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Introduction

There are over 279,000 oil and wells and over 54,000 injection wells in Texas. Wells where hydraulic fracking takes place accounts for about half of all oil produced and two-thirds of all gas. Texas has parts of the Anadarko Basin, the Palo Duro Basin, the Permian Basin, the Barnett Shale, the Eagle Ford Shale, and the Haynesville-Bossier Shale.

The shale plays are typically fracked to aid in recovery of the oil and gas in the formations. A frack well typically uses between 2-8 million gallons and almost all of it is returned as wastewater. Where to get this water is a huge problem, but of equal concern is what to do with it once it becomes contaminated. There are plenty of injection wells, but which one do you use? Does it make more sense to return the water to the city? How do you move the water? All these are questions that fracking producers have to wrestle with.

To help solve this problem I create a mixed integer program that investigates how to optimize the wastewater from a small number of wells which have sinks in nearby cities and injection wells. There is plenty of literature that investigates this such as Zhang, Sun, and Duncan,2016 who use a stochastic mixed integer formulation and Yang, Manno, and Grossmann,2014 who use a two-stage mixed integer formulation. My model will follow the same principles but with a simplified sample set.

Methods

I developed a mixed integer program that takes into account each of the steps involved with producing wastewater, shipping it to a treatment facility, and then either selling it or disposing of it in an injection well. I first develop a simple model that does not consider choosing the location of treatment facilities. I then expand on that model to take into account location selection of the treatment facilities. I solved both instances using GAMS/CPLEX with an optimality gap of 0. The following sections apply to both models.

Wastewater Production

Wastewater produced at each of the production nodes *i* is determined exogenously and is specified by W_i. All the wastewater produced at each node must be removed and that is shown by Equation 1.

Municipal Water

The produced waste water can be sold to municipalities only after it has been cleaned to drinking standards. This will be enforced by the cost parameters of shipping water to cities rather than explicitly. However, the maximum amount of water a city can accept is restricted explicitly by G_k and is shown by Equation 2.

Injection Water

Produced waste water can also be disposed of at injection wells after it has been cleaned to any standard. It is shipped from the cleaning facility to a injection well (b_k) only if that injection well's infrastructure has been built this is enforced with Equation 3.

Wastewater Treatment

All wastewater must be treated, and this is enforced by the indices. Water must leave from the production node *i* and go to a cleaning facility *j.* Furthermore, water can only go to cleaning facilities (d_j) that have been built and this is enforced by Equation 4. Furthermore, wastewater can only be shipped from cleaning facilities to sinks *k* if the cleaning facility actually received that volume of wastewater. Equation 5 ensures the mass balance of the model.

Shipping Wastewater via Trucks or Pipes

The producer gets to determine how to ship the wastewater from the production site to the cleaning facility and eventually to the injection site or city. If they choose trucks, they must purchase the trucks and pay the variable costs (gas, drivers, etc.) which is enforced by Equation 6. If they choose pipes, they must build the infrastructure and pay the variable costs (pumps) and this is enforced by Equation 7.

Objective Function

The cost of shipping to treatment facility depends on whether you use a truck or pipe which production node you are coming from and which treatment facility you are going to. Furthermore, the decision to use a truck or a pipe comes with a fixed cost. These costs are associated with the first four terms of the objective function.

The cost of cleaning water depends on the volume of water processed and is dependent on the type of cleaning facility. Furthermore, each cleaning facility has a fixed cost. These costs are associated with the fifth through seventh terms of the objective function.

The cost of shipping from the treatment facility to the sink (either injection well or city) depends on whether your use a truck or pipe and what sink you send the water too. If you send it to an injection well that has a fixed cost. These costs are accounted for with the eighth through twelfth terms.

Modifications for Economies of Scale and Facility Location

In order to account for economies of scale the cost functions for the treatment facilities and the injection wells their associated cost functions could be non-linear. However, since this is a mixed integer program, I can simply discretize the size of the facilities and wells, but I found that for this quantity of water no matter the economy of scale the model wants to process all the water at one facility. Therefore, I ignored modeling different sizes of treatment plants and their varied cost per gallon because the result would be the same; build the biggest facility that could handle all the water.

To account for being able to place the treatment facilities anywhere I can also discretize placements. I can place injection treatment facilities at every injection well, centered along the cities, and centered along production wells. I can apply the same logic to drinking treatment facilities as well, but with them at every city instead of injection well. There are infinitely more possibilities, but for this level of model these distinctions should be more than enough.

