Student Information

Full Name : Musa Alper Yaman
Id Number : 2581155

Answer 1

There are 2^n vertices in Q_n cubegraph and each of them have $degree(n)$. Thus by using "Handshaking Theorem ($2|E| = \sum_{v \in V} degree(v)$)", $n.2^n = 2.E_n$. For Q_{n+1},
$2.|E_{n+1}| = (n+1).2^{n+1} \implies E_{n+1} = \frac{n.2^{n+1}+2^{n+1}}{2} = \frac{2(n.2^n+2^n)}{2} = n.2^n + 2^n = E_n + 2^n$.
Hence, $E_{n+1} = E_n + 2^n$,for $n \geq 0$ and $E_0 = 0$

Answer 2

$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots$ as in textbook pp.542 Table 1 line 5
$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty}(k+1)x^k = 1 + 2x + 3x^2 + \cdots$ as in textbook pp.542 Table 1 line 8
$\frac{x}{(1-x)^2} = \sum_{k=0}^{\infty}(k+1)x^{k+1} = 0 + x + 2x^2 + \cdots$
$\frac{3x}{(1-x)^2} = 3.\sum_{k=0}^{\infty}(k+1)x^{k+1} = 0 + 3x + 6x^2 + \cdots$
$\frac{1}{1-x} + \frac{3x}{(1-x)^2} = \sum_{k=0}^{\infty} x^k + 3.\sum_{k=0}^{\infty}(k+1)x^{k+1} = 1 + 4x + 7x^2 + \cdots$
$\frac{1}{1-x} + \frac{3x}{(1-x)^2} = \frac{1+2x}{(1-x)^2} =< 1, 4, 7, 10, 13, ... >$

Answer 3

$a_n = a_{n-1} + 2^n, n \geq 1, a_0 = 1$

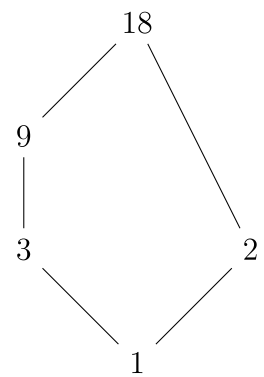
$$\begin{aligned}
f(x) &= \sum_{n=1}^{\infty} a_n x^n \\
&= \sum_{n=1}^{\infty} (a_{n-1} + 2^n) x^n \\
&= \sum_{n=1}^{\infty} a_{n-1} x^n + \sum_{n=1}^{\infty} 2^n x^n \\
&= x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + \sum_{n=1}^{\infty} 2^n x^n \\
&= x \sum_{n=0}^{\infty} a_n x^n + \sum_{n=1}^{\infty} 2^n x^n
\end{aligned}$$

$f(x) = x \cdot f(x) + \frac{1}{1-2x}$, since $\sum_{n=1}^{\infty} 2^n x^n = \frac{1}{1-2x}$ as in textbook pp.542 Table 1 line 6.
$f(x) = \frac{1}{(1-2z).(1-x)} = \frac{A}{1-2x} + \frac{B}{1-x}$

$A - Ax + B - 2Bx = 1 \implies -A - 2B = 0$ and $A + B = 1 \implies A = 2, B = -1$
$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots$ as in textbook pp.542 Table 1 line 5
$\sum_{n=1}^{\infty} 2^n x^n = \frac{1}{1-2x}$ as in textbook pp.542 Table 1 line 6 $\implies \sum_{n=1}^{\infty} 2^{n+1} x^n = \frac{2}{1-2x}$
Hence, $f(x) = \frac{2}{1-2x} - \frac{1}{1-x} =< 2^1 - 1, 2^2 - 1, 2^3 - 1, ..., 2^{n+1} - 1, ... >$
Thus,$a_n = 2^{n+1} - 1$

Answer 4

a)



b)

$$R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Firstly we should check is R partially ordered set. It is partially ordered because it is reflective,antisymmetric and transitive.It is lattice since for every pair we have chosen there exits a unique Least Upper Bound (LUB) and Greatest Lower Bound (GLB).

d) Symmetric closure of $R = R_s = R \cup R^{-1}$

$$R \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \bigcup R^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = R_s$$

e) x,y are comparable if and only if $(x, y) \in R$ or $(y, x) \in R$
2 and 9 are not comparable because $(2, 9)$ or $(9, 2)$ are not in the set R
3 and 18 are comparable because $(3, 18)$ is in the set of R

Answer 5

a) Let A be a nxn matrix representation of both reflexive and symmetric relations. For reflexive property the main diagonal must be all 1's. Every pair of $M_{i,j}$ and $M_{j,i}$ must be equal for symmetric property. There are total n^2 slots to choose, n times 1's in the main diagonal, and $\frac{n^2-n}{2}$ (because in pairs we choose one of them and other one is the same) slots to choose. Every slot has 2 options.
The total number is $2^{\frac{n^2-n}{2}}$

b)Let A be a nxn matrix representation of both reflexive and asymmetric relations. For reflexive property the main diagonal must be all 1's. Every pair of $M_{i,j}$ and $M_{j,i}$ have 3 possibilities, $(0,1),(1,0),(0,0)$. There are total n^2 slots to choose, n times 1's in the main diagonal, and $\frac{n^2-n}{2}$ (because in pairs we choose one of them and other one is related to first one) slots to choose. Every slot has 3 options.
The total number is $3^{\frac{n^2-n}{2}}$

Answer 6

Transitive closure of an antisymmetric relation is not always antisymmetric.
Consider a set $A = \{x,y,z\}$. Let Relation set $R = \{(x,z),(y,x),(z,y)\}$. R is antisymmetric but it is not transitive. Transitive closure is $R^+ = \{(x,z),(y,x),(z,y),(x,y),(z,x),(y,z)\}$.
R^+ is transitive but it is not antisymmetric since it contains pairs $\{(x,z),(z,x)\}$ or $\{(y,x),(x,y)\}$ or $\{(z,y),(y,z)\}$