Enhancing Portfolio Performance: A Comparative Analysis of Optimization Techniques for S&P500 Constituents

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2022DMF08

A project report submitted

in partial fulfillment of the requirement for the award of the

POST GRADUATE DIPLOMA IN MANAGEMENT

IN

(FINANCIAL ENGINEERING)

Madras School of Economics

(MSE Business School)

May 2024

Chennai, India

Degree and Branch: **POSTGRADUATE OF DIPLOMA IN MANAGEMENT**

(FINANCIAL ENGINEERING)

Month and year of submission: **MAY 2024**

Title of the project work: **Enhancing Portfolio Performance: A Comparative Analysis of Optimization Techniques for S&P500 Constituents**

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ACKNOWLEDGEMENT

I would like to take this opportunity to thank all those people without whom I could not have completed my dissertation. I would like to express my sincere gratitude to my guide Dr. Poorna for her able guidance, and support and for providing me with all the necessary resources and facilities without which it would not have been possible to complete this dissertation. I would like to thank my parents for their constant support and all other resources utilized in the process. I also thank all my friends for helping throughout the last two years.

-Yamini Devi

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Abstract

The S&P500 Index, a cornerstone of the financial landscape, has historically epitomized the performance of 500 prominent US companies since its inception in 1928. Utilizing a market capitalization-weighted methodology, it has delivered an average annual return of approximately 11.85%. However, the emergence of portfolio optimization techniques offers avenues for refining traditional strategies and potentially elevating portfolio performance.

This study conducts a comprehensive investigation into four distinct optimization strategies in comparison to the classic market-cap weighted S&P500 methodology. These methodologies encompass equal weighting, minimum variance portfolio construction, eigen portfolio optimization, and the innovative application of random matrix filtering to the latter two approaches. Drawing upon extensive price data spanning from 2017 to 2019 as the training dataset, this research aims to discern optimal portfolio compositions grounded in historical performance.

Subsequently, the evaluation extends to the period from Jan 2019 to Jan 2021, serving as a window for assessing the efficacy of each optimization technique. Leveraging an array of financial metrics, the analysis encompasses not only returns but also risk-adjusted performance measures to provide a comprehensive view of portfolio effectiveness.

This study contributes significantly to portfolio management literature by offering insights into alternative methodologies for constructing diversified investment portfolios. Findings from this research serve as a guide for investors navigating the complexities of financial markets, illuminating pathways toward potentially superior risk-adjusted returns amidst dynamic market conditions.

Chapter 1
Introduction

1.1 Background:

The S&P 500, often referred to simply as the S&P or Standard & Poor's 500, stands as one of the most renowned and influential stock market indices globally. Created by the American financial services company Standard & Poor's, the S&P 500 provides a comprehensive snapshot of the performance of the largest and most established publicly traded companies in the United States. It encompasses a broad spectrum of industries, including technology, healthcare, finance, consumer goods, and energy, among others, making it a diversified representation of the U.S. economy.

The index tracks the stock prices of 500 companies listed on either the New York Stock Exchange (NYSE), Nasdaq, or CBOE BZX Exchange, selected based on various criteria including market capitalization, liquidity, and sector representation. To be included in the S&P 500, a company must meet stringent requirements, such as having a market capitalization above a certain threshold, a sufficient public float, profitability, and positive earnings over the preceding quarters. The S&P 500 is weighted by market capitalization, meaning that companies with higher market values exert greater influence on the index's performance. This methodology reflects the relative importance of each constituent company within the broader market.

Investors and financial professionals often utilize the S&P 500 as a benchmark for assessing the performance of investment portfolios and evaluating the overall health of the U.S. stock market. It serves as a gauge of investor sentiment, economic conditions, and corporate profitability. Additionally, the S&P 500 index is frequently referenced in financial news and analysis, providing insights into market trends and sentiment. The historical performance of the S&P 500 has been robust, with an average annual return of around 10% over the long term. However, like any investment, it is subject to market volatility and fluctuations influenced by various factors, including economic indicators, geopolitical events, corporate earnings reports, and monetary policy decisions.

This study represents a pioneering effort in exploring alternative portfolio optimization strategies within the context of the S&P500 Index. By investigating four distinct methodologies – equal weighting, minimum variance portfolio construction, eigen portfolio optimization, and the application of random matrix filtering – alongside the traditional market-cap weighted approach, this research seeks to elucidate avenues for enhancing portfolio performance. The study aims to provide valuable insights into optimal portfolio compositions grounded in empirical evidence. Through a rigorous analysis encompassing both returns and risk-adjusted measures, this research contributes to the ongoing dialogue on portfolio management, offering investors actionable insights for navigating the complexities of financial markets and potentially achieving superior risk-adjusted returns.

1.2 Purpose of the Study:

The purpose of studying S&P 500 portfolio optimization is to explore strategies and methodologies that can potentially enhance investment performance within the context of one of the most prominent stock market indices globally. By examining various optimization techniques such as equal weighting, minimum variance portfolio construction, eigen portfolio optimization, and the application of random matrix filtering, the aim is to identify approaches that may offer improved risk-adjusted returns compared to the traditional market-cap weighted methodology. Through empirical analysis using historical price data and evaluation of performance metrics, the study seeks to provide insights into optimal portfolio compositions and contribute to the ongoing discourse on portfolio management. Ultimately, the goal is to offer investors actionable insights for constructing diversified portfolios within the S&P 500 universe, thereby potentially achieving superior investment outcomes in the dynamic landscape of financial markets.