Equations

$$\min \sum_i \sum_j c_{rt} t_{ij} + \sum_i \sum_j c_{rp} p_{ij} + \sum_i \sum_j f_{rt} e_{ij} + \sum_i \sum_j f_{rp} g_{ij} + \sum_j c_{cj} \sum_i t_{ij} + \sum_j c_{cj} \sum_i p_{ij} + \sum_j f_{cj} d_j + \sum_j f_{sk} b_k + \sum_j \sum_k c_{rt} t_{jk} + \sum_j \sum_k c_{rp} p_{jk} + \sum_j \sum_k f_{rt} e_{jk} + \sum_j \sum_k f_{rp} g_{jk}$$

s.t.

$$(\mathbf{1}) \quad \textstyle\sum_j t_{ij} + \sum_j p_{ij} = W_i \; for \; all \; i$$

$$(\mathbf{2}) \; \textstyle\sum_j t_{jk} + \sum_j p_{jk} \leq G_k \; for \; all \; k$$

$$(\mathbf{3}) \textstyle\sum_j t_{jk} + \sum_j p_{jk} \leq Mb_k \; for \; all \; k$$

$$(\mathbf{4}) \sum_i t_{ij} + \sum_i p_{ij} \leq Md_j \; for \; all \; j$$

$$(\mathbf{5}) \quad \sum_i t_{ij} + \sum_i p_{ij} = \sum_k t_{jk} + \sum_k p_{jk} \;\; for \; all \; j$$

$$(\mathbf{6}) \; t_{ij/jk} \leq M \, e_{ij/jk} \; for \; all \; i, j, and \; k$$

$$(\mathbf{7}) \; p_{ij/jk} \leq M \, g_{ij/jk} \; for \; all \; i, j, and \; k$$

Symbol	DV or Parameter	Description
$\boldsymbol{W_i}$	Parameter	Water produced at i
$\boldsymbol{c_{rt}}$	Parameter	Cost of Shipping by truck along route r
$\boldsymbol{c_{rp}}$	Parameter	Cost of Shipping by pipe along route r
$\boldsymbol{c_{cj}}$	Parameter	Cost of cleaning water at j
$\boldsymbol{c_{sk}}$	Parameter	Cost of injecting or selling water at k
$\boldsymbol{f_{rt}}$	Parameter	Fixed Cost of purchasing trucks for route r
$\boldsymbol{f_{rp}}$	Parameter	Fixed Cost of pipes for route r
$\boldsymbol{f_{cj}}$	Parameter	Fixed Cost of facility at j
$\boldsymbol{f_{sk}}$	Parameter	Fixed cost of injecting or selling water at k
$\boldsymbol{M}$	Parameter	Extremely Large Number
$\boldsymbol{G_k}$	Parameter	Max Volume of Municipal Drinking Water
$\boldsymbol{t_{ij}}$	Decision Variable	Water transported from node i to facility j by truck
$\boldsymbol{p_{ij}}$	Decision Variable	Water transported from node i to facility j by pipe
$\boldsymbol{t_{jk}}$	Decision Variable	Water from facility j to node k by truck
$\boldsymbol{p_{jk}}$	Decision Variable	Water from facility j to node k by truck
$\boldsymbol{b_k}$	Decision Variable	Decision to inject or sell water at node k
$\boldsymbol{d_j}$	Decision Variable	Decision to use facility j
$\boldsymbol{e_{ij}}$	Decision Variable	Decision to truck between i and j
$\boldsymbol{g_{ij}}$	Decision Variable	Decision to pipe between i and j

Parameter Values Model 1

The general layout for this model is that all producing wells are in a row about 5 miles from each other. The injection cleaning facilities are located 5 miles below the production wells centered along all the production wells. The injection wells are 10 miles below production wells and 5 miles below the injection cleaning facilities also centered along the production wells. The drinking water cleaning facilities are 5 miles below the injection wells and 15 miles below the production wells also centered along the production wells. The cities are 20 miles below the production wells and 5 miles below the drinking water cleaning facilities also centered along the production wells.

The costs of transporting water from production node to and from cleaning facilities are determined by distance and transportation medium. The costs are listed below for both trucks and pipelines in the Appendix.

The costs of treating the produced water to injection standards water is about $1/barrel and it cost another $50,000 to set an injection well up for disposal. The cost of treatment to drinking standards seems to vary wildly, I use $2/barrel, but even values as high as $4/barrel don't change the optimal decisions. Karangwa,2004 cites the cost of a water cleaning facility from $10 million - $18 million. I use $10 million for injection cleaning and $11 million for a drinking water cleaning facility.

The cost for a water/oil pipeline is generally $300,000 per mile, so I fix the cost of the pipelines at $3 million per route. A tank truck typically costs about $250,000, so I assume that each route would need 10 trucks. This would require multiple trips and is addressed in the shipping costs. So, for a given route with 10 trucks that would require $2.5 million in an initial investment.

Karangwa 2004 uses values of 4500 – 7000 barrels of water per well, and I have 6 wells that start at 4500 barrels and increase by 500 barrels until well 6 produces 7000 barrels. The injection wells have capacities of 20, 25 and 30 thousand barrels respectively. Each city can handle 35,000 barrels which translates into about 1.5 million gallons of water. Note each well cannot hold all the produced water, but each city can.

Parameter Values Model 2

The main difference between this parameterization and the previous one is the addition of choices for the cleaning facilities. Although, this model does add a few choices for picking the location of the cleaning facilities it doesn't provide unlimited choices (i.e. you can not put any cleaning facility anywhere in the sample space). The choices for the injection cleaning facility are restricted to the original locations, the other cleaning facilities original location, and the injection well sites. The choices for the drinking water cleaning facilities are similarly restricted, but to the cities instead of the injection wells. Modified Parameters Listed in the Appendix.

