Compression of Biomedical Signals With Mother Wavelet Optimization and Best-Basis Wavelet Packet Selection

Laurent Brechet, Marie-Françoise Lucas, Christian Doncarli, and Dario Farina*

***Abstract*—We propose a novel scheme for signal compression based on the discrete wavelet packet transform (DWPT) decompositon. The mother wavelet and the basis of wavelet packets were optimized and the wavelet coefficients were encoded with a modified version of the embedded zerotree algorithm. This signal dependant compression scheme was designed by a two-step process. The first (internal optimization) was the best basis selection that was performed for a given mother wavelet. For this purpose, three additive cost functions were applied and compared. The second (external optimization) was the selection of the mother wavelet based on the minimal distortion of the decoded signal given a fixed compression ratio. The mother wavelet was parameterized in the multiresolution analysis framework by the scaling filter, which is sufficient to define the entire decomposition in the orthogonal case. The method was tested on two sets of ten electromyographic (EMG) and ten electrocardiographic (ECG) signals that were compressed with compression ratios in the range of 50%–90%. For 90% compression ratio of EMG (ECG) signals, the percent residual difference after compression decreased from (mean ±SD) 48.6±9.9% (21.5±8.4%) with discrete wavelet transform (DWT) using the wavelet leading to poorest performance to 28.4±3.0% (6.7±1.9%) with DWPT, with optimal basis selection and wavelet optimization. In conclusion, best basis selection and optimization of the mother wavelet through parameterization led to substantial improvement of performance in signal compression with respect to DWT and randon selection of the mother wavelet. The method provides an adaptive approach for optimal signal representation for compression and can thus be applied to any type of biomedical signal.**

***Index Terms*—Embedded zerotree, wavelet design, wavelet packet.**

I. Introduction

BIOMEDICAL signal compression is of increasing interest due to the need of storing or transmitting large amount of multichannel data. The biomedical areas where most compression methods have been developed are medical images, electrocardiogram (ECG), and electroencephalogram (EEG) [6], [21]. Since lossless compression is limited to relatively small compression ratios, lossy compression schemes are often implemented. In the latter case, the quality of a compression algorithm is assessed from the distortion rate on the coded signal for a given compression ratio.

Many transform techniques have been previously proposed for biomedical signal compression (e.g., [7], [21]). Among these methods, wavelet transform has shown promising results in various areas [5], [19]. Discrete wavelet transform (DWT) provides a representation of the signal on coefficients partly localized in time and frequency, is not redundant, and invertible [11]. Processing of the coefficients may allow their representation on a lower number of bits than needed for representing the original signal. The coefficients of the DWT represent the projection of the signal over a set of basis functions generated as translation and dilatation of a protype function, called mother wavelet. The selection of the mother wavelet determines the signal representation. Many algorithms based on the DWT have been proposed for signal compression. In all cases, however, the mother wavelet is selected from a library of functions or chosen by comparing the results of a few wavelets on a set of experimental signals (e.g., [14], [15], [19]). However, it is expected that different mother wavelets provide different performance depending on the signal characteristics.

Nielsen *et al.* [13] recently proposed a compression method based on DWT and embedded zerotree wavelet (EZW) coding in which the mother wavelet was parameterized [10] and optimized with respect to the signal. The optimization criterion was the minimization of the distorsion rate in the coded signal for a fixed compression ratio. For electromyographic (EMG) signals, wavelet optimization determined a substantial reduction of the distorsion rate with respect to random selection of the mother wavelet.

The DWT represents the signal in a dyadic subband decomposition. Generalization of the DWT in wavelet packets allows subband analysis without the constraint of dyadic decomposition. The discrete wavelet packet transform (DWPT) performs an adaptive decomposition of the frequency axis. The specific decomposition (pruned wavelet packet tree) may be selected according to an optimization criterion.

In this paper, we propose a novel scheme for signal compression based on DWPT decompositon with optimization of the mother wavelet (after parameterization) and of the basis of wavelet packets. The method is an extension of that previously proposed by Nielsen *et al.* [13] and is applied to sets of EMG

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and ECG recordings. The two types of signals chosen for validation of the method provide representative applications. The ECG case has been considered as a signal type where large research efforts in compression have been devoted while less compression techniques have been proposed for EMG signals. Moreover, the two signals have very distinct statistical characteristics. The proposed technique allows the adaptation of both the shape of the mother wavelet and the frequency subbands on the basis of the signal characteristics and thus provides an optimal representation of the signal in frequency bands. The approach is the first that combines optimization of the prototype function and of the wavelet packet tree for compression.