1.3 Significance of Study:

The significance of this study lies in its potential to offer valuable insights and practical implications for investors and financial professionals operating within the realm of the S&P 500 index. By comprehensively investigating and comparing various portfolio optimization strategies, including both traditional and innovative methodologies, the study contributes to expanding the toolkit available for constructing investment portfolios. This research may provide investors with alternative approaches to diversifying their portfolios, potentially leading to improved risk-adjusted returns and enhanced portfolio performance. Furthermore, the study's empirical analysis and evaluation of performance metrics offer evidence-based guidance for decision-making in portfolio management, aiding investors in navigating the complexities of financial markets. Ultimately, the findings of this study have the potential to inform investment strategies, optimize portfolio allocations, and ultimately contribute to more informed and effective investment decision-making within the dynamic landscape of the S&P 500 index.

Chapter 2
Literature Review

The S&P 500 index stands as a cornerstone in financial markets, serving as a benchmark for investment performance and a representation of the broader U.S. economy. Over the years, numerous studies have explored portfolio optimization techniques within the context of the S&P 500, aiming to identify strategies that can potentially enhance investment returns and mitigate risk. This literature review provides an overview of key findings and insights from existing research on portfolio optimization strategies for the S&P 500 index.

Traditional Approaches:

Traditional portfolio optimization approaches, such as mean-variance optimization and the capital asset pricing model (CAPM), have been extensively studied within the context of the S&P 500. These approaches emphasize diversification to reduce risk while seeking to maximize returns. Studies by Markowitz (1952) and Sharpe (1964) laid the foundation for modern portfolio theory (MPT) and the Sharpe ratio, respectively, which remain influential in portfolio construction methodologies.

Equal Weighting:

Equal weighting, where each stock in the S&P 500 is given the same weight, has been explored as an alternative to market-cap weighting. Arnott and Hsu (2005) found that equal-weighted portfolios often outperformed market-cap weighted portfolios over certain periods, suggesting that this approach may offer benefits in terms of diversification and potentially higher returns.

Minimum Variance Portfolio:

Minimum variance portfolio construction aims to create a portfolio with the lowest possible volatility, or variance, given the constraints and characteristics of the S&P 500 constituents. Studies by Choueifaty and Coignard (2008) and Clarke et al. (2013) demonstrated the efficacy of minimum variance portfolios in reducing risk while maintaining competitive returns, particularly during turbulent market conditions.

Eigen portfolio Optimization:

Eigen portfolio optimization involves constructing portfolios based on principal component analysis (PCA) of historical stock returns. Meucci (2009) proposed eigen portfolio optimization as a method to extract diversified portfolios with reduced dimensionality from a large universe of assets. This approach has been shown to effectively capture the underlying structure of the S&P 500 and improve portfolio diversification.

Random Matrix Theory (RMT):

Random matrix theory (RMT) has emerged as a powerful tool for portfolio optimization, particularly when applied in conjunction with eigen portfolio techniques. Laloux et al. (1999) applied RMT to analyze the correlation structure of stock returns in the S&P 500 and demonstrated its utility in reducing noise and improving portfolio performance.

Factor-Based Approaches:

Factor-based investing involves selecting stocks based on specific factors such as value, momentum, or quality. Fama and French (1992) introduced the three-factor model, which considers market risk, size, and value factors in explaining stock returns. Subsequent research by Asness et al. (2013) and others explored additional factors and their implications for portfolio construction within the S&P 500 universe.

The literature on portfolio optimization strategies for the S&P 500 index offers a rich body of research that continues to evolve. While traditional approaches remain influential, there is growing interest in exploring alternative methodologies such as equal weighting, minimum variance portfolio construction, eigen portfolio optimization, and factor-based investing. These studies collectively contribute to a deeper understanding of portfolio management within the dynamic landscape of the S&P 500, offering valuable insights for investors seeking to optimize their investment portfolios

Research Gap:

Despite the extensive research conducted on portfolio optimization strategies for the S&P 500 index, several notable research gaps persist. Primarily, there is a lack of focus on contemporary techniques such as machine learning algorithms and deep learning models, which could offer novel insights into portfolio construction. Additionally, the literature predominantly overlooks dynamic asset allocation strategies and fails to consider practical constraints such as transaction costs and implementation issues, thereby limiting the applicability of optimization models in real-world investment scenarios. Furthermore, the underrepresentation of alternative asset classes and the scant examination of behavioral finance aspects contribute to a less comprehensive understanding of portfolio management within the S&P 500 context. Addressing these research gaps could provide valuable insights for investors and financial professionals seeking to optimize their portfolios within the dynamic landscape of the S&P 500 index.

Chapter 3
Research Methodology

3.1 Problem Statement

The challenge of portfolio optimization for the S&P 500 involves constructing diversified investment strategies that leverage various methodologies such as minimum variance, eigen portfolio, filtered eigen portfolio, and filtered minimum variance with equal weights. This entails selecting a combination of assets from the S&P 500 index to achieve objectives like minimizing portfolio volatility, maximizing returns, and reducing correlation among holdings. Incorporating techniques like eigen portfolios and filtered approaches adds complexity by integrating advanced statistical analyses and filtering mechanisms to refine portfolio selection. The overarching goal is to develop robust investment portfolios that balance risk and return characteristics while adapting to changing market conditions, investor preferences, and constraints.

3.2 Research Design

In this work we have collected the S&P 500 companies' data from date 2017-01-01 to 2021-12-01. We collected the data prices and returns data from yfinance library in Python. We have divided the data in training and testing data. Training data is from 2017-01-01 to 2018-12-01. Testing data is from 2019-01-01 to 2021-12-01. The cumulative returns over the test period is analyzed for various methods.

We have used 0.2% fee for equal weights, 0.5% fee for min var portfolio and 0.8% fee for eigen portfolios. We have passed the data to 'get_cumulative_returns_over_time' function to get the cumulative returns over time given series of price values over time and weights. The output is cumulative returns over time.

