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Note to self: Combinations are written in-line, but they are meant to be expressed as binomial coefficients. I can't find a hack to do so in google docs.

Test 1

- **Counting**
  - *Alphabet size* = m, *length* = n;
    - How many ways with repetition allowed? → m^n
    - How many ways when repetition is not allowed? → P(m,n) = $\frac{m!}{(m-n)!}$
    - How many subsets of an *n* element set? 2^n
  - *Lattice Paths*
    - Total paths from (0,0) to (m,n)? → $C(m+n,\ m)\ =\ C(m+n,\ n)$
    - From (0,0) to (n,n) that never go above diagonal: Catalan # → $\frac{C(2n,\,n)}{n+1}$. You do not use the catalan # when passing through specific points. Do the Combination thing.
  - *Stars & Bars*
    - No Empty Cells: → $C(m-1,\ n-1)$
    - Empty Cells Permitted: → $C(m+n-1,\ n-1)$
    - Add a slack variable when inequality is like $x_1 + x_2 + x_3 < 42;\ x_i > 0$
    - Total = Good - Bad
    - MISSISSIPPI → $C(\#\ of\ different\ chars,\ \#\ of\ the\ repetition\ of\ each\ char)$
    - If you are going to need any letters/objects to stay together or occur consecutively, count them as 1 object.
  - *Euclidean Algorithm*: m >= n positive integers; $m\ =\ qn\ +\ r$
    - gcd(306,1190) → n is the smaller one & m is the larger one.
      - m = n on subsequent steps after step 1 and n = r later.
  - *Binomial Theorem:*
    - $(x+y)^n\ =\ \sum_{k=0}^{n} C(n,\ k)\, x^{n-k}\, y^k$
    - Example: $a^{14}\, b^{18}\ (3a^2\ -\ 5b)^{25}$ = $C(25, 18)\ 3^7\ (-5)^{18}$

        - Explanation: k = 18 b/c that is the exponent on b. Additionally, n = 25 because that is the exponent for the parenthesis. Remember, C(25,18) = C(25, 7) b/c of the Binomial Coefficient Identity. DON'T forget to place <u>parentheses around negative numbers!</u>

    - *Binomial Coefficient Identity:*
        - Complement: $C(m,n) = C(m, m-n)$ $when$ $0 \leq n \leq m$

    - *Multinomial Theorem:*
        - $(x_1 + x_2 + x_3 + x_4)^n = C(n, k_1, k_2, k_3, k_4)\ x_1^{k1}\ x_2^{k2}\ x_3^{k3}\ x_4^{k4}$
        - Example: What is the coefficient of $a^6\ b^8\ c^6\ d^6\ in\ (4a^3 - 5b + 9c^2 + 7d)^{19}$?
            - Answer: $C(19,\ 2, 8, 3, 6)\ 4^2\ (-5)^8\ (9)^3\ (7)^6$

    - *Erdos-Szekeres Theorem*
        - Any sequence of $mn + 1$ distinct #'s has an increasing subsequence of length $m + 1$ or a decreasing subsequence of length $n + 1$
            - Example: $37 = 1 + 36 = 1 + 4 \cdot 9$

    - *Labels on Trees*
        - Algorithm: *TO D*
        - *Adhoc fashion to solve the problem...brute force*

- ## Graph Terminology for Test 1
    - <u>Path:</u> A sequence of *distinct* vertices and each consecutive pair is connected by an edge. Any sequence of 1 vertex is a path. *Size = n vertices*

    - <u>Cycle:</u> A path in which the last vertex is incident/adjacent to the first. In a cycle, *vertices = edges*

    - If q represents the # of edges in a graph, then $\sum deg_G(x) = 2q$

    - <u>Hamiltonian Path:</u> A path that visits every vertex exactly once

- Hamiltonian Cycle: A cycle that visits every vertex and the last vertex is adjacent to the first one. (It is implied the last vertex in the set goes back to the first one so we don't re-list the first vertex)
  - NP hard problem. A graph *could* have this property if Dirac's Theorem is true.

- Dirac's Theorem: If G has n vertices and every vertex has at least $ceiling\ of \left[\frac{n}{2}\right]$ neighbors, *then G has a hamiltonian cycle.*

- Subgraph: $H \subseteq G \leftrightarrow H \subseteq V \ \ \&\& \ H \subseteq E\ where\ V\ corresponds\ to\ the\ vertex\ set\ and\ E\ is\ the\ edge\ set.$

- Induced Subgraph: A subgraph where it must have all the edges for a subset vertex set of the original graph

- Complete Graph: Every pair of vertices is joined by an edge. CLIQUE is a synonym (no difference).

