Relationship between Household Electricity Consumption and Surface Air Temperature in Singapore

Lecturer: Ms. Catherine Khaw

Table of Contents

1. Introduction 1

2. Hypothesis 1

3. Data Sources 1

4. Analysis of Input Series 2

4.1 Pre-whitening of Input Series 2

5. Analysis of Output Series 3

5.1 Pre-whitening of Output Series 3

5.2 Verification of white noise residuals on Input Series 6

6. Transfer Function Modelling 7

6.1 Cross-correlation between Pre-Whitened Series 7

6.2 Transfer Function Model Parameters 8

6.3 Transfer Function Model Evaluation 8

7. Relationship between Household Electricity Consumption and Mean Surface Air Temperature 10

8. Conclusion 10

9. References 11

1. Introduction

According to the National Environment Agency (NEA), Singapore's household electricity consumption has increased by about 17 per cent over the past decade [1]. A 2017 NEA survey of 550 households found that air-conditioners were largely to blame, accounting for about 24 per cent of the consumption of a typical home [1]. However, it remains unknown if households currently use their air-conditioners out of habit (e.g. turning on the air-conditioner every day), or when the need arises. In addition, when referenced against the surface air temperature, there might be a delay in turning on the air-conditioner as our sensation of heat depends not only on temperature, but other factors such as humidity, cloud cover, sun intensity and wind [2]. Hence, the intent of this study is to investigate if there is a relationship between household electricity consumption and the surface air temperature in Singapore.

2. Hypothesis

This study hypothesises that:

1. Household Electricity Consumption in Singapore is influenced by the Surface Air Temperature of Singapore
2. There are no appreciable lags between the two variables

3. Data Sources

Table 1: Data Sources

Variable	Frequency	Source	URL
Mean Surface Air Temperature	Monthly	National Environment Authority	https://data.gov.sg/dataset/surface-air-temperature-monthly-mean
Household Electricity Consumption	Monthly	Energy Market Authority	https://data.gov.sg/dataset/monthly-electricity-consumption-by-sector-total

To perform analysis on these variables, data sources with the same time resolution was first obtained from data.gov.sg. The Mean Surface Air Temperature was obtained from the "Surface Air Temperature - Monthly Mean" dataset published by NEA, whereas the Household Electricity Consumption was obtained from the "Monthly Electricity Consumption by Sector" dataset published by the Energy Market Authority (EMA) of Singapore.

It was noted that "Monthly Household Electricity Consumption" records are only available till September 2015. Hence, this study utilised data from October 2010 to September 2015 (5 years), providing a total of 60 data points for analysis.

4. Analysis of Input Series

4.1 Pre-whitening of Input Series

Based on our hypothesis indicated in section 2, the input series was taken to be Mean Surface Air Temperature. Initial inspection of the data reveals that the data was stationary. However, based on the time series and the Autocorrelation Function (ACF) plots, a 12-month seasonality was observed. This is consistent with the knowledge that weather in Singapore follows a 12-months cycle, largely characterised by two monsoon seasons – the Northeast Monsoon (December to early March) and the Southwest Monsoon (June to September) [3]. As such, the first step is to apply a first order seasonal differencing of 12 months to the input series.