We have used the following types of optimizations in our case

- Equal Weight S&P 500: Assigning equal weights to all stocks in the S&P 500 index.

•Minimum Variance Portfolio: Constructing a portfolio with the lowest possible volatility.

•Filtered Minimum Variance Portfolio: Applying random matrix theory to filter the covariance matrix and construct a minimum variance portfolio.

•Eigen portfolio: Using eigen decomposition to create a portfolio based on the principal components of the covariance matrix.

•Filtered Eigen portfolio: Applying random matrix theory to filter the covariance matrix and construct an eigen portfolio.

We have visualized and compared the cumulative returns over time for all the methods above.

We have also looked at the max weight, cumulative returns, and correlation with classic S&P500 (Equal weight).

Chapter 4
Analysis and Interpretation

4.1 Assumptions and Consideration:

To simplify the optimization process, certain assumptions will be made. Firstly, short selling will not be allowed in our portfolios. This is because short selling involves borrowing stock before selling it, which adds complexity. While selling borrowed stock generates additional cash for reinvestment, it also requires returning the stock, often incurring borrowing costs and necessitating specific return arrangements. By excluding short selling, a constraint will be imposed where all portfolio weights must total 1, thereby simplifying the optimization and analysis tasks.

Assuming no additional fees besides a fixed expense ratio for our custom portfolios. The expense ratio, a fixed percentage of total value charged by Exchange-Traded Fund (ETF) providers for managing the ETF, is considered. ETFs employing more complex investment strategies often have higher charges. Higher fees will be simulated for portfolios involving more intricate optimization to enhance realism. Moreover, complex and precise strategies are expected to incur higher transaction costs and face trading challenges such as frequent rebalancing. Transaction costs will not be simulated for educational purposes and to maintain simplicity and clarity.

Companies sometimes get added to and removed from the S&P500. Over time companies get delisted, merge together or are removed from the S&P500 for other reasons. In order to avoid survivorship, bias we are going to work with only the companies from the S&P500 for January 1st, 2000. We also might not be able to pull every asset from our financial data API. Therefore, there are 370 stocks we will optimize for instead of 500.

4.2 S&P500 vs. Equal weighted S&P500

Before delving into complex mathematics, let's initially examine the contrast between the market capitalization-weighted S&P500 and its equal-weighted counterpart. Historically, the equal-weight S&P500 has shown a tendency for higher returns, albeit with increased volatility. There exist several Exchange-Traded Funds (ETFs) such as the "Invesco S&P500 Equal Weight ETF (RSP)" with an expense ratio of 0.2%. This expense ratio surpasses that of a traditional S&P500 ETF like SPY, which boasts an expense ratio as low as 0.09%. The elevated expense ratio of the equal-weight portfolio can be attributed, in part, to the necessity of periodic rebalancing to maintain equal weighting. Conversely, the traditional S&P500 undergoes adjustments as new assets are added or removed, but it does not necessitate periodic rebalancing akin to an equal-weighted S&P500. This distinction arises because, for the companies within the traditional S&P500, their weight naturally increases with rising stock prices (i.e., market capitalization increases) and decreases with declining stock prices (i.e., market capitalization decreases).

Rebalancing incurs costs due to transaction expenses and slippage associated with each trade. Moreover, there must be adequate liquidity for each asset, though liquidity poses less concern for S&P500 companies. This is because every company within the S&P500 is extensively traded daily, with numerous market makers ensuring liquidity.

Despite the added expenses, the visualization below illustrates that the equal-weighted portfolio typically surpasses the familiar classic S&P500. Nonetheless, investors may still favor investing in the market capitalization-weighted S&P500 due to lower expense ratios, reduced volatility, and broader trading availability. Furthermore, there is no guarantee that the equal-weighted S&P500 will consistently outperform its market capitalization-weighted counterpart.

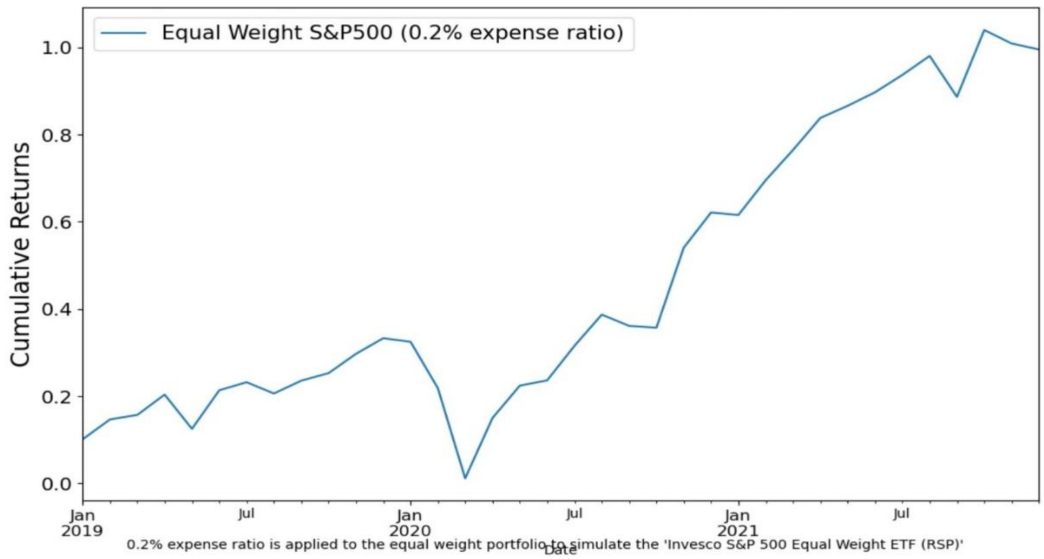

Figure 4.1: Equal Weighted S&P500

This line graph plots the performance of an Equal Weight S&P 500 strategy, spanning from January 2019 to beyond July 2021. The plot indicating the Equal Weight S&P 500 with a 0.2% expense ratio, fluctuates over time, reflecting the strategy's cumulative returns. The expense ratio applied is simulating the Invesco S&P 500 Equal Weight ETF (RSP), suggesting that the graph aims to reflect the real-world performance of this investment product after accounting for management expenses.

