Student Information

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Answer 1

a)

By using the formula:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(X,Y)\,dx\,dy = 1$$

Since our function is 0 except $0 < x < 1$ and $0 < y < 1$. we should use:

$$\int_{0}^{1}\int_{0}^{1} f_{X,Y}(X,Y)\,dx\,dy = 1$$

$$\int_{0}^{1}\int_{0}^{1} x + ky^3\,dx\,dy = 1$$

$$\int_{0}^{1}\left[\frac{x^2}{2} + kxy^3\right]_{x=0}^{x=1} dy = 1$$

$$\int_{0}^{1}\frac{1}{2} + ky^3\,dy = 1$$

$$\left[\frac{y}{2} + \frac{ky^4}{4}\right]_{y=0}^{y=1} = 1$$

$$\frac{1}{2} + \frac{k}{4} = 1$$

Thus, $k = 2$.

b)

To calculate $P\{x = \frac{1}{2}\}$, we should use:

$$\int_0^1 \int_{\frac{1}{2}}^{\frac{1}{2}} x + 2y^3 \, dx \, dy$$

Since at the inner integral we are trying to calculate from $\frac{1}{2}$ to $\frac{1}{2}$ the result equals to 0:

$$\int_0^1 \int_{\frac{1}{2}}^{\frac{1}{2}} x + 2y^3 \, dx \, dy = 0$$

c)

For $0 < x < \frac{1}{2}$ and $0 < y < \frac{1}{2}$ we should calculate $P\{0 < x < \frac{1}{2}, 0 < y < \frac{1}{2}\}$:

$$P\{0 < x < \frac{1}{2}, 0 < y < \frac{1}{2}\} = \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} x + 2y^3 \, dx \, dy$$

$$P\{0 < x < \frac{1}{2}, 0 < y < \frac{1}{2}\} = \int_0^{\frac{1}{2}} \left[\frac{x^2}{2} + 2xy^3\right]_{x=0}^{x=\frac{1}{2}} dy$$

$$P\{0 < x < \frac{1}{2}, 0 < y < \frac{1}{2}\} = \int_0^{\frac{1}{2}} (\frac{1}{8} + y^3) \, dy$$

$$P\{0 < x < \frac{1}{2}, 0 < y < \frac{1}{2}\} = \left[\frac{1}{8} + \frac{y^4}{4}\right]_{y=0}^{y=\frac{1}{2}}$$

$$P\{0 < x < \frac{1}{2}, 0 < y < \frac{1}{2}\} = \frac{1}{16} + \frac{1}{64}$$

$$P\{0 < x < \frac{1}{2}, 0 < y < \frac{1}{2}\} = \frac{5}{64}$$

Thus, the result is $\frac{5}{64}$

Answer 2

a)

To find the marginal pdf of Y, we should integrate the function with x for all values at y:

$$\int_0^\infty f_{XY}(XY) \, dx = \int_0^\infty \frac{e^{-y-\frac{x}{y}}}{y} \, dx$$

To calculate integral, we use substitution where, $u = y + \frac{x}{t}$ and $du = \frac{1}{y}\,dx$

$$\int_0^\infty \frac{e^{-y-\frac{x}{y}}}{y}\,dx = -\int_0^\infty e^{-u}\,du$$
$$= \Big[-e^{-u}\Big]_{u=0}^{u\to\infty}$$
$$= 0 - (-e^{-y})$$
$$= e^{-y}$$

So, marginal probability density function Y is e^{-y} for $0 < y < \infty$ and belongs to Exponential Distribution Family.

b)

Since Exponential Distribution has a template of $f(x) = \lambda e^{-\lambda x}$, in our function we have $f(x) = e^{-y}$. By comparing those two functions, we can see that in our function $\lambda = 1$. Since Expected value of Y is given by the formula $E(Y) = \frac{1}{\lambda}$

$$E(Y) = \frac{1}{\lambda} = \frac{1}{1} = 1$$

Answer 3

a)

We use binomial distribution for this question. Given that %10 of the soldiers are naval forces, so $p = 0.1$ and we pick 1000 soldiers, thus, $n = 1000$. At least %9 means we are looking for at least 90 in 1000 soldiers:

$$P\{X \geq 90\} = 1 - P\{X < 90\}$$
$$P\{X \geq 90\} = 1 - P\{X \leq 89\}$$
$$= 1 - \sum_{n=0}^{89} \binom{1000}{n} p^n (1-p)^{1000-n}$$
$$= 1 - 0.133$$
$$= 0.867$$

The result is 0.867 calculated by octave.

b)

If we change n to 2000, we should look for %9 of 2000 and that means at least 180 naval forces:

$$P\{X \geq 180\} = 1 - P\{X < 180\}$$
$$P\{X \geq 180\} = 1 - P\{X \leq 179\}$$
$$= 1 - \sum_{n=0}^{179} \binom{2000}{n} p^n (1-p)^{2000-n}$$
$$\cong 1 - 0.062$$
$$= 0.978$$

By calculating the probability we obtain that, the probability is increased because as the sample size increases the probability of at least %9 naval soldiers more likely to occur, since %10 of Turkish Armed Forces belongs to the naval forces.