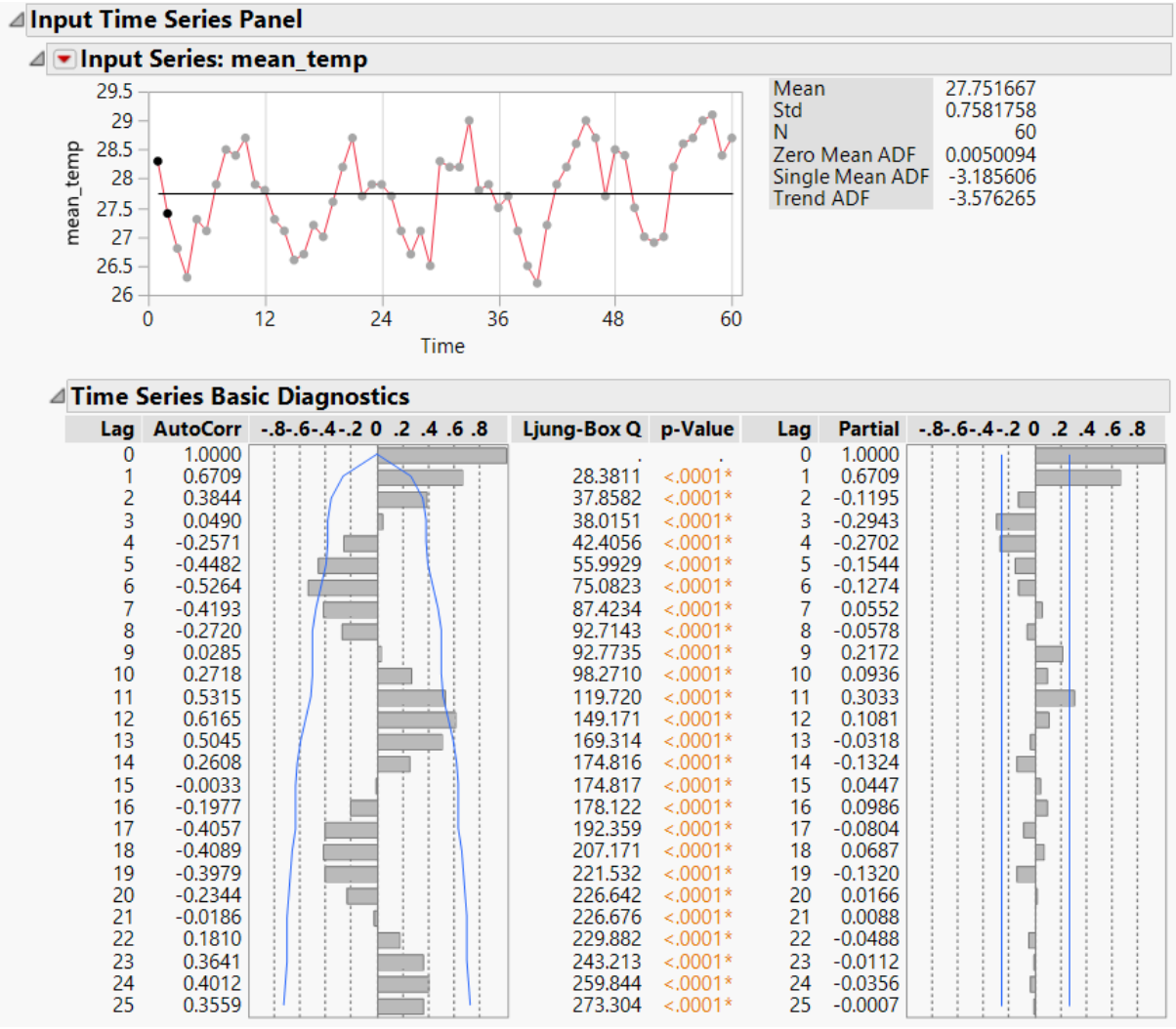
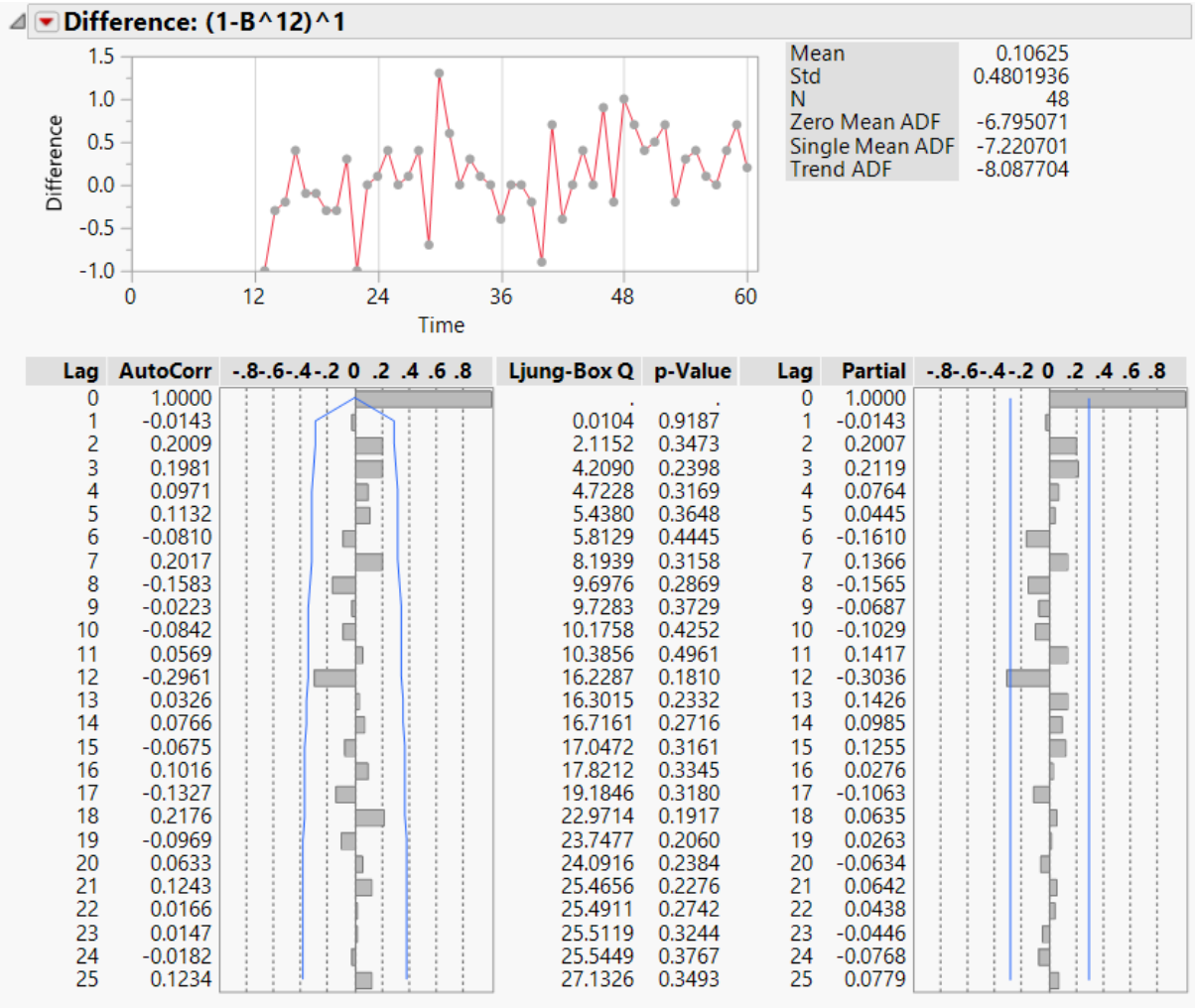