4.3 Minimum Variance Portfolio

When discussing portfolio optimization in finance, Modern Portfolio Theory often springs to mind as one of the most straightforward optimization techniques. In this approach, variance (i.e., volatility) serves as the primary measure of risk, with the optimization process aiming to minimize volatility while maximizing returns. It operates under the assumption that stock returns follow a roughly (log-)normally distributed pattern. An implication of this assumption is that increasing diversification invariably results in less volatile portfolios.

However, there are numerous limitations to consider when employing Modern Portfolio Theory. Building long-term wealth and assessing risk necessitate broader considerations beyond solely minimizing volatility. For instance, prioritizing the avoidance of permanent capital loss might be more prudent. Metrics such as Max. Drawdown and Conditional Value at Risk (CVaR), which focus on catastrophic loss, could also be pertinent. Additionally, prominent investors like Peter Lynch, Warren Buffett, and Charlie Munger argue that excessive diversification often leads to unfavorable risk-return tradeoffs. They caution against what they term "Diworsification," which refers to excessive and careless diversification.

Optimization of the weights for the minimum variance portfolio begins with the computation of the covariance matrix using all available stock return data.

$$C = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)(x_i - \mu)^T$$

Covariance matrix formula. mu denotes the mean of our return vector x. x has a length of N.

Subsequently, we calculate the Moore-Penrose (pseudo) inverse of the covariance matrix. This computation bears strong resemblance to both matrix inversion and least squares methods, as it facilitates the determination of the best linear fit.

$$\mathbf{C}^{+} = (C^T C) C^{-1} C^T$$

Moore-Penrose (pseudo) inverse

Finally, to ensure valid portfolio weights, it is imperative to confirm that the weights sum up to 1. This involves normalizing the pseudo inverse matrix we obtained.

This process yields the weights for an optimized portfolio designed to minimize variance while maximizing returns within the specified constraints. However, it may result in negative weights, indicating short sales of certain stocks. To eliminate short sales, we set all negative weights to 0 and then renormalize the weights to ensure they sum up to 1.

Interestingly, the minimum variance portfolio appears to outperform the market-cap weighted S&P500, yet it experiences significant drawdowns. Despite minimizing portfolio variance based on historical stock returns, it tends to suffer more severe crashes compared to other portfolios. This suggests an overemphasis on historical data without effective generalization. Remarkably, the equal weighted S&P500 appears to outperform the minimum variance portfolio over the long term.

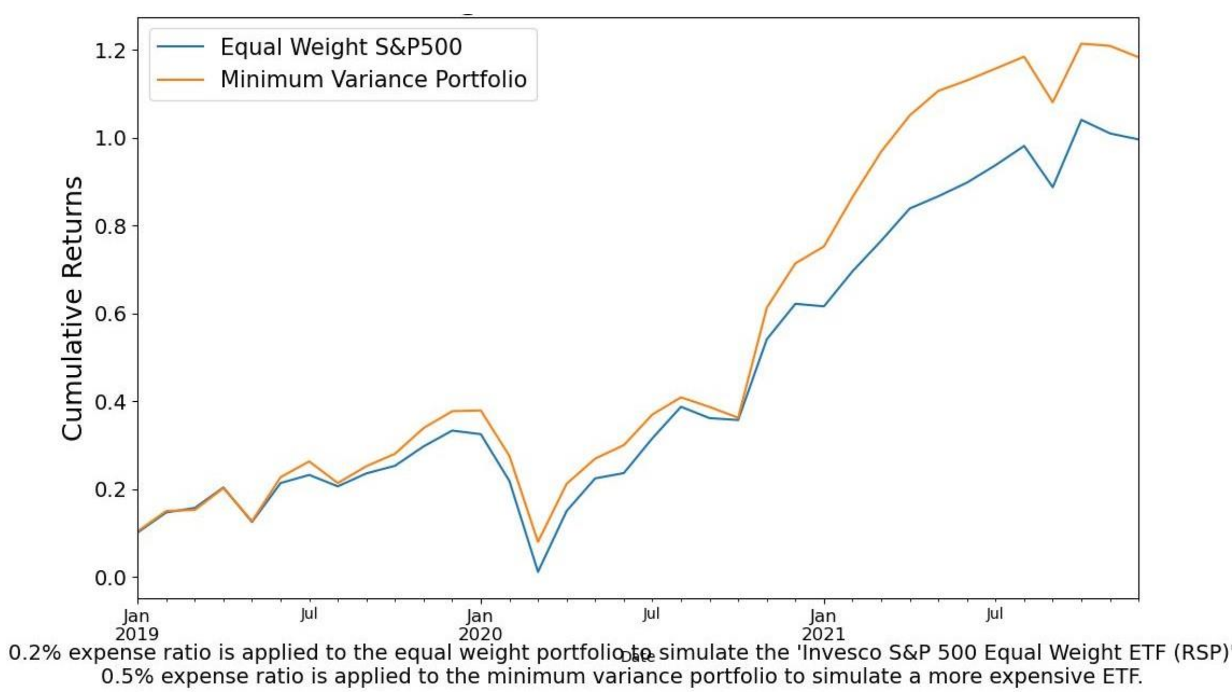

Figure 4.2: Evaluating a Minimum Variance S&P500

Figure 4. 2presents the cumulative returns of two investment strategies: 'Equal Weight S&P500' and 'Minimum Variance Portfolio.' The graph shows that the minimum variance portfolio, designed to have lower risk, underperforms the equal weight S&P 500 at certain intervals, but overtakes it in others, suggesting that the minimum variance strategy may offer

a better risk-adjusted return in the long run. However, the performance gap between the two narrows once a 0.5% expense ratio is applied to the minimum variance portfolio, indicating that cost considerations are crucial when evaluating investment performance.

