STAT_ML_HW1

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1 Statistical ML - Homework 1

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[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split, cross_val_score,␣
 ↪GridSearchCV
from sklearn.linear_model import LinearRegression, Ridge, Lasso, ElasticNet,␣
 ↪ElasticNetCV, RidgeCV
from sklearn.preprocessing import StandardScaler
from sklearn.feature_selection import RFE
from sklearn.metrics import mean_squared_error
from itertools import combinations
```

1. Data Analysis

[2]:
```
# Loading the data
url = "https://archive.ics.uci.edu/ml/machine-learning-databases/communities/
 ↪communities.data"
df = pd.read_csv(url, header=None, na_values = '?')

# Checking out the structure
df.shape, df.head()
```

[2]:
```
((1994, 128),
    0     1        2                    3    4     5     6     7     8     9  \
 0   8   NaN      NaN         Lakewoodcity    1  0.19  0.33  0.02  0.90  0.12
 1  53   NaN      NaN          Tukwilacity    1  0.00  0.16  0.12  0.74  0.45
 2  24   NaN      NaN         Aberdeentown    1  0.00  0.42  0.49  0.56  0.17
 3  34   5.0  81440.0  Willingborotownship    1  0.04  0.77  1.00  0.08  0.12
 4  42  95.0   6096.0     Bethlehemtownship    1  0.01  0.55  0.02  0.95  0.09

    …   118   119   120   121   122  123  124   125   126   127
 0  …  0.12  0.26  0.20  0.06  0.04  0.9  0.5  0.32  0.14  0.20
```

```
1  …  0.02  0.12  0.45   NaN   NaN  NaN  NaN  0.00   NaN  0.67
2  …  0.01  0.21  0.02   NaN   NaN  NaN  NaN  0.00   NaN  0.43
3  …  0.02  0.39  0.28   NaN   NaN  NaN  NaN  0.00   NaN  0.12
4  …  0.04  0.09  0.02   NaN   NaN  NaN  NaN  0.00   NaN  0.03

[5 rows x 128 columns])
```

[3]:
```
# Naming the columns. Went through dataset documentation to pick which columns␣
↪to name and include in the dataframe
columns = [
    "state", "county", "community", "communityname", "fold",
    "population", "householdsize", "racepctblack", "racePctWhite",␣
↪"racePctAsian",
    "racePctHisp", "agePct12t21", "agePct16t24", "agePct65up", # Removed␣
↪agePct12t29
    "numbUrban", "pctUrban", "medIncome", "pctWWage", "pctWFarmSelf",
    "pctWInvInc", "pctWSocSec", "pctWPubAsst", "pctWRetire", "medFamInc",
    "perCapInc", "whitePerCap", "blackPerCap", "indianPerCap", "AsianPerCap",
    "HispPerCap", "NumUnderPov", "PctPopUnderPov", "PctLess9thGrade",␣
↪"PctNotHSGrad",
    "PctBSorMore", "PctUnemployed", "PctEmploy", "PctEmplManu",␣
↪"PctEmplProfServ",
    "PctOccupManu", "PctOccupMgmtProf", "MalePctDivorce", "MalePctNevMarr",␣
↪"FemalePctDiv",
    "TotalPctDiv", "PersPerFam", "PctFam2Par", "PctKids2Par",␣
↪"PctYoungKids2Par",
    "PctTeen2Par", "PctWorkMomYoungKids", "PctWorkMom", "NumIlleg", "PctIlleg",
    "NumImmig", "PctImmigRecent", "PctImmigRec5", "PctImmigRec8",␣
↪"PctImmigRec10",
    "PctRecentImmig", "PctRecImmig5", "PctRecImmig8", "PctRecImmig10",␣
↪"PctSpeakEnglOnly",
    "PctNotSpeakEnglWell", "PctLargHouseFam", "PctLargHouseOccup",␣
↪"PersPerOccupHous",
    "PersPerOwnOccHous", "PersPerRentOccHous", "PctPersOwnOccup",␣
↪"PctPersDenseHous", "PctHousLess3BR",
    "MedNumBR", "HousVacant", "PctHousOccup", "PctHousOwnOcc",␣
↪"PctVacantBoarded",
    "PctVacMore6Mos", "MedYrHousBuilt", "PctHousNoPhone", "PctWOFullPlumb",␣
↪"OwnOccLowQuart",
    "OwnOccMedVal", "OwnOccHiQuart", "OwnOccQrange", "RentLowQ", "RentMedian",
    "RentHighQ", "RentQrange", "MedRent", "MedRentPctHousInc",␣
↪"MedOwnCostPctInc",
    "MedOwnCostPctIncNoMtg", "NumInShelters", "NumStreet", "PctForeignBorn",␣
↪"PctBornSameState",
    "PctSameHouse85", "PctSameCity85", "PctSameState85", "LemasSwornFT",␣
↪"LemasSwFTPerPop",
```