Figure 1: Basis for first order seasonal differencing, season = 12 months



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.0143	0.0104	0.9187	1	-0.0143
2	0.2009	2.1152	0.3473	2	0.2007
3	0.1981	4.2090	0.2398	3	0.2119
4	0.0971	4.7228	0.3169	4	0.0764
5	0.1132	5.4380	0.3648	5	0.0445
6	-0.0810	5.8129	0.4445	6	-0.1610
7	0.2017	8.1939	0.3158	7	0.1366
8	-0.1583	9.6976	0.2869	8	-0.1565
9	-0.0223	9.7283	0.3729	9	-0.0687
10	-0.0842	10.1758	0.4252	10	-0.1029
11	0.0569	10.3856	0.4961	11	0.1417
12	-0.2961	16.2287	0.1810	12	-0.3036
13	0.0326	16.3015	0.2332	13	0.1426
14	0.0766	16.7161	0.2716	14	0.0985
15	-0.0675	17.0472	0.3161	15	0.1255
16	0.1016	17.8212	0.3345	16	0.0276
17	-0.1327	19.1846	0.3180	17	-0.1063
18	0.2176	22.9714	0.1917	18	0.0635
19	-0.0969	23.7477	0.2060	19	0.0263
20	0.0633	24.0916	0.2384	20	-0.0634
21	0.1243	25.4656	0.2276	21	0.0642
22	0.0166	25.4911	0.2742	22	0.0438
23	0.0147	25.5119	0.3244	23	-0.0446
24	-0.0182	25.5449	0.3767	24	-0.0768
25	0.1234	27.1326	0.3493	25	0.0779

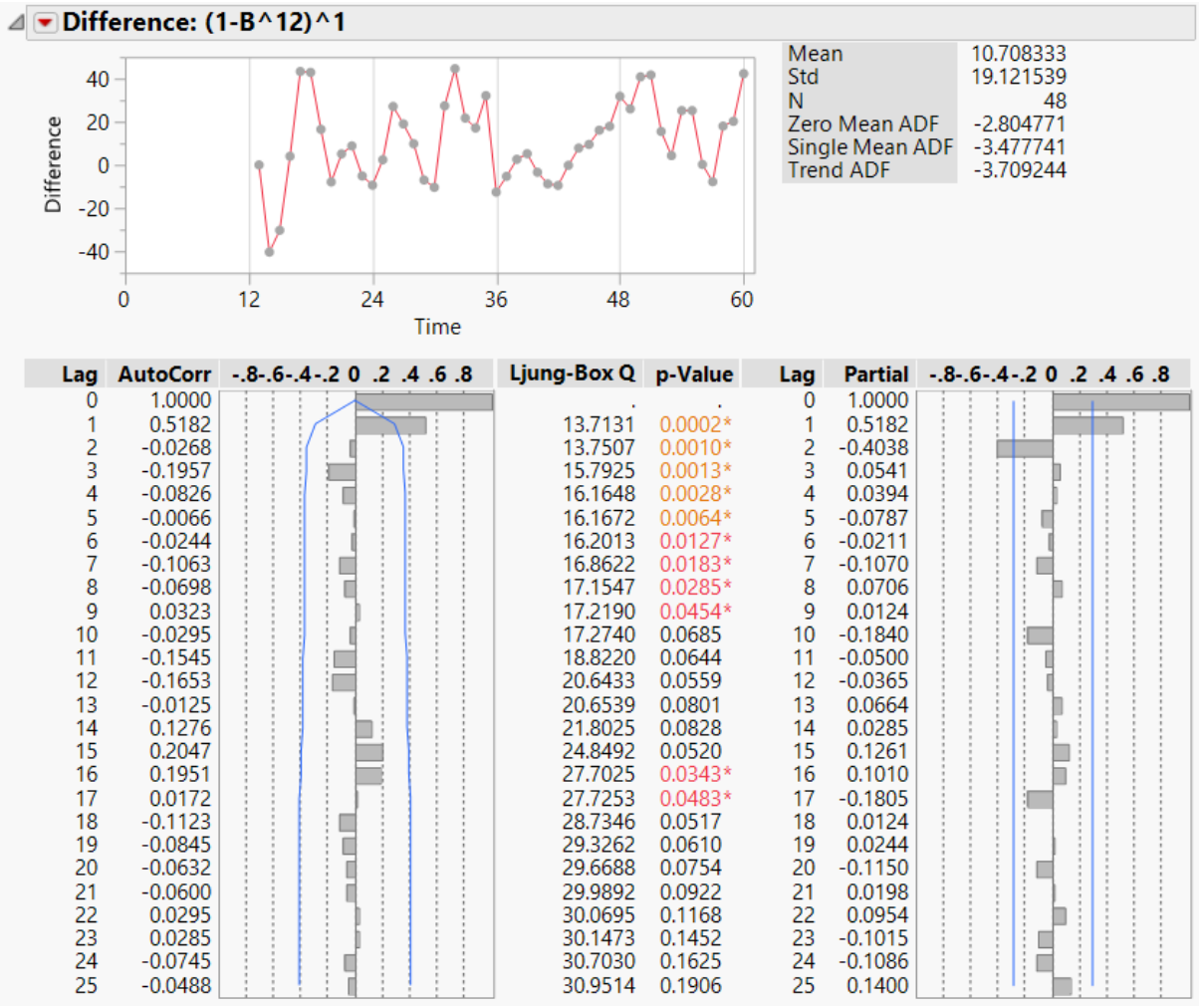
Figure 2: ACF and PACF plots after first order seasonal differencing, season = 12 months

The resulting Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots (Figure 2) indicated that the resulting residuals are white noise. Hence, the Seasonal Difference model built for the input series is taken to be $(0,0,0),(0,1,0)_{12}$ at this stage.