II. METHODS

We propose the use of DWPT with optimization of both the mother wavelet and of the frequency subbands (best basis) and adaptation of the embedded zerotree algorithm (EZW) to wavelet packet.

A. Discrete Wavelet Transform

In the discrete wavelet transform (DWT) a signal is represented by inner products with basis functions that are temporal shifts and dilatation of a prototype function ψ (mother wavelet). In the orthonormal case, the mother wavelet is directly deduced from the scaling function ϕ which is used to approximate the signal at different scales. This function and its shifted versions belong to the space V_0 which is decomposed into a lower resolution approximation space V_1 and a detail space W_1. The resulting space V_1 is further decomposed into two subspaces, and so on. If $\{\phi(t-k)\}_{k\in Z}$ is an orthonormal basis of V_0, then $\{\phi_{j,k}(t)=(1)/(\sqrt{2^j})\phi(2^{-j}t-k)\}_{k\in Z}$ is an orthonormal basis of V_j and $\{\psi_{j,k}(t)=(1)/(\sqrt{2^j})\psi(2^{-j}t-k)\}_{k\in Z}$ is an orthonormal basis of W_j. These embedded subspaces define the multiresolution analysis (MRA) of a signal. In practice, the inner products between the signal and the basis functions are computed using low-pass and high-pass filters h and g that are directly related to the functions ϕ and ψ. In the case of an orthogonal MRA, the filter g is deduced from h from the relation $g[k]=(-1)^{1-k}h[1-k]$ and the reconstruction filters $\tilde{h}$ and $\tilde{g}$ are also deduced from h. Consequently, h defines the entire MRA.

B. Wavelet Packets

The DWPT generalizes the MRA by decomposing the detail spaces W_j (noted in that case W_j^p) into two orthogonal subspaces W_{j+1}^{2p} and W_{j+1}^{2p+1} which can be further decomposed into two orthogonal spaces. The resulting basis functions are not only shifts and dilatations of the mother wavelet but also modulated versions of it. Defining $\psi_0^0=\varphi$ and $W_0^0=V_0$, then $\{\psi_j^p(t-2^jk)\}_{k\in Z}$ is an orthonormal basis of W_j^p [20], where

$$\begin{aligned}\psi_{j+1}^{2p}(t)&=\sum_k h[k]\psi_j^p(t-2^jk)\\ \psi_{j+1}^{2p+1}(t)&=\sum_k g[k]\psi_j^p(t-2^jk).\end{aligned}\tag{1}$$

Equation (1) represents the basic stationary wavelet packets and the subspaces verify $W_j^p=W_{j+1}^{2p}\oplus W_{j+1}^{2p+1}$. The DWPT of a discrete signal $(x[k])_{1\leq k<N}$ is then computed using this discrete sequence as if it was the approximation, belonging to the space V_0, of a continuous signal

$$\begin{aligned}w_0^0[k]&=x[k]\\ w_{j+1}^{2p}[k]&=\downarrow 2(\bar{h}*w_j^p)[k]\\ &\qquad k=1,\ldots,\tfrac{N}{2^j}\quad p=0,\ldots,2^j-1\\ w_{j+1}^{2p+1}[k]&=\downarrow 2(\bar{g}*w_j^p)[k]\end{aligned}$$

with $\bar{h}[k]=h[-k]$ and $\downarrow h[k]=h[2k]$. The full wavelet packet transform is achieved when $j=1,\ldots,\log_2(N)-1$. The parameters p,j,k in $w_j^p[k]$ have a natural interpretation as frequency, scale, and position, respectively. This decomposition allows more flexibility than the DWT (deduced from an MRA) in the representation of the signal since the DWPT adaptively segments the frequency axis whereas DWT uses a predefined subband decomposition.