```
    "LemasSwFTFieldOps", "LemasSwFTFieldPerPop", "LemasTotalReq",␣
  ↪"LemasTotReqPerPop", "PolicReqPerOffic",
    "PolicPerPop", "RacialMatchCommPol", "PctPolicWhite", "PctPolicBlack",␣
  ↪"PctPolicHisp",
    "PctPolicAsian", "PctPolicMinor", "OfficAssgnDrugUnits",␣
  ↪"NumKindsDrugsSeiz", "PolicAveOTWorked",
    "LandArea", "PopDens", "PctUsePubTrans", "PolicCars", "PolicOperBudg",
    "LemasPctPolicOnPatr", "LemasGangUnitDeploy", "LemasPctOfficDrugUn",␣
  ↪"PolicBudgPerPop",
    "ViolentCrimesPerPop"
]

len(columns), df.shape[1]

df.columns = columns
```

[4]:
```
print(f"Number of columns in df: {df.shape[1]}")  # Should be 128
print(f"Number of column names assigned: {len(columns)}")  # Should be 128
```

```
Number of columns in df: 128
Number of column names assigned: 128
```

[5]:
```
# Replaced "?" with NaN and dropped columns that are entirely NaN (except␣
  ↪communityname)
communityname = df['communityname']

df.loc[:, df.columns != "communityname"] = df.loc[:, df.columns !=␣
  ↪"communityname"].replace("?", pd.NA)
df = df.dropna(axis=1, how="all")
df.isna().sum()
```

[5]:
```
state                     0
county                 1174
community              1177
communityname             0
fold                      0
                       …
LemasPctPolicOnPatr    1675
LemasGangUnitDeploy    1675
LemasPctOfficDrugUn       0
PolicBudgPerPop        1675
ViolentCrimesPerPop       0
Length: 128, dtype: int64
```

[6]:
```
threshold = len(df) * 0.5 # 50% of dataset size
df.dropna(axis = 1, thresh = threshold, inplace = True)
df.shape[1]
```

[6]: 104

[7]:
```
numeric_cols = df.columns.drop(['communityname'])  # Excluding column 3
df[numeric_cols] = df[numeric_cols].apply(pd.to_numeric, errors="coerce")
df[numeric_cols] = df[numeric_cols].fillna(df[numeric_cols].mean())  # Filling␣
↪missing values with mean

df['communityname'] = communityname
```

[8]:
```
df.dtypes
```

[8]:
```
state                    int64
communityname           object
fold                     int64
population             float64
householdsize          float64
                        …
LandArea               float64
PopDens                float64
PctUsePubTrans         float64
LemasPctOfficDrugUn    float64
ViolentCrimesPerPop    float64
Length: 104, dtype: object
```

[9]:
```
df.isna().sum().sort_values(ascending=False).head(10)
```

[9]:
```
state               0
communityname       0
PctVacMore6Mos      0
PctVacantBoarded    0
PctHousOwnOcc       0
PctHousOccup        0
HousVacant          0
MedNumBR            0
PctHousLess3BR      0
PctPersDenseHous    0
dtype: int64
```

[10]:
```
df.head()
```

[10]:
```
   state         communityname  fold  population  householdsize  racepctblack  \
0      8          Lakewoodcity     1        0.19           0.33          0.02
1     53           Tukwilacity     1        0.00           0.16          0.12
2     24          Aberdeentown     1        0.00           0.42          0.49
3     34  Willingborotownship     1        0.04           0.77          1.00
4     42     Bethlehemtownship     1        0.01           0.55          0.02
```