- Connected Graph:
  - Graphs that are connected are either 2-connected or they are not

- Bridge:

- Disconnected Graph: When you have at least two vertices that are missing a path

- Component: A connected subgraph in a disconnected graph (Go to lecture 11, Eulers' formula video and ask him how he got 3 components?)

- Isomorphism: There is a bijection (1-1) between the vertex sets of 2 graphs (same edges are present, you have to go back and forth with both graphs to make sure they're the same)

- Clique: Every pair of vertices is joined by an edge. Every 2 element subset is an edge.

- Max Clique: The largest complete subgraph. Denoted by omega, $\omega(G)$
  - If you have a bunch of loose points, $\omega(G) = 1$

- Graph Coloring: Assigns integers to the vertices and no 2 vertices connected by an edge can have the same integer.

- Chromatic Number: The coloring of a graph. Denoted by chi, $\chi(G)$. In general, $\chi(G) \geq \omega(G)$

- Leaf: A vertex of degree 1

- Tree: Doesn't have any cycles and is connected. Has a unique path between any 2 vertices. Has *n-1 edges.*

- Trail/Walk: A sequence of 2 adjacent vertices. You can repeat edges & vertices. On the other hand, a path never repeats vertices.

- Circuit: If the last vertex in a trail is adjacent to the 1st vertex. Don't write the root vertex at the end of the set b/c it's implied.

- Euler Circuit: In order for a graph to have an euler circuit, every vertex must have an even degree and the graph has to be connected. You can only walk on each edge once.
  - ~~Greedy Algorithm:~~
  - *Not necessarily true in regards to the greedy algorithm.*

Test 2

- ## Graph Terminology for Test 2
  - Forest: A graph that doesn't have any cycles.
    - Girth of a forest = $Infinity$

  - Girth: Size of the smallest cycle.

  - Triangle: $\omega(G) = 3 \textit{ aka } K_3 \textit{ which is a } 3 \textit{ cycle}$
    - Hence, Triangle-Free graphs are ones with $\omega(G) \leq 2$.
  - Complement: Take non-edges and make them into edges. Take edges and make them into non-edges.

  - Shift Graphs: Triangle-free graphs. We want to show that $\chi(G)$ is the least t so that $2^t \geq n$. The set (i,j) and (j,k) where j has to be less than k so and if it is, then you don't have an edge between your 2 element subsets. Remember that vertices are 2 element subsets.
    - $\chi(S_n) \ = \ ceiling\,[lognk] \ when\ n \geq 3,\ \omega(S_n) = 2$
      - if n = 1962, then you need at least 11 colors because 1024 is not enough (2^10)
    - # of vertices = $C(n,\ 2)\ when\ n \geq 2$

- # of edges = $C(n,\ 3)\ when\ n \geq 3$

- Planar Graphs: A graph that can be drawn without edge crossings
  - Max # of edges = $3v - 6\ when\ v \geq 3$. (Note: A implies B, but B doesn't imply A)
  - $\chi(G) \leq 4\ b/c\ of\ 4 - color\ theorem$
  - Any subgraph of a planar graph is planar
  - $K_5\ and\ K_{3,3}\ are\ non-planar$.
  - For a 2-colorable planar graph, the maximum number of edges is $2v - 4\ when\ v \geq 3$

- Homeomorph: When you add vertices on *some* of the edges.
  - Any graph that is a homeomorph to $K_{3,3}\ or\ K_5$ is non-planar
  - Kurtowski's Theorem is when a graph is non planar if it a homeomorph of k3,3 or k5

- Bipartite Graphs/2-colorable Graphs: $\chi(G) = 2\ and\ \omega(G) = 2$
  - For $m,\ n \geq 1,\ complete\ bipartite\ graphs,\ K_{m,n}\ ,\ has\ m + n\ vertices\ and\ mn\ edges.$
  - You can characterize a 2-colorable graph by the absence of odd cycles.

- Interval Graphs:
  - For an on-line interval graph, $\chi(G)\ \leq 3w - 2\ where\ w\ is\ the\ max\ clique\ size$
  - If their endpoints are parallel with each other, then they intersect
  - Use First Fit to color the graph
  - When coloring an unknown interval graph online, the maximum number of colors used will be no more than $3\omega - 2$. This is known as Kierstead & WTT.

- Perfect Graphs: Interval graphs and Bipartite Graphs
  - Perfect if $\chi(G) = \omega(G)\ for\ every\ induced\ subgraph\ H\ of\ G\ if\ the\ subgraph\ has\ an\ odd\ cycle\ \geq 5$

Perfect graphs are not planar because all complete graphs are perfect graphs. K5 is not perfect but it is perfect

- Euler's Formula:
  - $V - E + F\ = 2\ for\ only\ one\ component$
  - $V - E + F\ = t + 1\ for\ more\ than\ one\ component$