C. Wavelet Parametrization

In both the DWT and DWPT, the basis functions are determined by the selection of the mother wavelet. For compression, the mother wavelet is usually chosen within a given library of well-known wavelets (e.g., [5], [6], [8], [14], [15], [19]). As opposed to this approach, we propose a signal-based wavelet parametrization to compute DWT and DWPT. The wavelets are parametrized by the scaling filter h, which is sufficient to define the DWT and DWPT in the orthogonal case. For a finite impulse response filter of length L, there are $L/2+1$ conditions to ensure that the wavelets define an orthogonal DWT (or DWPT) and thus there are $L/2-1$ degrees-of-freedom to design the scaling filter h. The lattice parameterization described by Vaidyanathan [18] offers the opportunity to design orthogonal wavelet filters via unconstrained parameters. For example, if $L=6$, the design parameter vector θ has two components α and β, and h is given by

$$\begin{aligned}i=0,1:h[i]&=\frac{1}{4\sqrt{2}}\\ &\quad\times[(1+(-1)^i\cos\alpha+\sin\alpha)(1-(-1)^i\cos\beta-\sin\beta)\\ &\qquad+(-1)^i2\sin\beta\cos\alpha]\\ i=2,3:h[i]&=\frac{1}{2\sqrt{2}}[(1+\cos(\alpha-\beta)+\sin\alpha\\ &\qquad+(-1)^i\sin(\alpha-\beta))]\\ i=4,5:h[i]&=\frac{1}{\sqrt{2}}-h(i-4)-h(i-2).\end{aligned}$$

The optimal parameter set (i.e., mother wavelet) can be chosen according to a criterion specific for the application. In the case of signal compression, a natural criterion is the distortion of the signal after decoding given a compression rate.

D. Best Basis Selection in DWPT

The DWPT decomposition can be seen as a binary tree in which each node represents either a space W_j^p or the related coefficients $C_j^P=\{w_j^p[k]\}_{1\leq k\leq N/2^j}$ (see Fig. 1). This signal representation is highly redundant since each node space is the direct sum of its two children. To obtain a nonredundant and still invertible representation of the signal, the full tree can be pruned by selecting a set of subspaces within the tree. This selection is

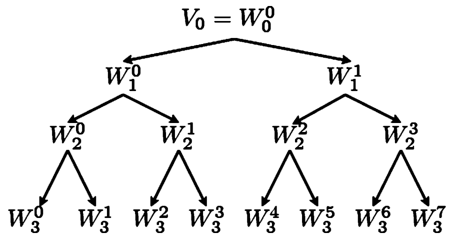

Fig. 1. Four-level wavelet packet tree. W_j^p are the projection spaces indexed by scale j and frequency p. V_0 is the original signal space, as in the MRA.

performed by choosing two children between their parent. The leaves of the resulting tree constitute a decomposition of the initial space V_0. For a tree of depth J, the number of possible decompositions of V_0 is $N_J = N_{J-1}^2 + 1$ with $N_0 = 1$.

This large collection of wavelet packet bases must be explored to find a pertinent basis for a given signal and a given mother wavelet. An exhaustive search is not feasible, given the large number of possible decompositions. Wickerhauser and Coifman [3] proposed a fast algorithm along with an additive information cost function to prune the full wavelet packet tree. A map M from sequences $\{x_i\}$ to R is called an additive information cost function if $M(0) = 0$ and $M\{x_j\} = \sum_j M(x_j)$. Given such a map, a full wavelet packet tree can be fast pruned [order $O(N \log N)$] to find the basis B that minimizes $M(Bx)$ with a bottom-up series of decisions to keep children nodes (split) or to keep parent node (merge). Numerous additive cost functions are possible. In this paper, we compared the following:

- *energy entropy:* $M(x) = -\sum_j |x_j|^2 \log |x_j|^2$ (proposed in [3]);
- *l^1 norm:* $M(x) = \sum_j |x_j|$ [20];
- *log:* $M(x) = \sum_j \log |x_j|$.

For a given cost function, the resulting tree will be different for different mother wavelets. Besides, once selected, the best basis needs to be transmitted from the encoder to the decoder and the inverse transform. This requires at least $\log_2(N_J)$ bits that are added to the output bitstream.