```
   racePctWhite  racePctAsian  racePctHisp  agePct12t21  …  PctForeignBorn  \
0          0.90          0.12         0.17         0.34  …            0.12
1          0.74          0.45         0.07         0.26  …            0.21
2          0.56          0.17         0.04         0.39  …            0.14
3          0.08          0.12         0.10         0.51  …            0.19
4          0.95          0.09         0.05         0.38  …            0.11

   PctBornSameState  PctSameHouse85  PctSameCity85  PctSameState85  LandArea  \
0              0.42            0.50           0.51            0.64      0.12
1              0.50            0.34           0.60            0.52      0.02
2              0.49            0.54           0.67            0.56      0.01
3              0.30            0.73           0.64            0.65      0.02
4              0.72            0.64           0.61            0.53      0.04

   PopDens  PctUsePubTrans  LemasPctOfficDrugUn  ViolentCrimesPerPop
0     0.26            0.20                 0.32                 0.20
1     0.12            0.45                 0.00                 0.67
2     0.21            0.02                 0.00                 0.43
3     0.39            0.28                 0.00                 0.12
4     0.09            0.02                 0.00                 0.03

[5 rows x 104 columns]
```

[11]:
```python
X = df.drop(['communityname', 'ViolentCrimesPerPop'], axis = 1) # Features
y = df['ViolentCrimesPerPop'] # Target: violent crime rate
```

[12]:
```python
X.shape, y.shape
```

[12]: ((1994, 102), (1994,))

[13]:
```python
# 4. Split data
X_train, X_temp, y_train, y_temp = train_test_split(X, y, test_size=0.4,
 ↪random_state=42)
X_val, X_test, y_val, y_test = train_test_split(X_temp, y_temp, test_size=0.5,
 ↪random_state=42)

# 5. Standardize features
scaler = StandardScaler()
X_train_scaled = pd.DataFrame(scaler.fit_transform(X_train), columns=X_train.
 ↪columns)
X_val_scaled = pd.DataFrame(scaler.transform(X_val), columns=X_val.columns)
X_test_scaled = pd.DataFrame(scaler.transform(X_test), columns=X_test.columns)

X_train.shape, X_val.shape, X_test.shape
```

[13]: ((1196, 102), (399, 102), (399, 102))

[14]:
```python
non_numeric_cols = X_train.select_dtypes(include = ['object']).columns
non_numeric_cols
```

[14]:
```
Index([], dtype='object')
```

[15]:
```python
X_train = X_train.drop(columns = non_numeric_cols)
X_val = X_val.drop(columns = non_numeric_cols)
X_test = X_test.drop(columns = non_numeric_cols)
```

(a) What are the most important features?

i. Compare and contrast the top features as determined by:

[16]:
```python
# OLS
ols = LinearRegression()
ols.fit(X_train, y_train)

importance = abs(ols.coef_)

feature_ranking = pd.Series(importance, index = X_train.columns).
 ↪sort_values(ascending = False)

print("Top 10 OLS Features:")
feature_ranking.head(10)
```

```
Top 10 OLS Features:
```

[16]:
```
PctHousLess3BR       0.892275
PctVacMore6Mos       0.725661
blackPerCap          0.368323
pctUrban             0.359191
PersPerOwnOccHous    0.317947
MedRent              0.295365
NumImmig             0.291203
FemalePctDiv         0.281053
RentMedian           0.273867
MedNumBR             0.263495
dtype: float64
```

[17]:
```python
# RFE (for stepwise approach)
rfe = RFE(estimator=LinearRegression(), n_features_to_select=10)
rfe.fit(X_train, y_train)
rfe_importance = pd.Series(rfe.ranking_, index=X_train.columns)
rfe_selected = rfe_importance[rfe_importance == 1].index