4.4 Eigen Portfolio

In the section discussing the minimum variance portfolio, we have observed the pivotal role played by the covariance matrix in optimization, containing substantial information on the interactions among stock returns. In this section, we apply a covariance matrix decomposition known as eigen decomposition. It is important to note that while this approach extends Modern Portfolio Theory, it shares similar limitations. However, owing to its sophistication, we anticipate an enhancement over the minimum variance portfolio.

The eigen decomposition for our covariance matrix *A* appears as follows:

$$A = Q\Lambda Q^{-1}$$

Eigen Decomposition

where $\boldsymbol{Q}$ contains the eigenvectors and $\boldsymbol{\Lambda}$ is a diagonal matrix containing the eigenvalues on the diagonal line.

Specifically, our focus lies on the normalized eigenvector corresponding to the largest eigenvalue, often referred to as the "Market Eigenvalue," as it exhibits a robust correlation with other portfolios. While we select the eigenvector associated with the highest eigenvalue, you could also choose, for instance, the eigenvector corresponding to the 2nd or 3rd highest eigenvalue. These alternative eigenvectors represent portfolios that demonstrate less correlation with other S&P500 portfolios. Each eigenvector presents a potential portfolio, with the eigenvalue characterizing the "risk" associated with the respective portfolio. It is worth noting that a portfolio constructed from the eigenvector with the lowest corresponding eigenvalue may significantly underperform, suggesting an overfitting issue due to minimal variance in the training data.

The decomposition can be derived from the fundamental property of eigenvectors:

$$Av = \lambda v$$

Fundamental property of eigenvectors

where the ***lambda*** symbol is the eigenvalue that corresponds with eigenvector ***v***.

For a more realistic assessment, let us assume the existence of an Exchange-Traded Fund (ETF) representing this portfolio, albeit with a higher expense ratio of 0.8% compared to standard S&P500 ETFs. This strategy surpasses all previous methodologies. As depicted in the plot below, it is evident that while its performance remains highly correlated with other approaches, it consistently exhibits superior performance, even surpassing that of the equal-weighted S&P500.

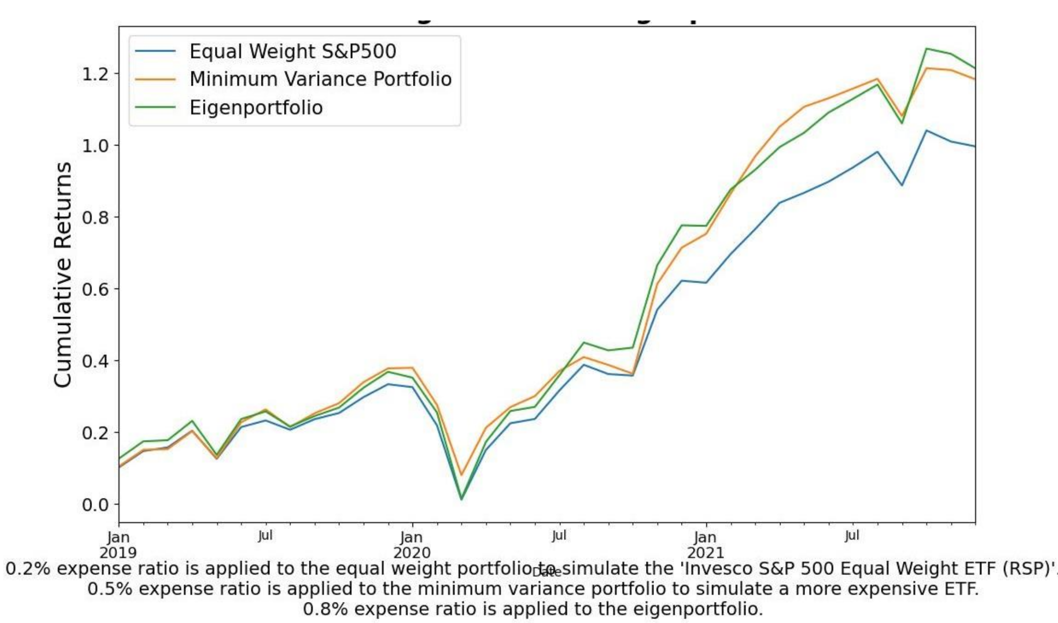

Figure4.3: Evaluating and Eigen portfolio

Figure 4.3 compares the cumulative returns of three different investment strategies: Equal Weight S&P500, Minimum Variance Portfolio, and Eigen portfolio from January 2019 to beyond July 2021. Each line represents one of these strategies, factoring in varying expense ratios, which are fees associated with managing these portfolios. Specifically, the Equal

Weight S&P500 has a 0.2% expense ratio, the Minimum Variance Portfolio has a 0.5% expense ratio, and the Eigen portfolio has the highest at 0.8%, simulating more expensive ETFs. The graph indicates that while all three portfolios generally follow the same market trends, there are distinct periods where their performance diverges due to their respective strategies and costs. For example, the Eigen portfolio, despite its higher expense ratio, sometimes provides higher returns, though there are points where it underperforms relative to the others. This suggests that portfolio performance can vary significantly based on the investment strategy and associated costs.