E. Embedded Zerotree Wavelet Packet

After transforming the signal, the coefficients of the transformation should be coded. The EZW algorithm, first introduced by Shapiro [17], enables to quantize and encode the coefficients of the DWT. This compression algorithm is based on two main concepts: the prediction of the absence of significant information across scale by exploiting self-similarity inherent in the signal and an entropy-coded successive-approximation quantization (SAQ). This quantization scheme outputs an embedded bit stream by extracting at each step (i.e., each decreasing threshold) another significant bit for each coefficient which has been found to be significant (i.e., with a magnitude superior to the threshold). It allows to stop the compression whenever the desired compression ratio or distortion rate is met.

If a coefficient of the transform is insignificant at a coarse scale, the coefficients at the same temporal position but at a finer scale will likely be insignificant too (zerotree). This assumption yields a parent–child relation between subbands that can be easily extracted from a natural hierarchical subband decomposition, such as the DWT. The concept of zerotree can be extended to the DWPT in order to apply a EZW-like coder to a wavelet packet representation of the signal (EZWP). The extension is done by building a temporal orientation tree [8], [16] (i.e., a tree containing the parent–child relations between subbands) for wavelet packet coefficients.

F. Compression Scheme

The overall compression algorithm is separated in two stages. The DWPT of the signal is first computed for a given vector value θ (mother wavelet parameterization). Then, using a cost function, the wavelet packet tree is pruned according to the best basis algorithm presented before. This stage is lossless and its output is another representation of the signal. However, for transmission purposes, the size of the output of the DWPT is larger than the original number of samples since the tree structure corresponding to the best basis must also be transmitted together with the transformation coefficients. If we assume that each basis is equiprobable, then for N_j possible bases, $\log_2(N_j)$ bits will be needed to code the basis. For example, considering a 12-bit resolution signal of 1024 samples to be compressed with 90% ratio, half of the bits of the resulting compressed signal would be occupied by the basis. To avoid this problem, we restricted the maximum depth of the basis for DWPT to 7. With this choice, 75 bits are needed to transmit the basis while 602 are needed with the maximal depth of 10. Other choices for maximum depth are not tested in this paper.

The EZWP block (second stage) encodes the coefficients to achieve a predefined target compression ratio CR, defined as:

$$\mathrm{CR}(\%) = \frac{Os - Hs}{Os} \cdot 100 \tag{2}$$

where Os is the original data size and Hs is the first-order Shannon entropy of the output of the coding block. Hence, an ideal first-order lossless compression is assumed.

The decoding and inverse transformation blocks perform the inverse of these two stages to reconstruct the signal. The distortion metric used to quantify the difference between the original signal $x[k]$ and the reconstructed signal $\hat{x}[k]$ after decoding is the percent residual difference (PRD)

$$\mathrm{PRD}(\%) = \sqrt{\frac{\sum_k (x[k] - \hat{x}[k])^2}{\sum_k x[k]^2}} \cdot 100.$$

G. Optimization

DWPT and EZWP are repeated for all values of a predefined set of values of the parameter vector θ and the θ value leading to the minimum PRD is chosen as representing the best mother wavelet (wavelet optimization). Thus, in the case of DWPT, two signal-based optimization processes are implemented. The first (internal optimization) is the best basis selection that is performed for each mother wavelet and the second (external optimization) is the selection of the mother wavelet. The coding/decoding scheme for DWPT/EZWP is reported in Fig. 2.

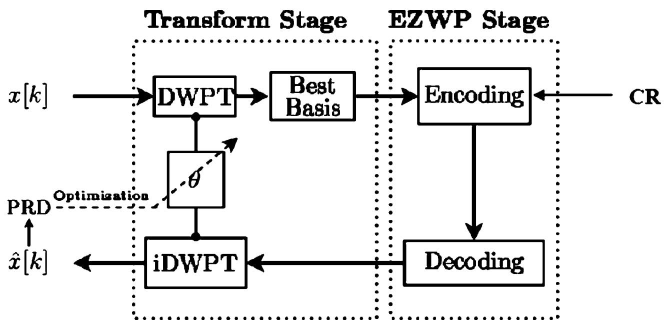

Fig. 2. Compression algorithm diagram. The best basis is selected (internal optimization) for each mother wavelet, parameterized through θ. The signal is then encoded with EZWP. The optimal mother wavelet is selected to minimize the percent residual difference (PRD) computed from the decoded signal. The best basis and parameter θ are transmitted to the decoder together with the encoded signal. CR = compression ratio.