print("\nFeatures selected by RFE:")
rfe_selected
```

```
Features selected by RFE:
```

[17]:
```
Index(['population', 'racepctblack', 'pctUrban', 'whitePerCap', 'PctBSorMore',
       'PctUnemployed', 'PctWorkMomYoungKids', 'MedNumBR', 'PctHousOwnOcc',
       'NumStreet'],
      dtype='object')
```

[18]:
```python
# Lasso
from sklearn.linear_model import LassoCV

lasso = LassoCV(cv=50, random_state=42)
lasso.fit(X_train, y_train)
lasso_importance = pd.Series(abs(lasso.coef_), index=X_train.columns)
lasso_features = lasso_importance.sort_values(ascending=False)

print("Top 10 Lasso features:")
lasso_features.head(10)
```

```
Top 10 Lasso features:
```

[18]:
```
PctImmigRecent         0.203366
racepctblack           0.171315
FemalePctDiv           0.153681
NumStreet              0.137228
PctWorkMomYoungKids    0.113342
PctHousOwnOcc          0.077554
MedNumBR               0.071715
MedYrHousBuilt         0.064361
PctTeen2Par            0.061865
PctVacantBoarded       0.061485
dtype: float64
```

[19]:
```python
# Elastic Net
elastic_net = ElasticNetCV(cv=5, random_state=42)
elastic_net.fit(X_train, y_train)
elastic_importance = pd.Series(abs(elastic_net.coef_), index=X_train.columns)
elastic_features = elastic_importance.sort_values(ascending=False)

print("\nTop 10 Elastic Net features:")
elastic_features.head(10)
```

```

Top 10 Elastic Net features:
```

[19]:
```
PctImmigRecent         0.189986
racepctblack           0.158529
FemalePctDiv           0.143410
```

```
NumStreet              0.126532
PctWorkMomYoungKids    0.104400
PctHousOwnOcc          0.080582
MedYrHousBuilt         0.066296
MedNumBR               0.065641
PctVacantBoarded       0.060143
PctTeen2Par            0.056994
dtype: float64
```

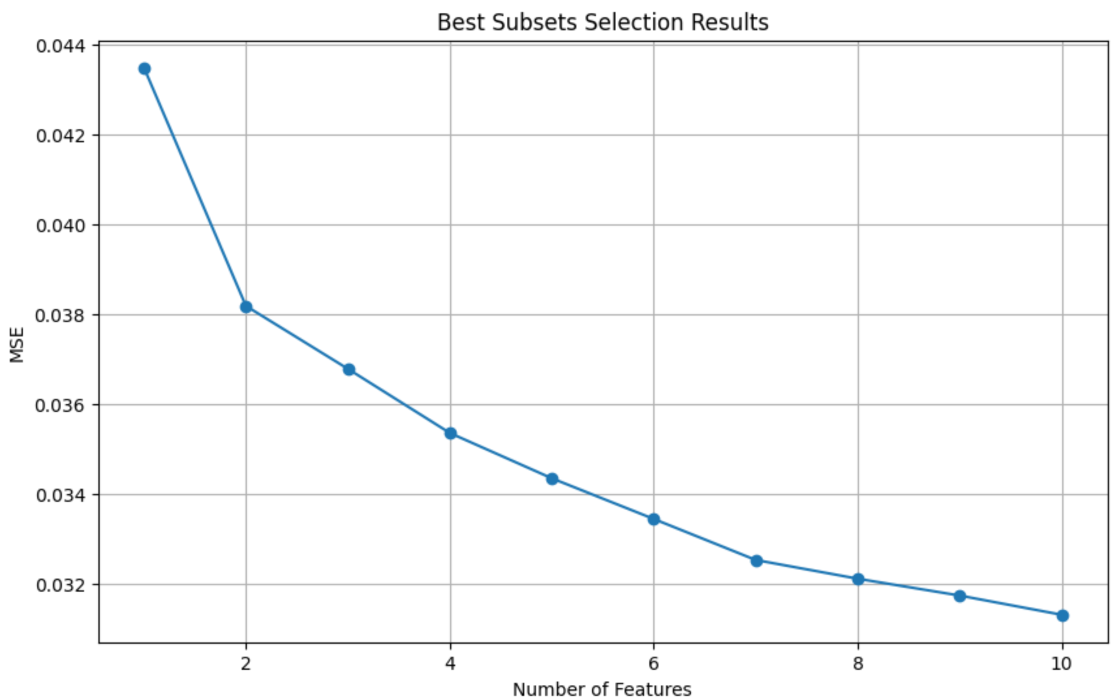
[20]:

```python
def best_subset_selection(X, y, max_features=10):
    # 1. First, pre-screen features based on correlation with target
    correlations = abs(X.corrwith(pd.Series(y)))
    top_features = correlations.sort_values(ascending=False).head(20).index
    X_reduced = X[top_features]

    n_features = len(top_features)
    best_models = {}

    # 2. Now do best subsets on reduced feature set
    for k in range(1, min(max_features + 1, n_features + 1)):
        print(f"\nTesting {k} features...")
        best_score = float('inf')
        best_combo = None

        # Try combinations from pre-screened features
        for combo in combinations(range(n_features), k):
            X_subset = X_reduced.iloc[:, list(combo)]

            # 3. Use simple train/validation split instead of full CV
            model = LinearRegression()
            model.fit(X_subset, y)
            mse = mean_squared_error(y, model.predict(X_subset))

            if mse < best_score:
                best_score = mse
                best_combo = combo

        selected_features = [X_reduced.columns[i] for i in best_combo]
        best_models[k] = {
            'features': selected_features,
            'score': best_score
        }

        print(f"Best {k} features: {selected_features}")
        print(f"MSE: {best_score:.4f}")

    return best_models
```

```python
# Run faster version
print("Starting Best Subsets Selection...")
best_models = best_subset_selection(X_train_scaled, y_train, max_features=10)

# Visualize results
plt.figure(figsize=(10, 6))
mses = [model['score'] for model in best_models.values()]
plt.plot(range(1, len(mses) + 1), mses, marker='o')
plt.xlabel('Number of Features')
plt.ylabel('MSE')
plt.title('Best Subsets Selection Results')
plt.grid(True)
plt.show()
```

```
Starting Best Subsets Selection…

Testing 1 features…
Best 1 features: ['PctEmplManu']
MSE: 0.0435

Testing 2 features…
Best 2 features: ['PctEmplManu', 'pctUrban']
MSE: 0.0382

Testing 3 features…
Best 3 features: ['perCapInc', 'PctEmplManu', 'pctUrban']
MSE: 0.0368

Testing 4 features…
Best 4 features: ['MedRent', 'pctWWage', 'PctEmplManu', 'pctUrban']
MSE: 0.0354

Testing 5 features…
Best 5 features: ['MedRent', 'medIncome', 'pctWWage', 'PctEmplManu',
'population']
MSE: 0.0343

Testing 6 features…
Best 6 features: ['medFamInc', 'medIncome', 'perCapInc', 'PctEmplManu',
'pctWFarmSelf', 'population']
MSE: 0.0334

Testing 7 features…
Best 7 features: ['MedRent', 'medFamInc', 'medIncome', 'perCapInc',
'PctEmplManu', 'pctWFarmSelf', 'population']
MSE: 0.0325
```

```
Testing 8 features…
Best 8 features: ['MedRent', 'medFamInc', 'medIncome', 'perCapInc',
'PctEmplManu', 'PctUsePubTrans', 'pctWFarmSelf', 'population']
MSE: 0.0321

Testing 9 features…
Best 9 features: ['MedRent', 'medFamInc', 'medIncome', 'perCapInc',
'PctEmplManu', 'RentMedian', 'PctUsePubTrans', 'pctWFarmSelf', 'population']
MSE: 0.0317