5. Analysis of Output Series

5.1 Pre-whitening of Output Series

With the Seasonal Difference Model for the input series identified in section 4.1, the output series (Household Electricity Consumption) was pre-whitened with the same model. Based on the ACF and PACF plots in Figure 3, it's observed that the PACF plot is dying down, while the ACF plot cuts off at the first lag. As such, a MA(1) term was added to the Seasonal Difference model in an attempt to achieve white noise residuals.



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	0.5182	13.7131	0.0002*	1	0.5182
2	-0.0268	13.7507	0.0010*	2	-0.4038
3	-0.1957	15.7925	0.0013*	3	0.0541
4	-0.0826	16.1648	0.0028*	4	0.0394
5	-0.0066	16.1672	0.0064*	5	-0.0787
6	-0.0244	16.2013	0.0127*	6	-0.0211
7	-0.1063	16.8622	0.0183*	7	-0.1070
8	-0.0698	17.1547	0.0285*	8	0.0706
9	0.0323	17.2190	0.0454*	9	0.0124
10	-0.0295	17.2740	0.0685	10	-0.1840
11	-0.1545	18.8220	0.0644	11	-0.0500
12	-0.1653	20.6433	0.0559	12	-0.0365
13	-0.0125	20.6539	0.0801	13	0.0664
14	0.1276	21.8025	0.0828	14	0.0285
15	0.2047	24.8492	0.0520	15	0.1261
16	0.1951	27.7025	0.0343*	16	0.1010
17	0.0172	27.7253	0.0483*	17	-0.1805
18	-0.1123	28.7346	0.0517	18	0.0124
19	-0.0845	29.3262	0.0610	19	0.0244
20	-0.0632	29.6688	0.0754	20	-0.1150
21	-0.0600	29.9892	0.0922	21	0.0198
22	0.0295	30.0695	0.1168	22	0.0954
23	0.0285	30.1473	0.1452	23	-0.1015
24	-0.0745	30.7030	0.1625	24	-0.1086
25	-0.0488	30.9514	0.1906	25	0.1400

Figure 3: $(0,0,0),(0,1,0)_{12}$ I model applied on output series

From Figure 4, it was observed that the MA(1) term added was significant. The Akaike's 'A' Information Criterion (AIC) was deemed acceptable, and the model managed to accurately model the 60 points to project a forecast that not only repeats the trend, but also has a tight confidence interval. The residuals of this $(0,0,1),(0,1,0)_{12}$ IMA Model has also achieved white noise as evidenced in the ACF and PACF plots in Figure 5. Hence, the IMA model built for the output series is taken to be $(0,0,1),(0,1,0)_{12}$ at this stage.

Model: Seasonal ARIMA(0, 0, 1)(0, 1, 0)12

Model Summary

DF	46	Stable	Yes
Sum of Squared Errors	10830.0678	Invertible	Yes
Variance Estimate	235.436256		
Standard Deviation	15.3439322		
Akaike's 'A' Information Criterion	400.969736		
Schwarz's Bayesian Criterion	404.712139		
RSquare	0.90200383		
RSquare Adj	0.89987347		
MAPE	2.05353933		
MAE	11.5086422		
-2LogLikelihood	396.969736		

Parameter Estimates

Term	Factor	Lag	Estimate	Std Error	t Ratio	Prob>|t|
MA1,1	1	1	-0.68959	0.099304	-6.94	<.0001*
Intercept	1	0	11.31522	3.635002	3.11	0.0032*

Constant	Mu
Estimate	11.3152241
11.3152241	

Forecast

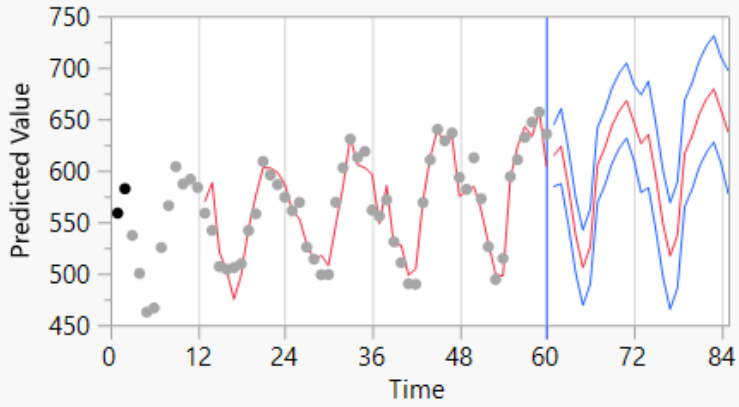


Figure 4: $(0,0,1),(0,1,0)_{12}$ IMA Model (Output Series)

