# Student Information

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# Answer 1

**a)** Let $P(n)$ be the function that $2^{3n} - 3^n$ and $P(n)$ is divisible by 5 for all integers $n \geq 1$ to prove that we will first prove $P(1)$ is true then $P(n) \implies P(n+1)$ is true for all integers $n \geq 1$
**Basis Step:** $n = 1 \implies 2^{3n} - 3^n = 2^3 - 3 = 5 \implies 5|5 = 1$
**Inductive Step:** $n = k, k \geq 1$ Assume that $P(k) = 2^{3k} - 3^k$ is divisible by 5. Then $n = k+1 \implies P(k+1) = 2^{3(k+1)} - 3^{(k+1)} = 2^3 \cdot 2^{3k} - 3 \cdot 3^k = 8 \cdot 2^{3k} - 8 \cdot 3^k + 5 \cdot 3^k = 8(2^{3k} - 3^k) + 5 \cdot 3^k$.
Since left side $8(2^{3k} - 3^k) = 8 \cdot P(k)$ and $P(k)$ is divisible by 5, right side $5 \cdot 3^k$ is divisible by 5 since it is a multiple of 5. Hence, $P(k+1) = 8(2^{3k} - 3^k) + 5 \cdot 3^k$ is divisible by 5.

**b)** Let $P(n)$ be the function that $4^n - 7n - 1$ and $P(n) > 0$ for all integers $n \geq 2$ to prove that we will first prove $P(1) > 0$ is true then $P(n) > 0 \implies P(n+1) > 0$ is true for all integers $n \geq 2$
**Basis Step:** $n = 2 \implies 4^2 - 7.(2) - 1 = 16 - 14 - 1 = 1 > 0$
**Inductive Step:** $n = k \wedge n \geq 2$ Assume that $P(k) > 0$. Then for $n = k+1$
$P(k+1) = 4^{k+1} - 7 \cdot (k+1) - 1 = 4 \cdot 4^k - 7k - 7 - 1 = 4 \cdot 4^k - 7k - 8 = P(k) + 3 \cdot 4^k - 7$
$\forall k \geq 2 \implies 3 \cdot 4^k > 7$ . This completes the inductive step by mathematical induction.

# Answer 2

**a)** We are looking for bit strings of length 10 and at least with 7 1's $<\_,\_,\_,\_,\_,\_,\_,\_,\_,\_>$.
There are 4 possibilities:

- 7 1's $= C(10,7) = 120$
- 8 1's $= C(10,8) = 45$
- 9 1's $= C(10,9) = 10$
- 10 1's $= C(10,10) = 1$

By summation all possibilities are $C(10,7)+C(10,8)+C(10,9)+C(10,10) = 120+45+10+1 = 176$.

**b)** As we are making a collection of books, sorting does not matter and since we must choose at least 1 Discrete Mathematics textbook, 1 Statistical Methods textbook we have 2 slots to choose. Thus since book are identical between each other we have 3 possibilities:

- 2 Discrete Mathematics textbooks.
- 2 Statistical Methods textbooks.
- 1 Discrete Mathematics textbooks, 1 Statistical Methods textbooks.

These are all possible ways of making a collection of 4 books between 4 identical Discrete Mathematics textbooks and 5 identical Statistical Methods textbooks.
**c)**Since our function is onto all elements in the image set should have a equivalent in the pre-image set. Thus all possibilities are:
$C(3,1) \cdot C(5,3) \cdot C(2,1) \cdot C(1,1) + C(3,1) \cdot C(5,2) \cdot C(3,2) \cdot C(1,1) = 60 + 90 = 150$

# Answer 3

If we divide equilateral triangular circus with each side length of 500 meters into 4 equal small equilateral triangles with side of length 250 meters we have 5 kids and we have 4 equal triangles by Pigeonhole Principle at least two of them must be in the same small triangle. If two kids are in the small triangle with side length 250m their distance to each other is at most 250 meters. This proofs how much they wander away from each other, so long as they stay in the triangle-shaped circus, there are two of them within 250 meters each other.

# Answer 4

**a)** $a_n^h : a_n - 3a_{n-1} = 0$
characteristic equation: $\alpha - 3 = 0 \implies \alpha = 3$
$a_n^h = A(3)^n$
**b)**$a_n^p = B(5)^{n-1}$ place the $a_n^p$ in the main function $\implies B.5^{n-1} = 3B.5^{n-2} + 5n - 1$
$\frac{2}{5}B^{n-1} = 5^{n-1} \implies B = \frac{1}{2}$
$a_n^p = \frac{5^n}{2}$
$a_n = A.3^n + \frac{5}{2}5^{n-1}, a_1 = 4 \implies 3A + \frac{5}{2} = 4 \implies A = \frac{1}{2}$
$a_n = \frac{3^n+5^n}{2}$
**c)** Basis Step: n = 1 $\implies \frac{3^1+5^1}{2} = \frac{8}{2} = 4$
Inductive Step: $n = k$ Assume that $a_k = \frac{3^k+5^k}{2}$ is true
for n = k+1
$$\begin{aligned} a_{k+1} &= \frac{3^{k+1}+5^{k+1}}{2} \\ &= \frac{3.3^k+5.5^k}{2} \\ &= 3a_k + 5^k \end{aligned}$$
Thus $P(k) \implies P(k+!)$