Answer 4

a)

We can use Standard Normal Distribution in this question. It is given that mean $\mu = 65$ and standard derivation $\sigma = 6$ in the question. To calculate the probability that a randomly selected elephant will live more than 60 years, but less than 75 years we pick $60 < x < 75$:

$$P\{60 < x < 75\} = P\{x < 75\} - P\{x < 60\}$$
$$= P\left\{\frac{75-65}{6}\right\} - P\left\{\frac{60-65}{6}\right\}$$
$$\Phi(1.667) - \Phi(-0.833)$$
$$\cong 0.952 - 0.202$$
$$= 0.750$$

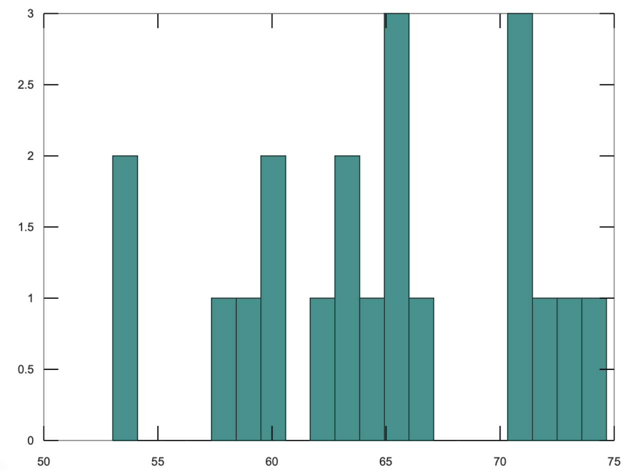
The result is 0.154 calculated by octave.

b)

Here are the codes and graphs:

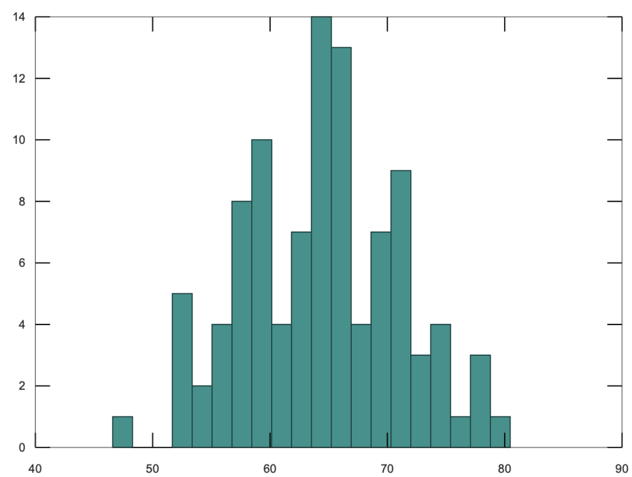
```
octave:1> mu = 65;
octave:2> sigma = 6;
octave:3> size = 20;
octave:4> span = normrnd(mu,sigma,[1,size]);
```

```
octave:5> hist(span,20);
```

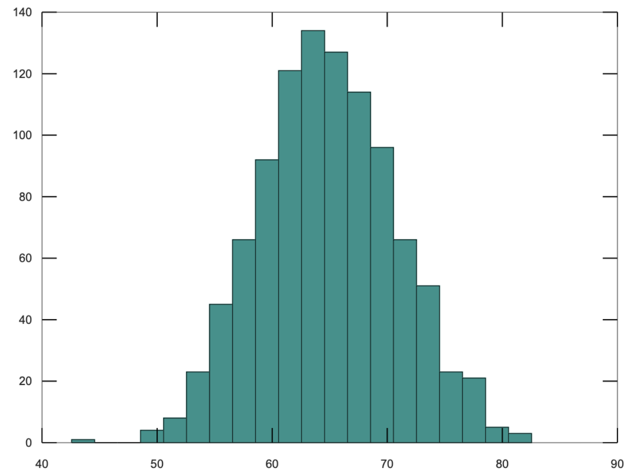


The only change is size.

```
octave:6> size = 100;
octave:7> span = normrnd(mu,sigma,[1,size]);
octave:8> hist(span,20);
```



```
octave:9> size = 1000;
octave:10> span = normrnd(mu,sigma,[1,size]);
octave:11> hist(span,20);
```



c)

Here are the code and outputs:

```
octave:17> mu=65;
octave:18> sigma=6;
octave:19> minyear=60;
octave:20> maxyear=75;
octave:21> counter70=0;
octave:22> counter85=0;
octave:23>
octave:23> size=100;
octave:24> iterations=1000;
octave:31> for iter= 1:iterations
    sample=normrnd (mu, sigma, [size, 1]) ;
    countinrange=sum(sample>= minyear & sample <= maxyear);
    if countinrange > 0.7* size
        counter70=counter70+1;
    end
    if countinrange>= 0.85* size
        counter85=counter85 +1;
    end
end
```

```
octave:16> disp([' At least %70 of the elephants had a lifespan in given range: ' num2str(counter70)]);
disp([' At least %85 of the elephants had a lifespan in given range: ' num2str(counter85)]);
 At least %70 of the elephants had a lifespan in given range: 823
 At least %85 of the elephants had a lifespan in given range: 12
```

In part a we found that %75 percent of African bush elephants have a lifespan of 60-75 years. In the iteration, we found that 823 of 1000 have at least %70 of elephants had a lifespan in the given range which is a similar result to part a. It is close to our expected possibility %75. However, only 14 of 1000 have at least %85 of elephants had a lifespan in the given range. This is not an expected result. In total the simulation worked well.