Testing 10 features…
Best 10 features: ['MedRent', 'medFamInc', 'medIncome', 'perCapInc',
'PctEmplManu', 'pctWPubAsst', 'RentMedian', 'PctUsePubTrans', 'pctWFarmSelf',
'population']
MSE: 0.0313
```

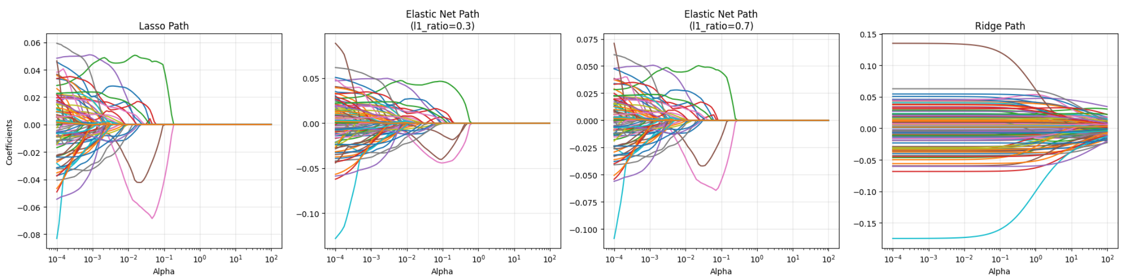


ii. Fit and visualize regularization paths for the following methods:

[21]:
```python
# Create range of alphas for regularization path
alphas = np.logspace(-4, 2, 100)

# Store coefficients for each path
lasso_coefs = []
elastic_coefs1 = []
```

```python
elastic_coefs2 = []
ridge_coefs = []

# Calculate paths with increased max_iter and better convergence settings
for alpha in alphas:
    # Lasso with increased iterations
    lasso = Lasso(alpha=alpha, max_iter=5000, tol=1e-4)
    lasso.fit(X_train_scaled, y_train)
    lasso_coefs.append(lasso.coef_)

    # Elastic Net with l1_ratio = 0.3
    elastic1 = ElasticNet(alpha=alpha, l1_ratio=0.3, max_iter=5000, tol=1e-4)
    elastic1.fit(X_train_scaled, y_train)
    elastic_coefs1.append(elastic1.coef_)

    # Elastic Net with l1_ratio = 0.7
    elastic2 = ElasticNet(alpha=alpha, l1_ratio=0.7, max_iter=5000, tol=1e-4)
    elastic2.fit(X_train_scaled, y_train)
    elastic_coefs2.append(elastic2.coef_)

    # Ridge
    ridge = Ridge(alpha=alpha)
    ridge.fit(X_train_scaled, y_train)
    ridge_coefs.append(ridge.coef_)

# Convert to arrays
lasso_coefs = np.array(lasso_coefs)
elastic_coefs1 = np.array(elastic_coefs1)
elastic_coefs2 = np.array(elastic_coefs2)
ridge_coefs = np.array(ridge_coefs)

# Create visualization with improved styling
plt.figure(figsize=(20, 5))

# Lasso path
plt.subplot(141)
plt.semilogx(alphas, lasso_coefs)
plt.xlabel('Alpha')
plt.ylabel('Coefficients')
plt.title('Lasso Path')
plt.grid(True, alpha=0.3)

# Elastic Net path (l1_ratio=0.3)
plt.subplot(142)
plt.semilogx(alphas, elastic_coefs1)
plt.xlabel('Alpha')
plt.title('Elastic Net Path\n(l1_ratio=0.3)')
```

```python
plt.grid(True, alpha=0.3)

# Elastic Net path (l1_ratio=0.7)
plt.subplot(143)
plt.semilogx(alphas, elastic_coefs2)
plt.xlabel('Alpha')
plt.title('Elastic Net Path\n(l1_ratio=0.7)')
plt.grid(True, alpha=0.3)

# Ridge path
plt.subplot(144)
plt.semilogx(alphas, ridge_coefs)
plt.xlabel('Alpha')
plt.title('Ridge Path')
plt.grid(True, alpha=0.3)

plt.tight_layout()
plt.show()
```



b. Which linear method is best for prediction?

(i) Compare the average prediction MSE on the test set for the following methods:

[22]:
```python
def best_subset_selection_for_prediction(X_train, y_train, X_val, y_val, X_test, y_test):
    max_features = 5  # Keep small for computational ease
    best_val_error = float('inf')
    best_test_error = None