4.5 Random Matrix Filtering

Applying random matrix theory involves refining the covariance matrix. As discussed earlier, eigenvectors associated with the lowest eigenvalues often demonstrate overfitting to the training data. By leveraging random matrix theory, a reasonable threshold for filtering the matrix can be established. Eigenvalues below this threshold are reset to 0, following which the covariance matrix is reconstructed. This filtered covariance matrix serves as the basis for creating both a new minimum variance portfolio and a new eigen portfolio.

Random matrix theory, pioneered by Alan Edelman at MIT, offers a framework for analyzing matrices comprised of elements sampled from random variables, making it well-suited for addressing challenges in financial engineering.

To facilitate the reconstruction of a filtered covariance matrix, standard deviations are initially computed from logarithmic stock returns. Subsequently, a correlation matrix undergoes filtration.
The next step involves determining a threshold for filtering eigenvalues. This threshold is established by fitting a Marchenko-Pastur distribution to the eigenvalues derived from our correlation matrix.

For a normally distributed matrix of dimensions 500 by 1000, the eigenvalue distribution exhibits characteristics consistent with the Marchenko-Pastur distribution. The correct lambda parameter is calculated as 1000 / 5000 = 2. The objective is to filter out all eigenvalues falling within this Marchenko-Pastur range.

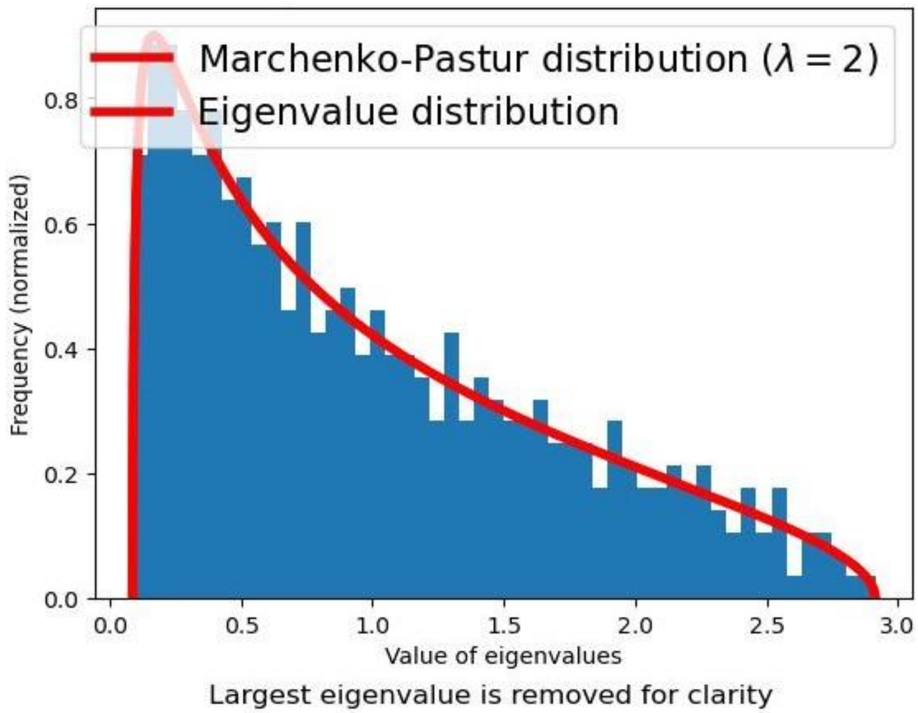

Figure 4.4: Distribution of eigen values with Marchenko-Pasture fit

Figure 4.4 Depicts the frequency distribution of eigenvalues from a correlation matrix in comparison to the theoretical Marchenko-Pastur (M-P) distribution. The histogram bars represent the observed distribution of eigenvalues, while the red curve signifies the expected M-P distribution with a specific lambda ($\lambda = 2$). This graph is from a financial context where Random Matrix Theory (RMT) is used to distinguish signal from noise in the eigenvalues of correlation matrices of asset returns. The note that the largest eigenvalue is removed for clarity indicates that this outlier has been omitted to better fit the bulk of the data to the M-P distribution, a common practice to focus on the 'bulk' of the eigenvalue spectrum.

The actual stock returns from S&P500 companies exhibit a somewhat messier pattern, yet a similar trend can still be observed. Determining the lambda parameter is slightly more intricate in this case, but a value of 2 appears to provide a reasonable fit. It's important to note that in this example, the largest eigenvalue is excluded because it significantly exceeds the others (approximately 122 in our data). Stock returns typically exhibit fat-tailed distributions, resulting in an eigenvalue distribution that differs from that of normally distributed values. This difference is characterized by a much longer tail of large eigenvalues.

With the fitted parameter of 2, a threshold (max_theoretical_eval) can be calculated. Eigenvalues below this threshold are considered too noisy and are filtered out. All eigenvalues below the max_theoretical_eval threshold are eliminated, and a new matrix is constructed. Jim Gatheral's presentation underscores the importance of filling the diagonal with 1s, although the rationale behind this is not entirely clear. Alternative methods of filling the diagonal can be explored. The filtered matrix can be utilized to reconstruct a filtered covariance matrix. This process involves employing the standard deviations computed at the outset of this section.

It is worth noting that the new covariance matrix maintains a high correlation (98.5%) with the original covariance matrix. Nevertheless, the filtered covariance matrix is expected to contain a significantly higher signal for portfolio optimization, given that noisy eigenvalues have been filtered out. Utilizing the filtered covariance matrix, we can proceed to construct new portfolios. For instance, we can develop a new eigen portfolio that leverages the filtered covariance matrix.