Note: The Appendix isn't included in the print out but is available via email. Its long and not too interesting other than for completeness.

Results

Model 1

The parameterization of Model 1 yields an ***objective value of $28.78 million***. **All production wells ship their produced water by truck to drinking water cleaning facility 2** for a cost of $15.18 million. Drinking water cleaning facility cleans all the water for a cost of $11.1 million and then **ships all its water to city 2**. The cost of shipping the clean water to city 2 is a relatively small $2.535 million.

Model 2

The parameterization of Model 2 yields an **objective value of $28.72 million** only slightly less than the original results. The difference is that the **drinking water cleaning facility is located closer to the production wells** in the site the original injection cleaning facility 2 was located. This **lowers the shipping cost to the cleaning facility** by about $100,000 to $15.08 million. However, since the cleaning facility is now further away from City 2 **the cost for shipping the cleaned water to that city increased** by $34,000 to $2.569 million. So, the net savings of being able to pick the location of the cleaning facility is only about $66,000.

Discussion

The results of this case study show the value of being able to purify water and send it to cities, the value of being able to locate cleaning facilities, and the value of the lower capital costs of trucks. However, all these results could vary wildly with only minor adjustments to the parameters. Sensitivity analysis of these results show that under only moderately different conditions the optimal medium of travel, optimal cleaning facility, and optimal sinks can all change dramatically. The following table shows the changes to parameter and the corresponding changes to the objective value and optimal decisions.

Old Parameterization	New Parameterization	Old Optimal Decision Variable	New Optimal Decision Variable	Objective Value
$\mathrm{f_{rt}} = \$2.5\ \mathrm{MM}$	$\mathrm{f_{rt}} = \$2.99\ \mathrm{MM}$	$\mathrm{e_{ij}} = 1, \mathrm{g_{ij}} = 0$	$\mathrm{e_{ij}} = 0, g_{\mathrm{ij}} = 1$	$32.03 MM
$\mathrm{f_{cdrink}} = \$11\ MM$	$\mathrm{f_{cdrink}} = \$15\ MM$	$\mathrm{d_j} = \mathrm{Dclean4}$	$\mathrm{d_j} = \mathrm{Iclean7}$	$30.24 MM
$\mathrm{f_{cdrink}} = \$11\ MM$	$\mathrm{f_{cdrink}} = \$15\ MM$	$\mathrm{b_k} = \mathrm{City\ 2}$	$\mathrm{b_k} = \mathrm{IWell2,3}$	$30.24 MM
$\mathrm{G}_{City} = 35{,}000\ \mathrm{gal}$	$\mathrm{G}_{City} = 34{,}000\ \mathrm{gal}$	$\mathrm{b_k} = \mathrm{City\ 2}$	$\mathrm{b_k} = \mathrm{IWell2,3}$	$30.24 MM
$\sum_k W_{\mathrm{k}} < \mathrm{City\ 1}$	$\sum_k W_{\mathrm{k}} > \mathrm{City\ 1,2}$	$\mathrm{b_k} = \mathrm{City\ 2}$	$\mathrm{b_k} = \mathrm{IWell1,2,3}$	$33.03 MM

The results of the sensitivity analysis show that the optimum medium of travel is largely dependent on the capital costs of that medium. Although, the operating costs of pipes is much lower than the trucks until the trucks have nearly the same capital costs for this volume of water it makes since to invest the least up front. Changing the capital costs causes the largest shift in objective function.

The optimal cleaning facility once again changes only once the capital costs of the drinking water cleaning facility became large enough to justify using only injection standards for water treatment. I also modified the cost of treating water, but at this volume the main driver is the capital costs of the system.

The optimal location only changes (assuming all other costs are constant) if the number of cities that can hold all of the drinking water that comes out of a facility is equal to or greater than the number of wells that can hold the injection water. The program tries to minimize the number of routes the water takes. Note, even for more than double the amount of water (the last sensitivity case) the objective function doesn't increase by much compared to if a capital cost changed.

The model is clearly limited by the parameterization as even small changes can drastically change the optimal decisions. Furthermore, not having the complete freedom to choose the cleaning facility location is also a limitation, though the results show that this doesn't have a dramatic effect on the objective value. Also, this model only evaluates one time period and doesn't take into account the uncertainty of the cost of injection or how much the city would be willing to pay for clean water. The operational costs are likely to change although the sensitivity analysis shows they don't really affect the decisions. The capital costs and the capacity constraints are the main drivers of decisions and their values could change wildly for instance if the operator decides to not invest in as many trucks, the cleaning facilities costs shrink dramatically, cities only accept so much water, etc. The model doesn't take into account these dynamics even though they are the biggest driver of decisions.

Nonetheless, the main value of this model is that it allows you to quickly determine if your system is close to operating optimally. Although different parameterizations can yield different optimal decisions, the objective value doesn't change dramatically, so even a near optimal solution will suffice. And the simplicity of parameterizing the model and the ability to set variables to certain values can allow you to evaluate your system and determine if you are close enough to the "optimal" value or if adjustments can lower your costs even without the certainty of future costs.

Sources

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