    # Correlations
    correlations = np.array([abs(np.corrcoef(X_train[:, i], y_train)[0, 1])
                             for i in range(X_train.shape[1])])

    # Indices of top correlated features
    top_feature_indices = np.argsort(correlations)[-15:]  # Get top 15 features

    for k in range(1, max_features + 1):
```

```python
        print(f"Testing {k} features...")
        best_combo_error = float('inf')
        best_combo = None

        # Combinations from top features
        for combo in combinations(top_feature_indices, k):
            # Fit model
            model = LinearRegression()
            model.fit(X_train[:, list(combo)], y_train)

            # Validate
            val_pred = model.predict(X_val[:, list(combo)])
            val_error = mean_squared_error(y_val, val_pred)

            if val_error < best_val_error:
                best_val_error = val_error
                test_pred = model.predict(X_test[:, list(combo)])
                best_test_error = mean_squared_error(y_test, test_pred)

    return best_test_error

def evaluate_models(X, y, n_trials=10):
    results = {
        'Least Squares': [],
        'Ridge': [],
        'Lasso': [],
        'Elastic Net': [],
        'RFE': [],
        'Best Subsets': []
    }

    for trial in range(n_trials):
        print(f"Trial {trial + 1}/{n_trials}")

        # Split data
        X_temp, X_test, y_temp, y_test = train_test_split(X, y, test_size=0.2,␣
↪random_state=trial)
        X_train, X_val, y_train, y_val = train_test_split(X_temp, y_temp,␣
↪test_size=0.25, random_state=trial)

        # Scale features
        scaler = StandardScaler()
        X_train_scaled = scaler.fit_transform(X_train)
        X_val_scaled = scaler.transform(X_val)
        X_test_scaled = scaler.transform(X_test)

        # 1. Least Squares
```

```
        lr = LinearRegression()
        lr.fit(X_train_scaled, y_train)
        y_pred = lr.predict(X_test_scaled)
        results['Least Squares'].append(mean_squared_error(y_test, y_pred))

        # 2. Ridge
        ridge_cv = RidgeCV(alphas=np.logspace(-4, 4, 100))
        ridge_cv.fit(X_train_scaled, y_train)
        y_pred = ridge_cv.predict(X_test_scaled)
        results['Ridge'].append(mean_squared_error(y_test, y_pred))

        # 3. Lasso
        lasso_cv = LassoCV(alphas=np.logspace(-4, 4, 100), max_iter=10000)
        lasso_cv.fit(X_train_scaled, y_train)
        y_pred = lasso_cv.predict(X_test_scaled)
        results['Lasso'].append(mean_squared_error(y_test, y_pred))

        # 4. Elastic Net
        enet_cv = ElasticNetCV(alphas=np.logspace(-4, 4, 20),
                               l1_ratio=[.1, .3, .5, .7, .9],
                               max_iter=10000)
        enet_cv.fit(X_train_scaled, y_train)
        y_pred = enet_cv.predict(X_test_scaled)
        results['Elastic Net'].append(mean_squared_error(y_test, y_pred))

        # 5. RFE
        rfe = RFE(estimator=LinearRegression(), n_features_to_select=10)
        rfe.fit(X_train_scaled, y_train)
        y_pred = rfe.predict(X_test_scaled)
        results['RFE'].append(mean_squared_error(y_test, y_pred))

        # 6. Best Subsets
        results['Best Subsets'].append(best_subset_selection_for_prediction(
            X_train_scaled, y_train, X_val_scaled, y_val, X_test_scaled,␣
↪y_test))

    return results

# Evaluation
results = evaluate_models(X, y, n_trials=10)

# Average results
avg_results = {method: round(float(np.mean(scores)), 5) for method, scores in␣
↪results.items()}
std_results = {method: round(float(np.std(scores)), 5) for method, scores in␣
↪results.items()}
```