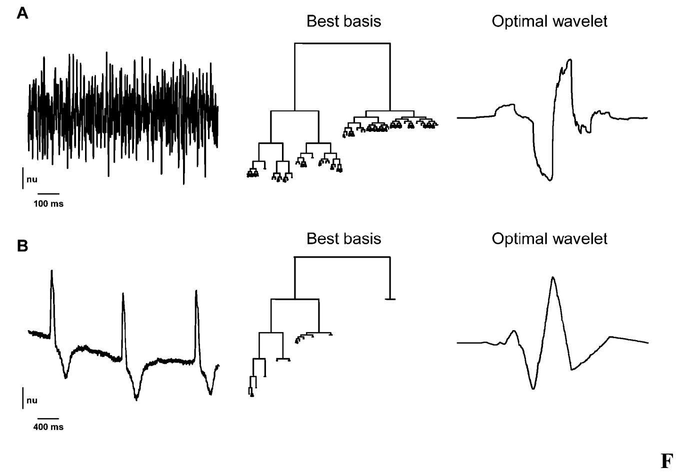

Fig. 3. Representative examples of (A) EMG and (B) ECG signals, their best basis (entropy cost function) representation, and the optimal mother wavelet. Compression ratio was 80% in both cases. Nu = normalized units.

H. Experimental Data

The compression algorithm was tested on two sets of biomedical signals, consisting of surface electromyographic (EMG) and electrocardiographic (ECG) signals recorded from two samples of ten subjects each. Surface EMG signals were recorded from the upper trapezius muscle of ten healthy male volunteers (age, mean $\pm$SD, 25.6$\pm$2.4 year). The subjects were in an erect posture with both arms abducted at 90°. The bipolar electrodes used for recording (Neuroline 720–01-k, Ølstykke, Denmark) were placed 31 mm apart, 20 mm lateral with respect to the midpoint between the acromion and the seventh cervical vertebra. The signals were amplified 2000 times, band-pass filtered (2–500 Hz), digitized on 12 bit and sampled at 1 kHz. Details on data collected can be found in [9].

ECG signals were obtained from the MIT-BIH ECG Compression Test Database [4]. The ten signals selected to test the compression algorithm were the channel 1 of 08730_01, 11442_01, 11950_01, 12247_01, 12431_01, 12490_01, 12531_01, 12621_01, 12713_01 and 12921_01. Each strip is 4-s long (the first 1024 points of the database signals) and was digitized with the following specifications: 250 samples per second per channel with 12-bit resolution over a $\pm$10 mV range, bandpass filtered from 0.1 to 100 Hz to limit analog-to-digital converter saturation and for anti-aliasing (see [12] for details).

I. Data Analysis

The two sets of signals were compressed with the proposed scheme at compression rates in the range of 50–90% (10% increment). The resulting PRD was considered as an index of performance. PRD was compared between DWPT/EZWP with optimized mother wavelet, DWPT/EZWP with classic mother wavelets (Daubechies and Coiflet), DWT/EZW with optimized mother wavelet, and DWT/EZW with classic wavelets. For the DWPT, the three cost functions defined before were compared. For wavelet parameterization, a filter length $L = 6$ was used in all cases, with step size for both parameters of the vector θ equal to $\pi/7$. Results are reported as mean $\pm$SD over the ten signals (for each dataset) for signal segments of 1024 samples.

III. Results

A. EMG Signals

Best basis selection and optimal wavelet for a representative EMG recording are shown in Fig. 3(a). PRD for DWPT/EZWP (using three cost functions for best basis selection) on the ten EMG recordings is reported in Table I. Fig. 4 shows the results obtained with DWPT/EZWP as compared to those obtained with DWT/EZW. Results from DWT/EZW are presented in detail in [13]. The optimal mother wavelet and the best basis were different among all EMG signals tested.

B. ECG Signals

Fig. 3(b) reports an example of best basis selection and optimal wavelet for a representative ECG recording. Tables II and III show the PRD resulting from DWT/EZW and DWPT/EZWP, respectively. The optimal mother wavelet was different among all ECG signals tested. Moreover, although with a smaller impact on PRD, also the best basis differed among all ECG signals.