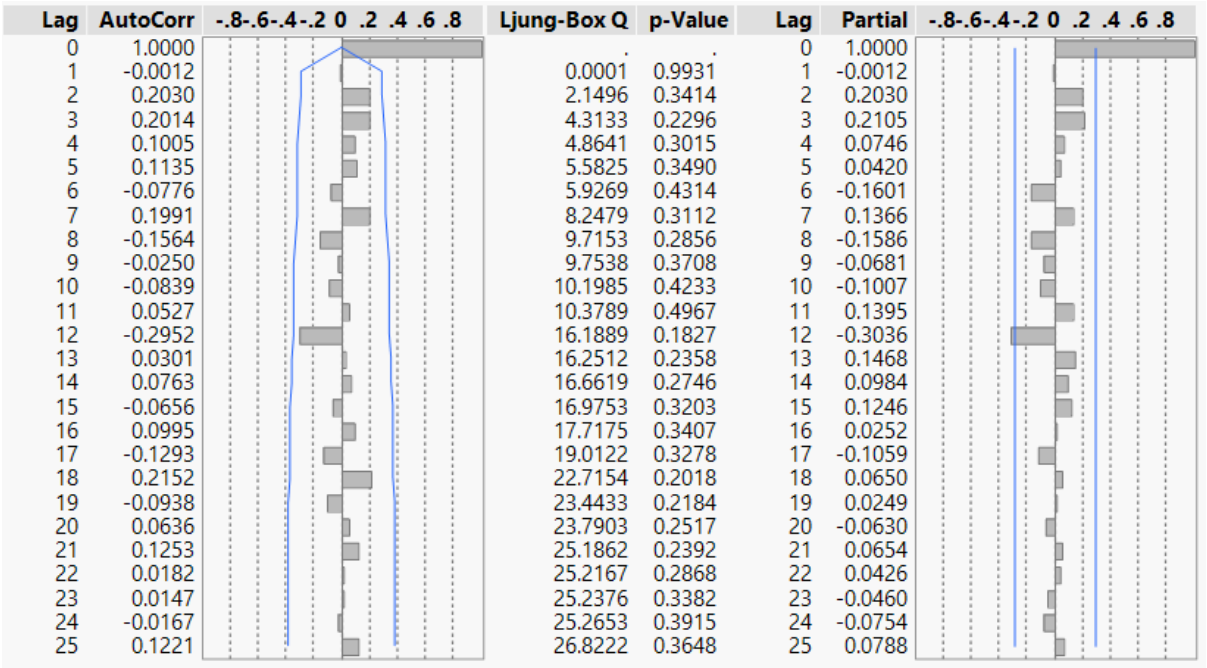
Lag	AutoCorr	-.8 -.6 -.4 -.2 0 .2 .4 .6 .8	Ljung-Box Q	p-Value	Lag	Partial	-.8 -.6 -.4 -.2 0 .2 .4 .6 .8
0	1.0000		.	.	0	1.0000	
1	0.1426		1.0382	0.3083	1	0.1426	
2	0.0185		1.0560	0.5898	2	-0.0019	
3	-0.2256		3.7700	0.2874	3	-0.2327	
4	0.0072		3.7728	0.4376	4	0.0776	
5	-0.0487		3.9050	0.5632	5	-0.0541	
6	0.0567		4.0890	0.6646	6	0.0176	
7	-0.0794		4.4578	0.7258	7	-0.0745	
8	-0.0833		4.8742	0.7709	8	-0.0930	
9	0.0914		5.3882	0.7992	9	0.1598	
10	-0.0086		5.3928	0.8634	10	-0.0975	
11	-0.0421		5.5080	0.9041	11	-0.0691	
12	-0.1743		7.5328	0.8205	12	-0.1132	
13	0.0723		7.8916	0.8506	13	0.1039	
14	0.0516		8.0796	0.8851	14	0.0353	
15	0.1429		9.5638	0.8462	15	0.0252	
16	0.1434		11.1050	0.8030	16	0.1998	
17	-0.0105		11.1136	0.8506	17	-0.0690	
18	-0.1121		12.1196	0.8410	18	-0.0845	
19	-0.0542		12.3630	0.8695	19	0.0136	
20	-0.0310		12.4455	0.8999	20	-0.0505	
21	-0.0903		13.1700	0.9026	21	-0.0570	
22	0.0551		13.4498	0.9199	22	0.0448	
23	0.0409		13.6101	0.9375	23	0.0502	
24	-0.0897		14.4156	0.9367	24	-0.1687	
25	-0.0580		14.7664	0.9467	25	-0.0139	

Figure 5: ACF & PACF Plots for $(0,0,1),(0,1,0)_{12}$ IMA Model (Output Series)

5.2 Verification of white noise residuals on Input Series

Since the input series was previously pre-whitened with a $(0,0,0),(0,1,0)_{12}$ Seasonal Difference model, the IMA model $(0,0,1),(0,1,0)_{12}$ developed for the output series was applied to the input series to verify that pre-whitening the input series with the output IMA model would still result in white noise residuals.

Based on Figure 6, it was observed that the residuals remained as white noise. In addition, it was observed that the additional MA(1) term added is not significant. This was assessed to be reasonable, as the original input series didn't require this MA(1) term during the previous pre-whitening stage to achieve white noise. Hence, the final model chosen is a $(0,0,1),(0,1,0)_{12}$ IMA Model.



Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	-0.0012	0.0001	0.9931	1	-0.0012
2	0.2030	2.1496	0.3414	2	0.2030
3	0.2014	4.3133	0.2296	3	0.2105
4	0.1005	4.8641	0.3015	4	0.0746
5	0.1135	5.5825	0.3490	5	0.0420
6	-0.0776	5.9269	0.4314	6	-0.1601
7	0.1991	8.2479	0.3112	7	0.1366
8	-0.1564	9.7153	0.2856	8	-0.1586
9	-0.0250	9.7538	0.3708	9	-0.0681
10	-0.0839	10.1985	0.4233	10	-0.1007
11	0.0527	10.3789	0.4967	11	0.1395
12	-0.2952	16.1889	0.1827	12	-0.3036
13	0.0301	16.2512	0.2358	13	0.1468
14	0.0763	16.6619	0.2746	14	0.0984
15	-0.0656	16.9753	0.3203	15	0.1246
16	0.0995	17.7175	0.3407	16	0.0252
17	-0.1293	19.0122	0.3278	17	-0.1059
18	0.2152	22.7154	0.2018	18	0.0650
19	-0.0938	23.4433	0.2184	19	0.0249
20	0.0636	23.7903	0.2517	20	-0.0630
21	0.1253	25.1862	0.2392	21	0.0654
22	0.0182	25.2167	0.2868	22	0.0426
23	0.0147	25.2376	0.3382	23	-0.0460
24	-0.0167	25.2653	0.3915	24	-0.0754
25	0.1221	26.8222	0.3648	25	0.0788

Figure 6: ACF & PACF Plots for $(0,0,1),(0,1,0)_{12}$ IMA Model (Input Series)

Model: Seasonal ARIMA(0, 0, 1)(0, 1, 0)12

Model Summary

DF	46	Stable	Yes
Sum of Squared Errors	11.0663755	Invertible	Yes
Variance Estimate	0.24057338		
Standard Deviation	0.4904828		
Akaike's 'A' Information Criterion	69.7883104		
Schwarz's Bayesian Criterion	73.5307124		
RSquare	0.61156191		
RSquare Adj	0.6031176		
MAPE	1.33930082		
MAE	0.37025556		
-2LogLikelihood	65.7883104		

Parameter Estimates

Term	Factor	Lag	Estimate	Std Error	t Ratio	Prob>|t|
MA1,1	1	1	0.01087503	0.1236148	0.09	0.9303
Intercept	1	0	0.10647904	0.0686150	1.55	0.1276

Constant Estimate	Mu
0.10647904	0.10647904

Forecast

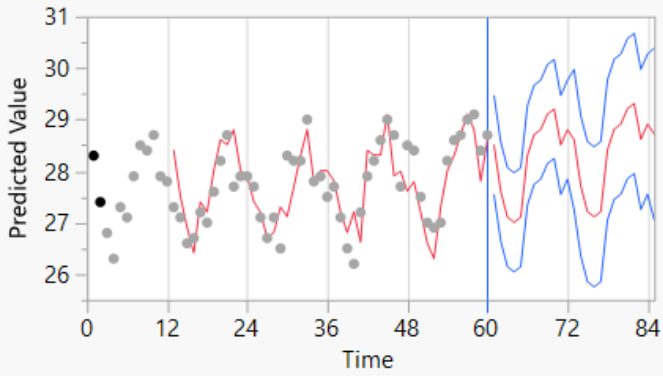


Figure 7: $(0,0,1),(0,1,0)_{12}$ IMA Model (Input Series)

6. Transfer Function Modelling

6.1 Cross-correlation between Pre-Whitened Series

With the $(0,0,1),(0,1,0)_{12}$ IMA model finalised in section 5.2, the cross correlation between the pre-whitened input and output series was computed. From the results shown in Figure 8, significant correlation was observed between lag 0 to 2. Hence, the transfer function model will resemble the following equation:

$$y_t = v_0 x_t + v_1 x_{t-1} + v_2 x_{t-2} + n_t$$

where v_0, v_1 and v_2 are the transfer function weights to be determined.