Additionally, we use the new covariance matrix to create a filtered minimum variance portfolio. Filtering smaller eigenvalues and reconstructing the covariance matrix doesn't significantly alter the final eigenvector, as observed with the filtered eigen portfolio, which closely resembles the normal eigen portfolio. However, employing the filtered covariance matrix to construct a new minimum variance portfolio result in a noticeably distinct outcome (green line vs. red line), consistently outperforming the unfiltered minimum variance portfolio.

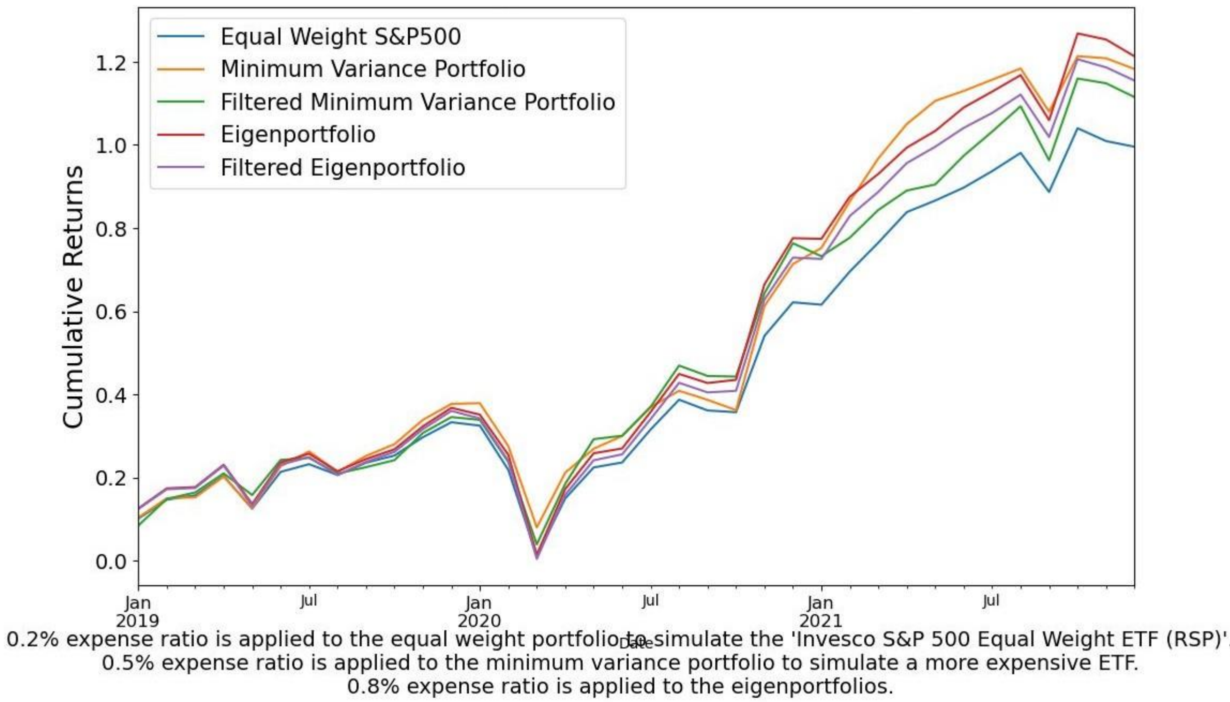

Figure 4.5: Evaluating Random Matrix Filtering

This graph compares cumulative returns of different portfolio strategies—'Equal Weight S&P500,' 'Minimum Variance Portfolio,' 'Filtered Minimum Variance Portfolio,' 'Eigen portfolio,' and 'Filtered Eigen portfolio'—over time from January 2019 to July 2021. The performance of each portfolio is tracked to assess the impact of Random Matrix Theory-based filtering on portfolio construction. The graph shows that while all strategies generally move together, there are periods where the filtered approaches either outperform or underperform their non-filtered counterparts, indicating that filtering may have both benefits and drawbacks depending on market conditions.

When we zoom in on the difference between an unfiltered and filtered minimum variance portfolio, the unfiltered portfolio may outperform sometimes, but this is very rare. There is a ~86% correlation between these two minimum variance portfolios.