IV. Discussion

A compression algorithm based on wavelet packets and mother wavelet optimization has been proposed. The algorithm is an extension of the DWT optimization proposed in [13] with the inclusion of the best wavelet packet tree to represent the signal. This constitutes a more flexible mapping of the frequency plane than the DWT and is the first approach that combines optimization of the scaling filter and of the wavelet packet for compression. Depending on the signal type, the parameterization of the mother wavelet and the best basis selection were shown to have an impact in reducing the PRD with fixed compression ratio.

A. Optimization of the Mother Wavelet

The results presented in [13] were confirmed in this paper that showed that wavelet optimization with DWT improves the quality of the decoded signal with respect to classic wavelets

TABLE I
PRD (MEAN ±SD, OVER THE TEN EMG SIGNALS) FOR DWPT/EZWP USING DAUBECHIES 3 (DB3), COIFLET 1 (COIF1), AND BEST/WORST PARAMETERIZED WAVELETS AND THREE COST FUNCTIONS (ENERGY ENTROPY, LOG, AND l^1 NORM) FOR BEST BASIS SELECTION. THE "WORST" WAVELET IS THE ONE LEADING TO THE HIGHEST PRD AMONG THE TESTED WAVELETS

	Energy Entropy			
CR(%)	db3	coif1	worst	Optimal
50	4.0 ± 0.7	4.2 ± 0.9	10.9 ± 1.7	3.3 ± 0.6
60	6.4 ± 1.1	7.1 ± 1.4	16.7 ± 2.2	5.1 ± 0.6
70	10.8 ± 1.9	10.3 ± 1.3	27.6 ± 4.2	9.0 ± 1.5
80	18.4 ± 1.9	17.9 ± 2.1	41.8 ± 4.5	15.3 ± 2.5
90	32.6 ± 3.9	31.5 ± 3.9	60.3 ± 6.5	29.0 ± 2.0

	Log			
CR(%)	db3	coif1	worst	Optimal
50	4.0 ± 0.7	3.9 ± 0.7	9.8 ± 1.1	3.3 ± 0.5
60	6.1 ± 1.3	6.7 ± 1.2	15.2 ± 1.2	5.1 ± 0.8
70	10.3 ± 2.1	10.7 ± 1.9	24.3 ± 3.7	8.4 ± 1.2
80	18.0 ± 4.0	17.1 ± 2.6	40.8 ± 4.7	14.5 ± 2.9
90	32.5 ± 6.0	31.8 ± 5.7	58.7 ± 5.3	28.5 ± 2.8

	l^1 norm			
CR(%)	db3	coif1	worst	Optimal
50	4.1 ± 0.8	4.0 ± 0.8	10.1 ± 1.0	3.4 ± 0.5
60	6.7 ± 1.3	7.0 ± 1.1	15.9 ± 1.8	5.1 ± 0.4
70	10.9 ± 2.1	10.6 ± 1.6	26.5 ± 4.1	8.7 ± 1.1
80	17.9 ± 2.5	17.6 ± 2.5	41.4 ± 5.0	15.0 ± 2.8
90	32.7 ± 5.2	32.5 ± 5.2	59.3 ± 5.8	28.4 ± 3.0

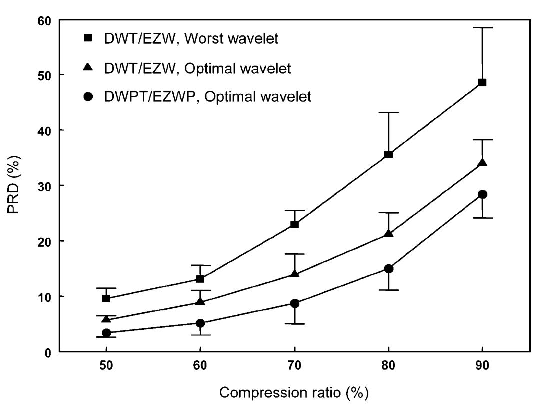

Fig. 4. Percent residual difference (PRD) versus compression ratio for EMG signals (mean ±SD over the ten subjects) for DWT/EZW with the wavelet leading to the poorest performance, DWT/EZW with optimized wavelet, and DWT/DWPT with optimized mother wavelet and best basis selection. The results with DWT/EZW are the same presented in [13], here reported again for comparison with the proposed approach. Note the improvement due to optimization of the mother wavelet with respect to random wavelet selection and the further improvement gained with best basis and optimal wavelet. SD bars are reported only on one side for each graph for clarity.