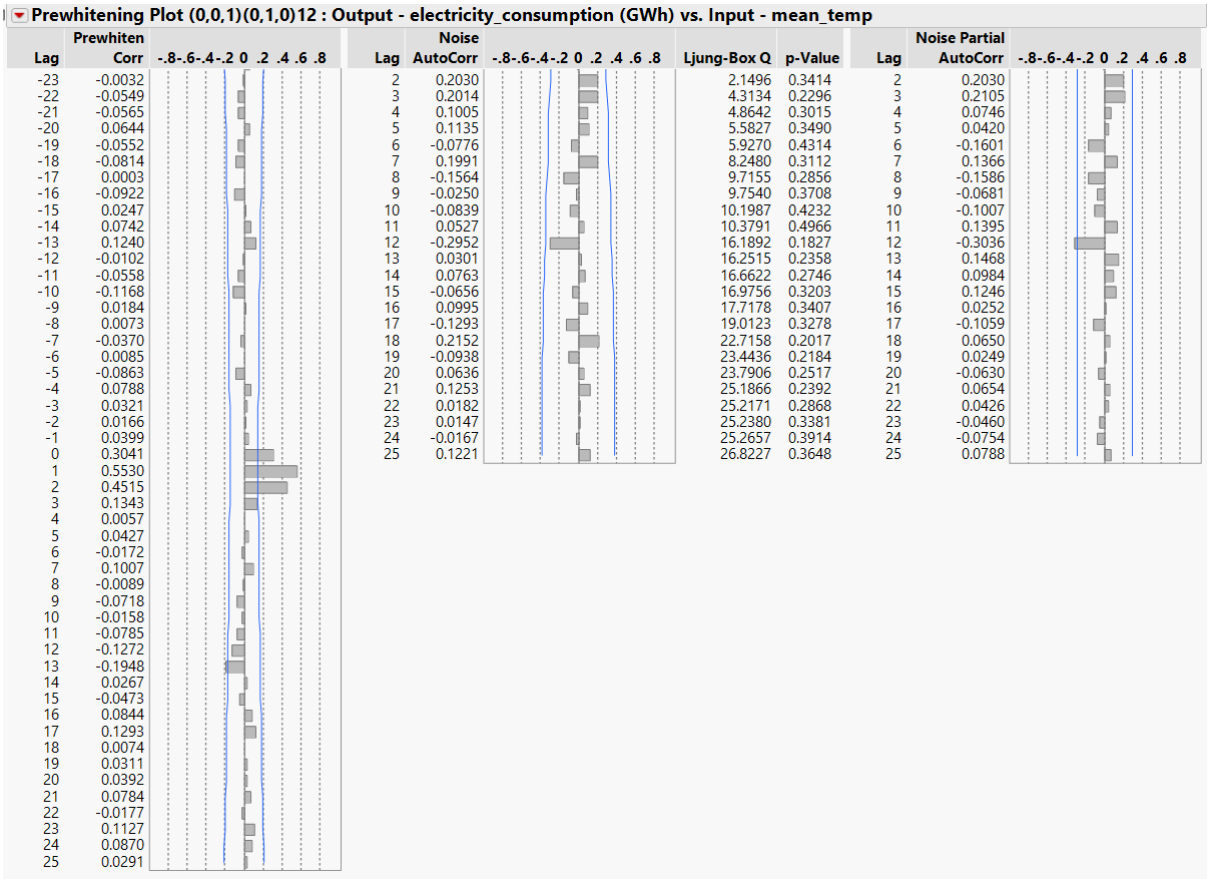


Figure 8: Pre-whitening Plot (0,0,1),(0,1,0)$_{12}$

6.2 Transfer Function Model Parameters

An alternate method to represent the model would be as follows:

$$y_t = v(B)x_t + n_t$$

where

$$v(B) \approx \frac{\omega_0 - \omega_1 + \cdots - \omega_s B^s}{1 - \delta_1 B - \cdots - \delta_r B^r} B^b$$

In order to determine the transfer function model, the parameters *b*, *s* and *r* must first be identified. From Figure 8, it was shown that the first non-zero correlation starts at lag 0, which means that the input and output series are in sync. As such, $b = 0$. Meanwhile, the cross correlation peak was observed at lag 1. Hence, $s = 1$. Lastly, it was noted that the cross correlation seemed to be oscillating about 0 in the pre-whitening plot. Hence, $r = 2$ was selected.

6.3 Transfer Function Model Evaluation

With the transfer function model parameters selected in the previous section, the model was fitted and evaluated. From Figure 9, it was observed that both the AIC and Schwarz's Bayesian Criterion (SBC) are reasonably small, and the parameters used in fitting the models were all

statistically significant. Figure 10 also shows that the residuals of this transfer function model was white noise. Compared with an alternate transfer function model where $r = 1$ ($b = 0$, $s = 1$, $r = 1$), this model was assessed to be better in terms of AIC and SBC. Hence, the final transfer function model parameters selected was ($b = 0$, $s = 1$, $r = 2$).

Transfer Function Model (1)

Model Summary

DF	40
Sum of Squared Errors	8309.61056
Variance Estimate	207.740355
Standard Deviation	14.4132007
Akaike's 'A' Information Criterion	382.559338
Schwarz's Bayesian Criterion	393.531186
RSquare	0.52652841
RSquare Adj	0.47016275
MAPE	6.88709949
MAE	38.1029702
-2LogLikelihood	370.559338

Failed: Cannot Decrease Objective Function

Parameter Estimates

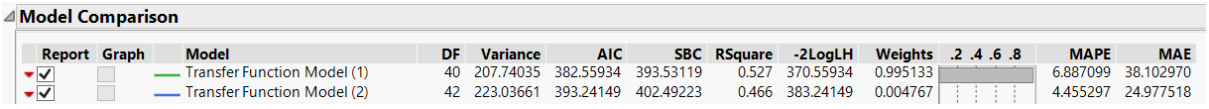
Variable	Term	Factor	Lag	Estimate	Std Error	t Ratio	Prob>|t|
mean_temp	Num0,0	0	0	5.709171	3.2311e-6	1.8e+6	<.0001*
mean_temp	Num1,1	1	1	5.713323	3.2335e-6	1.8e+6	<.0001*
mean_temp	Den1,1	1	1	0.742687	0.001683	441.21	<.0001*
mean_temp	Den1,2	1	2	0.725338	0.002189	331.37	<.0001*
electricity_consumption (GWh)	MA1,1	1	1	-0.789603	0.101560	-7.77	<.0001*
	Intercept	0	0	9.575651	3.842517	2.49	0.0169*

$$\left(1 - B^{12}\right) \bullet \text{electricity_consumption (GWh)}_t = 9.5757 + \left(\frac{\left(5.7092 - 5.7133 \bullet B\right)}{\left(\left(1 - 0.7427 \bullet B\right) - 0.7253 \bullet B^2\right)} \right) \bullet \text{mean_temp}_t + \left(1 + 0.7896 \bullet B\right) \bullet e_t$$

