Student Information

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Answer 1

a)

p	q	$p \to q$	$p \wedge \neg q$	$(p \to q) \oplus (p \wedge \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

TAUTOLOGY

b)

$$
\begin{aligned}
p \to ((q \vee \neg p) \to r) &\equiv p \to (\neg(q \vee \neg p) \vee r) && \textit{Table } 7, \textit{Equivalence } 1 \\
&\equiv p \to ((\neg q \wedge \neg\neg p) \vee r) && \textit{Table } 6, \textit{De Morgan's laws} \\
&\equiv p \to ((\neg p \wedge p) \vee r) && \textit{Table } 6, \textit{Double negation law} \\
&\equiv (p \to (\neg q \wedge p)) \vee (p \to r) && \textit{Table } 7, \textit{Equivalence } 8 \\
&\equiv (\neg p \vee (\neg q \wedge p)) \vee (p \to r) && \textit{Table } 7, \textit{Equivalance } 8 \\
&\equiv ((\neg p \vee \neg q) \wedge (\neg p \vee p)) \vee (p \to r) && \textit{Table } 6, \textit{Distributive laws} \\
&\equiv ((\neg p \vee \neg q) \wedge \mathbf{T}) \vee (p \to r) && \textit{Table } 6, \textit{Negation laws} \\
&\equiv (\neg p \vee \neg q) \vee (p \to r) && \textit{Table } 6, \textit{Identity law} \\
&\equiv (\neg p \vee \neg q) \vee (\neg p \vee r) && \textit{Table } 6, \textit{De Morgan's laws} \\
&\equiv \neg p \vee (\neg q \vee r) && \textit{Table } 6, \textit{Distributive laws} \\
&\equiv (\neg p \vee \neg q) \vee r && \textit{Table } 6, \textit{Associative laws} \\
&\equiv \neg(p \wedge q) \vee r && \textit{Table } 6, \textit{De Morgan's laws} \\
&\equiv (p \wedge q) \to r && \textit{Table } 7, \textit{Equivalence } 1
\end{aligned}
$$

c)

- False
- False
- False
- True
- True

Answer 2

a) $P(Can, x) \land T(x, L)$
b) $\forall x(T(x, S) \to \exists y(N(y, Turkish) \land P(y, x)))$
c) $\forall x(T(x, S) \to \exists y(R(x, y) \land \forall z(z \to y = z)))$
d) $\forall x(W(x, M) \to \exists y(N(y, \text{English}) \land P(y, x)))$
e) $\exists y \exists z \forall a(N(y, \text{Turkish}) \land N(z, \text{Turkish}) \land P(y, G) \land P(z, G) \land y \neq z \land (P(a, G) \to \neg N(a, \text{Turkish})))$
f) $\exists x(\exists y T(x, y) \land \exists z T(x, z) \land y \neq z)$

Answer 3

1.	$p \to q$	*premise*
2.	$(r \land s) \to p$	*premise*
3.	$r \land \neg q)$	*premise*
4.	q	*assumed*
5.	r	$\land e, 3$
6.	$(r \land s)$	$\land i, 4, 5$
7.	p	$\to e, 2, 6$
8.	q	$\to e, 1, 7$
9.	$\neg q$	$\land e, 3, 5$
10.	$\bot$	$\neg e, 8, 9$
11.	$\neg s$	$\neg i, 4 - 10$

Answer 4

a)

- Some students need to study for the exam in order to pass $\equiv \exists x(P(x) \rightarrow S(c))$
- Every student passed the exam. $\equiv \forall x P(x)$
- There is at least one student that studied for the exam $\equiv \exists x S(x)$

b)

1.	$\exists x(P(x) \rightarrow S(x))$	*premise*
2.	$\forall x P(x)$	*premise*
3.	$P(c) \rightarrow S(c)$	*Assumed*
4.	$P(c)$	$\forall e, 2$
5.	$S(c)$	$\rightarrow e, 3, 4$
6.	$\exists x S(x)$	$\exists i, 5$
7.	$\exists x S(x)$	$\exists i, 3-6$