```
avg_results, std_results
```

```
Trial 1/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 2/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 3/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 4/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 5/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 6/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 7/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 8/10
Testing 1 features…
Testing 2 features…
```

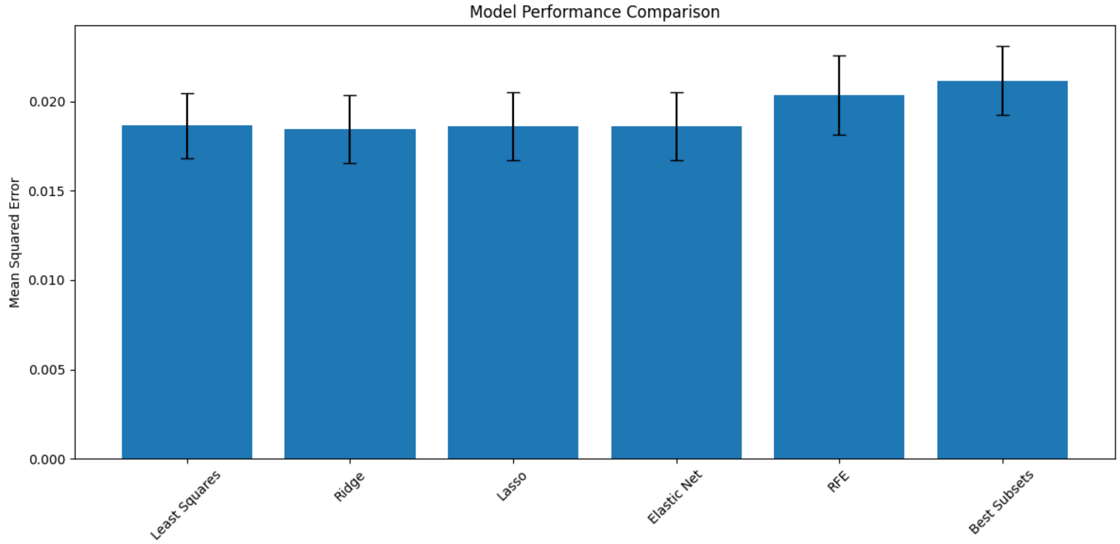
```
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 9/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
Trial 10/10
Testing 1 features…
Testing 2 features…
Testing 3 features…
Testing 4 features…
Testing 5 features…
```

[22]:
```
({'Least Squares': 0.01865,
  'Ridge': 0.01845,
  'Lasso': 0.01859,
  'Elastic Net': 0.01862,
  'RFE': 0.02033,
  'Best Subsets': 0.02116},
 {'Least Squares': 0.00182,
  'Ridge': 0.00191,
  'Lasso': 0.0019,
  'Elastic Net': 0.0019,
  'RFE': 0.00221,
  'Best Subsets': 0.00192})
```

(ii) Visualize the comparison results

[23]:
```
# Create visualization
plt.figure(figsize=(12, 6))
methods = list(avg_results.keys())
means = list(avg_results.values())
stds = list(std_results.values())

plt.bar(methods, means, yerr=stds, capsize=5)
plt.xticks(rotation=45)
plt.ylabel('Mean Squared Error')
plt.title('Model Performance Comparison')
plt.tight_layout()
plt.show()

# Numerical results
for method in methods:
    print(f"{method}:")
    print(f"  Mean MSE: {avg_results[method]:.4f}")
```

```
    print(f"  Std MSE: {std_results[method]:.4f}")
```




```
Least Squares:
  Mean MSE: 0.0186
  Std MSE: 0.0018
Ridge:
  Mean MSE: 0.0185
  Std MSE: 0.0019
Lasso:
  Mean MSE: 0.0186
  Std MSE: 0.0019
Elastic Net:
  Mean MSE: 0.0186
  Std MSE: 0.0019
RFE:
  Mean MSE: 0.0203
  Std MSE: 0.0022
Best Subsets:
  Mean MSE: 0.0212
  Std MSE: 0.0019
```

[ ]:

```

```