(see Fig. 4 and Table II). Optimization of the mother wavelet also resulted in reduced PRD in case of DWPT (see Tables I and III). Since the distortion rate cannot be predicted, a mother wavelet could not be selected *a priori* but should be based on the signal characteristics. Different signals resulted in different optimal wavelets (see Fig. 3), adaptively designed to minimize the distorsion rate.

TABLE II
PRD (MEAN ±SD, OVER THE TEN ECG SIGNALS) FOR DWT/EZW USING DAUBECHIES 3 (DB3), COIFLET 1 (COIF1), AND BEST/WORST PARAMETERIZED WAVELETS. THE "WORST" WAVELET IS THE ONE LEADING TO THE HIGHEST PRD AMONG THE TESTED WAVELETS

CR(%)	db3	coif1	worst	Optimal
50	0.9 ± 0.2	0.9 ± 0.2	2.7 ± 1.0	0.8 ± 0.2
60	1.3 ± 0.5	1.3 ± 0.5	4.0 ± 1.7	1.2 ± 0.4
70	2.1 ± 0.5	2.2 ± 0.7	6.5 ± 2.9	1.9 ± 0.4
80	3.6 ± 0.9	3.4 ± 0.8	11.9 ± 5.0	3.0 ± 0.9
90	7.5 ± 2.8	7.3 ± 2.4	21.5 ± 8.4	6.2 ± 1.4

TABLE III
PRD (MEAN ±SD, OVER THE TEN ECG SIGNALS) FOR DWPT/EZWP USING DAUBECHIES 3 (DB3), COIFLET 1 (COIF1), AND BEST/WORST PARAMETERIZED WAVELETS AND THREE COST FUNCTIONS (ENERGY ENTROPY, LOG, AND l^1 NORM) FOR BEST BASIS SELECTION. THE "WORST" WAVELET IS THE ONE LEADING TO THE HIGHEST PRD AMONG THE TESTED WAVELETS

	Energy Entropy			
CR(%)	db3	coif1	worst	Optimal
50	0.8 ± 0.2	1.0 ± 0.3	3.7 ± 1.6	0.7 ± 0.2
60	1.3 ± 0.4	1.4 ± 0.4	5.9 ± 2.9	1.2 ± 0.4
70	2.1 ± 0.7	2.2 ± 0.6	9.9 ± 5.4	1.8 ± 0.5
80	3.8 ± 1.2	4.0 ± 1.4	15.2 ± 6.5	3.2 ± 1.0
90	9.0 ± 3.5	8.6 ± 4.2	30.6 ± 13.0	7.0 ± 2.0

	Log			
CR(%)	db3	coif1	worst	Optimal
50	0.9 ± 0.3	0.9 ± 0.3	3.2 ± 1.5	0.7 ± 0.2
60	1.4 ± 0.5	1.3 ± 0.5	5.4 ± 2.3	1.2 ± 0.4
70	2.1 ± 0.7	2.3 ± 0.8	8.7 ± 4.1	1.8 ± 0.5
80	3.8 ± 1.3	3.9 ± 1.4	14.5 ± 5.5	3.3 ± 1.0
90	9.1 ± 3.9	9.4 ± 4.0	30.1 ± 11.6	7.0 ± 2.0

	l^1 norm			
CR(%)	db3	coif1	worst	Optimal
50	0.9 ± 0.2	1.0 ± 0.3	3.5 ± 1.7	0.7 ± 0.2
60	1.3 ± 0.4	1.4 ± 0.4	5.4 ± 2.5	1.2 ± 0.4
70	2.1 ± 0.7	2.3 ± 0.7	8.6 ± 4.6	1.8 ± 0.5
80	3.7 ± 1.2	4.0 ± 1.4	14.2 ± 5.1	3.2 ± 1.0
90	9.0 ± 4.0	9.3 ± 4.0	29.5 ± 11.4	6.7 ± 1.9

It should be noted that optimization of the mother wavelet in both DWT/EZW and DWPT/EZWP increases the computational time in the encoding part, as previously discussed [13]. For example, a step size of $\pi/7$ for two filter parameters leads to 196 mother wavelets to be tested, compared to the use of a single mother wavelet in classic schemes. This problem is however only related to the coding part since the decoding implies the same number of operations as without wavelet parameterization.