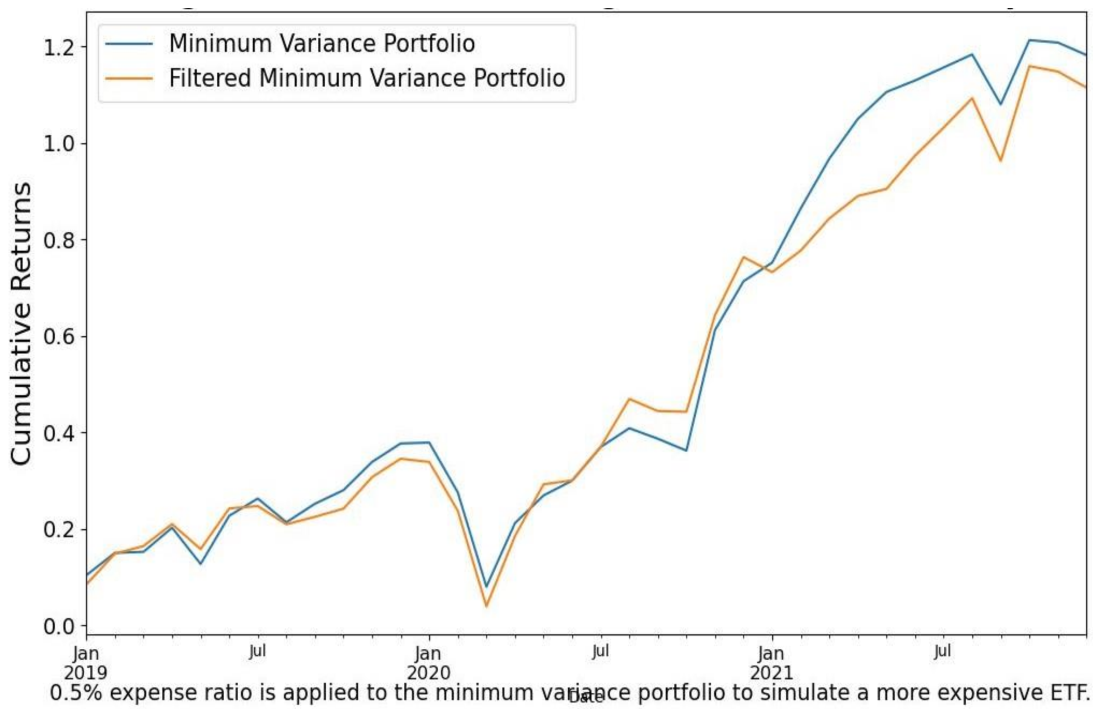

Figure 4.6: Evaluating Random Matrix Filtering on minimum variance portfolio

The graph zooms in on the comparison between 'Minimum Variance Portfolio' and 'Filtered Minimum Variance Portfolio.' This focused comparison illustrates the effect of filtering on a strategy designed to minimize volatility. Over time, the filtered portfolio appears to provide slightly smoother cumulative returns, suggesting that the filtering process may be beneficial in reducing portfolio variance, but it also experiences periods of underperformance relative to the unfiltered minimum variance portfolio, indicating a trade-off between risk and return.

CHAPTER 5

Findings & Conclusions

5.1 Findings:

5.1.1 Correlation Matrices:

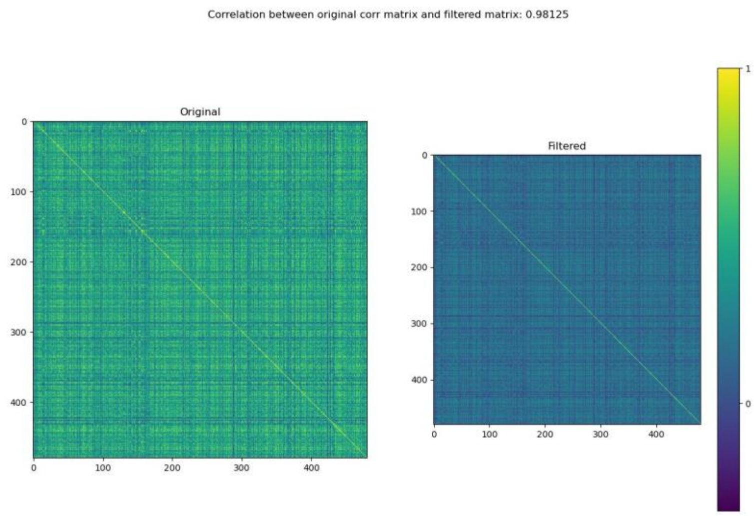

Figure 5.1: Correlation between original corr matrix and filtered matrix

The figure 5.1 compares two correlation matrices labeled "Original" and "Filtered". These matrices represent correlations between asset returns within portfolios.

- Both matrices have a strong diagonal line, indicating perfect correlation of an asset with itself. The filtered matrix appears smoother and less noisy compared to the original, suggesting that a filtering process has been applied to reduce noise and possibly enhance the important relationships between assets.

- The correlation between the original and filtered matrices is very high (0.98125), indicating that the filtering process preserved much of the structural information in the data.

5.1.2 Portfolio Performance Comparison:

Final overview sorted by cumulative returns

	max_weight	cumulative_return (%)	corr_with_classic_sp500
Eigenportfolio	0.0073	121.32	0.9963
Filtered Eigenportfolio	0.0078	117.28	0.9962
Equal Weight S&P500	0.0021	99.56	1.0000
Minimum Variance Portfolio	0.0152	96.64	0.9905
Filtered Minimum Variance Portfolio	0.0359	72.11	0.9669

Table 5.1: Final overview sorted by cumulative returns

Table 5.2 provides a table comparing different investment strategies: Eigen portfolio, Filtered Eigen portfolio, Equal Weight S&P500, Minimum Variance Portfolio, and Filtered Minimum Variance Portfolio.

- The metrics shown include the maximum weight allocated to any single asset within the portfolio, cumulative returns over a specific period, and correlation with a classic S&P500 portfolio.

- The Equal Weight S&P500 strategy shows a correlation of 1.0000 with the classic S&P500, which is expected since it mirrors the index.

- The "Eigen portfolio" yields the highest return (121.32%), followed closely by the "Filtered Eigen portfolio" (117.28%).

- Filtering seems to impact the returns slightly but generally maintains a high correlation with the classic S&P500.

5.2 Conclusion

Filtering techniques applied to correlation matrices can help in reducing noise without significantly altering the underlying structure, which is beneficial for robust portfolio optimization.

The Eigen portfolio and its filtered variant perform the best in terms of returns, suggesting that these might be effective strategies for maximizing returns while still maintaining a high degree of correlation with the broader market index.

The "Minimum Variance Portfolio" and its filtered version aim to reduce risk, as indicated by lower maximum weights, but this comes at the cost of lower returns compared to other strategies.

When evaluating cumulative returns over the assessment period, we observe that the eigen portfolio outperforms other portfolios. It's noteworthy that while its maximum drawdown is slightly worse compared to the market cap-weighted and equal-weight S&P 500, the disparity is not substantial.

Incorporating advanced optimization techniques, such as eigen decomposition and random matrix theory, can lead to improved portfolio performance. However, careful consideration of fees and risk factors is essential in portfolio construction and management. Moreover, continuous monitoring and adjustment of portfolios based on evolving market conditions are crucial for long-term success.

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