - 2^h = 2^h$
If we say n = total number of vertices $= 2^{h+1} - 1 \implies$ number of leafs $= 2^h = \frac{(2^{h+1}-1)-1}{2} = \frac{n+1}{2}$

b) Chromatic number of a tree is $X(G) = 2$. Since tree consists of parent and children and there is no any connection between children, it is enough to have different colors for parent and children which is 2 different color.

c) In full m-ary trees, every node have m or 0 children. To maximize height, in each level every node except one should be leaf node. So total number of nodes is $1+m+m+m+\cdots+m = 1+h.m$ where h is height of the tree. If we say n is total number of nodes then $n = 1 + h.m \implies h = \frac{n-1}{m}$