B. Joint Optimization

The representation of the signal with wavelet packet corresponds to the distribution of signal energy over subbands. The separation between subbands is performed by filters whose selectivities depend on the mother wavelet and whose bandwidths depend on the packet selected. Joint optimization of the mother wavelet and of the wavelet packet corresponds to the optimal subband analysis in terms of filter transfer function and bandwidths for the purpose of compression.

The best basis selection resulted in a further decrease in PRD with respect to DWT for EMG signals. In this case, DWPT with Daubechies and Coiflet wavelets performed better than DWT with optimal wavelet (compare Fig. 4 and Table I), indicating that the best basis for EMG compression is very different from the dyadic wavelet basis [see also Fig. 3(a)]. A basis optimization is thus necessary. It is noted that for EMG signals, PRD in the case of, e.g., 90% compression ratio was reduced from ~48% with DWT/EZW and the wavelet leading to poorest performance (see Fig. 4) to ~28% with DWPT/EZWP and optimized wavelet (see Table I), which is a substantial improvement in performance. Table I shows that the cost function did not significantly influence the results, thus any of the three cost functions compared can be used.

For ECG signals, the best basis selection in DWPT was less effective than for EMG signals, although all ECG signals resulted in a different best basis. As shown in Fig. 3(b), the best basis for the ECG signals analyzed was close to the dyadic wavelet basis and, due to the need to transmit the basis, the PRD was similar for DWT/EZW and DWPT/EZWP (see Tables II and III). Thus, as expected, the gain in performance strongly depended on the signal type. The chosen representative signal sets allow to show different impacts of the two optimization procedures on performance. However, it is underlined that the proposed optimized DWPT/EZWP method does not result in significant worsening of performance even when the optimal basis is close to the dyadic one (the set of ECG recordings chosen in this study). Its application may provide large or more limited improvement over classic DWT depending on the signal type. In particular, the DWPT/EZWP approach with best basis selection performs consistently worse than the DWT/EZW in the case of worst wavelet.

C. Methods Comparison

The method proposed (DWPT/EZWP) has been compared with DWT/EZW with classic wavelets, which has been used in previous work (e.g., [14], [15], [19]). We have also shown the improvement in performance due separately to optimization of the mother wavelet and to best basis selection, comparing a number of possible wavelet approaches (see Tables I–III and Fig. 4). Comparison with methods not based on wavelets was not reported since in most cases it is difficult to compare different encoding schemes. For example, the algebraic code excited linear prediction (ACELP) paradigm, widely used for speech signal coding [1] and recently also applied for EMG compression [2], allows only a few choices of compression ratios corresponding to bit rates ranging from 4.75 to 12.2 kb/s (in the case 12.2 kb/s, the compression ratio is 87.3%). For this reason, this paper focused only on wavelet compression schemes. In addition, the encoder chosen (EZWP) is naturally adapted to the hierarchical DWPT transformation but any other encoding scheme could be used with the proposed optimization. The current study aimed mainly at proposing a subband signal decomposition optimized for the purpose of compression while the specific encoding scheme was of smaller interest.

V. Conclusion

In conclusion, it was shown that best basis selection and optimization of the mother wavelet through parameterization lead to improvement of performance in signal compression with respect to random selection of the mother wavelet and DWT. The method provides an adaptive approach for optimal signal representation for the purpose of compression and can thus be applied to any type of 1-D biomedical signal.

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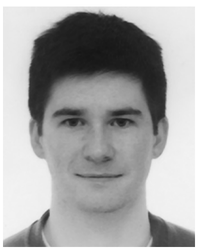

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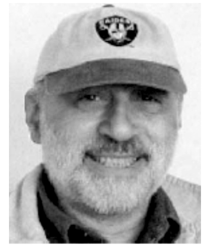

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