Figure 9: Transfer Function Model Summary ($b = 0$, $s = 1$, $r = 2$)

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	0.0317	0.0512	0.8211	1	0.0317
2	-0.0775	0.3643	0.8335	2	-0.0786
3	-0.1252	1.1998	0.7530	3	-0.1209
4	0.0818	1.5651	0.8150	4	0.0848
5	-0.0773	1.8988	0.8630	5	-0.1039
6	-0.0714	2.1902	0.9014	6	-0.0703
7	-0.1465	3.4459	0.8409	7	-0.1396
8	-0.0780	3.8111	0.8737	8	-0.1174
9	-0.0195	3.8346	0.9220	9	-0.0470
10	-0.1170	4.6987	0.9104	10	-0.1836
11	-0.1419	6.0055	0.8730	11	-0.1881
12	-0.1410	7.3317	0.8349	12	-0.2445
13	0.0902	7.8893	0.8507	13	-0.0710
14	0.0339	7.9705	0.8909	14	-0.1382
15	0.1823	10.3883	0.7946	15	0.0493
16	0.0311	10.4609	0.8415	16	-0.0622
17	0.1514	12.2343	0.7857	17	0.0424
18	-0.0276	12.2953	0.8316	18	-0.1084
19	0.0393	12.4229	0.8668	19	-0.0740
20	-0.0331	12.5168	0.8971	20	-0.0831
21	0.1176	13.7454	0.8803	21	0.0426
22	-0.0711	14.2117	0.8938	22	-0.1107
23	0.0743	14.7413	0.9037	23	0.0868
24	-0.1402	16.7067	0.8609	24	-0.1458
25	-0.1085	17.9352	0.8451	25	-0.0987

Figure 10: ACF & PACF Plot of Transfer Function Model ($b = 0$, $s = 1$, $r = 2$)



Model Comparison

Report	Graph	Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights	.2 .4 .6 .8	MAPE	MAE
☑		Transfer Function Model (1)	40	207.74035	382.55934	393.53119	0.527	370.55934	0.995133		6.887099	38.102970
☑		Transfer Function Model (2)	42	223.03661	393.24149	402.49223	0.466	383.24149	0.004767		4.455297	24.977518

Figure 11: Comparison between Transfer Function Models (*b* = 0, *s* = 1, *r* = 2) and (*b* = 0, *s* = 1, *r* = 1)

7. Relationship between Household Electricity Consumption and Mean Surface Air Temperature

Based on the pre-whitening plot in Figure 8, it was initially concluded that there was no lag between Household Electricity Consumption (*y*) and Mean Surface Air Temperature (*x*). However, expansion of the transfer function model shown in figure 9 yields the following relationship:

$$(y_t - 0.7427y_{t-1} - 0.7253y_{t-2}) - (y_{t-12} - 0.7427y_{t-13} - 0.7253y_{t-14}) = 5.7092x_t - 5.7133x_{t-1} + e_t - 1.5323e_{t-1} - 0.1389e_{t-2} + 0.5723e_{t-3} + 9.5757$$

This shows that while there is no lag between *x* and *y*, past values of both variables also impact y. More importantly, it was observed that the coefficients of each *y* term are the same with itself 12 months ago. Rearranging the above formula gives the following:

$$y_t - 0.7427y_{t-1} - 0.7253y_{t-2} = 5.7092x_t - 5.7133x_{t-1} + e_t - 1.5323e_{t-1} - 0.1389e_{t-2} + 0.5723e_{t-3} + 9.5757 + (y_{t-12} - 0.7427y_{t-13} - 0.7253y_{t-14})$$

This means that while y_{t-1}, y_{t-2}, x_t, x_{t-1}, e_t, e_{t-1}, e_{t-2} and e_{t-3} may explain the variability in *y*, an offset from last year's set of *y* terms is required for it to fully explain y_t. This indicates an upward trend in household electricity consumption, assuming mean surface air temperature remains the same every season.

8. Conclusion

This study hypothesizes that there is an instantaneous unidirectional relationship between the mean monthly surface air temperature of Singapore, and the monthly household electricity consumption in Singapore. While some believe that human behaviour such as being cost conscious or simply the lack of awareness of rising temperatures may influence and possibly delay the decision to switch on the air-conditioner, time series transfer function modelling of these two variables confirms that (1) household electricity consumption is influenced by mean surface air temperature and (2) they are moving in phase. It was further observed that there is a rising trend in the household electricity consumption in Singapore, as household electricity consumption is autocorrelated with its previous season.

9. References

[1] A. Zhaki, “Singapore’s household electricity consumption up 17 per cent over past decade,” *The Straits Times*, Singapore, 05-May-2018.

[2] AccuWeather, “The AccuWeather RealFeel Temperature,” 2011. [Online]. Available: https://www.accuweather.com/en/outdoor-articles/outdoor-living/the-accuweather-realfeel-tempe/55627. [Accessed: 18-Sep-2018].

[3] Meteorological Service Singapore, “Climate of Singapore.” [Online]. Available: http://www.weather.gov.sg/climate-climate-of-singapore/. [Accessed: 18-Sep-2018].